1C. Slope and derivative

1a) $A(r) = \pi r^2$

$$A'(r) = \lim_{\Delta r \to 0} \frac{A(r+\Delta r) - A(r)}{\Delta r} = \lim_{\Delta r \to 0} \frac{\pi (r+\Delta r)^2 - \pi r^2}{\Delta r}$$

$$= \lim_{\Delta r \to 0} \frac{\pi (r^2 + 2r\Delta r + \Delta r^2) - \pi r^2}{\Delta r} = \lim_{\Delta r \to 0} (2\pi r + \Delta r)$$

$$= \boxed{2\pi r}$$

3a) $f(x) = \frac{1}{2x+1}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{1}{\Delta x} \left(\frac{1}{2(x+\Delta x)+1} - \frac{1}{2x+1} \right)$$

$$= \lim_{\Delta x \to 0} \frac{1}{\Delta x} \left(\frac{2x+1-2x-2\Delta x-1}{(2(x+\Delta x)+1)(2x+1)} \right)$$

$$= \lim_{\Delta x \to 0} \frac{1}{\cancel{\Delta x}} \left(\frac{\cancel{-2\Delta x}}{(2(x+\Delta x)+1)(2x+1)} \right) = \boxed{-\frac{2}{(2x+1)^2}}$$

3b) $f(x) = 2x^2 + 5x + 4$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2(x+\Delta x)^2 + 5(x+\Delta x) + 4 - 2x^2 - 5x - 4}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{2x^2} + 4x\Delta x + 2\Delta x^2 + \cancel{5x} + 5\Delta x \cancel{+4} \cancel{-2x^2} \cancel{-5x} \cancel{-4}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} (4x + 2\Delta x + 5) = \boxed{4x + 5}$$

3e)

a. $f'(x) = -\frac{2}{(2x+1)^2}$, $\boxed{f'(x) < 0 \text{ for all } x}$

b. $f'(x) = 4x + 5$

$$\boxed{\begin{aligned} 1 &= 4x + 5 \Rightarrow x = -1 \\ 0 &= 4x + 5 \Rightarrow x = -\tfrac{5}{4} \\ -1 &= 4x + 5 \Rightarrow x = -\tfrac{3}{2} \end{aligned}}$$

