

Mathematical Framework for Intra-Pair Hedging in Automated Trading Systems

An Approach to Dynamic Risk Management

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Abstract

This paper develops a mathematical framework for intra-pair hedging in automated trading systems. Using a vector-space formulation, we derive conditions under which hedge sizes can be computed consistently for positions exhibiting different drawdown profiles. The method identifies subsets of positions that contribute most to risk and determines the hedge volume needed to offset their combined exposure. Formal propositions and examples show how the framework can be applied in practice to manage risk across multiple simultaneous positions in a transparent and operationally simple manner.

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1 Notation Summary

Symbol	Meaning
n	Number of open positions
E	Finite-dimensional normed vector space over \mathbb{R}
d_i	Drawdown of position i
l_i	Lot size of position i
$d = (d_1, \dots, d_n)$	Drawdown vector (sorted in decreasing order)
$l = (l_1, \dots, l_n)$	Lot size vector
$\ d\ _1^E$	ℓ^1 norm of d (total portfolio drawdown)
$f_i : \mathbb{R} \rightarrow \mathbb{R}$	Linear function mapping lot size to drawdown for position i
$\mathcal{F}_E : E \rightarrow \mathbb{R}$	Aggregate function: $\mathcal{F}_E(u) = \sum_{i=1}^n f_i(u_i)$
$x_i = \frac{d_i}{\ d\ _1^E}$	Normalized drawdown (proportion of total)
$\alpha \in]0, 1[$	Coverage parameter (proportion of drawdown to hedge)
$\beta \in]0, 1[$	Concentration parameter (threshold for position selection)
m	Cutoff index (number of positions to hedge)
$L_{1:m}$	Truncated lot size vector: (l_1, \dots, l_m)
$X_{1:m}$	Truncated normalized drawdown vector: (x_1, \dots, x_m)
$F = \text{Vect}_{\mathbb{R}}(L_{1:m})$	Vector space spanned by first m lot sizes
$h = (h_1, \dots, h_m)$	Hedge vector (optimal hedge lot sizes)
$[u, v]$	Hadamard (component-wise) product: $[u, v]_i = u_i v_i$

Table 1: Summary of mathematical notation

2 Introduction

2.1 Context

Automated FX trading systems often hold several positions simultaneously, which creates a portfolio-level exposure that is not trivially decomposable into independent risks. Drawdown management becomes challenging because individual positions can exhibit markedly different loss profiles depending on entry level, size, and local market dynamics.

Empirically, a small subset of positions typically accounts for most of the unrealized loss, and their contribution evolves dynamically as prices move. Since hedging all positions uniformly is capital-inefficient, effective risk control requires a method for identifying the positions that dominate drawdown at any given time and for determining the minimal hedge needed to neutralize their combined exposure.

2.2 Intra-pair hedging

An intra-pair hedge is a trading position opened in the opposite direction to an existing position in the same instrument, with a lot size calculated to offset a specified proportion of the unrealized loss (drawdown) of the original position. Unlike traditional portfolio hedging, which might use correlated instruments or derivatives, intra-pair hedging involves direct offsetting positions in the identical market.

An intra-pair hedge offers several operational advantages: because the hedge is taken in the exact same instrument, its correlation with the underlying position is effectively perfect, eliminating basis risk and avoiding the need for proxy instruments. The mechanism is operationally simple, since hedge and underlying can be managed independently, and it remains flexible in that partial hedges allow the trader to reduce downside exposure while preserving a controlled level of directional risk.

2.3 Problem statement

Consider an automated trading system where n open positions experiencing a positive drawdown (unrealized loss). Let d_i denote the drawdown of position i , and let l_i denote its opening lot size. We assume, without loss of generality, that positions are ordered by decreasing drawdown: $d_1 \geq d_2 \geq \dots \geq d_n \geq 0$.

The total portfolio drawdown is:

$$D_{total} = \sum_{i=1}^n d_i$$

The risk manager faces two interrelated questions:

1. Which subset of positions should be hedged to efficiently reduce risk?
2. For each selected position, what hedge lot size will achieve a target drawdown reduction?

3 Preliminary Results: Ordered Vector Decomposition

3.1 Motivation

Before developing our hedging framework, we establish a fundamental result about decomposing a probability-like vector with respect to a threshold. This result will enable us to identify which positions must be hedged based on their contribution to total drawdown.

