Properties of Shortest Paths and Relaxation

- Triangle inequality (Lemma 24.10)
- Upper-bound property (Lemma 24.11)
- No-path property (Corollary 24.12)
- Convergence property (Lemma 24.14)
- Path-relaxation property (Lemma 24.15)
- Predecessor-subgraph property (Lemma 24.17)

Triangle Inequality (Lemma 24.10)

- Let $G=(V,E)$ be a weighted, directed graph with weight function $w : E \rightarrow R$ and source vertex s. Then, for all edges $(u,v) \in E$, we have $\delta(s,v) \leq \delta(s,u)+w(u,v)$

Triangle Inequality (Lemma 24.10)

- Let $G=(V,E)$ be a weighted, directed graph with weight function $w : E \rightarrow R$ and source vertex s. Then, for all edges $(u,v) \in E$, we have $\delta(s,v) \leq \delta(s,u)+w(u,v)$
- Proof:
  - Suppose that there is a shortest path p from source s to v. Then p has no more weight than any other path from s to v. Specifically, path p has no more weight than the particular path that takes a shortest path from source s to vertex u and then takes edge (u,v).
  - Otherwise, (there is no shortest path from s to v). Exercise 24.5-3.

Upper-bound Property (Lemma 24.11)

- Let
  - $G=(V,E)$ be a weighted, directed graph with weight function $w : E \to R$
  - $s \in V$ the source vertex
- The graph G is initialized by INITILIZE-SINGLE-SOURCE(G,s).
- Then, we have $v.d \geq \delta(s,v)$ for all $v \in V$. This invariant is maintained over any sequence of relaxation steps on the edges of G. Moreover, once $v.d$ achieves its lower bound $\delta(s,v)$, it never changes.

Upper-bound Property (Lemma 24.11)

- Proof by induction over the number of relaxation steps
  - Basis case (0 relaxation)
    - $v.d = \infty \geq \delta(s,v)$ for all vertices $v \in V-\{s\}$
    - $s.d = 0 = \delta(s,s)$

Upper-bound Property (Lemma 24.11)

- Proof by induction over the number of relaxation steps
  - Induction step
    - Induction hypothesis: $v.d \geq \delta(s,v)$ for all $v \in V$ prior to the relaxation of an edge (u,v)
    - The only d value that may change is v.d. If it changes, we have
      $v.d = u.d + w(u,v) \geq \delta(s,u) + w(u,v) \geq \delta(s,v)$
  - To see that the value of v.d never changes once $v.d = \delta(s,v)$
    - v.d cannot decrease because $v.d \geq \delta(s,v)$
    - v.d cannot increase because relaxation steps do not increase d values

No-path Property (Corollary 24.12)

- Suppose that in a weighted, directed graph $G=(V,E)$ with weight function $w:E \rightarrow R$, no path connects a source s to a given vertex v.
- Then, after the graph is initialized by INITIALIZE-SINGLE-SOURCE(G,s), we have $v.d = \delta(s,v) = \infty$ and this equality is maintained as an invariant over any sequence of relaxation steps on the edges of G.
- Proof: By the upper-bound property, we always have $\infty = \delta(s,v) \leq v.d$ and thus we have $v.d = \infty = \delta(s,v)$.

Lemma 24.13

- Let $G=(V,E)$ be a weighted, directed graph with weight function $w:E \to R$, and let $(u,v) \in E$.
- Then, immediately after relaxing edge (u,v) by executing RELAX(u,v,w), we have $v.d \leq u.d+w(u,v)$.
- Proof: if, just prior to relaxing edge (u,v), we have $v.d>u.d+w(u,v)$, then before $v.d=u.d+w(u,v)$ afterward. If, instead, $v.d \leq u.d+w(u,v)$ just before the relaxation, then neither $u.d$ nor $v.d$ changes, and so $v.d \leq u.d+w(u,v)$ afterward.

Convergence Property (Lemma 24.14)

- Let $G=(V,E)$ be a weighted, directed graph with weight function $w : E \rightarrow R$.
- Let $s \in V$ the source vertex.
- Let $s \rightsquigarrow u \rightarrow v$ be a shortest path in G for some vertices $u,v \in V$.
- Let the graph is initialized by INITILIZE-SINGLE-SOURCE(G,s) and then a sequence of relaxation steps that includes the call RELAX(u,v,w) is executed on the edge of G.
- If $u.d = \delta(s,u)$ at any time prior to the call, then we have $v.d = \delta(s,v)$ at all times after the call.

Convergence Property (Proof)

- By the upper-bound property, if $u.d = \delta(s,u)$ at some point prior to relaxing edge (u,v), this equality holds thereafter.
- In particular, after relaxing edge (u,v), we have
  $v.d \le u.d + w(u,v) = \delta(s,u) + w(u,v) = \delta(s,v)$.
- By the upper-bound property, $v.d \ge \delta(s,v)$ from which we conclude that $v.d = \delta(s,v)$.
- Thus, this equality is maintained thereafter.

Shortest-paths Properties

- Triangle Inequality
  - For any edge $(u,v) \in E$, we have $\delta(s,v) \leq \delta(s,u)+w(u,v)$.
- Upper-bound property
  - We always have $v.d \geq \delta(s,v)$ for all $v \in V$, and once $v.d$ achieves the value $\delta(s,v)$, it never changes.
- No-path property
  - If there is no path from s to v, then we always have $v.d = \infty = \delta(s,v)$.
- Convergence property
  - If $s \leadsto u \rightarrow v$ be a shortest path in G for some vertices $u,v \in V$, and if $u.d = \delta(s,u)$ at any time prior to relaxing edge (u,v), then $v.d = \delta(s,v)$ at all times afterward.

Path-relaxation Property (Lemma 24.15)

- If $p=<v_0,v_1,\ldots,v_k>$ is a shortest path from $s=v_0$ to v_k, and the edges of p are relaxed in the order $(v_0,v_1),(v_1,v_2),\ldots,(v_{k-1},v_k)$, then $v_k.d=\delta(s,v_k)$.
- This property holds regardless of any other relaxation steps that occur, even if they are intermixed with relaxations of the edges of p.

Path-relaxation Property (Proof)

- We show by induction that after i-th edge of path $p=\langle v_0,v_1,\ldots,v_k\rangle$ is relaxed, we have $v_i.d=\delta(s,v_i)$.
- For the basis, i=0, and before any edges of p have been relaxed, we have from the initialization that $v_0.d=s.d=0=\delta(s,s)$. By the <u>upper-bound property</u>, the value of s.d never changes after initialization.
- For the induction step, we assume that $v_{i-1}.d = \delta(s,v_{i-1})$, and we examine the relaxation of edge (v_{i-1},v_i). By the convergence property, after relaxation, we have $v_i.d=\delta(s,v_i)$, and this equality is maintained at all times thereafter.

Predecessor-subgraph Property (Lemma 24.17)

- Let G=(V,E) be a weighted, directed graph with weight function w:E→R, let s ∈ V be a source vertex, and assume that G contains no negative-weight cycles that are reachable from s.
- Let us call INITIALIZE-SINGLE-SOURCE(G, s) and then execute any sequence of relaxation steps on edges of G that produces v.d=δ(s,v) for all v ∈ V.
- Then, the predecessor subgraph G is a shortest-path tree rooted at s.
- Proof is omitted.

Bellman-Ford Algorithm

Bellman-Ford Algorithm

```
BELLMAN-FORD(G, w, s)
    INITIALIZE-SINGLE-SOURCE(G,s)
    for i=1 to |G.V|-1
        for each edge (u,v) ∈ G.E
            RELAX(u, v, w)
    for each edge (u,v) ∈ G.E
        if  v.d > u.d + w(u,v)
            return FALSE
    return TRUE
```

Bellman-Ford Algorithm

BELLMAN-FORD(G, w, s)

1. INITIALIZE-SINGLE-SOURCE(G,s)
2. **for** i=1 **to** |G.V|-1
3.   **for** each edge (u,v) $\in$ G.E
4.   RELAX(u, v, w)
5. **for** each edge (u,v) $\in$ G.E
6.   **if** v.d > u.d + w(u,v)
7.   **return** FALSE
8. **return** TRUE

Relaxation:
Make |V|-1 passes,
relaxing each edge

Test whether negative
-weight cycle exists

- **Edge weights may be negative**

Bellman-Ford Algorithm

```
BELLMAN-FORD(G, w, s)
    INITIALIZE-SINGLE-SOURCE(G,s)      ← O(|V|)
    for i=1 to |G.V|-1                 ← O(|V||E|)
        for each edge (u,v) ∈ G.E
            RELAX(u, v, w)
    for each edge (u,v) ∈ G.E          ← O(|E|)
        if  v.d > u.d + w(u,v)
            return FALSE
    return TRUE
```

- **Running time O(|V||E|)**

Lemma 24.2

- Let $G=(V,E)$ be a weighted, directed graph with source s and weight function $w:E \rightarrow R$.
- Assume that G contains no negative-weight cycles that are reachable from s.
- Then, after the $|V|-1$ iterations of the **for** loop of lines 2 - 4 of BELLMAN-FORD, we have $v.d=\delta(s,v)$ for all vertices that are reachable from s.

