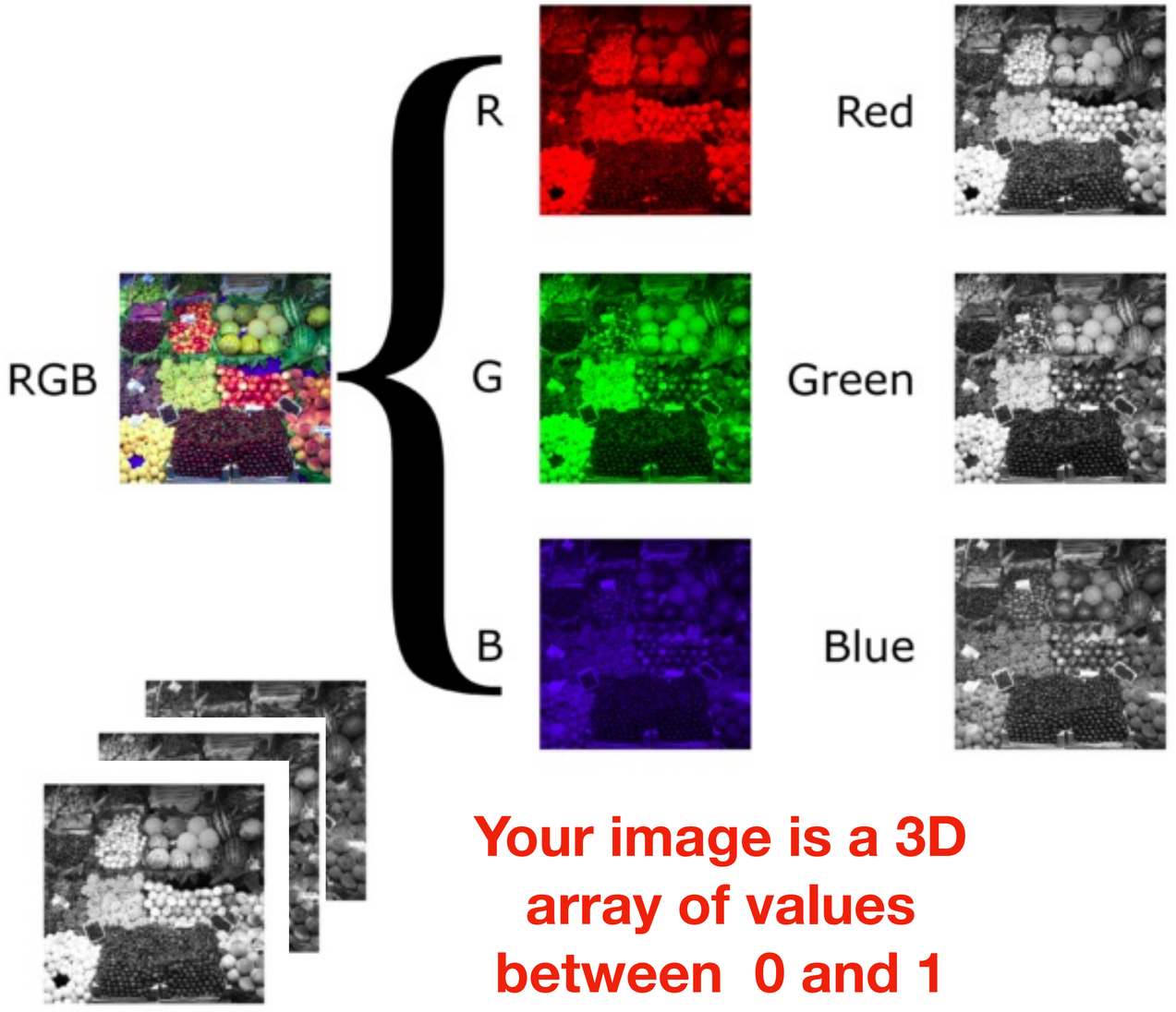
Lecture 4: Convolutional Neural Networks

Program for Today

Date	Topic	Tutorial
Sep 17	Intro & class description	Linear Algebra in a nutshell + prob and stat
Sep 24	Basic of machine learning + Dense NN	*Basic jupyter + DNN on mnist (give jet dnn as homework)*
Oct 1	Convolutional NN	*Convolutional NNs with MNIST*
Oct 8	Training in practice: regularization, optimization, etc	*Practical methodology*
Oct 15	*Tensor Flow tutorial (tbc)*	
Oct 22	Recurrent NN	*Tutorial on RNNs*
Oct 29	Graph NNs	*Tutorial on Graph NNs*
Nov 5	Unsupervised learning and anomaly detection	*Autoencoders with MNIST*
Nov 12	Generative models: GANs, VAEs, etc	Normalizing flows
Nov 19	Spiking Neural Network	*Tutorial on neuromorphic chips & spiking NNs (tbc)*
Nov 26	Network compression (pruning, quantization, Knoweldge Distillation)	
Dec 3	Tutorial on hls4ml/qkeras	
Dec 10	Transformers	
Dec 17	tbd	

