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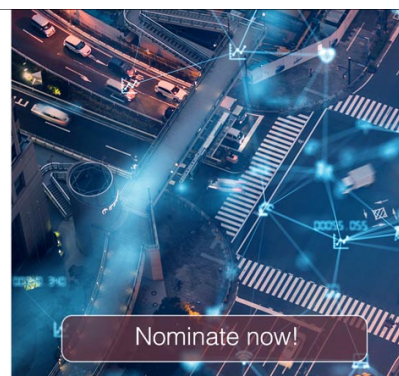


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Mathematical Modelling of Hover Board

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Abstract. A self-balancing scootera (also "hoverboard", self-balancing system on board) is a self-balancing transport system consisting of motorized wheels have pad on which the rider places her or his feet and stands on pad to drive. The driver or a person controls the velocity of Hoverboard by leaning forwards or backwards, and provide direction with a steering command. The study of balancing of a person on Hoverboard can be explained with the help of a complex computer algorithm that stabilizes the under-actuated system. The methodology or The Concepts used for controlling the Hover board mechanism through mathematical modelling can be studied by explaining through kinematic model, the dynamic model (using Lagrange approach) that are presented to control the Two-Wheel personal Balancing Transporter (TWPBT) as follows.

1. Introduction

A self-balancing scootera (also "hover board", self-balancing system on board) is a self-balancing transport system with wheel-variants connected to a self-balancing control mechanism using built-in gyroscopic and a sensor pad on which the rider places her or his feet and stands on pad to drive. In 2019, hover boards now feature a self-balancing mode, in which the motors automatically engage the gyroscope in the opposite direction. This way, when the rider leans forward or backward the board is always attempting to level itself, making it easier to ride than its 2016 predecessors. The device will accelerate forward when the rider leans forward and decelerate when the rider leans back. And provide direction with a steering command. The study of balancing of a person on Hover board can be explained with the help of a complex computer algorithm that stabilizes the under-actuated system.

Since 2001, Segway PT are available in the market used as a two-wheeled self-balancing vehicles and recognized as a powerful personal transporter and commercial vehicle as a versions. Another successful example is -so called- Hover board: .The Hover board may be consider as an evolution of the first, it has driven mechanism and has the advantage of being minimum weight, portable and smaller in size.

The device has three 6.5 inches (170 mm), 8 inches (200 mm), 10 inches (250 mm) diameter wheel variants connected to a self-balancing control mechanism using built-in gyroscopic and a sensor pad. By tilting the pad the rider can control the speed and direction of travel achieving speeds from 6 to 15 miles per hour (9.7 to 24 km/h) with a range of up to 15 miles (24 km) dependent on model, terrain, rider weight and other factors

Another section The problem of controlling the Two-Wheel Personal Balancing Transporter (TWPBT) starts with the mathematical modelling that controls the energy (i.e. both kinetic energy and potential energy) which are govern by kinematic modelling, the dynamic modelling that are explained through Lagrange mathematical Equation .



A subsection Mathematical Modelling of Hover board

The methodology or The Concepts used for controlling the Hover board mechanism through mathematical modelling can be studied as follows. The problem of controlling the Two-Wheel Personal Balancing Transporter (TWPBT) starts with the mathematical modelling that controls the energy (i.e. kinetic energy) which are govern by kinematic modelling, and energy (i.e. potential energy) which are govern by the dynamic modelling and an energy (i.e. both kinetic energy and potential energy) which are govern and explained through Lagrange mathematical Equation that are presented and explained in details as follows;

2. Kinematic modelling of hoverboard

The Nomenclature used for this Model are

Let

m_p Mass/weight of the Driver [kg]/N

m_w Mass/weight of the Right AND Left wheel (SAME) [kg/N]

$J_{\theta p}$ Mass moment of inertia of the Driver, w.r.t. pitch rotation [kgm²]

$J_{\delta p}$ Mass moment of inertia of the Driver w.r.t. yaw rotation [kgm²]

J_w Mass moment of inertia of the ROTOR (wheels) [kgm²]

α_m, β_m Angular position of the (Right, Left) motor (w.r.t to the base) [rads]

α, β Angular position of the (Right, Left) ROTOR (wheels) (w.r.t to the ground) [rads]

