Chapter 5

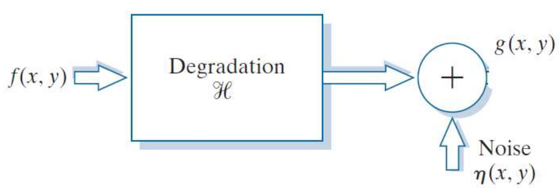
Image Restoration

Introduction

- The goal of Image Restoration techniques is to **improve an image**.
- Image Restoration attempts to **recover an image that has been degraded** over the time.
- Restoration techniques are based on **modeling the degradation** and **applying the inverse process** to recover the original image.
- Image Restoration has applications in many field like **computed tomography (CT)**, **commercial applications** of image processing and **health care**.

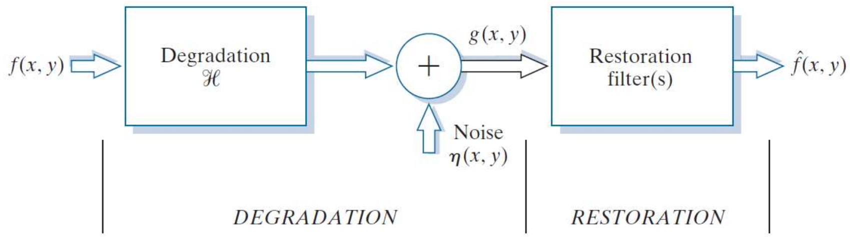
Image Degradation and Restoration

- Image degradation is represented by an **operator** $\mathcal{H}$.
- This operator $\mathcal{H}$ together with a **noise** $\eta(x, y)$, operates on an **input image** $f(x, y)$.
- This produces a **degraded image** $g(x, y)$.



- Suppose we are **given a degraded image** $g(x, y)$.
- Suppose we have some knowledge about factor causing this **degradation** $\mathcal{H}$ and **noise** $\eta(x, y)$.
- Then the objective of Image Restoration techniques is to **obtain an estimated original image** $\hat{f}(x, y)$.
- We want the estimate to be as close as possible to the original image.
- The more we know about $\mathcal{H}$ and η, the closer $\hat{f}(x, y)$ will be to $f(x, y)$.

- Image Restoration and Degradation Model:



Noise Models

- Noise in digital images arise during **image acquisition** & **transmission**.
- The performance of imaging sensors is affected by
  1. a **variety of environmental factors** during image acquisition,
  2. the **quality of the sensing elements**.
- **Light levels** and **sensor temperature** are major factors creating a noise in the resulting image.
- Noise are also created by **transmission channel**.
- An image transmitted using a wireless network might be **corrupted by lightning or other atmospheric disturbance**.

Some Important Noise Probability Density Functions

- Noise components $\eta(x, y)$ are **random variables**.
- Hence Noise can be modelled using various **probability density functions (PDF)**.
- The noise component $\eta(x, y)$ is nothing but **an image** of the same size as the input image.
- We can create a noise image by generating an array whose intensity values are **random numbers** with a specified **probability density function**.

Most common noise PDFs

1. Gaussian noise
2. Rayleigh noise
3. Exponential noise
4. Erlang (Gamma) noise
5. Uniform noise

1. Gaussian Noise

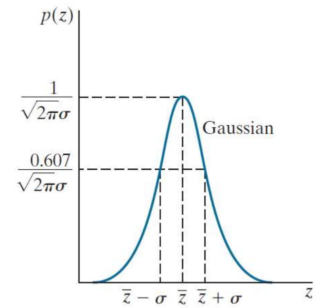
- Gaussian noise models are used frequently in practice.
- The PDF of a Gaussian random variable, z, is defined by

$$p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z-\bar{z})^2}{2\sigma^2}} \qquad -\infty < z < \infty$$

- z is intensity.
- $\bar{z}$ is the mean (average) value of z
- σ is its standard deviation.

