Processamento e Análise de Imagens

Convolutional Neural Networks

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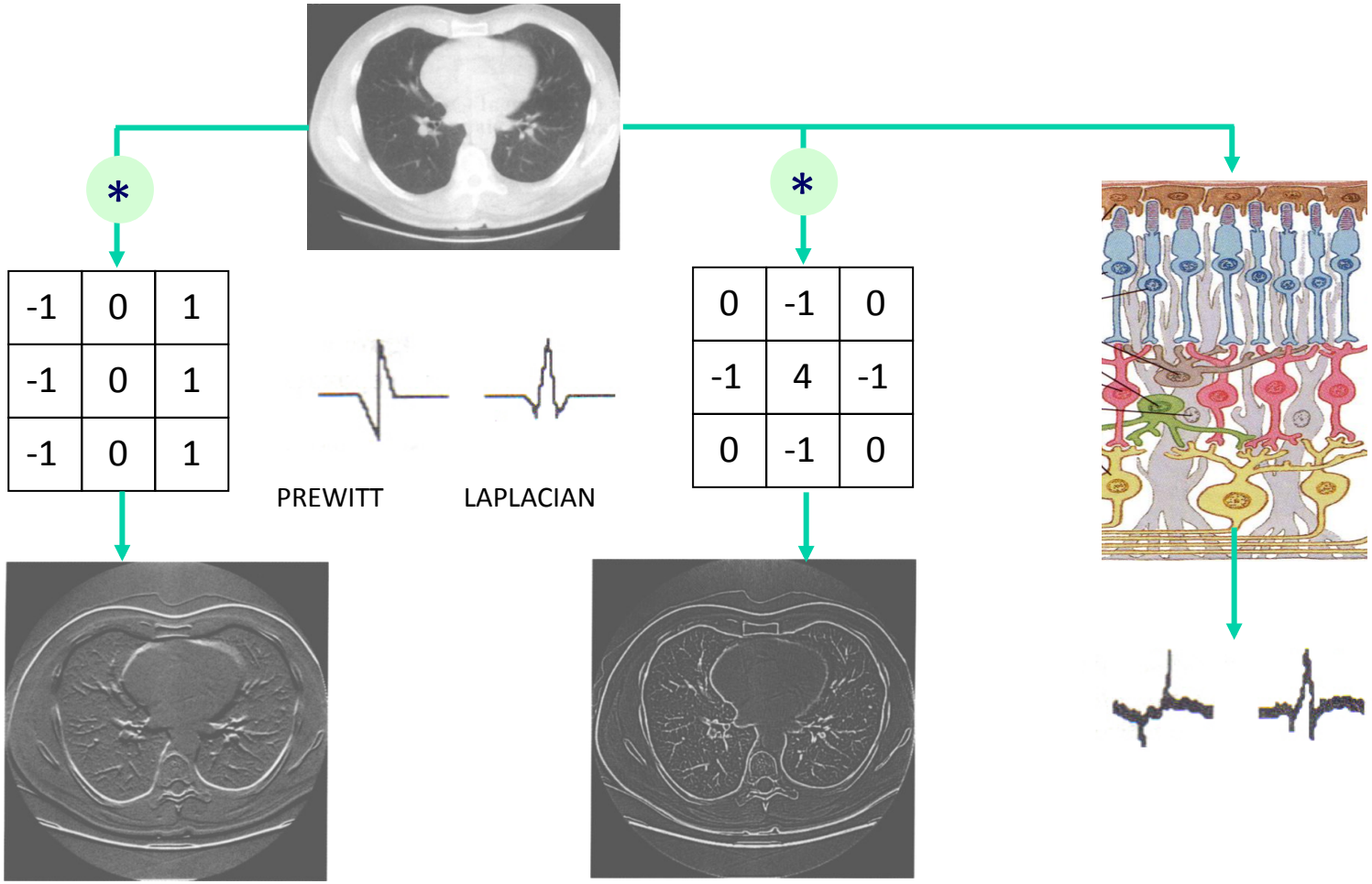
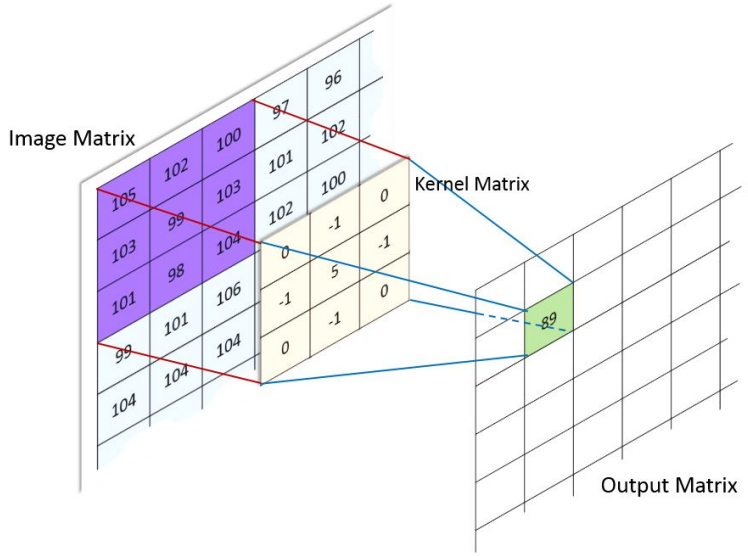
PUC Minas

Convolutional Neural Networks

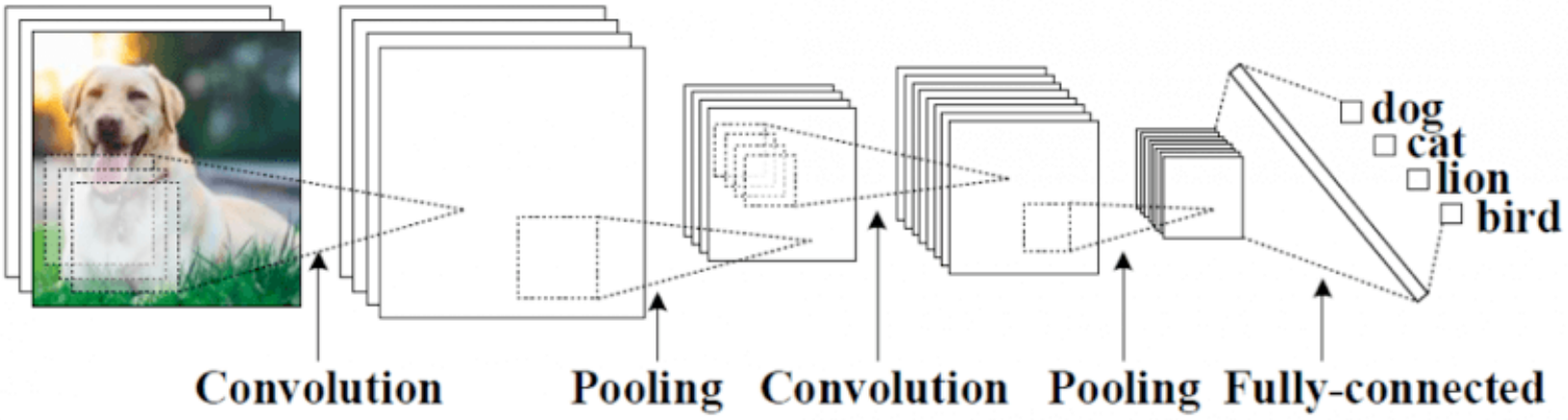


Convolutional Neural Networks



$$f[x,y] * g[x,y] = \sum_{n_1=-\infty}^{\infty} \sum_{n_2=-\infty}^{\infty} f[n_1,n_2] \cdot g[x-n_1, y-n_2]$$