θ_p Angular position of the Driver (w.r.t .the ground, where 0 is the upper position in the vertical direction) [rads]

v_L, v_R Linear Velocity of the (Right, Left) centre of the ROTOR (wheels) [m/s]

x_b, v_b X-axis position and Linear Velocity of the centre of the base (origin) [m],[m/s]

x_p, y_p, z_p – X-Y-Z-coordinates of the Driver's centre of Mass [m]

L Length between centre of Mass of the Driver and Base [m]

D Distance between ROTORS (wheels) [m]

R Radius of the ROTOR (wheels) [m]

C_L, C_R Torques acted on the ROTOR (wheels), by Motor (than to gearbox) [Nm]

ρ Velocity ratio of the motor and the wheel

τ_L, τ_R Torque of the (Right, Left) motor [Nm]

Ψ Frictional viscosity coefficient

ABOVE Nomenclature gives Meaning with units for Hoverboard

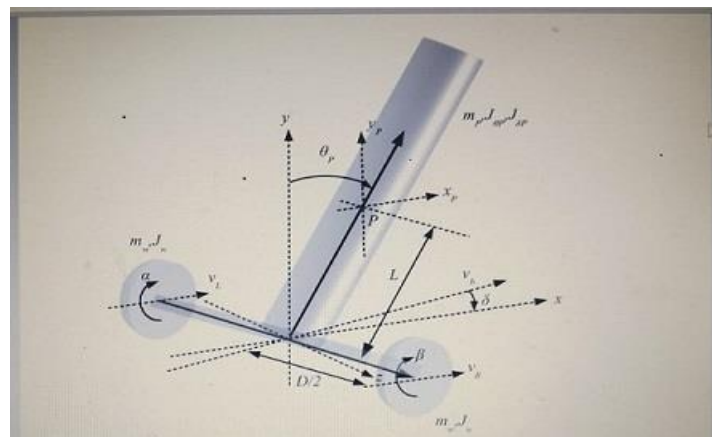


Figure 1. Graphical representation of Hover board showing above parameters.

3. Assume suitable data

Let's assume suitable data:

- 1 The frictional parameter is assumed to be linear and directly proportional to the motor's velocity, other than reality.
2. Assuming Driver as a rigid body (cylinder) of "2L" height
3. Efficiency of Gearbox = 1
- 4 No Elasticity to Velocity ratio
- 5 Neglecting Air friction
6. System's vertical origin is treated as vertical coordinate of the base

The given below equation relates motor's position (declared to the Driver's angle) with the ROTOR's (wheels) angle (declared to the ground) are:

$$\begin{aligned}\alpha &= \theta_p + \rho \alpha_m \\ \beta &= \theta_p + \rho \beta_m \\ \dot{\alpha} &= \dot{\theta}_p + \rho \dot{\alpha}_m \\ \dot{\beta} &= \dot{\theta}_p + \rho \dot{\beta}_m\end{aligned}\quad (1)$$

The relation between the ROTOR (wheels) and base are:

$$\begin{aligned}v_L &= r \dot{\alpha} \\ v_R &= r \dot{\beta} \\ v_b &= v_L + v_R \quad 2r = r \dot{\alpha} + \dot{\beta} \quad 2r \dot{\delta} \\ \delta &= v_L - v_R \quad D = r \dot{\alpha} - \dot{\beta} \quad D\end{aligned}\quad (2)$$

Referring to Driver's centre of mass:

$$\begin{aligned}x_p &= x_b + L \sin \theta_P \cos \delta \\ y_p &= L \cos \theta_P \\ z_p &= z_b + L \sin \theta_P \sin \delta \\ \dot{x}_p &= \dot{x}_b + L \dot{\theta}_P \cos \theta_P \cos \delta - L \dot{\delta} \sin \theta_P \sin \delta \\ \dot{y}_p &= L \dot{\theta}_P \sin \theta_P \\ \dot{z}_p &= \dot{z}_b + L \dot{\theta}_P \cos \theta_P \sin \delta + L \dot{\delta} \sin \theta_P \cos \delta \\ v^2 &= \dot{x}^2 + \dot{y}^2 + \dot{z}^2 = \\ &= v_b^2 + L^2 \dot{\theta}_P^2 + 2L \dot{\theta}_P [\dot{x}_b \cos \theta_P \cos \delta + \dot{z}_b \cos \theta_P \sin \delta] + 2L \dot{\delta} [-\dot{x}_b \sin \theta_P \sin \delta + \dot{z}_b \sin \theta_P \cos \delta]\end{aligned}\quad (3)$$

