Lab 8 - Solving Ax=b

A. Generating Random Matrices and Vectors

The rmat.m function creates a random matrix of m by n and the rvect.m function creates a random vector of m.

```
rand('seed', 2397);
```

B. Ax=b when A is square

```
A = rmat(5,5);
disp(A);
```

```
     4     0     5     1     8
     7     9     8     1     5
     8     1     3     4     3
     8     0     6     8     4
     5     6     1     1     8
```

```
b = rvect(5);
disp(b);
```

```
     0
     5
     7
     4
     6
```

```
Arank = rank(A);
disp(Arank);
```

```
     5
```

Based on the result of the rank command, which resulted in 5, there will be one solution for vector b since all the columns of matrix A are independent.

```
A_aug = [A b];
disp(A_aug);
```

```
     4     0     5     1     8     0
     7     9     8     1     5     5
     8     1     3     4     3     7
     8     0     6     8     4     4
     5     6     1     1     8     6
```

```
R = rref(A_aug);
disp(R);
```

```
    1.0000         0         0         0         0    1.2048
         0    1.0000         0         0         0    0.3349
         0         0    1.0000         0         0   -0.6965
         0         0         0    1.0000         0   -0.1054
         0         0         0         0    1.0000   -0.1539
```

```
x_rref = R(:,6);
disp(x_rref);
```

```
    1.2048
    0.3349
   -0.6965
   -0.1054
   -0.1539
```

```
b_check = cast(A*x_rref,'int8');
if (all(b_check==b))
    disp('x_rref is the solution to Ax=b');
else
    disp('x_rref is not the solution to Ax=b');
end
```

```
x_rref is the solution to Ax=b
```

```

A_inv = inv(A);
if(det(A)==0)
    error('Matrix A has no inverse')
end
I = eye(5);
if (A_inv*A~=I)
    error('Matrix A has no inverse')
end
x_inv = A_inv*b;
disp(x_inv);
```

```
    1.2048
    0.3349
   -0.6965
   -0.1054
   -0.1539
```

```
% the solutions are the same
disp('Same solution as found using rref');
```

```
Same solution as found using rref
```

```

a5 = A(:,1) + A(:,2) + A(:,3) + A(:,4);
A1 = [A(:,1) A(:,2) A(:,3) A(:,4) a5];
disp(A1);
```

```
     4     0     5     1    10
     7     9     8     1    25
     8     1     3     4    16
     8     0     6     8    22
     5     6     1     1    13
```

```
% hyperplane is the linear combination of all columns, so
% copy all columns into A1 for the first four columns and
% then the linear combo of the columns into the fifth column of A1
A1rank = rank(A1);
disp(A1rank);
```

```
     4
```

```
%new rank should be 4
```

```
if (A1rank ==4)
    disp('A1 rank is correct')
else
    disp('A1 rank is wrong')
end
```

```
A1 rank is correct
```

```
A_aug1 = [A1 b];
R1 = rref(A_aug1);
disp(A_aug1);
```

```
     4     0     5     1    10     0
     7     9     8     1    25     5
     8     1     3     4    16     7
     8     0     6     8    22     4
     5     6     1     1    13     6
```

```
disp(R1);
```

```
     1     0     0     0     1     0
     0     1     0     0     1     0
     0     0     1     0     1     0
     0     0     0     1     1     0
     0     0     0     0     0     1
```

Does the resulting R1 matrix show there is a solution or that there is not a solution? Since the matrix has dependent columns, there is no solution because vector b is not in the column space.

```
x1 = rvect(5);
disp(x1);
```

```
     9
     5
     1
     5
     1
```

```
b1 = A1*x1;
disp(b1);
```

```
    56
   146
   116
   140
    94
```

Since b1 is created from A1*x1, the b1 will be in the column space of the matrix, resulting in a solution.

```
A_aug2 = [A1 b1];
disp(A_aug2);
```

```
     4     0     5     1    10    56
     7     9     8     1    25   146
     8     1     3     4    16   116
     8     0     6     8    22   140
     5     6     1     1    13    94
```

```
R2 = rref(A_aug2);
```

```
disp(R2);
```

```
     1     0     0     0     1    10
     0     1     0     0     1     6
     0     0     1     0     1     2
     0     0     0     1     1     6
     0     0     0     0     0     0
```

```
x2 = R2(:,6);
disp(x2);
```

```
    10
     6
     2
     6
     0
```

What is the solution that can be extracted from R2? The x2 above is the extracted solution.

```
b2check = A1*x2;
if (all(b1==b2check))
    disp('x2 is a solution to b1')
else
    disp('x2 is not a solution to be b1')
end
```

```
x2 is a solution to b1
```

Perform a computation to verify that it is a solution. Perform that same computation to verify that x1 is also a solution! x1 is used to calculate b1, so it is already a solution and the computation to check x2 is performed above.