Convolutional Neural Networks



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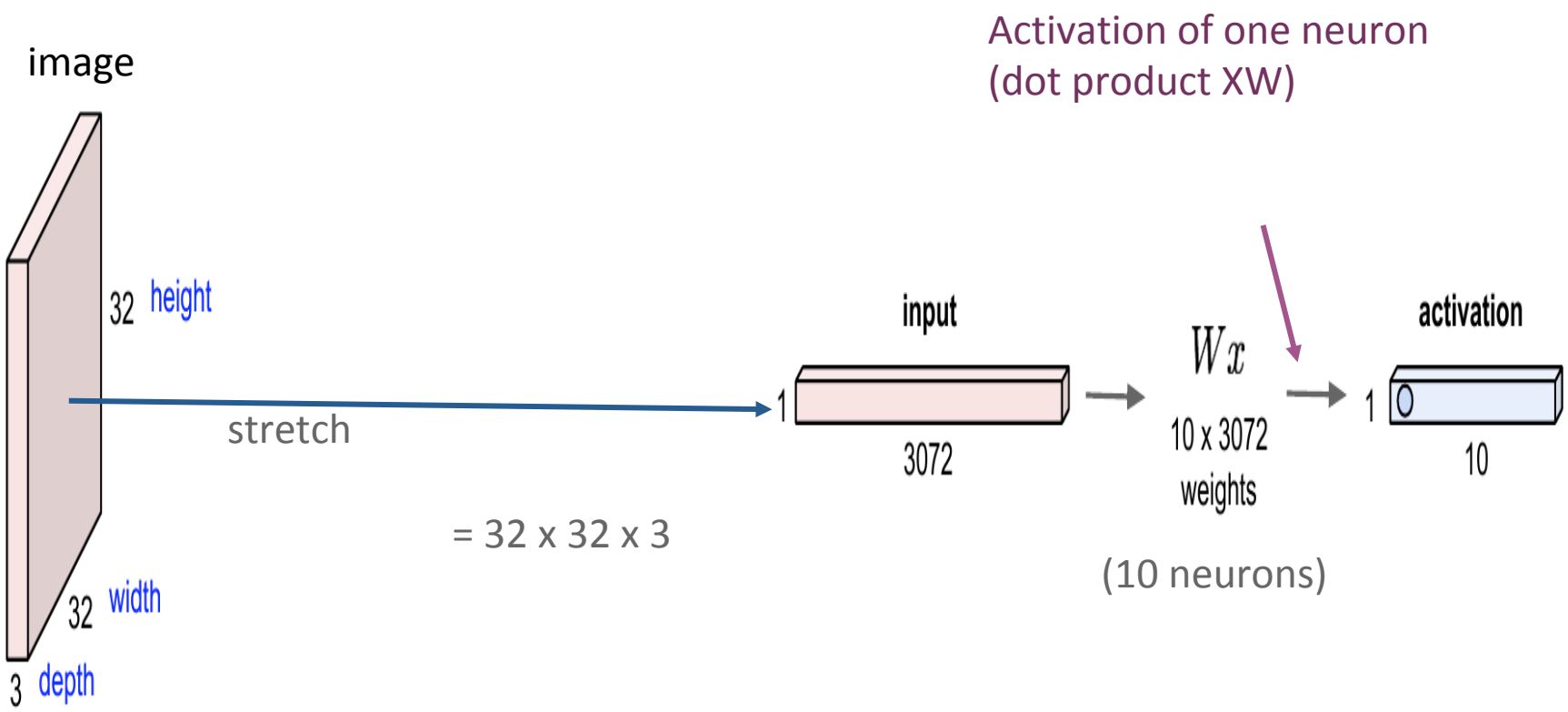
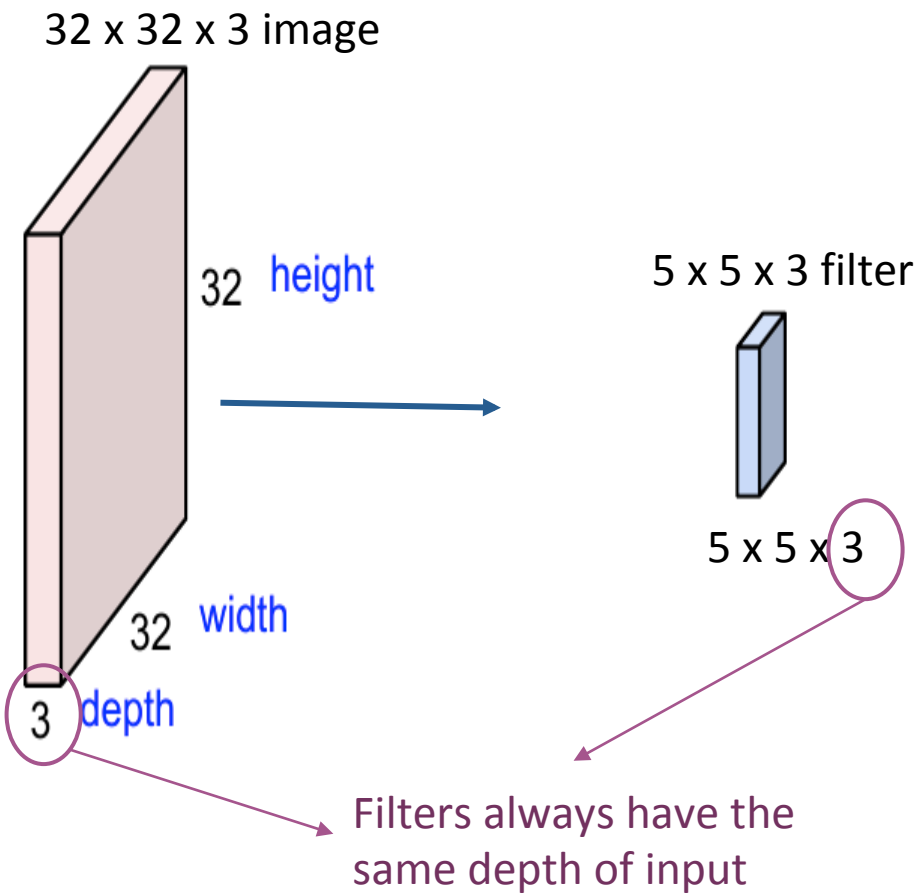
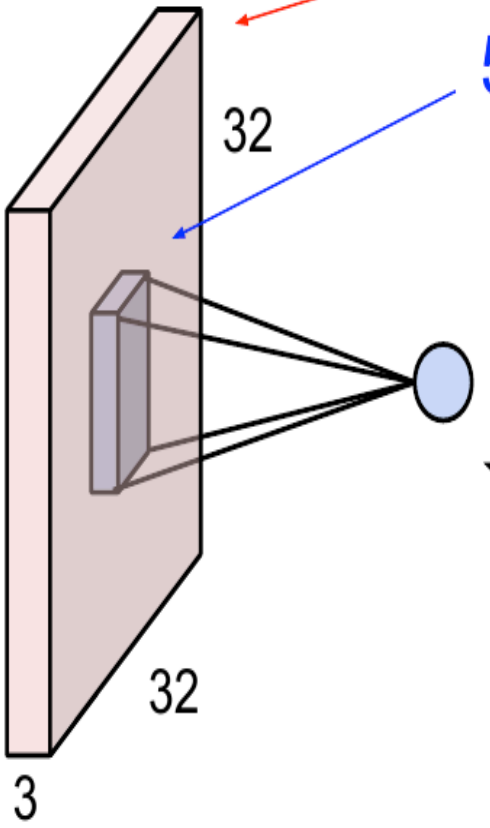
Fully-connected layers