Digital Image Processing

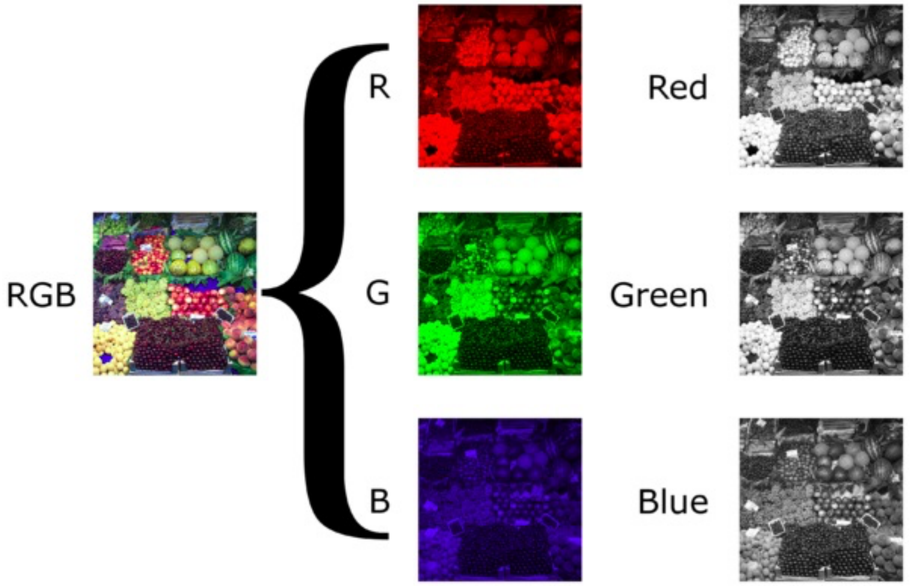
- *Each image is a matrix of pixels*
- *Each pixel comes with a color*
- *In RGB scheme, these are three colour values defined in [0,1]*
- *Each image becomes a 3D tensor*
  - *3 channels of 2D pixelated images*
- *Digital images can be processed with Convolutional Neural Networks*

