Ensemble Methods (Adaboost and GBDT)

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2. Adaboost
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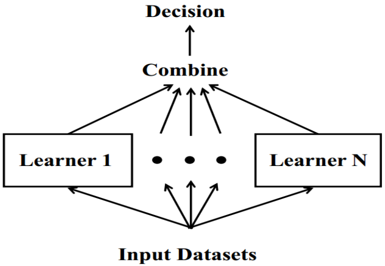
Ensemble Learning

- Ensemble learning: Combine numerous weak learners to a strong learner
- Main methods: Boosting, Bagging



Boosting Method

Algorithm 1: Adaboost

Input: $D = \{(\mathbf{x}_1, y_1), ..., (\mathbf{x}_n, y_n)\}$, **where** $\mathbf{x}_i \in X, y_i \in \{-1, 1\}$
Initialize: Sample weight distribution $D_1 = \frac{1}{n}$

```
Train a base learner h_1(x) with D_1
for m=2,3,...,M do
    Update the sample distribution D_m, to make the wrong
      predictive samples more important
    Train a new base learner h_m(x) with D_m
end
```

Output: $H(\mathbf{x}) = \sum_{m=1}^{M} \alpha_m h_m(\mathbf{x})$

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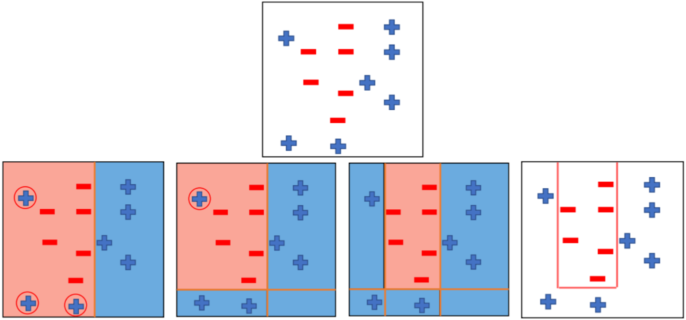
Adaboost

How to train the base leaner?

Make the wrong predictive samples more important, and handle it in next round:



Adaboost

Sample weight updating formula

$$w_{m+1}(i) = \frac{w_m(i)}{z_m} e^{-\alpha_m y_i h_m(\mathbf{x}_i)}$$

$z_m = \sum_{i=1}^{n} w_m(i) e^{-\alpha_m y_i h_m(\mathbf{x}_i)}$ is normalization term, makes $w_m(i)$ become probability distributions

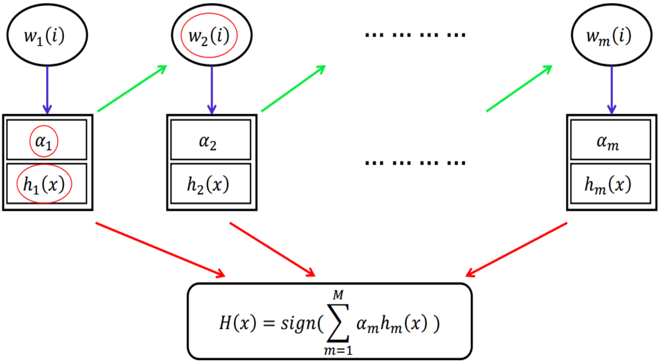
$$w_{m+1}(i) = \begin{cases} \dfrac{w_m(i)}{z_m} e^{-\alpha_m} & \text{for right predictive sample} \\ \dfrac{w_m(i)}{z_m} e^{\alpha_m} & \text{for wrong predictive sample} \end{cases}$$

so in next round, $\frac{w_{wrong}(i)}{w_{right}(i)} = e^{2\alpha_m} = \dfrac{1-\epsilon_m}{\epsilon_m}$ and $\epsilon_m < 0.5$, wrong samples will be more important

Adaboost

How to combine the base learner?

Every iteration generates a new base learner $h_m(\mathbf{x})$ and its importance score α_m



Adaboost

Evaluate the performance of the base learner

- Base learner

$$h_m(\mathbf{x}) : \mathbf{x} \mapsto \{-1, 1\}$$

- Error rate

$$\epsilon_m = p(h_m(\mathbf{x}_i) \neq y_i) = \sum_{i=1}^{n} w_m(i)\mathbb{I}(h_m(\mathbf{x}_i) \neq y_i)$$

ϵ_m¡0.5, or the performance of Adaboost is weaker than random classfication.

Adaboost

Importance score of base learner

Make the base learner with lower ϵ_m more important

$$\alpha_m = \frac{1}{2} \log \frac{1 - \epsilon_m}{\epsilon_m}$$

Adaboost

Additive model

- Final learner

$$H(\mathbf{x}) = \text{sign}(\sum_{m=1}^{M} \alpha_m h_m(\mathbf{x}))$$

Note: $h_m(\mathbf{x}) = \text{sign}(\mathbf{w}^\top \mathbf{x})$ is a nonlinear function, so the Adaboost can deal with nonlinear problem

Algorithm

Algorithm 2: Adaboost

Input: $D = \{(\mathbf{x}_1, y_1), ..., (\mathbf{x}_n, y_n)\}$, **where** $\mathbf{x}_i \in X, y_i \in \{-1, 1\}$
Initialize: Sample distribution w_m
Base learner: $\mathcal{L}$

```
w_1(i) = 1/n
for m=1,2,...,M do
    h_m(x) = L(D, w_m)
    ε_m = Σ_{i=1}^{n} w_m(i) I(h_m(x_i) ≠ y_i)
    if ε_m > 0.5 then
        break
    end
    α_m = 1/2 log((1 - ε_m)/ε_m)
    w_{m+1}(i) = (w_m(i)/z_m) e^{-α_m y_i h_m(x_i)}, where i = 1, 2, ..., n and
      z_m = Σ_{i=1}^{n} w_m(i) e^{-α_m y_i h_m(x_i)}
end
```

