1 Punkt przecięcia trzech sfer

Punkt przecięcia trzech sfer jest rozwiązaniem poniższego układu równań:

$$\left\{\begin{array}{lcl} a^2 & = & (x-x_a)^2+(y-y_a)^2+(z-z_a)^2 \\ b^2 & = & (x-x_b)^2+(y-y_b)^2+(z-z_b)^2 \\ c^2 & = & (x-x_c)^2+(y-y_c)^2+(z-z_c)^2 \end{array}\right. \tag{1}$$

1.1 Przykład

$$\left\{\begin{array}{lcl} 3^2 & = & (x-1)^2+(y-1)^2+(z-4)^2 \\ 4^2 & = & (x-2)^2+(y-2)^2+(z-5)^2 \\ 5^2 & = & (x-2)^2+(y+1)^2+(z-6)^2 \end{array}\right. \tag{1}$$

$$\left\{\begin{array}{rcl} 9 & = & x^2-2x+1+y^2-2y+1+z^2-8z+16 \\ 16 & = & x^2-4x+4+y^2-4y+4+z^2-10z+25 \\ 25 & = & x^2-4x+4+y^2+2y+1+z^2-12z+36 \end{array}\right. \tag{2}$$

$$\left\{\begin{array}{rcl} 9+2x-1+2y-1+8z-16 & = & x^2+y^2+z^2 \\ 16+4x-4+4y-4+10z-25 & = & x^2+y^2+z^2 \\ 25+4x-4-2y-1+12z-36 & = & x^2+y^2+z^2 \end{array}\right. \tag{3}$$

$$\left\{\begin{array}{rcl} 2x+2y+8z-9 & = & x^2+y^2+z^2 \\ 4x+4y+10z-17 & = & x^2+y^2+z^2 \\ 4x-2y+12z-16 & = & x^2+y^2+z^2 \end{array}\right. \tag{4}$$

$$\left\{\begin{array}{l} 2x+2y+8z-9=x^2+y^2+z^2 \\ 2x+2y+2z-8=0 \Rightarrow y(x,z)=-x-z+4 \\ 2x-4y+4z-7=0 \Rightarrow z(x,y)=-\frac{1}{2}x+y+\frac{7}{4} \Rightarrow z(x)=-\frac{3}{4}x+\frac{23}{8} \end{array}\right. \tag{5}$$

$$\left\{\begin{array}{l} 2x+2y+8z-9=x^2+y^2+z^2 \\ y=-x-z+4 \Rightarrow y=-\frac{1}{4}x+\frac{9}{8} \\ z=-\frac{3}{4}x+\frac{23}{8} \end{array}\right. \tag{6}$$

$$2x+2(-\frac{1}{4}x+\frac{9}{8})+8(-\frac{3}{4}x+\frac{23}{8})-9 = x^2+(-\frac{1}{4}x+\frac{9}{8})^2+(-\frac{3}{4}x+\frac{23}{8})^2 \tag{7}$$

$$2x-0.5x+\frac{9}{4}-6x+23-9 = x^2+\frac{1}{16}x^2-\frac{9}{16}x+\frac{81}{64}+\frac{9}{16}x^2-\frac{69}{16}x+\frac{529}{64} \tag{8}$$

$$-\frac{9}{2}x+\frac{65}{4} = \frac{13}{8}x^2-\frac{39}{8}x+\frac{305}{32} \tag{9}$$

$$-144x+520 = 52x^2-156x+305 \tag{10}$$

$$-52x^2+12x+215 = 0 \tag{11}$$

1.2 Próba rozwiązania analitycznego

$$\left\{\begin{array}{lcl} a^2 & = & (x-x_a)^2+(y-y_a)^2+(z-z_a)^2 \\ b^2 & = & (x-x_b)^2+(y-y_b)^2+(z-z_b)^2 \\ c^2 & = & (x-x_c)^2+(y-y_c)^2+(z-z_c)^2 \end{array}\right. \tag{1}$$

$$\left\{\begin{array}{rcl} a^2+2x_ax-x_a^2+2y_ay-y_a^2+2z_az-z_a^2 & = & x^2+y^2+z^2 \\ b^2+2x_bx-x_b^2+2y_by-y_b^2+2z_bz-z_b^2 & = & x^2+y^2+z^2 \\ c^2+2x_cx-x_c^2+2y_cy-y_c^2+2z_cz-z_c^2 & = & x^2+y^2+z^2 \end{array}\right. \tag{2}$$

$$\left\{\begin{array}{rcl} a^2+2x_ax-x_a^2+2y_ay-y_a^2+2z_az-z_a^2 & = & x^2+y^2+z^2 \\ 2x(x_b-x_a)+2y(y_b-y_a)+2z(z_b-z_a) & = & t-b^2+x_b^2+y_b^2+z_b^2 \\ 2x(x_c-x_a)+2y(y_c-y_a)+2z(z_c-z_a) & = & t-c^2+x_c^2+y_c^2+z_c^2 \\ t & = & a^2-x_a^2-y_a^2-z_a^2 \end{array}\right. \tag{3}$$

$$
\begin{cases}
a^2 + 2x_a x - x_a^2 + 2y_a y - y_a^2 + 2z_a z - z_a^2 = x^2 + y^2 + z^2 \\
x = \frac{t-b^2+x_b^2+y_b^2+z_b^2}{2(x_b-x_a)} - \frac{y_b-y_a}{x_b-x_a} y - \frac{z_b-z_a}{x_b-x_a} z \\
(\frac{t-b^2+x_b^2+y_b^2+z_b^2}{2(x_b-x_a)} - \frac{y_b-y_a}{x_b-x_a} y - \frac{z_b-z_a}{x_b-x_a} z)(x_c - x_a) + y(y_c - y_a) + z(z_c - z_a) = \frac{t-c^2+x_c^2+y_c^2+z_c^2}{2} \Rightarrow \\
\Rightarrow ((z_c - z_a) - \frac{(z_b-z_a)(x_c-x_a)}{x_b-x_a})z = (\frac{(y_b-y_a)(x_c-x_a)}{x_b-x_a} - (y_c - y_a))y + \frac{t-c^2+x_c^2+y_c^2+z_c^2}{2} - \frac{(t-b^2+x_b^2+y_b^2+z_b^2)(x_c-x_a)}{2(x_b-x_a)} \\
t = a^2 - x_a^2 - y_a^2 - z_a^2
\end{cases}
\tag{4}
$$

Może i wykonalne, ale zbyt skomplikowane...