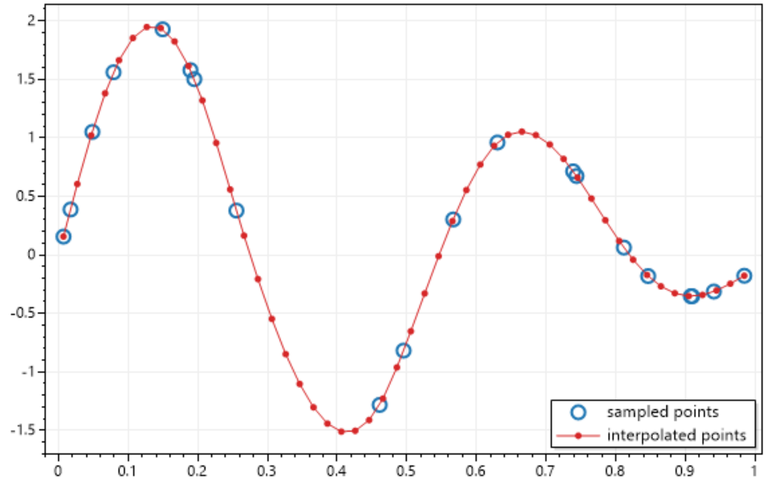


Your image is a 3D array of values between 0 and 1

What is a CNN

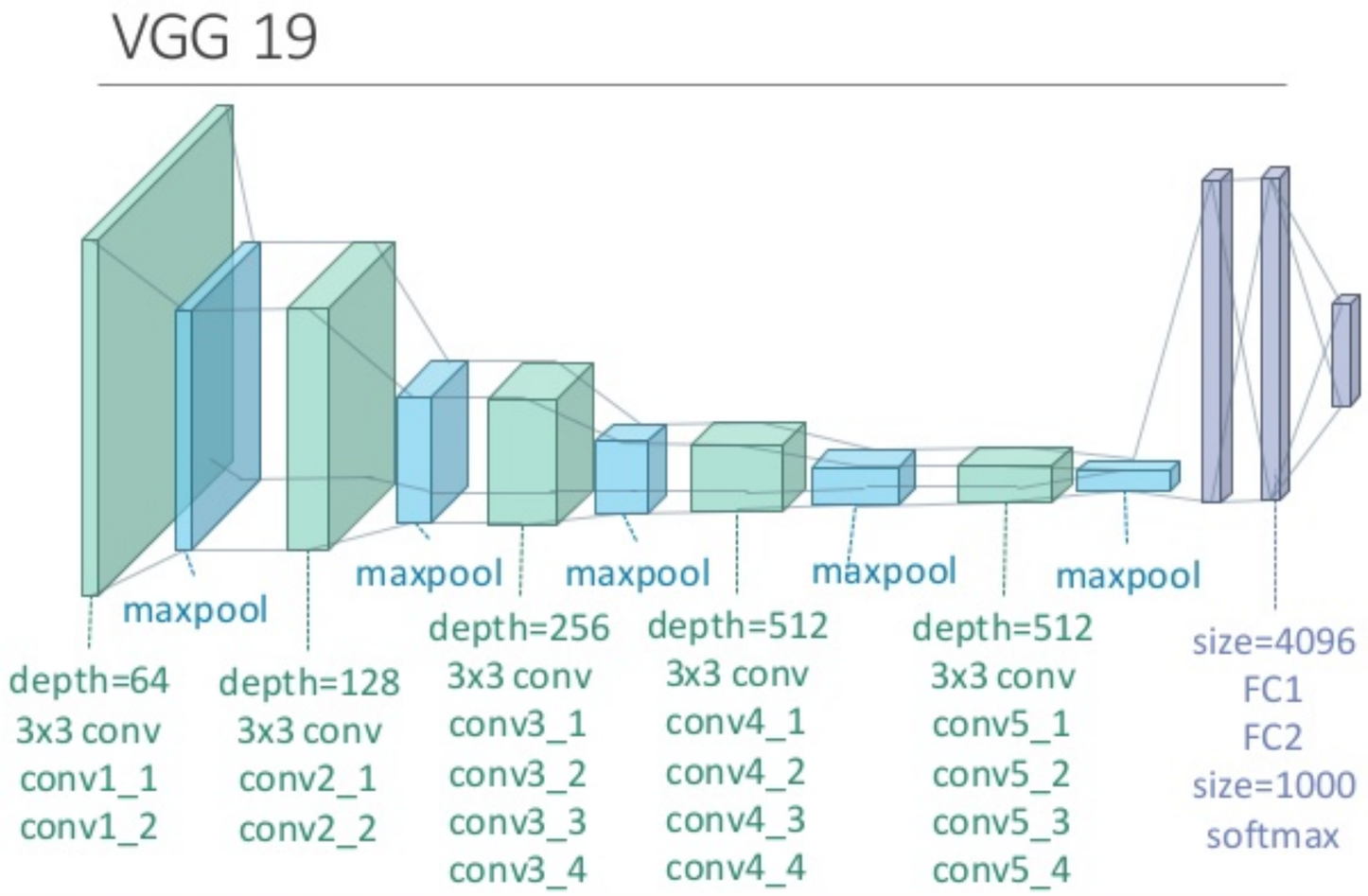
- *A special kind of NN designed process data coming in the form of a regular array*
  - *An image, coming in RGB format*
  - *A time series, i.e., the recording of a given readout at fixed intervals of time*
  - *…*
- *The name of the network comes from the mathematical operation that the network applies to the data: Convolutional networks use convolution in place of general matrix multiplication in at least one of their layers.*





A three-step process

- *A CNN processing comes in three steps. You could see it as a sequence of three layers*
  - *Convolution: the actual image processing (inspired in structure by pre-Deep-Learning image processing)*
  - *Activation: same as DNN*
  - *Pooling: an array manipulation to reduce the image dimensionality, typically compensated by channel increase*



