Regra de l'Hôspital Suponha que f e g sejam deriváveis e $g'(x) \neq 0$ em um intervalo aberto I que contém a (exceto possivelmente em a). Suponha que

$$\lim_{x \to a} f(x) = 0 \qquad \text{e} \qquad \lim_{x \to a} g(x) = 0$$

ou que

$$\lim_{x \to a} f(x) = \pm\infty \qquad \text{e} \qquad \lim_{x \to a} g(x) = \pm\infty$$

(Em outras palavras, temos uma forma indeterminada do tipo $\frac{0}{0}$ ou ∞/∞.) Então

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

se o limite do lado direito existir (ou for ∞ ou $-\infty$).

$$\lim_{x\to 1}\frac{\overbrace{x^2-1}^{\to 0}}{\underbrace{x-1}_{\to 0}} = \lim_{x\to 1}\frac{\cancel{(x-1)}(x+1)}{\cancel{x-1}} = \lim_{x\to 1} x+1 = 2$$

ou

$$\lim_{x\to 1}\frac{x^2-1}{x-1} \underset{\substack{\uparrow\\ \text{RH}}}{=} \lim_{x\to 1}\frac{2x}{1} = 2$$

pg 278

17. $\lim_{x \to \infty} \frac{\ln x}{\sqrt{x}}$ → $\frac{\infty}{\infty}$

$\sqrt{x} = x^{1/2}$

$\frac{1}{2} x^{-1/2} = \frac{1}{2x^{1/2}} = \frac{1}{2\sqrt{x}}$

$$\lim_{x \to \infty} \frac{\ln x}{\sqrt{x}} \underset{RL}{=} \lim_{x \to \infty} \frac{\frac{1}{x}}{\frac{1}{2\sqrt{x}}} = \lim_{x \to \infty} \frac{1}{x} \cdot 2\sqrt{x} =$$

$$\lim_{x \to \infty} 2 \frac{\sqrt{x}}{x} \underset{RL}{=} \lim_{x \to \infty} 2 \cdot \frac{\frac{1}{2\sqrt{x}}}{1} = \lim_{x \to \infty} \cancel{2} \frac{1}{\cancel{2}\sqrt{x}} = \lim_{x \to \infty} \frac{1}{\sqrt{x}} = 0$$

19. $\displaystyle\lim_{x \to 0^+} \frac{\ln x}{x}$ ∞

∞ ∞

$$\lim_{x \to \infty} \frac{\ln x}{x} = \lim_{x \to \infty} \frac{\frac{1}{x}}{1} = \lim_{x \to \infty} \frac{1}{x} = 0$$

EXEMPLO:

$$\lim_{x \to \infty} \frac{\overset{\to \infty}{2x^2 + 1}}{\underset{\to \infty}{5x^2 + 3x}} = \lim_{x \to \infty} \frac{\overset{\to \infty}{4x}}{\underset{\to \infty}{10x + 3}} = \lim_{x \to \infty} \frac{4}{10} = \frac{2}{5}$$

Outras formas indeterminadas

1) $0 . \infty$

45. $\lim\limits_{x \to \infty} x^3 e^{-x^2}$

$$\lim_{x \to \infty} \underbrace{x^3}_{\infty} \cdot \underbrace{\frac{1}{e^{x^2}}}_{0} = \lim_{x \to \infty} \frac{x^3 \nearrow \infty}{e^{x^2} \to \infty} = \lim_{x \to \infty} \frac{3x^{\cancel{2}}}{e^{x^2} \cdot 2\cancel{x}} =$$

$$\lim_{x \to \infty} \frac{3x \nearrow \infty}{2e^{x^2} \searrow \infty} = \lim_{x \to \infty} \frac{3}{2e^{x^2} \cdot 2x} = \lim_{x \to \infty} \frac{3}{4xe^{x^2}} = 0$$

2) $\infty - \infty$

49. $\lim\limits_{x \to 1} \left(\dfrac{x}{x-1} - \dfrac{1}{\ln x} \right)$ $= \lim\limits_{x \to 1} \dfrac{x \ln x - (x-1)}{(x-1)\ln x} =$

$\lim\limits_{x \to 1} \dfrac{x \ln x - x + 1}{x \ln x - \ln x} = \lim\limits_{x \to 1} \dfrac{\ln x + \cancel{x} \cdot \frac{1}{\cancel{x}} - 1}{\ln x + x \cdot \frac{1}{x} - \frac{1}{x}} =$

$\lim\limits_{x \to 1} \dfrac{\ln x}{\ln x + 1 - \frac{1}{x}} = \lim\limits_{x \to 1} \dfrac{\frac{1}{x}}{\frac{1}{x} + \frac{1}{x^2}} = \dfrac{1}{2}$

3) 0^0

55. $\lim_{x \to 0^+} x^{\sqrt{x}} = l$

$$\ln\left(\lim_{x\to 0^+} x^{\sqrt{x}}\right) = \ln l$$

$$\ln l = 0$$

$$e^{\ln l} = e^0 \Rightarrow l = 1$$

$$\ln\left(\lim_{x\to 0^+} x^{\sqrt{x}}\right) = \lim_{x\to 0^+} \ln x^{\sqrt{x}}$$

$$= \lim_{x\to 0^+} \sqrt{x} \cdot \ln x = \lim_{x\to 0^+} \frac{\ln x}{\frac{1}{\sqrt{x}}} =$$

$$\lim_{x\to 0^+} \frac{\frac{1}{x}}{-\frac{1}{2}x^{-3/2}} = \lim \frac{\frac{1}{x}}{-\frac{1}{2x^{3/2}}} =$$

$$\lim_{x\to 0^+} \frac{1}{x} \cdot -2x^{3/2} = \lim_{x\to 0^+} -2x^{1/2} = 0$$

4) 1^{∞}

59. $\lim_{x \to 1^+} x^{1/(1-x)} = e^{-1}$

$$\lim_{x \to 1^+} \ln x^{1/1-x} = \lim_{x \to 1^+} \frac{1}{1-x} \cdot \ln x = \lim_{x \to 1^+} \frac{\ln x \to 0}{1-x \to 0} =$$

$$\lim_{x \to 1^+} \frac{\frac{1}{x}}{-1} = -1$$

5) ∞^0

61. $\lim_{x \to \infty} x^{1/x}$