Grafos Hamiltonianos e Grafos Eulerianos

Zenilton Patrocínio

Grafo Hamiltoniano

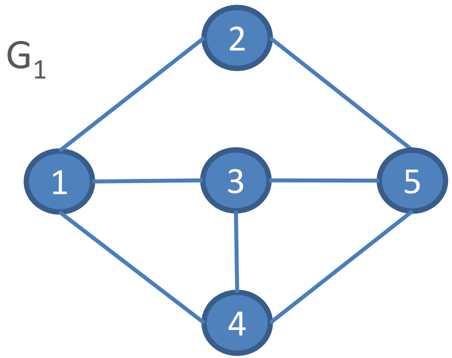
Um caminho hamiltoniano é um caminho que passa por cada vértice de um grafo exatamente uma vez.

Um ciclo hamiltoniano é um caminho hamiltoniano que retorna ao vértice inicial (isto é, um caminho fechado).

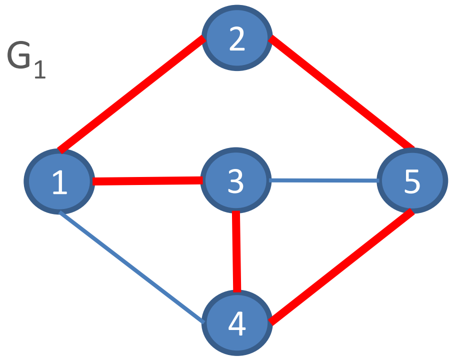
Um grafo é dito hamiltoniano se possuir um **ciclo hamiltoniano**.

Um grafo é dito semi-hamiltoniano se possuir um **caminho hamiltoniano**. Logo, um grafo hamiltoniano é também semi-hamiltoniano.

Grafo Hamiltoniano – Exemplo

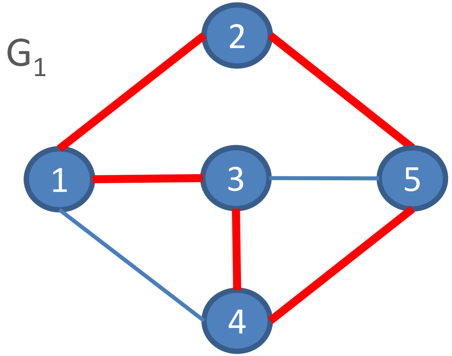


Grafo Hamiltoniano – Exemplo

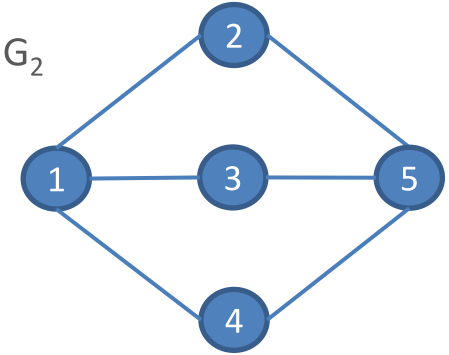


Hamiltoniano

Grafo Hamiltoniano – Exemplo

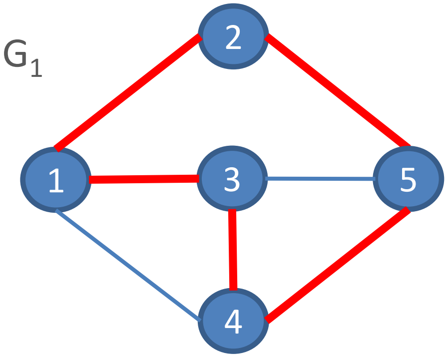


Hamiltoniano



Grafo Hamiltoniano – Exemplo

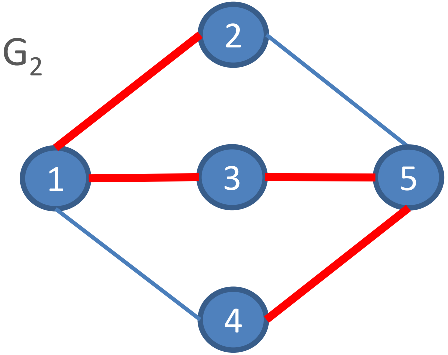
G_1



Hamiltoniano

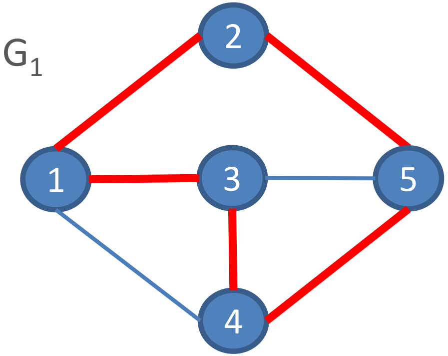
Semi-hamiltoniano

G_2



Grafo Hamiltoniano – Exemplo

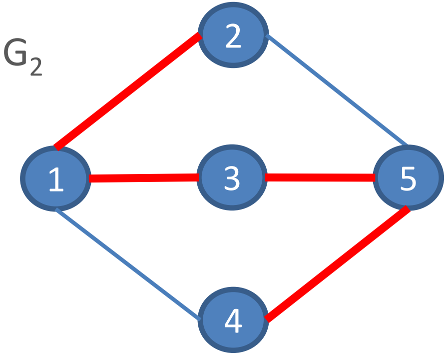
G_1



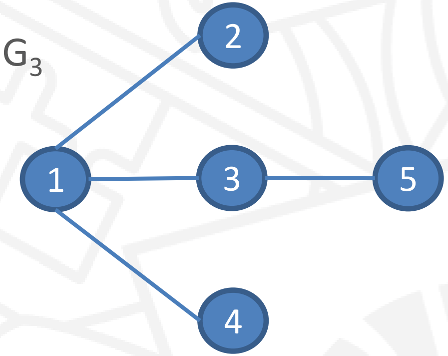
Hamiltoniano

Semi-hamiltoniano

G_2

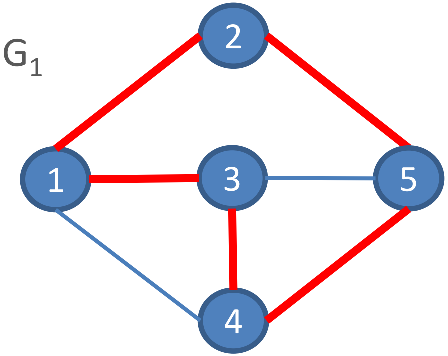


G_3



Grafo Hamiltoniano – Exemplo

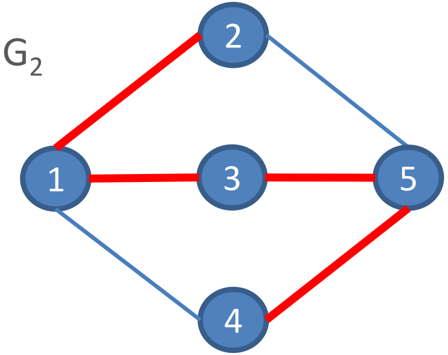
G_1



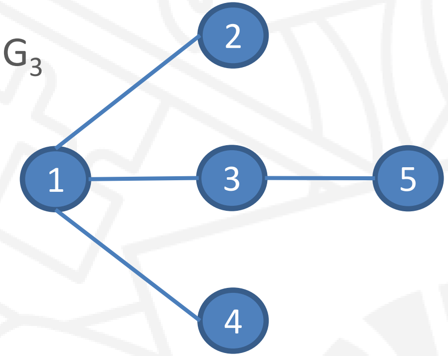
Hamiltoniano

Semi-hamiltoniano

G_2



G_3



Não-hamiltoniano

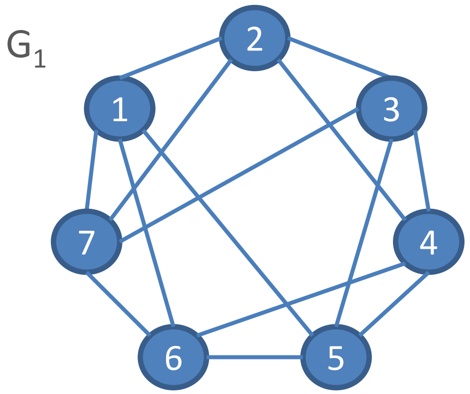
Grafo Hamiltoniano – Condição Suficiente

Um grafo simples G com n (≥ 3) vértices é hamiltoniano, se o grau de cada um de seus vértices $d(v) \geq n/2$, $\forall v \in V(G)$. (Teorema de Dirac)

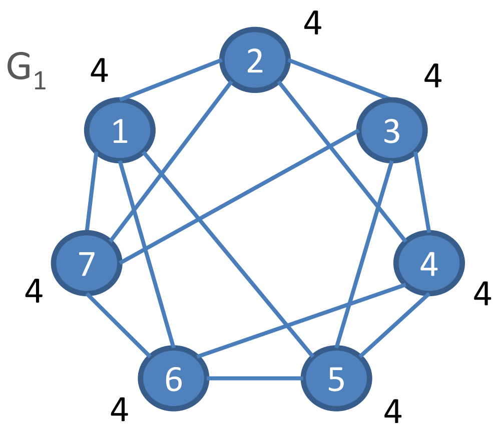
Um grafo simples G com n (≥ 3) vértices é hamiltoniano, se, para cada par de vértices não adjacentes v e w, a soma de seus graus $d(v) + d(w) \geq n$, $\forall \{v,w\} \notin E(G)$. (Teorema de Ore)

Se o fecho hamiltoniano de G for um grafo completo, então G é hamiltoniano. Fecho hamiltoniano de uma grafo é obtido adicionando-se arestas, enquanto for possível, entre vértices não adjacentes cuja soma de graus $\geq n$. (Teorema de Bondy & Chvátal)