Why are there two different solutions?

There are two different solutions because R2 results in infinitely many solutions since b1 is in the column space of A1 and A1 has dependent columns.

C. Ax=b when A is Short Rectangular

```
clear;

A = rmat(3,5);
b = rvect(3);
Arank = rank(A);
disp(Arank);
```

```
     3
```

There are infinitely many solutions for matrix A because the matrix is full rank and the b lies in the column space.

```
A_aug = [A b];
disp(A_aug);
```

```
     3     7     0     3     8     2
     0     7     0     2     9     5
     9     1     4     4     9     3
```

```
R = rref(A_aug);
disp(R);
```

```
    1.0000         0         0    0.3333   -0.3333   -1.0000
         0    1.0000         0    0.2857    1.2857    0.7143
         0         0    1.0000    0.1786    2.6786    2.8214
```

```
x1 = [R(:,6);0;0];
disp(x1);
```

```
   -1.0000
    0.7143
    2.8214
         0
         0
```

```
b1 = cast(A*x1,'int8');
if (all(b1 == b))
    disp('x1 is the solution to Ax=b');
else
    disp('x1 is not the solution to Ax=b');
end
```

```
x1 is the solution to Ax=b
```

```
x0 = [R(:,4);-1;0];
disp(x0);
```

```
    0.3333
    0.2857
    0.1786
   -1.0000
         0
```

```
b0 = A*x0;
b0 = cast(b0,'int8');
disp(b0);
```

```
   0
   0
   0
```

```
if (all(b0==0))
    disp('Result is approximately 0');
else
    disp('Result is not approximately 0');
end
```

```
Result is approximately 0
```

```
b_check = A*(x1+x0);
b_check = cast(b_check,'int8');
if (all(b_check==b))
    disp('x1+x0 is also a solution');
```

```
else
    disp('x1+x0 is not a solution');
end
```

```
x1+x0 is also a solution
```

```

for a = 2:3:10
    disp(a);
    b_check = A*(x1+(a*x0));
    b_check = cast(b_check,'int8');
    if (all(b_check==b))
        disp('Above a works as a solution for x1+a*x0');
    else
        disp('Above a does not work as a solution for x1+a*x0');
    end
end
```

```
     2
Above a works as a solution for x1+a*x0
     5
Above a works as a solution for x1+a*x0
     8
Above a works as a solution for x1+a*x0
```

Distributive Property

A(x1 + a*x0) = b

Using distributive property => A*x1 +a(A*x0) = b

Since A*x0 is approximately 0, the equation becomes A*x1 = b.

Therefore, any number a is also a solution through the distributive property.

D. Ax=b when A is Tall Rectangular

```
clear;
A = rmat(5,3);
Arank = rank(A);
disp(Arank);
```

```
     3
```

Since A is a tall matrix with full rank, it needs the vector b to be in the column space for there to be a unique solution.

```
b = A(:,1)+A(:,2)+A(:,3);

A_aug = [A b];
disp(A_aug);
```

```
     3     5     3    11
     8     6     1    15
     4     1     6    11
     5     5     2    12
     3     1     5     9
```

```
R = rref(A_aug);
disp(R);
```

```
     1     0     0     1
     0     1     0     1
     0     0     1     1
     0     0     0     0
     0     0     0     0
```

```
x_sol = R(1:3,4);
disp(x_sol);
```

```
     1
     1
     1
```

```
b_check = A*x_sol;
b_check = cast(b_check,'int8');
if (all(b_check==b))
    disp('x is a solution');
else
    disp('x is not a solution');
end
```

```
x is a solution
```

E. Summary

The lab analyzes three different matrix cases: Square Matrix A, Short Rectangular Matrix A, and Tall Rectangular Matrix A. For square matrix A, there were two different scenarios. The first scenario was a full rank square matrix A, which had one solution because the b was in the column space. The second scenario was a not full rank square matrix A that had no solution when b was not in the column space and infinitely many solutions when b was in the column space. For short rectangular matrix A, there will be infinitely many solutions when the matrix is full rank and b is in the column space. If you take an x from a column that is not the last column of the reduced row echelon form of matrix A, you can find an x that will result in an approximate 0 vector. For tall rectangular matrix A, there will be an unique solution when the matrix is full rank and the b is in the column space.