

TADI: Pratical work – Scale space

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Exercise 1

In the 2D case, the CFL condition is:

$$\frac{\Delta t}{\Delta x^2} + \frac{\Delta t}{\Delta y^2} \leq \frac{1}{2}.$$

As $\Delta x^2 = 1$ and $\Delta y^2 = 1$, the condition is:

$$\Delta t < \frac{1}{4}.$$

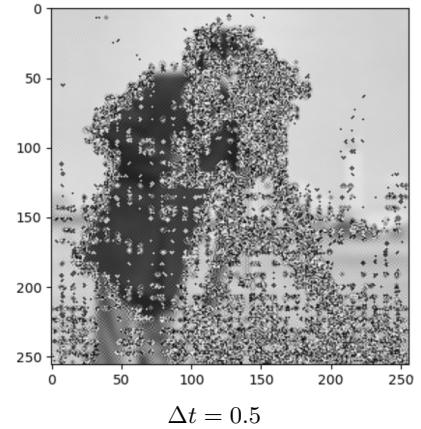
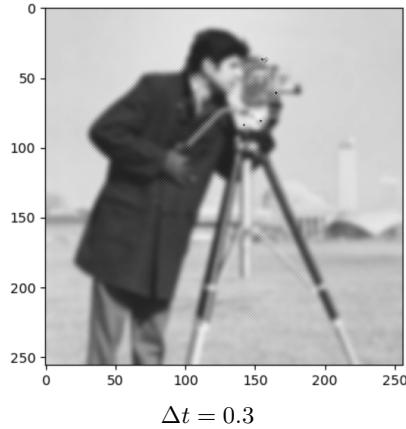
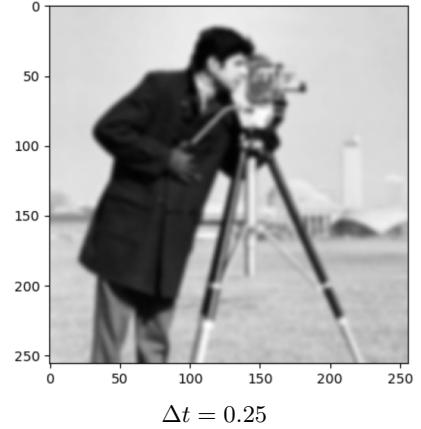
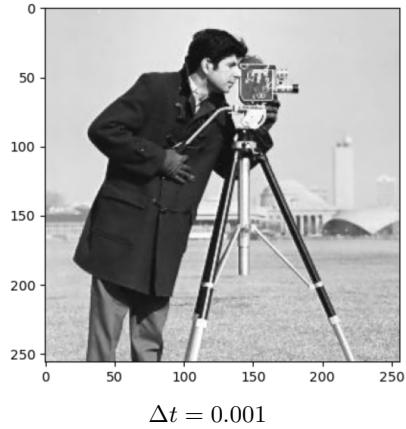


Figure (1) Different results obtained with the 2-D forward and centered numerical scheme of the heat equation.

As we can see in Figure 1:

- when the CFL condition is not satisfied the results are not good.
- the application of the forward and centered numerical scheme of the heat equation allows a smoothing of the image.

