Programa de Pós-graduação em
INFORMÁTICA

PUC Minas

Teoria dos Grafos e Computabilidade

— Sets on graphs —

(Professor version)

Silvio Jamil F. Guimarães

Graduate Program in Informatics – PPGINF
Image and Multimedia Data Science Laboratory – IMScience
Pontifical Catholic University of Minas Gerais – PUC Minas

Feb 2021

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— Independent sets —

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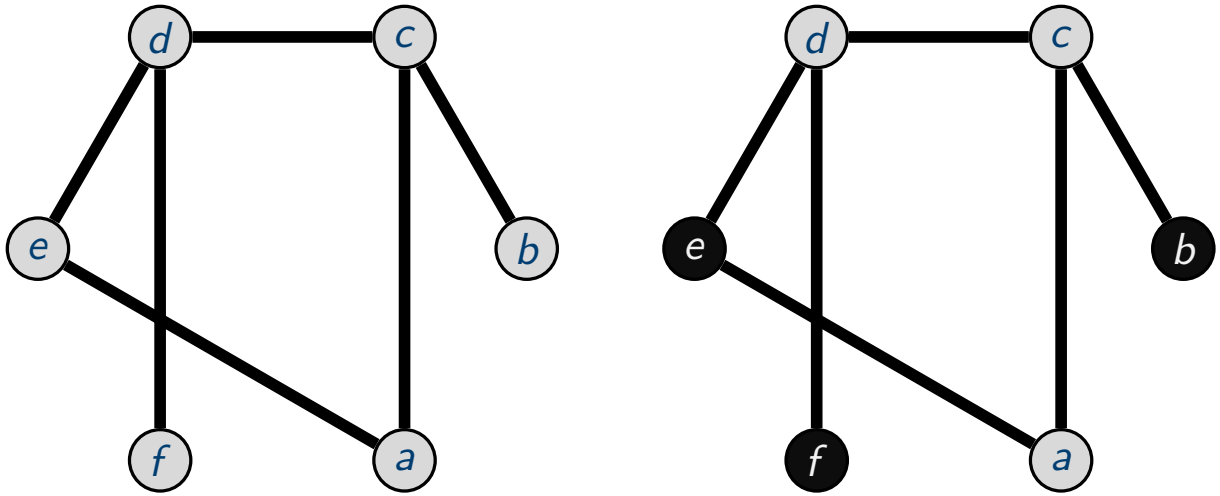
Independent sets

Let $G = (V, E)$ be an undirected connected graph.

- A subset $S \subseteq V$ is an **independent set** if $\forall u, v \in S$ there is no ~~exist an~~ edge $(u, v) \in E$.
- Independent sets have also been called internally stable sets.



Independent sets

Let $G = (V, E)$ be an undirected connected graph, and S an independent set of G

- We say that the subset $S \subseteq V$ is a **maximal independent set** if there is no other independent set A in which $S \subset A$;
- The number of **internal stability** $\beta(G)$ is equal to the cardinality of the largest maximal independent set.

As S is an independent set of G, then S is a **clique** in the complement graph.

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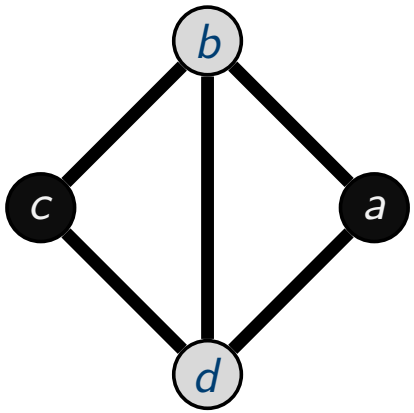

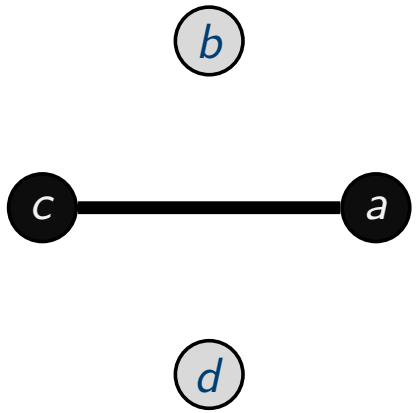

Independent sets

Let $G = (V, E)$ be an undirected connected graph. **Design** a method for computing an independent set of G

Algorithm: A method for computing an independent set

input : A graph $G = (V, E)$.
output: A independent set S

```
1 S = ∅;
2 while V ≠ ∅ do
3 |   u = vertex with the smallest degree in G;
4 |   V = V − {u} − Γ(u);
5 |   S = S ∪ {u};
6 end
7 return S;
```

Questions?

