16. Higher derivatives

4)

$$y = (x+1)^{-1}$$

$$\boxed{\frac{d^n y}{dx^n} = n!(-1^n)(x+1)^{-1-n}}$$

0	$(x+1)^{-1}$
1	$-1(x+1)^{-2}\cdot 1$
2	$2(x+1)^{-3}\cdot 1$
3	$-6(x+1)^{-4}\cdot 1$
4	$24(x+1)^{-5}\cdot 1$
5	$-120(x+1)^{-6}\cdot 1$
$\vdots$	

5b)

$$y = u(x)v(x) = uv$$

$$\frac{d^n y}{dx^n} = \sum_{k=0}^{n} \binom{n}{k} \cdot \frac{d^{n-k} u}{dx^{n-k}} \cdot \frac{d^k v}{dx^k}$$

$$= \sum_{k=0}^{n} \left(\frac{n!}{k!(n-k)!}\right) \cdot \frac{d^{n-k} u}{dx^{n-k}} \cdot \frac{d^k v}{dx^k}$$

general Leibniz product rule

$$y = x^p(1+x)^q = u^p v^q$$

$$p + q = n$$

$$\frac{d^n y}{dx^n} = \sum_{k=0}^{n} \binom{n}{k} \cdot \underbrace{\frac{d^{n-k}}{dx^{n-k}} u^p}_{\text{evaluates to 0 when } (n-k) > p} \cdot \overbrace{\frac{d^k}{dx^k} v^q}^{\text{evaluates to 0 when } k > q}$$

$$= \sum_{k=0}^{p+q} \left(\frac{(p+q)!}{k!(p+q-k)!}\right) \cdot \overbrace{\frac{d^{p+q-k}}{dx^{p+q-k}} x^p \cdot \frac{d^k}{dx^k}(1+x)^q}^{k=q \text{ is the only value of } k \text{ that results in a nonzero term of the sum}}$$

$$= \frac{(p+q)!}{q!\,p!} \cdot \frac{d^p}{dx^p} x^p \cdot \frac{d^q}{dx^q} (1+x)^q$$

$$= \frac{(p+q)!}{q!\,p!} p!\,q!$$

$$= \boxed{(p+q)!}$$