Equilibrium configurations of rotating magnetized self-gravitating tori

Influence of self-gravity and the organized magnetic field

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We present a toy model to study equilibrium configurations of magnetized rotating self-gravitating gaseous tori in the context of gaseous/dusty tori surrounding supermassive black holes in galactic nuclei. The rotating torus is modelled by a perfect fluid with barotropic equation of state, is embedded in a spherical gravitational and dipolar magnetic fields. While the central black hole dominates the gravitational field and it remains electrically neutral, the surrounding material has a non-negligible self-gravitational effect on the torus structure. By charging mechanisms it also acquires non-zero electric charge density, so the two influences need to be taken into account to achieve a self-consistent picture. The vertical and radial structure of the torus are influenced by the balance between the gravitational and the magnetic force. Firstly, the existence of the solutions is possible for certain values of the model parameters, such as the rotation law, the polytropic index, the magnetic field and self-gravity intensity. By comparison with a previous work without self-gravity, we show that the conditions can be different. Secondly, we will show that the self-gravity produced by the charged torus can have an impact on the morphology of the solutions and on the one-ring sequence.

Introduction

Studies of equilibrium of toroidal structures of a perfect fluid are important to understand the physics of accretion disks in active galactic nuclei – AGN [1] and the dense self-gravitating tori around stellar mass black holes, which can be the result of the merger of a black hole – neutron star binary or a remnant after the collapse of a massive star [2]. Nuclei of galaxies contain dusty tori and a central compact body that is frequently associated with a supermassive black hole (the mass typically $M_c \simeq 10^6$–$10^9 M_\odot$ [3]). At a distance of 10^4–10^5 gravitational radii ($R_g \equiv GM_c/c^2 \approx 1.5(M_c/M_\odot)$ km, where G is the gravitational constant) these tori become self-gravitating [4]. At the same time this distance is large enough to reduce the effects of General Relativity (essential near the center) to negligible level [5]. Therefore, an adequate and accurate description of the torus can be made using the fluid equations with Newtonian gravity [6]. This subject has been studied in great detail [7, 8, 9] also within the framework of General Relativity. In this work, we apply the Newtonian hydrodynamical approach. The torus is modelled by a perfect fluid with some net electric charge spread through the fluid. This model represents a different limit to the well-known ideal magnetohydrodynamics (MHD) with zero resistivity and vanishing volume charge density.

Set-up of the model and hypotheses

We present a convenient formalism and we give examples of a viable physical set-up where both the self-gravity of the torus material and non-vanishing electric charge density interact to define the radial and vertical structure of an equilibrium configuration torus. The idea is to use the same method as [10] but we include effects of self-gravity, which has been previously neglected for simplicity. The tori equilibrium condition is governed by the Euler equation ([11], equation (2.3)) in its stationary form, in which we have added two terms - the first one describing the self-gravitation and the second one corresponding to the Lorentz force density (and describing the electromagnetic interaction of charged fluid with external (electro)magnetic field. In the Newtonian limit, the equation adopts the following form:

$$\rho_{\rm m}(\partial_t v_i + v^j \nabla_j v_i) = -\nabla_i P - \rho_{\rm m}\nabla_i \Psi - \rho_{\rm m}\nabla_i \Psi_{\rm Sg} + \rho_e(E_i + \epsilon_{ijk} v^j B^k), \tag{1}$$

where P, Ψ_{Sg}, Ψ_c, v, $\rho_{\rm m}$ and $\rho_{\rm e}$ are, respectively, pressure, the self-gravitational potential of the torus, the central mass potential, velocity of the fluid, the mass density and the charge density. All quantities are functions of cylindrical coordinates R and Z. The electromagnetic field is described by its electric part E and magnetic part B. The last term on the right hand side of equation (1) corresponds to the Lorentz force. The charged gas velocity is assumed to be the same as the fluid velocity. In our work, the fluid is assumed to be stationary, axially symmetrical, self-gravitating, and embedded in a spherical gravitational and dipolar magnetic field, so $E_i = 0, i = (R, \phi, Z), \partial_t = 0$ and the conservation of mass and electric charge are fulfilled automatically. Due to our opposite approach to the ideal MHD (i.e. assuming vanishing conductivity), it is reasonable to prescribe the azimuthal motion of the fluid only (and no meridional or radial one), so $v_R = v_Z = 0, v_\phi = v_\phi(R, Z)$. Equation (1) can be rewritten as:

$$-\frac{1}{\rho_{\rm m}}\vec{\nabla}P - \vec{\nabla}\Psi_{Sg} - \vec{\nabla}\Psi_c - \vec{\nabla}\Phi + \frac{\vec{\mathcal{L}}}{\rho_{\rm m}} = 0, \quad \text{and can be integrated as} \quad aH + d_t\Psi_{Sg} + b\Psi_c + \Phi + e\mathcal{M} = \text{Const}, \tag{2}$$

where Φ is the centrifugal potential, $\vec{\mathcal{L}}$ the Lorentz force, H is the enthalpy and $\mathcal{M}$ the "magnetic potential". We assumed that (i) the fluid is incompressible, $\rho_{\rm m} = $ Const, so $H/\rho_{\rm m}$, (ii) The integrability condition of the equation (2) leads to two unknown functions: the orbital velocity v_ϕ, linked to Φ, i.e. the way of rotation of the fluid, and the specific charge q linked to $\mathcal{M}$, (iii) the fluid is embedded in an external dipolar magnetic field. The parameter a, b, c, d_t and e determine respectively, the maximum of the enthalpy, the way of rotation, the surface where the pressure is equal to zero, the ratio of the torus mass over the central mass and the strength of the dipolar field.

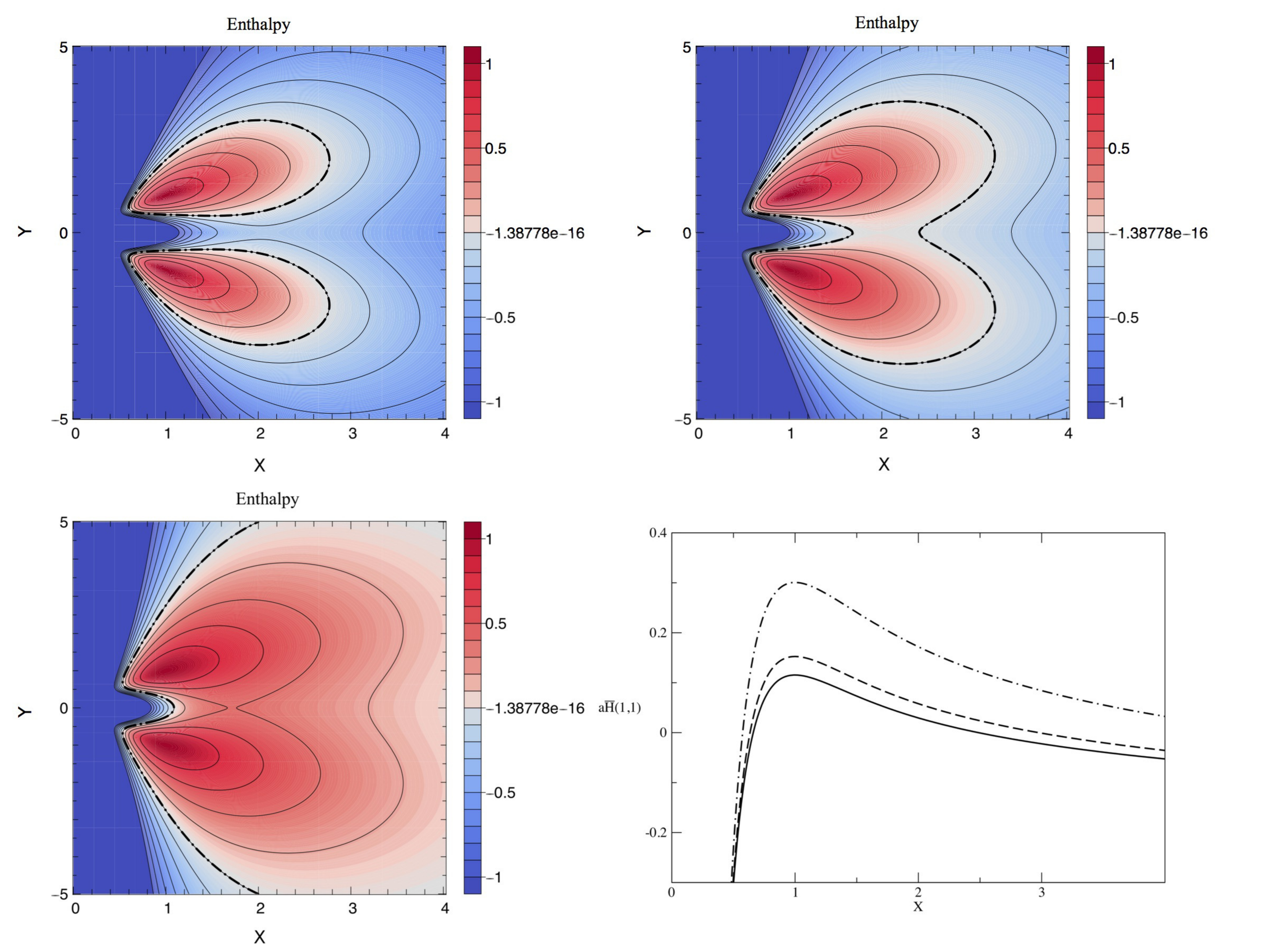

Fig. 2: Same as in Figure , but for off-equatorial tori. Two differences are included: 1) all the tori are negatively charged, 2) according the value of d_t the morphology of the torus is not the same, the lobes can be linked or not with the equatorial plane.

