

1)

$$\ln(y+1) + \ln(y-1) = 2x + \ln(x)$$

$$\ln(y^2 - 1) = 2x + \ln(x)$$

$$\ln\left(\frac{y^2 - 1}{x}\right) = 2x$$

$$\frac{y^2 - 1}{x} = e^{2x}$$

$$y = (xe^{2x} + 1)^{1/2}$$

2)

$$\log_b(y+1) = x^2 + \log_b(y-1)$$

$$\log_b\left(\frac{y+1}{y-1}\right) = x^2$$

$$\frac{y+1}{y-1} = b^{x^2}$$

$$y+1 = yb^{x^2} - b^{x^2}$$

$$y - yb^{x^2} = -b^{x^2} - 1$$

$$y = \frac{-b^{x^2} - 1}{1 - b^{x^2}}$$

3)

$$2\ln(y) = \ln(y+1) + x$$

$$\ln(y^2) = \ln(y+1) + x$$

$$\ln\left(\frac{y^2}{y+1}\right) = x$$

$$\frac{y^2}{y+1} = e^x$$

$$y^2 = ye^x + e^x$$

$$y^2 - ye^x - e^x = 0$$

$$y = \frac{-e^x \pm \sqrt{e^{2x} - 4e^x}}{2}$$

4)

$$\frac{e^x + e^{-x}}{e^x - e^{-x}} = y$$

$$\frac{e^x + \frac{1}{e^x}}{e^x - \frac{1}{e^x}} = y$$

$$\left. \frac{u + \frac{1}{u}}{u - \frac{1}{u}} = y \right\} u = e^x$$

$$u + \frac{1}{u} = yu - \frac{y}{u}$$

$$u(1-y) + \frac{1+y}{u} = 0$$

$$u^2(1-y) + 1+y = 0$$

$$u^2(1-y) = -1-y$$

$$e^{2x} = \frac{-1-y}{1-y}$$

$$2x = \ln\left(\frac{-1-y}{1-y}\right)$$

$$x = \frac{1}{2} \ln\left(\frac{-1-y}{1-y}\right)$$

5)

$$y = e^x + e^{-x}$$

$$y = e^x + \frac{1}{e^x}$$

$$y = u + \frac{1}{u} \quad \left. \vphantom{y = u + \frac{1}{u}} \right\} u = e^x$$

$$u - y + \frac{1}{u} = 0$$

$$u^2 - uy + 1 = 0$$

$$e^x = u = \frac{y \pm \sqrt{y^2 - 4}}{2}$$