Grafo Hamiltoniano – Condição Suficiente



Grafo Hamiltoniano – Condição Suficiente



Atende ao Teorema de Dirac

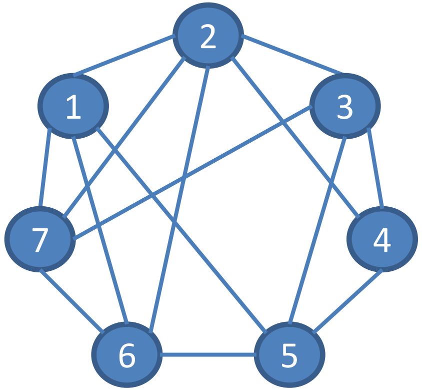
Atende ao Teorema de Ore

Atende ao Teorema de Bondy & Chvátal

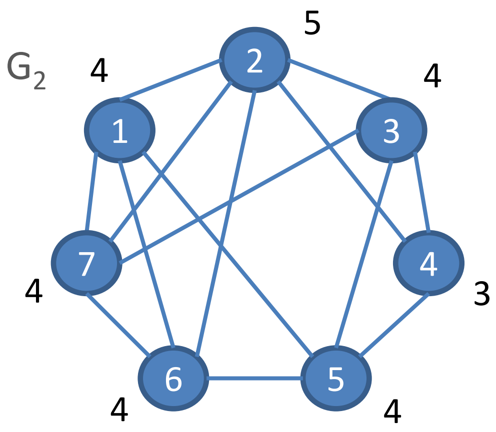
G_1 é hamiltoniano

Grafo Hamiltoniano – Condição Suficiente

G_2



Grafo Hamiltoniano – Condição Suficiente



Não atende ao Teorema de Dirac

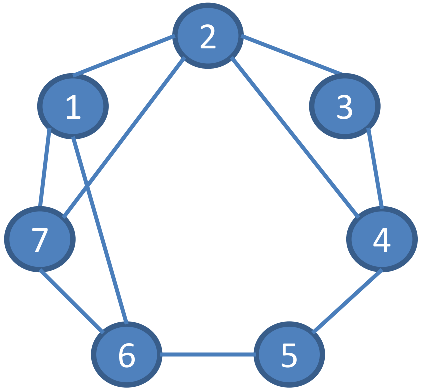
Atende ao Teorema de Ore

Atende ao Teorema de Bondy & Chvátal

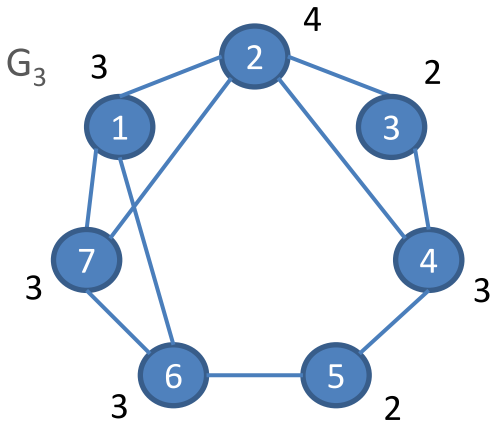
G_2 é hamiltoniano

Grafo Hamiltoniano – Condição Suficiente

G_3



Grafo Hamiltoniano – Condição Suficiente



Não atende ao Teorema de Dirac

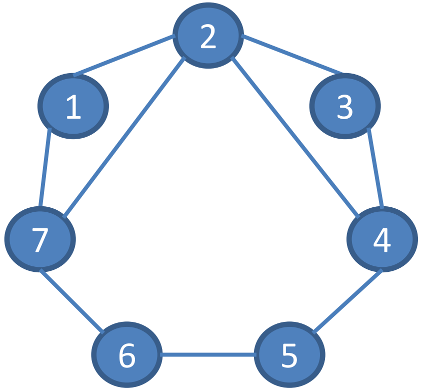
Não atende ao Teorema de Ore

Atende ao Teorema de Bondy & Chvátal

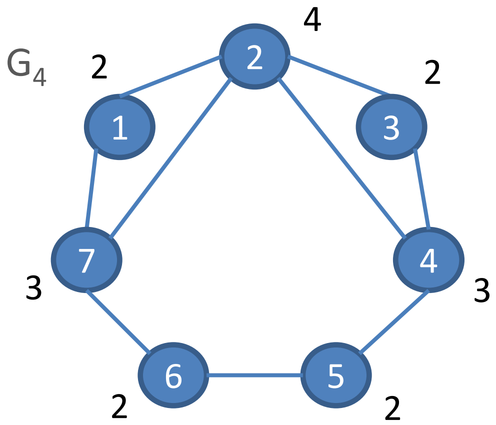
G_3 é hamiltoniano

Grafo Hamiltoniano – Condição Suficiente

G_4



Grafo Hamiltoniano – Condição Suficiente



Não atende ao Teorema de Dirac

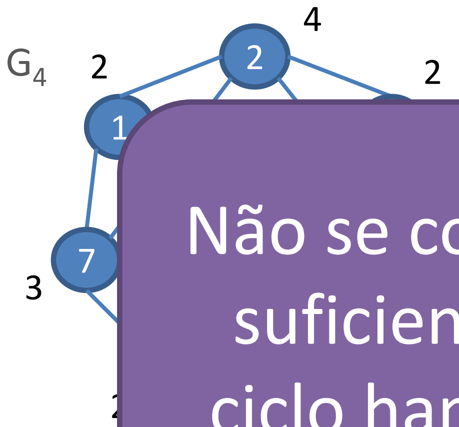
Não atende ao Teorema de Ore

Não atende ao Teorema de Bondy & Chvátal

PORÉM

G_4 é hamiltoniano

Grafo Hamiltoniano – Condição Suficiente



Não se conhece uma condição necessária e suficiente trivial para a existência de um ciclo hamiltoniano em um grafo qualquer.

Grafo Euleriano

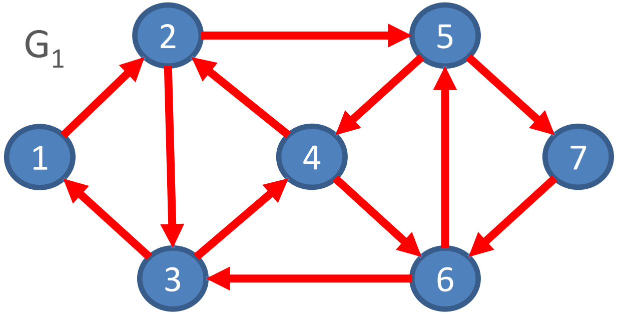
Um trajeto euleriano é um trajeto que passa por cada aresta de um grafo exatamente uma vez.

Um ciclo euleriano é um trajeto euleriano que começa e termina no mesmo vértice (isto é, um trajeto fechado).

Um grafo é dito euleriano se possuir um **ciclo euleriano**.

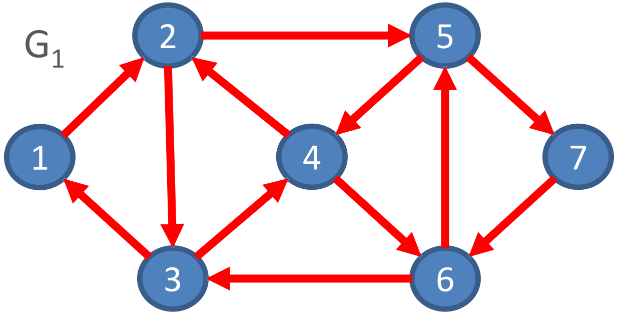
Um grafo é dito semi-euleriano se possuir um **trajeto euleriano**. Logo, um grafo euleriano é também semi-euleriano.

Grafo Euleriano – Exemplo



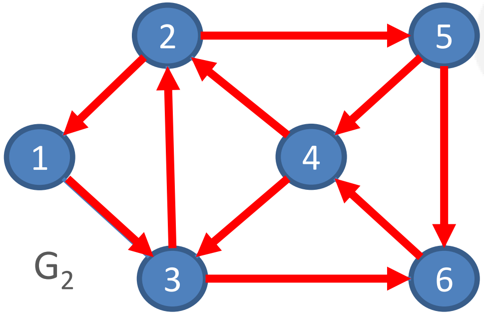
Euleriano

Grafo Euleriano – Exemplo

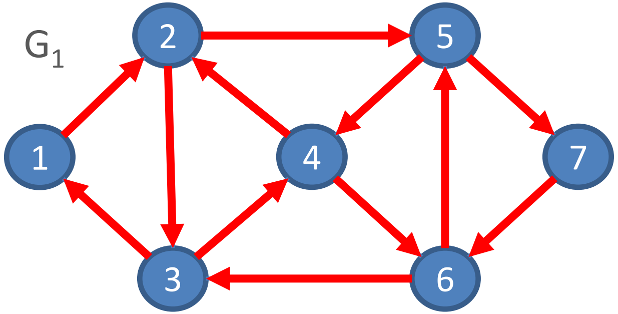


Euleriano

Semi-euleriano

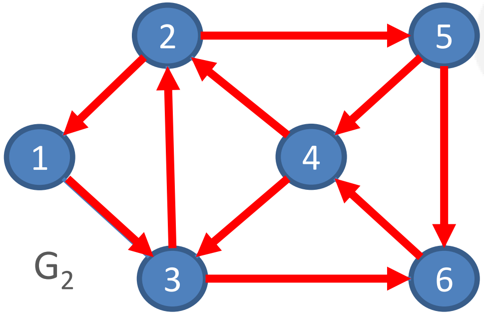


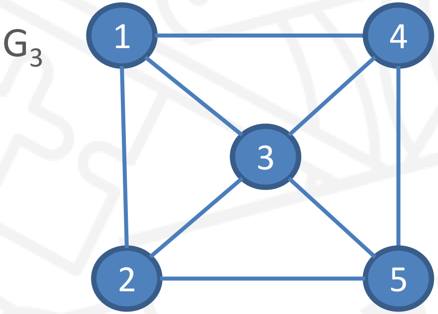
Grafo Euleriano – Exemplo



Euleriano

Semi-euleriano





Não-euleriano

Grafo Euleriano – Condição Suficiente

Um grafo conexo é euleriano se e somente se todos os seus vértices possuírem grau par. (Teorema de Euler)

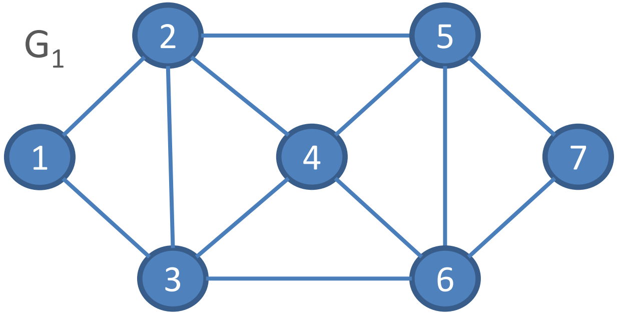
Um grafo conexo é não-euleriano se existirem dois os mais vértices de grau ímpar.

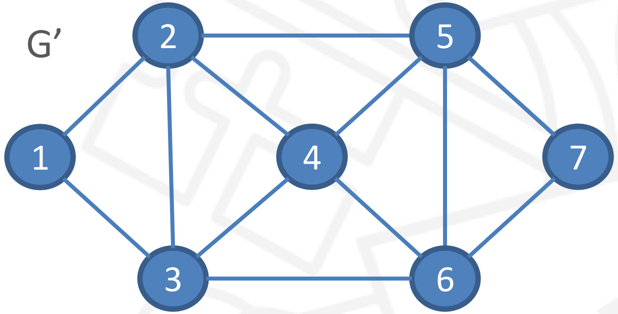
Um grafo conexo é semi-euleriano se e somente se existem exatamente dois vértices de grau ímpar.