Convolutional layers



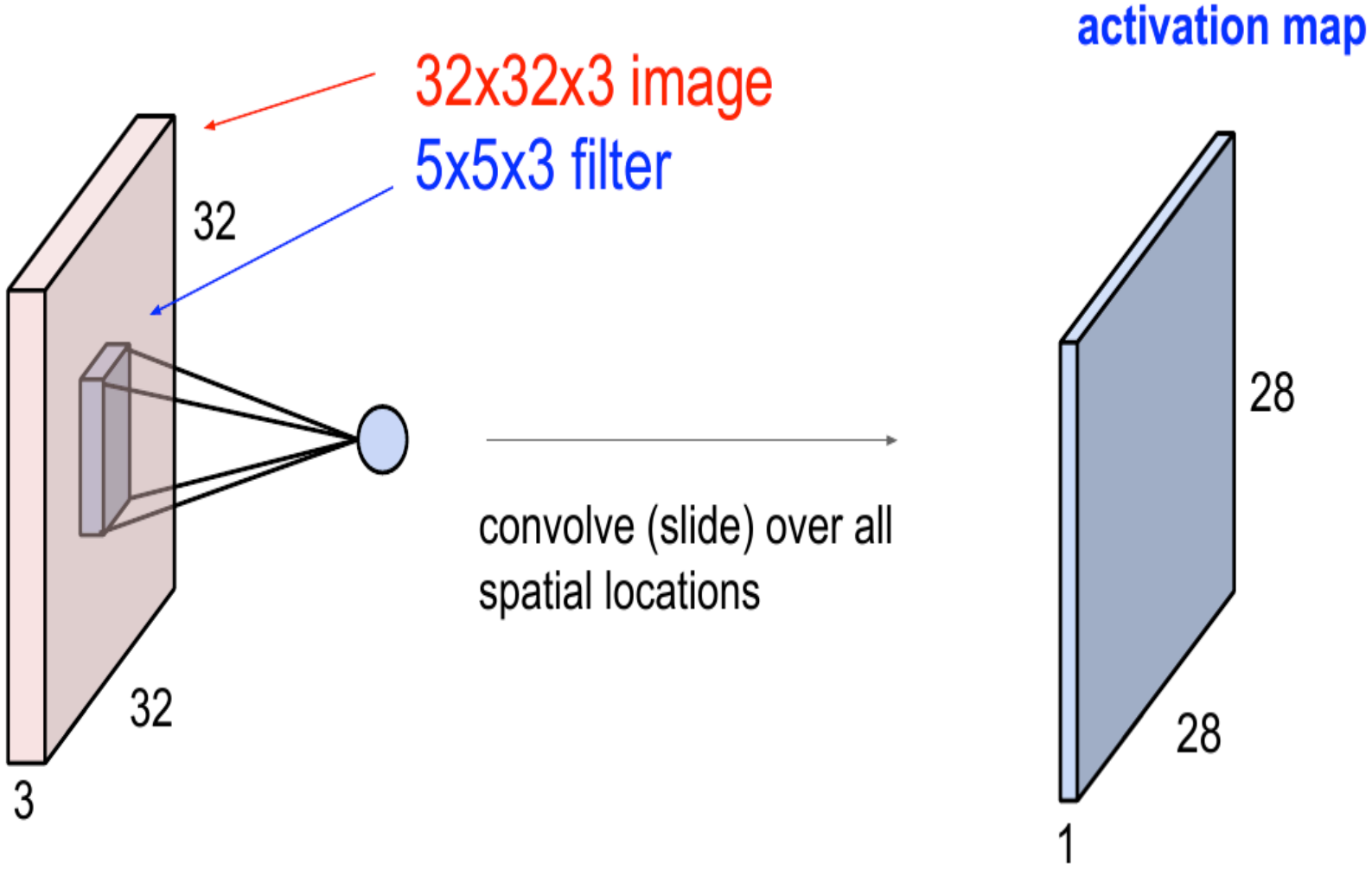
Convolutional layers



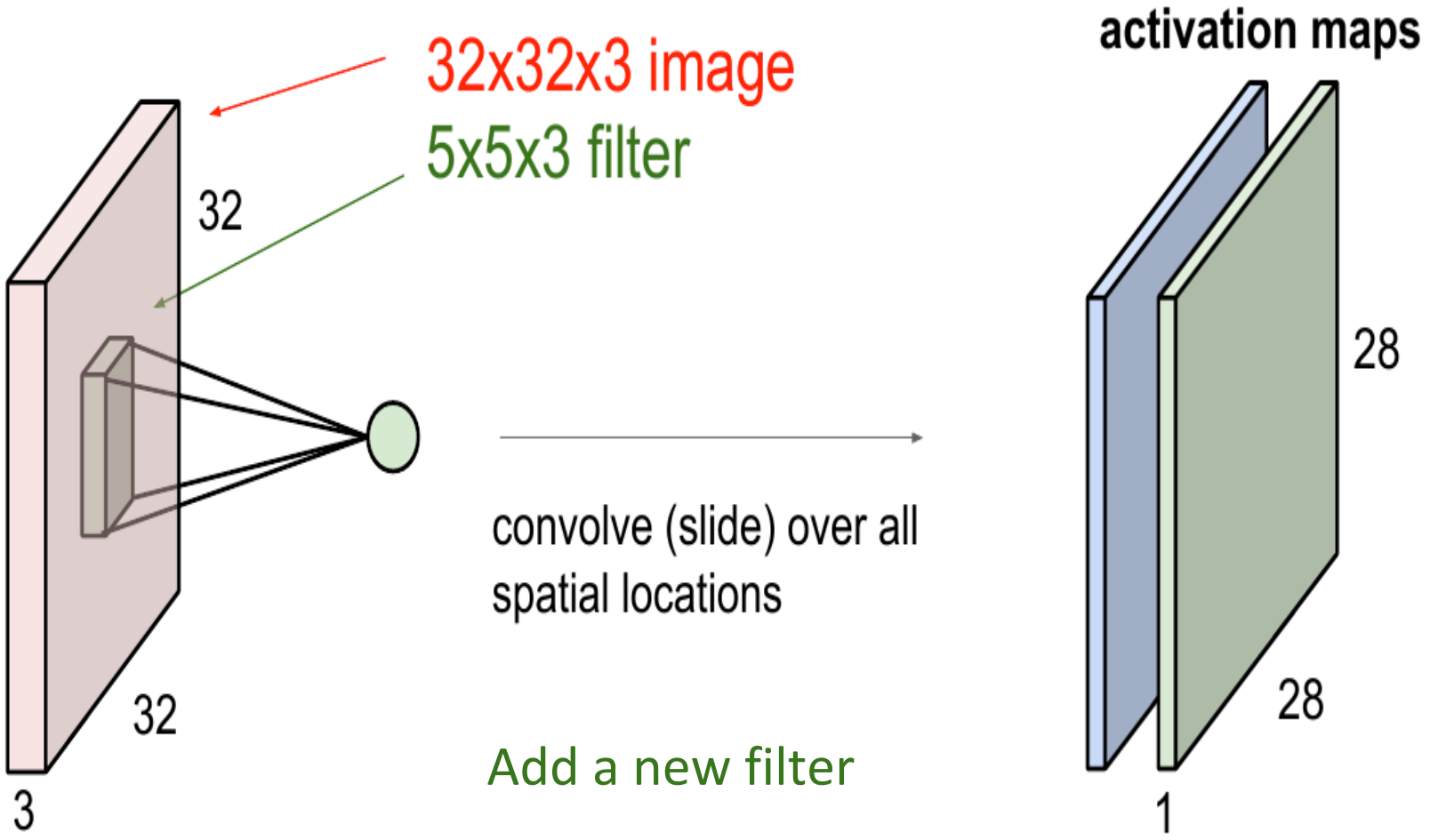
1 number:
the result of taking a dot product between the filter and a small 5x5x3 chunk of the image
(i.e. 5*5*3 = 75-dimensional dot product + bias)

$$w^T x + b$$

Convolutional layers

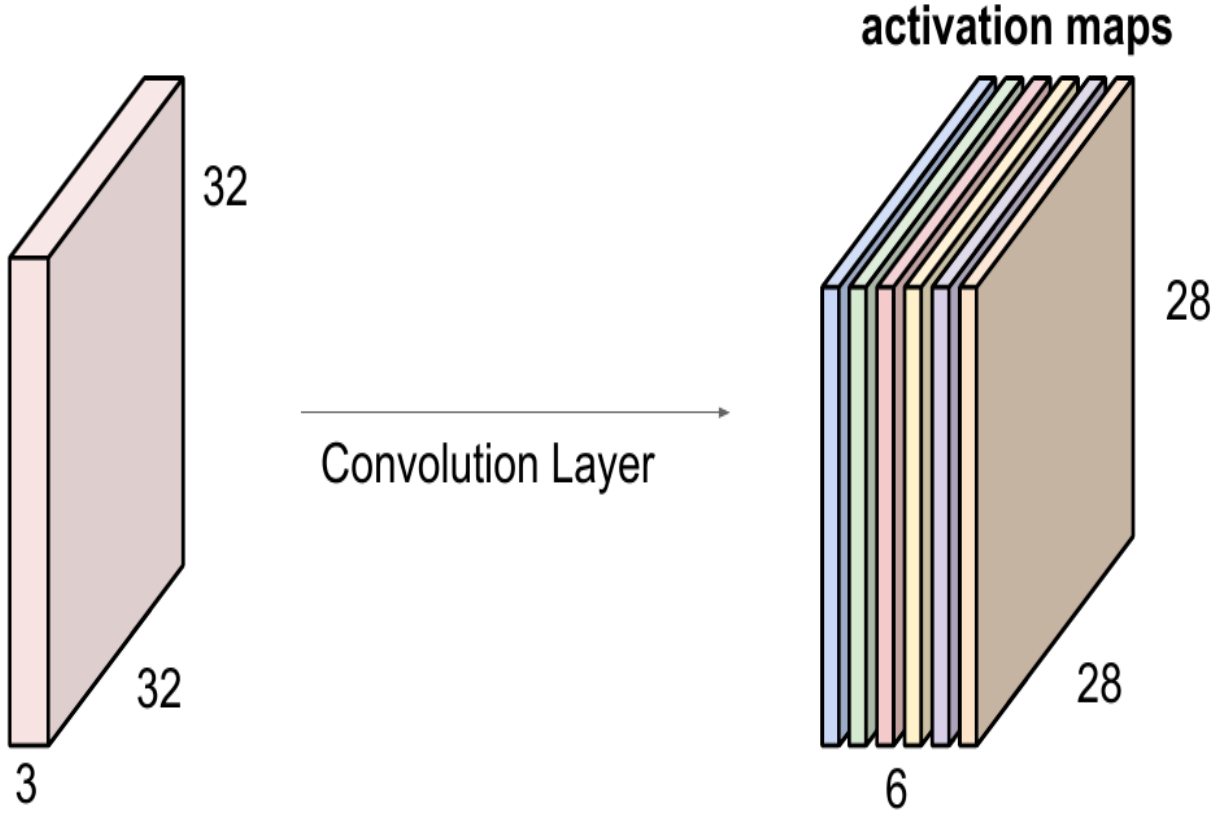


Convolutional layers



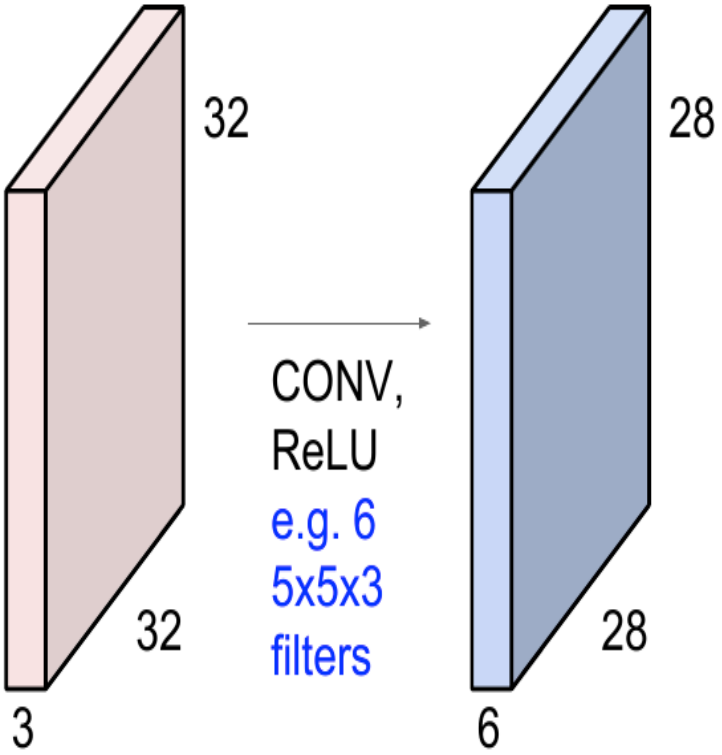
Convolutional layers

If we have 6 filters (5x5x3), there will be 6 activation maps:



Convolutional layers

A ConvNet is a sequence of convolutional layers, each followed by an activation function:



Convolutional layers

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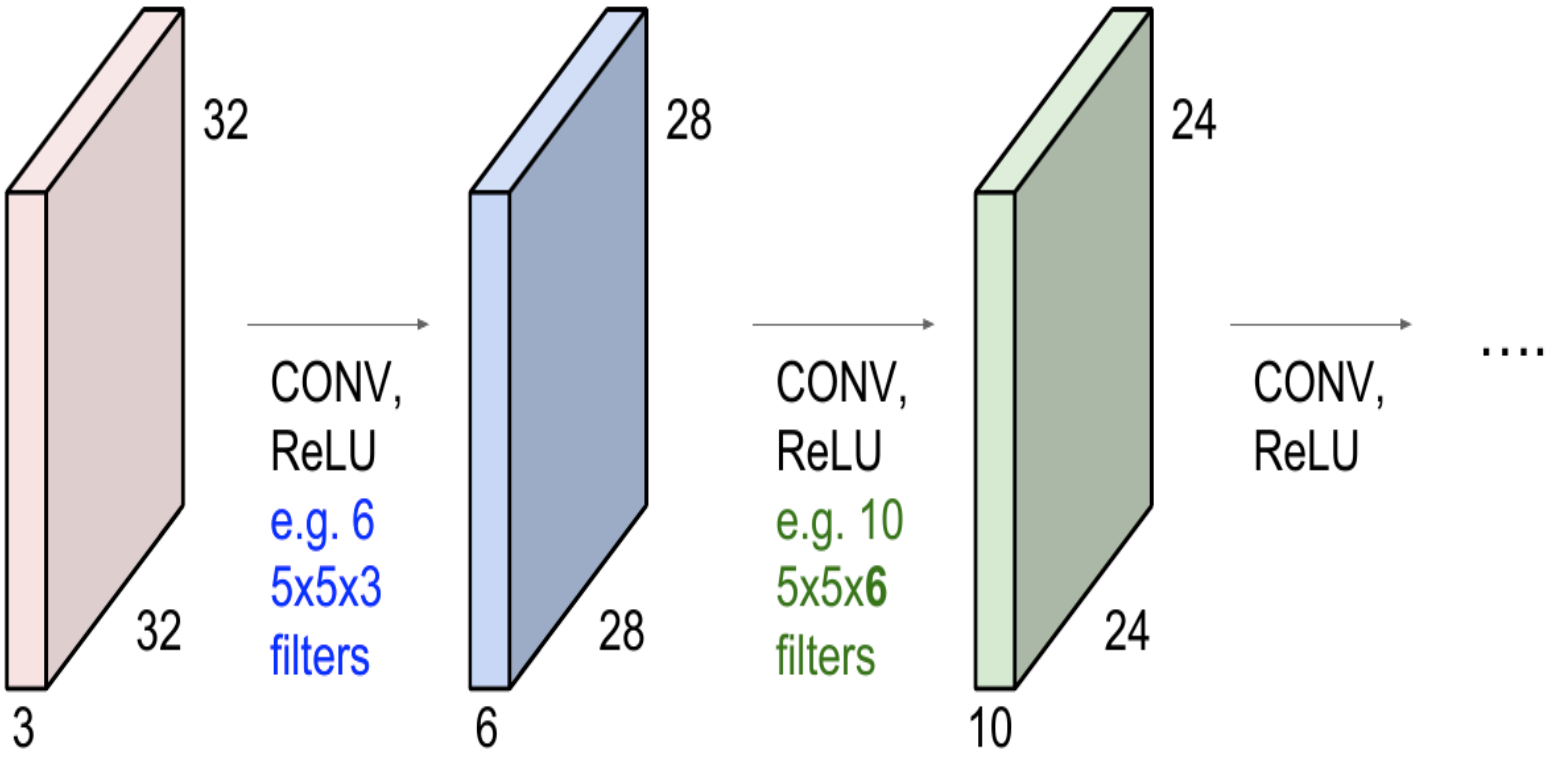
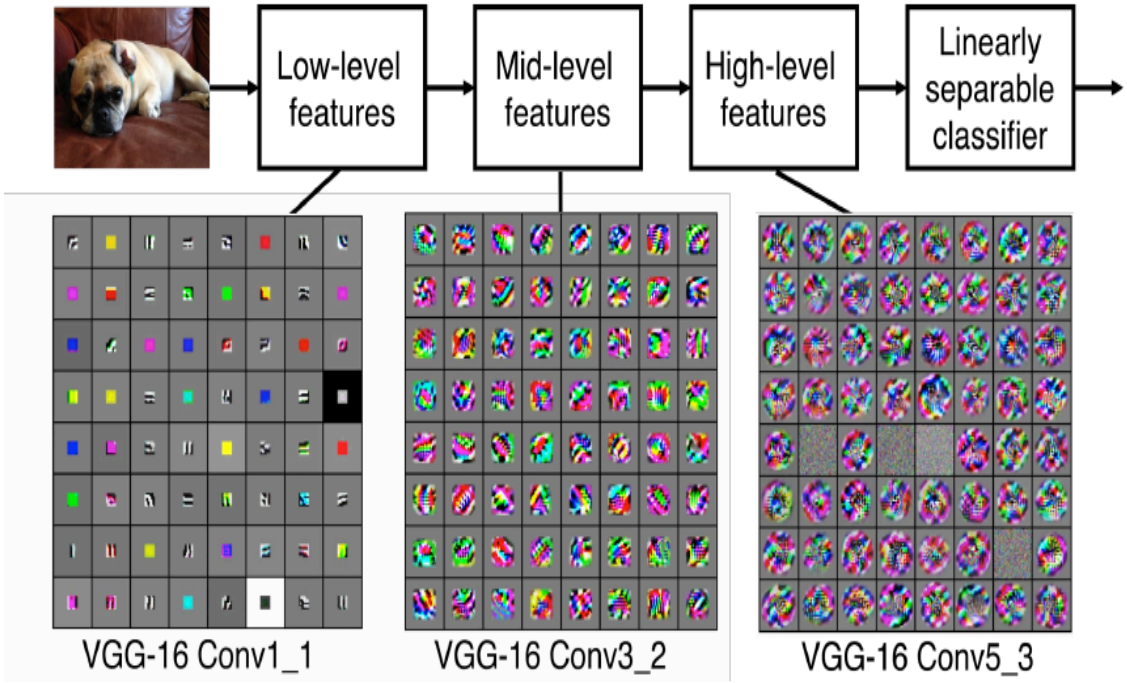
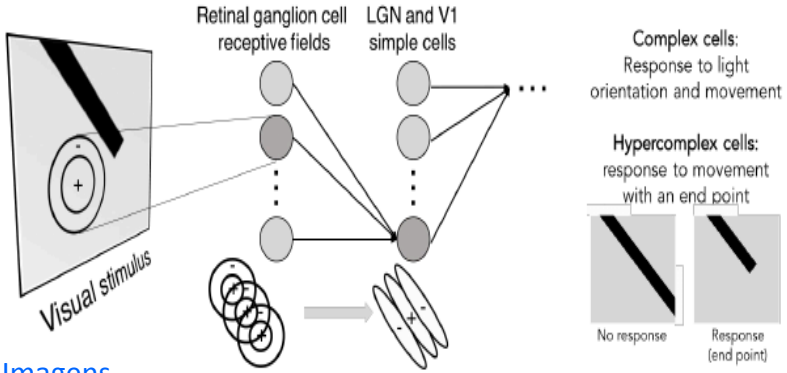
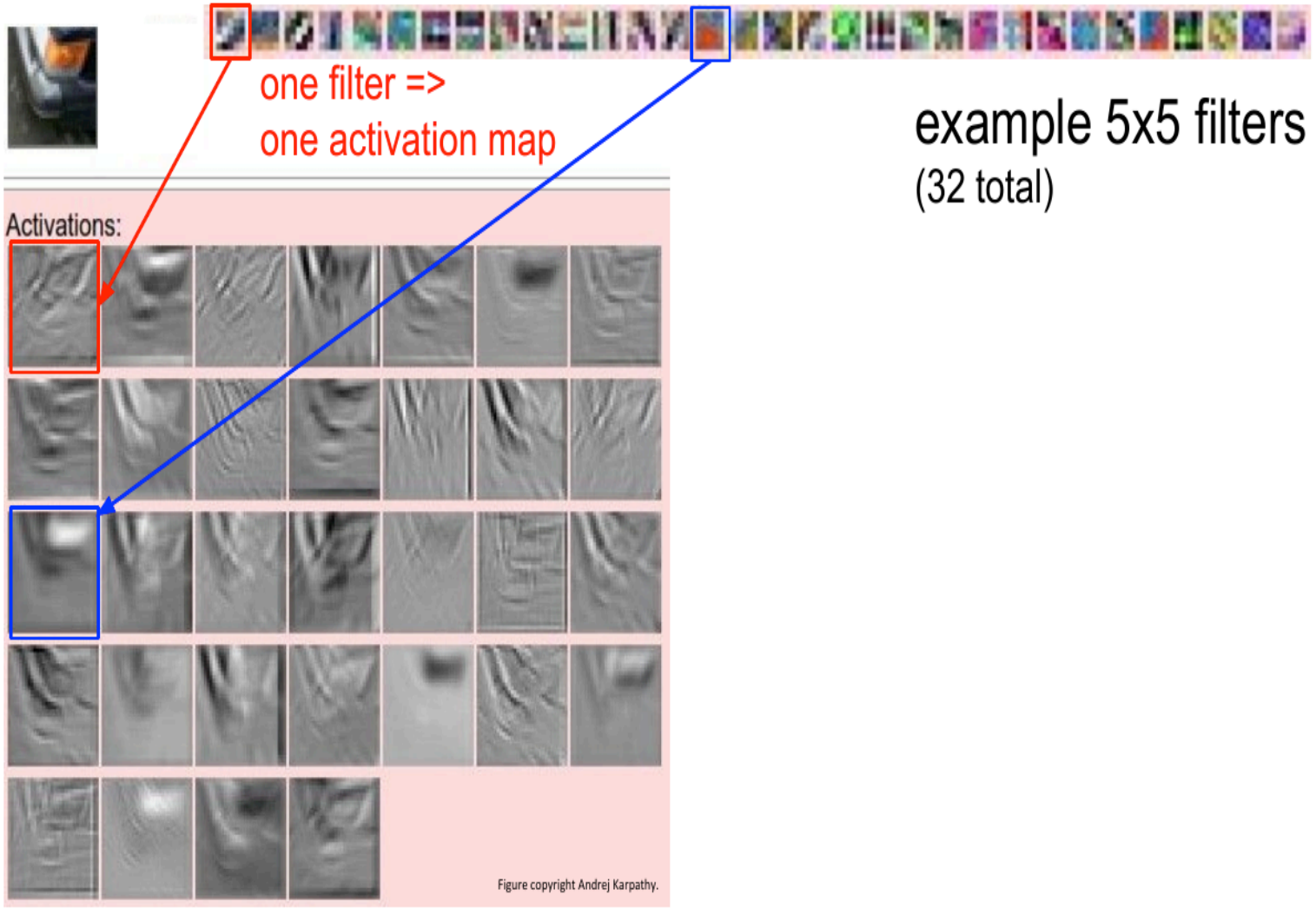