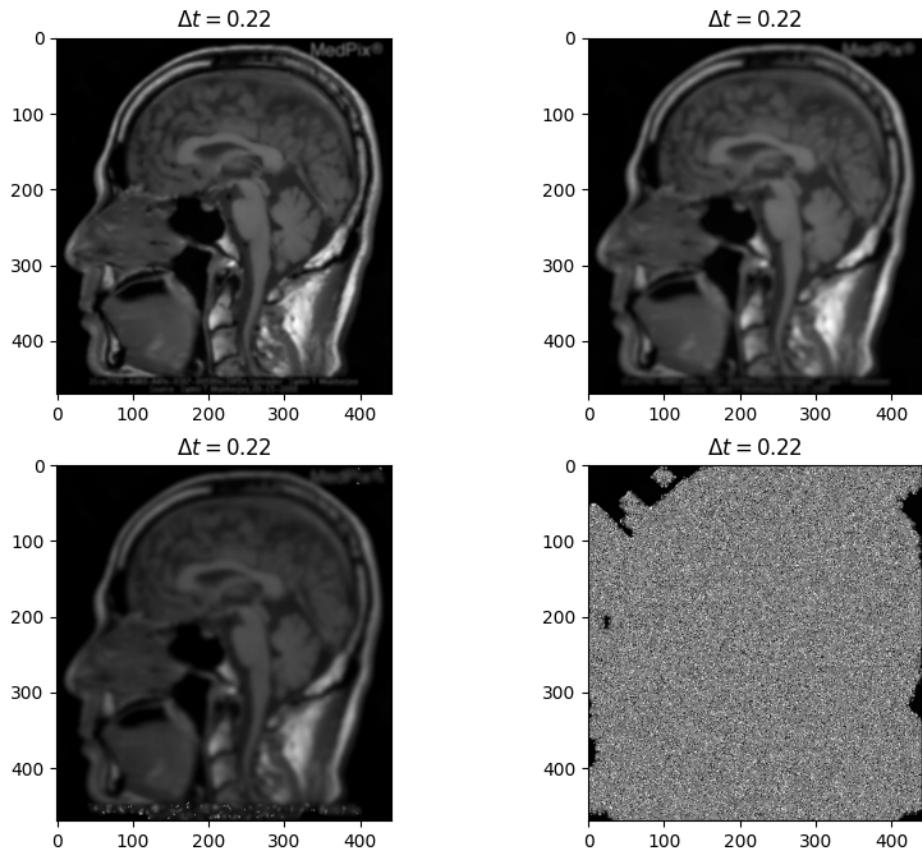
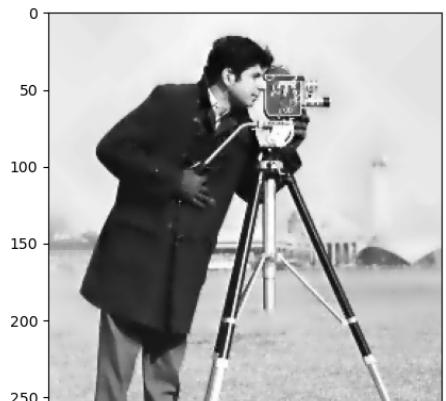
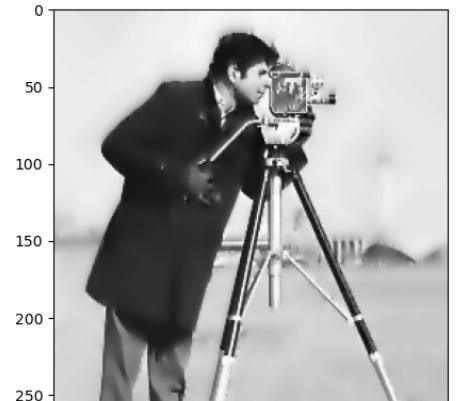


Figure (2) Different results obtained with the 2-D forward and centered numerical scheme of the heat equation.

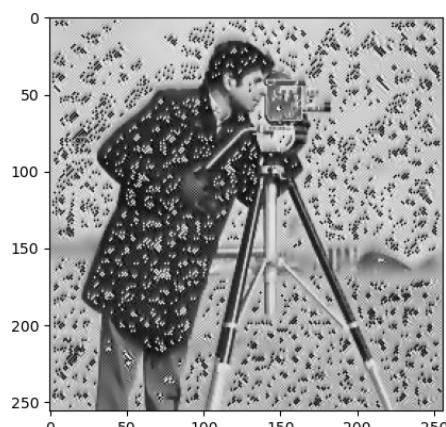
Exercise 2



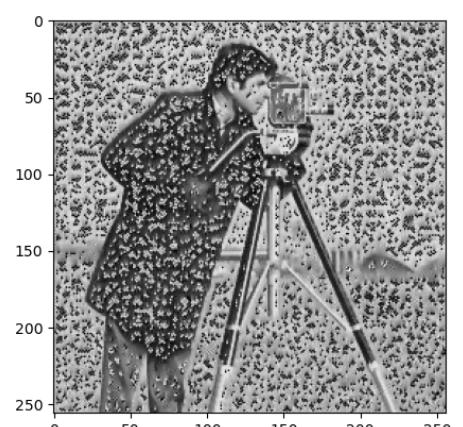
$\Delta t = 0.001$



$\Delta t = 0.25$



$\Delta t = 0.3$



$\Delta t = 0.5$

Figure (3) **Different results obtained with the Perona-Malik scheme using the linear interpolation.**
The results were obtained with $K = 15$ and $\alpha = 5$.

