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Equilibrium structures of fluid charged tori around a compact object in rotation

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We are presenting an analytical approach for the equilibrium of electrically charged perfect fluid surrounding a rotating charged compact object ($Q \neq 0$), embedded in an asymptotically large scale uniform magnetic field ($B \neq 0$). The vertical and radial structure of the torus are influenced by the balance between the gravitational and the magnetic force. Following our previous study of rotating charged test fluids around a non rotating black hole, this time, we show that according to the spin of the black hole the existence of such structures change. In this work, we focus our attention on orbiting structures in permanent rigid rotation. We prove, in this rotating case also, the existence of equilibrium configurations in the equatorial plane (tori) and on the polar axis such as charged polar clouds. The equilibrium highly depends on the model parameters, such as the spin and the charge of the black hole, the uniform magnetic field and the angular velocity. We are also showing that in the case ($B = 0$, $Q \neq 0$) and ($B \neq 0$, $Q = 0$), the rotation of the black hole allows the possibility of equilibrium fluid in both configurations.

Introduction

Our purpose is the study of the equilibrium of charged perfect fluid encircling a charged Kerr black hole. These equilibrium toroidal configurations are important to understand the physics of accretion discs around compact objects [1, 2, 3]. Many studies of perfect fluid tori (equilibrium/dynamics) have been done with various spacetimes, from the Schwarzschild, the Reissner-Nordström-de Sitter to the Kerr background [4, 5, 6]. Other works have been done by adding the impact of a magnetic field (dipolar, uniform,...)[7, 8, 9, 10]. All of these works were done in the relativistic framework, but many others exist under the Newtonian description. Astrophysical fluids can be immersed in various systems. According to the forces involved in the system, fluids can exhibit various different behaviours. We focus our study on two different configurations: a regular one, located in the equatorial plane (toroidal configurations) and a unique one, located on the polar axis of the black hole (polar clouds configurations). The whole system is immersed in an asymptotically uniform magnetic field.

Set-up of the model and hypotheses

We present a convenient formalism and we give examples of a viable physical set-up where the gravity of the central mass, its rotation and electric charge, the uniform B-field, and the non-vanishing electric charge density of the torus interact to define the radial and vertical structure of an equilibrium configuration torus. We have the same background as in [9] but we include effects of the rotation of the black hole. The self-gravitational and the electromagnetic field produced by the fluid are neglected. We assumed that the fluid has a polytropic equation of state ($p = \kappa\rho^\Gamma$) and has an organized rotational motion. In addition, we are working with a constant angular velocity ω (permanent rigid rotation). Moreover the turbulence and the viscosity are neglected and the conductivity of the medium is set to zero. The pressure equations, describing a perfect fluid in orbital (azimuthal) motion, are derived from the conservation laws and Maxwell's equations,

$$\partial_r p = -(p+\epsilon)\left(\partial_r \ln|U_t| - \frac{\omega\partial_r l}{1-\omega l}\right) + q\left(U^t\partial_r A_t + U^\phi \partial_r A_\phi\right), \ \partial_\theta p = -(p+\epsilon)\left(\partial_\theta \ln|U_t| - \frac{\omega\partial_\theta l}{1-\omega l}\right) + q\left(U^t\partial_\theta A_t + U^\phi \partial_\theta A_\phi\right),$$

where p is the pressure of the fluid, ϵ is the energy density, l is the specific angular momentum ans q is the charge density. The components of the vector potential, A_t and A_ϕ, are described by the Wald's test-field solution of Maxwell equations [12, 13],

$$A_t = \frac{B}{2}\left[g_{t\phi} + 2ag_{tt} - e\left(g_{tt} - 1\right)\right], \quad A_\phi = \frac{B}{2}\left[g_{\phi\phi} + 2ag_{t\phi} - eg_{t\phi}\right], \tag{1}$$

with $e = Q/B$ and $g_{\alpha\beta}$ and a being components of the Kerr metric and black hole spin. B and Q represent, respectively, the strength of the external uniform magnetic field and the charge of the black hole. They are test-field parameters and do not influence the spacetime geometry. Using the following simplification $\partial_{r,\theta} h = \frac{\Gamma-1}{\Gamma}\frac{\partial_{r,\theta} p}{p+\epsilon}$, and the integrability conditions of the pressure equations, we can write

$$h = -\ln|U_t| - \ln(1-\omega l) + \int_S f(S)dS + h_0, \quad \text{with} \quad S = A_t + \omega A_\phi, \tag{2}$$

and $f(S)$ is an arbitrary function that determines the charge density distribution. According to [9] we choose:

- Polar clouds: $f(S) = -2\mu S/eB^2$ with $\mu = k_0 B$, where k_0 is a constant representing the effect of charge.
- Equatorial tori: $f(S) = \mu B/4S$ with $\mu = k_0/B$.

