SGN – Assignment #2

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Exercise 1: Uncertainty propagation

You are asked to analyze the state uncertainty evolution along a transfer trajectory in the Planar Bicircular Restricted Four-Body Problem, obtained as optimal solution of the problem stated in Section 3.1 (Topputo, 2013)*. The mean initial state $\mathbf{x}_i$ at initial time t_i with its associated covariance $\mathbf{P}_0$ and final time t_f for this optimal transfer are provided in Table 1.

1. Propagate the initial mean and covariance within a time grid of 5 equally spaced elements going from t_i to t_f, using both a Linearized Approach (LinCov) and the Unscented Transform (UT). We suggest to use $\alpha = 1$ and $\beta = 2$ for tuning the UT in this case. Plot the mean and the ellipses associated with the position elements of the covariances obtained with the two methods at the final time.

2. Perform the same uncertainty propagation process on the same time grid using a Monte Carlo (MC) simulation †. Compute the sample mean and sample covariance and compare them with the estimates obtained at Point 1). Provide the following outputs.

   - Plot of the propagated samples of the MC simulation, together with the mean and the covariance obtained with all methods in terms of ellipses associated with the position elements at the final time.
   - Plot of the time evolution (for the time grid previously defined) for all three approaches (MC, LinCov, and UT) of $3\sqrt{\max(\lambda_i(P_r))}$ and $3\sqrt{\max(\lambda_i(P_v))}$, where P_r and P_v are the 2x2 position and velocity covariance submatrices.
   - Plot resulting from the use of the MATLAB function `qqplot`, for each component of the previously generated MC samples at the final time.

   Compare the results, in terms of accuracy and precision, and discuss on the validity of the linear and Gaussian assumption for uncertainty propagation.

Table 1: Solution for an Earth-Moon transfer in the rotating frame.

Parameter	Value
Initial state $\mathbf{x}_i$	$\mathbf{r}_i$ = [-0.011965533749906, -0.017025663128129] $\mathbf{v}_i$ = [10.718855256727338, 0.116502348513671]
Initial time t_i	1.282800225339865
Final time t_f	9.595124551366348
Covariance $\mathbf{P}_0$	$\begin{bmatrix} +1.041e-15 & +6.026e-17 & +5.647e-16 & +4.577e-15 \\ +6.026e-17 & +4.287e-18 & +4.312e-17 & +1.855e-16 \\ +5.647e-16 & +4.312e-17 & +4.432e-16 & +1.455e-15 \\ +4.577e-15 & +1.855e-16 & +1.455e-15 & +2.822e-14 \end{bmatrix}$

*F. Topputo, "On optimal two-impulse Earth–Moon transfers in a four-body model", Celestial Mechanics and Dynamical Astronomy, Vol. 117, pp. 279–313, 2013, DOI: 10.1007/s10569-013-9513-8.

†Use at least 1000 samples drawn from the initial covariance

Write your answer here

- Develop the exercises in <u>one MATLAB script</u> per each exercise; name the file `lastname123456_Assign2_ExN.m`
- Organize the script in sections, one for each exercise; use local functions if needed.
- Download the <u>PDF</u> from the Main menu.
- <u>Create a single .zip file</u> containing both the <u>report in PDF</u> and the <u>MATLAB files</u>. The name shall be `lastname123456_Assign2.zip`.
- Red text indicates where answers are needed; be sure there is no red stuff in your report.
- In your answers, <u>be concise</u>: to the point.
- **Deadline for the submission: Dec 20 2024, 23:59**.
- **Load the compressed file to the Assignments folder on Webeep**.

Exercise 2: Batch filters

The Soil Moisture and Ocean Salinity (SMOS) mission, launched on 2 November 2009, is one of the European Space Agency's Earth Explorer missions, which form the science and research element of the Living Planet Programme.

You have been asked to track SMOS to improve the accuracy of its state estimate. To this aim, you shall schedule the observations from the three ground stations reported in Table 2.

1. *Compute visibility windows.* The Two-Line Elements (TLE) set of SMOS are reported in Table 3 (and in WeBeep as 36036.3le). Compute the osculating state from the TLE at the reference epoch t_{ref}, then propagate this state assuming Keplerian motion to predict the trajectory of the satellite and compute all the visibility time windows from the available stations in the time interval from $t_0 =$ `2024-11-18T20:30:00.000` (UTC) to $t_f =$ `2024-11-18T22:15:00.000` (UTC). Consider the different time grid for each station depending on the frequency of measurement acquisition. Report the resulting visibility windows and plot the predicted Azimuth and Elevation profiles within these time intervals.