Método de Fleury – Algoritmo

1. se V(G) possuir 3 ou mais vértices de grau ímpar então **PARE**;
2. Seja $G' = (V', E')$ tal que $V' \leftarrow V(G)$e $E' \leftarrow E(G)$; // Inicializar grafo auxiliar
3. Selecionar vértice inicial $v \in V'$ (escolher v cujo grau seja ímpar, se houver)
4. enquanto $E' \neq \varnothing$ efetuar
   a. se $d(v) > 1$ então
      Selecionar aresta $\{v, w\}$ que não seja ponte em G'
   b. senão
      Selecionar a única aresta $\{v, w\}$ disponível em G'
   c. $v \leftarrow w$; $E' \leftarrow E' - \{v, w\}$; // Caminhar de v para w e eliminar aresta

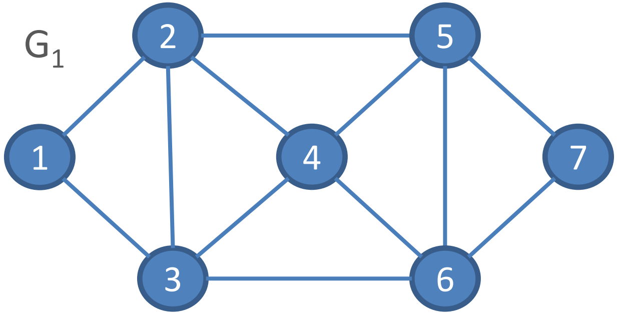
Método de Fleury – Exemplo 1



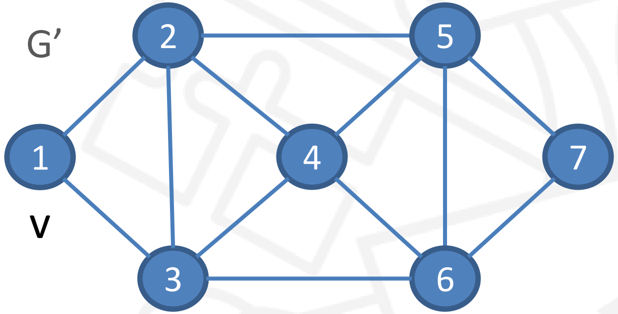


Método de Fleury – Exemplo 1

G_1

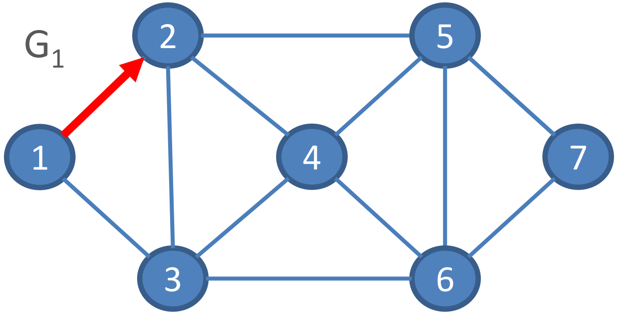


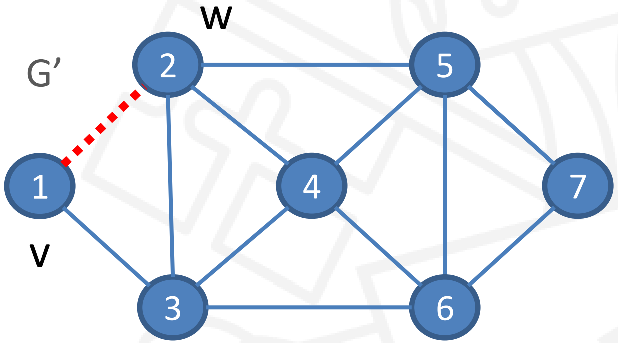
G’



1

Método de Fleury – Exemplo 1





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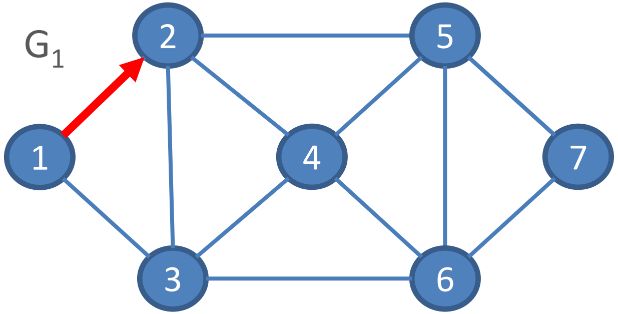
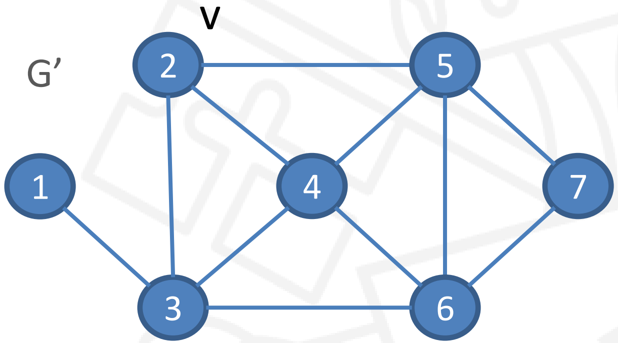
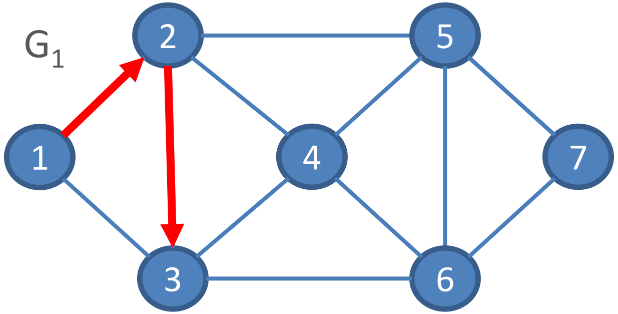
Método de Fleury – Exemplo 1

