5A. Inverse trigonometric functions; Hyperbolic functions

1a)

$\arctan(\sqrt{3}) = 1.047$

1b)

$\arcsin\left(\frac{\sqrt{3}}{2}\right) = 1.047$

1c)

$\arctan(5) = 1.373$ rad

$\sin 1.373 = 0.981$

$\cos 1.373 = 0.197$

$$\sec 1.373 = \frac{1}{\cos 1.373} = 5.089$$

3f)

$$\frac{d}{dx}\arcsin\left(\frac{a}{x}\right) = \frac{\frac{d}{dx}\left(\frac{a}{x}\right)}{\left(1-\left(\frac{a}{x}\right)^2\right)^{1/2}} = \frac{-\frac{a}{x^2}}{\left(1-\left(\frac{a}{x}\right)^2\right)^{1/2}} = \boxed{-\frac{a}{x^2\left(1-\left(\frac{a}{x}\right)^2\right)^{1/2}}}$$

3g)

$$\frac{d}{dx}\arctan\left(\frac{x}{(1-x^2)^{1/2}}\right) = \frac{\frac{d}{dx}\left(\frac{x}{(1-x^2)^{1/2}}\right)}{1+\left(\frac{x}{(1-x^2)^{1/2}}\right)^2} = \frac{\frac{d}{dx}\left(\frac{x}{(1-x^2)^{1/2}}\right)}{1+\frac{x^2}{1-x^2}}$$

$$= \frac{1\cdot(1-x^2)^{1/2} + x\cdot\frac{1}{2}(1-x^2)^{-1/2}\cdot 2x}{1-x^2}\cdot\frac{1}{1+\frac{x^2}{1-x^2}}$$

$$= (1-x^2)^{1/2} + x^2(1-x^2)^{-1/2} = \boxed{\frac{1}{(1-x^2)^{1/2}}}$$

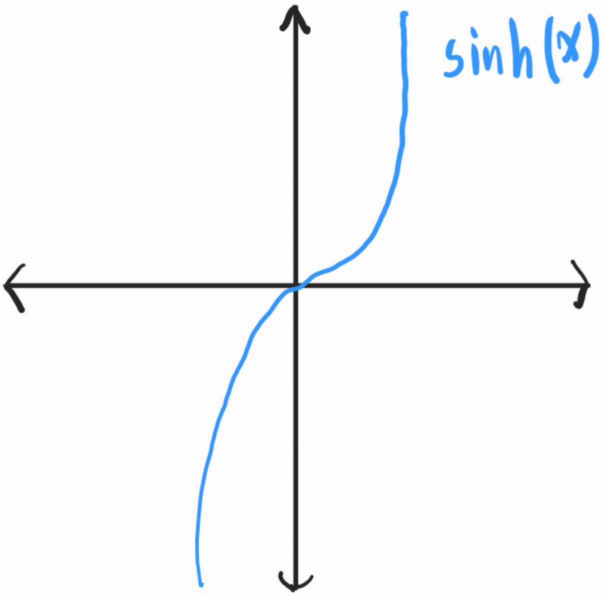
3h)

$$\frac{d}{dx}\arcsin\left(\sqrt{1-x}\right)=\frac{\frac{d}{dx}\left(\sqrt{1-x}\right)}{\left(1-\left(\sqrt{1-x}\right)^2\right)^{1/2}}=\frac{\frac{1}{2}(1-x)^{-1/2}\cdot -1}{x^{1/2}}=\frac{-1}{2x^{1/2}(1-x)^{1/2}}$$

$$=\boxed{\frac{-1}{2(x-x^2)^{1/2}}}$$

5a)

$$\sinh(x)=\frac{1}{2}\left(e^{x}-e^{-x}\right)$$



5c)

$$x=\frac{1}{2}\left(e^{y}-e^{-y}\right)$$

$$x=\frac{1}{2}\left(u-\frac{1}{u}\right)$$

$$2x=u-\frac{1}{u}$$

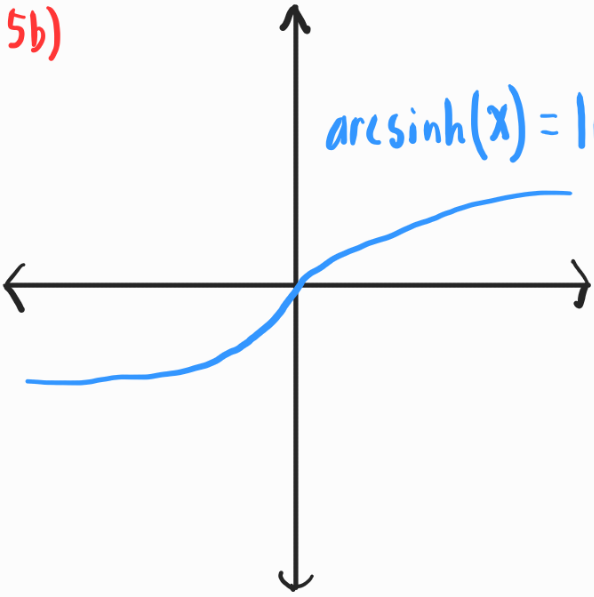
$$u^2-2xu-1=0$$

$$e^{y}=\frac{2x\pm\left(4x^2+4\right)^{1/2}}{2}=\frac{2x\pm 2\left(x^2+1\right)^{1/2}}{2}=x\pm\left(x^2+1\right)^{1/2}$$

$$\boxed{y=\ln\left(x+\left(x^2+1\right)^{1/2}\right)}$$

we abandon the minus sign because no real solutions exist for $y=\ln\left(x-\left(x^2+1\right)^{1/2}\right)$

5b)



$\text{arcsinh}(x) = \ln\left(x + (x^2+1)^{1/2}\right)$