Filter hierarchy



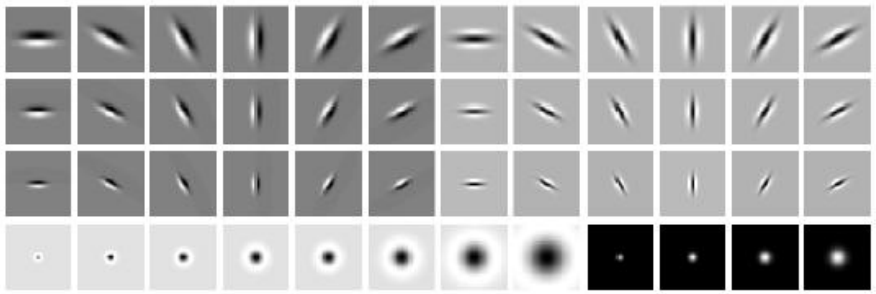


Activation maps



Convolution as feature extraction

bank of K filters

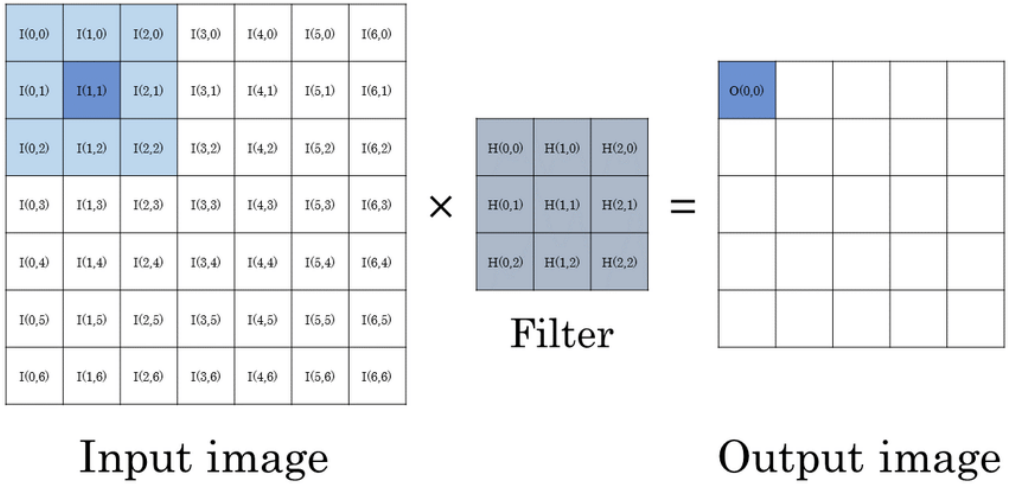


K feature maps

image

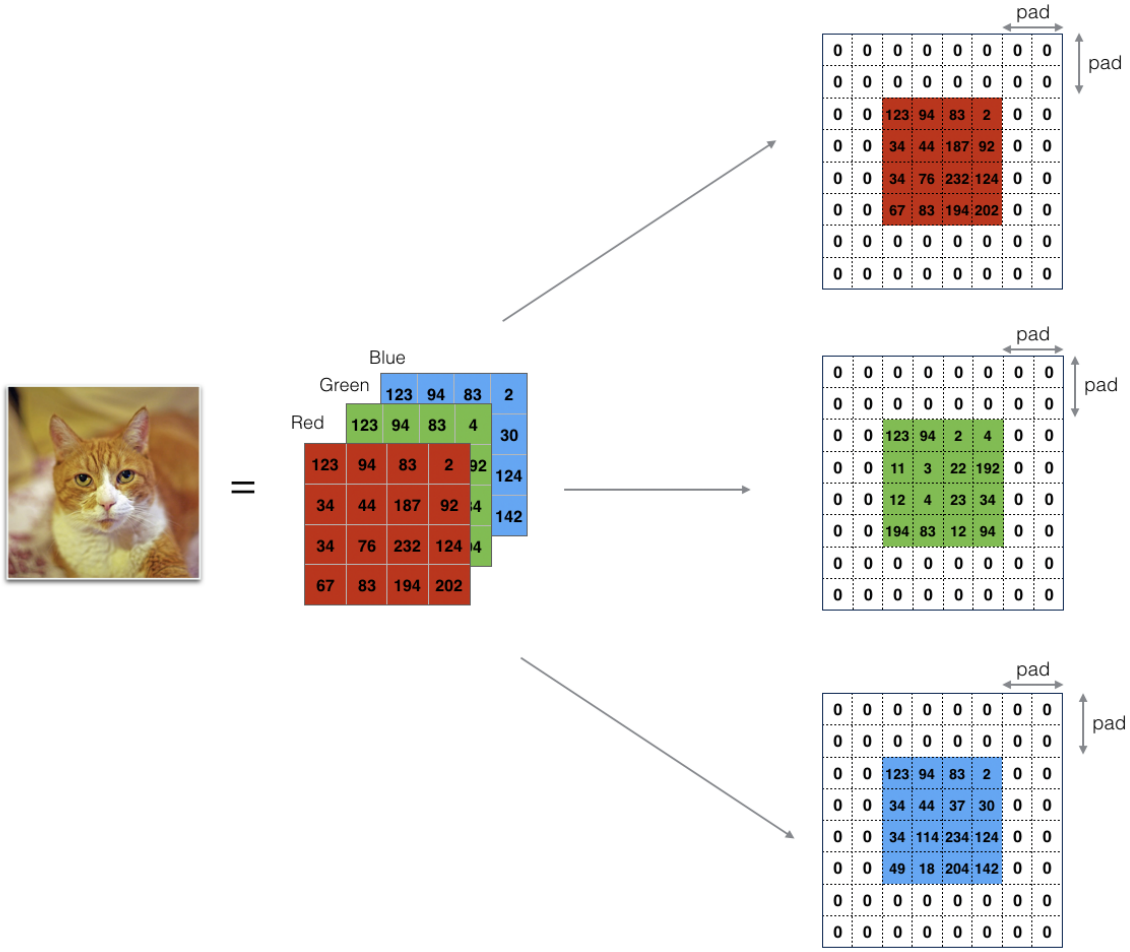
feature map

Padding

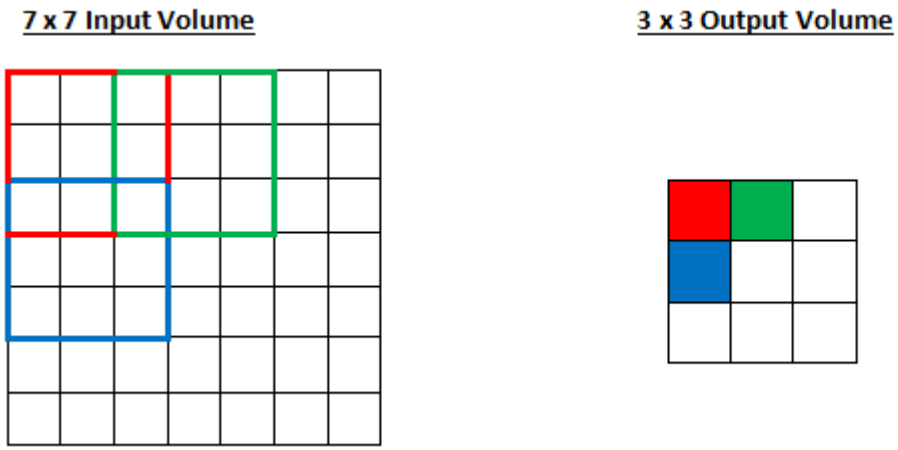


- When an image nxn is convolved by a filter fxf, it loses rows and columns, resulting in a n-f+1 x n-f+1 image
- In order for the image to keep the same dimensions, p columns and rows must be added prior to convolution, resulting in a n+2p-f+1 x n+2p-f+1 image

Padding


pad
pad
Blue
Green
Red
123 94 83 2
123 94 83 4
123 94 83 2
34 44 187 92
34 76 232 124
67 83 194 202
30
124
142
pad
pad
pad
pad

Striding



- When an image nxn is convolved by a filter fxf using stride s, it loses rows and columns, resulting in a (n-f)/s+1 x (n-f)/s+1 image

Padding and striding

- When an image nxn is convolved by a filter fxf using stride s and pad p the resulting image is

$$(n+2p-f)/s+1 \times (n+2p-f)/s+1$$

- In order for the image to keep original dimensions ("same" padding):

$$p = (ns - n + f - s) / 2$$

Convolution over volumes

- A convolutional layer may receive an image with multiple channels to be convolved by multiple filters
- The number of channels of the image and the filters must be the same
- Example: Input image of **6x6x3** and **10** filters **3x3x3** resulting in a **4x4x10** output (with **p = 0**, **s = 1**)

Convolution over volumes

- Each layer receives a volume of $\mathbf{W_1}$ x $\mathbf{H_1}$ x $\mathbf{D_1}$
- Requires 4 hyperparameters:
  - Number of filters $\mathbf{n_c}$
  - Size of filter **f**
  - *Stride* **s**
  - *Padding* **p**
- Outputs a volume of $\mathbf{W_2}$ x $\mathbf{H_2}$ x $\mathbf{D_2}$, where:
  - $\mathbf{W_2} = (\mathbf{W_1} - \mathbf{f} + 2\mathbf{p})/\mathbf{s} + 1$
  - $\mathbf{H_2} = (\mathbf{H_1} - \mathbf{f} + 2\mathbf{p})/\mathbf{s} + 1$
  - $\mathbf{D_2} = \mathbf{n_c}$
- Must compute **f** x **f** x $\mathbf{D_1}$ weights per filter, a total of **f** x **f** x $\mathbf{D_1}$ x $\mathbf{n_c}$ weights and $\mathbf{n_c}$ *biases*
- In the output volume, the d-th slice of size $\mathbf{W_2}$ x $\mathbf{H_2}$ is the result of the convolution between the d-th filter and the image with *stride* **s**, and applying an offset related to the d-th *bias*

Summary of notation

f[l] = filter size

p[l] = padding

s[l] = stride

nc[l] = number of filters

Input: n[l-1] x n[l-1] x nc[l-1] **Or** nH[l-1] x nW[l-1] x nc[l-1]

Output: n[l] x n[l] x nc[l] **Or** nH[l] x nW[l] x nc[l]

Where n[l] = (n[l-1] + 2p[l] - f[l] / s[l]) + 1

Each filter is: f[l] x f[l] x nc[l-1]

Activations: a[l] is nH[l] x nW[l] x nc[l]

A[l] is m x nH[l] x nW[l] x nc[l]

Weights: f[l] * f[l] * nc[l-1] * nc[l]

bias: (1, 1, 1, nc[l])