Lemma 24.2 (Proof)

- We prove the lemma by appealing to the path-relaxation property.
- Consider any vertex that is reachable from s, and let $p=\langle v_0,v_1,\ldots,v_k \rangle$, where $v_0 = s$ and $v_k = v$, be any shortest path from s to v.
- Because shortest paths are simple, p has at most $|V|-1$ edges, and so $k \leq |V|-1$. Each of the $|V|-1$ iterations of the **for** loop of lines 2–4 relaxes all $|E|$ edges.
- The edge (v_{i-1}, v_i) is one among the edges relaxed in the i-th iteration, for $i = 1, 2, \ldots, k$.
- By the path-relaxation property, therefore, $v.d=v_k.d=\delta(s,v_k)=\delta(s,v)$.

Corollary 24.3

- Let $G=(V,E)$ be a weighted, directed graph with source vertex s and weight function $w:E \rightarrow R$.
- Assume that G contains no negative-weight cycles that are reachable from s.
- Then, for each vertex $v \in V$, there is a path from s to v if and only if BELLMAN-FORD terminates with $v.d < \infty$ when it is run on G.
- The proof is left as Exercise 24.1-2.

Correctness of the Bellman-Ford Algorithm

- Let BELLMAN-FORD be run on a weighted, directed graph $G=(V,E)$ with source s and weight function $w:E\rightarrow R$.
- If G contains no negative-weight cycles that are reachable from s, then the algorithm returns TRUE, we have $v.d=\delta(s,v)$ for all vertices $v \in V$, and the predecessor subgraph G is a shortest-paths tree rooted at s.
- It G does contain a negative-weight cycle reachable from s, then the algorithm returns FALSE

Correctness of the Bellman-Ford Algorithm (Proof)

- Suppose that G contains no negative-weight cycles that are reachable from s.
- We first prove the claim that at termination, $v.d = \delta(s,v)$ for all vertices $v \in V$.
  - If vertex v is reachable from s, then Lemma 24.2 proves this.
  - If v is not reachable from s, then the claim follows from the no-path property.
  - Thus, the claim is proven.
- The predecessor-subgraph property, along with the claim, implies that $G.\pi$ is a shortest-paths tree.
- Now we use the claim to show that BELLMAN-FORD returns TRUE.
- At termination, we have for all edges $(u,v) \in E$,
  - $v.d = \delta(s,v)$

    $\leq \delta(s,u) + w(u,v)$ (by the triangle inequality)

    $= u.d + w(u,v)$
- and so none of the tests in line 6 causes BELLMAN-FORD to return FALSE.
- Thus, it returns TRUE.

Correctness of the Bellman-Ford Algorithm (Proof)

- Suppose G contains a negative-weight cycle that is reachable from s.
- Let this cycle c=<v_0,v_1,…,v_k>, where v_0=v_k and $\boldsymbol{\sum_{i=1}^{k} w(v_{i-1}, v_i) < 0}$ **(24.1)**
- Assume Bellman-Fold algorithm returns TRUE.

  $v_i.d \le v_{i-1}.d + w(v_{i-1}, v_i)$ for $i = 1,2,\ldots,k$
- Summing the inequalities around cycle c

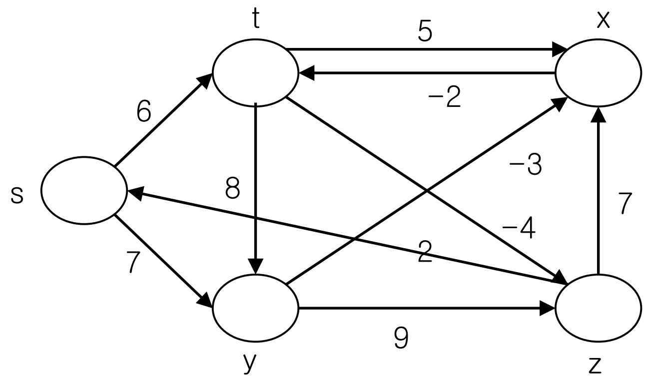
$$\sum_{i=1}^{k} v_i.d \le \sum_{i=1}^{k} \left(v_{i-1}.d + w(v_{i-1}, v_i)\right)$$

$$= \sum_{i=1}^{k} v_{i-1}.d + \sum_{i=1}^{k} w(v_{i-1}, v_i)$$

- Since v_0=v_k, each vertex in c appears exactly once in each of the summations, $\sum_{i=1}^{k} v_i.d = \sum_{i=1}^{k} v_{i-1}.d$.
- Moreover, by Corollary 24.3, v_i.d is finite for i=1,2,…,k.
- Thus, $0 \le \sum_{i=1}^{k} w(v_{i-1}, v_i)$ which contradicts inequality **(24.1)**

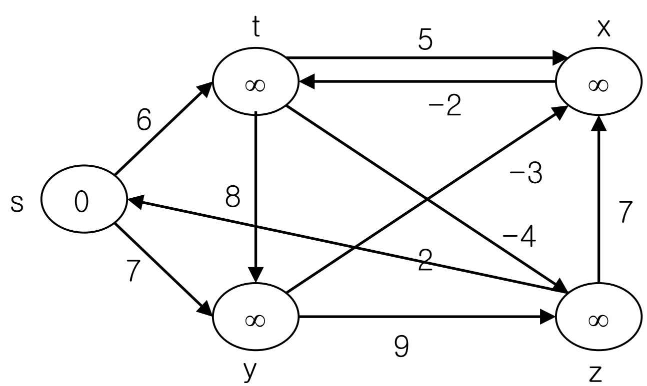
Bellman-Ford Algorithm

```
BELLMAN-FORD(G, w, s)
INITIALIZE-SINGLE-SOURCE(G,s)
for i=1 to |G.V|-1
    for each edge (u,v) ∈ G.E
        RELAX(u, v, w)
for each edge (u,v) ∈ G.E
    if  v.d > u.d + w(u,v)
        return FALSE
return TRUE
```



Bellman-Ford Algorithm

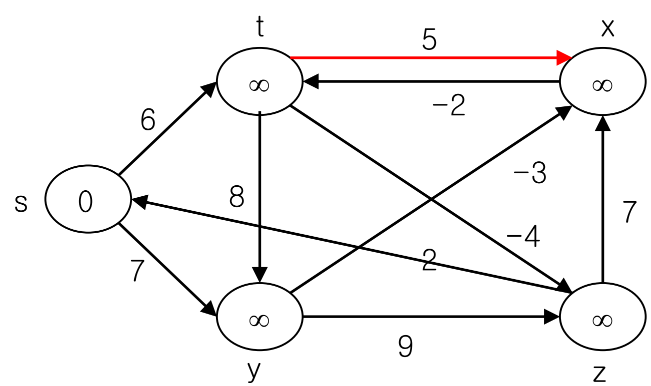
```
BELLMAN-FORD(G, w, s)
    INITIALIZE-SINGLE-SOURCE(G,s)
    for i=1 to |G.V|-1
        for each edge (u,v) ∈ G.E
            RELAX(u, v, w)
    for each edge (u,v) ∈ G.E
        if  v.d > u.d + w(u,v)
            return FALSE
    return TRUE
```



Bellman-Ford Algorithm

```
BELLMAN-FORD(G, w, s)
    INITIALIZE-SINGLE-SOURCE(G,s)
    for i=1 to |G.V|-1
        for each edge (u,v) ∈ G.E
            RELAX(u, v, w)
    for each edge (u,v) ∈ G.E
        if  v.d > u.d + w(u,v)
            return FALSE
    return TRUE
```

i = 1

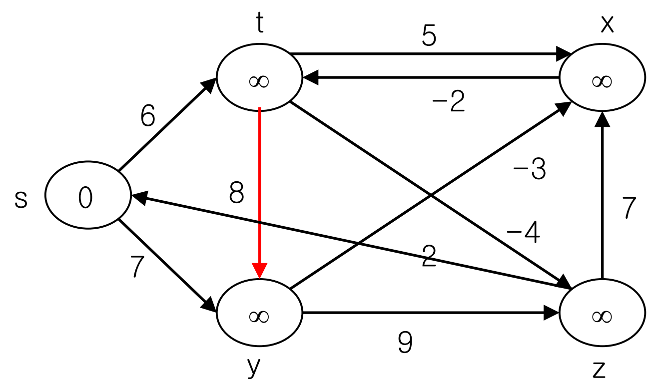


G.E	(t,x)	(t,y)	(t,z)	(x,t)	(y,x)	(y,z)	(z,x)	(z,s)	(s,t)	(s,y)

