a) $f(x) = \dfrac{x^3}{x+1}$

$$f'(x) = \frac{3x^2(x+1) - x^3}{(x+1)^2} = \boxed{\frac{2x^3 + 3x^2}{(x+1)^2}}$$

b) $f(x) = x^3(x+1)^{-1}$

$$f'(x) = \boxed{3x^2(x+1)^{-1} - x^3(x+1)^{-2}}$$

equiv

$$\frac{2x^3 + 3x^2}{(x+1)^2} = 3x^2(x+1)^{-1} - x^3(x+1)^{-2}$$

$$2x^3 + 3x^2 = 3x^2(x+1) - x^3$$

$$2x^3 + 3x^2 = 2x^3 + 3x^2$$

c) $f(x) = u(x)\left(v(x)\right)^{-1}$

$$f'(x) = u'(x)\left(v(x)\right)^{-1} - u(x)\left(v(x)\right)^{-2}v'(x)$$

$$= \frac{u'(x)}{v(x)} - \frac{u(x)v'(x)}{v(x)^2}$$

$$= \frac{u'(x)}{v(x)}\left(\frac{v(x)}{v(x)}\right) - \frac{u(x)v'(x)}{v(x)^2}$$

$$= \boxed{\frac{u'(x)v(x) - u(x)v'(x)}{v(x)^2}}$$