Forward propagation

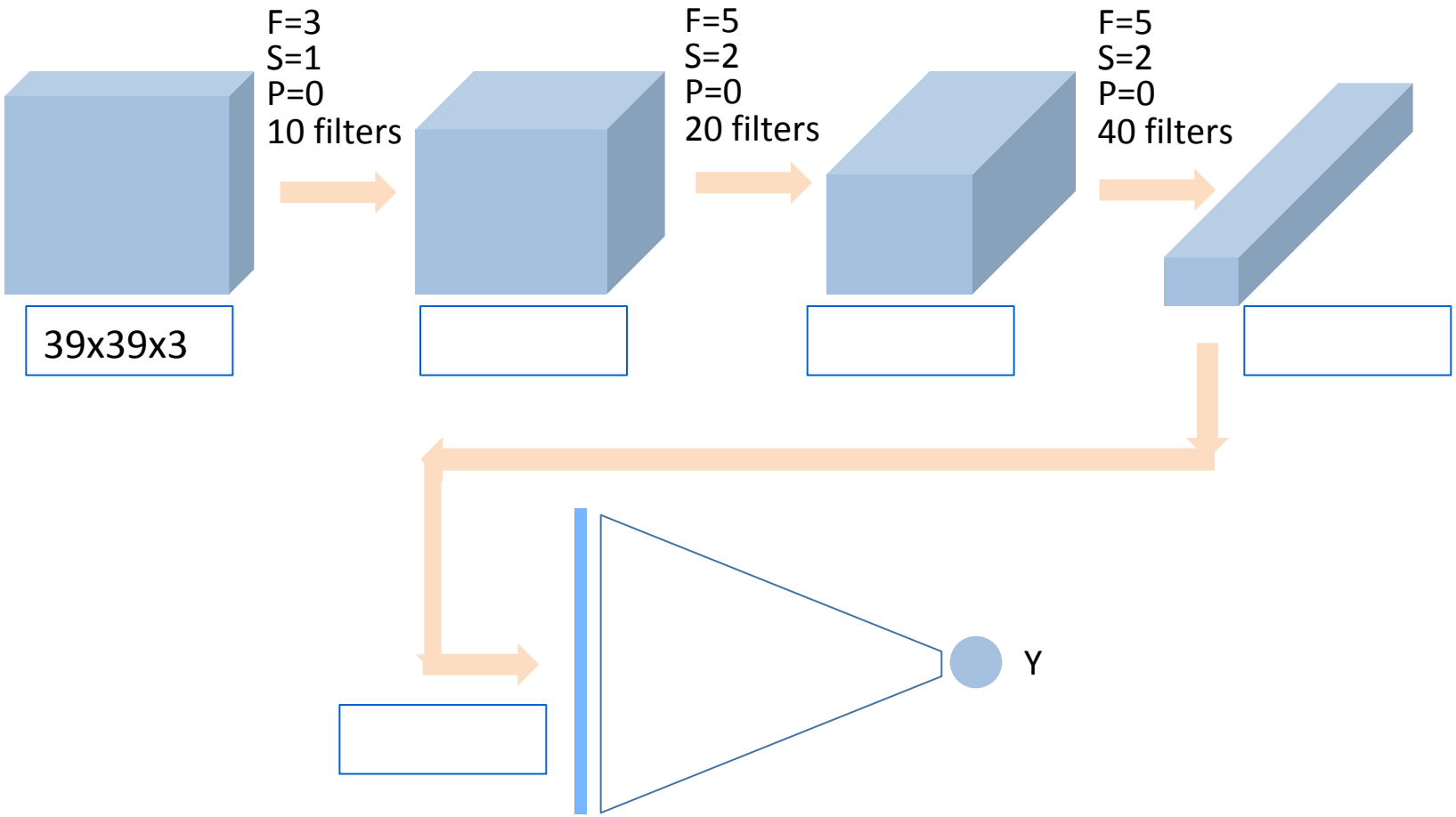
- Convolve the filters with the image
- Add a bias to the output of each filter
- Apply ReLU to the result

Forward propagation

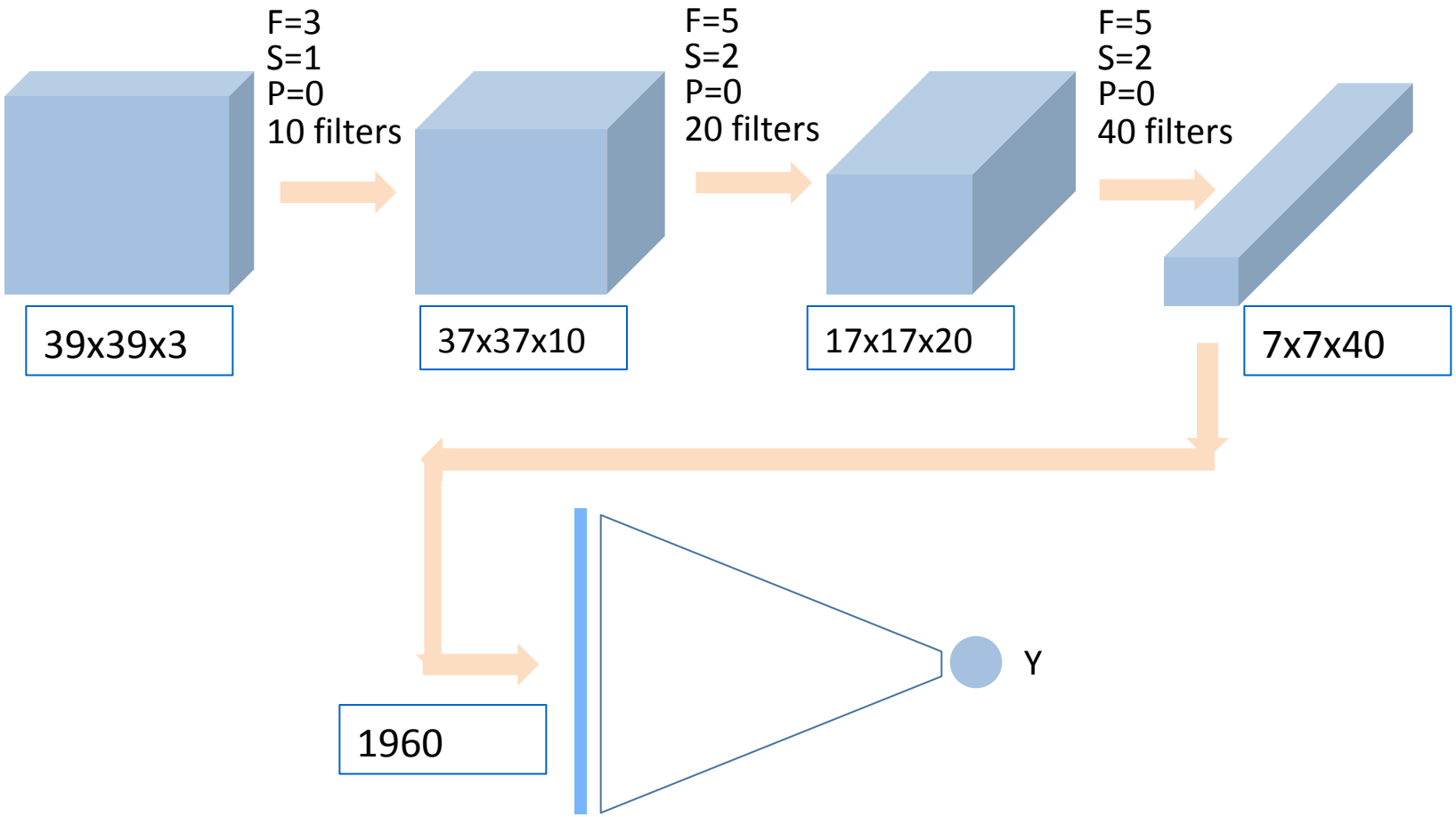
Example:

- Convolve the filters with the image:
  - Input image: **6x6x3** (a0)
  - 10 filters: **3x3x3** (W1)
  - Result: **4x4x10** (W1a0)
- Add a bias to the output of each filter
  - **b**, **10x1**: image **4x4x10** (W1a0 + b)
- Apply ReLU to the result
  - imagem **4x4x10** (A1 = RELU (W1a0 + b))
- Number of parameters: **(3x3x3x10) + 10 = 280**

Example of ConvNet



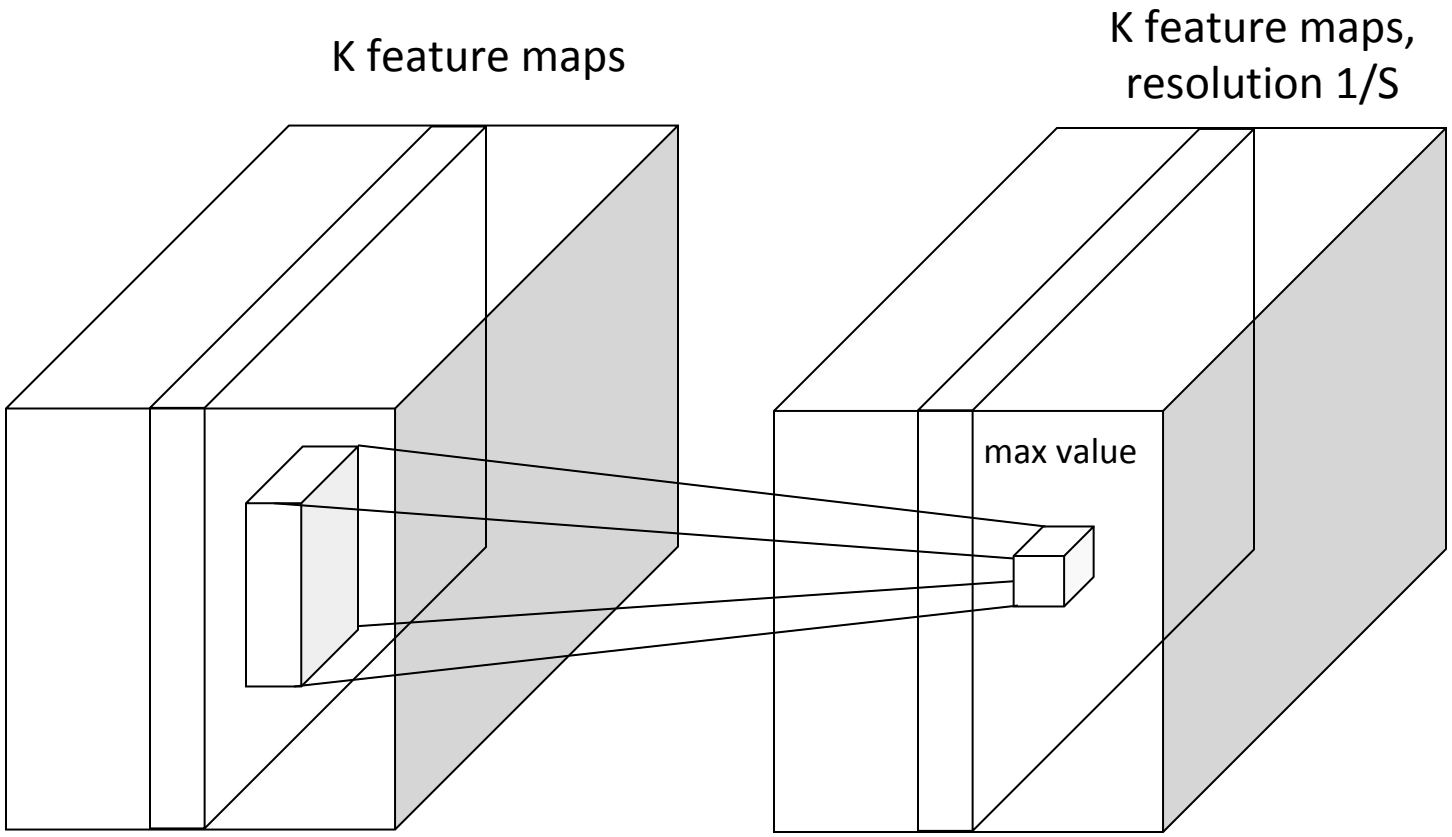
Example of ConvNet



Pooling layers

- In addition to the convolutional layers, CNNs usually use pooling layers to
  - Reduce the size of entries
  - Speed up computation
  - Make some detected features more robust
- Pooling layers have no parameteres to learn!