3.2 Main result

Proposition 1

Let $n \in \mathbb{N}$ and $(x_1, \dots, x_n) \in \mathbb{R}^n$ such that:

- For all $i, j \in \{1, \dots, n\}$ with $i < j$, we have $x_i > x_j$,
- $0 \leq x_i \leq 1$ for all $i \in \{1, \dots, n\}$,
- $\sum_{i=1}^n x_i = 1$.

Then, for all $\beta \in]0, 1[$, there exists a unique $m \in \{1, \dots, n\}$ such that:

$$\sum_{i=1}^{m-1} x_i < \beta \quad \text{and} \quad \sum_{i=1}^m x_i \geq \beta.$$

3.3 Interpretation

This proposition states that for any strictly decreasing probability distribution (x_1, \dots, x_n) and any threshold $\beta \in]0, 1[$, there exists a unique "cutoff index" m such that:

- The first $m - 1$ components account for strictly less than β of the total mass.
- The first m components account for at least β of the total mass.

In the context of trading, if x_i represents the proportion of total drawdown contributed by position i (with positions ordered by decreasing drawdown), then m identifies the minimal number of positions that must be considered to cover at least $\beta \times 100\%$ of total portfolio drawdown.

3.4 Proof

Let $n \in \mathbb{N}^*$ and $(x_1, x_2, \dots, x_n) \in \mathbb{R}^n$ satisfying the stated conditions. Let $\beta \in]0, 1[$. Define the partial sums:

$$S_k = \sum_{i=1}^k x_i, \quad \text{for all } k \in \{1, \dots, n\}.$$

We prove existence and uniqueness of $m \in \{1, \dots, n\}$ such that:

$$S_{m-1} < \beta \leq S_m.$$

Existence of m : By hypothesis, (x_i) is a strictly decreasing sequence of non-negative reals. Therefore, the partial sums (S_k) form a strictly increasing sequence.

Moreover, $S_1 = x_1 > 0$ (since $x_1 > 0$ as the largest component of a probability distribution) and $S_n = \sum_{i=1}^n x_i = 1$ by hypothesis.

Since $\beta \in]0, 1[$, we have $0 < \beta < S_n = 1$. The sequence (S_k) is strictly increasing from $S_1 > 0$ to $S_n = 1$, so by the intermediate value theorem for discrete sequences, there exists an index $m \in \{1, \dots, n\}$ such that:

$$S_{m-1} < \beta \leq S_m.$$

(Note: We adopt the convention that $S_0 = 0$.)

Uniqueness of m : Suppose by contradiction that there exist $m_1, m_2 \in \{1, \dots, n\}$ with $m_1 < m_2$ such that both satisfy:

$$S_{m_1-1} < \beta \leq S_{m_1} \quad \text{and} \quad S_{m_2-1} < \beta \leq S_{m_2}.$$

Since (S_k) is strictly increasing and $m_1 < m_2$, we have:

$$S_{m_1} < S_{m_1+1} \leq S_{m_2-1}.$$

From the first condition, $\beta \leq S_{m_1}$. From the second condition, $\beta > S_{m_2-1}$. This implies:

$$S_{m_2-1} < \beta \leq S_{m_1},$$

which contradicts $S_{m_1} < S_{m_2-1}$ derived above.

Conclusion: There exists a unique $m \in \{1, \dots, n\}$ such that $S_{m-1} < \beta \leq S_m$, which completes the proof. \square

3.5 Corollary: Minimal covering set

Corollary 3.1. Under the conditions of Proposition 1, the set $\{1, 2, \dots, m\}$ is the unique minimal subset $I \subseteq \{1, \dots, n\}$ such that:

$$\sum_{i \in I} x_i \geq \beta \quad \text{and} \quad |I| \text{ is minimal.}$$

Proof. Proposition 1 establishes that $\sum_{i=1}^m x_i \geq \beta$ and $\sum_{i=1}^{m-1} x_i < \beta$. Any subset with fewer than m elements cannot achieve the threshold β . Due to the strict ordering $x_1 > x_2 > \dots > x_n$, any other subset of size m not consisting of the first m elements would have a smaller sum, thus failing to meet the threshold. \square

4 Hedging Framework: Fundamental Properties

4.1 Mathematical setup

Let E be a finite-dimensional normed vector space over \mathbb{R} with dimension n . Consider a vector of linear functions (f_1, \dots, f_n) where each $f_i : \mathbb{R} \rightarrow \mathbb{R}$ is linear.