Bellman-Ford Algorithm

BELLMAN-FORD(G, w, s)
1. INITIALIZE-SINGLE-SOURCE(G,s)
2. **for** i=1 **to** |G.V|-1
3. **for** each edge (u,v) $\in$ G.E
4. RELAX(u, v, w)
5. **for** each edge (u,v) $\in$ G.E
6. **if** v.d > u.d + w(u,v)
7. **return** FALSE
8. **return** TRUE

i = 1



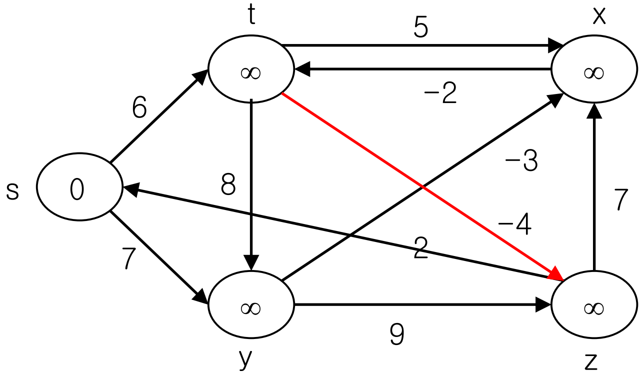
G.E	(t,x)	(t,y)	(t,z)	(x,t)	(y,x)	(y,z)	(z,x)	(z,s)	(s,t)	(s,y)

Bellman-Ford Algorithm

BELLMAN-FORD(G, w, s)

1. INITIALIZE-SINGLE-SOURCE(G,s)
2. **for** i=1 **to** |G.V|-1
3. **for** each edge (u,v) $\in$ G.E
4. RELAX(u, v, w)
5. **for** each edge (u,v) $\in$ G.E
6. **if** v.d > u.d + w(u,v)
7. **return** FALSE
8. **return** TRUE

i = 1

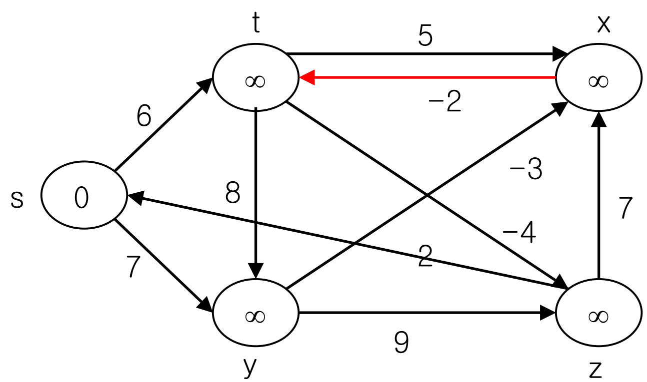


G.E	(t,x)	(t,y)	(t,z)	(x,t)	(y,x)	(y,z)	(z,x)	(z,s)	(s,t)	(s,y)

Bellman-Ford Algorithm

```
BELLMAN-FORD(G, w, s)
    INITIALIZE-SINGLE-SOURCE(G,s)
    for i=1 to |G.V|-1
        for each edge (u,v) ∈ G.E
            RELAX(u, v, w)
    for each edge (u,v) ∈ G.E
        if  v.d > u.d + w(u,v)
            return FALSE
    return TRUE
```

i = 1

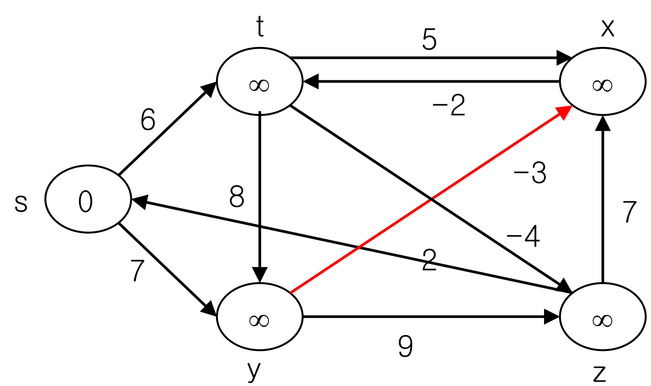


G.E	(t,x)	(t,y)	(t,z)	(x,t)	(y,x)	(y,z)	(z,x)	(z,s)	(s,t)	(s,y)

Bellman-Ford Algorithm

```
BELLMAN-FORD(G, w, s)
    INITIALIZE-SINGLE-SOURCE(G,s)
    for i=1 to |G.V|-1
        for each edge (u,v) ∈ G.E
            RELAX(u, v, w)
    for each edge (u,v) ∈ G.E
        if  v.d > u.d + w(u,v)
            return FALSE
    return TRUE
```

i = 1



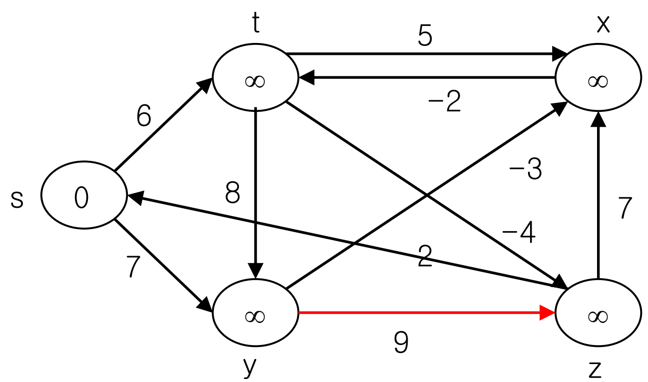
G.E	(t,x)	(t,y)	(t,z)	(x,t)	(y,x)	(y,z)	(z,x)	(z,s)	(s,t)	(s,y)

Bellman-Ford Algorithm

BELLMAN-FORD(G, w, s)

1. INITIALIZE-SINGLE-SOURCE(G,s)
2. **for** i=1 **to** |G.V|-1
3. **for** each edge (u,v) ∈ G.E
4. RELAX(u, v, w)
5. **for** each edge (u,v) ∈ G.E
6. **if** v.d > u.d + w(u,v)
7. **return** FALSE
8. **return** TRUE

i = 1

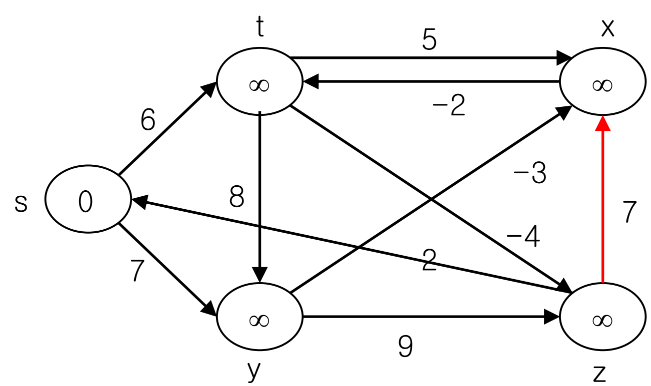


G.E	(t,x)	(t,y)	(t,z)	(x,t)	(y,x)	(y,z)	(z,x)	(z,s)	(s,t)	(s,y)

Bellman-Ford Algorithm

```
BELLMAN-FORD(G, w, s)
    INITIALIZE-SINGLE-SOURCE(G,s)
    for i=1 to |G.V|-1
        for each edge (u,v) ∈ G.E
            RELAX(u, v, w)
    for each edge (u,v) ∈ G.E
        if  v.d > u.d + w(u,v)
            return FALSE
    return TRUE
```

i = 1



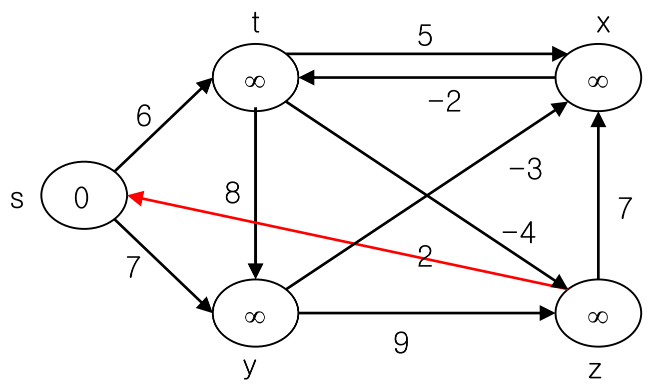
G.E	(t,x)	(t,y)	(t,z)	(x,t)	(y,x)	(y,z)	(z,x)	(z,s)	(s,t)	(s,y)

Bellman-Ford Algorithm

BELLMAN-FORD(G, w, s)

1. INITIALIZE-SINGLE-SOURCE(G,s)
2. **for** i=1 **to** |G.V|-1
3.   **for** each edge (u,v) ∈ G.E
4.   RELAX(u, v, w)
5. **for** each edge (u,v) ∈ G.E
6.   **if** v.d > u.d + w(u,v)
7.   **return** FALSE
8. **return** TRUE

i = 1

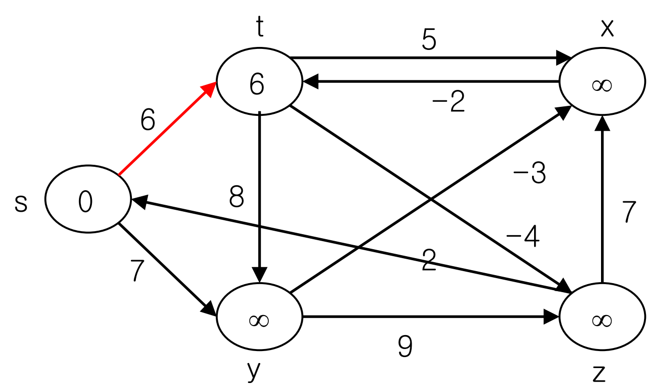


G.E	(t,x)	(t,y)	(t,z)	(x,t)	(y,x)	(y,z)	(z,x)	(z,s)	(s,t)	(s,y)

