Logical Equivalences

LOGIC

False, True, Statements

Logical Operators

Operator	Symbol	Usage
Negation	$\neg$, $\sim$	not
Conjunction	$\wedge$	and
Disjunction	$\vee$	or
Exclusive or	$\oplus$	xor
Conditional	$\rightarrow$	if,then
Biconditional	$\leftrightarrow$	iff

Negation – truth table

p	$\neg p$
F	T
T	F

Conjunction – truth table

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction – truth table

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Exclusive-Or – truth table

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Conditional -- truth table

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Bi-Conditional -- truth table

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Theories
Logical Equivalences

Tautology.

Logical equivalence is denoted by $\Leftrightarrow$ or $\equiv$

The way to check for logical equivalences

- Truth tables
- Derivational Proof Techniques

EXAMPLE

Truth tables

The easiest way to check for logical equivalence is to see if the truth tables of both variants have identical last columns.

Logical Equivalence of
$p \rightarrow q$ and $\neg q \rightarrow \neg p$

p	q	$p \rightarrow q$	$\neg q \rightarrow \neg p$	$(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$
T	T	**T**	**T**	**T**
T	F	**F**	**F**	**T**
F	T	**T**	**T**	**T**
F	F	**T**	**T**	**T**

Logical Non-Equivalence of
$p \rightarrow q$ and $q \rightarrow p$

p	q	$p \rightarrow q$	$q \rightarrow p$	$(p \rightarrow q) \leftrightarrow (q \rightarrow p)$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

Tables of Logical Equivalences

- Identity laws
  - Like adding 0
- Domination laws
  - Like multiplying by 0
- Idempotent laws
  - Delete redundancies
- Double negation
  - "I don't like you, not"
- Commutativity
  - Like "$x+y = y+x$"
- Associativity
  - Like "$(x+y)+z = y+(x+z)$"
- Distributivity
  - Like "$(x+y)z = xz+yz$"
- De Morgan

Equivalence	***Name***
$p \wedge \mathbf{T} \Leftrightarrow p$ $p \vee \mathbf{F} \Leftrightarrow p$	Identity laws
$p \vee \mathbf{T} \Leftrightarrow \mathbf{T}$ $p \wedge \mathbf{F} \Leftrightarrow \mathbf{F}$	Domination laws
$p \vee p \Leftrightarrow p$ $p \wedge p \Leftrightarrow p$	Idempotent laws
$\neg(\neg p) \Leftrightarrow p$	Double negation law
$p \vee q \Leftrightarrow q \vee p$	Commutative laws
$(p \vee q) \vee r \Leftrightarrow p \vee (q \vee r)$ $(p \wedge q) \wedge r \Leftrightarrow p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$ $\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$	De Morgan's laws

Some Useful Logical Equivalences

Equivalence	***Useful Logical Equivalences* (ULE)**
$p \vee \neg p \Leftrightarrow \mathbf{T}$	ULE 1
$p \wedge \neg p \Leftrightarrow \mathbf{F}$	ULE 2
$p \rightarrow q \Leftrightarrow \neg p \vee q$	ULE 3

Derivational Proof Techniques

EXAMPLE

Tautology by proof

$[\neg p \wedge (p \vee q)] \rightarrow q$	
$\Leftrightarrow [(\neg p \wedge p) \vee (\neg p \wedge q)] \rightarrow q$	Distributive
$\Leftrightarrow [\mathrm{F} \vee (\neg p \wedge q)] \rightarrow q$	ULE
$\Leftrightarrow [\neg p \wedge q] \rightarrow q$	Identity
$\Leftrightarrow \neg[\neg p \wedge q] \vee q$	ULE
$\Leftrightarrow [\neg(\neg p) \vee \neg q] \vee q$	DeMorgan
$\Leftrightarrow [p \vee \neg q] \vee q$	Double Negation
$\Leftrightarrow p \vee [\neg q \vee q]$	Associative
$\Leftrightarrow p \vee [q \vee \neg q]$	Commutative
$\Leftrightarrow p \vee \mathrm{T}$	ULE
$\Leftrightarrow \mathrm{T}$	Domination

Example

$(p \lor q \lor r) \land (p \lor t \lor \neg q) \land (p \lor \neg t \lor r) \Leftrightarrow p \lor [r \land (t \lor \neg q)]$

