

$$\overrightarrow{P_1 P_2} = \vec{P}_2 - \vec{P}_1 = \langle -1, 1, 0 \rangle$$

$$\overrightarrow{P_1 P_3} = \vec{P}_3 - \vec{P}_1 = \langle 0, 1, -1 \rangle$$

$$\vec{N} = \overrightarrow{P_1 P_2} \times \overrightarrow{P_1 P_3} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & 0 \\ 0 & 1 & -1 \end{vmatrix} = \hat{i} \begin{vmatrix} 1 & 0 \\ 1 & -1 \end{vmatrix} - \hat{j} \begin{vmatrix} -1 & 0 \\ 0 & -1 \end{vmatrix} + \hat{k} \begin{vmatrix} -1 & 1 \\ 0 & 1 \end{vmatrix} = \langle -2, 1, -1 \rangle$$

$$\vec{P} = \langle x, y, z \rangle$$

$$\overrightarrow{P_1 P} = \vec{P} - \vec{P}_1 = \langle x-1, y, z-1 \rangle$$

$$\vec{N} \cdot \overrightarrow{P_1 P} = 0$$

$$\Rightarrow \langle -2, 1, -1 \rangle \cdot \langle x-1, y, z-1 \rangle = 0$$

$$\Rightarrow -2x + 2 - 2y - 2z + 2 + x - 1 + y + z - 1 - x + 1 - y - z + 1 = 0$$

$$\Rightarrow -2x - 2y - 2z + 4 = 0$$

$$\Rightarrow \boxed{x + y + z = 2}$$