Bellman-Ford Algorithm

```
BELLMAN-FORD(G, w, s)
    INITIALIZE-SINGLE-SOURCE(G,s)
    for i=1 to |G.V|-1
        for each edge (u,v) ∈ G.E
            RELAX(u, v, w)
    for each edge (u,v) ∈ G.E
        if  v.d > u.d + w(u,v)
            return FALSE
    return TRUE
```

i = 1



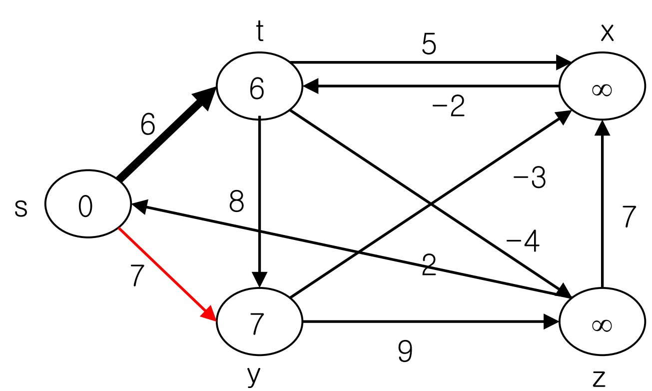
G.E	(t,x)	(t,y)	(t,z)	(x,t)	(y,x)	(y,z)	(z,x)	(z,s)	(s,t)	(s,y)

Bellman-Ford Algorithm

BELLMAN-FORD(G, w, s)

1. INITIALIZE-SINGLE-SOURCE(G,s)
2. **for** i=1 **to** |G.V|-1
3.   **for** each edge (u,v) $\in$ G.E
4.   RELAX(u, v, w)
5. **for** each edge (u,v) $\in$ G.E
6.   **if** v.d > u.d + w(u,v)
7.   **return** FALSE
8. **return** TRUE

i = 1



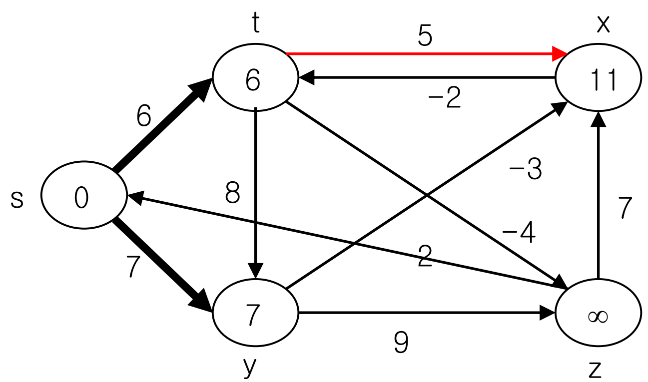
G.E	(t,x)	(t,y)	(t,z)	(x,t)	(y,x)	(y,z)	(z,x)	(z,s)	(s,t)	(s,y)

Bellman-Ford Algorithm

BELLMAN-FORD(G, w, s)

1. INITIALIZE-SINGLE-SOURCE(G,s)
2. **for** i=1 **to** |G.V|-1
3.   **for** each edge (u,v) $\in$ G.E
4.   RELAX(u, v, w)
5. **for** each edge (u,v) $\in$ G.E
6.   **if** v.d > u.d + w(u,v)
7.   **return** FALSE
8. **return** TRUE

i = 2

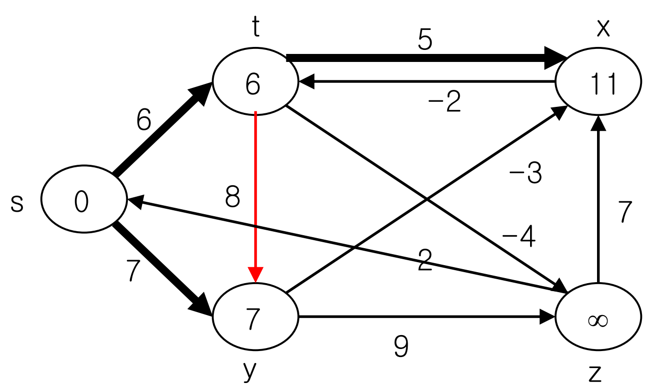


G.E	(t,x)	(t,y)	(t,z)	(x,t)	(y,x)	(y,z)	(z,x)	(z,s)	(s,t)	(s,y)

Bellman-Ford Algorithm

```
BELLMAN-FORD(G, w, s)
    INITIALIZE-SINGLE-SOURCE(G,s)
    for i=1 to |G.V|-1
        for each edge (u,v) ∈ G.E
            RELAX(u, v, w)
    for each edge (u,v) ∈ G.E
        if  v.d > u.d + w(u,v)
            return FALSE
    return TRUE
```

i = 2

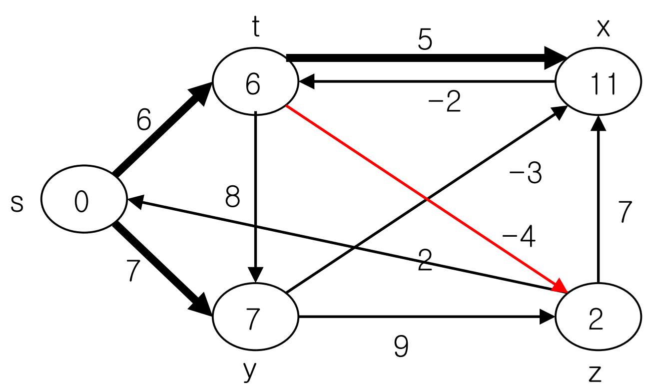


G.E	(t,x)	(t,y)	(t,z)	(x,t)	(y,x)	(y,z)	(z,x)	(z,s)	(s,t)	(s,y)

Bellman-Ford Algorithm

```
BELLMAN-FORD(G, w, s)
    INITIALIZE-SINGLE-SOURCE(G,s)
    for i=1 to |G.V|-1
        for each edge (u,v) ∈ G.E
            RELAX(u, v, w)
    for each edge (u,v) ∈ G.E
        if  v.d > u.d + w(u,v)
            return FALSE
    return TRUE
```

i = 2



G.E	(t,x)	(t,y)	(t,z)	(x,t)	(y,x)	(y,z)	(z,x)	(z,s)	(s,t)	(s,y)

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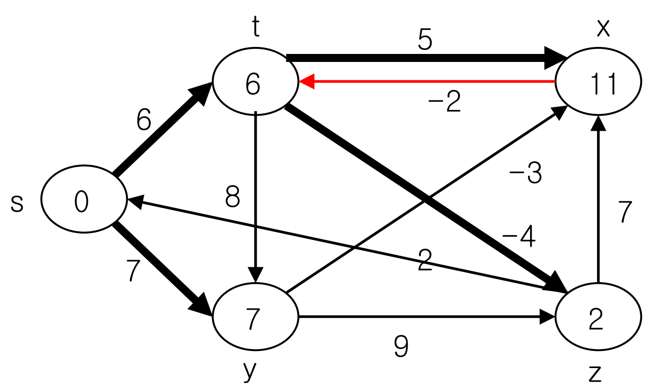

G.E	(t,x)	(t,y)	(t,z)	(x,t)	(y,x)	(y,z)	(z,x)	(z,s)	(s,t)	(s,y)

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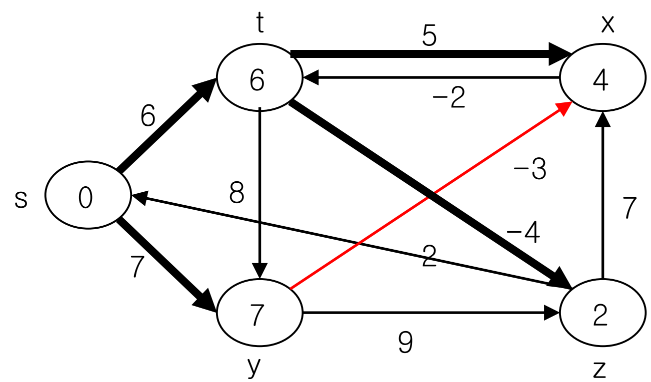

G.E	(t,x)	(t,y)	(t,z)	(x,t)	(y,x)	(y,z)	(z,x)	(z,s)	(s,t)	(s,y)