Physical solutions can be found using the mathematical conditions of existence. It allows also to constraint the parameters ω, μ. All others are arbitrarily fixed.

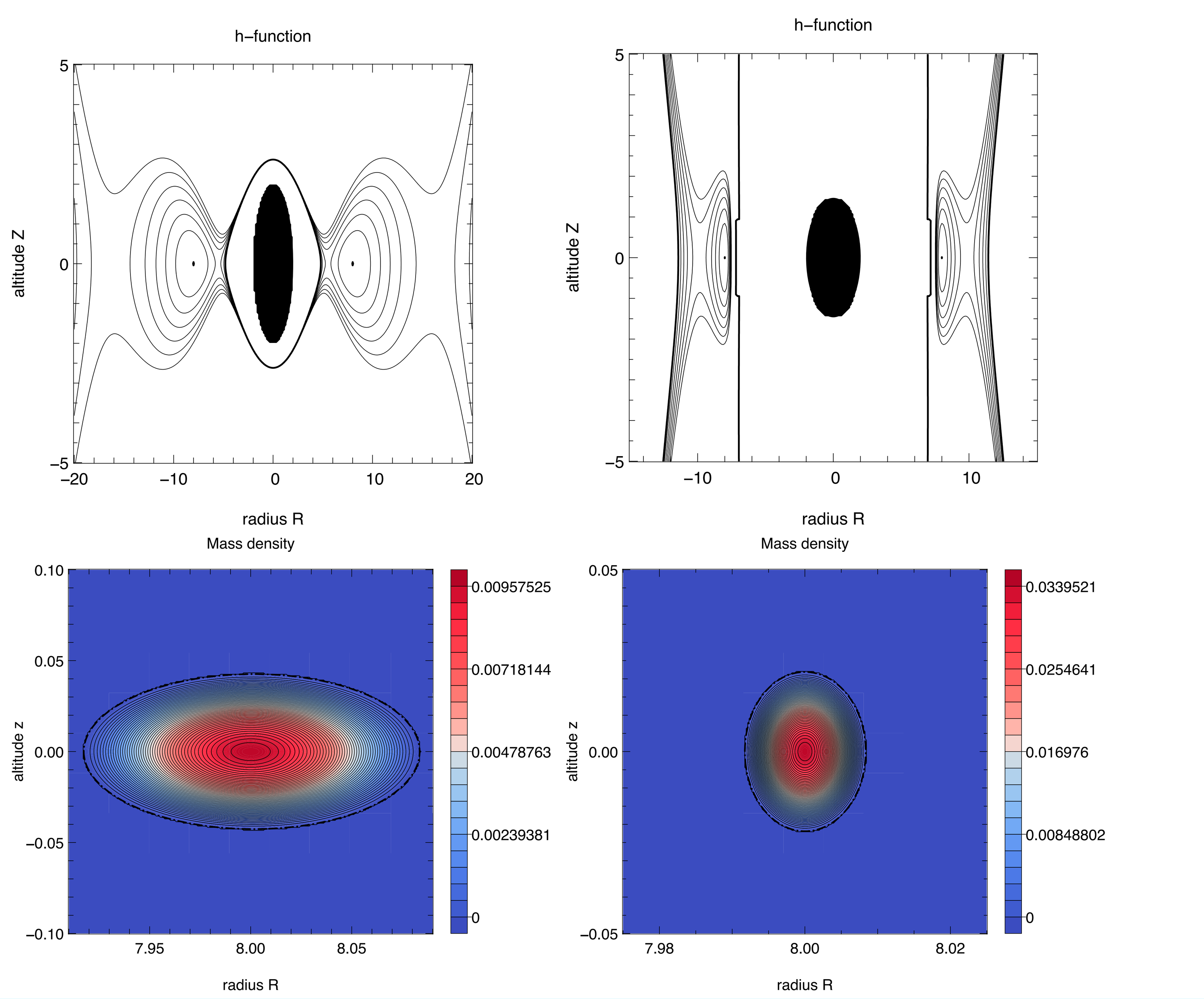

Fig. 2: At the top, equatorial tori configurations for a slowly rotating black hole at the left ($a = 0.01$), and a fast one at the right ($a = 0.9$). At the bottom, mass density profiles ($\rho/10^{-20}$) of the corresponding equatorial tori shown above are presented.