4a) $f(x) = \frac{1}{2x+1}$

$f'(x) = -(2x+1)^{-2} \cdot 2 = -\frac{2}{(2x+1)^2}$

$f'(1) = -\frac{2}{9}$

$\frac{1}{3} = -\frac{2}{9}(1) + b$

$b = \frac{5}{9}$

$$\boxed{y = -\frac{2}{9}x + \frac{5}{9}}$$

4b) $f(x) = 2x^2 + 5x + 4$

$f'(x) = 4x + 5$

$f'(a) = 4a + 5$

$2a^2 + 5a + 4 = a(4a+5) + b$

$b = 4 - 2a^2$

$$\boxed{y = x(4a+5) - 2a^2 + 4}$$

5) $y = 1 + (x-1)^2$

$$\frac{dy}{dx} = 2(x-1)$$

$$y_{tan} = 2(x-1)\cdot x + 0$$

$$2(x-1)\cdot x = 1 + (x-1)^2$$

$$2x = \frac{1}{x-1} + x - 1$$

$$\frac{1}{x-1} + x - 1 - 2x = 0$$

$$\frac{1}{x-1} - x - 1 = 0$$

$$1 - (x+1)(x-1) = 0$$

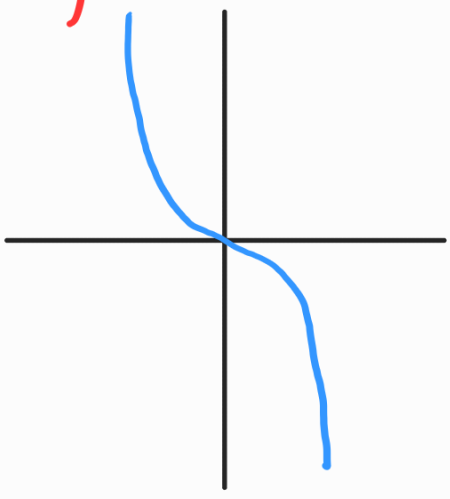
$$1 - x^2 + 1 = 0$$

$$x = 2^{\frac{1}{2}}$$

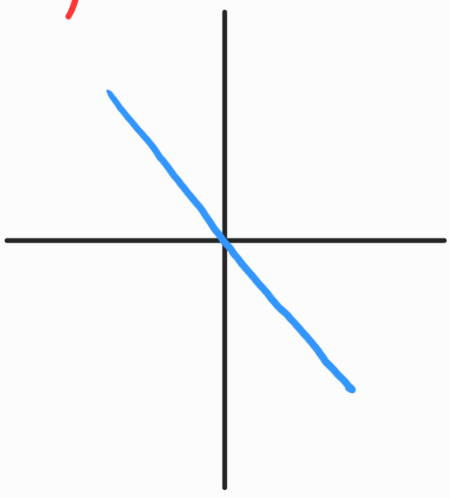
$$m = 2\left(2^{\frac{1}{2}} - 1\right)$$

$$\boxed{y_{tan} = 2\left(2^{\frac{1}{2}} - 1\right)\cdot x}$$

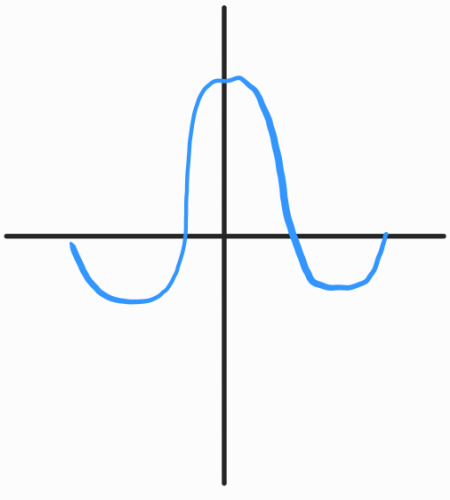
6a)



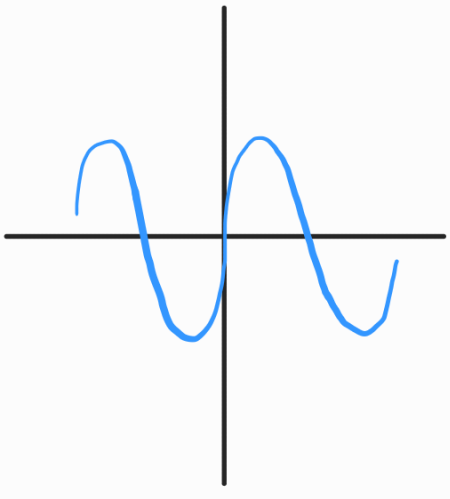
6b)



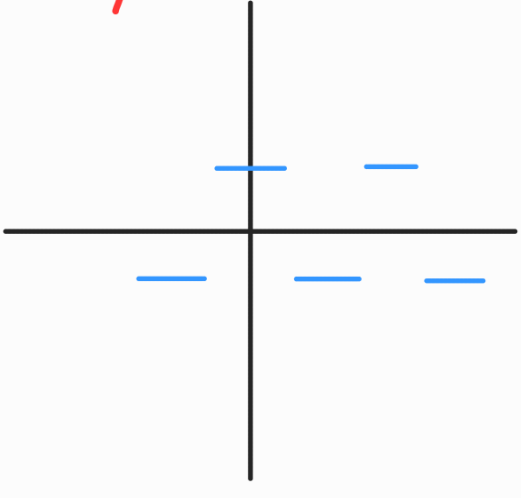
6c)



6d)



6e)



2) $f(x) = (x-a)g(x)$

$$f'(a) = \lim_{x \to a} \frac{f(a) - f(x)}{a - x}$$

$$f(a) = (a-a)g(a) = 0 \cdot g(a) = 0$$

$$f'(a) = \lim_{x \to a} \frac{0 - f(x)}{a - x} = \frac{-(x-a)g(x)}{-(x-a)}$$

$$\boxed{f'(a) = \lim_{x \to a} g(x) = g(a)}$$