Bellman-Ford Algorithm

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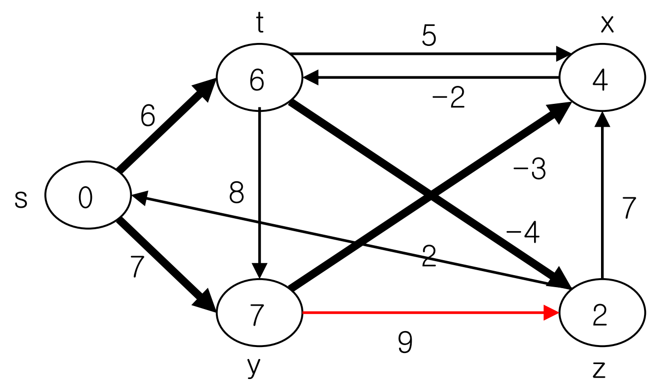
7. **return** FALSE
8. **return** TRUE

i = 2



G.E	(t,x)	(t,y)	(t,z)	(x,t)	(y,x)	(y,z)	(z,x)	(z,s)	(s,t)	(s,y)

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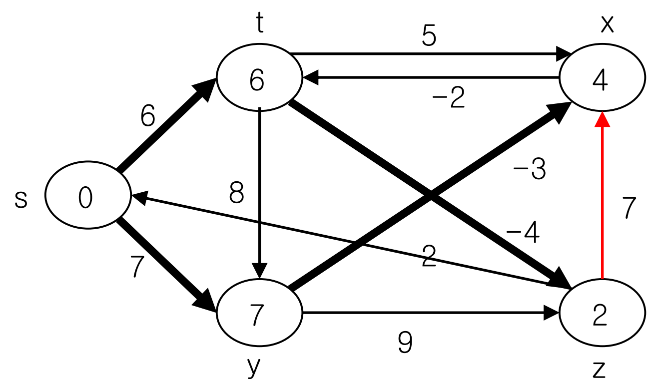
i = 2



G.E	(t,x)	(t,y)	(t,z)	(x,t)	(y,x)	(y,z)	(z,x)	(z,s)	(s,t)	(s,y)

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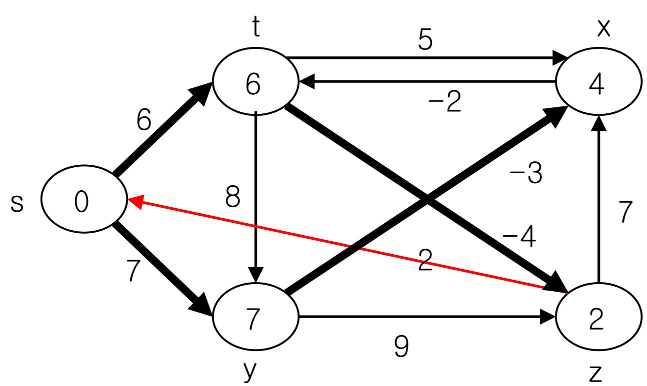

G.E	(t,x)	(t,y)	(t,z)	(x,t)	(y,x)	(y,z)	(z,x)	(z,s)	(s,t)	(s,y)

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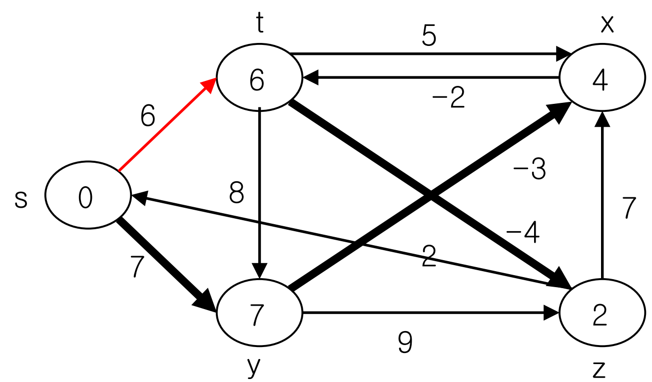

G.E	(t,x)	(t,y)	(t,z)	(x,t)	(y,x)	(y,z)	(z,x)	(z,s)	(s,t)	(s,y)

Bellman-Ford Algorithm

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7. **return** FALSE
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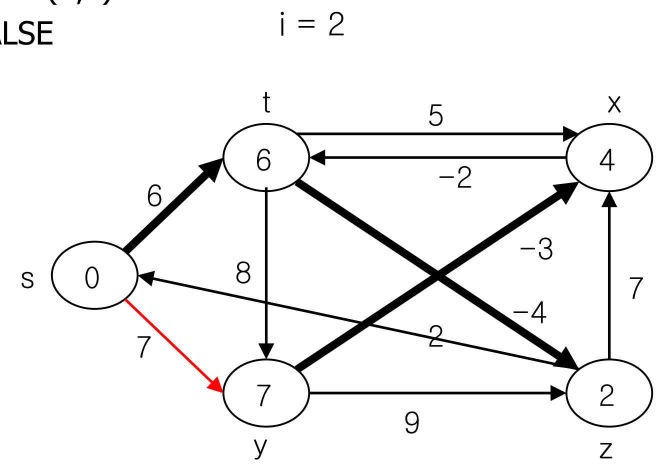

G.E	(t,x)	(t,y)	(t,z)	(x,t)	(y,x)	(y,z)	(z,x)	(z,s)	(s,t)	(s,y)