Where:

$$\dot{x}_b = v_b \cos \delta \quad \dot{z}_b = v_b \sin \delta \quad (4)$$

Thus:

$$\begin{aligned}v^2_P &= v_b^2 + L^2 \dot{\theta}_P^2 + 2L \dot{\theta}_P [v_b \cos \theta_P \cos 2\delta + v_b \cos \theta_P \sin 2\delta] = \\ &= v_b^2 + L^2 \dot{\theta}_P^2 + 2L \dot{\theta}_P v_b \cos \theta_P\end{aligned}\quad (5)$$

4. Dynamic modelling of hover board

The Resultant torque, of motor after the gearbox, due to Frictional viscosity coefficient can be modelled as:

$$\begin{aligned} C_L &= 1/\rho (\tau_L - \psi \cdot \alpha_m) = 1/\rho \tau_L - \psi \rho^2 (\dot{\alpha} - \dot{\theta}_p) \\ C_R &= 1/\rho (\tau_R - \psi \cdot \beta_m) = 1/\rho \tau_R - \psi \rho^2 (\dot{\beta} - \dot{\theta}_p) \end{aligned} \quad (6)$$

4.1. kinetic energy of the Rotors (wheel)

Neglecting the kinetic energy of rotor (wheel) about its vertical axis, the equations given below referred both translational (linear) and rotational parameters of the rotor (wheel) motion.

$$\begin{aligned} KE_L &= 1/2 m w v^2_L + 1/2 J w \dot{\alpha}^2 = 1/2 (m w r^2 + J w) \dot{\alpha}^2 \\ KE_R &= 1/2 m w v^2_R + 1/2 J w \dot{\beta}^2 = 1/2 (m w r^2 + J w) \dot{\beta}^2 \\ KE_w &= T_L + T_R = 1/2 (m w r^2 + J w) (\dot{\alpha} + \dot{\beta})^2 \end{aligned} \quad (7)$$

4.2. kinetic energy of the driver

The **kinetic energy** of the Driver is compounded with three parameters as:

Thus **translational or the Linear kinetic energy** is written as:

$$\begin{aligned} KE_P &= 1/2 m_P v_P^2 = 1/2 m_P (v_b^2 + L^2 \dot{\theta}_P^2 + 2L \dot{\theta}_P v_b \cos \theta_P) \\ &= 1/2 m_P [r \dot{\alpha} + \dot{\beta}/2]^2 + L^2 \dot{\theta}_P^2 + 2L \dot{\theta}_P r \dot{\alpha} + \dot{\beta}/2 \cos \theta_P \end{aligned} \quad (8)$$

The **rotational or circular kinetic energy** w.r.t. rotation of the Driver around the rotor's (wheel's) centre (θ_P) is written as:

$$KE_{\theta_P} = 1/2 J_{\theta_P} \dot{\theta}_P^2 \quad (9)$$

The **rotational or circular kinetic energy** w.r.t the rotation around the vertical axis (y) is written as

$$KE_{y_P} = 1/2 (J_{\delta_P} + m_P L^2 \sin^2 \theta_P) \dot{\delta}^2 = 1/2 (J_{\delta_P} + m_P L^2 \sin^2 \theta_P) \{r \dot{\alpha} - \dot{\beta}/D\}^2 \quad (10)$$

Therefore Total **kinetic energy of the Driver** is given by:

$$\begin{aligned} KE_P &= KE_{t_P} + KE_{\theta_P} + KE_{y_P} \\ &= 1/2 m_P [r \dot{\alpha} + \dot{\beta}/2]^2 + L^2 \dot{\theta}_P^2 + 2L \dot{\theta}_P r \dot{\alpha} + \dot{\beta}/2 \cos \theta_P + 1/2 J_{\theta_P} \dot{\theta}_P^2 + \\ &\quad 1/2 (J_{\delta_P} + m_P L^2 \sin^2 \theta_P) \{r \dot{\alpha} - \dot{\beta}/D\}^2 \end{aligned} \quad (11)$$