Definition 4.1 (Aggregate function). We define the aggregate function $\mathcal{F}_E : E \rightarrow \mathbb{R}$ by:

$$\mathcal{F}_E(u) = \sum_{i=1}^n f_i(u_i).$$

Definition 4.2 (Drawdown vector). Let $d \in E$ be the *drawdown vector* satisfying:

- For all $i, j \in \{1, \dots, n\}$, if $i < j$ then $d_i > d_j$ (strict ordering),
- $d_i \neq 0$ for all i (non-degeneracy).

Definition 4.3 (Lot size vector). Let $l \in E$ be the *lot size vector* such that for all $i \in \{1, \dots, n\}$:

$$f_i(l_i) = d_i.$$

This means that the function f_i maps the lot size l_i to the observed drawdown d_i .

Consequently, we have:

$$\mathcal{F}_E(l) = \sum_{i=1}^n f_i(l_i) = \sum_{i=1}^n d_i = \|d\|_1^E,$$

where $\|d\|_1^E$ denotes the ℓ^1 norm of d in E .

4.2 Interpretation in trading context

In the context of trading:

- d_i : The drawdown (unrealized loss) of position i , measured in base currency.
- l_i : The opening lot size of position i .
- f_i : A linear function relating lot size to drawdown for position i . The linearity assumption means that drawdown scales proportionally with lot size, which is valid for constant price movements.
- $\mathcal{F}_E(u)$: The total drawdown of a portfolio represented by vector u .
- $\|d\|_1^E$: The total portfolio drawdown.

The strict ordering condition $d_1 > d_2 > \dots > d_n$ ensures that positions are indexed in decreasing order of drawdown severity.

4.3 Bounded hedge construction

Proposition 2

For all $\alpha \in]0, 1[$ and $\beta \in]0, 1[$, there exists a unique $m \in \{1, \dots, n\}$ such that there exists a vector $h \in F = \text{Vect}_{\mathbb{R}}(L_{1:m})$ satisfying:

$$\alpha \|d\|_1^E \leq \mathcal{F}_F(h) < \alpha \frac{\beta \|d\|_1^E + d_m}{\beta},$$

where $L_{1:m}$ denotes the vector l with the last $n - m$ components removed.

4.4 Interpretation

This proposition establishes that:

1. There exists a unique minimal subset of positions (the first m positions) such that a hedge portfolio h can be constructed within their span.
2. This hedge portfolio achieves coverage of at least $\alpha \|d\|_1^E$ (i.e., $\alpha \times 100\%$ of total drawdown).
3. The coverage is bounded above by a quantity involving both α , β , and the drawdown of the m -th position.
4. The parameter α controls the *coverage level* (what proportion of total drawdown we hedge).
5. The parameter β controls the *concentration threshold* (what proportion of drawdown we require the selected positions to represent).

4.5 Proof of Proposition 2

Let $\alpha, \beta \in]0, 1[$. Define the normalized drawdown vector $x \in E$ by:

$$x_i = \frac{d_i}{\|d\|_1^E}, \quad \forall i \in \{1, \dots, n\}.$$

The vector x satisfies:

- $0 < x_i \leq 1$ for all i (since $d_i > 0$ and $d_i \leq \|d\|_1^E$),
- For all $i, j \in \{1, \dots, n\}$ with $i < j$, we have $x_i > x_j$ (inherited from the ordering of d),
- $\|x\|_1^E = \sum_{i=1}^n x_i = 1$ (by construction).

By Proposition 1, there exists a unique $m \in \{1, \dots, n\}$ such that:

$$\sum_{i=1}^{m-1} x_i < \beta \quad \text{and} \quad \sum_{i=1}^m x_i \geq \beta.$$

Define $F = \text{Vect}_{\mathbb{R}}(L_{1:m})$, the vector space spanned by the first m components of the lot size vector l .

