Stochastic analysis of tipping points

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1 Introduction and Overview

The project consists in the implementation of a stochastic version of DICE able to integrate phenomena of uncertain nature as the occurrence of tipping points. Specifically, in our work, two generic types of tipping points have been considered: one that modifies the climate sensitivity, thus leading to a larger temperature increase for the same emissions, while another that directly increases the amount of emissions. Although we have remained on a general level, without including data referring to specific tipping points, these two cases can be easily associated to real catastrophic phenomena such as the ice-sheets disintegration, the rainforest dieback and the more frequent droughts or the permafrost melting. Furthermore, this more flexible approach chosen allowed us to perform sensitivity analyses including in this way a wider range of phenomena and better capturing the uncertainty that characterizes tipping points.

2 Stochastic Model

The starting point is the DICE model which we chose for its computational simplicity and because it's sufficient for our analysis. DICE maximize the following objective function:

$$Objective = \sum_{t=1}^{T_{max}} Utility(t) \tag{1}$$

Instead, when considering a stochastic model, the utility function does not depend only on time but also on the state of the world in which you are. To better understand how to calculate the objective function we can consider the simple case with only one tipping point. As shown in fig. 1, at the time instant $t = 2055$ the timeline divides in two branches, the catastrophic red one and the optimistic green one. We are considering that the tipping point can occur with probability H. In this case the stochastic model therefore maximize the expected utility:

$$Objective = \mathbb{E}\left[\sum_{t=1}^{T_{max}} Utility(t,w)\right] = \sum_{w} \mathbb{P}(w) \sum_{t=1}^{T_{max}} Utility(t,w) \tag{2}$$

More specifically to fig. 1:

$$\begin{aligned} Objective &= ... + u(9,0) + u(10,0) + (1-H)(u(11,0) + u(12,0) + ..) + H(u(11,1) + u(12,1) + ..) + ... = \\ &= ... + (1-H)(u(9,0) + u(10,0)) + H(u(9,0) + u(10,0)) + ... = \\ &= (1-H)(UpperBranchUtility) + H(LowerBranchUtility) \end{aligned}$$

Specifically, in our work, we are restricted due to computational complexity to consider that only every 10 timesteps (50 years) there can happen the two generic types of tipping points.
This results in 4 branching possibilities every 10 timesteps depending on which of those happens: only the first one, only the second one, both or none, leading to a total number of 25 paths.
As far as concern the probability H of the tipping point to occur, called hazard rate, depends on the temperature since higher average temperature will raise the possibility of catastrophe. In order to evaluate it we refer to the formula used in Derek Lemoine (2014) smoothing it using an hyperbolic tangent:

$$H(t,w) = \frac{tanh(\beta * (TATM(t,w) - T_{Ref})) - 1}{2} \tag{3}$$

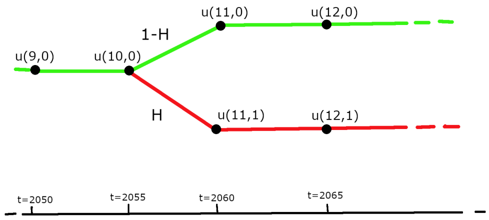

Figure 1: The two states of the world as time changes, the red is catastrophic while the green is optimistic.

This formula depends on two parameters, β and T_{Ref} , which respectively affect how abruptly the probability jumps from zero to one and around which temperature it does so, as it can be seen in fig. 2.

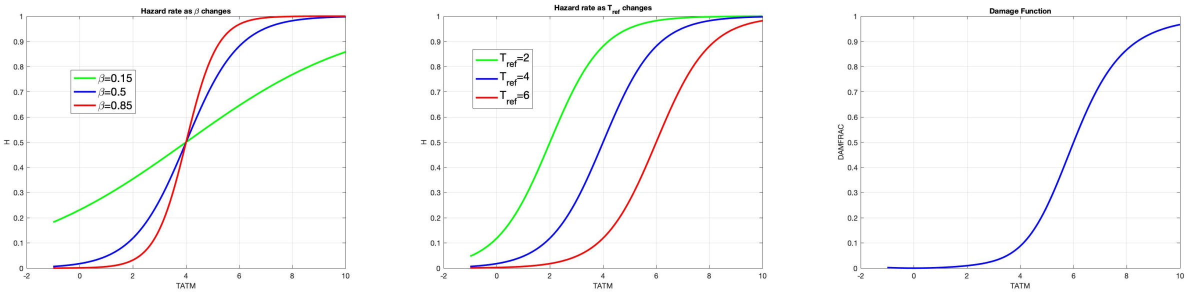

Figure 2: Hazard Rate respectively by varying β and T_{ref} and Damage Function

The other change made compared to DICE is the expression of DAMFRAC. In order to capture better the damage of tipping points, penalizing more especially with high increases in temperature (> 3 C°), we introduced the following damage function proposed in Weitzman (2012):

$$DAMFRAC(t,w) = 1 - \frac{1}{1 + (\frac{TATM(t,w)}{20.64})^2 + (\frac{TATM(t,w)}{6.081})^{6.754}} \tag{4}$$

In particular, inspired by Loïc Berger (2016), we assumed that the tipping point would have an impact on the economy damages and we implemented this by multiplying this damage function by a parameter $k \in [1.1, 1.3]$ if any catastrophy happens and $k \in [1.3, 1.6]$ if both happen.
For the specific effects of a single tipping point we modified, referring to Garth Heutel (2016), also the parameter regulating the sensitivity to carbon $tcre \in [1.8, 3]$ and the absorption of industrial emissions that was increased by 27 %.