2 ways for proving

Truth table

Proof

Truth table

(p ∨ q ∨ r) ∧ (p ∨ t ∨ ¬q) ∧ (p ∨ ¬t ∨ r)

p	q	r	t	¬q	¬t	(p ∨ q ∨ r)	(p ∨ t ∨ ¬q)	(p ∨ ¬t ∨ r)	(p ∨ q ∨ r) ∧ (p ∨ t ∨ ¬q)	(p ∨ q ∨ r) ∧ (p ∨ t ∨ ¬q) ∧ (p ∨ ¬t ∨ r)
T	T	T	T	F	F	T	T	T	T	T
T	T	T	F	F	T	T	T	T	T	T
T	T	F	T	F	F	T	T	T	T	T
T	T	F	F	F	T	T	T	T	T	T
T	F	T	T	T	F	T	T	T	T	T
T	F	T	F	T	T	T	T	T	T	T
T	F	F	T	T	F	T	T	T	T	T
T	F	F	F	T	T	T	T	T	T	T
F	T	T	T	F	F	T	T	T	T	T
F	T	T	F	F	T	T	F	T	F	F
F	T	F	T	F	F	T	T	F	T	F
F	T	F	F	F	T	T	F	T	F	F
F	F	T	T	T	F	T	T	T	T	T
F	F	T	F	T	T	T	T	T	T	T
F	F	F	T	T	F	F	T	F	F	F
F	F	F	F	T	T	F	T	T	F	F

Truth table

p ∨ [r ∧ (t ∨ ¬q)]

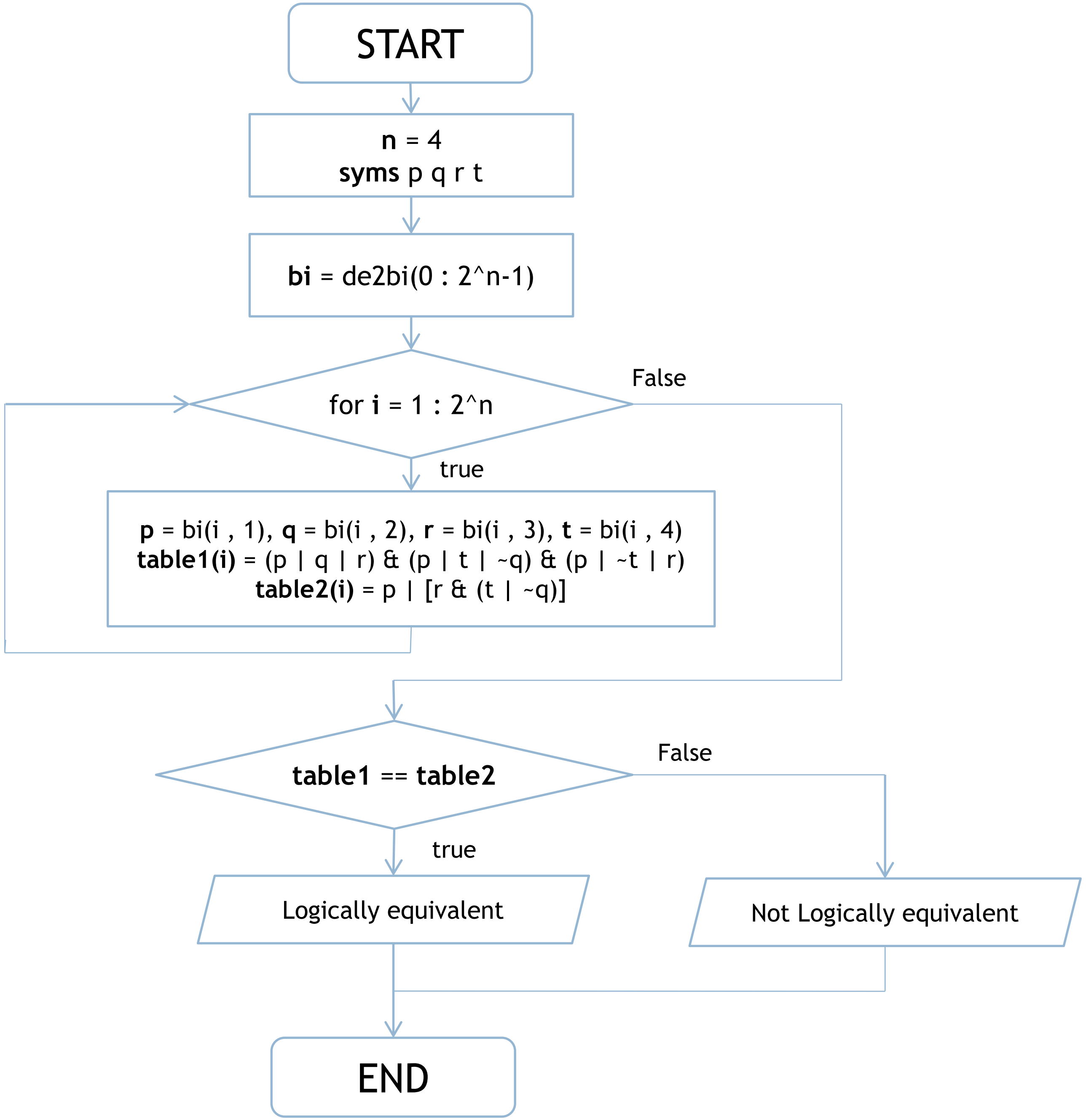
p	q	r	t	¬q	(t ∨ ¬q)	[r ∧ (t ∨ ¬q)]	p ∨ [r ∧ (t ∨ ¬q)]
T	T	T	T	F	T	T	T
T	T	T	F	F	F	F	T
T	T	F	T	F	T	F	T
T	T	F	F	F	F	F	T
T	F	T	T	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	T	T	T	F	T
T	F	F	F	T	T	F	T
F	T	T	T	F	T	T	T
F	T	T	F	F	F	F	F
F	T	F	T	F	T	F	F
F	T	F	F	F	F	F	F
F	F	T	T	T	T	T	T
F	F	T	F	T	T	T	T
F	F	F	T	T	T	F	F
F	F	F	F	T	T	F	F