Lower bound: From $\sum_{i=1}^m x_i \geq \beta$, we obtain:

$$\sum_{i=1}^m d_i \geq \beta \|d\|_1^E.$$

Multiplying both sides by $\frac{\alpha}{\beta}$:

$$\frac{\alpha}{\beta} \sum_{i=1}^m d_i \geq \alpha \|d\|_1^E.$$

Since $f_i(l_i) = d_i$ and each f_i is linear:

$$\frac{\alpha}{\beta} \sum_{i=1}^m f_i(l_i) \geq \alpha \|d\|_1^E.$$

By linearity of f_i :

$$\sum_{i=1}^m f_i\left(\frac{\alpha}{\beta} l_i\right) \geq \alpha \|d\|_1^E.$$

Upper bound: From $\sum_{i=1}^{m-1} x_i < \beta$, we derive:

$$\sum_{i=1}^m x_i < \beta + x_m.$$

Therefore:

$$\sum_{i=1}^m d_i < (\beta + x_m) \|d\|_1^E = \beta \|d\|_1^E + d_m.$$

Multiplying by $\frac{\alpha}{\beta}$:

$$\frac{\alpha}{\beta} \sum_{i=1}^m d_i < \frac{\alpha}{\beta} (\beta \|d\|_1^E + d_m) = \alpha \|d\|_1^E + \frac{\alpha d_m}{\beta}.$$

By linearity:

$$\sum_{i=1}^m f_i \left(\frac{\alpha}{\beta} l_i \right) < \alpha \left(\|d\|_1^E + \frac{d_m}{\beta} \right) = \alpha \frac{\beta \|d\|_1^E + d_m}{\beta}.$$

Construction of h : Define:

$$h = \left(\frac{\alpha}{\beta} l_i \right)_{i \in \{1, \dots, m\}}.$$

Then $h \in F$ and satisfies:

$$\alpha \|d\|_1^E \leq \mathcal{F}_F(h) < \alpha \frac{\beta \|d\|_1^E + d_m}{\beta}.$$

This completes the proof. □

4.6 Exact hedge formula

While Proposition 2 provides bounds, in practice we desire an exact hedge that achieves precisely $\alpha \|d\|_1^E$ coverage. The next proposition addresses this.

Proposition 3

Let $m \in \mathbb{N}$ be fixed. For all $\alpha \in]0, 1[$, there exists a unique $h \in F = \text{Vect}_{\mathbb{R}}(L_{1:m})$ such that:

$$\mathcal{F}_F(h) = \alpha \|d\|_1^E,$$

where $L_{1:m}$ is defined as in the previous propositions.

4.7 Explicit formula

The unique hedge vector h is given by:

$$h = \frac{\alpha}{m} [L_{1:m}, X_{1:m}^{-1}],$$

where:

- $L_{1:m} = (l_1, \dots, l_m)$ is the truncated lot size vector,
- $X_{1:m} = (x_1, \dots, x_m)$ with $x_i = \frac{d_i}{\|d\|_1^E}$ is the truncated normalized drawdown vector,
- $X_{1:m}^{-1} = \left(\frac{1}{x_1}, \dots, \frac{1}{x_m} \right)$ is the component-wise inverse,

- $[L_{1:m}, X_{1:m}^{-1}]$ denotes the Hadamard (component-wise) product: $[L_{1:m}, X_{1:m}^{-1}]_i = l_i \cdot \frac{1}{x_i}$.

Explicitly, the i -th component of the hedge vector is:

$$h_i = \frac{\alpha}{m} \cdot l_i \cdot \frac{\|d\|_1^E}{d_i} = \frac{\alpha \|d\|_1^E}{m \cdot d_i} \cdot l_i.$$

4.8 Proof of Proposition 3

We proceed by analysis-synthesis.

Analysis: We seek $h \in F$ such that $\mathcal{F}_F(h) = \alpha \|d\|_1^E$.

Define $x_i = \frac{d_i}{\|d\|_1^E}$. Note that $d_i \neq 0$ for all i by hypothesis.

We can write:

$$\alpha \|d\|_1^E = \alpha \|d\|_1^E \cdot \frac{1}{m} \sum_{i=1}^m \frac{d_i}{d_i}$$

Since $f_i(l_i) = d_i$ and $d_i = x_i \|d\|_1^E$:

$$\alpha \|d\|_1^E = \alpha \|d\|_1^E \cdot \frac{1}{m} \sum_{i=1}^m \frac{f_i(l_i)}{d_i} = \alpha \|d\|_1^E \cdot \frac{1}{m} \sum_{i=1}^m \frac{f_i(l_i)}{x_i \|d\|_1^E}.$$

Simplifying:

$$\alpha \|d\|_1^E = \alpha \cdot \frac{1}{m} \sum_{i=1}^m \frac{f_i(l_i)}{x_i}.$$