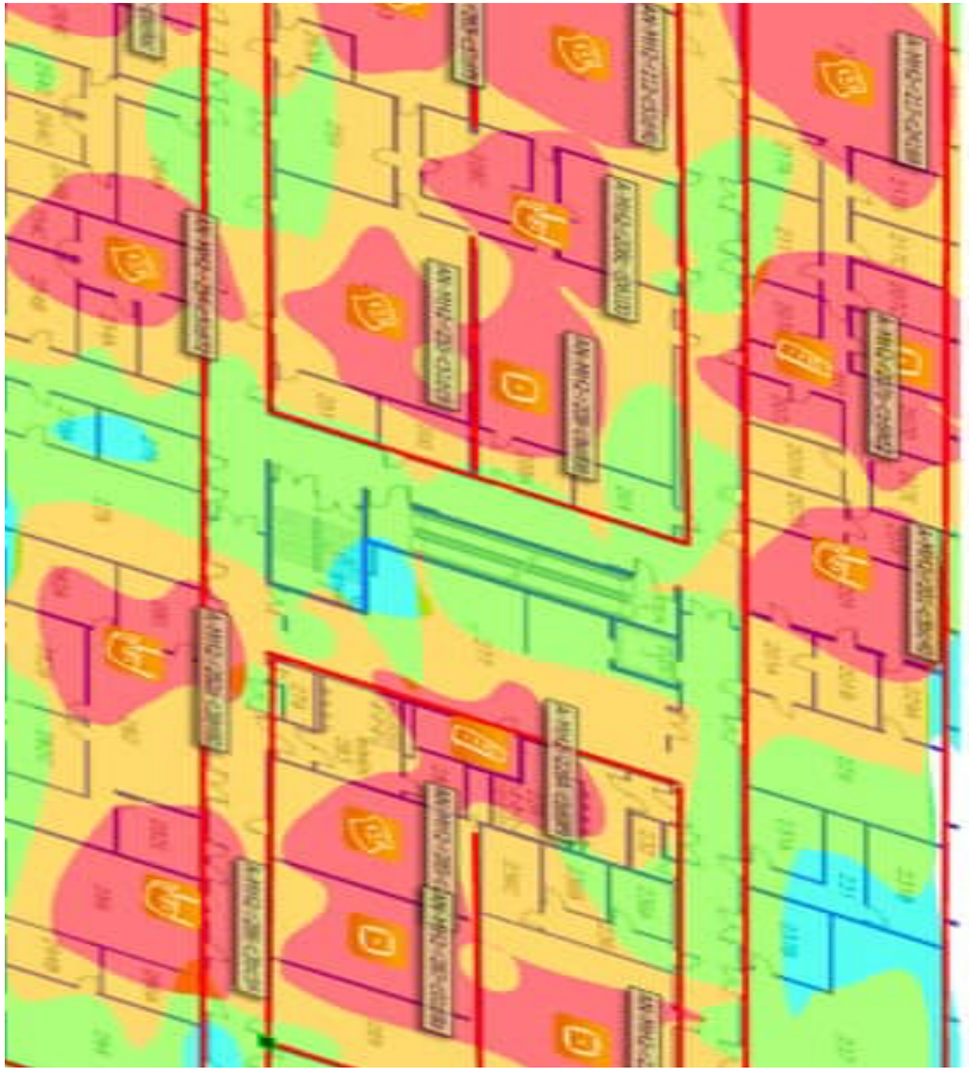
What is a convolution

- *Example: you want to know how good is the signal on your cell phone*
  - *You know the signal strength as a function of position* $f(x)$
  - *You know the phone position through the coordinates returned by the GPS*
    - *But the GPS has a Gaussian resolution uncertainty* σ*. For a given value of* x*, the GPS returns some* x'*, distributed according to* $G(x - x'|\sigma)$
  - *To know the signal strength, you need to average across all the values of* x*, weighted by the probability that for that given value of* x *the GPS returns* x'

The input **The convolution kernel**

$$S(x') = \int dx\, f(x) * G(x' - x) \equiv (f * G)(x')$$

This is the convolution of f with G
G has to be a positive defined function (a pdf)



A discrete convolution

- Our data is a discrete (1D, 2D, …, nD) array of values
  - For a 1D array, need to define the discrete equivalent of the convolutional integral, which trivially is

$$(f * g)(t_i) = \sum_j f(a_j) g(t_i - a_j)$$

  - At 2D, for an image I of pixel I(i,j) and a kernel given by some function K(i,j) of the same pixel coordinates (i,j),

$$S(i,j) = (I * K)(i,j) = \sum_k \sum_l I(k,l) K(i-k, j-l) = \sum_k \sum_l I(i-k, j-l) K(k,l)$$

You can convince yourself of the last identity defining $k = i - k'$, $l = j - l'$, and then redefining $k' \to k$ and $l' \to l$. Notice that for a given point (i,j) the convolution moves towards lower coordinates while one moves fwd in the kernel coordinates

CNN use Cross correlation

- *Typically, CNNs don't apply the convolution we just described*
- *Instead, they apply a similar expression, called cross correlation*

$$S(i,j) = (I * K)(i,j) = \sum_k \sum_l I(i+k, j+l) K(k,l)$$

- *In this case, one moves coherently across the input image and the kernel*

$I(i,j)$

$i \in [0,6]$

$j \in [0,6]$

0	3	5	6	2	4	5
7	4	7	3	6	3	4
9	1	2	1	9	6	0
9	2	1	1	7	3	5
8	0	4	7	6	8	0
8	3	4	5	5	3	4
7	9	4	6	5	2	6

$K(k,l)$

$k \in [0,2]$

$l \in [0,3]$

4	-1	4
-2	2	-5
3	1	-6

S(0,0) = 0x4 - 3x1 + 5x4 + -7x2 + 4x2 - 7x5 + 9x3 + 1x1 - 2x6 = -8

-8				

CNN use Cross correlation

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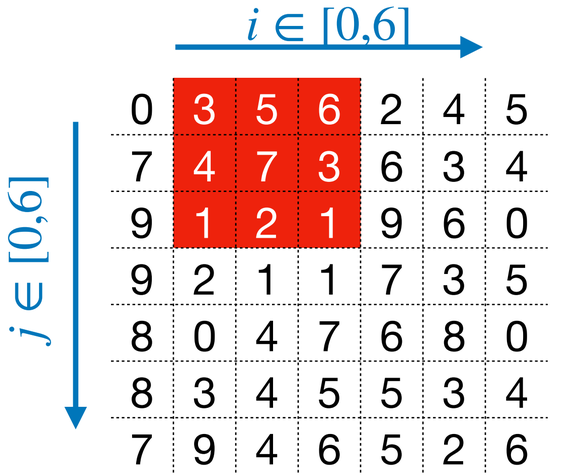
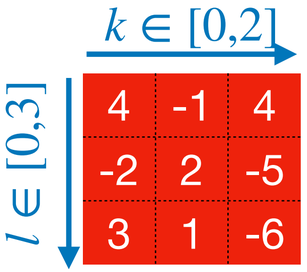


