1a)

$$f(x) = \frac{x}{1-x^2}$$

$$f'(x) = \frac{1\cdot(1-x^2) - x\cdot -2x}{(1-x^2)^2} = \boxed{\frac{1+x^2}{(1-x^2)^2}}$$

1b)

$$f(x) = \ln(\cos x) - \frac{1}{2}\sin^2(x)$$

$$f'(x) = \frac{1}{\cos x}\cdot -\sin x - \sin x \cdot \cos x = \boxed{-\tan x - \sin(x)\cos(x)}$$

1c)

$$\frac{d^5}{dx^5} xe^x = \boxed{5e^x + xe^x}$$

1 $\quad e^x + xe^x$

2 $\quad e^x + e^x + xe^x$

$\vdots$

2)

$$x^{2/3} + y^{2/3} = 4$$

$$\frac{2}{3x^{1/3}} + \frac{dy}{dx}\cdot\frac{2}{3y^{1/3}} = 0$$

$$\frac{dy}{dx} = \frac{y^{1/3}}{-x^{1/3}} = \underbrace{\frac{-x^{-1/3}}{y^{-1/3}}}$$

we need it in this form so that $m = \frac{1}{3^{1/2}}$ instead of $-\frac{1}{27^{1/6}}$

$$y_{tan} = mx + b$$

$$1 = \frac{1}{3^{1/2}} \cdot -27^{1/2} + b$$

$$b = 1 + \frac{1^{3/2}}{3^{1/2}} = 4$$

$$\boxed{y_{tan} = \frac{1}{3^{1/2}}x + 4}$$

3)

$$y(t) = t^3 - 3t + 3$$

$$y'(t) = 3t^2 - 3$$

$y(1) - y(0) = -2 \}$ negative velocity from $t=0$ to $t=1$

$y(3) - y(1) = 20 \}$ Positive velocity for $t > 1$

$$\boxed{\text{total distance} = 22\text{ m}}$$

4)

Product rule: $\frac{d}{dx} fg = f'g + fg'$

$$\frac{d}{dx} fg = \lim_{\Delta x \to 0} \frac{f(x+\Delta x)g(x+\Delta x) - f(x)g(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{ab - cd}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{ab - bc + bc - cd}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{b(a-c) + c(b-d)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \left(b \cdot \frac{a-c}{\Delta x}\right) + \lim_{\Delta x \to 0} \left(c \cdot \frac{b-d}{\Delta x}\right)$$

$$= \lim_{\Delta x \to 0} \left(g(x+\Delta x) \cdot \frac{f(x+\Delta x) - f(x)}{\Delta x}\right) + \lim_{\Delta x \to 0} \left(f(x+\Delta x) \cdot \frac{g(x+\Delta x) - g(x)}{\Delta x}\right)$$

$$= \boxed{g(x)f'(x) + f'(x)g(x)}$$

5)

$$f(x) = \begin{cases} \arctan x & \text{for } x \leq 0 \\ ax^2 + bx + c & \text{for } 0 < x < 2 \\ x^3 - \frac{1}{4}x^2 + 5 & \text{for } x \geq 2 \end{cases}$$

$$f'(x) = \begin{cases} \frac{1}{1+x^2} & \text{for } x \leq 0 \\ 2ax + b & \text{for } 0 < x < 2 \\ 3x^2 - \frac{1}{2}x & \text{for } x \geq 2 \end{cases}$$

Necessary conditions to be true

1a) $\frac{1}{1+x^2} = 2ax + b$ at $x = 0$

1b) $\arctan x = ax^2 + bx + c$ at $x = 0$

2a) $2ax + b = 3x^2 - \frac{1}{2}x$ at $x = 2$

2b) $ax^2 + bx + c = x^3 - \frac{1}{4}x^2 + 5$ at $x = 2$

$b = 1$ (and a and c = any real number) satisfies condition 1a

$b = 1$ and $a = \frac{5}{2}$ satisfies condition 2a

$b = 1$, $a = \frac{5}{2}$ and $C = 0$ satisfies conditions 1b and 2b

√ there exists a combination of a, b, c such that f is differentiable everywhere

6a)

if $\lim_{x \to 0} \frac{f(x)}{x} = 1$ then $f(x) \to \sin(x)$ as $x \to 0$, thus

$\boxed{f(0) = \sin 0 = 0}$

6b)

$\boxed{f'(0) = \cos 0 = 1}$

6c)

?