The purpose of this algorithm is to solve for x and y given integer constants a, b, c when the equation is in this form:

$ax + by = c$

Algorithm

STEP 1: Does gcd(a,b) evenly divide c? (notation: gcd(a,b) | c)

if NO,

NO SOLUTION

if YES,

carry on to step 2

STEP 2: b | a? (Does b evenly divide a?)

if YES,

$x = 0$
$y = c/b$
and you are done

if NO,
carry on to step 3

STEP 3:
Run the algorithm on
$bu + rv = c$
if you get a solution for this equation, (values for u and v), then the solution for x and y:

$\mathbf{x = v}$
$\mathbf{y = u - qx}$

Example:

#1. $12x + 8y = 40$

Step 1: gcd(12,8) | 40 ?

4 | 40?

YES

Step 2: 8 | 12 ?

NO

Step 3: $b = 8$

$r = a \% b$

$= 12 \% 8 = 4$

solve for u, v

8u + 4v = 40

#2. (recursive call) 8x + 4y = 40

Step 1: gcd(8,4) | 40 ?

4 | 40 ?

YES

Step 2: 4 | 8?

YES

x = 0

y = c/b = 40/4 = 10

x = 0 and y = 10 are the solutions for #2. We will now use the solution for #2 to get the solution to #1 (the original problem)

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back to #1...

we now know that u = 0 and v = 10

therefore,

$x = v$

$y = u - qx$

$x = 10$

$y = 0 - q(10) = -q(10)$

$q = a/b = 12/8 = 1$

so, $y = -1(10) = -10$

$x = 10$ and $y = -10$ is our solution.

CHECK THE ANSWER:

$12(10) + 8(-10) = 40$

$40 = 40$