Pooling layers



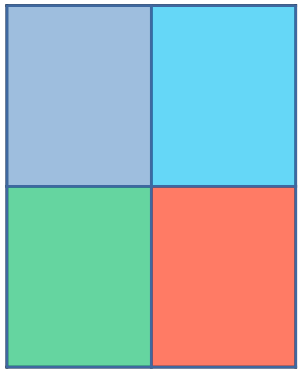
F x F pooling filter, stride S

Usually: F=2 or 3, S=2

Backward pass: gradient from next layer is passed back only to the unit with max value

Max pooling layer

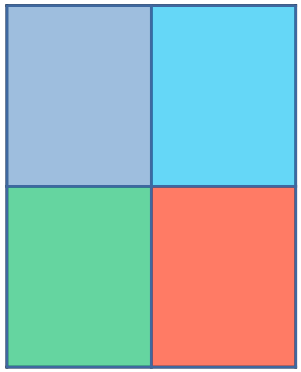
1	3	2	1
2	9	1	1
1	3	2	3
5	6	1	2



Max pooling says that if a feature is detected anywhere in the filter, keep it high to the next layer

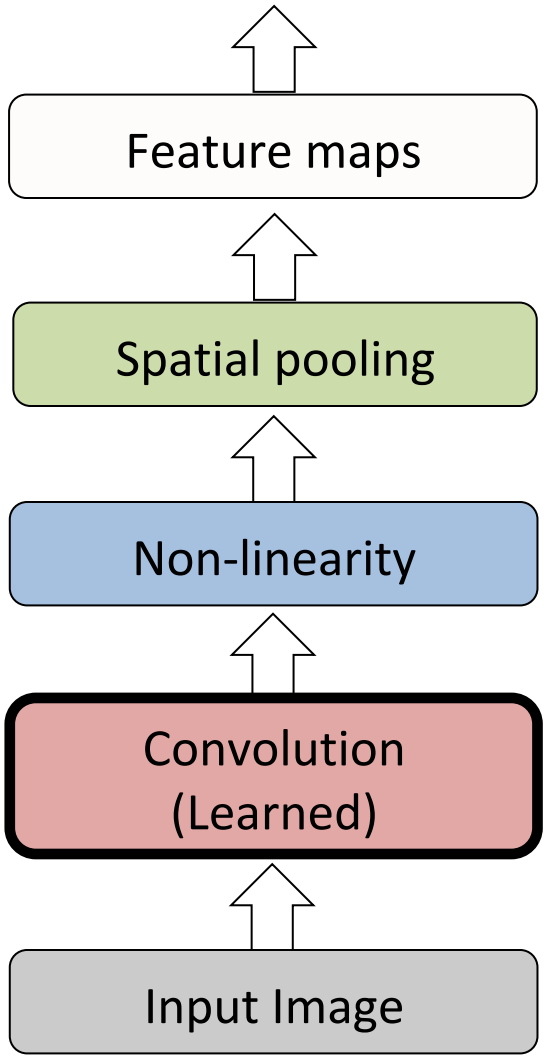
Average pooling layer

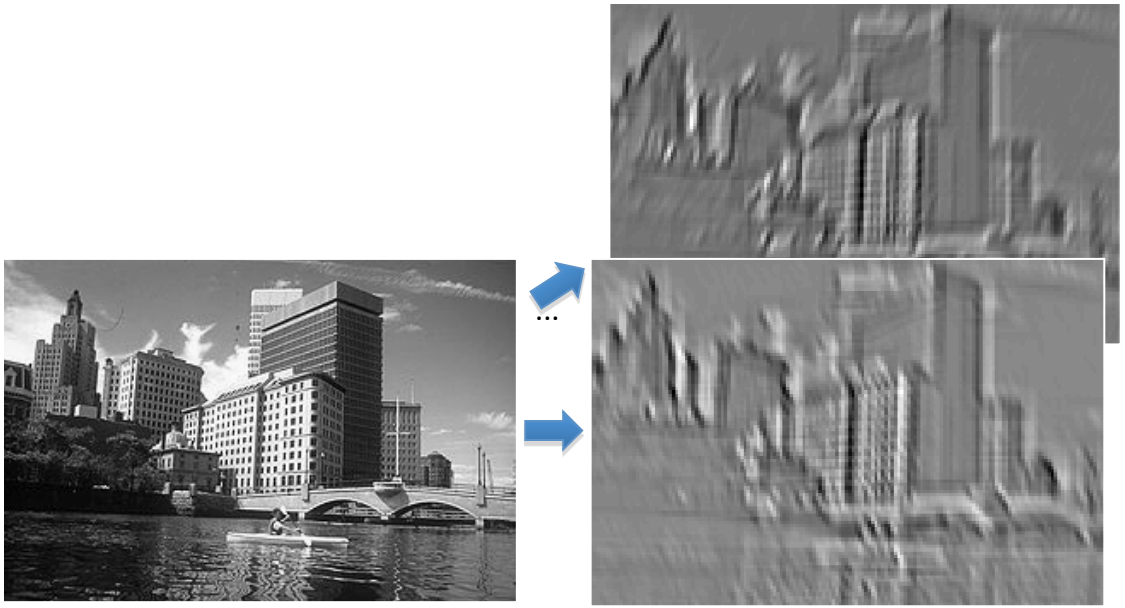
1	3	2	1
2	9	1	1
1	3	2	3
5	6	1	2



Average pooling takes the average of values and is less used than max pooling

Summary: CNN pipeline

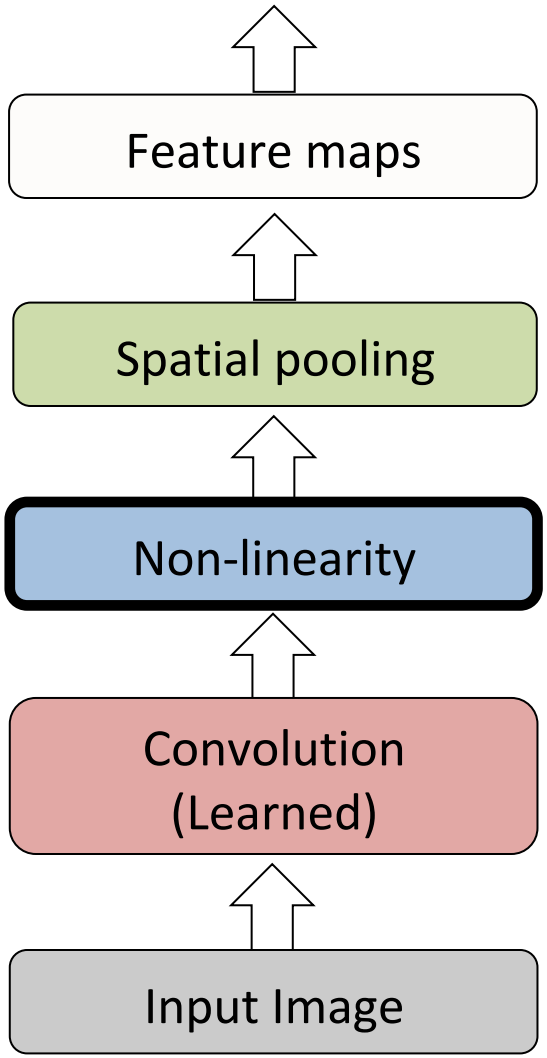




Input Feature Map

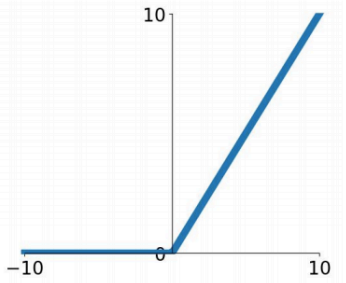
Source: R. Fergus, Y. LeCun

Summary: CNN pipeline



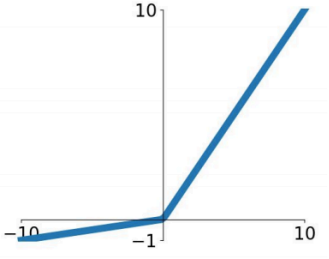
