

$$1a) \langle 1, 3, 1 \rangle \times \langle 2, -1, 5 \rangle = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & 1 \\ 2 & -1 & 5 \end{vmatrix} = \begin{vmatrix} 3 & 1 \\ -1 & 5 \end{vmatrix} \hat{i} - \begin{vmatrix} 1 & 1 \\ 2 & 5 \end{vmatrix} \hat{j} + \begin{vmatrix} 1 & 3 \\ 2 & -1 \end{vmatrix} \hat{k} = \boxed{16\hat{i} - 3\hat{j} - 7\hat{k}}$$

$$1b) (\hat{i} + 2\hat{j}) \times (2\hat{i} - 3\hat{j}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 0 \\ 2 & -3 & 0 \end{vmatrix} = \begin{vmatrix} 2 & 0 \\ -3 & 0 \end{vmatrix} \hat{i} - \begin{vmatrix} 1 & 0 \\ 2 & 0 \end{vmatrix} \hat{j} + \begin{vmatrix} 1 & 2 \\ 2 & -3 \end{vmatrix} \hat{k} = \boxed{-7\hat{k}}$$

$$2) A = \left| \left(\langle 1, 2, 1 \rangle - \langle 3, 1, 6 \rangle \right) \times \left(\langle 2, 5, 2 \rangle - \langle 1, 2, 1 \rangle \right) \right|$$

$$= \left| \langle -2, 1, -5 \rangle \times \langle 1, 3, 1 \rangle \right|$$

$$= \left| \det \begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 1 & -5 \\ 1 & 3 & 1 \end{pmatrix} \right|$$

$$= \left| \begin{vmatrix} 1 & -5 \\ 3 & 1 \end{vmatrix} \hat{i} - \begin{vmatrix} -2 & -5 \\ 1 & 1 \end{vmatrix} \hat{j} + \begin{vmatrix} -2 & 1 \\ 1 & 3 \end{vmatrix} \hat{k} \right|$$

$$= \left| \langle 16, -3, -7 \rangle \right|$$

$$= \sqrt{16^2 + (-3)^2 + (-7)^2}$$

$$= \boxed{\sqrt{314}}$$