2. *Simulate measurements.* Use SGP4 and the provided TLE to simulate the measurements acquired by the sensor network in Table 2 by:

   (a) Computing the spacecraft position over the visibility windows identified in Point 1 and deriving the associated expected measurements.

   (b) Simulating the measurements by adding a random error to the expected measurements (assume a Gaussian model to generate the random error, with noise provided in Table 2). Discard any measurements (i.e., after applying the noise) that does not fulfill the visibility condition for the considered station.

3. *Solve the navigation problem.* Using the measurements simulated at the previous point:

   (a) Find the least squares (minimum variance) solution to the navigation problem without a priori information using
      - the epoch t_0 as reference epoch;
      - the reference state as the state derived from the TLE set in Table 3 at the reference epoch;
      - the simulated measurements obtained for the KOROU ground station only;
      - pure Keplerian motion to model the spacecraft dynamics.

   (b) Repeat step 3a by using all simulated measurements from the three ground stations.

   (c) Repeat step 3b by using a J2-perturbed motion to model the spacecraft dynamics.

   Provide the results in terms of navigation solution[‡], square root of the trace of the estimated covariance submatrix of the position elements, square root of the trace of the estimated covariance submatrix of the velocity elements. Finally, considering a linear mapping of the estimated covariance from Cartesian state to Keplerian elements, provide the standard deviation associated to the semimajor axis, and the standard deviation associated to the inclination. Elaborate on the results, comparing the different solutions.

4. *Trade-off analysis.* For specific mission requirements, you are constrained to get a navigation solution within the time interval reported in Point 1. Since the allocation of antenna time has a cost, you are asked to select the passes relying on a budget of 70.000 €. The cost per pass of each ground station is reported in Table 2. Considering this constraint,

[‡]Not just estimated state or covariance

and by using a J2-perturbed motion for your estimation operations, select the best combination of ground stations and passes to track SMOS in terms of resulting standard deviation on semimajor axis and inclination, and elaborate on the results.

5. *Long-term analysis.* Consider a nominal operations scenario (i.e., you are not constrained to provide a navigation solution within a limited amount of time). In this context, or for long-term planning in general, you could still acquire measurements from multiple locations but you are tasked to select a set of prime and backup ground stations. For planning purposes, it is important to have regular passes as this simplifies passes scheduling activities. Considering the need to have *reliable* orbit determination and *repeatable* passes, discuss your choices and compare them with the results of Point 4.

Table 2: Sensor network to track SMOS: list of stations, including their features.

Station name	KOUROU	TROLL	SVALBARD
Coordinates	LAT = 5.25144° LON = -52.80466° ALT = -14.67 m	LAT = -72.011977° LON = 2.536103° ALT = 1298 m	LAT = 78.229772° LON = 15.407786° ALT = 458 m
Type	Radar (monostatic)	Radar (monostatic)	Radar (monostatic)
Measurements type	Az, El [deg] Range (one-way) [km]	Az, El [deg] Range (one-way) [km]	Az, El [deg] Range (one-way) [km]
Measurements noise (diagonal noise matrix R)	$\sigma_{Az,El}$ = 125 mdeg σ_{range} = 0.01 km	$\sigma_{Az,El}$ = 125 mdeg σ_{range} = 0.01 km	$\sigma_{Az,El}$ = 125 mdeg σ_{range} = 0.01 km
Minimum elevation	6 deg	0 deg	8 deg
Measurement frequency	60 s	30 s	60 s
Cost per pass	30.000 €	35.000 €	35.000 €

Table 3: TLE of SMOS.

```
1␣36036U␣09059A␣␣␣24323.76060260␣␣.00000600␣␣00000-0␣␣20543-3␣0␣␣9995
2␣36036␣␣98.4396␣148.4689␣0001262␣␣95.1025␣265.0307␣14.39727995790658
```

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Exercise 3: Sequential filters

An increasing number of lunar exploration missions will take place in the next years, many of them aiming at reaching the Moon's surface with landers. In order to ensure efficient navigation performance for these future missions, space agencies have plans to deploy lunar constellations capable of providing positioning measurements for satellites orbiting around the Moon.