9

- Figure shows a plot of this function.
- The probability that values of z are in the range $z \pm \sigma$ is approximately 0.68.
- This probability is about 0.95 that the values of z are in the range $z \pm \sigma$.



10

2. Rayleigh Noise

- The PDF of Rayleigh noise is given by

$$p(z) = \begin{cases} \frac{2}{b}(z-a)e^{-(z-a)^2/b} & z \geq a \\ 0 & z < a \end{cases}$$

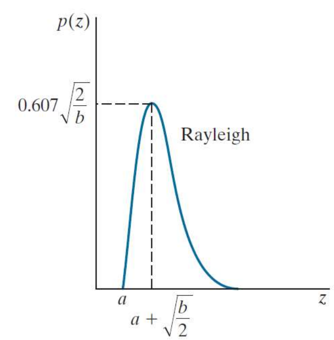
- Here, mean and variance of z are

$$\bar{z} = a + \sqrt{\pi b/4}$$

$$\sigma^2 = \frac{b(4-\pi)}{4}$$

11

- Figure shows a plot of the Rayleigh density.
- There is a small displacement of curve from the origin.
- The basic shape of the density is skewed to the right.
- The Rayleigh density can be quite useful for modeling the shape of skewed histograms.



12

3. Exponential Noise

- The PDF of Exponential noise is given by

$$p(z) = \begin{cases} ae^{-az} & z \geq 0 \\ 0 & z < 0 \end{cases}$$

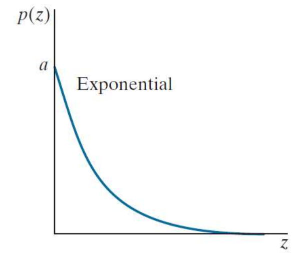
- Here $a > 0$. The mean and variance of z are

$$\bar{z} = \frac{1}{a}$$

$$\sigma^2 = \frac{1}{a^2}$$

13

- Figure shows a plot of the exponential density function.



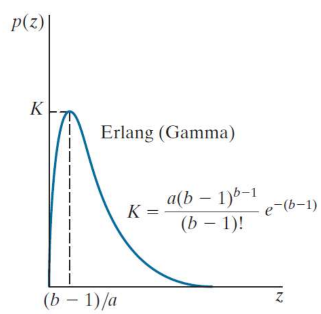
14

4. Erlang (Gamma) Noise

$$p(z) = \begin{cases} \dfrac{a^b z^{b-1}}{(b-1)!} e^{-az} & z \geq 0 \\ 0 & z < 0 \end{cases}$$

$$\bar{z} = \frac{b}{a}$$

$$\sigma^2 = \frac{b}{a^2}$$



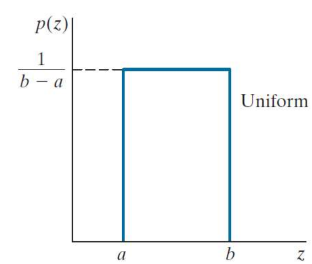
15

5. Uniform Noise

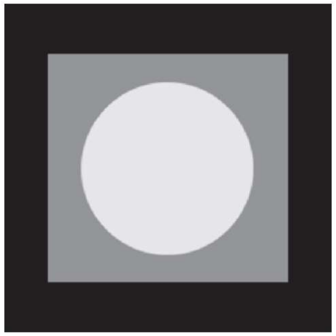
$$p(z) = \begin{cases} \dfrac{1}{b-a} & a \leq z \leq b \\ 0 & \text{otherwise} \end{cases}$$

