# TP1 - Mathematical Morphology

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October 19, 2022

## 1 Application on gray-scale images

**Question 1 & 2:**
The following images demonstrates the result of each morphological operations using a disk of radius 2 as structuring element: From the image resulting from

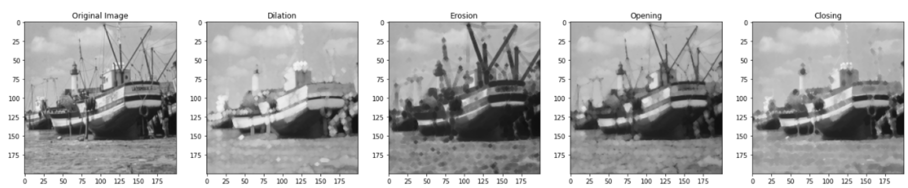


Figure 1: Morphological Operators (disk r = 2)

dilation, the spreading of high intensity values can be seen. The image resulting from erosion, shows how low intensities are spread. From the erosion the anti-extensive property can be seen. An opening is simply an erosion followed by a dilation. The peaks (local maxima) are attenuated. The closing is a dilation followed by an erosion. The "valleys" (local minima) are filled. The size of the structuring element can be changed, *e.g.* to a disk of radius 5: The choice of the

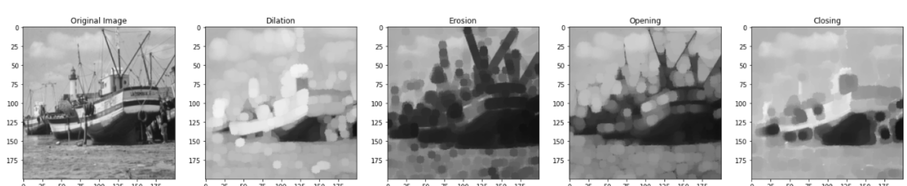


Figure 2: Effect of different morphological operators (disk r = 5)

shape of the structuring element determines the shapes of the areas of the image that are modified by the morphological operation. These affected areas tend to have the same shape as the structuring element. By increasing the structuring element this effect is simply more visible. This is the increasing property with respect to the structuring element, which is evident in the image representing dilation and erosion. We can also change the shape of the structuring element, *e.g.* with a line of length 10 with an angle of -45 degrees. Clearly, the choice of shape for the structuring element and its size will have an effect on the resulting image:

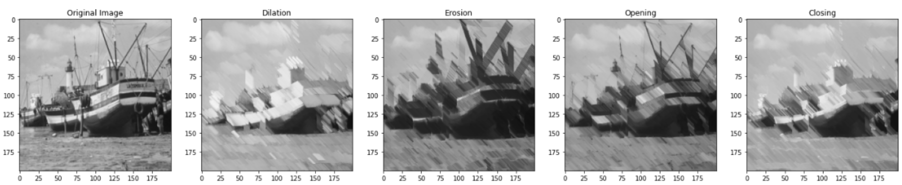


Figure 3: Effect of different morphological operators (line $l = 10$ & $\alpha = -45$)

**Question 3:**
A dilation by a square of *3 by 3* pixels, followed with a dilation by a square of *5 by 5* pixels, is simply a single dilation by a square of size *7 by 7* pixels. This is the iterative property. As can be seen with the following images, both methods results in the same image: An opening by a square of *3 by 3* pixels,

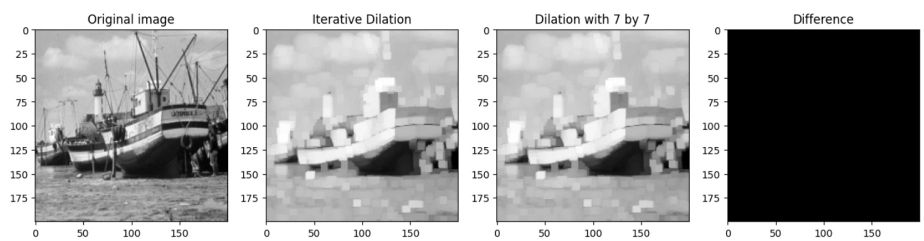


Figure 4: Iteration of dilation

followed by an opening by a square of *5 by 5* pixels, is a single opening by a square with the size of the largest square of the initial iteration, *i.e.* 5. As can be seen with the following images, both methods results in the same image:

**Question 4:**
The top-hat transform is defined by the following expression:

$$f_T = f - f_B \tag{1}$$

The top-hat transform will result in an image which preserves the intensity peaks of the image: Its dual expression will be the difference between an erosion and the original image:

$$f_{Td} = f^B - f \tag{2}$$

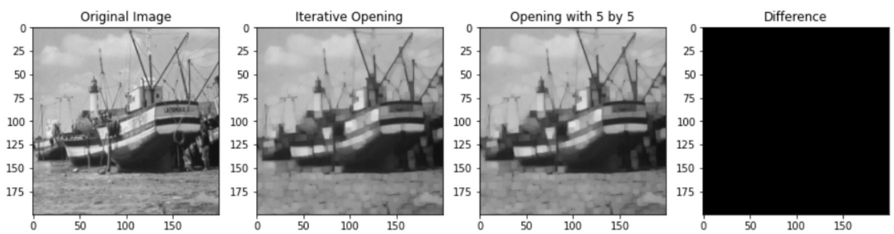


Figure 5: Iteration of openings

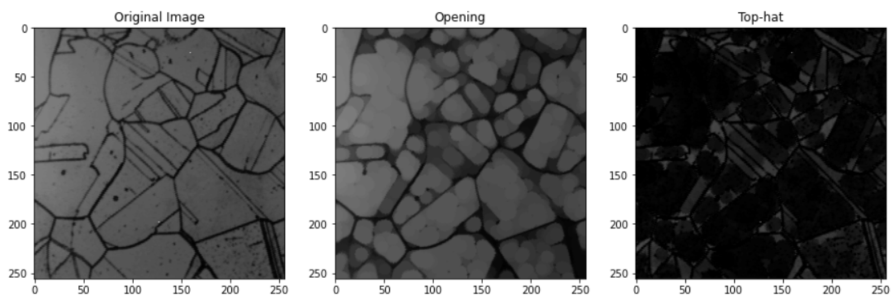


Figure 6: Top-Hat

The dual expression will preserve local minimums. As can be seen in the following image, the dark lines are preserved. For both the top-hat and dual top-hat

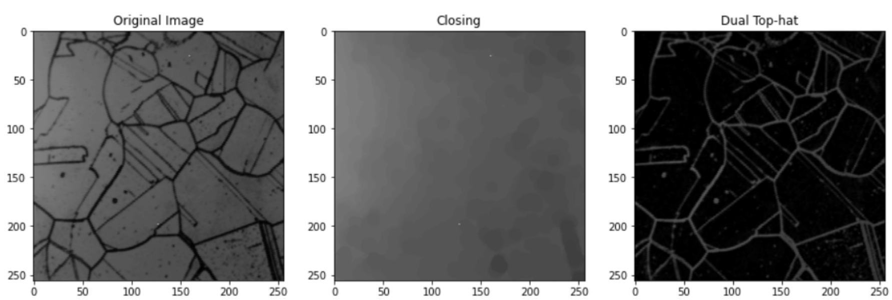


Figure 7: Dual Top-Hat

implementations a disk of radius 6 was chosen as structuring element.

**Question 5:**
For this task, four structuring elements have been chosen. They are lines of length 10 but with varying angles. The resulting opening will depend on the structuring element applied. For instance, if a detail is eroded at first, this detail will simply not be reconstructed by the dilation. The following images show the