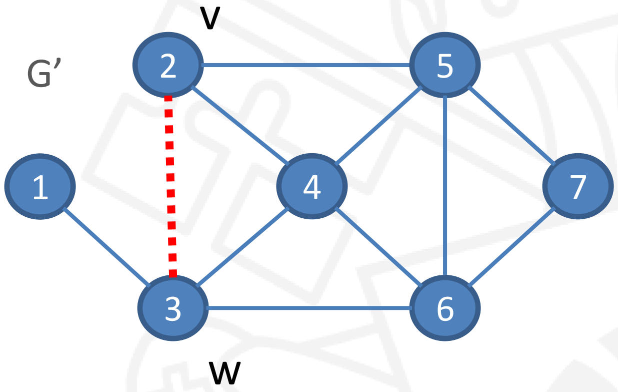




1 / 2

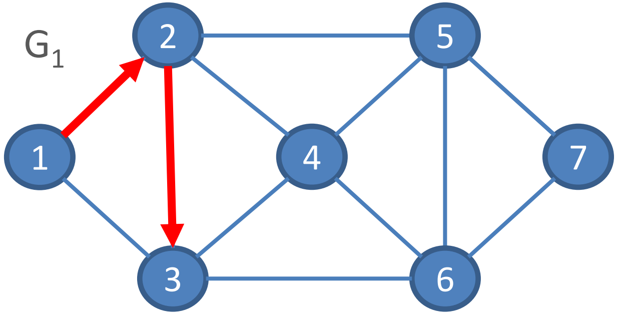
Método de Fleury – Exemplo 1

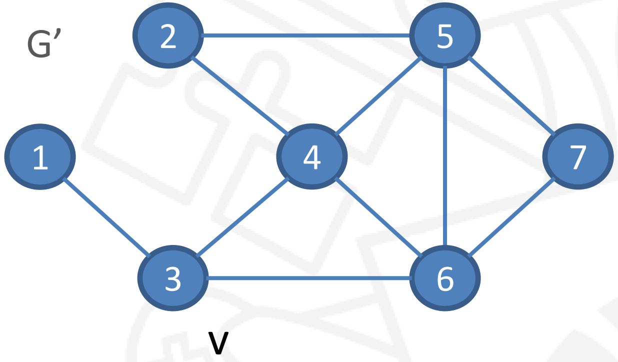




1 / 2

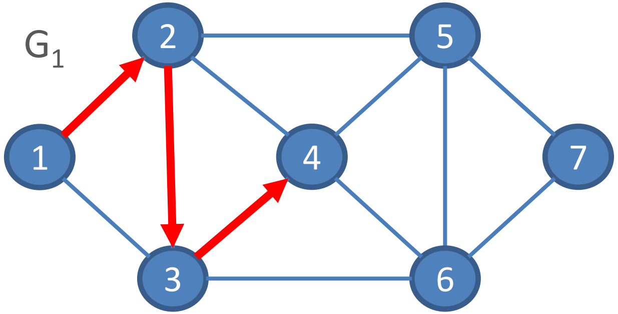
Método de Fleury – Exemplo 1

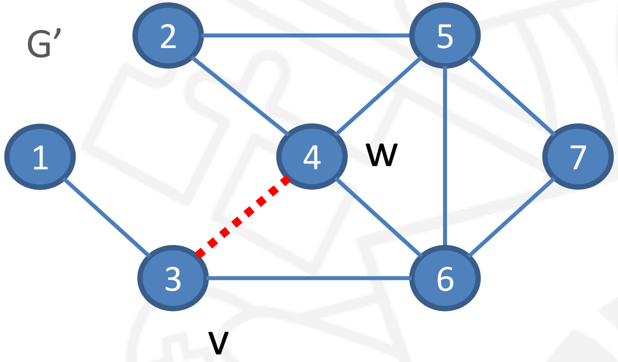




1 / 2 / 3

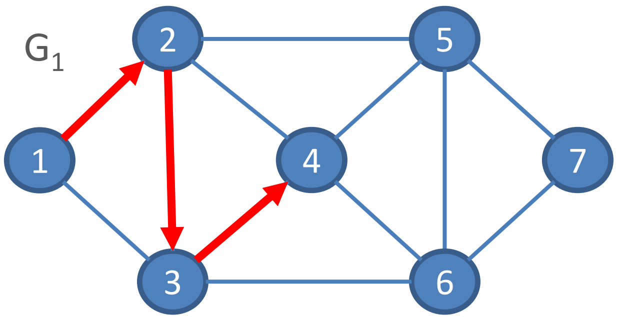
Método de Fleury – Exemplo 1

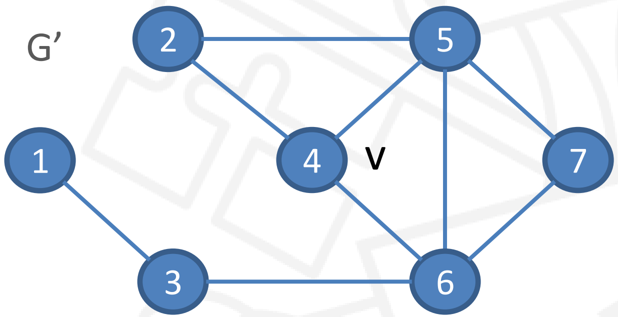




1 / 2 / 3

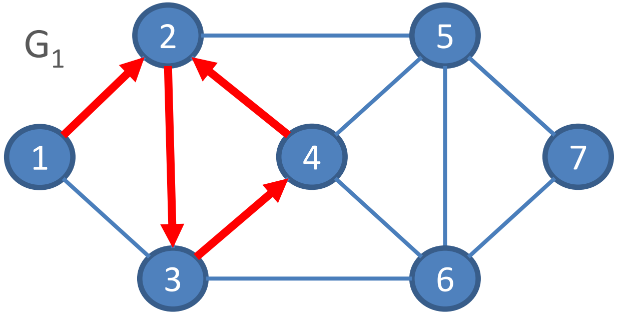
Método de Fleury – Exemplo 1

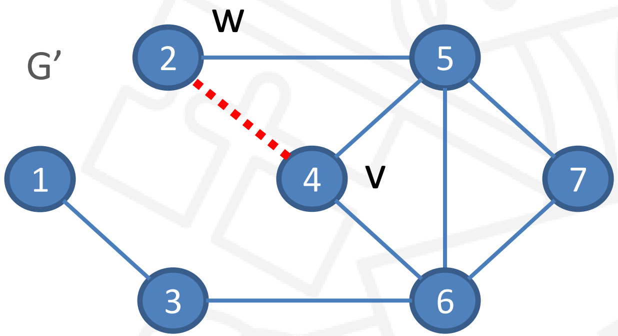




1 / 2 / 3 / 4

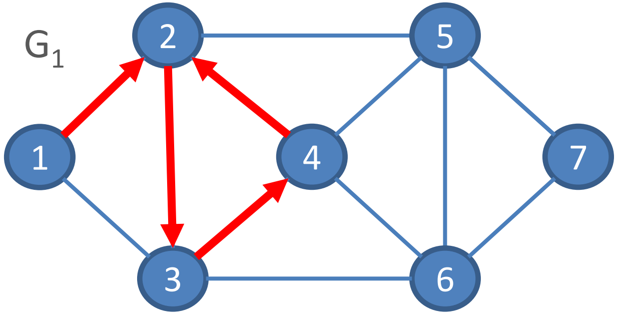
Método de Fleury – Exemplo 1

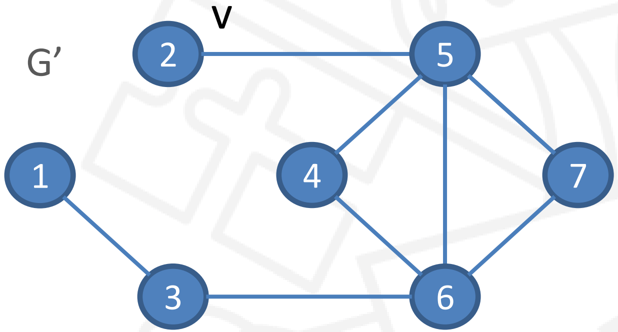




1 / 2 / 3 / 4

Método de Fleury – Exemplo 1





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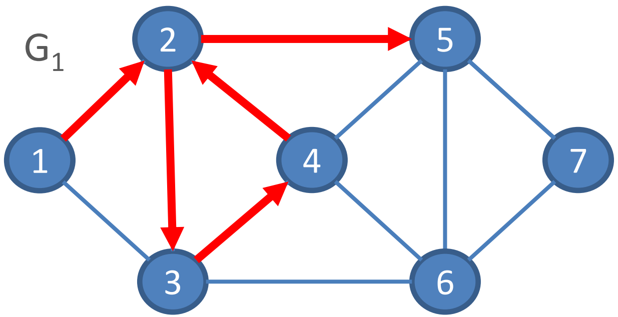
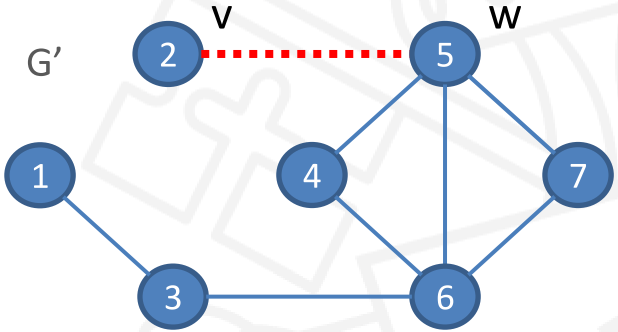
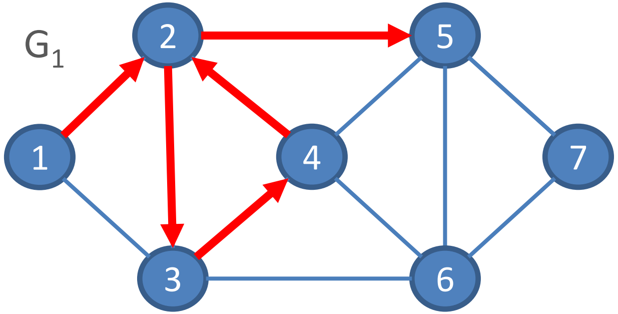
Método de Fleury – Exemplo 1

