Processamento e Análise de Imagens

The Perceptron

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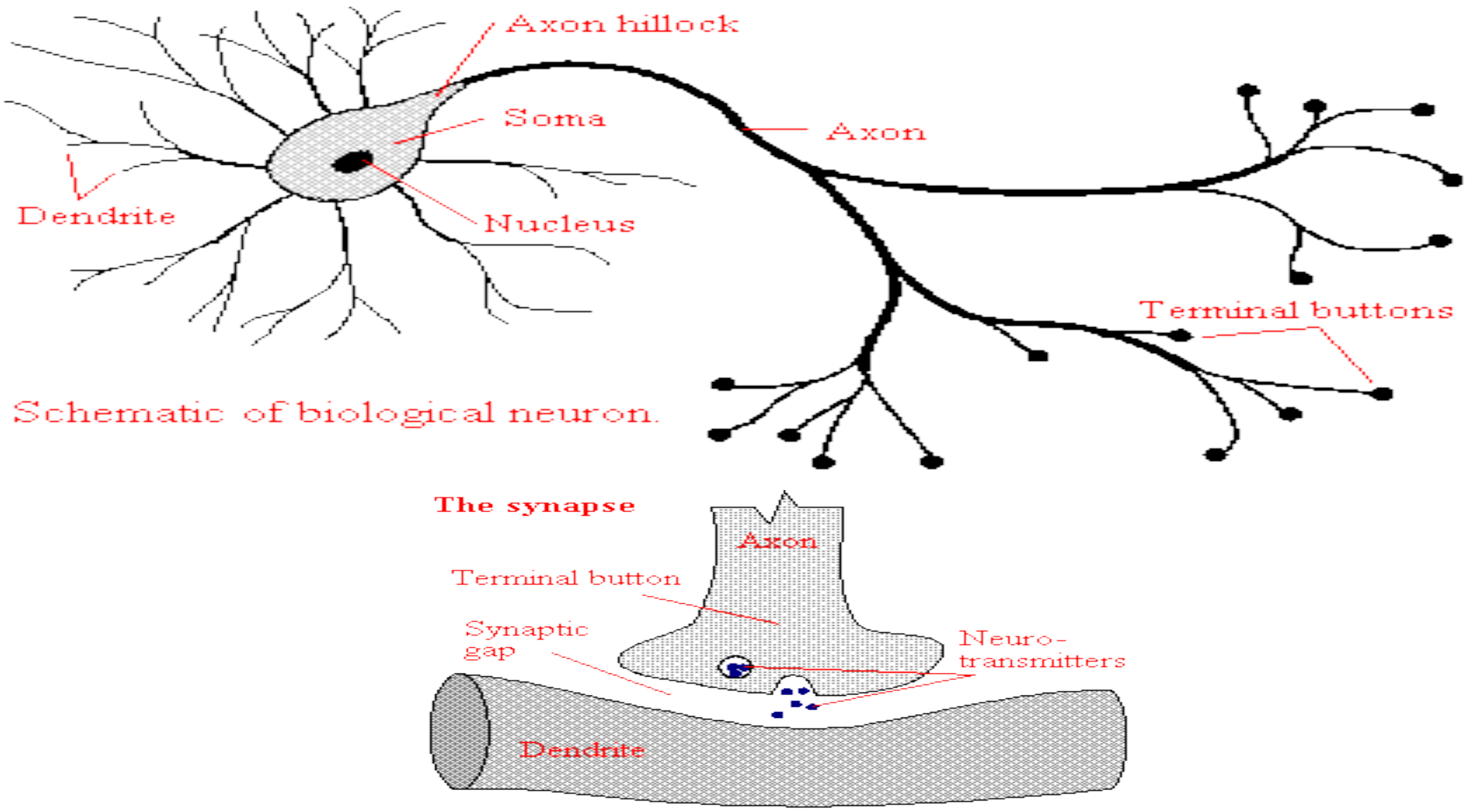
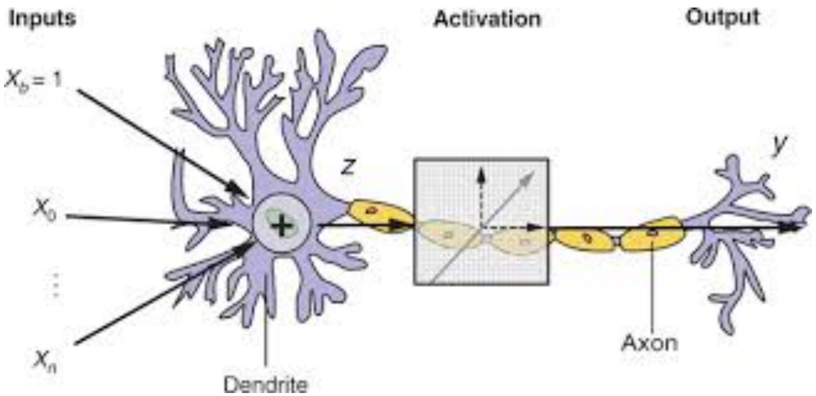
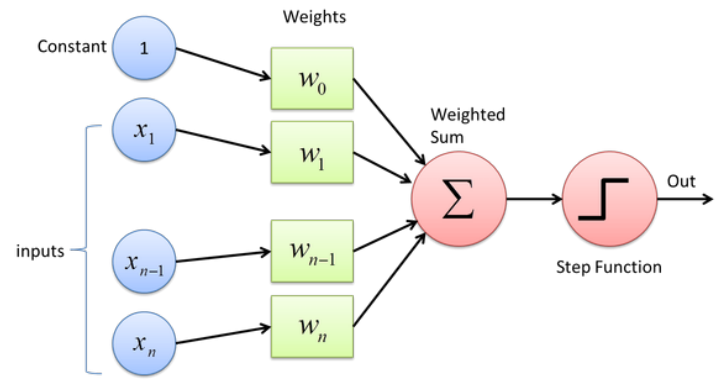
The Neuron


