2A. Approximation

1)

$$f(x) = (a+bx)^{1/2}$$

$$\approx (a+b\cdot 0)^{1/2} + \frac{1}{2}(a+b\cdot 0)^{-1/2}\cdot b\cdot(x-0)$$

$$\approx \boxed{\sqrt{a} + \frac{bx}{2\sqrt{a}}}$$

3)

$$f(x) = \frac{(1+x)^{3/2}}{1+2x} = (1+x)^{3/2}(1+2x)^{-1} = ab$$

$$a \approx 1 + \frac{3}{2}(1+0)^{1/2}\cdot x$$

$$\approx 1 + \frac{3}{2}x$$

$$b \approx 1 - (1+2\cdot 0)^{-2}\cdot 2\cdot x$$

$$\approx 1 - 2x$$

$$f'(x) \approx \left(1+\frac{3}{2}x\right)(1-2x)$$

$$\approx 1 - 2x + \frac{3}{2}x - 3x^2$$

$$\approx \boxed{1 - \frac{1}{2}x}$$

6)

$$f(x) = \tan\theta$$

$$\approx \tan 0 + \sec^2 0\cdot\theta + 2\sec^2 0\cdot\tan 0\cdot\theta^2$$

$$\approx 0 + 1\cdot\theta + 0$$

$$\approx \boxed{\theta}$$

11)

$$PV^k = C$$

$$P = CV^{-k}$$

$$P = C(V_0 + \vec{D}v)^{-k}$$

$$\frac{dp}{dv} = -kC(V_0+\vec{D}v)^{-k-1} \qquad \frac{dp^2}{d^2v} = kC(k+1)(V_0+\vec{D}v)^{-k-2}$$

$$\approx CV_0^{-k} - kCV_0^{-k-1}\cdot\vec{D}v + \frac{1}{2}\vec{D}v^2k^2CV_0^{-k-2} + \frac{1}{2}\vec{D}v^2kV_0^{-k-2}$$

$$\approx CV_0^{-k} - kCV_0^{-k}V_0^{-1}\vec{D}v + \frac{1}{2}\vec{D}v^2k^2CV_0^{-k}V_0^{-2} + \frac{1}{2}\vec{D}v^2kCV_0^{-k}V_0^{-2}$$

$$\approx \boxed{cV_0^{-k}\left(1 - kV_0^{-1}\vec{D}_v + \frac{1}{2}\vec{D}_v^2 k^2 V_0^{-2} + \frac{1}{2}\vec{D}_v^2 kV_0^{-2}\right)}$$

12a)

$$f(x) = e^x(1+x)^{-1} = ab$$

$$a \approx 1 + x + \frac{1}{2}x^2$$

$$b \approx 1 - x + x^2$$

$$f(x) \approx \left(1 + x + \frac{1}{2}x^2\right)\left(1 - x + x^2\right)$$

$$\approx 1 - x + x^2 + x - x^2 + \cancel{x^3} + \frac{1}{2}x^2 - \cancel{\frac{1}{2}x^3} + \cancel{\frac{1}{2}x^4}$$

$$\approx \boxed{1 + \frac{1}{2}x^2}$$

12d)

$$f(x) = \ln\cos x = \ln(a+1)$$

$$\ln(1+a) \approx a - \frac{a^2}{2}$$

$$\cos x \approx 1 - \frac{x^2}{2}$$

$$a = \cos x - 1$$

$$\boxed{f(x) \approx -\frac{x^2}{2} - \frac{x^4}{8}}$$

12e)

$$f(x) = x\ln x = (1+a)\ln(1+a)$$

$$1 + a \approx a$$

$$\ln(1+a) \approx a - 1 - \frac{(a-1)^2}{2}$$

$$a = x - 1$$

$$f(x) \approx a\left(a - 1 - \frac{(a-1)^2}{2}\right)$$

$$\approx a^2 - a - \frac{a(a-1)^2}{2}$$

$$\approx (x-1)^2 - x + 1 - \frac{(x-1)(x-2)^2}{2}$$

$$\approx x^2-2x-1-x+1-\frac{(x-1)(x^2-4x+4)}{2}$$

$$\approx x^2-3x-\frac{x^3-4x^2+4x-x^2+4x-4}{2}$$

$$\approx \boxed{x^2-3x-\frac{-5x^2+8x-4}{2}}$$