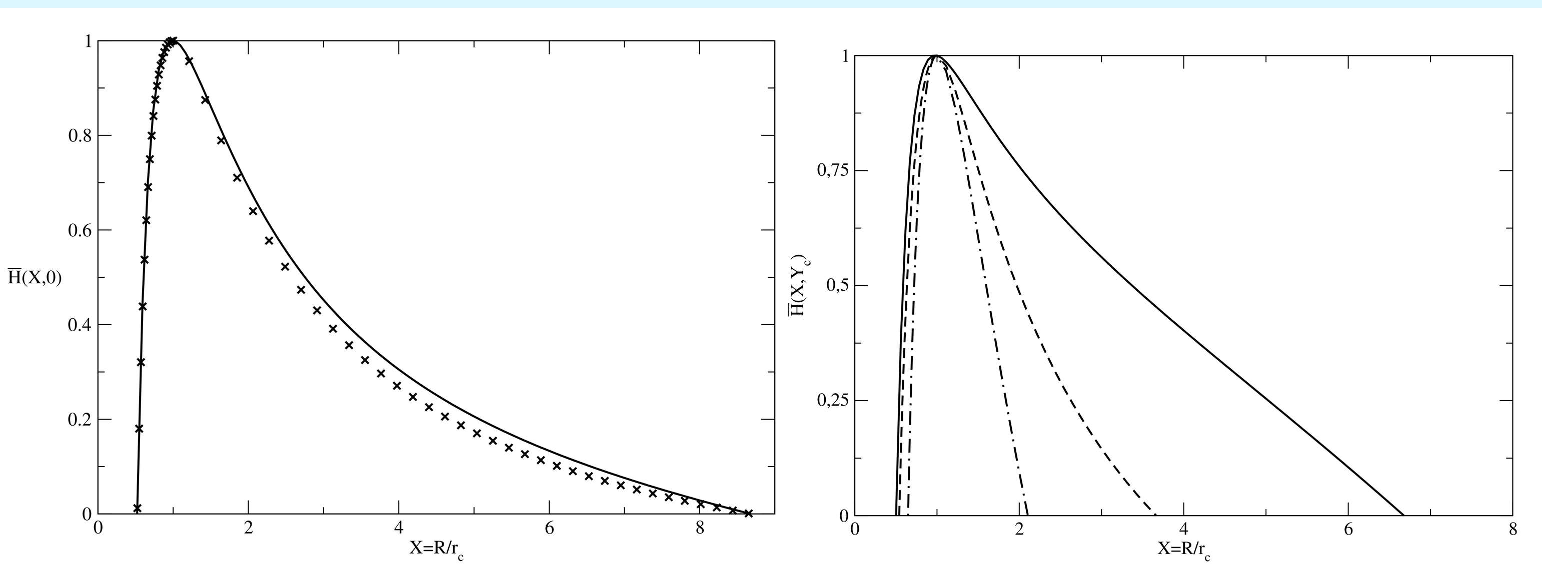

Fig. 3: At the left: the equatorial radial profile of enthalpy is shown for the two approaches. The result from SCF method in plotted by solid line, and the points from the analytical approach are indicated by crosses. At the right: Analogically to the left one, the enthalpy profile is shown but for the case of two off-equatorial lobes. The result from SCF method is plotted by solid line, the one ring approach by dot-dashed line and the double-ring approach is indicated by dashed line.