We can see in Figure 9 that the application of the Perona-Malik method allows a smoothing of the homogeneous regions while preserving the edges.



Figure (4) Different results obtained with the Perona-Malik scheme using the simplification. The results were obtained with $K = 15$ and $\alpha = 5$.

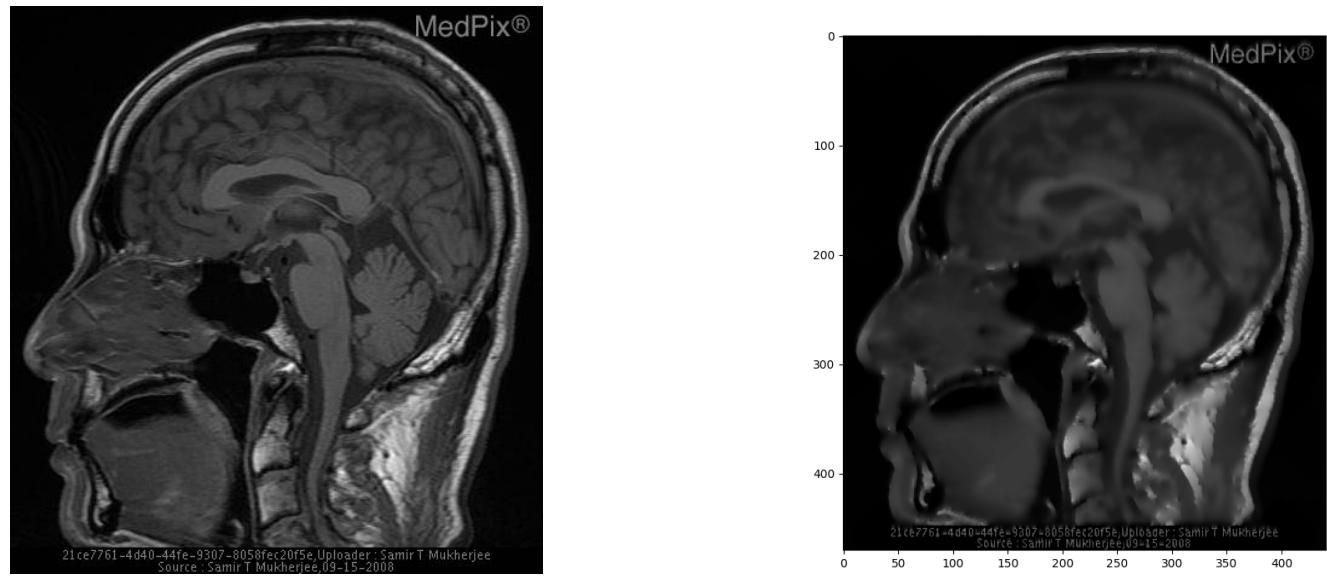
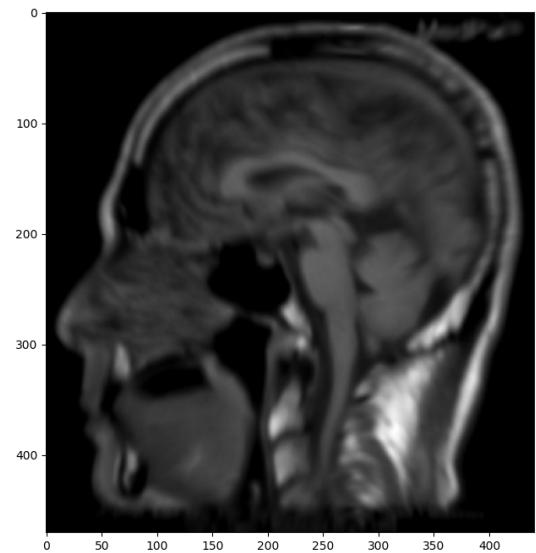


Figure (5) Images of a head scan. The result was obtained with $\Delta t = 0.22$, $k = 15$ and $\alpha = 20$.