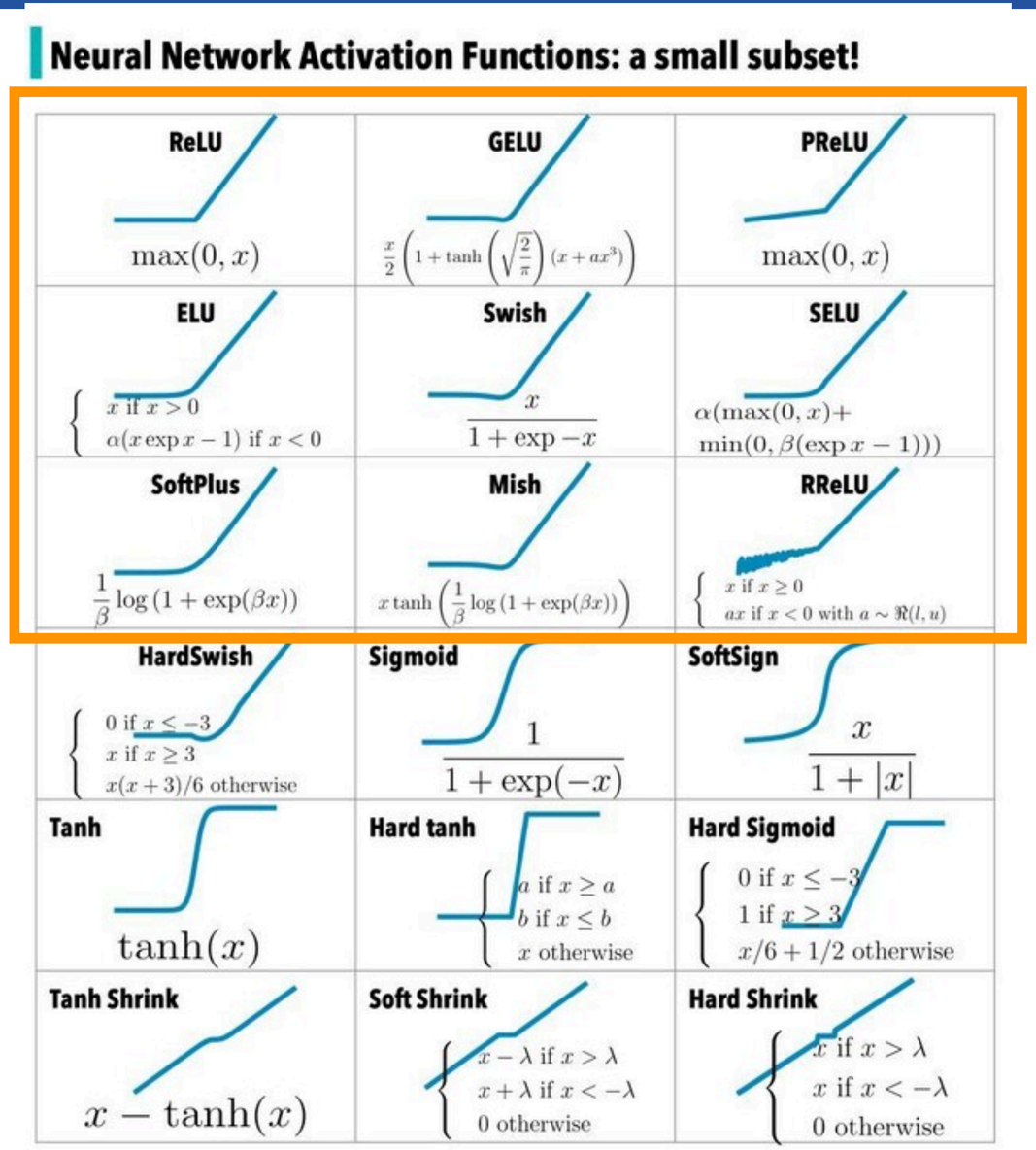
$K(k,l)$



S(1,0) = 3x4 - 5x1 + 6x4 + -4x2 + 7x2 - 3x5 + 1x3 + 2x1 - 1x6 = 21

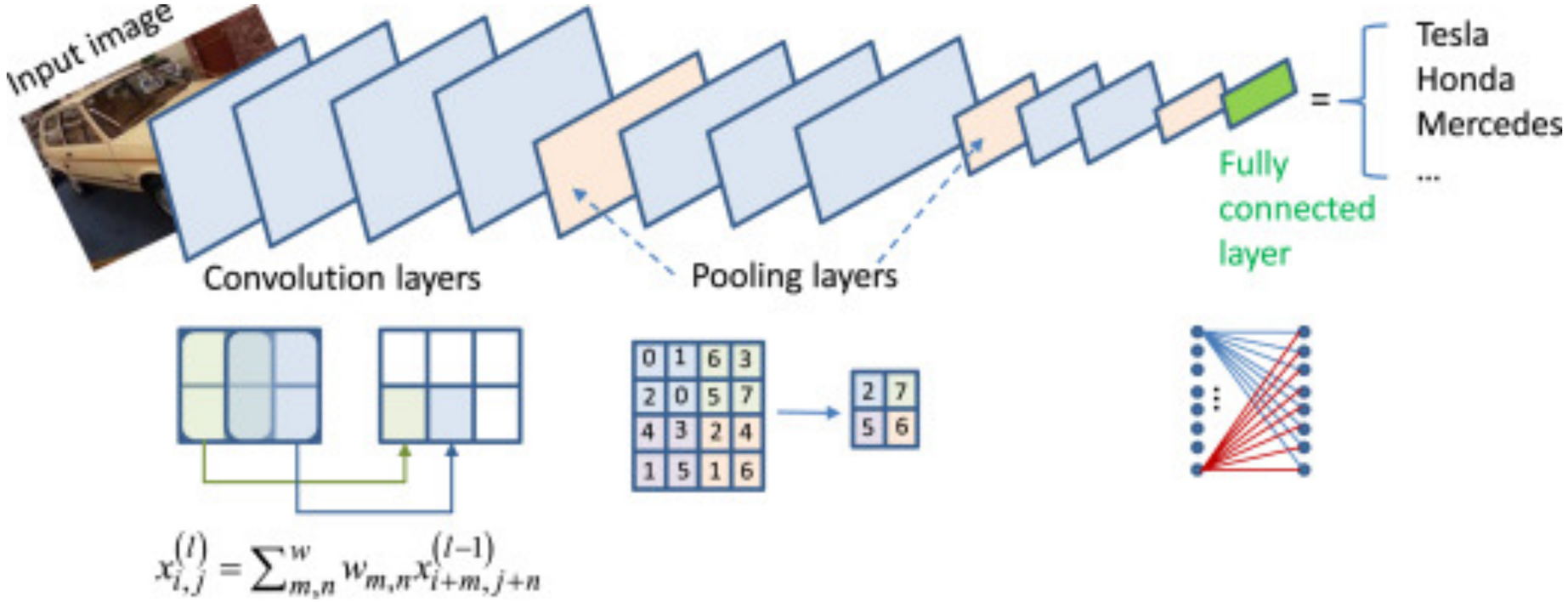
After convolution: activation

- *Activation functions are the essential ingredient to NNs complexity*
  - *Bring non-linearity in NN processing. Crucial to increase complexity*
  - *The choice of the activation function is a hyperparameter*
  - *In practice, the fastest is the function, the more efficient is the learning*
    - *Recent development on functions with fast gradient calculation*