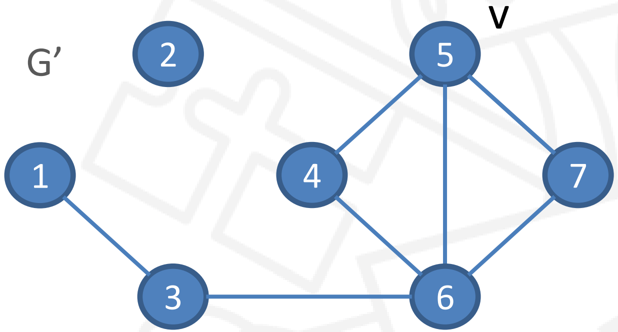




1 / 2 / 3 / 4 / 2

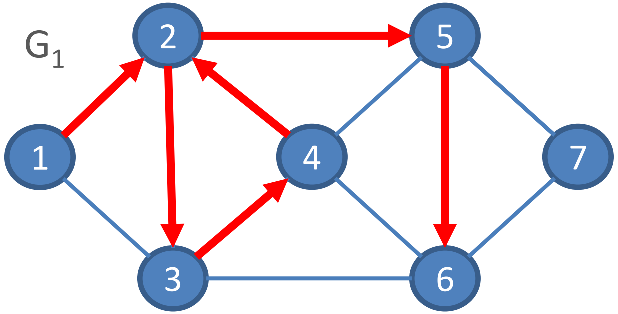
Método de Fleury – Exemplo 1

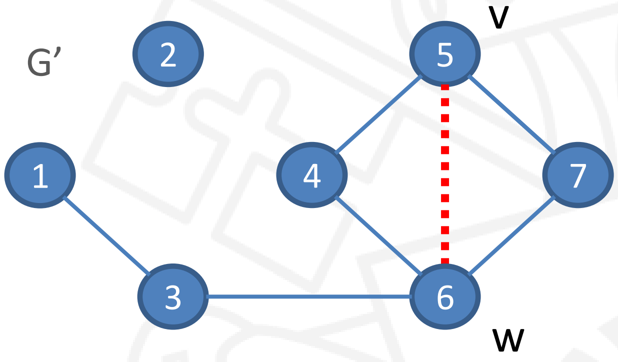




1 / 2 / 3 / 4 / 2 / 5

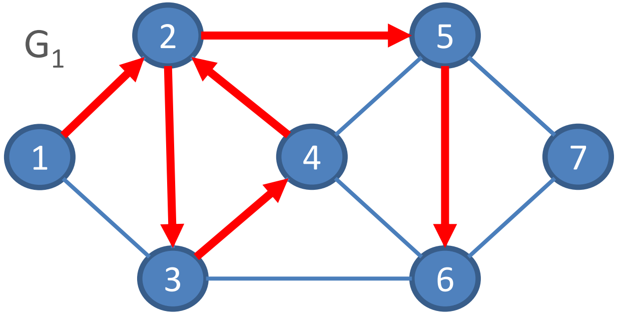
Método de Fleury – Exemplo 1

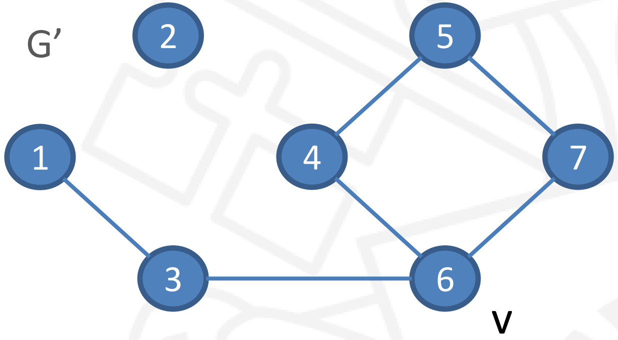




1 / 2 / 3 / 4 / 2 / 5

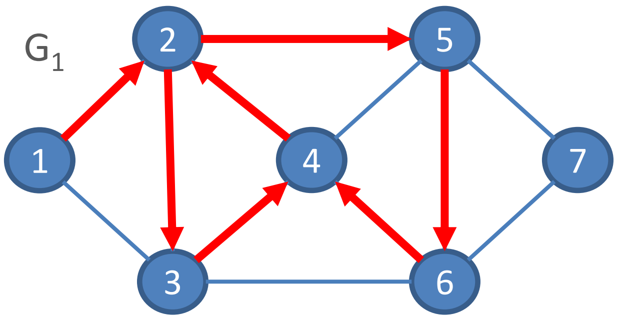
Método de Fleury – Exemplo 1

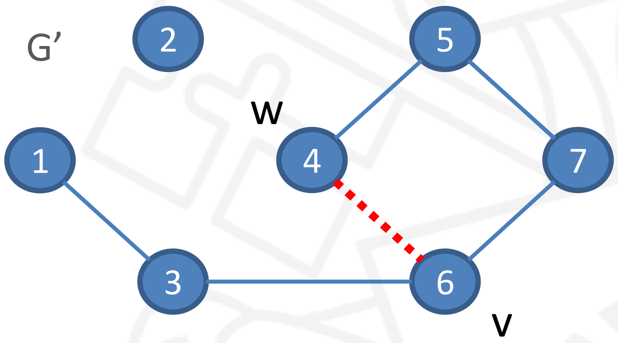




1 / 2 / 3 / 4 / 2 / 5 / 6

Método de Fleury – Exemplo 1





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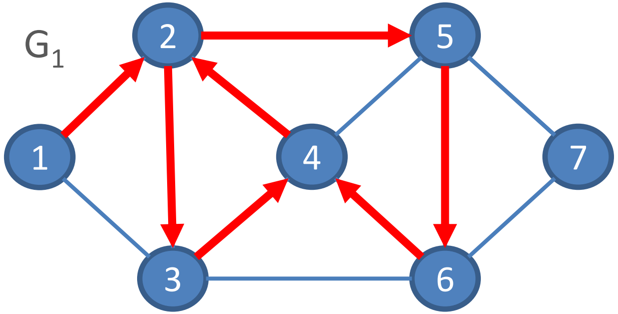
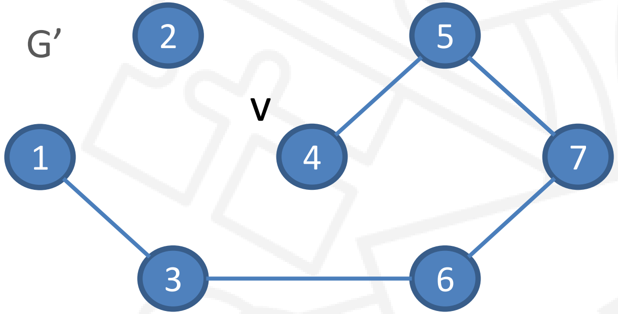
Método de Fleury – Exemplo 1





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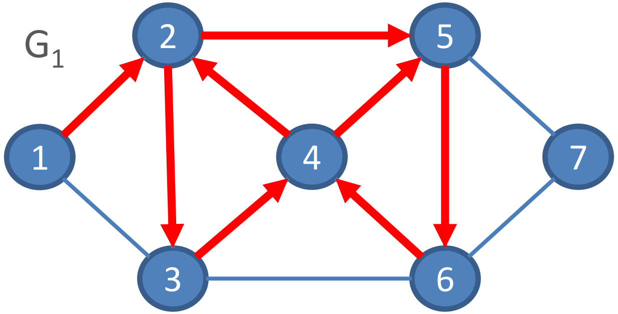
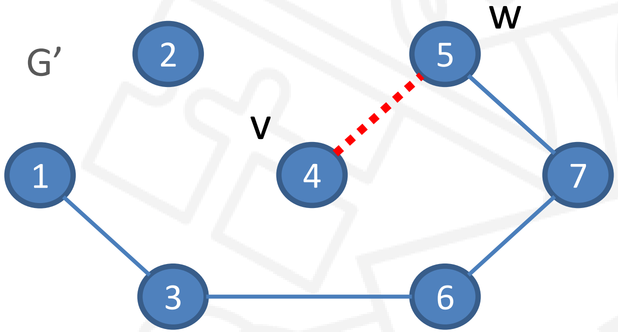
Método de Fleury – Exemplo 1





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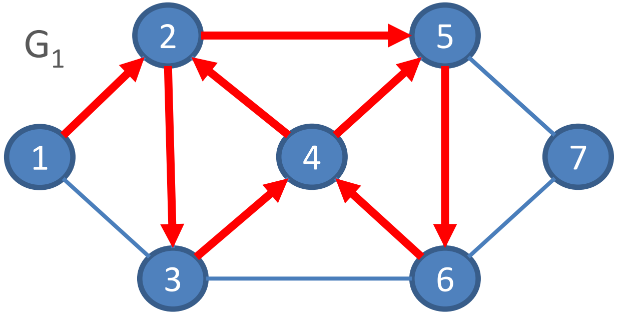
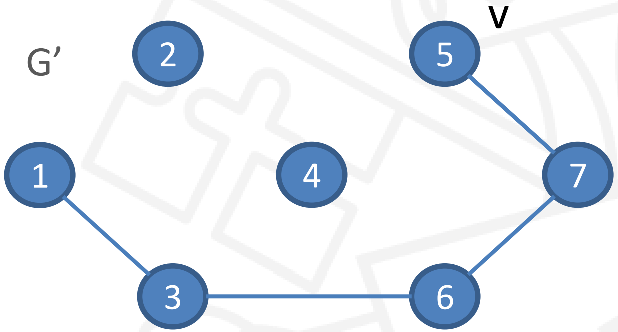
Método de Fleury – Exemplo 1





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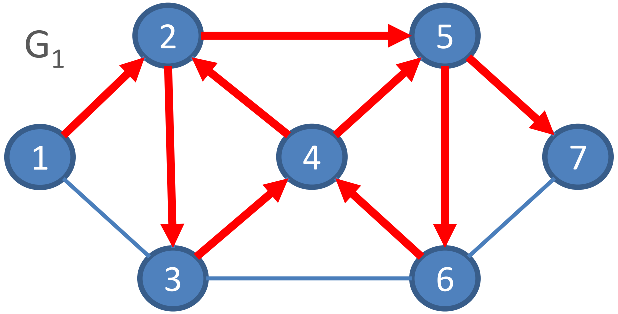
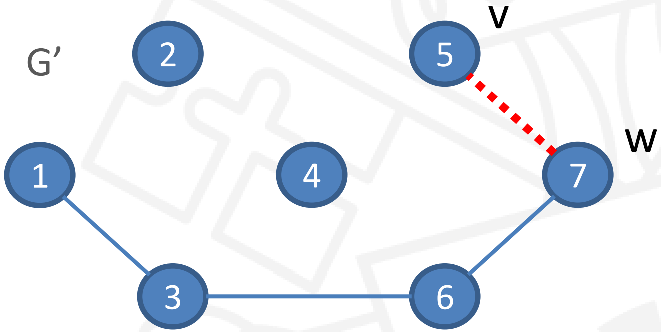
Método de Fleury – Exemplo 1





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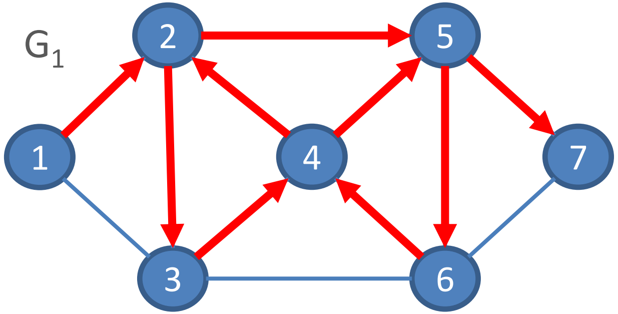
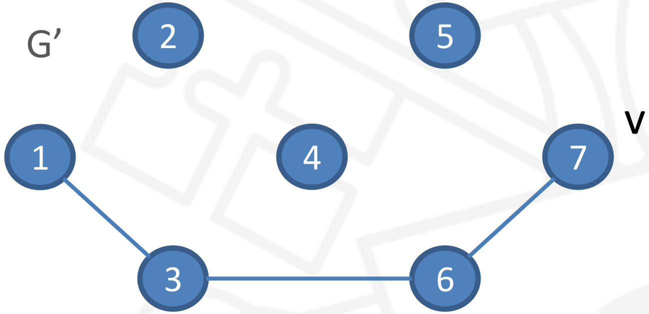
Método de Fleury – Exemplo 1





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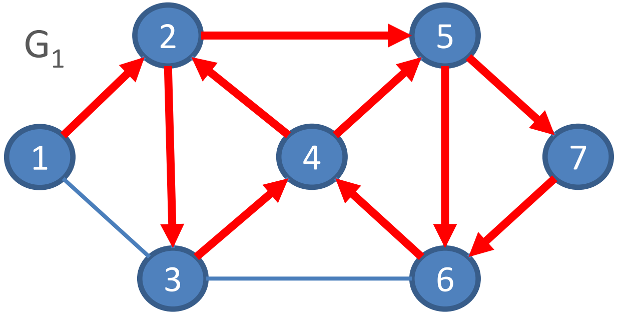

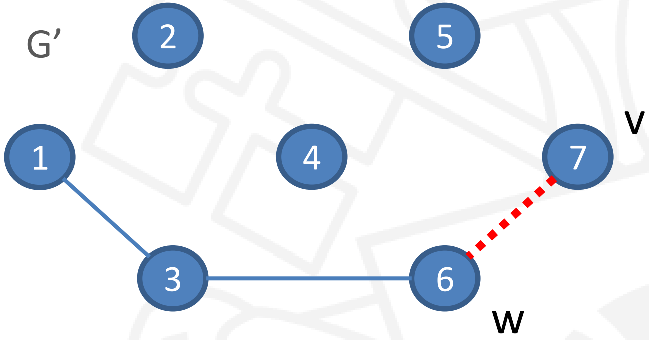

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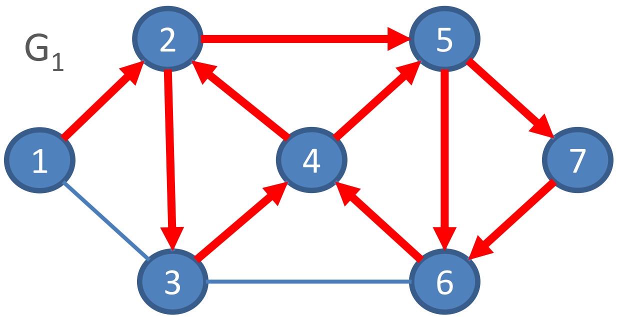
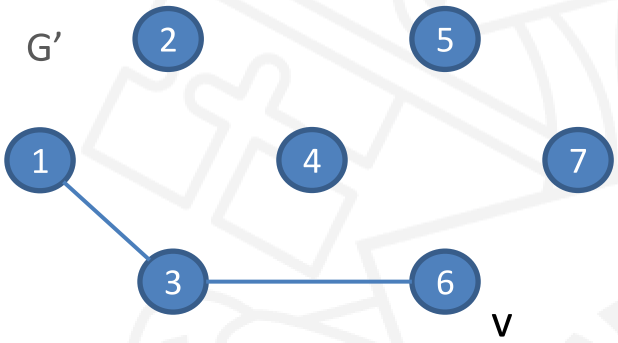
Método de Fleury – Exemplo 1