Exercise 3



Original image



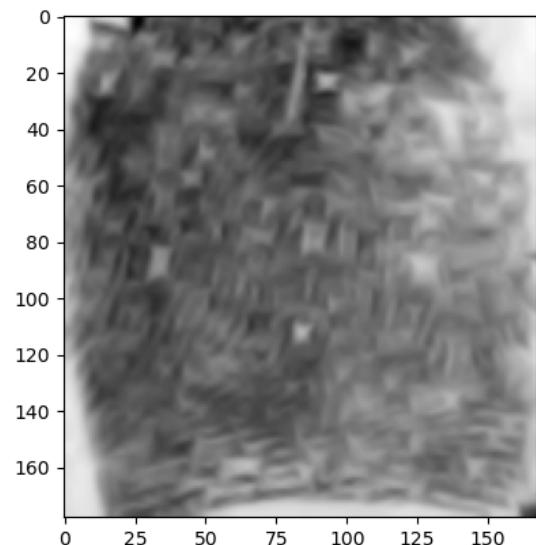
Processed image

Figure (6) **Images of a head scan.** The result were obtained with $\Delta t = 0.25$, $k = 100$ and $\sigma = 10$.

We notice that the edge enhancing smooths the homogeneous regions as expected but it also struggles a little to keep sharp edges. This might be due to the fact that some edges are not clearly defined in the original image as the intensity difference is not so big and the gradient is thus not so important.



Original image



Processed image

Figure (7) **Images of a fingerprint.** The result was obtained with $\Delta t = 0.3$, $K = 300$ and $\sigma = 2$.

The figure above illustrates the fact that, when opposed gradients are close, they compensate each other. When they describe neighboring edges, they thus cancel each other out. This results in a blurred image.