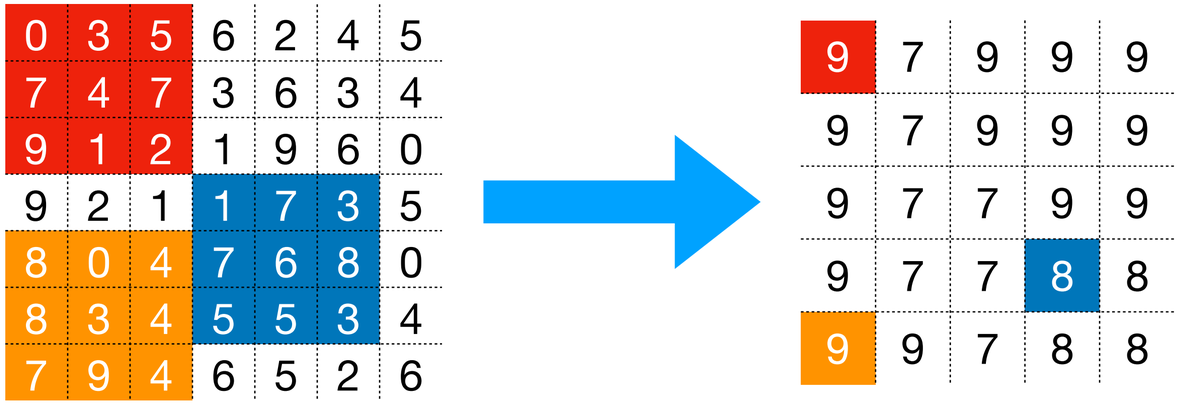
Pooling

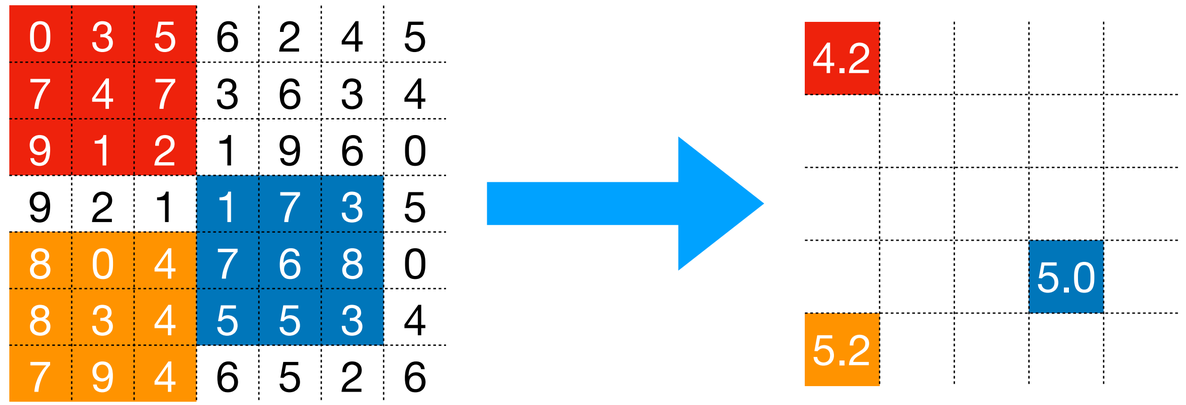
- *A pooling function is a manipulation of an array, in which the content of a pixel is replaced by a function of the nearby pixels*
- *As a result, the image size is reduced.*



Pooling

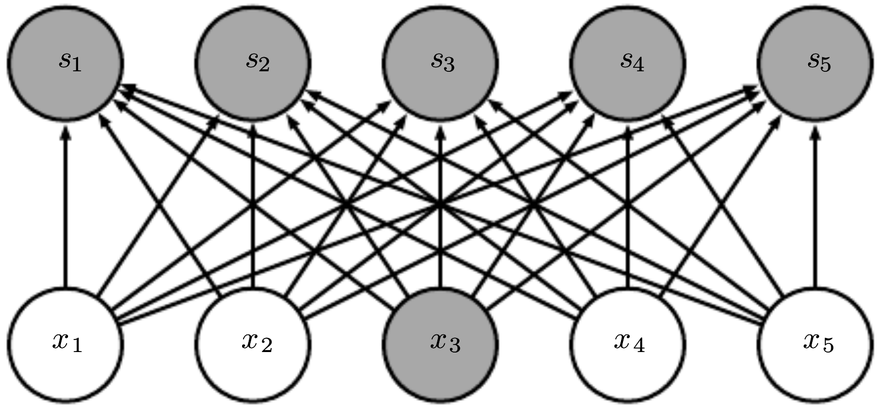
- *MaxPooling: Given an image and a filter of size k x k', scans the image and replaces each k x k' patch with its maximum*
- *AveragePooling: Given an image and a filter of size k x k', scans the image and replaces each k x k' patch with its average*
- *…*



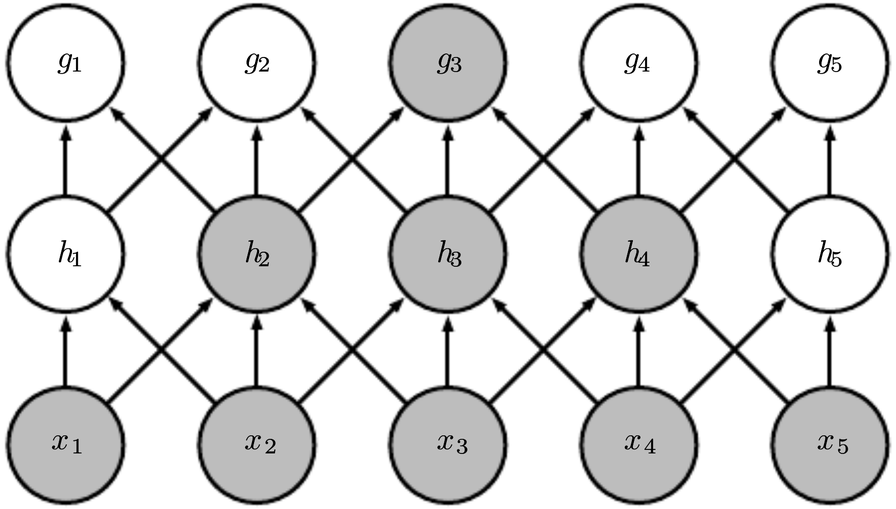


Why do CNN work?

- *Sparse interactions: in a DNN, each output unit interacts with every input unit, increasing the number of parameters. In a CNN instead, the kernel is smaller than the image. A kernel with a few parameters might identify a given feature (an edge) over millions of input pixels.*
  - *less parameters to reach large complexity*
  - *better computational efficiency (e.g., during training)*