four resulting openings. By iterating over the images and taking the maximum

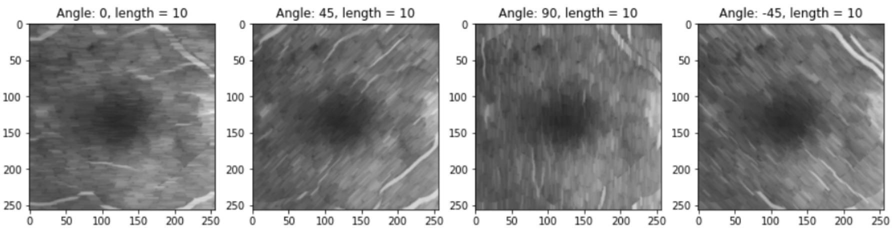


Figure 8: Openings with 4 angles

value of the corresponding pixels a new image can be constructed. The result

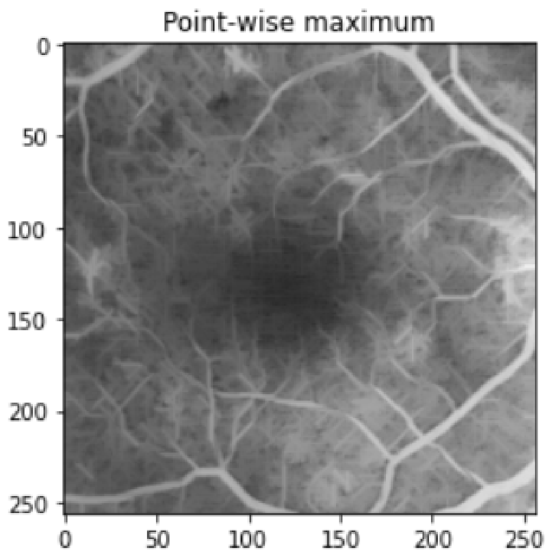


Figure 9: Point-wise max

seems to be very similar to a reconstruction by dilation. This will be explored further in the section on reconstruction.

# 2 Alternate sequential filter

The alternate sequential filter implemented here is a succession of a same operation that consists of taking the opening of an image and then the closing of the resulting opening. This operation is repeated such that at each iteration the size of the structuring element is increased by one (otherwise there would not be any effect since these operations are idempotent). This incremental increasing allows to preserve some details. Indeed, if we would apply this operation only once with a big structuring element we would lose too much details, which is not desirable.

The alternate sequential filter implemented here, takes a disk with $r = 1$. The filter does this for nine iterations. The following images show the original image at first followed by the intermediate filtering steps and then the final image: If

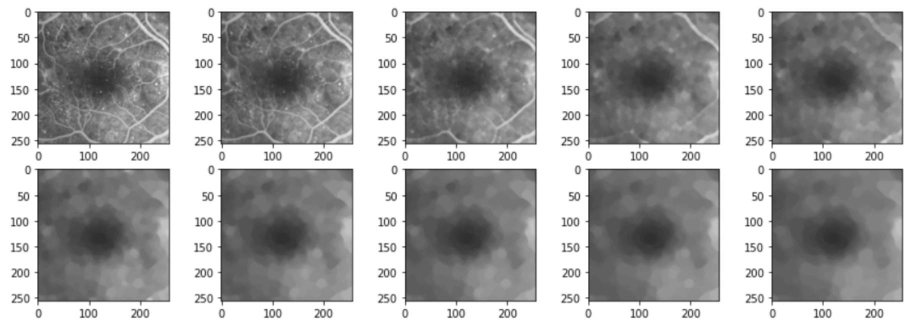

Figure 10: From left to right and top to bottom: the resulting images of the alternate sequential filter.

we continue with increasingly larger elements we would tend towards a uniform image in gray level. Interestingly, after a sufficiently high number of applied operations we would obtain flat regions that provide a kind of pre-segmentation.