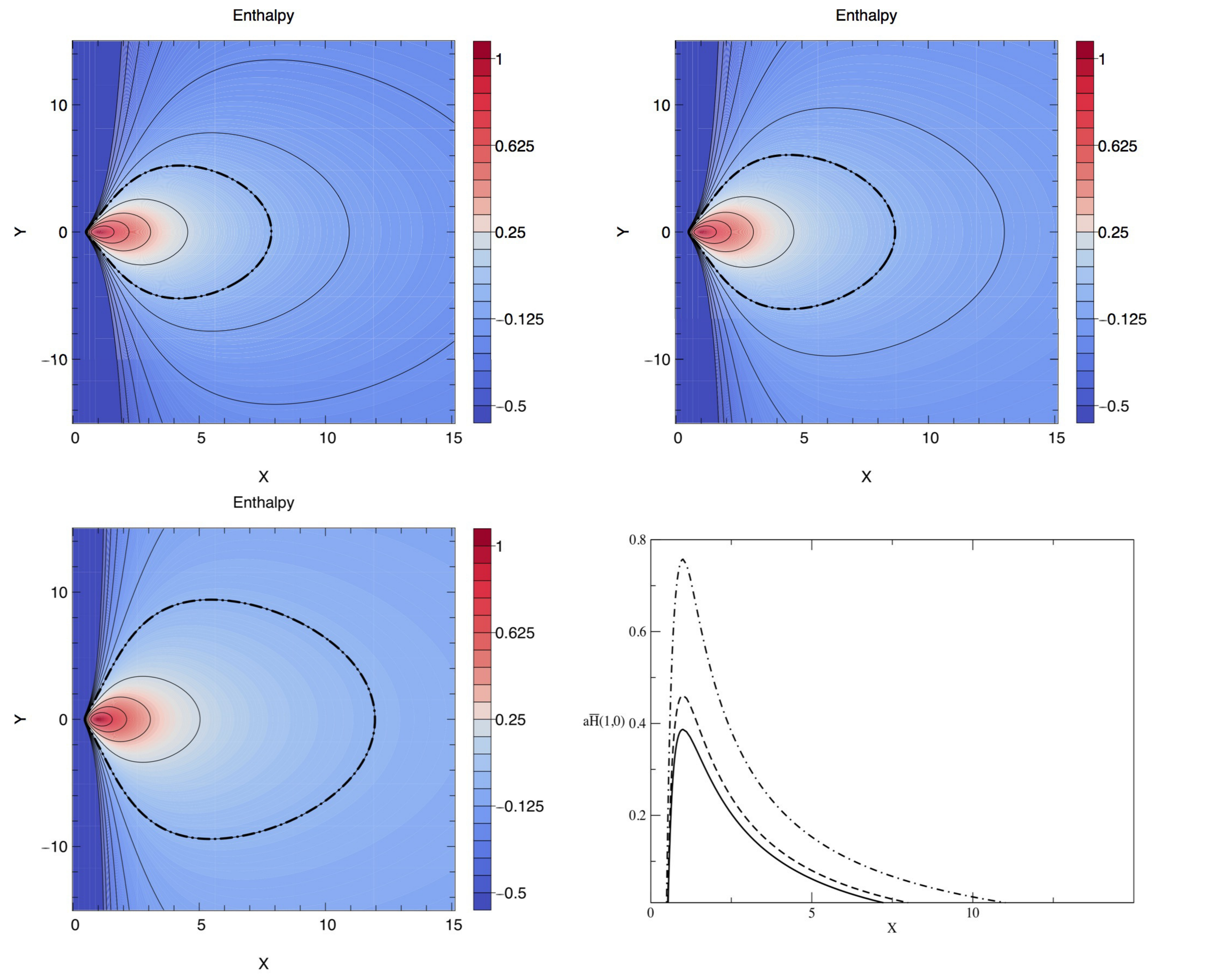

Fig. 1: Maps of enthalpy distribution in positively charged tori for $d_t = 0$ (top left), $d_t = 0.1$ (top right) and in negatively charged tori for $d_t = 0.5$ (bottom left). At the bottom right, the corresponding equatorial pressure profiles are shown (full line for $d_t = 0$, dashed line for 0.1 and dash-dot line for 0.5).

Results: Influence of the self-gravity for fluid with a constant angular momentum rotation

An equilibrium solution exists if there is a local pressure maximum, or a local enthalpy maximum. This is the case for fluid with constant angular momentum and for some values of the parameters presented below. We show that it can exists equatorial and off-equatorial configurations. To study the influence of the self-gravity, we chose the same parameters for each test but we change the strength of the self-gravity via the parameter d_t.

Figure 1 shows the equilibrium configurations obtain with three different values of d_t (0,0.1,0.5) via the enthalpy distribution. We can see that in this case the morphology of the solution does not change in function of d_t. The pressure field has a toroidal shape for all the three figures. The main change appears in the charge of the torus. For $d_t = 0$ and $d_t = 0.1$ the torus is positively charged but it is negatively charged for $d_t = 0.5$. Another interesting effect is that the maximum of pressure increases with the value of d_t. The torus grows with the strength of the self-gravity.

In **Figure 5**, we show the results for off-equatorial tori. We perform the same study as previously. We can see that for $d_t = 0$ (top right), there are toroidal off-equatorial structures located above and under the equatorial plane. And by increasing d_t, the morphology of the solutions changes. The off-equatorial toroidal structures are linked to each other by the equatorial plane. Moreover, in that case all the solutions are negatively charged.

Whereas the above-described approximation has enabled us to develop a systematic classification across the parameter space of the constraints for the existence of topologically different toroidal configurations, the adopted limit of an infinitesimally narrow gravitating ring is an idealisation. In order to relax the mentioned restriction we can compare the analytical description with a corresponding spatially extended configuration constructed numerically. To this end we employ the Self-Consistent Field method (SCF) that was developed initially by [12]. This comparison is shown in **Figure 3**. The equatorial enthalpy of equatorial tori obtained with our approximation method is close to the corresponding profile reached by the SCF method. But for off-equatorial tori, as expected, the analytical profile from the one ring approximation comes out quite inaccurate, however, the accuracy is much improved with the double-ring approach.

Conclusions

To conclude, we can say that the self-gravity has an impact on the charge of the equilibrium torus and on the morphology. On the other hand, the morphologies of tori remain similar to the non-self-gravitating case shown in [10]. We found the toroidal configuration, the closed isobars with cusps, and the toroidal off-equatorial configurations. The above described approach in this paper can serve as a useful test bed for comparisons with other methods. In particular, it provides us with a better insight into conditions that define the form of the electrically charged configurations. The method allows us to produce a relatively precise approximation to their structure, taking self-gravity of the fluid into account. See [13] for additional details.

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