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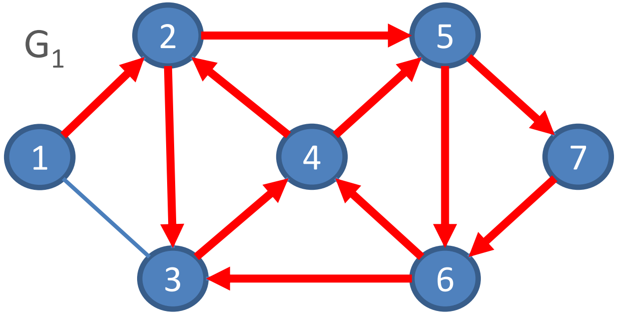
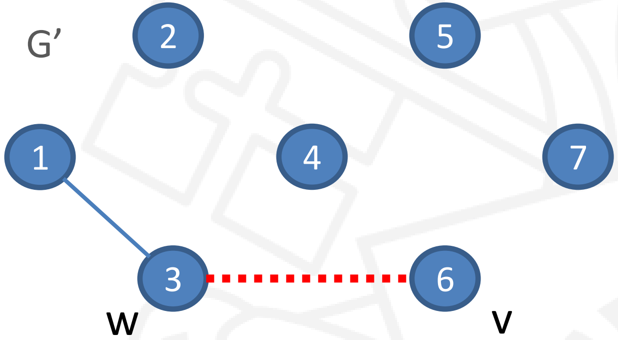
Método de Fleury – Exemplo 1





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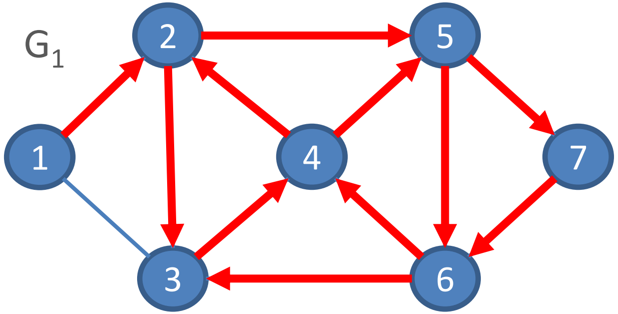
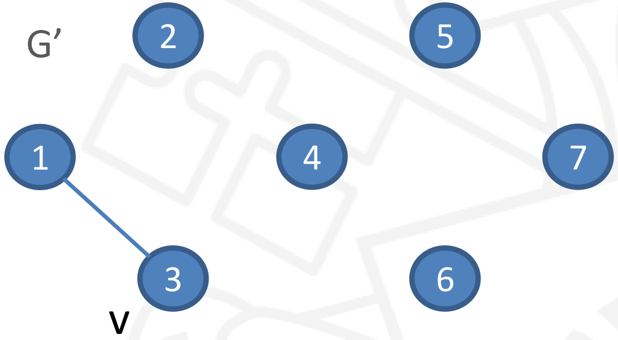
Método de Fleury – Exemplo 1





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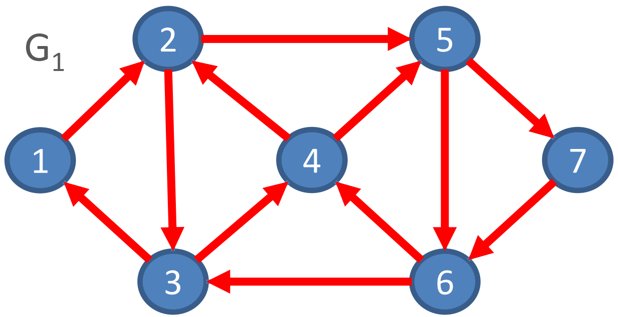
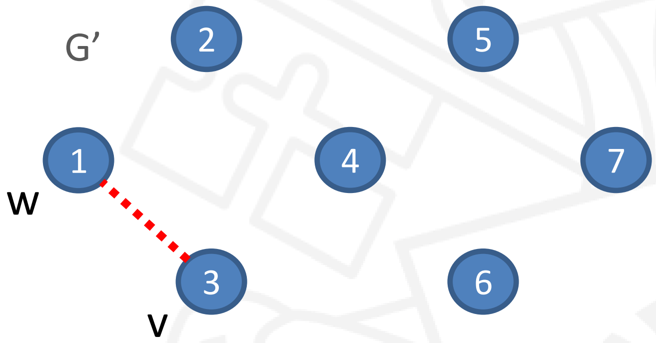
Método de Fleury – Exemplo 1





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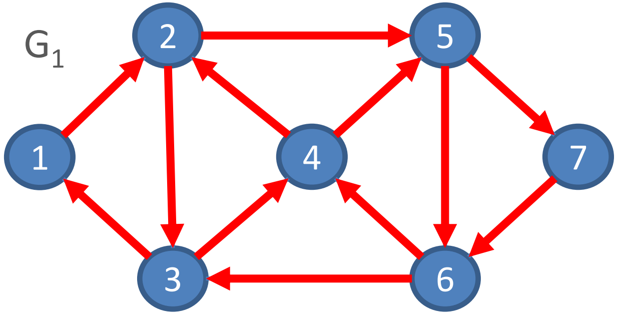
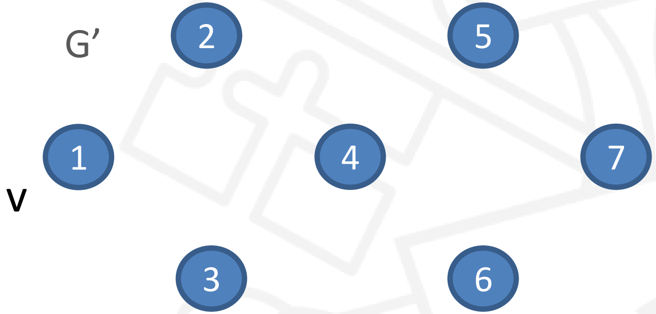
Método de Fleury – Exemplo 1





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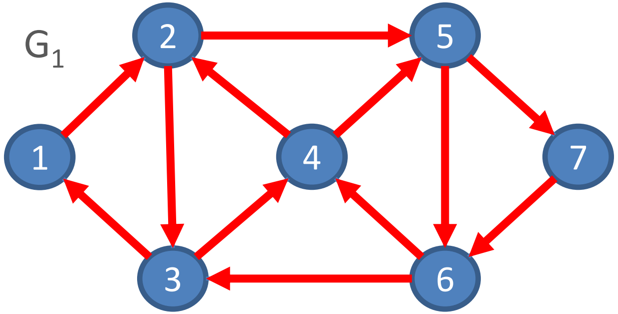
Método de Fleury – Exemplo 1



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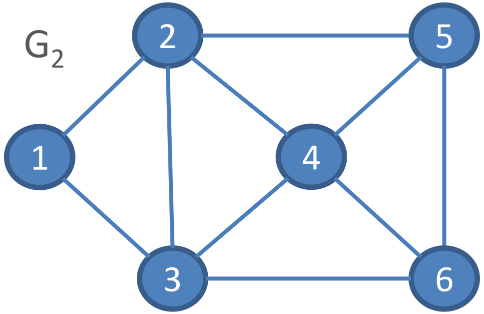
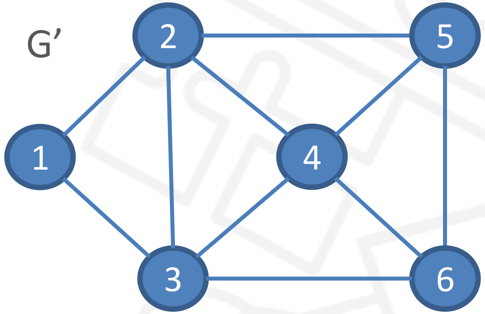
Método de Fleury – Exemplo 2





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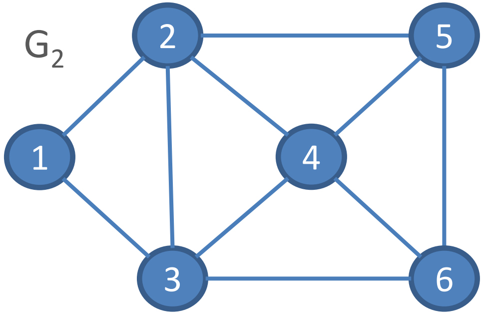

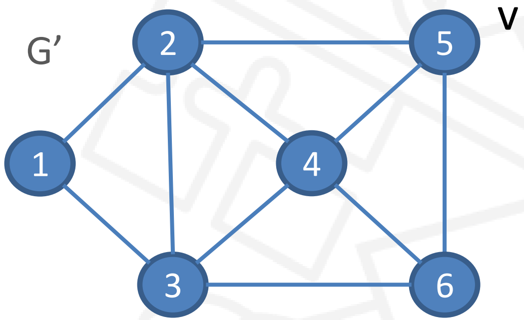

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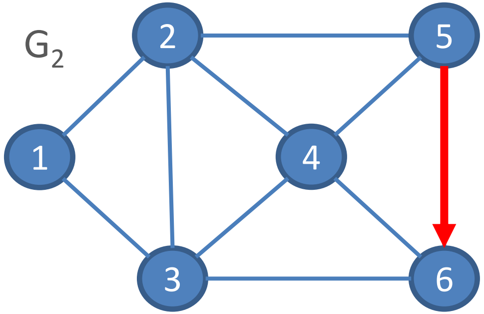
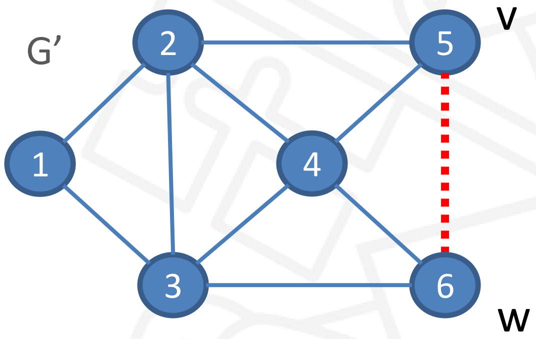
Método de Fleury – Exemplo 2