# 3 Reconstruction

**Question 1:**
The opening of our retina image is taken. The opening will preserve some of the features of the original image. We can use this opening as a marker to reconstruct our original image: The reconstruction by dilation is simply a geodesic

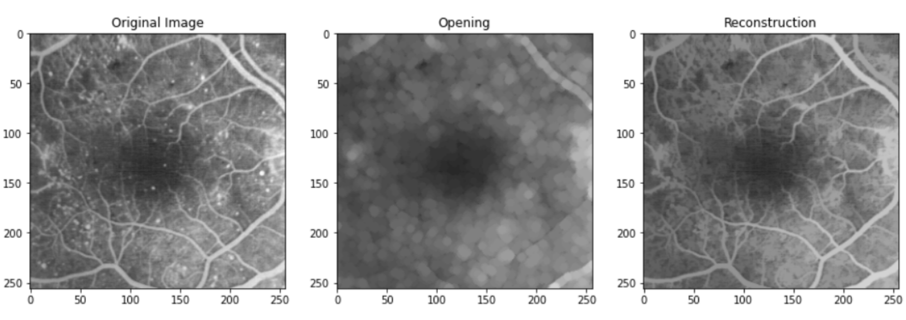


Figure 11: Reconstruction (dilation)

dilation of the marker function, *i.e.* our opening, within the mask, *i.e.* the original image.

**Question 2:**
Thus, a reconstruction by erosion is simply a geodesic erosion of the marker function. The reconstruction by erosion can be implemented with the following sequence of operations:

$$\begin{aligned} m_0 &= m \vee I \\ m_1 &= E(m_0, B_1) \vee I \\ &\vdots \\ m_n &= E(m_{n-1}, B1) \vee I \end{aligned}$$

where, according to the duality principle, we have:

$$E(m, B_1) = \left[D\left(m^C, B_1\right)\right]^C.$$

Therefore instead of each dilation we can use this algorithm:

1. Take the complementary of the marker $m$: each intensity $x$ is replaced by $255 - x$;
2. Perform the dilation on the inversed marker: $D\left(m^C, B_1\right)$;
3. Return $\left[D\left(m^C, B_1\right)\right]^C$ which is the complementary of the previous result.

**Question 3:**
The following images show the alternate sequential filter with applied reconstructions in the intermediate steps. A reconstruction by dilation is performed after each opening and a reconstruction by erosion is performed after each closing. Clearly, more information is preserved after the filtering due to the reconstruction.

# 4 Segmentation

**Question 1:**
The morphological gradient of an image is the difference between the dilation and erosion of the image. To get a thin gradient line, the structuring element must not be large. Here a disk of radius 1 has been applied for both the dilation and erosion.