Bellman-Ford Algorithm

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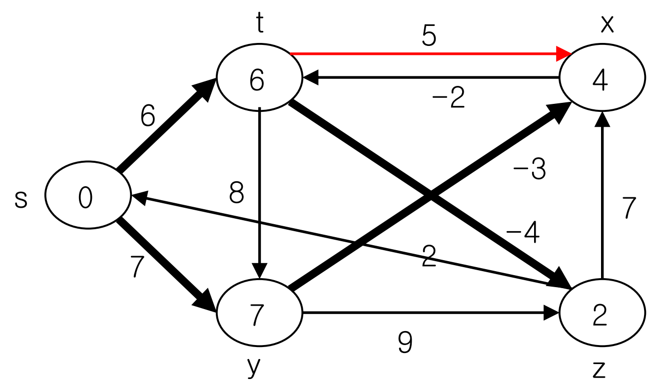
        if  v.d > u.d + w(u,v)
            return FALSE
    return TRUE
```

i = 3



G.E	(t,x)	(t,y)	(t,z)	(x,t)	(y,x)	(y,z)	(z,x)	(z,s)	(s,t)	(s,y)

Bellman-Ford Algorithm

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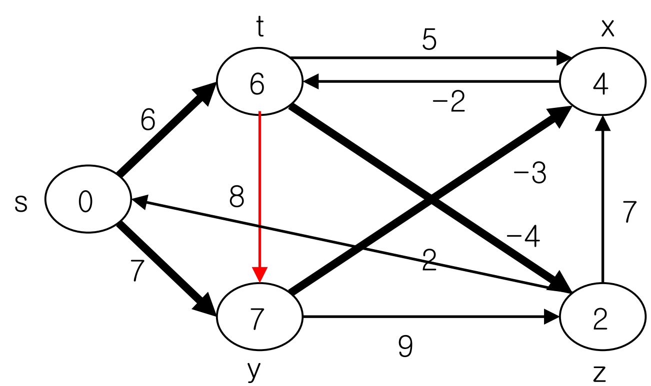

G.E	(t,x)	(t,y)	(t,z)	(x,t)	(y,x)	(y,z)	(z,x)	(z,s)	(s,t)	(s,y)

Bellman-Ford Algorithm

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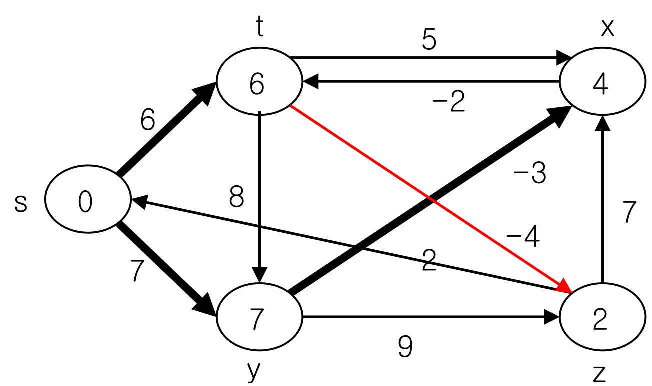

G.E	(t,x)	(t,y)	(t,z)	(x,t)	(y,x)	(y,z)	(z,x)	(z,s)	(s,t)	(s,y)

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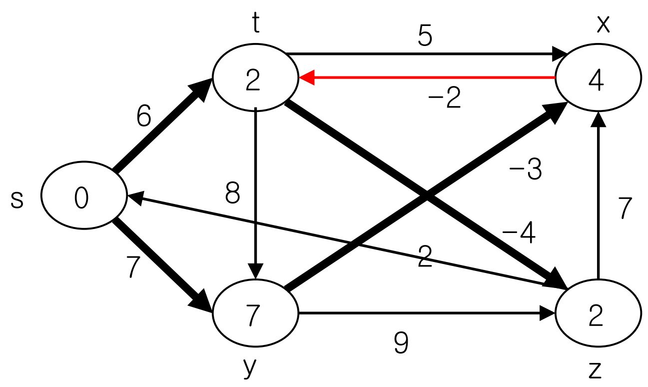

G.E	(t,x)	(t,y)	(t,z)	(x,t)	(y,x)	(y,z)	(z,x)	(z,s)	(s,t)	(s,y)

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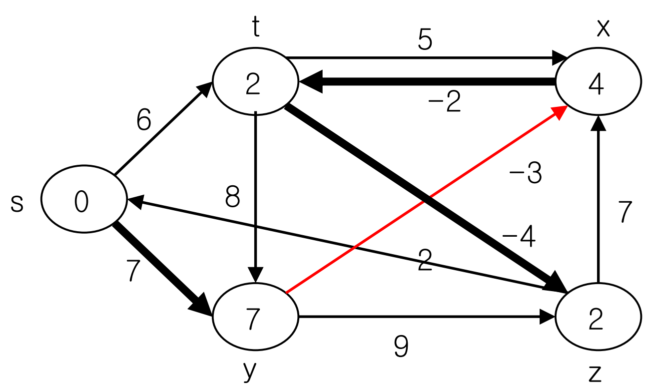

G.E	(t,x)	(t,y)	(t,z)	(x,t)	(y,x)	(y,z)	(z,x)	(z,s)	(s,t)	(s,y)

Bellman-Ford Algorithm

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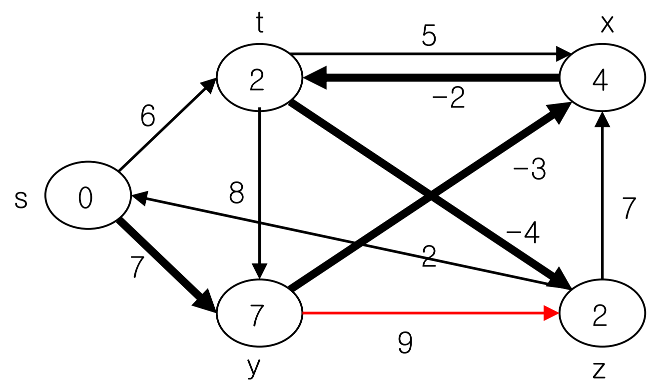

G.E	(t,x)	(t,y)	(t,z)	(x,t)	(y,x)	(y,z)	(z,x)	(z,s)	(s,t)	(s,y)

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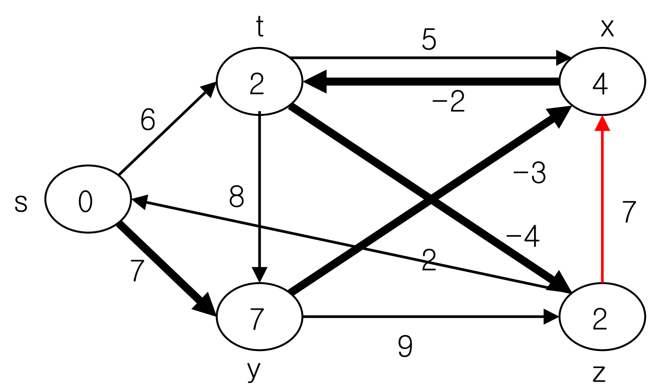

G.E	(t,x)	(t,y)	(t,z)	(x,t)	(y,x)	(y,z)	(z,x)	(z,s)	(s,t)	(s,y)

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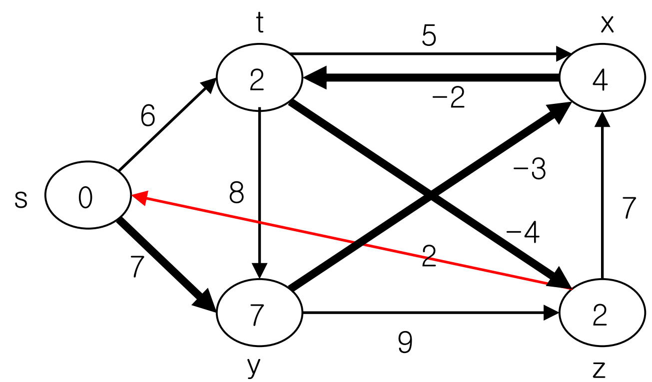

G.E	(t,x)	(t,y)	(t,z)	(x,t)	(y,x)	(y,z)	(z,x)	(z,s)	(s,t)	(s,y)

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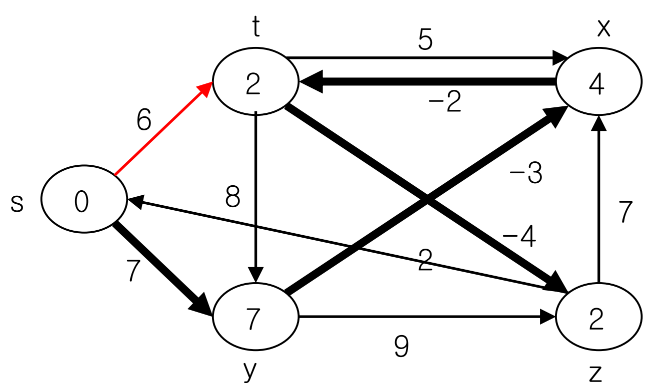

G.E	(t,x)	(t,y)	(t,z)	(x,t)	(y,x)	(y,z)	(z,x)	(z,s)	(s,t)	(s,y)

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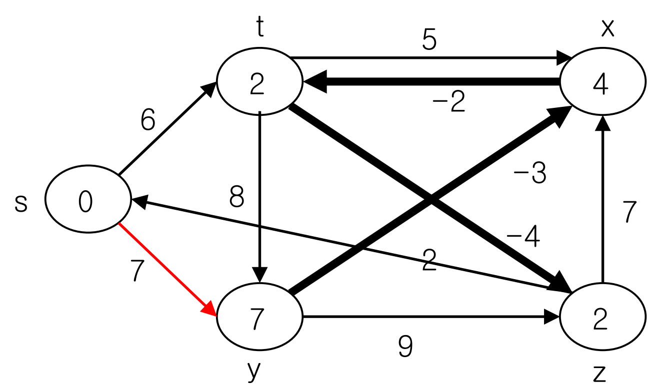

G.E	(t,x)	(t,y)	(t,z)	(x,t)	(y,x)	(y,z)	(z,x)	(z,s)	(s,t)	(s,y)

Bellman-Ford Algorithm

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    for each edge (u,v) ∈ G.E
        if  v.d > u.d + w(u,v)
            return FALSE
    return TRUE
```
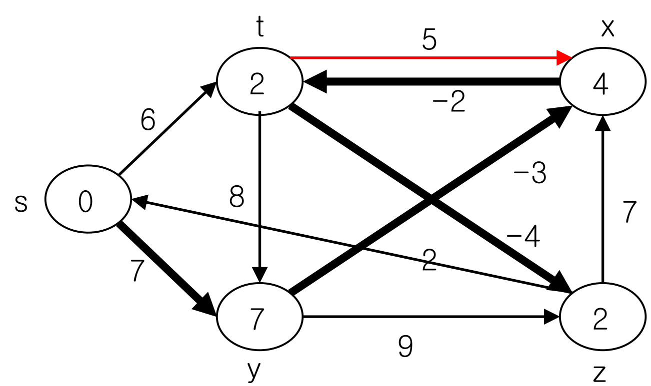

i = 4



G.E	(t,x)	(t,y)	(t,z)	(x,t)	(y,x)	(y,z)	(z,x)	(z,s)	(s,t)	(s,y)

Bellman-Ford Algorithm

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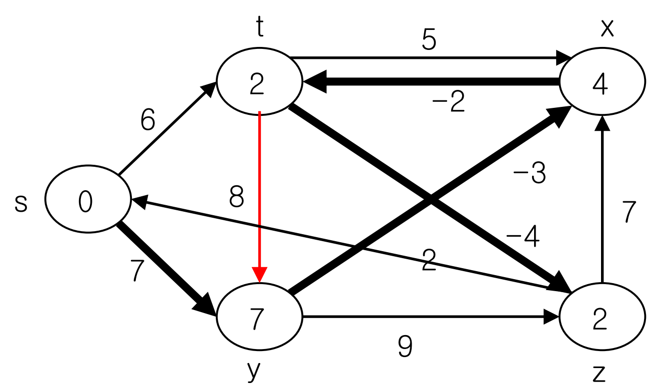

G.E	(t,x)	(t,y)	(t,z)	(x,t)	(y,x)	(y,z)	(z,x)	(z,s)	(s,t)	(s,y)

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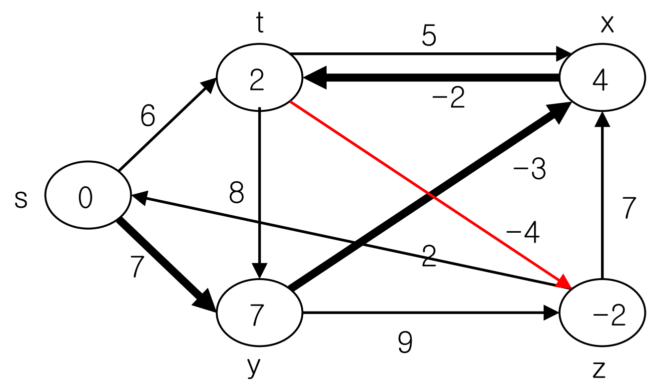