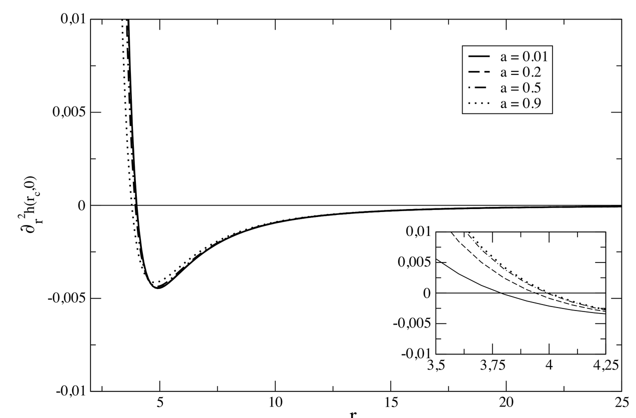

Fig. 3: Behaviour of $\partial_r^2 h(r_c, 0)$ for four values of the spin $a = 0.01, 0.2, 0.5, 0.9$, for $Q \neq 0, B = 0$ and $f(S) = -2k_0 S$. Solutions are possible when $\partial_r^2 h(r_c, 0) < 0$.

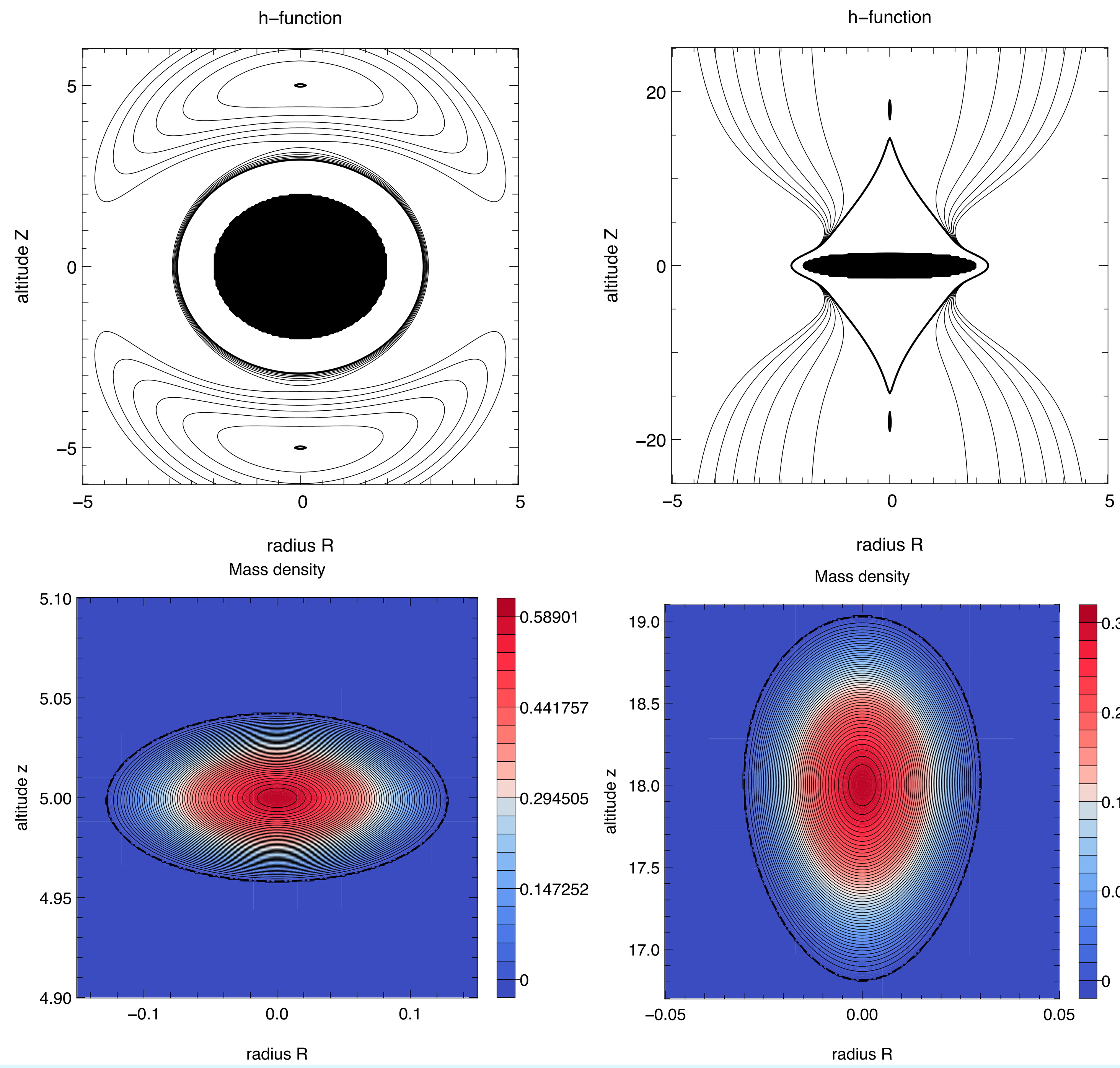

Fig. 1: At the top, polar cloud configurations for a slowly rotating black hole at the left ($a = 0.01$), and a fast one at the right ($a = 0.9$). At the bottom, mass density profiles ($\rho/10^{-19}$) of the corresponding polar cloud shown above are presented.

Results: Influence of the rotation of the black hole on the fluid equilibrium

An equilibrium solution exists if there is a local pressure maximum i.e. a local enthalpy maximum. This is the case for fluid with constant angular velocity and for some values of the parameters $(a, e, r_c, f(S))$. We show that both equatorial and polar clouds configurations exist. To study the influence of rotation of the black hole, we play with the parameter a.

Figure 1 shows, via the enthalpy function, the equilibrium configuration of a polar-cloud type for two different values of a: (i) $a = 0.01$ (slowly rotating black hole), (ii) $a = 0.9$ (fast rotating one). We can see that in the slow case the morphology is basically the same as in the non rotating case, i.e. ellipsoidal shape. But in the case of a fast rotating black hole, we can note that the morphology move to an oblate spheroid. Moreover, in the polar cloud case, we showed that increasing the value of a, increase the distance from the central mass, where a solution is possible. In the fast case, equilibrium configurations can be found only relatively far away from the black hole in comparison with the slow case.

In **Figure 2**, we show the results for equatorial tori. We perform the same study as previously. We can note that the same effect are observable. The morphology in the fast rotating case move to an oblate spheroid. Moreover on the conditions of existence, we observe the same effect that the maximum of pressure where a solution can be found is pushed far away from the central compact object.