Proof

- $(p \vee q \vee r) \wedge (p \vee t \vee \neg q) \wedge (p \vee \neg t \vee r)$
  $\Leftrightarrow p \vee [r \wedge (t \vee \neg q)]$
- proof
- $\Leftrightarrow p \vee [\, (q \vee r) \wedge (t \vee \neg q) \wedge (\neg t \vee r)]$
- $\Leftrightarrow p \vee [\, (r \vee (\, q \wedge \neg t)) \wedge (t \vee \neg q)]$
- $\Leftrightarrow p \vee [\, ((r \wedge (t \vee \neg q)) \vee ((\, q \wedge \neg t) \wedge (t \vee \neg q))]$
- $\Leftrightarrow p \vee [\, ((r \wedge (t \vee \neg q)) \vee (\neg(t \vee \neg q) \wedge (t \vee \neg q))]$
- $\Leftrightarrow p \vee [\, ((r \wedge (t \vee \neg q)) \vee \mathbf{F}]$
- $\Leftrightarrow p \vee [r \wedge (t \vee \neg q)]$

Flowchart

Flowchart


START
n = 4
syms p q r t
bi = de2bi(0 : 2^n-1)
for i = 1 : 2^n
False
true
p = bi(i , 1), q = bi(i , 2), r = bi(i , 3), t = bi(i , 4)
table1(i) = (p | q | r) & (p | t | ~q) & (p | ~t | r)
table2(i) = p | [r & (t | ~q)]
table1 == table2
False
true
Logically equivalent
Not Logically equivalent
END

MATLAB

de2bi function

```
bi_0 = de2bi(0)
bi_1 = de2bi(1)
bi_3 = de2bi(3)
bi_4 = de2bi(4)
bi_3bit = de2bi(0:2^3-1)
```

Command Window

```
bi_0 =

     0


bi_1 =

     1


bi_3 =

     1     1


bi_4 =

     0     0     1


bi_3bit =

     0     0     0
     1     0     0
     0     1     0
     1     1     0
     0     0     1
     1     0     1
     0     1     1
     1     1     1

>>
```

Main Code

```matlab
%Number of logical variable
n = 4;

%Name of logical variable
syms p q r t

%Loop all possible truth value for each equation
bi = de2bi(0:2^n-1);
for i = 1:2^n

    %Assign more variable here
    %Ex."(name of variable) = bi(i,(order of variable))"
    p = bi(i,1);
    q = bi(i,2);
    r = bi(i,3);
    t = bi(i,4);

    %Enter equation here
    table1(i) = (p | q | r) & (p | t | ~q) & (p | ~t | r);
    table2(i) =  p | [r & (t | ~q)];
end

table1
table2

%Is Logically equivalent or not
if table1 == table2
    disp('Logically equivalent')
else
    disp('Not logically equivalent')
end
```

Command Window

```
table1 =

  1×16 logical array

   0   1   0   1   1   1   0   1   0   1   0   1   1   1   1   1


table2 =

  1×16 logical array

   0   1   0   1   1   1   0   1   0   1   0   1   1   1   1   1

Logically equivalent
>>
```

Main Code

```
%Problem
%(p ∨ q ∨ r) ∧ (p ∨ t ∨ ¬q) ∧ (p ∨ ¬t ∨ r)
%⇔ p ∨ [r ∧ (t ∨ ¬q)]
e1 = '(p ∨ q ∨ r) ∧ (p ∨ t ∨ ¬q) ∧ (p ∨ ¬t ∨ r)';
e2 = 'p ∨ [r ∧ (t ∨ ¬q)]';
```

```
%Show problem infomation
    disp('Problem')
    fprintf('Equation 1: ')
    disp(e1)
    fprintf('Equation 2: ')
    disp(e2)
    fprintf('\n')

    disp('Truth Table of equation 1')
    disp(table1)
    disp('Truth Table of equation 2')
    disp(table2)

    fprintf('Answer: ')
```

Command Window

```
Problem
Equation 1: (p ∨ q ∨ r) ∧ (p ∨ t ∨ ¬q) ∧ (p ∨ ¬t ∨ r)
Equation 2: p ∨ [r ∧ (t ∨ ¬q)]

Truth Table of equation 1
   0   1   0   1   1   1   0   1   0   1   0   1   1   1   1   1

Truth Table of equation 2
   0   1   0   1   1   1   0   1   0   1   0   1   1   1   1   1

Answer: Logically equivalent
>> 
```