G.E	(t,x)	(t,y)	(t,z)	(x,t)	(y,x)	(y,z)	(z,x)	(z,s)	(s,t)	(s,y)

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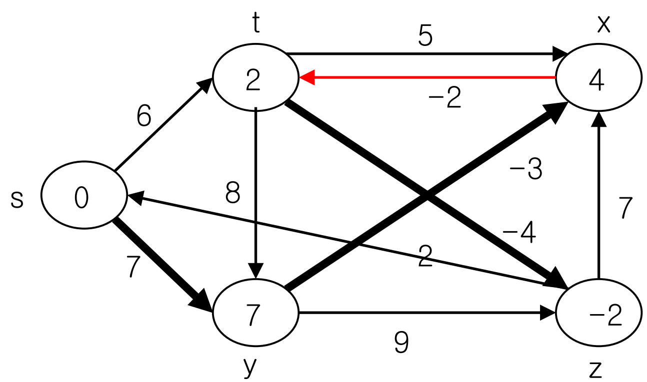

G.E	(t,x)	(t,y)	(t,z)	(x,t)	(y,x)	(y,z)	(z,x)	(z,s)	(s,t)	(s,y)

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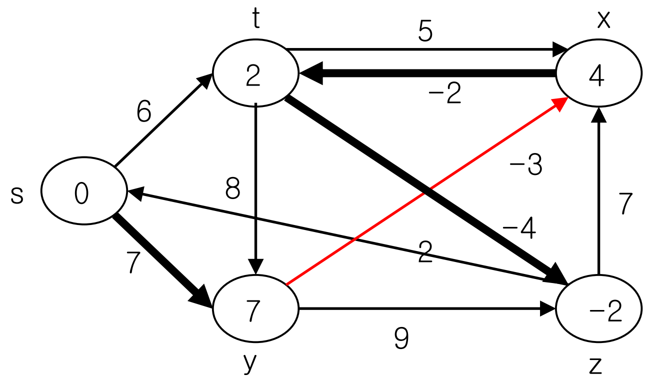

G.E	(t,x)	(t,y)	(t,z)	(x,t)	(y,x)	(y,z)	(z,x)	(z,s)	(s,t)	(s,y)

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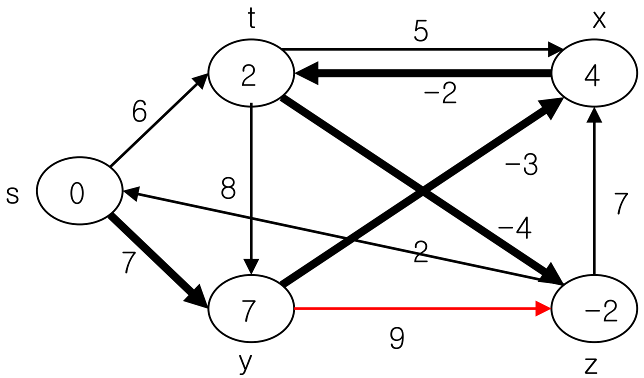

G.E	(t,x)	(t,y)	(t,z)	(x,t)	(y,x)	(y,z)	(z,x)	(z,s)	(s,t)	(s,y)

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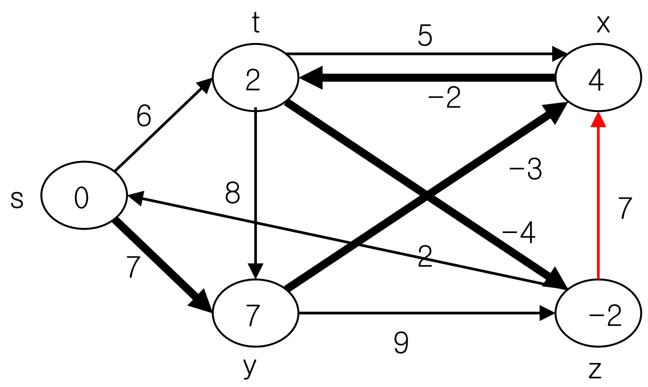

G.E	(t,x)	(t,y)	(t,z)	(x,t)	(y,x)	(y,z)	(z,x)	(z,s)	(s,t)	(s,y)

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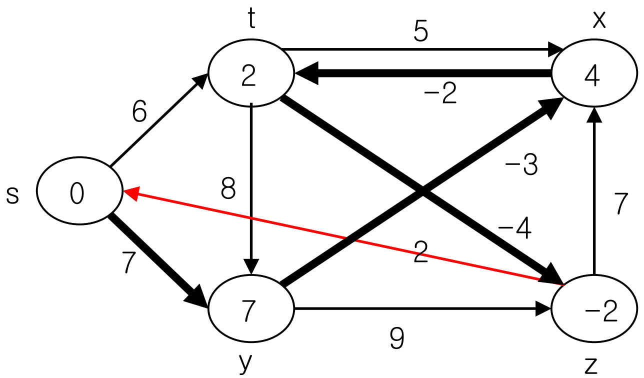

G.E	(t,x)	(t,y)	(t,z)	(x,t)	(y,x)	(y,z)	(z,x)	(z,s)	(s,t)	(s,y)

Bellman-Ford Algorithm

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BELLMAN-FORD(G, w, s)
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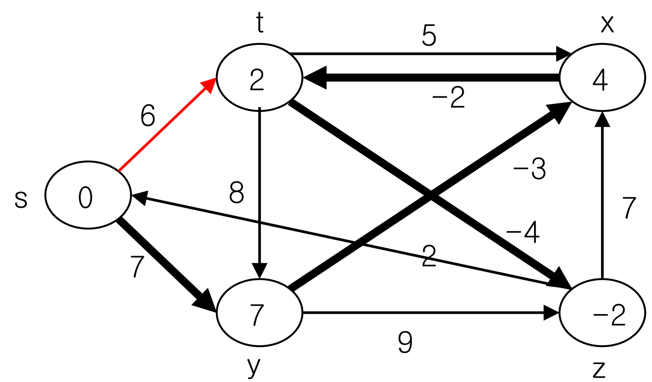

G.E	(t,x)	(t,y)	(t,z)	(x,t)	(y,x)	(y,z)	(z,x)	(z,s)	(s,t)	(s,y)

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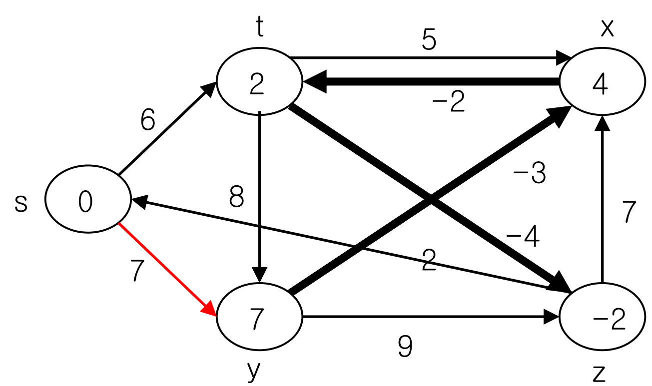

G.E	(t,x)	(t,y)	(t,z)	(x,t)	(y,x)	(y,z)	(z,x)	(z,s)	(s,t)	(s,y)

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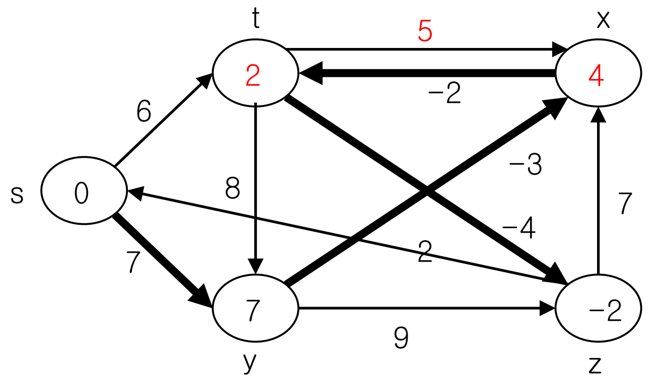

G.E	(t,x)	(t,y)	(t,z)	(x,t)	(y,x)	(y,z)	(z,x)	(z,s)	(s,t)	(s,y)

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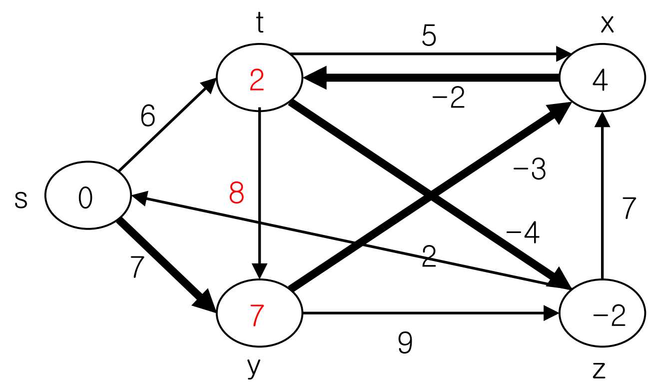

