Formulae for Diffractive DIS Fit

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1 Cross-section

$$\frac{d\sigma}{d\beta\, dQ^2\, d\xi\, dt} = \frac{2\pi\alpha^2}{\beta Q^4} \left(1+(1-y)^2\right) \tilde{\sigma}^{D(4)}(\beta, Q^2, \xi, t) \tag{1a}$$

$$\equiv \frac{2\pi\alpha^2}{\beta Q^4} \left(\left[1+(1-y)^2\right] F_2^{D(4)}(\beta, Q^2, \xi, t) - y^2 F_{\mathrm{L}}^{D(4)}(\beta, Q^2, \xi, t)\right) \tag{1b}$$

where the 'reduced cross-section' is defined as

$$\tilde{\sigma} = F_2 - \frac{y^2}{1+(1-y)^2} F_{\mathrm{L}} = F_{\perp} + \frac{2(1-y)}{1+(1-y)^2} F_{\mathrm{L}} \tag{2}$$

Nb. ξ is denoted by $x_{I\!P}$ in the H1 and ZEUS papers.
The dimension of $F_2^{D(4)}(\beta, Q^2, \xi, t)$, $F_{\mathrm{L}}^{D(4)}(\beta, Q^2, \xi, t)$ and $\tilde{\sigma}^{D(4)}(\beta, Q^2, \xi, t)$ is GeV^{-2}.
Thus the quantities integrated over t

$$A^{D(3)}(\beta, Q^2, \xi) \equiv \int_{t_{\min}}^{t_{\max}} dt A^{D(4)}(\beta, Q^2, \xi, t) \tag{3}$$

are dimensionless.
Maximum kinematically allowed value of t reads

$$t_{\mathrm{MAX}} = -\frac{\xi^2 m_p^2 + p_\perp^2}{1-\xi} \approx -\frac{\xi^2}{1-\xi} m_p^2 \tag{4}$$

where m_p is the proton mass.
As $x = \xi\beta$ we can 'normalize' to the standard DIS formula

$$\frac{d\sigma}{d\beta\, dQ^2\, d\xi\, dt} = \frac{2\pi\alpha^2}{x\, Q^4} \left(1+(1-y)^2\right) \xi\tilde{\sigma}^{D(4)}(\beta, Q^2, \xi, t) \tag{5}$$

which upon integration over t reads

$$\frac{d\sigma}{d\beta\, dQ^2\, d\xi} = \frac{2\pi\alpha^2}{\beta Q^4} \left(1+(1-y)^2\right) \tilde{\sigma}^{D(3)}(\beta, Q^2, \xi) \tag{6a}$$

$$= \frac{2\pi\alpha^2}{\beta Q^4} \left(\left[1+(1-y)^2\right] F_2^{D(3)}(\beta, Q^2, \xi) - y^2 F_{\mathrm{L}}^{D(3)}(\beta, Q^2, \xi)\right) \tag{6b}$$

$$\tag{6c}$$

Nb. the ZEUS data files contain $\xi\tilde{\sigma}^{D(3)}$.

2 Regge factorization

To describe the data we include a contribution from a secondary Reggeon $I\!R$,

$$F_j^{D(4)}(\beta, Q^2, \xi, t) = \phi_{I\!P}(\xi, t)\, F_j^{I\!P}(\beta, Q^2) + \phi_{I\!R}(\xi, t)\, F_j^{I\!R}(\beta, Q^2) \tag{7}$$

or

$$F_{2/\mathrm{L}}^{D(3)}(\beta, Q^2, \xi) = \Phi_{I\!P}(\xi)\, F_{2/\mathrm{L}}^{I\!P}(\beta, Q^2) + \Phi_{I\!R}(\xi)\, F_{2/\mathrm{L}}^{I\!R}(\beta, Q^2) \tag{8}$$

where

$$\Phi_{I\!P/I\!R}(\xi) = \int_{t_{\min}}^{t_{\max}} dt\, \phi_{I\!P/I\!R}(\xi, t) \tag{9}$$

Parametrization of the fluxes

$$\phi(\xi, t) = \frac{A\, e^{bt}}{\xi^{2\alpha(t)-1}} \tag{10a}$$

where

$$\alpha(t) = \alpha(0) + \alpha' t \tag{10b}$$

$F_j^{I\!R}(\beta, Q^2)$ are taken as that of pion — normalization factor being absorbed in $\phi_{I\!R}(\xi, t)$.

3 Pomeron parametrization

The Pomeron is parametrized at the initial Q_0^2 in terms of two singlet distributions, f_g and f_+.

$$\begin{aligned} \frac{d}{dt} f_+ &= \frac{\alpha_{\mathrm{s}}}{2\pi}\left[\mathcal{P}_{\mathrm{FF}} f_+ + \mathcal{P}_{\mathrm{FG}} f_g\right] & \text{(11a)} \\ \frac{d}{dt} f_g &= \frac{\alpha_{\mathrm{s}}}{2\pi}\left[\mathcal{P}_{\mathrm{GF}} f_+ + \mathcal{P}_{\mathrm{GG}} f_g\right] & \text{(11b)} \end{aligned}$$

As Pomeron is neutral, $f_q = f_{\bar{q}}$ for each flavour. Let's assume that all light quark PDFs are equal

$$f_d = f_u = f_s\,. \tag{12}$$

Thus

$$\begin{aligned} f_{q-} &\equiv 0 & \text{(13a)} \\ f_{q+} &\equiv 2 f_q & \text{(13b)} \end{aligned}$$

At $N_{\mathrm{f}} = 3$

$$f_{q+} = f_+/3,\ q = d, u, s\,. \tag{14}$$

i.e.

$$\tilde{f}_{q+} = 0,\ q = d, u, s\,, \tag{15}$$

where

$$\tilde{f}_{q+} \equiv f_{q+} - \frac{1}{N_{\mathrm{f}}} f_+\,. \tag{16}$$

This gives all PDFs for the FFNS, while for VFNS $f_{h+}, h = c, b, t$ are generated dynamically above the respective transition scales Q_h^2. Hence at $N_{\mathrm{f}} > 3$ the singlet has contributions from the heavy quarks and we get non-trivial nonsinglet distributions $\tilde{f}_{h+}$ satisfying

$$\frac{d}{dt}\tilde{f}_{h+} = \frac{\alpha_{\mathrm{s}}}{2\pi}\mathcal{P}_{(+)}\tilde{f}_{h+} \tag{17}$$

4 PDFs Parametrization at Q_0^2

Full PDFs are given according to (8)

$$f_k^{D(3)}(\beta, Q^2, \xi) = \Phi_{I\!P}(\xi)\, f_k^{I\!P}(\beta, Q^2) + \Phi_{I\!R}(\xi)\, f_k^{I\!R}(\beta, Q^2) \tag{18}$$

with the fluxes given by (10).

The Pomeron PDFs (omitting superscript $I\!P$) are parametrized as

$$f_N = A_1^{(N)} x^{A_2^{(N)}} (1-x)^{A_3^{(N)}} \, \exp\left(-\frac{d}{1.00001 - x}\right) , \tag{19}$$

where the 'dumping factor' d is taken as 0.01 or 0.001. $N = \mathrm{G}$ for gluon or S for 'singlet': $f_{\mathrm{S}} \equiv f_+(N_{\mathrm{f}} = 3)$, *cf.* (14).

The Reggeon PDFs $f_k^{I\!R}$ are taken from pion.