We also worked on two limiting cases ($B = 0$, $Q \neq 0$) and ($B \neq 0$, $Q = 0$). In our previous work **[9]**, in the polar cloud configuration, no solution was possible. The conditions of existence with $a = 0$ was not satisfied. Adding a rotating black hole to the system allows the existence of equilibrium solutions, for specific parameters. This is shown in **Figure 3**, where the condition of existence are plotted. Solutions exist if $\partial_r^2 h(r_c, 0) < 0$, which is the case here. We can note that, on contrary with the previous case, increasing the spin does not push the region of existence far away from the center. Solutions relatively close can also be found. This result is really interesting because it means that in absence of the charge or of the magnetic field, the rotation of the black hole is crucial and allows the possibility of polar equilibrium solution in rigid rotation.

Conclusions

The above-described approximation has enabled us to develop a systematic classification across the parameter space of the constraints for the existence of topologically different charged perfect fluid configurations. To conclude, we can say that the rotation of the black hole has an impact on the existence conditions of the equilibrium configurations and on their morphology. We showed that increasing the spin of the black hole seems to change the morphology from an prolate shape to an oblate spheroid. This effect is observable in both configurations. The above described approach provides us with a better insight into conditions that define the form of the electrically charged fluid configurations embedded in a uniform magnetic field and gives us information on the effect of the presence of a Kerr black hole.

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