# HW2

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## 1 Exercise 1

We can construct a function $f$ which on input $\langle M, w\rangle$ returns $\langle M', w\rangle$, such that:
$\langle M, w\rangle \in HALT_{TM}$ iff $\langle M', w\rangle \in A_{TM}$ The idea is to modify $M$ into $M'$ such that $M'$ accepts $w$ iff $M$ halted on $w$.
The following $M_f$ computes the function $f$.
$M_f$: On input $\langle M, w\rangle$:
1) Construct $M'$ such that: Run $M$ in the input and if $M$ accepts or reject, then $M'$ accepts.
2) Output $\langle M', w\rangle$.

So: $M_f$ computes the function $f$ so $HALT_{TM} \leq_m A_{TM}$. Therefore since $HALT_{TM}$ is undecidable also $A_{TM}$ is undecidable.

## 2 Exercise 2

No, this is not true.
For example, we can define 2 languages:
$A = \{0^n 1^n | n > 0\}$
$B = \{0\}$
We can define a function $f$ such that:
$f(w) = \begin{cases} 0 & w \in A \\ 1 & w \notin A \end{cases}$

Since A is a context-free language it's also Turing-decidable, therefore $f$ exists.
Also, $w \in A$ iff $f(w) = 0$ which is true iff $f(w) \in B$.
Therefore A $\leq_m$ B. But Language A is nonregular and B it's regular.