Axon hillock
Soma
Axon
Dendrite
Nucleus
Terminal buttons
Schematic of biological neuron.
The synapse
Axon
Terminal button
Synaptic gap
Neuro-transmitters
Dendrite

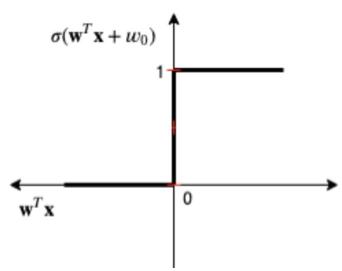
The Perceptron (Rosenblatt 1958)





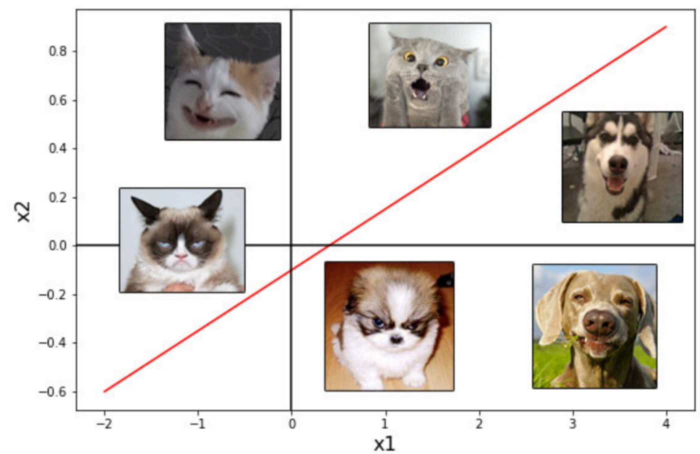
$$z = w_0 + w_1 x_1 + w_2 x_2 + \ldots + w_n x_n = b + \mathrm{w}^T \mathrm{x}$$

$$a = \hat{y} = \sigma(z)$$



The Perceptron (Rosenblatt 1958)

- The perceptron is a binary classifier: y=1 if z>0 otherwise y=0
- The perceptron defines a linear decision function



- Example with 2 variables: the OR operator
- b= -1 (bias)

$$z = -1 + 2x_1 + 3x_2$$

The Perceptron (Rosenblatt 1958)

- Solutions also can be found for the AND and NOT operator
- What about XOR?

The Perceptron (Rosenblatt 1958)

- How do we find a feasible solution?

1. Initialize W
2. Repeat until W is stable (convergence)

2.1. For each sample (x,*y*) in the dataset

2.1.1 Compute $\hat{y} = \sigma(b + w^T x)$

2.1.2. If $\hat{y} \neq y$ then adjust each weight w_i so that $\hat{y}$ *gets closer to y*

- I.e. We backpropagate the output error in order to get a better estimate of the weights
- If the classes are linearly separable, the algorithm will converge, otherwise it will not!

The Perceptron (Rosenblatt 1958)

Remarks:

- A Sum of Squared error fuction can be used to evaluate convergence (Be careful to overfitting!)
- If the classes are linearly separable, the algorithm will converge, otherwise it will not! (Duda, Hart e Stork)
- The algorithm finds ANY solution that makes it converge. The SVM is an evolution of the perceptron that finds a decision function with maximum separability (Krauth e Mezard, 1987)
- If some input variable is useless its weight should have a small magnitude
- If we increase the number of input variables, linear separabiity may be achieved (not always, of course!)