Sets on graphs
– Independent sets –

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Teoria dos Grafos e Computabilidade

— Dominating sets —

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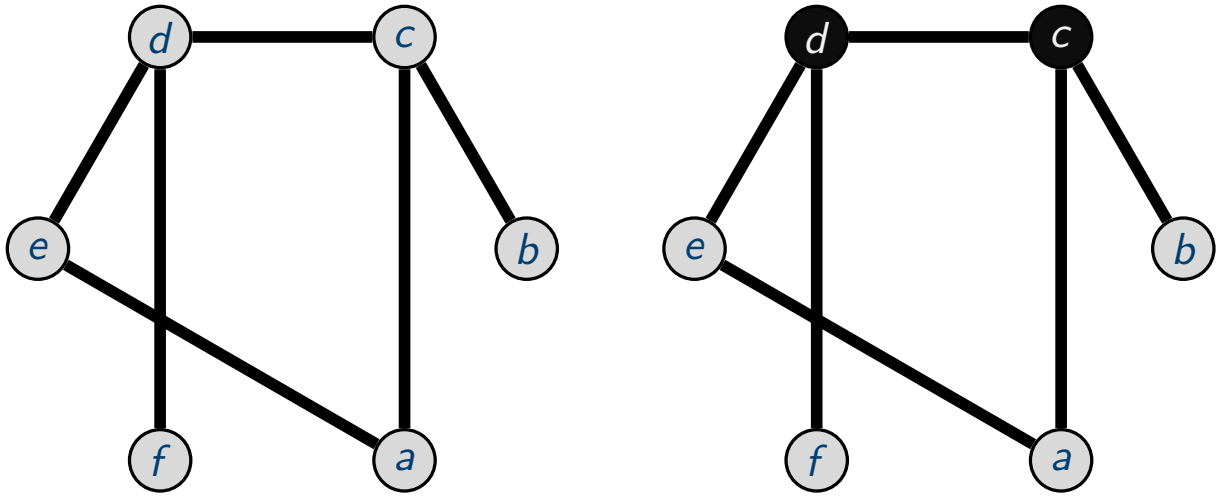
Dominating sets

Let $G = (V, E)$ be an undirected connected graph.

- A subset $S \subseteq V$ is an **dominating set** if $\forall u \in S$ there exist a $v \in V - S$ such that $(u, v) \in E$.
- Dominating sets have also been called externally stable sets.



Independent sets

Let $G = (V, E)$ be an undirected connected graph, and S a dominating set of G

- We say that the subset $S \subseteq V$ is a **minimal dominating set** if there is no other dominating set A in which $A \subset S$;
- The number of **external stability** $\beta(G)$ is equal to the cardinality of the smallest minimal dominating set.

Independent sets

Let $G = (V, E)$ be an undirected connected graph. **Design** a method for computing a dominance set of G

Algorithm: A method for computing a dominating set

input : A graph $G = (V, E)$.
output: A dominating set D

```
1 D = ∅;
2 while V ≠ ∅ do
3 |   u = vertex with the highest degree in G;
4 |   V = V − {u} − Γ(u);
5 |   D = D ∪ {u};
6 end
7 return D;
```

Questions?

Sets on graphs

– Dominating sets –

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Teoria dos Grafos e Computabilidade

— Vertex cover —

(Professor version)