4.3. Kinetic energy of the motors

The kinetic energy of the motors is:

$$\begin{aligned} KE_m &= KE_{mL} + KE_{mR} = 1/2 J_m (\dot{\alpha}^2 + \dot{\beta}^2) = \\ &= 1/2 J_m / \rho^2 (\dot{\alpha}^2 + \dot{\beta}^2 + 2 \dot{\alpha} \dot{\beta} \cos \theta_P - 2 \dot{\alpha} \dot{\theta}_P - 2 \dot{\beta} \dot{\theta}_P) \end{aligned} \quad (12)$$

Total kinetic energy:

Therefore Summation of all kinetic energy of the system is given as:

$$\begin{aligned} KE &= KE_P + KE_w + KE_m \\ &= 1/2 m_P [r \dot{\alpha} + \dot{\beta}/2]^2 + L^2 \dot{\theta}_P^2 + 2L \dot{\theta}_P r \dot{\alpha} + \dot{\beta}/2 \cos \theta_P + 1/2 (J_{\theta_P} \dot{\theta}_P^2 + \\ &\quad 1/2 (J_{\delta_P} + m_P L^2 \sin^2 \theta_P) \{r \dot{\alpha} - \dot{\beta}/D\}^2 + 1/2 (m w r^2 + J w) (\dot{\alpha} + \dot{\beta})^2 + \\ &\quad 1/2 J_m / \rho^2 (\dot{\alpha}^2 + \dot{\beta}^2 + 2 \dot{\alpha} \dot{\beta} \cos \theta_P - 2 \dot{\alpha} \dot{\theta}_P - 2 \dot{\beta} \dot{\theta}_P) \end{aligned} \quad (13)$$

4.4. Potential energy (of the Driver)

The potential energy w.r.t the vertical position of the Driver's centre of mass:

$$PE = m_P g L \cos \theta_P \quad (14)$$

5. Lagrangian equation

USING Lagrangian rule, the Equation for the system can be written as

$$T = KE - PE$$

Where T = Total Energy or Torque

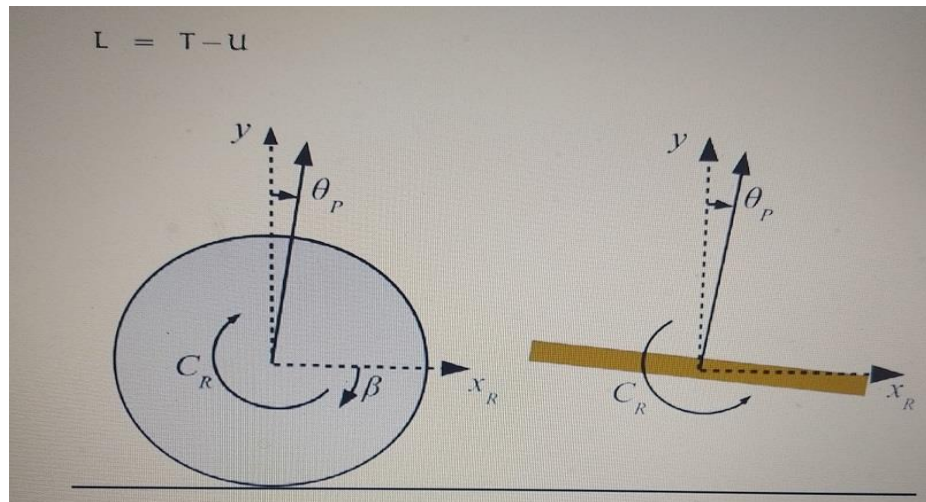


Figure 2. Torque acting on a wheel

The above figure gives the total idea how torques is applied on left wheel with complete references as shown., while in the left of the TWPBT base: and In the Right, the rotor(wheel),,,it is to be noted that the torque generated by the motor applies equal and opposite in the two bodies. The symmetric holds for the right rotor (wheel).

