Sodden*

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Abstract

If you need to walk 1*km in a downpour and don't like being wet, you'll be drier at the end if you get it over with as quickly as possible, conditional on you looking somewhat like a box.*

Keywords— water, rain, walking, perhaps biking

What does it mean to walk in the rain, *man*?

Toy model

First, let's lay down some reasonable assumptions:

- rain falls more or less straight down with speed w
- rain exists in a continuous uniform density over $\mathscr{R}^3$
- a person roughly resembles a box with top area A, front area B, and side area C
- this person unfortunately absorbs all water they touch

Thus, in the infinitesimal interval dt, a pedestrian collects a volume $A \cdot w \cdot dt$ of water atop their head, and similarly, a volume $B \cdot v \cdot dt$ on the front of their body, given they are moving at the speed v.

Integrating these two water contributions over the travel period (0 until $T = 1(km)/v$), we can find the total amount of water collected by the pedestrian,

$$V = \int_0^{\frac{1}{v}} (Aw + Bv)dt \tag{1}$$

We would like to minimize this volume by choosing the appropriate speed v.

*This paper provides a simplified version of the structural model-making process, from axioms to revelations. It could make for a nice homework problem in a dynamic optimization class, or something.

Taking a derivative

Taking a derivative of equation (1) is straightforward. Following Leibniz's rule,

$$\frac{\partial V}{\partial v} = \frac{\partial}{\partial v}\int_0^{\frac{1}{v}} (Aw + Bv)dt = -\frac{1}{v^2}(Aw + Bv) + \int_0^{\frac{1}{v}} Bdt = -\frac{1}{v^2}(Aw + Bv) + \frac{B}{v} \tag{2}$$

where the first term is the change in volume effected from a change in the total travel time T and the second term is the change in volume associated with a change in the rate at which water accumulates on the front of the pedestrian.

Simplifying the right-hand side of equation (2), we have

$$\frac{\partial V}{\partial v} = -\frac{Aw}{v^2}, \tag{3}$$

which is always negative. Thus increasing travel velocity v will decrease the amount of volume that drenches the pedestrian by the end of their trip.

Interpretation of this result

The right-hand side of equation (3) gives us the main incentive to move quickly. In every instant dt the pedestrian absorbs $A \cdot w \cdot dt$ on their head, and this has nothing to do with their speed. Thus they should attempt to avoid as many of these instances as possible, and increasing v decreases this accumulation by the quantity above.

More interestingly, the countervailing effects of moving more quickly and finishing early are a *wash* when we only consider water accumulated on the front of the pedestrian. To finish dt sooner means to scoop up a "sheet" of water $B \cdot v \cdot dt$ one instant sooner. Our pedestrian must travel through a great number of these sheets totaling volume $B \cdot (1km)$, which is not a function of speed; we should not be surprised that these effects "cancel out."

In a way, a simple intuition check would involve looking at two degenerates, one who teleports from location to location ($v = \infty$, a physical impossibility but mathematical limiting case), and another who forgets to move ($v = 0$), thus taking an eternity to finish their trip. Note that the teleporter fails to get wet and the sloth becomes infinitely waterlogged during their respective "trips." These limit results arise from the above model.

Validity

The exercise above likely contains enough detail to satisfy the pub-style question and nothing more. Any further model complications would change the spirit of the present investigation and should be ignored, like the one on the following page.

Appendix: Wind

If we choose to include wind with orthogonal component $u_\perp$ and parallel component $u_{||}$ (w.r.t. the direction of travel), equation (3) becomes

$$\frac{\partial V}{\partial v} = -\frac{1}{v^2}(Aw + Cu_\perp + B|v + u_{||}|) + \frac{B}{v}\frac{(v + u_{||})}{|v + u_{||}|} \tag{4}$$

where $u_{||} > 0$ denotes a headwind.

Note that the orthogonal component enters the derivative in a manner similar to the right-hand side of equation (2). In the case of a headwind, so does the parallel wind term, and the travel speed terms vanish as before.

However, in the case of a *tailwind* ($u_{||} < 0$), the result depends on the relative strength of the crosswinds (and the downward speed of rainfall) and size of the top (A) and side area (C), compared to the front area B. For weaker winds and lanky people, it becomes best to move at the speed of the tailwind, therefore avoiding any accumulation of rain on their front or back sides. For hurricanes and rounder folk, the strategy of walking as quickly as possible dominates. The proof of this is left as a numerical exercise for the reader.