Exercise 4

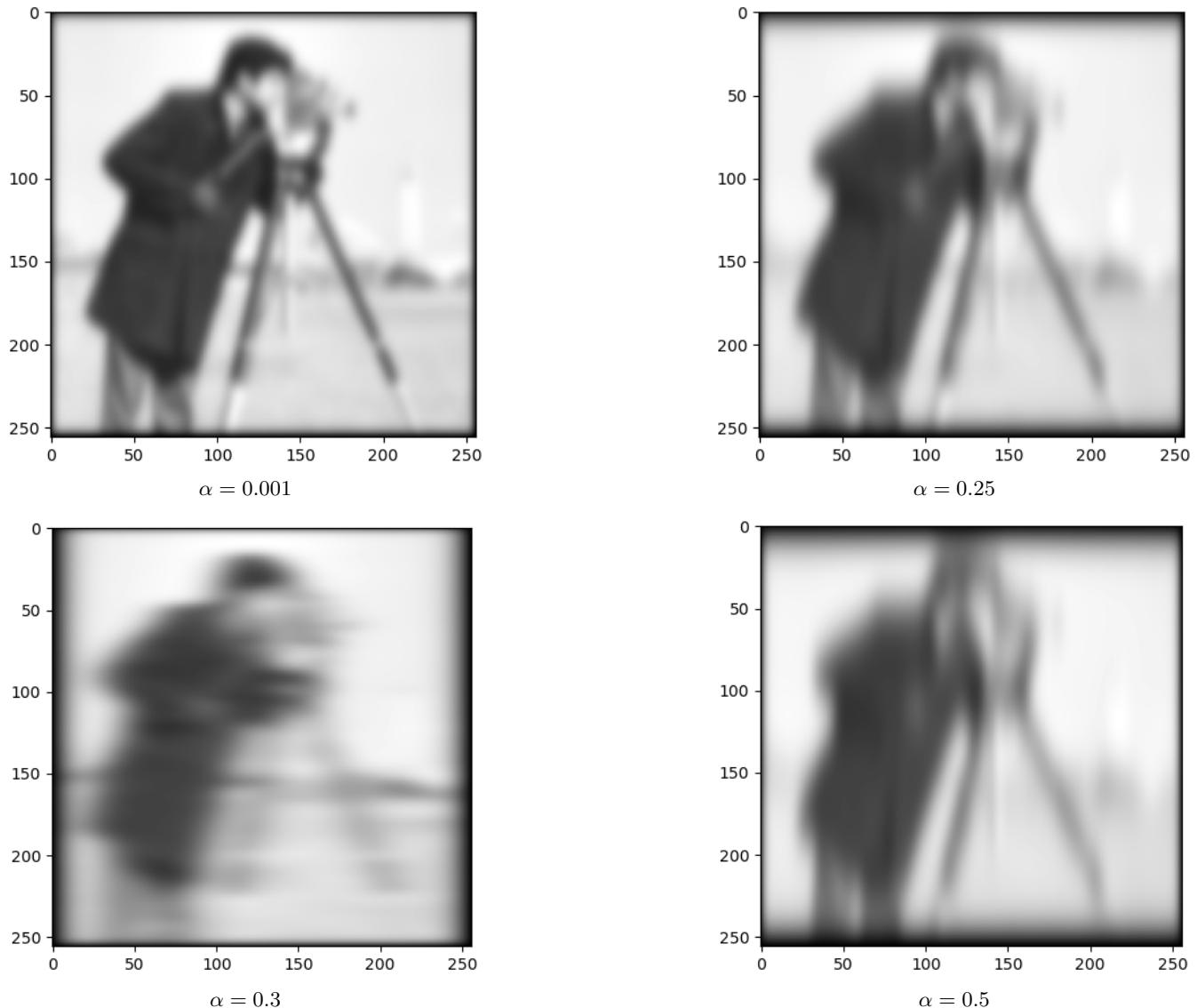
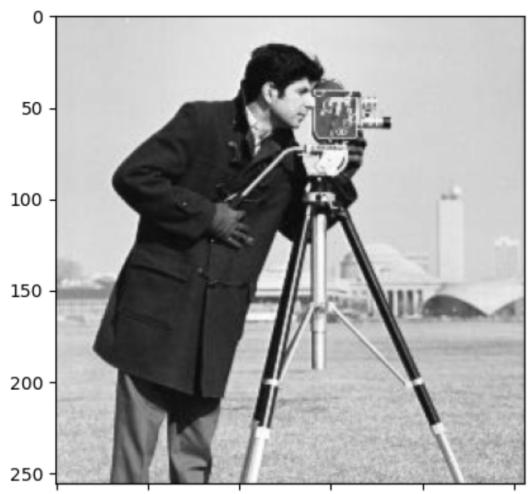
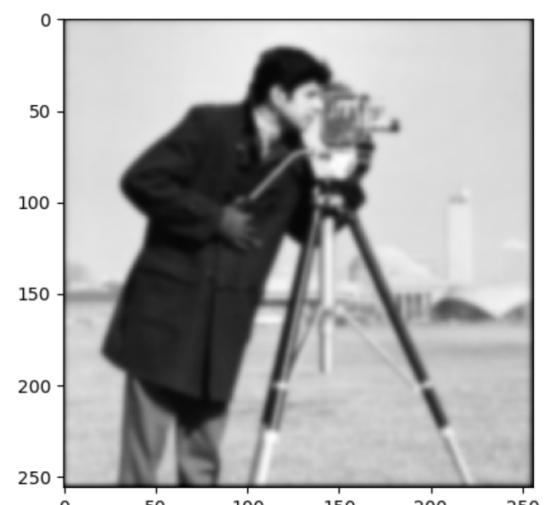