**Question 2:**
The watershed method segments the image with the contours obtained from

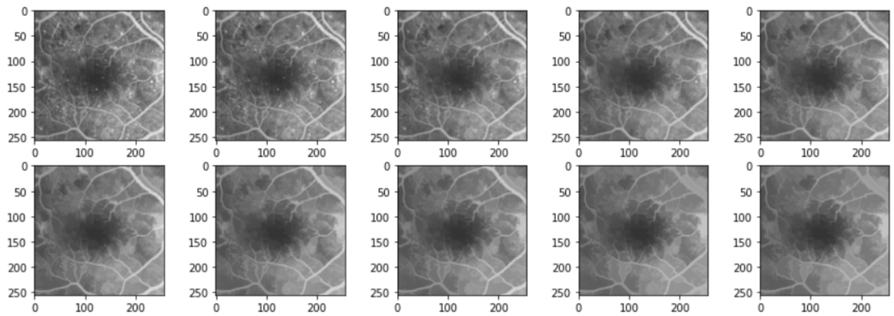

Figure 12: Filter with reconstruction

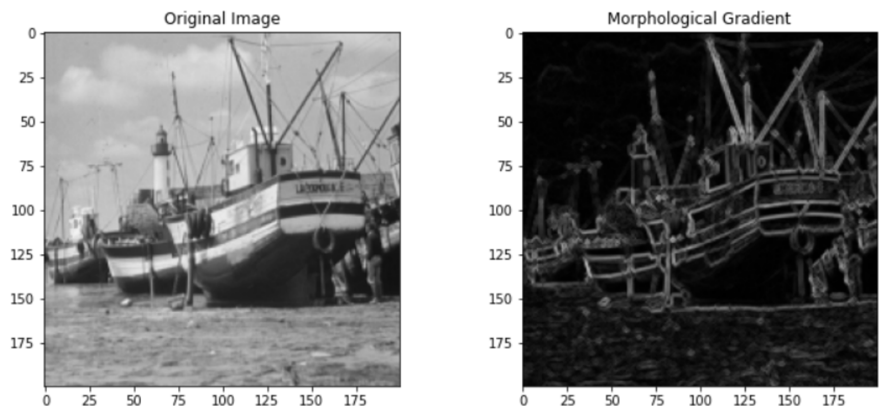


Figure 13: Morphological Gradient

the gradient as limits. Our gradient image is an image with large dark patches separated by the gradient lines. Of course there will be noise which must be filtered out. For instance, if two patches of low intensities are separated by a gradient line, the process consists of increasing the intensity of the patches until the intensity matches the gradient intensity. This allows the separation of the two patches. The following image shows a segmentation of the original image.

It would be preferable to remove some of the details of the gradient image to have a better segmentation. A morphological filter can remove some details. However, the number of iterations and the size of the structuring element is important.

**Question 3:**

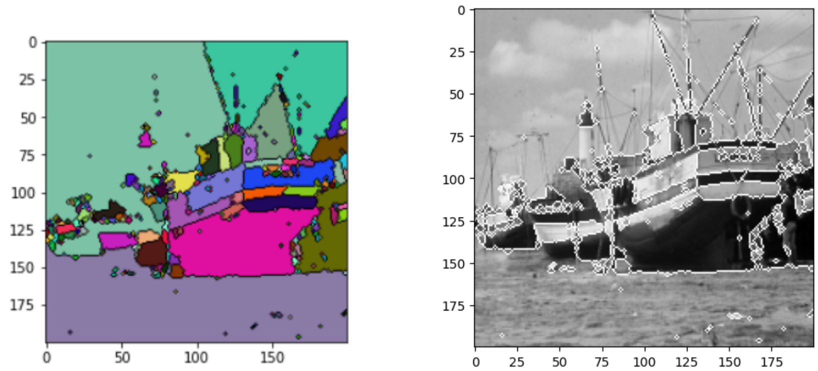


Figure 14: Watershed obtained from the gradient image of the Figure 13 (in which we kept only the values above 40) and the local minima that serve as markers.

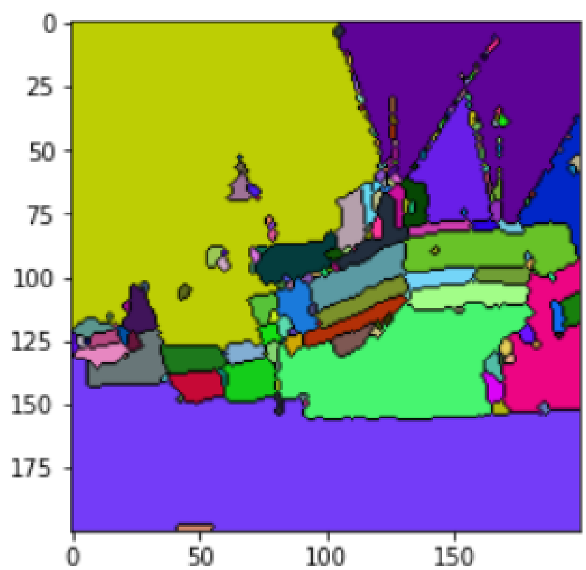


Figure 15: **Watershed obtained from a filtered image.** The filter used here is an alternate sequential filter with reconstruction by dilation and erosion.

**Question 4:**
In order to find robust regional minima with a dynamic less than some value $h$ we can perform, on the image $f^C$, this operation:

$$f^C - D_f^{\infty}(f^C - h).$$

where $D_f^\infty$ is the reconstruction operation.

**Question 5:**
We can use these regional minima $g$ to constrain the watershed applied on the image $f$ by performing by doing a reconstruction of $f$ in $g$ by erosion:

$$E_{f\wedge g}(g, B_\infty).$$

**Question 6:**
We could use the method proposed in the question 5 of the section 1 to eliminate the details that do not correspond to black lines, and then fill the "valleys" corresponding to the black lines such that they only produce a single region. Finally we can threshold the image to obtain segmentation masks.