By linearity of f_i :

$$\alpha \|d\|_1^E = \frac{1}{m} \sum_{i=1}^m f_i \left(\frac{\alpha l_i}{x_i} \right) = \sum_{i=1}^m f_i \left(\frac{\alpha l_i}{m x_i} \right).$$

Therefore, defining:

$$h_i = \frac{\alpha l_i}{m x_i} = \frac{\alpha \|d\|_1^E}{m d_i} \cdot l_i,$$

we have $h = (h_1, \dots, h_m) \in F$ and:

$$\mathcal{F}_F(h) = \sum_{i=1}^m f_i(h_i) = \alpha \|d\|_1^E.$$

In vector notation:

$$h = \frac{\alpha}{m} [L_{1:m}, X_{1:m}^{-1}],$$

where the bracket denotes component-wise multiplication.

Synthesis: The construction explicitly provides a vector $h \in F$ satisfying the desired equality, establishing existence.

Uniqueness: Suppose $h_1, h_2 \in F$ both satisfy $\mathcal{F}_F(h_1) = \mathcal{F}_F(h_2) = \alpha \|d\|_1^E$.

By linearity of \mathcal{F}_F :

$$\mathcal{F}_F(h_1 - h_2) = \mathcal{F}_F(h_1) - \mathcal{F}_F(h_2) = 0.$$

Thus $h_1 - h_2 \in \ker(\mathcal{F}_F)$.

However, we claim that \mathcal{F}_F is injective on F . To see this, note that for any $v = (v_1, \dots, v_m) \in F$:

$$\mathcal{F}_F(v) = \sum_{i=1}^m f_i(v_i) = \sum_{i=1}^m \frac{d_i}{l_i} v_i,$$

since $f_i(l_i) = d_i$ and f_i is linear implies $f_i(v_i) = \frac{d_i}{l_i} v_i$.

The coefficients $\frac{d_i}{l_i}$ are all positive and distinct (due to the strict ordering of d_i and non-degeneracy assumptions). Therefore, $\mathcal{F}_F(v) = 0$ implies $v = 0$, so \mathcal{F}_F is injective.

Hence $h_1 - h_2 = 0$, i.e., $h_1 = h_2$, establishing uniqueness. \square

5 Practical Application: Trading Example

5.1 Setup

Consider an automated trading system managing $n = 5$ open on different currencies in any of the two trade direction long or short with the following characteristics:

Position	Currency	Trade Direction	Lot Size (l_i)	Entry Price	Current Price	Drawdown (d_i)
1	EURUSD	Long	0.10	1.1200	1.1050	\$150
2	GBPUSD	Short	0.15	1.1180	1.1060	\$180
3	NZDJPY	Short	0.08	1.1150	1.1070	\$64
4	USDCHF	Long	0.12	1.1100	1.1080	\$24
5	AUDNZD	Long	0.05	1.1090	1.1085	\$2.50

Table 2: Portfolio positions (unsorted)

Total portfolio drawdown: $\|d\|_1 = 150 + 180 + 64 + 24 + 2.50 = \420.50 .

5.2 Step 1: Sort by drawdown

Reorder positions in decreasing order of drawdown:

Index i	Original Position	Lot Size (l_i)	Drawdown (d_i)
1	2	0.15	\$180
2	1	0.10	\$150
3	3	0.08	\$64
4	4	0.12	\$24
5	5	0.05	\$2.50

Table 3: Positions sorted by decreasing drawdown

5.3 Step 2: Compute normalized drawdowns

For each position i :

$$x_i = \frac{d_i}{\|d\|_1} = \frac{d_i}{420.50}$$

Index i	d_i	x_i	Cumulative $\sum_{j=1}^i x_j$
1	180	0.4280	0.4280
2	150	0.3567	0.7847
3	64	0.1522	0.9369
4	24	0.0571	0.9940
5	2.50	0.0059	1.0000

Table 4: Normalized drawdowns and cumulative sums

5.4 Step 3: Determine minimal position subset

Suppose we want to cover at least $\beta = 0.80$ (80%) of total drawdown. By Proposition 1, we seek the unique m such that:

$$\sum_{i=1}^{m-1} x_i < 0.80 \leq \sum_{i=1}^m x_i.$$

From the cumulative sums:

- $\sum_{i=1}^1 x_i = 0.4280 < 0.80$
- $\sum_{i=1}^2 x_i = 0.7847 < 0.80$
- $\sum_{i=1}^3 x_i = 0.9369 \geq 0.80 \checkmark$

Therefore, $m = 3$. The first 3 positions (with drawdowns \$180, \$150, and \$64) are the minimal subset covering at least 80% of total drawdown.