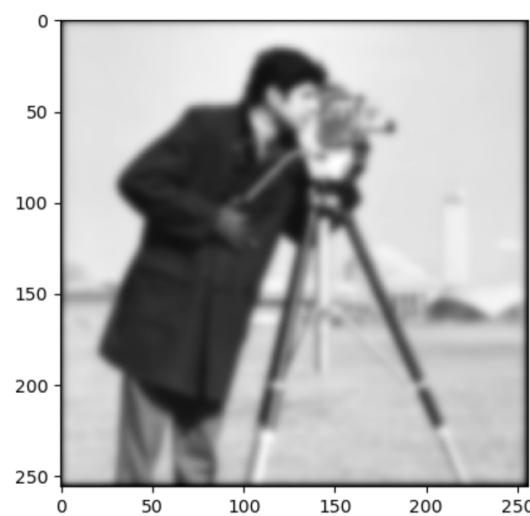
Figure (8) **Different results obtained with the implicit scheme.** The results were obtained with $\beta = 0.5$.



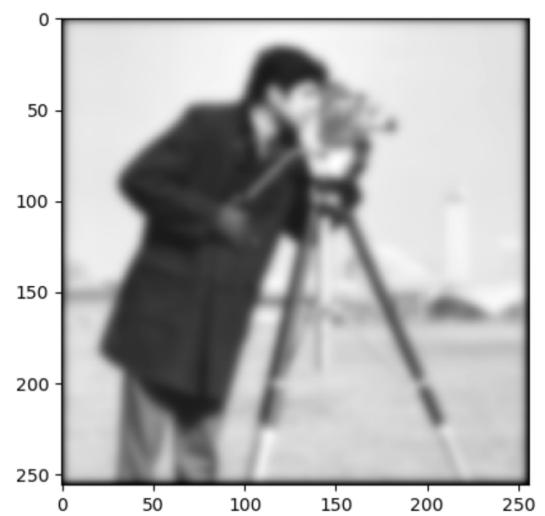
$$\alpha = \beta = 0.01$$



$$\alpha = \beta = 0.14$$



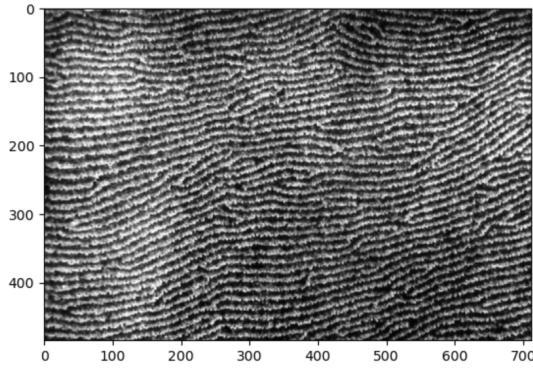
$$\alpha = \beta = 0.27$$



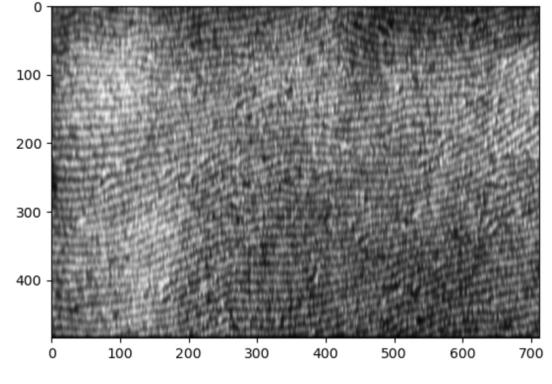
$$\alpha = \beta = 0.4$$

Figure (9) Different results obtained with the implicit scheme.

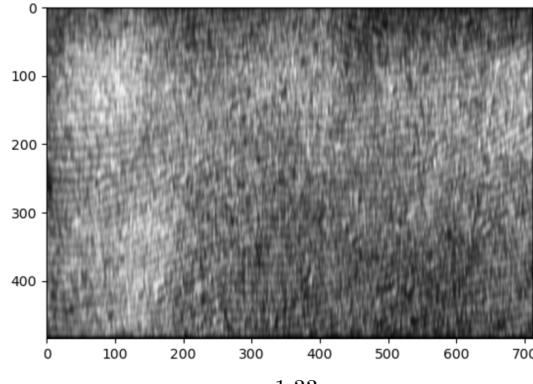
As we can see in Figure 10, the parameter α impacts the relation between the different rows of pixel. Therefore, the more we increase it the more the horizontal lines are blurred.