W.R.T ABOVE Situation, Therefore the Lagrange equations can be mathematically modelled as:

$$\begin{aligned} \frac{d}{dt}(\frac{\partial L}{\partial \dot{\alpha}}) - \frac{\partial L}{\partial \alpha} &= C_R \\ \frac{d}{dt}(\frac{\partial L}{\partial \dot{\beta}}) - \frac{\partial L}{\partial \beta} &= C_L \\ \frac{d}{dt}(\frac{\partial L}{\partial \dot{\theta_P}}) - \frac{\partial L}{\partial \theta_P} &= -(C_R + C_L) \end{aligned} \quad (15)$$

6. Conclusions

- 1) The right-side of the Lagrange equations state the active contribution to the dynamics.
- 2) The first and second equations' terms are torques of the motors,
- 3) the third equation, the active contribute is the sum of the torques of the two motors, as easily noticeable

Thus permits to write the complete Mathematical Modelled Equations of motion as

$$\begin{aligned} &[m_{pr}2/4 + r2(J\delta P + m_p L^2 \sin^2 \theta_P) / D^2 + m_w r^2 + J_w + J_m / \rho^2]'' \alpha + \\ &[m_{pr}2/4 - r2(J\delta P + m_p L^2 \sin^2 \theta_P) / D^2 + m_w r^2 + J_w]'' \beta + \\ &[m_p L \cos \theta_P / 2 - J_m / \rho^2]'' \theta_P + \psi / \rho^2 \dot{\alpha} - \psi / \rho^2 \dot{\theta_P} + \\ &- m_p L \dot{\theta_P} \sin \theta_P / 2 + 2 m_p L^2 r^2 \sin \theta_P \cos \theta_P / D^2 \dot{\theta_P} (\dot{\alpha} - \dot{\beta}) = \tau_L / \rho \\ &[m_{pr}2/4 - r2(J\delta P + m_p L^2 \sin^2 \theta_P) / D^2 + m_w r^2 + J_w]'' \alpha + \\ &[m_{pr}2/4 + r2(J\delta P + m_p L^2 \sin^2 \theta_P) / D^2 + m_w r^2 + J_w + J_m / \rho^2]'' \beta + \\ &[m_p L \cos \theta_P / 2 - J_m / \rho^2]'' \theta_P + \psi / \rho^2 \dot{\beta} - \psi / \rho^2 \dot{\theta_P} + \end{aligned}$$

$$\begin{aligned}
& -m_p L \cdot \theta^2 \Pr \sin \theta_p^2 + 2m_{PL} 2r^2 \sin \theta_P \cos \theta_P D^2 \cdot \theta_P \cdot \beta - \alpha = \tau R / \rho \\
& [m_{PL} r \cos \theta_p / 2 - J_m \rho^2] \ddot{\alpha} + [m_{PL} r \cos \theta_p / 2 - J_m / \rho^2] \ddot{\beta} + \\
& + [m_{PL} L + 2J_m \rho^2 + J \theta_p] \ddot{\theta}_p - m_p g L \sin \theta_P + \\
& + 2\psi / 2 \cdot \theta_p - \psi / \rho^2 \cdot \alpha - \psi / \rho^2 \cdot \beta - m_{PL} 2r^2 \sin \theta_P \cos \theta_P / D^2 (\ddot{\alpha} - \ddot{\beta})^2 = -\tau L / \rho - \tau R / \rho
\end{aligned}$$

Above are the **equations of motion** for the Hover board system.

In order to make a linear controller, it is possible to linearize system into a non-linear system

7. Future scope

Self-balancing scooter system like Hover board is the simplest mathematical model for the study. The Validation of above mathematical model can be made possible using simulation software like Simulink block model. And therefore the parameters related to systems, subsystem (example: stability, poles etc.) can easily be identify for making the model usable for design of suitable and proper controller. The system can be made more realistic if one can go for study related to model extensions, which will more exhaustive and interesting.

The evaluated model can also be analyzed from the above mathematical model parameters, where Driver will have few degree of freedom w.r.t. the angle of the base using above references.

Using model's parameters regarding the Driver and its nature of driving –A Block model can be prepared using physiological parameter. The mathematical model presented as a system model is used for the "self-balancing Robotised System" problems that consist of a rigid body erected on two wheels.

8. References

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