ReLU

$$\max(0, x)$$



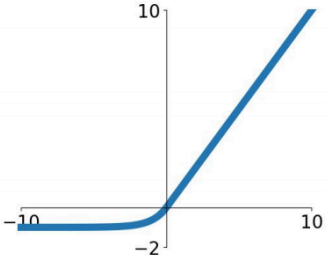
Leaky ReLU

$$\max(0.1x, x)$$



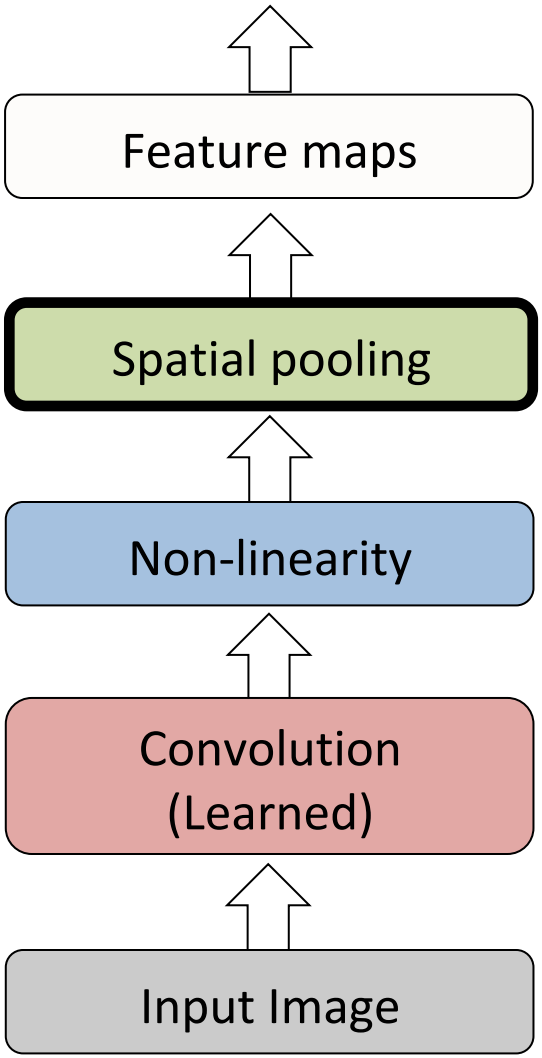
ELU

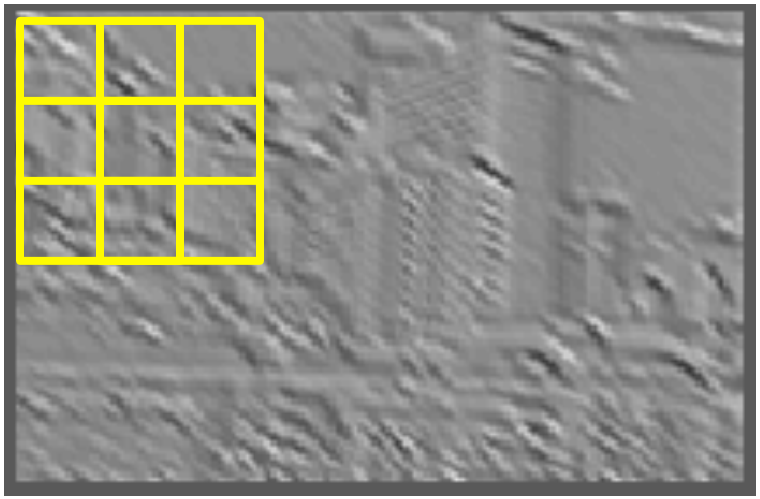
$$\begin{cases} x & x \geq 0 \\ \alpha(e^x - 1) & x < 0 \end{cases}$$



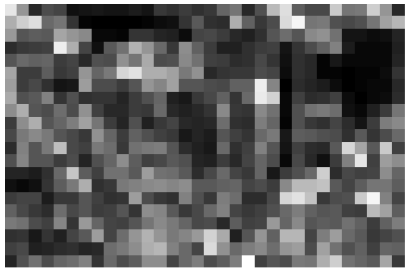
Source: R. Fergus, Y. LeCun

Summary: CNN pipeline



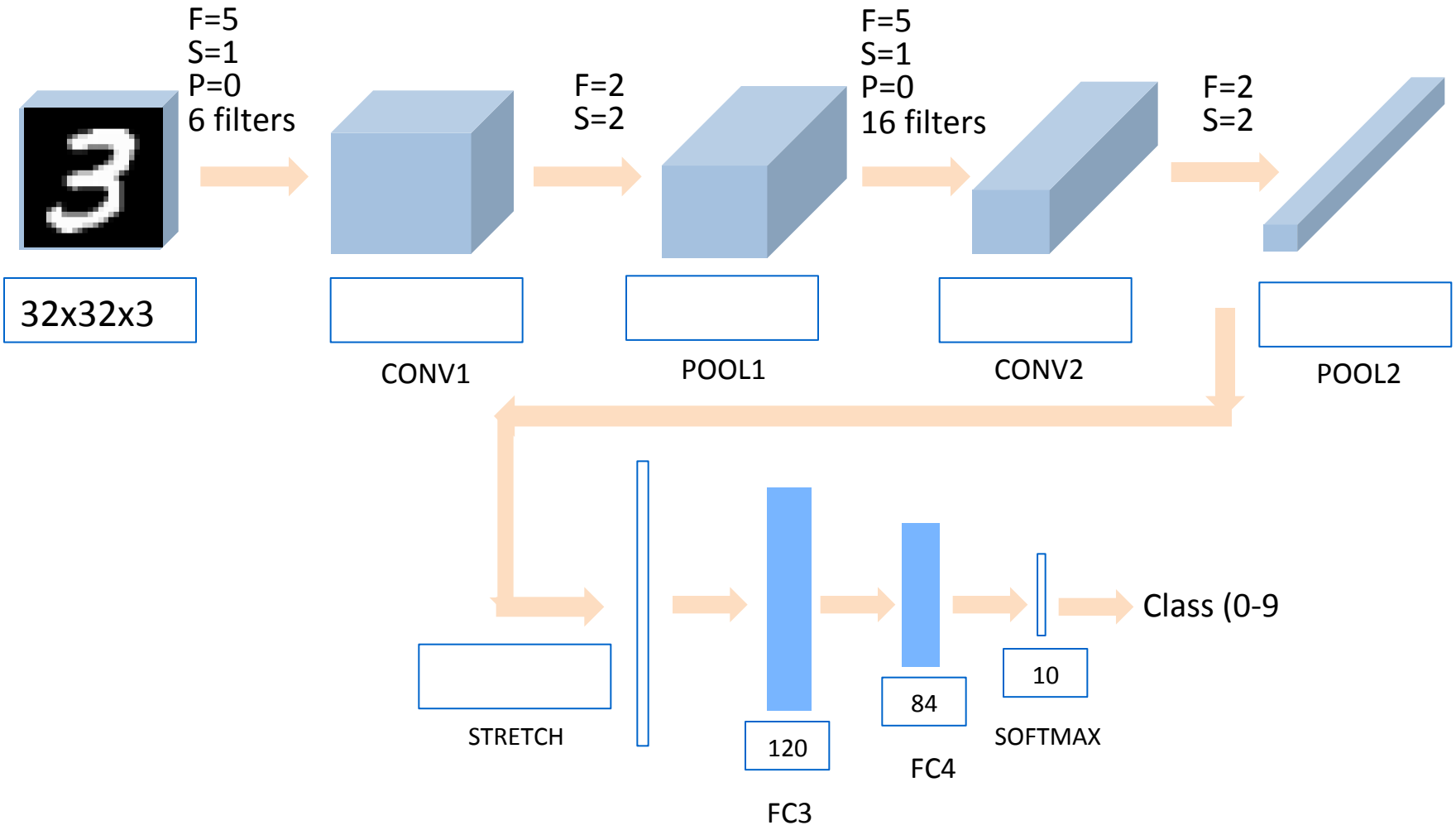


Max
(or Avg)

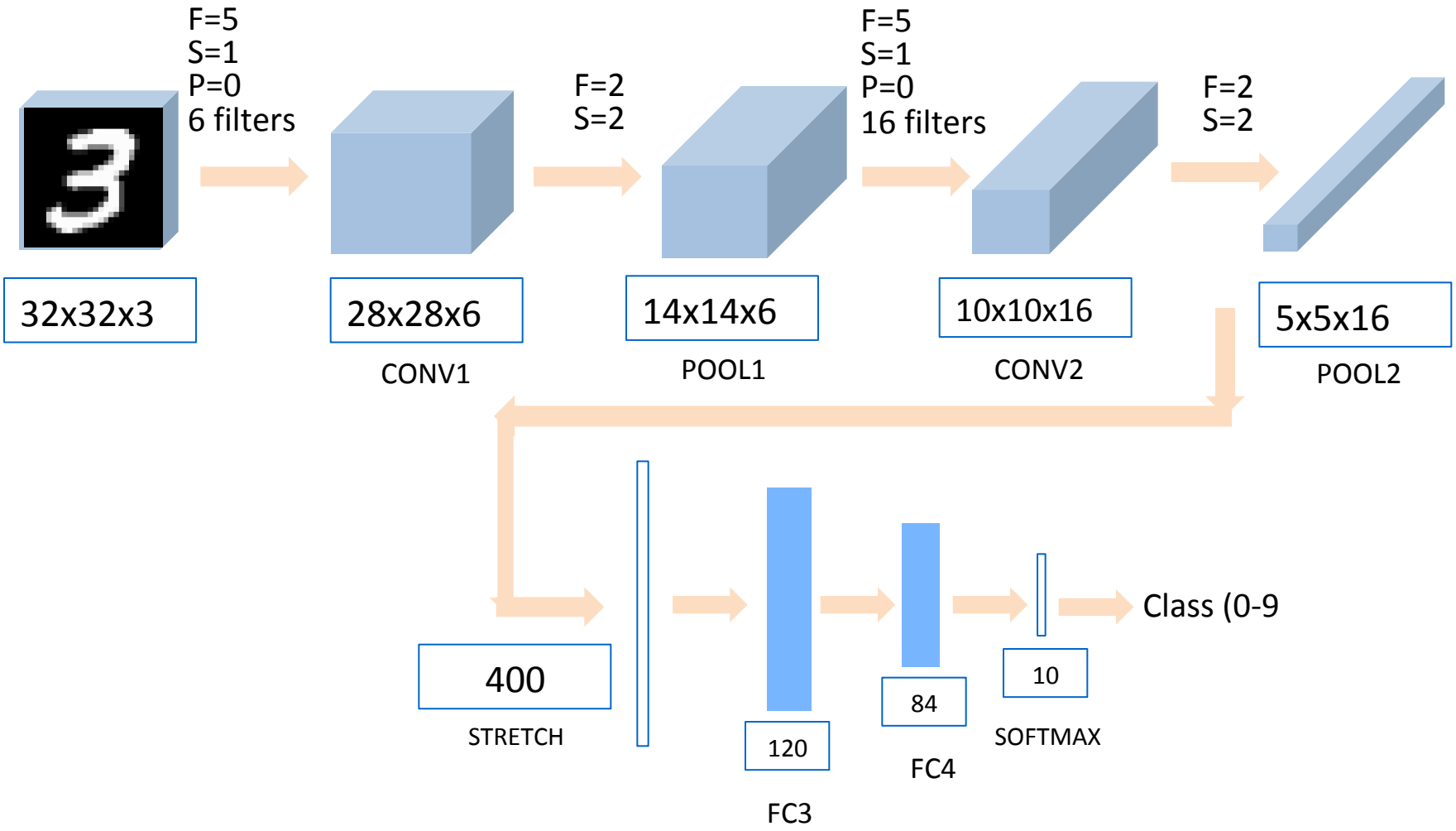


Source: R. Fergus, Y. LeCun

Example of CNN (LeNet-5)



Example of CNN (LeNet-5)



References and acknowledgements

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Reference courses include *Machine Learning* and *Deep Learning* CS230 and CS231 from Stanford University, *Deep Learning* and *Hands-on Deep Learning* from UFMG, *Deep Learning* CS498 from Un. Of Illinois.