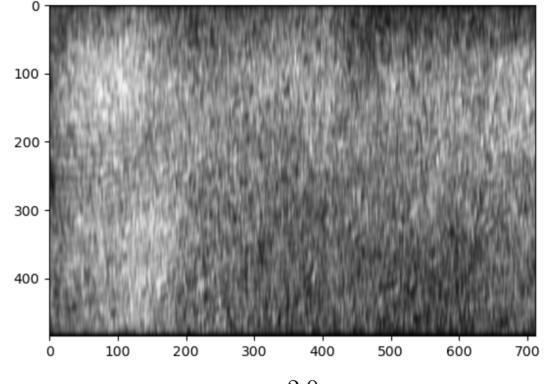
$$\alpha = 0.001$$



$$\alpha = 0.67$$



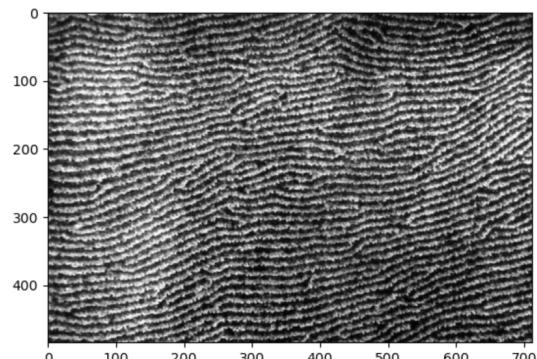
$$\alpha = 1.33$$



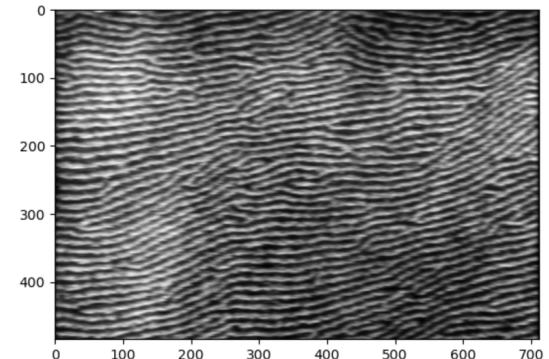
$$\alpha = 2.0$$

Figure (10) **Different results obtained with the implicit scheme.** The results were obtained with $\beta = 0$.

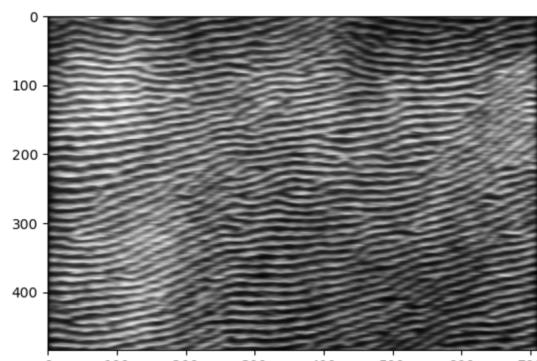
As we can see in Figure 11, the parameter β impacts only the relation between the different columns of pixel. Therefore, increasing the value of this parameter does not blur a lot the lines.



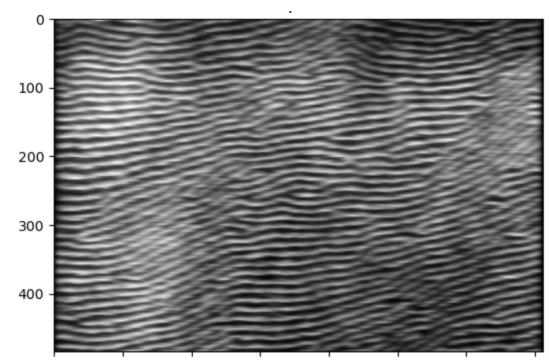
$$\beta = 0.001$$



$$\beta = 0.67$$



$$\beta = 1.33$$



$$\beta = 2.0$$

Figure (11) **Different results obtained with the implicit scheme.** The results were obtained with $\alpha = 0$.