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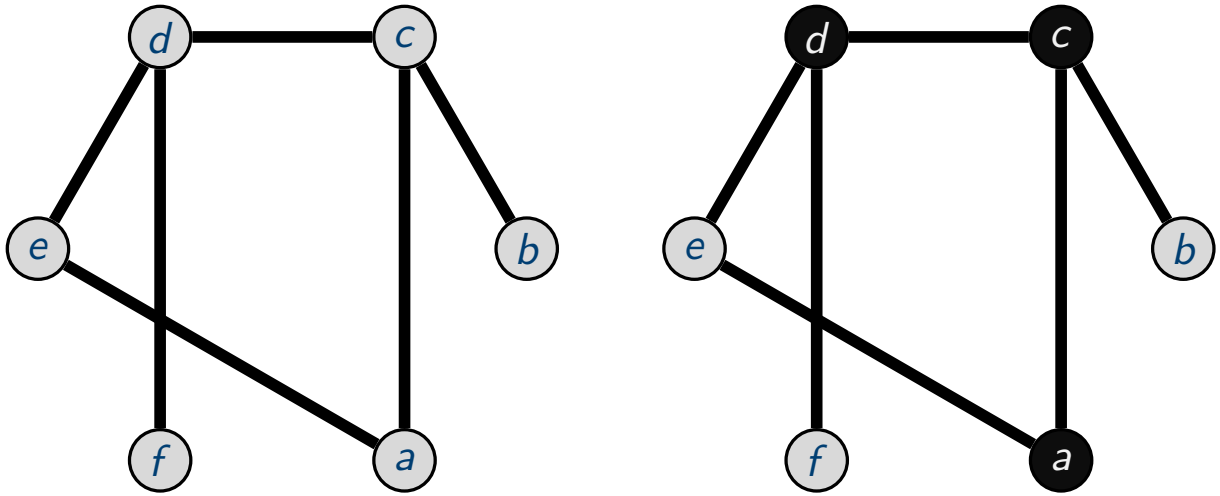
Vertex cover

Let $G = (V, E)$ be an undirected connected graph.

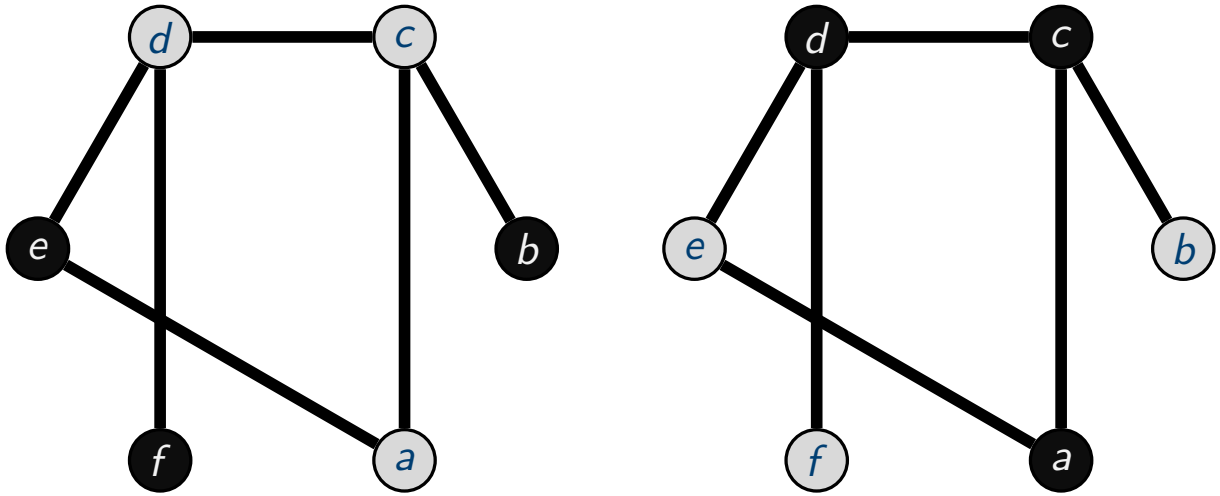
- A subset $S \subseteq V$ is an **vertex cover** if $\forall (u, v) \in E$, either $u \in S$ or $v \in S$.



Vertex cover

Let $G = (V, E)$ be an undirected connected graph, and S a vertex cover of G

As S is a vertex cover of G, then V-S is an independent set.



Vertex cover

Let $G = (V, E)$ be an undirected connected graph. **Design** a method for computing a vertex cover in G

Algorithm: A method for computing a ~~minimum~~ vertex cover

input : A graph $G = (V, E)$.
output: ~~A independent set~~ ~~S~~ A vertex cover S

```
1 S = ∅;
2 while E ≠ ∅ do
3 |   Let (u, v) an arbitrary edge of E;
4 |   Choose either u or v to be included to ~~C;~~ S
5 |   S = S ∪ {u} for instance;
6 |   V = V − u;
7 end
8 return S;
```

Questions?

Sets on graphs
– Vertex cover –