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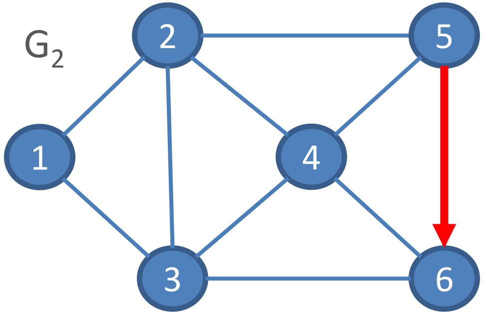
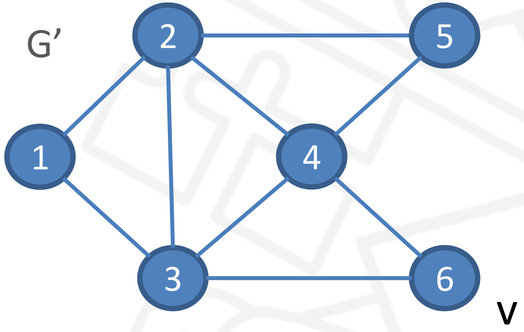
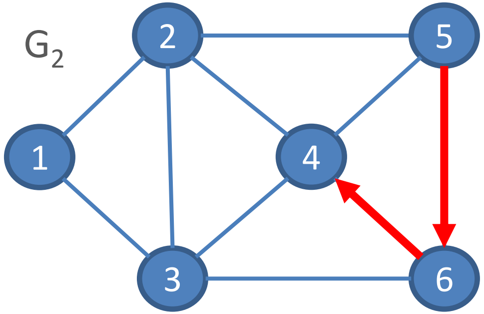
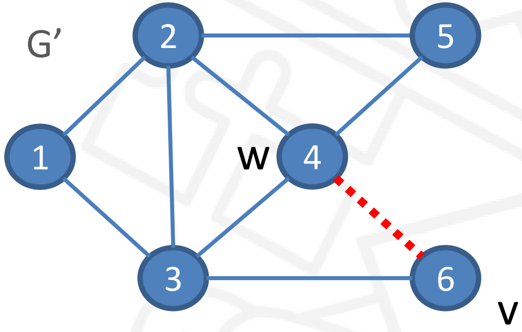




5 / 6

Método de Fleury – Exemplo 2





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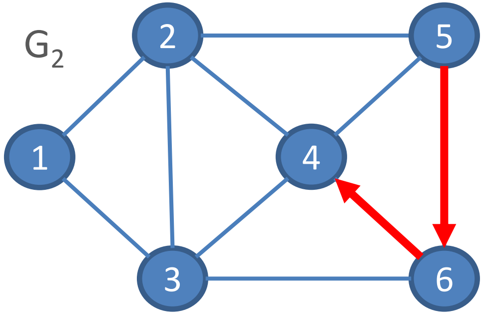
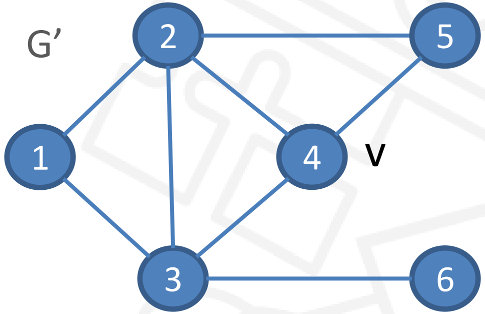
Método de Fleury – Exemplo 2





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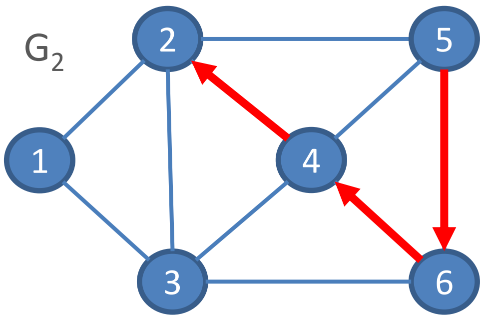
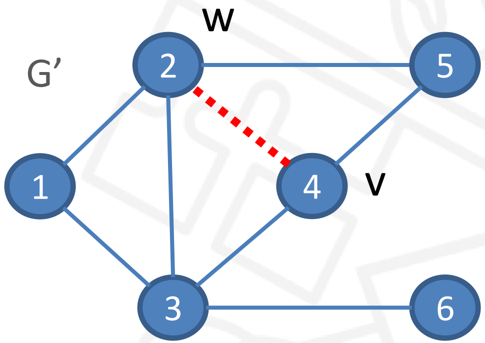
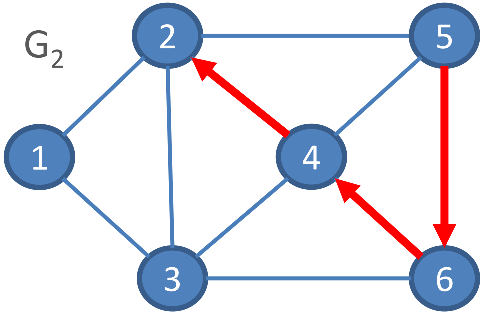
Método de Fleury – Exemplo 2

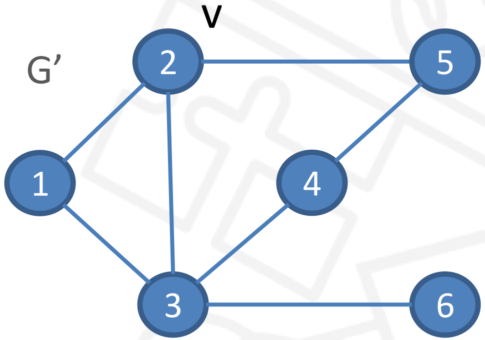




5 / 6 / 4

Método de Fleury – Exemplo 2





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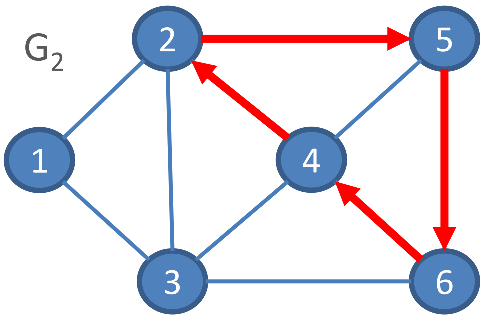

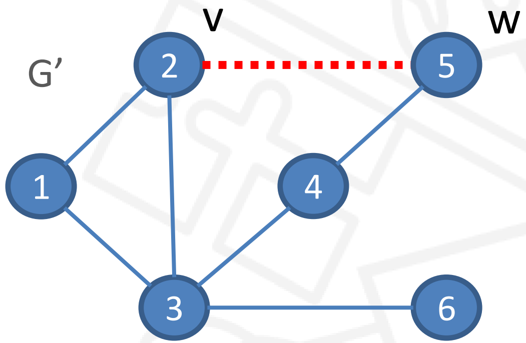

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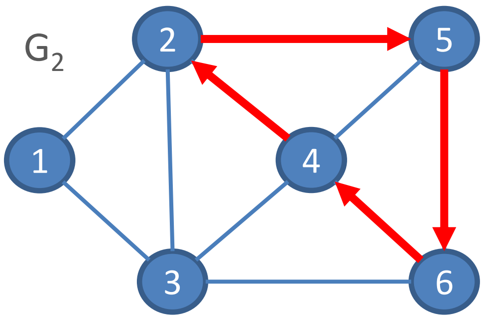
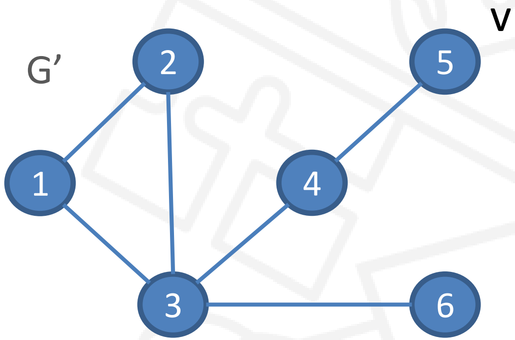
Método de Fleury – Exemplo 2





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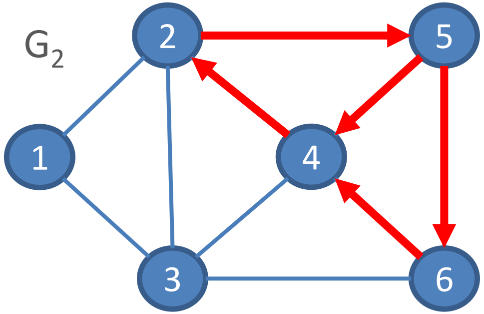
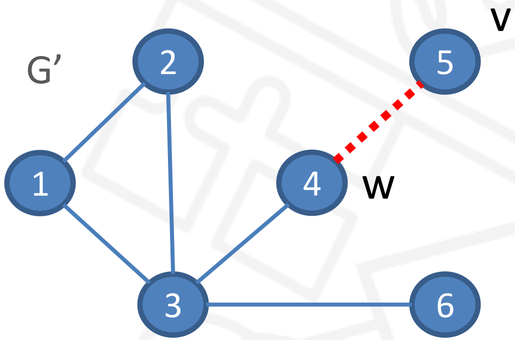
Método de Fleury – Exemplo 2





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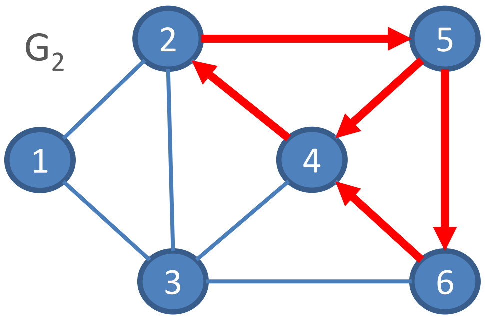
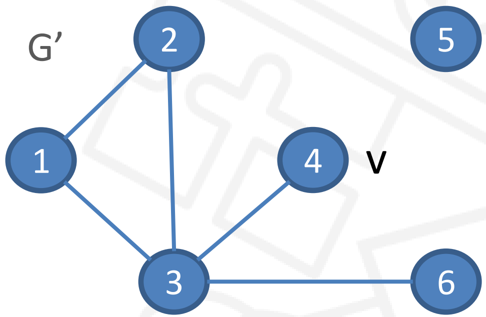
Método de Fleury – Exemplo 2





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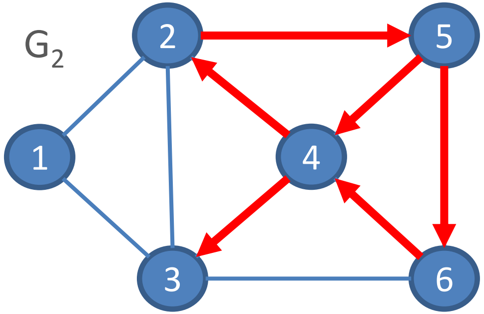
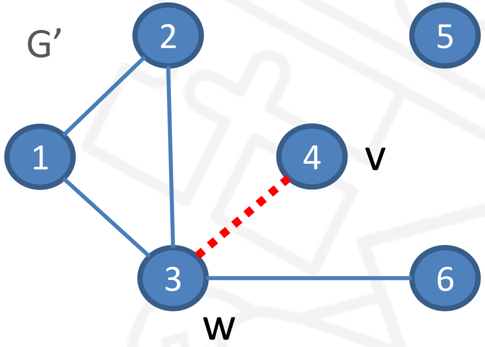
Método de Fleury – Exemplo 2