$w_1(i) = \frac{1}{n}$

for m=1,2,...,M **do**

- $h_m(x) = \mathcal{L}(D, w_m)$
- $\epsilon_m = \sum_{i=1}^{n} w_m(i)\mathbb{I}(h_m(\mathbf{x}_i) \neq y_i)$
- **if** $\epsilon_m > 0.5$ **then**
  - **break**
- **end**
- $\alpha_m = \frac{1}{2}\log\frac{1-\epsilon_m}{\epsilon_m}$
- $w_{m+1}(i) = \frac{w_m(i)}{z_m} e^{-\alpha_m y_i h_m(\mathbf{x}_i)}$,**where** $i = 1, 2, ..., n$ **and** $z_m = \sum_{i=1}^{n} w_m(i) e^{-\alpha_m y_i h_m(\mathbf{x}_i)}$

end

Output: $H(\mathbf{x}) = \sum_{m=1}^{M} \alpha_m h_m(\mathbf{x})$

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Gradient Boosting Decision Trees

GBDT is a decision tree algorithm with iteration

Example: What is the difference between regression tree and GBDT?

Suppose: There are 4 peoples A, B, C and D, whose age are 14, 16, 24, 26 respectively.

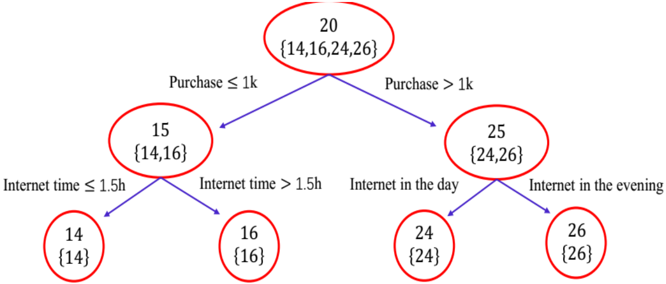
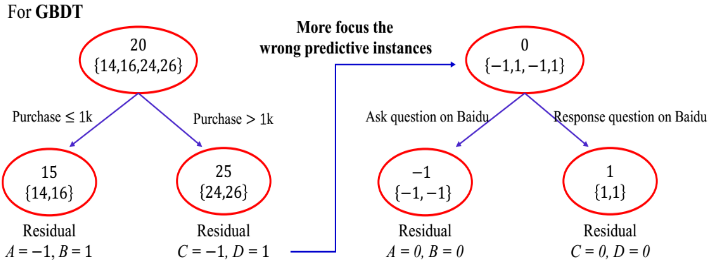


Figure: Single regression tree

Gradient Boosting Decision Trees

- The key of GBDT is that trees learn all the results and residuals of all trees before.
- The residual is the difference of predictive value and real value, so the predictive value is the sum of all results of trees.



- So,
  A=15+(-1)=14 B=15+1=16 C=25+(-1)=24 D=25+1=26

Gradient Boosting Decision Trees

Question

Q1:when results of these two algorithms are same, Why do we choose GBDT?

- The motivation of this algorithm is that every calculation of residual is to increase the weight of wrong predictive samples,and the residual of right predictive sample is zero.
- So in the next iteration, model can concentratively address these wrong predictive samples. Another function is to prevent over fitting.

Gradient Boosting Decision Trees

Question

Q2: Where does this algorithm reflect gradient boosting?

- In algorithm, residual is the gradient descent direction, which is the derivation of mean square error(MSE). Actually, MSE is the loss function of CART regression tree.

Setting	Loss Function	$-\partial L(y_i, f(x_i))/\partial f(x_i)$
Regression	$\frac{1}{2}[y_i - f(x_i)]^2$	$y_i - f(x_i)$
Regression	$\|y_i - f(x_i)\|$	$\text{sign}[y_i - f(x_i)]$
Regression	Huber	$y_i - f(x_i)$ for $\|y_i - f(x_i)\| \leq \delta_m$ $\delta_m \text{sign}[y_i - f(x_i)]$ for $\|y_i - f(x_i)\| > \delta_m$ where $\delta_m = \alpha$th-quantile$\{\|y_i - f(x_i)\|\}$
Classification	Deviance	kth component: $I(y_i = \mathcal{G}_k) - p_k(x_i)$

Algorithm

Algorithm 3: GBDT

Input: $D = \{(\mathbf{x}_1, y_1), ..., (\mathbf{x}_n, y_n)\}$, **where** $\mathbf{x}_i \in X, y_i \in \{-1, 1\}$

Initialize: $f_0(x) = \arg\min_\mu \sum_{i=1}^n L(y_i, \mu)$

for m=1,2,...,M **do**

    for i=1,2,...,n **do**

        $r_{im} = -\left[\frac{\partial L(y_i, f_{m-1}(\mathbf{x}_i))}{\partial f_{m-1}(\mathbf{x}_i)}\right]$

        Fit a regression tree to targets r_{im} giving terminal regions $R_{jm}, j = 1, 2, ..., J_m$

    end

    for j=1,2,...,J_m **do**

        $\mu_{jm} = \arg\min_\mu \sum_{\mathbf{x}_i \in R_{jm}} L(y_i, f_{m-1}(\mathbf{x}_i) + \mu), j = 1, 2, ..., J_m$

        Update $f_m(\mathbf{x}) = f_{m-1}(\mathbf{x}) + \sum_{j=1}^{J_m} \mu_{jm} \mathbb{I}(\mathbf{x} \in R_{jm})$

    end

end

Output: $\hat{f}(\mathbf{x}) = f_M(\mathbf{x})$

THANK YOU!