Considering a lander on the surface of the Moon, you have been asked to improve the accuracy of the estimate of its latitude and longitude (considering a fixed zero altitude). To perform such operation you can rely on the use of a lunar orbiter, which uses its Inter-Satellite Link (ISL) to acquire range measurements with the lander while orbiting around the Moon. At the same time, assuming the availability of a Lunar Navigation Service, you are also receiving measurements of the lunar orbiter inertial position vector components, such that you can also estimate the spacecraft state within the same state estimation process.
To perform the requested tasks you can refer to the following points.

1. *Check the visibility window.* Considering the initial state $\mathbf{x}_0$ and the time interval with a time-step of 30 seconds from t_0 to t_f reported in Table 4, predict the trajectory of the satellite in an inertial Moon-centered reference frame assuming Keplerian motion. Use the estimated coordinates given in Table 5 to predict the state of the lunar lander. Finally, check that the lander and the orbiter are in relative visibility for the entire time interval.

2. *Simulate measurements.* Always assuming Keplerian motion to model the lunar orbiter dynamics around the Moon, compute the time evolution of its position vector in an inertial Moon-centered reference frame and the time evolution of the relative range between the satellite and the lunar lander. Finally, simulate the measurements by adding a random error to the spacecraft position vector and to the relative range. Assume a Gaussian model to generate the random error, with noise provided in Table 4 for both the relative range and the components of the position vector. Verify (graphically) that the applied noise level is within the desired boundary.

3. *Estimate the lunar orbiter absolute state.* As a first step, you are asked to develop a sequential filter to narrow down the uncertainty on the knowledge of the lunar orbiter absolute state vector. To this aim, you can exploit the measurements of the components of its position vector computed at the previous point. Using an Unscented Kalman Filter (UKF), provide an estimate of the spacecraft state (in terms of mean and covariance) by sequentially processing the acquired measurements in chronological order. To initialize the filter in terms of initial covariance, you can refer to the first six elements of the initial covariance $\mathbf{P}_0$ reported in Table 4. For the initial state, you can perturb the actual initial state $\mathbf{x}_0$ by exploiting the MATLAB function `mvnrnd` and the previously mentioned initial covariance. We suggest to use $\alpha = 0.01$ and $\beta = 2$ for tuning the UT in this case. Plot the time evolution of the error estimate together with the 3σ of the estimated covariance for both position and velocity.

4. *Estimate the lunar lander coordinates.* To fulfill the goal of your mission, you are asked to develop a sequential filter to narrow down the uncertainty on the knowledge of the lunar lander coordinates (considering a fixed zero altitude). To this aim, you can exploit the measurements of the components of the lunar orbiter position vector together with the measurements of the relative range between the orbiter and the lander computed at the previous point. Using an UKF, provide an estimate of the spacecraft state and the lunar lander coordinates (in terms of mean and covariance) by sequentially processing the acquired measurements in chronological order. To initialize the filter in terms of initial covariance, you can refer to the initial covariance $\mathbf{P}_0$ reported in Table 4. For the initial state, you can perturb the actual initial state, composed by $\mathbf{x}_0$ and the latitude

and longitude given in Table 5, by exploiting the MATLAB function `mvnrnd` and the previously mentioned initial covariance. We suggest to use $\alpha = 0.01$ and $\beta = 2$ for tuning the UT in this case. Plot the time evolution of the error estimate together with the 3σ of the estimated covariance for both position and velocity.

Table 4: Initial conditions for the lunar orbiter.

Parameter	Value
Initial state $\mathbf{x}_0$ [km, km/s]	$\mathbf{r}_0$ = [4307.844185282820, -1317.980749248651, 2109.210101634011] $\mathbf{v}_0$ = [-0.110997301537882, -0.509392750828585, 0.815198807994189]
Initial time t_0 [UTC]	2024-11-18T16:30:00.000
Final time t_f [UTC]	2024-11-18T20:30:00.000
Measurements noise	$\sigma_p = 100$ m
Covariance $\mathbf{P}_0$ [km^2, km^2/s^2, rad^2]	`diag([10,1,1,0.001,0.001,0.001,0.00001,0.00001])`

Table 5: Lunar lander - initial guess coordinates and horizon mask

Lander name	MOONLANDER
Coordinates	LAT = 78° LON = 15° ALT = 0 m
Minimum elevation	0 deg

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