3 Results

We first look at the output of our model. In fig 3 we see the comparison between scenarios where a tipping point has happened at timestep 10 or at timestep 20. Before the CTP, the behaviour is common for all scenarios, whereas after we have 4 ramifications. One catastrophy has an immediate effect, and causes the temperature to quickly grow. The other one implements a more subtle change that influences only long term. We can then compare DICE with stochastic DICE. As it can be seen in fig. 4 emissions always stay below the Vanilla Dice case (except some boundary effects on the last years) and consequently the temperature, when no tipping points occurs, tends to stay below 2.5 C°, which is consistent to the Paris agreements. This fact highlights that policies are quite good while DICE is not strict enough.
It's interesting to notice some peaks in the emission graphic, this is due to the discretiziation step used for the times in which a tipping point can occur. The optimizer, before these steps, tries to keep the emissions as low as possible in order to prevent the CTP from occurring, but once the CTP is not verified it soon increases the emissions because the next possible happening of the CTP is in 50 years and so this gives more freedom. This problem would be easily solved by decreasing the discretization step, and

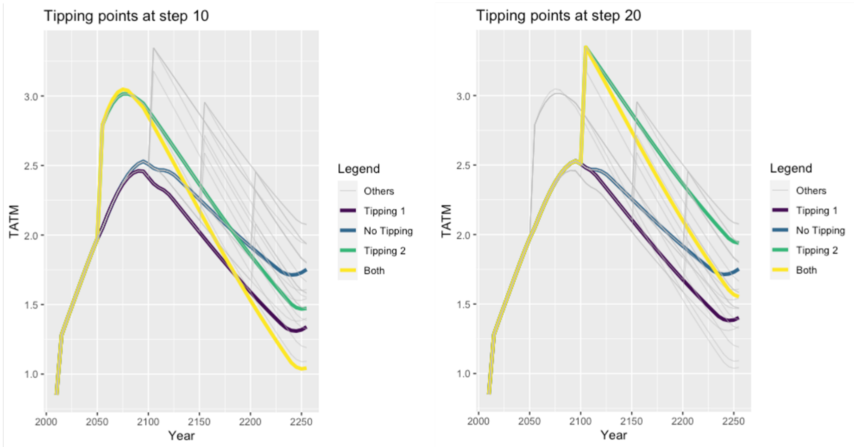

Figure 3: Comparison in the results between CTPs happen at timestep 10 and timestep 20

consequently the line in the graphic would be much smoother, but we decided to keep like this in order to avoid a huge computational effort (just decreasing to 5 time step instead of 10 would require hours to the solver to find the optimal solution).
Lastly we kept the same time horizon as in the vanilla version of DICE since tipping points are events with long term repercussions and effects.

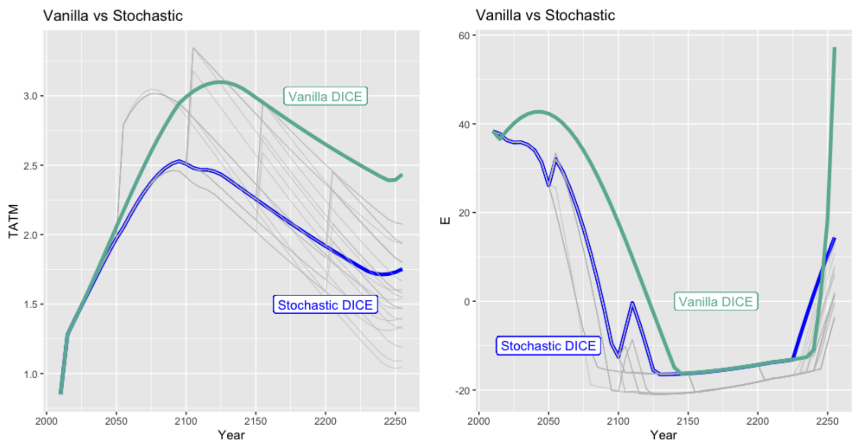

Figure 4: Comparison in the results between Vanilla Dice and the Stochastic one (especially the path with no CTP happening)

4 Sensitivity analysis

We perform finally a sensitivity analysis on the main parameters we used in the code such as the damage coefficient and the climate sensitivity coefficient. These arbitrary parameters as well as β and T_{ref} allowed us to perform an analyses capturing a wide range of results to make the prediction richer and more flexible, in accordance with the high rate of uncertainty that characterizes tipping points, as well as all phenomena concerning climate change. The model resulted as sufficiently robust, so we were very satisfied

5 Conclusions

The conclusions of our work are that scenarios predicted by Vanilla DICE should not be enough to avoid catastrophic consequences. Instead, the European objective of reaching carbon neutrality within 2050 are much more aligned with our simulation. The model could be further improved using a localised version and/or with a finer timestep for CTPs.

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