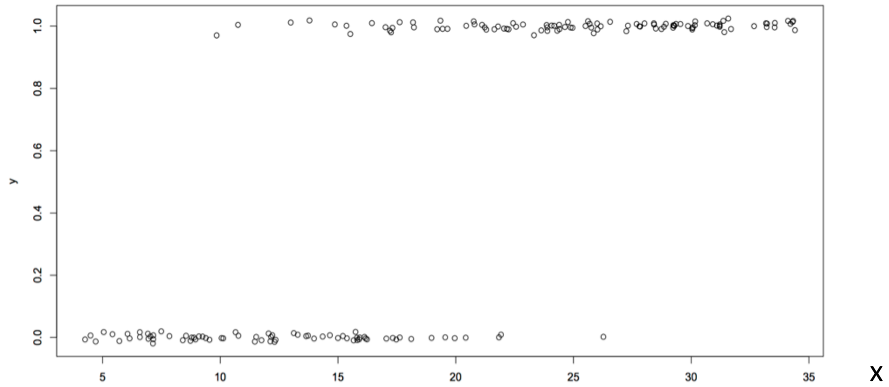
The Perceptron (Rosenblatt 1958)

Remarks:

- In 1969 Minsky and Pappert showed the weakness of the Perceptron to solve the XOR problem
- The research on Connectionism slowed down
- The perceptron model then receives 2 modifications: a different activation function and additional layers.

Logistic Regression

- In many cases, the behavior of a variable x does not change drastically from one class to the other:

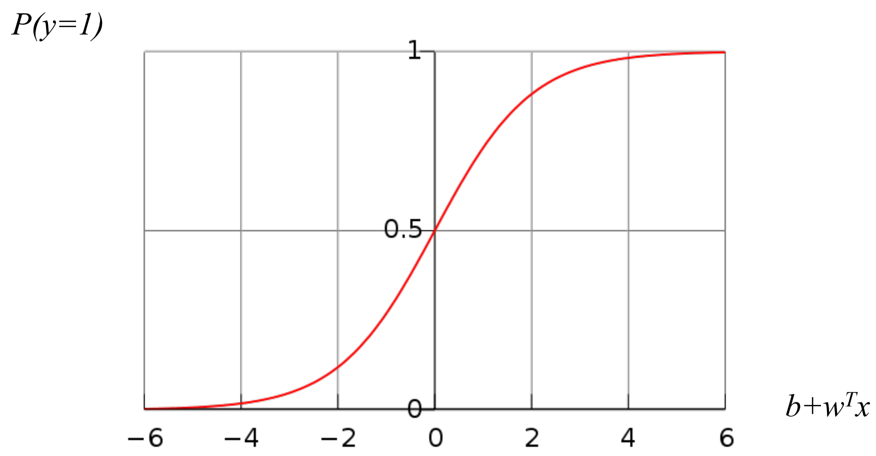


- Therefore we want the output of the classifier to give the probability of the class instead of being a 0/1
- The probability however is not properly defined by a linear function!

Logistic Regression

- We model the log of the odds ratio as a linear function, from which we get the sigmoid ativation function:

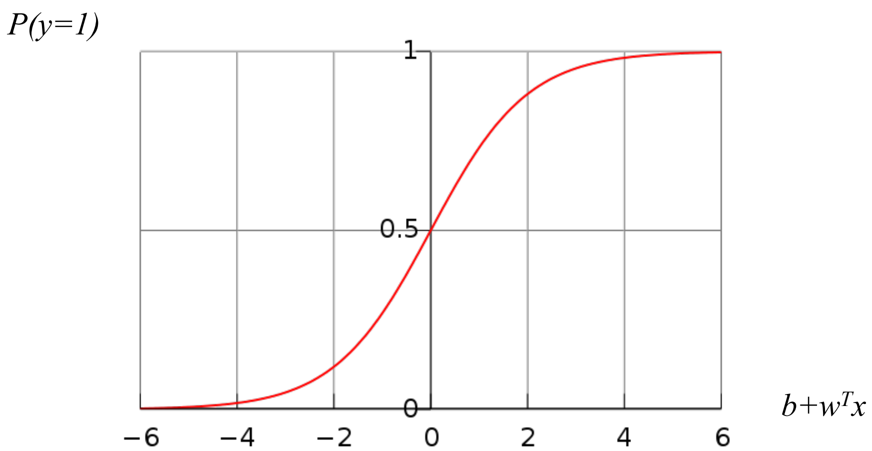
$$\ln \frac{P(y=1)}{1-P(y=1)} = b + \mathrm{w}^T \mathrm{x}$$