G.E	(t,x)	(t,y)	(t,z)	(x,t)	(y,x)	(y,z)	(z,x)	(z,s)	(s,t)	(s,y)

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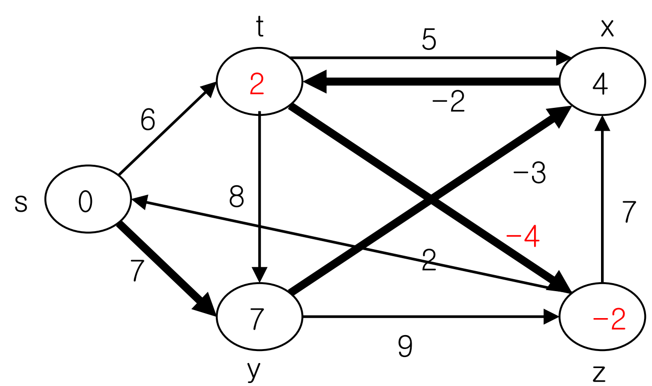

G.E	(t,x)	(t,y)	(t,z)	(x,t)	(y,x)	(y,z)	(z,x)	(z,s)	(s,t)	(s,y)

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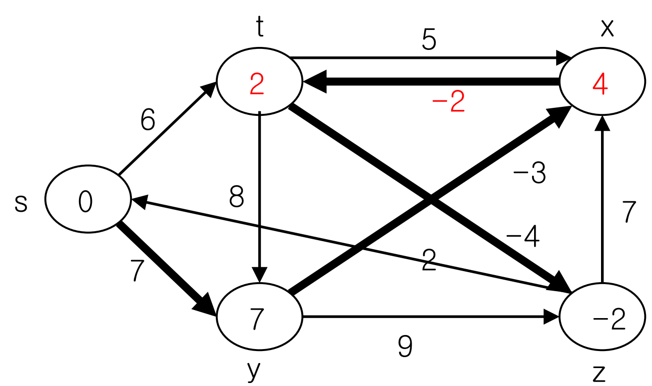

G.E	(t,x)	(t,y)	(t,z)	(x,t)	(y,x)	(y,z)	(z,x)	(z,s)	(s,t)	(s,y)

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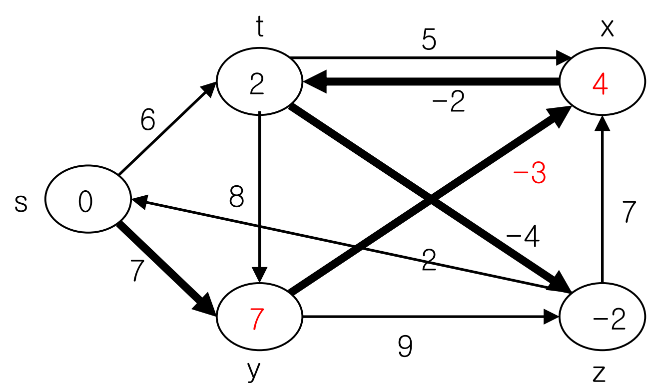

G.E	(t,x)	(t,y)	(t,z)	(x,t)	(y,x)	(y,z)	(z,x)	(z,s)	(s,t)	(s,y)

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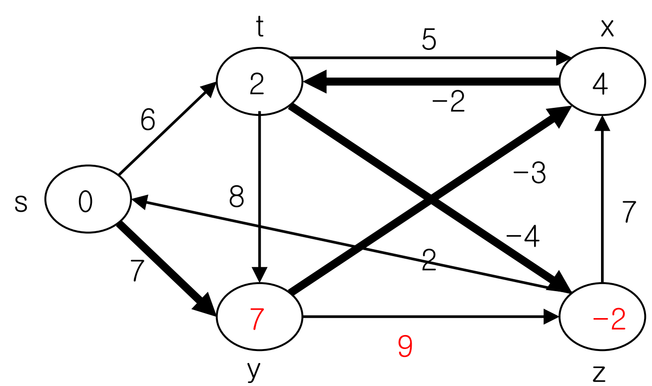

G.E	(t,x)	(t,y)	(t,z)	(x,t)	(y,x)	(y,z)	(z,x)	(z,s)	(s,t)	(s,y)

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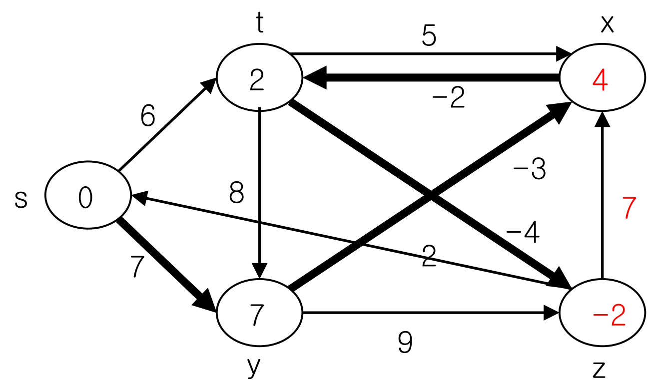

G.E	(t,x)	(t,y)	(t,z)	(x,t)	(y,x)	(y,z)	(z,x)	(z,s)	(s,t)	(s,y)

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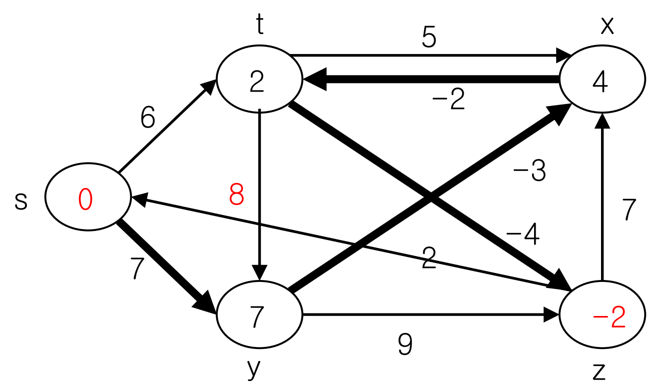

G.E	(t,x)	(t,y)	(t,z)	(x,t)	(y,x)	(y,z)	(z,x)	(z,s)	(s,t)	(s,y)

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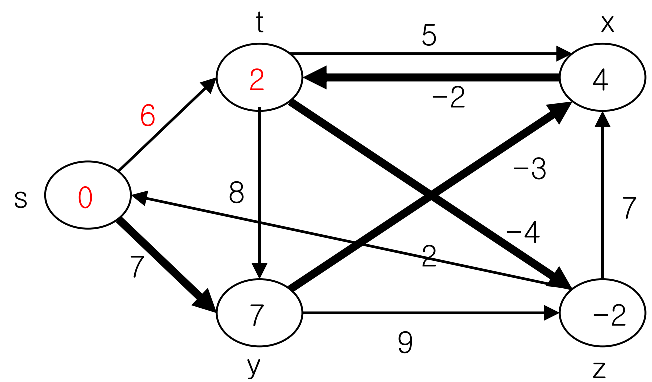

G.E	(t,x)	(t,y)	(t,z)	(x,t)	(y,x)	(y,z)	(z,x)	(z,s)	(s,t)	(s,y)

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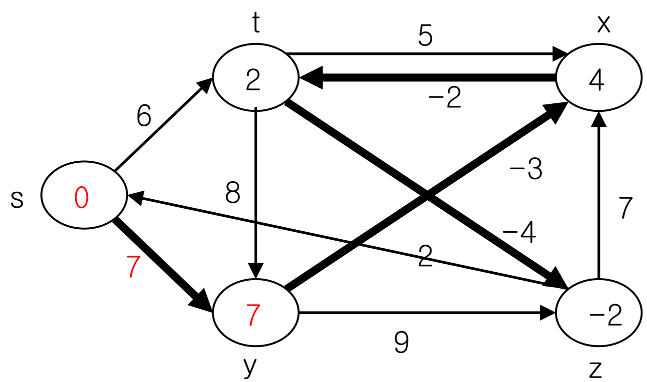

G.E	(t,x)	(t,y)	(t,z)	(x,t)	(y,x)	(y,z)	(z,x)	(z,s)	(s,t)	(s,y)

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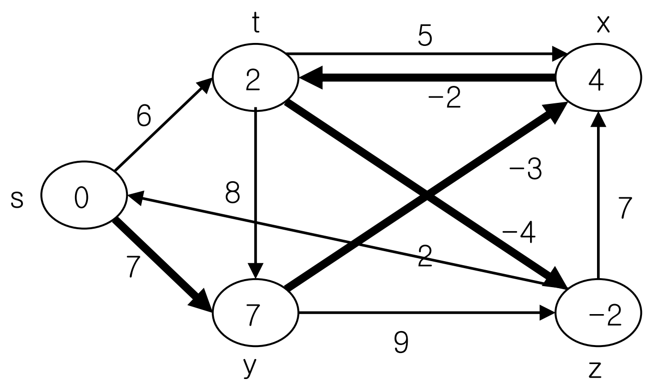

G.E	(t,x)	(t,y)	(t,z)	(x,t)	(y,x)	(y,z)	(z,x)	(z,s)	(s,t)	(s,y)