$$\bar{z} = \frac{a+b}{2}$$

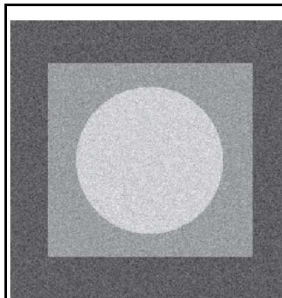
$$\sigma^2 = \frac{(b-a)^2}{12}$$



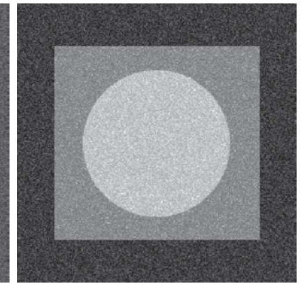
16



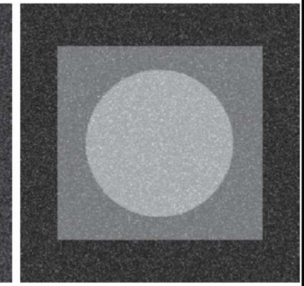
Test pattern used to illustrate the characteristics of the PDFs.



Gaussian



Rayleigh



Erlang noise

Restoration Process

- When an image is degraded by noise, In spatial domain we have
$$g(x,y) = f(x,y) + \eta(x,y)$$
  and in frequency domain we have
$$G(u,v) = F(u,v) + N(u,v)$$
- The noise terms generally are unknown.
- Subtracting noise term from $g(x,y)$ to obtain $f(x,y)$ is not an option.
- We need to apply various filters in spatial domain to remove noise.

Various filters used for Restoration process

- Arithmetic Mean Filter
- Geometric Mean Filter
- Harmonic Mean Filter
- Contraharmonic Mean Filter
- Median Filter
- Max and Min Filters
- Midpoint Filter

Recall that...

If a and b are postive numbers, then

$$\textbf{Arithmetic Mean (AM)} = \frac{a+b}{2}$$

$$\textbf{Geometric Mean (GM)} = \sqrt{ab}$$

$$\textbf{Harmonic Mean (HM)} = \frac{2ab}{a+b} = \frac{(GM)^2}{AM}$$

Arithmetic Mean Filter

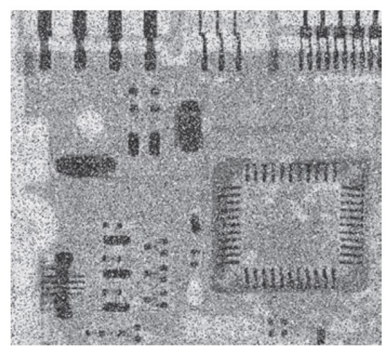
- The arithmetic mean filter is the simplest of the mean filters.
- It is same as the box filters.
- Let S_{xy} represent the set of coordinates in a rectangular subimage window (neighborhood) of size $m \times n$, centered on point (x, y).
- The arithmetic mean filter computes the average value of the corrupted image, $g(x, y)$, in the area defined by S_{xy}.
- The value of the restored image $\hat{f}$ at point (x, y) is the arithmetic mean computed using the pixels in the region defined by S_{xy}.

Arithmetic Mean Filter

- Hence we have

$$\hat{f}(x,y) = \frac{1}{mn} \sum_{(r,c) \in S_{xy}} g(r,c)$$

- r and c are the row and column coordinates of the pixels contained in the neighborhood S_{xy}.
- This operation can be implemented using a spatial kernel of size $m \times n$ in which all coefficients have value $\frac{1}{mn}$.
- A mean filter smooths local variations in an image, and noise is reduced as a result of blurring.



Degraded image

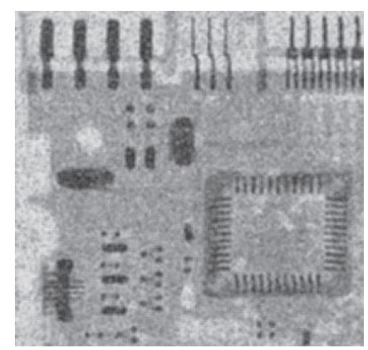


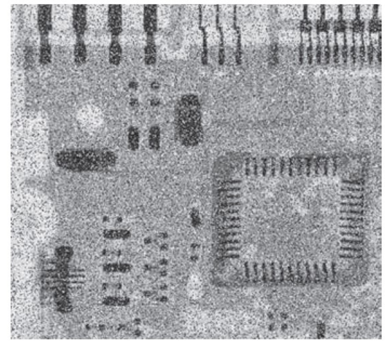
Image filtered with a 5×5 arithmetic mean filter

