1H. Exponentials and Logarithms: Algebra

1a)

$$y = y_0 e^{-kt}$$

$$\frac{y}{y_0} = e^{-kt}$$

$$-kt = \ln\left(\frac{y}{y_0}\right)$$

$$t = \frac{\ln\left(\frac{y}{y_0}\right)}{-k}$$

$$\boxed{\lambda = \frac{\ln\left(\frac{1}{2}\right)}{-k} = \frac{\ln 2}{k}}$$ substitute $\frac{y_0}{2}$ for y

1b)

$$y_1 = y_0 e^{-kt_1}$$

$$y_2 = y_0 e^{-k(t_1 + \lambda)}$$

$$= y_0 e^{(-kt_1 - \ln 2)}$$

$$= y_0 e^{-kt_1} \cdot \frac{1}{2} y_0$$

$$\boxed{y_2 = y_1 \cdot \frac{1}{2} y_0}$$

2)

$$pH_0 = -\log_{10}[H^+]$$

$$pH_1 = -\log_{10}\left(\frac{1}{2}[H^+]\right)$$

$$= -\log_{10}\frac{1}{2} - \log_{10}[H^+]$$

$$= -\log_{10}\frac{1}{2} + pH_0$$

$$pH_1 - pH_0 = -\log_{10}\frac{1}{2} = \boxed{0.301}$$

3a)

$$\ln(y+1)+\ln(y-1)=2x+\ln x$$

$$\ln\left(\frac{y^2-1}{x}\right)=2x$$

$$\frac{y^2-1}{x}=e^{2x}$$

$$\boxed{y=\left(xe^{2x}+1\right)^{1/2}}$$

5b)

$$y=e^x+\frac{1}{e^x}$$

$$y=u+\frac{1}{u} \Bigg\} \; u=e^x$$

$$u^2-yu+1=0$$

$$u=\frac{y\pm\left(y^2-4\right)^{1/2}}{-2}$$

$$e^x=\frac{y\pm\left(y^2-4\right)^{1/2}}{-2}$$

$$\boxed{x=\ln\left(\frac{y\pm\left(y^2-4\right)^{1/2}}{-2}\right)}$$