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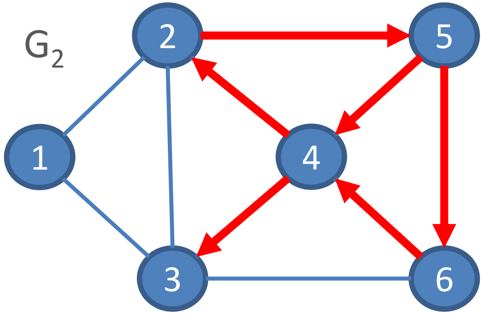
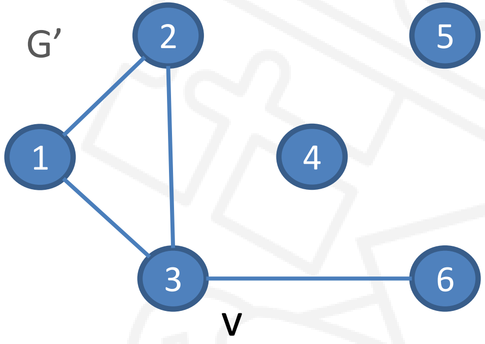
Método de Fleury – Exemplo 2





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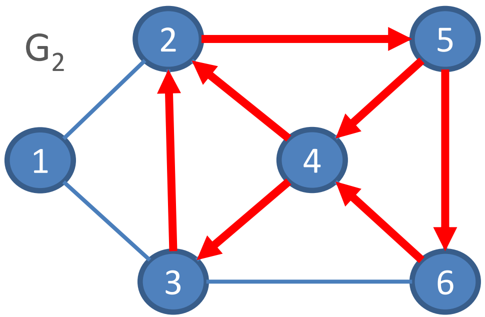
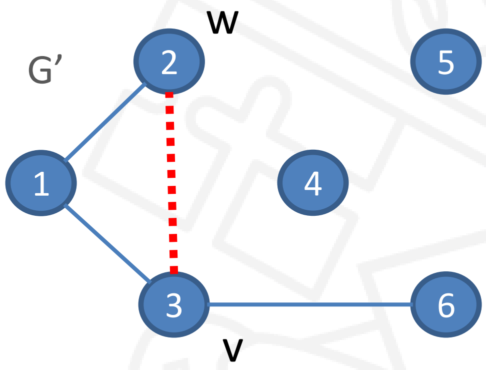
Método de Fleury – Exemplo 2





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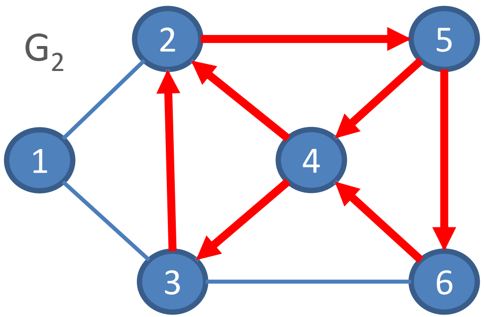
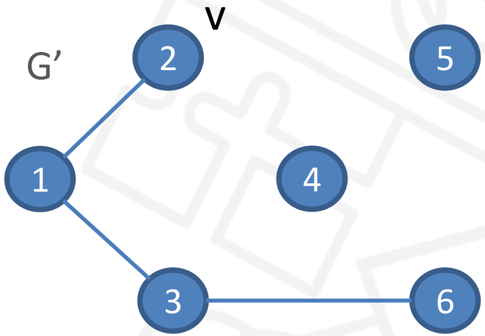
Método de Fleury – Exemplo 2





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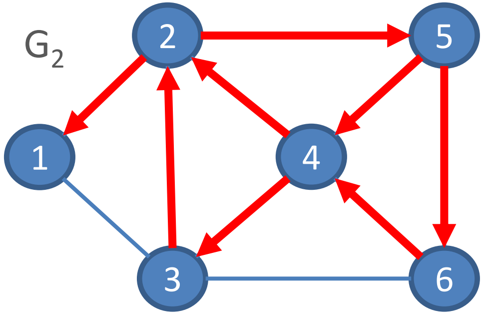

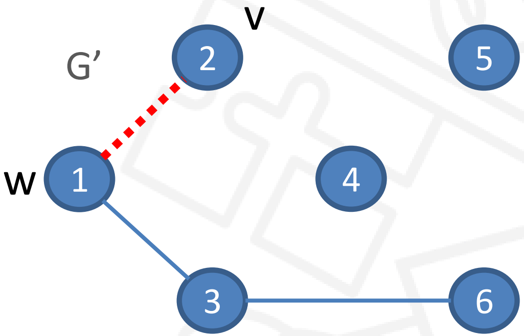

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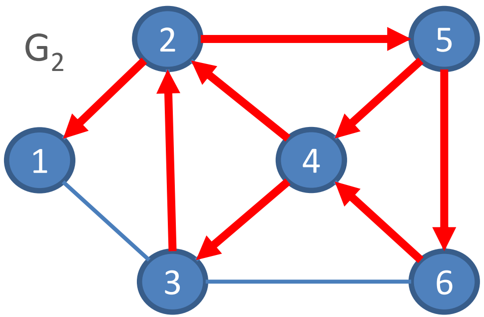
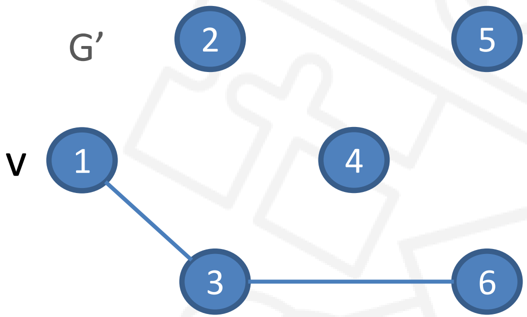
Método de Fleury – Exemplo 2





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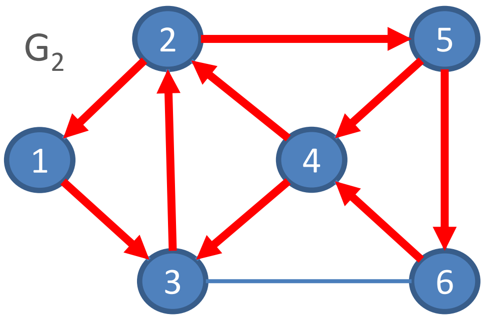
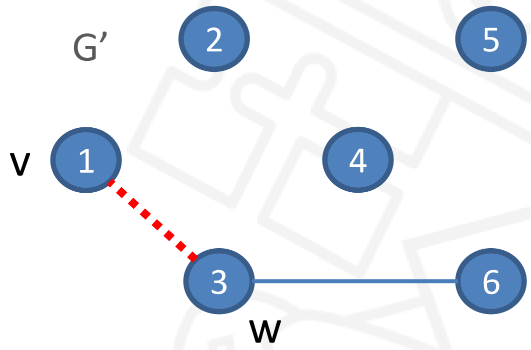
Método de Fleury – Exemplo 2





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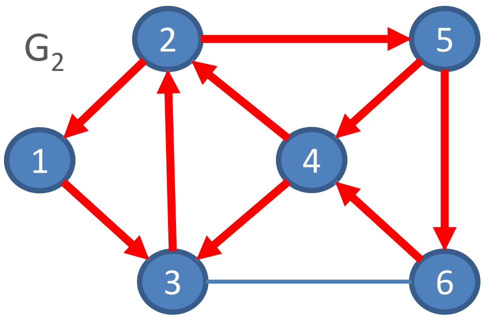
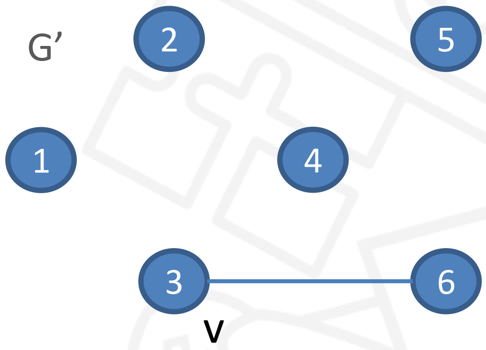
Método de Fleury – Exemplo 2





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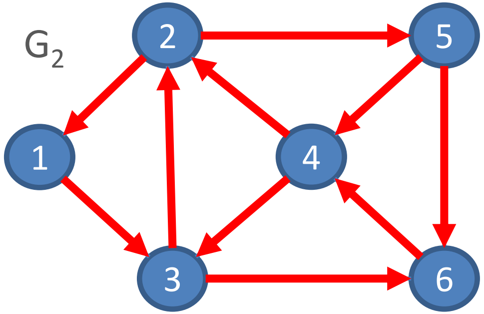
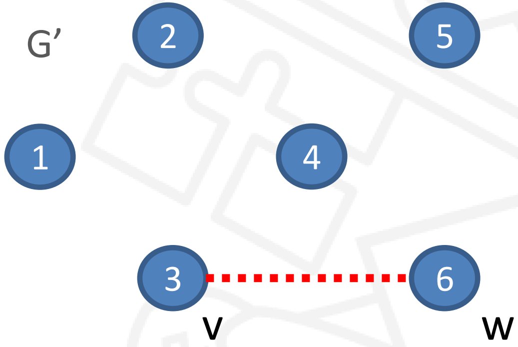
Método de Fleury – Exemplo 2





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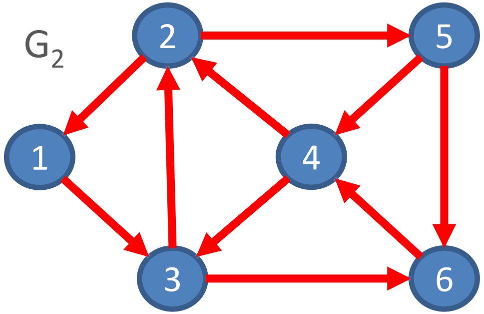
Método de Fleury – Exemplo 2



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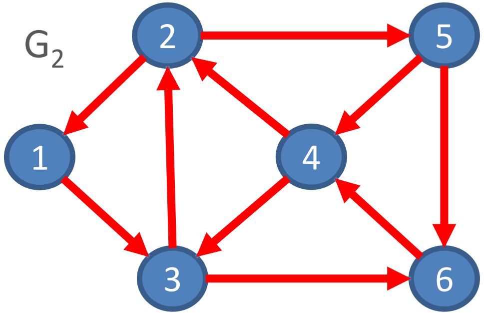
Método de Fleury – Exemplo 2



5 / 6 / 4 / 2 / 5 / 4 / 3 / 2 / 1 / 3 / 6 → **Trajeto euleriano**

Método de Fleury – Exemplo