Geometric Mean Filter

- An image restored using a geometric mean filter is given by the expression

$$\hat{f}(x,y) = \left[\prod_{(r,c)\in S_{xy}} g(r,c) \right]^{\frac{1}{mn}}$$

- Here Π indicates multiplication.
- Here, each restored pixel is given by the product of all the pixels in the subimage area, raised to the power $\frac{1}{mn}$.
- A geometric mean filter achieves smoothing similar to arithmetic mean filter, but it tends to lose less image detail in the process.



Degraded image

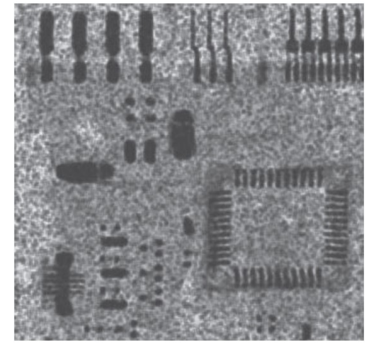


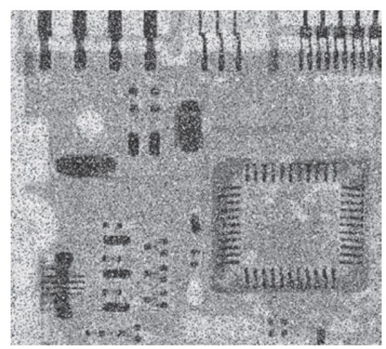
Image filtered with a 5×5 geometric mean filter

Harmonic Mean Filter

- The harmonic mean filtering operation is given by the expression.

$$\hat{f}(x,y) = \frac{mn}{\sum_{(r,c)\in S_{xy}} \frac{1}{g(r,c)}}$$

- The harmonic mean filter works well with Gaussian noise.



Degraded image

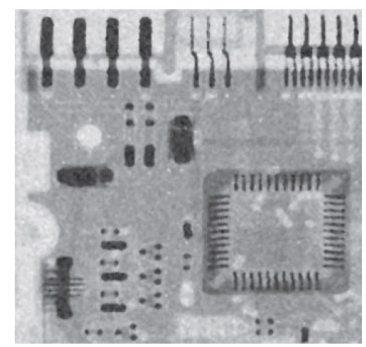


Image filtered with a 5×5 Harmonic mean filter

Contraharmonic Mean Filter

- The contraharmonic mean filter yields a restored image based on the expression

$$\hat{f}(x,y) = \frac{\sum_{(r,c)\in S_{xy}} g(r,c)^{Q+1}}{\sum_{(r,c)\in S_{xy}} g(r,c)^{Q}}$$

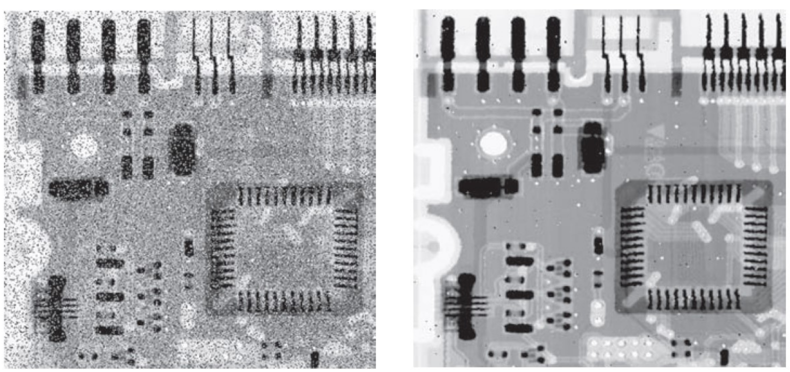
- Q is called the order of the filter.
- The contraharmonic filter reduces to the arithmetic mean filter if $Q = 0$, and to the harmonic mean filter if $Q = -1$.