5.5 Step 4: Calculate hedge sizes

We want to hedge $\alpha = 0.50$ (50%) of total drawdown: $\alpha \|d\|_1 = 0.50 \times 420.50 = \210.25 .

By Proposition 3, the hedge vector $h = (h_1, h_2, h_3)$ is given by:

$$h_i = \frac{\alpha}{m} \cdot l_i \cdot \frac{\|d\|_1}{d_i}$$

Computing each component:

$$\begin{aligned} h_1 &= \frac{0.50}{3} \cdot 0.15 \cdot \frac{420.50}{180} = 0.0584 \text{ lots} \\ h_2 &= \frac{0.50}{3} \cdot 0.10 \cdot \frac{420.50}{150} = 0.0467 \text{ lots} \\ h_3 &= \frac{0.50}{3} \cdot 0.08 \cdot \frac{420.50}{64} = 0.0876 \text{ lots} \end{aligned}$$

5.6 Step 5: Verification

We verify that our hedge achieves exactly $\alpha \|d\|_1 = \$210.25$ coverage.

For each position i , the drawdown per unit lot is $\frac{d_i}{l_i}$. The hedge h_i covers:

$$\text{Coverage}_i = h_i \cdot \frac{d_i}{l_i}.$$

Computing:

$$\begin{aligned}\text{Coverage}_1 &= 0.0584 \cdot \frac{180}{0.15} = 0.0584 \cdot 1200 = 70.08 \\ \text{Coverage}_2 &= 0.0467 \cdot \frac{150}{0.10} = 0.0467 \cdot 1500 = 70.05 \\ \text{Coverage}_3 &= 0.0876 \cdot \frac{64}{0.08} = 0.0876 \cdot 800 = 70.08\end{aligned}$$

Total coverage: $70.08 + 70.05 + 70.08 = 210.21 \approx \$210.25 \checkmark$
(Minor discrepancies due to rounding.)

5.7 Interpretation

The hedge strategy is as follows:

1. **Open opposite positions** (hedges) with the calculated lot sizes:
 - Position 1: Short 0.0584 lots GBPUSD
 - Position 2: Long 0.0467 lots EURUSD
 - Position 3: Short 0.0876 lots NZDJPY
2. **Result:** These three hedges collectively offset approximately \$210.25 (50%) of the total \$420.50 portfolio drawdown.
3. **Risk reduction:** The portfolio's net exposure is reduced, limiting further downside while maintaining some upside potential if the market reverses.
4. **Selective approach:** Only the 3 most significant positions are hedged, concentrating risk management efforts efficiently.

6 Conclusion

This paper has developed a rigorous mathematical framework for intra-pair hedging in automated trading systems. The problem has been formalized in finite-dimensional vector spaces and established the existence and uniqueness of optimal solutions under standard conditions (Propositions 1-3). The main contribution lies in the explicit formula

$$h_i = \frac{\alpha}{m} \cdot l_i \cdot \frac{\|d\|_1}{d_i}$$

which enables direct computation of hedge sizes with algorithmic complexity of $O(n \log n)$, suitable for real-time application.

The proposed framework offers several quantifiable advantages over ad-hoc methods. The precision of targeting the desired risk reduction level is exact by construction. Efficiency is optimized by concentrating hedges on positions contributing most to portfolio risk through the coefficient l_i/d_i . The parameters α and β allow fine control of the trade-off between drawdown reduction and capital preservation, with an established linear relationship between these quantities. Scalability is guaranteed by the sub-linear complexity of the algorithm.

For practitioners, this framework provides a systematic method to determine which positions to hedge and in what proportions, with transparent mathematical justification for each decision. The approach facilitates communication with stakeholders by providing a quantitative basis for risk management decisions and enables stress-testing of portfolio resilience under different hedging scenarios.

I am currently working on an extension of this framework to incorporate confidence intervals for each drawdown, reflecting the inherent uncertainty in estimations. This extension will formulate a robust optimization method that improves hedge quality by minimizing risk exposure in the worst case compatible with estimated confidence intervals.