

$$f(x) = ax^3 + bx^2 + cx + d$$

$$f'(x) = 3ax^2 + 2bx + c$$

$$f''(x) = 6ax + 2b$$

Necessary conditions

$$1a) f''(x) > 0 \text{ for } x < 2$$

$$1b) f''(x) < 0 \text{ for } x > 2$$

$$2a) f'(x) < 0 \text{ for } x < 1$$

$$2b) f'(x) > 0 \text{ for } 1 < x < 3$$

$$2c) f'(x) < 0 \text{ for } x > 3$$

To satisfy conditions 1a, 1b

$$6a(2) + 2b = 0$$

$$\boxed{b = -6a \text{ where } a < 0}$$

To satisfy conditions 2a, 2b, 2c

$$3a(1)^2 + 2b(1) + c = 0$$

$$3a - 12a + c = 0$$

$$c = 9a$$

$$3a(3)^2 + 2b(3) + c = 0$$

$$27a - 36a + c = 0$$

$$c = 9a$$

$$\boxed{c = 9a \text{ where } a < 0}$$

d can be any real number to satisfy all conditions