Logistic Regression

- We model the log of the odds ratio as a linear function, from which we get the sigmoid ativation function:

$$P(y=1) = \frac{1}{1+e^{-(b+\mathrm{w}^T\mathrm{x})}}$$



Logistic Regression

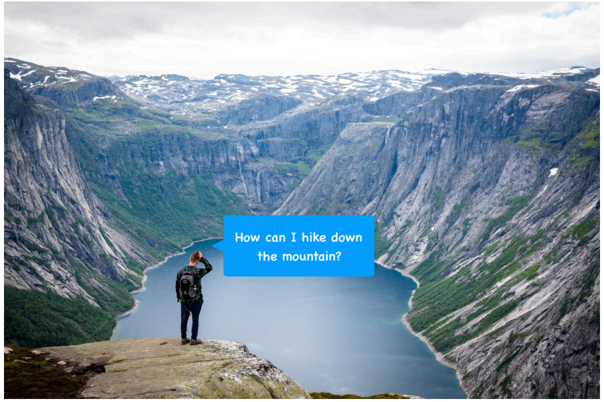
- How to adapt the perceptron's learning algorithm to the new activation function?
  - We need to properly update the weights
  - We need to compute a loss function and know when to stop iterating
- For the logistic activation function, the SSE loss function does not work well
- Since P(y=1|x)=y^ and P(y=0|x)=1-y^ ,we minimize the *cross-entropy loss function:*

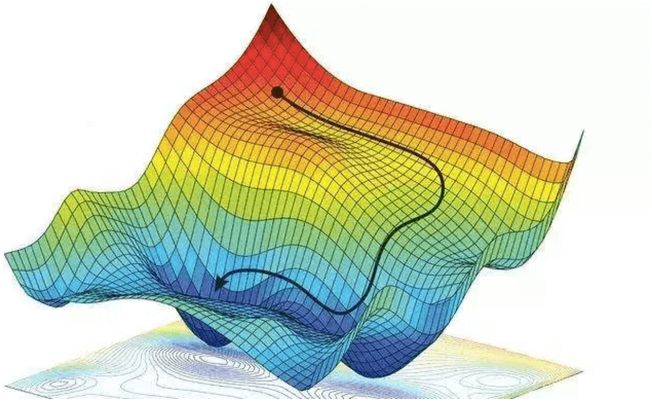
$$L(y,\hat{y}) = -\ln P(y\,|\,\mathrm{x}) = -\ln(\hat{y}^{y}(1-\hat{y})^{1-y}) = -(y\ln\hat{y} + (1-y)\ln(1-\hat{y}))$$

$$J(\mathrm{w},b) = \tfrac{1}{m}\sum_{i=1}^{m} L(y^{(i)},\hat{y}^{(i)})$$

Gradient Descent

- A man is lost in the mountains and is trying to get down to the road, which is in the lowest pat of the region.
- There is heavy fog such that visibility is extremely low, therefore, the path down the mountain is not visible.
- So the best he can do is to take the path that takes him to the lowest possible place from his current position.
- If we think that the geometry of the region gives altitude as a function of position, the gradient of it will point to the (locally) best path!





Gradient Descent

- In our case, the function that tells us how far we are from the best solution is the Loss function!
- But what is the gradient of the Loss function?

$$z = w^T x + b$$

$$\hat{y} = a = \sigma(z)$$

$$\mathcal{L}(a, y) = -(y \log(a) + (1 - y) \log(1 - a))$$

$$\boxed{\partial L(a,y) / \partial a = -\frac{y}{a} + \frac{1-y}{1-a}}$$

$$\boxed{\partial L(a,y) / \partial z = a - y}$$

$$\boxed{\partial L(a,y) / \partial w_i = x_i \partial L(a,y) / \partial z}$$

$$\boxed{\partial L(a,y) / \partial b = \partial L(a,y) / \partial z}$$

Gradient Descent

1. Initialize parameters / Define hyperparameters
2. Loop for num_iterations:
    a. Forward propagation
    b. Compute cost function
    c. Backward propagation
    d. Update parameters (using parameters, and grads from backprop)
4. Use trained parameters to predict labels

Gradient Descent

$J = 0,\ dw_1 = 0,\ dw_2 = 0,\ db = 0$

for i = 1 to m:

$$z^{(i)} = w^T x^{(i)} + b$$

$$a^{(i)} = \sigma(z^{(i)})$$

$$J \mathrel{+}= -[y^{(i)} \log a^{(i)} + (1 - y^{(i)}) \log(1 - a^{(i)})]$$

$$dz^{(i)} = a^{(i)} - y^{(i)}$$

$$dw_1 \mathrel{+}= x_1^{(i)} dz^{(i)}$$

$$dw_2 \mathrel{+}= x_2^{(i)} dz^{(i)}$$

$$db \mathrel{+}= dz^{(i)}$$

$J = J/m,\ dw_1 = dw_1/m,\ dw_2 = dw_2/m$

$db = db/m$

Complexity?

References and acknowledgements

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Reference courses include *Machine Learning* and *Deep Learning* CS230 and CS231 from Stanford University, *Deep Learning* and *Hands-on Deep Learning* from UFMG, *Deep Learning* CS498 from Un. Of Illinois.