Median Filter

- The median filter replaces the value of a pixel by the median of the intensity levels in a predefined neighborhood of that pixel.

$$\hat{f}(x,y) = \underset{(r,c)\in S_{xy}}{\text{median}}\{g(r,c)\}$$

- S_{xy} is a subimage (neighborhood) centered on point (x, y).
- The value of the pixel at (x, y) is included in the computation of the median.
- Median filters are quite popular because, for certain types of random noise, they provide excellent noise-reduction capabilities.
- It has considerably less blurring than linear smoothing filters of similar size.



Degraded image

Result of one pass with a median filter of size 3×3.

Max and Min Filters

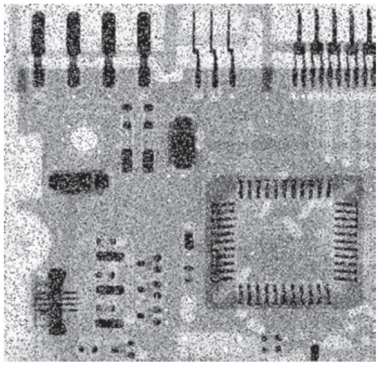
- Max filter is given by

$$\hat{f}(x,y) = \max_{(r,c)\in S_{xy}}\{g(r,c)\}$$

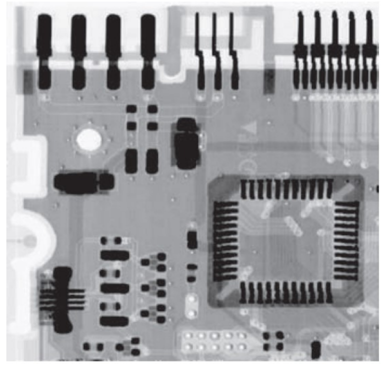
- This filter is useful for finding the brightest points in an image or for dark regions adjacent to bright areas.
- Min filter is given by

$$\hat{f}(x,y) = \min_{(r,c)\in S_{xy}}\{g(r,c)\}$$

- This filter is useful for finding the darkest points in an image or for light regions adjacent to dark areas.

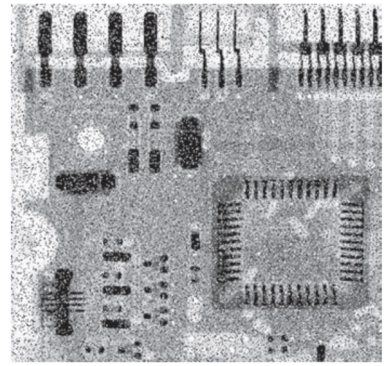


Degraded image

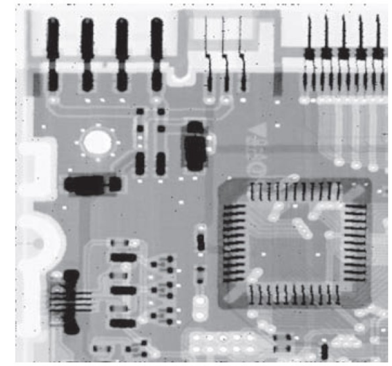


Result of one pass with a min filter of size 3×3.

33



Degraded image



Result of one pass with a max filter of size 3×3.

34

Midpoint Filter

- The midpoint filter computes the midpoint between the maximum and minimum values in the area encompassed by the filter:

$$\hat{f}(x,y) = \frac{1}{2}\left[\max_{(r,c)\in S_{xy}}\{g(r,c)\} + \min_{(r,c)\in S_{xy}}\{g(r,c)\}\right]$$

- It is used to remove randomly distributed noise

35