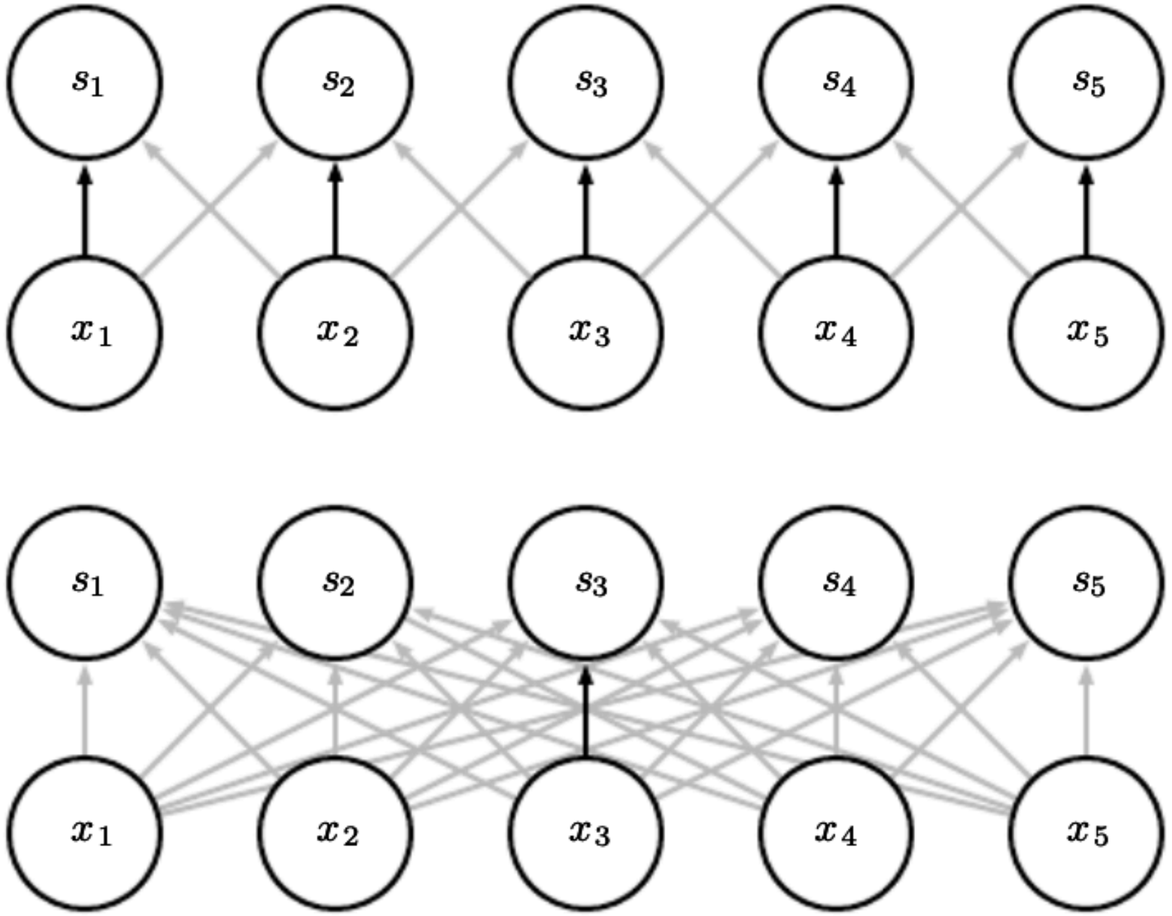


One extra layer but much less connections (i.e., parameters) reach the same receptive field



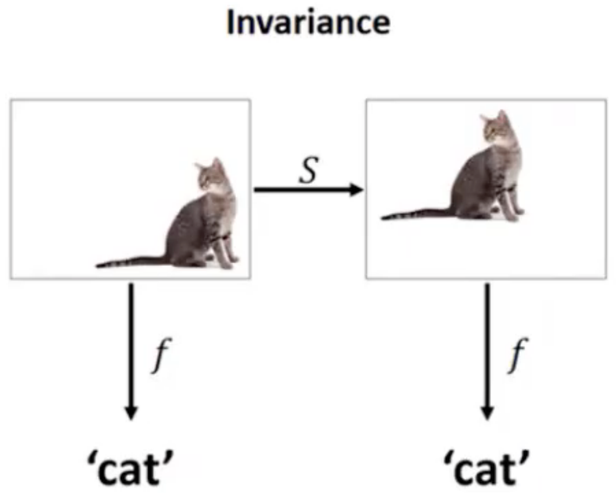
Why do CNN work?

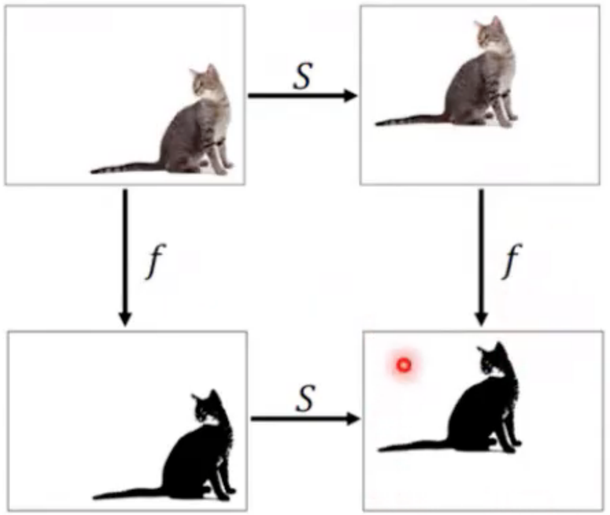
- *Parameter sharing: In a CNN, the same parameter acts on various parts of the network (dark arrows). In a DNN, each parameter is used once. While DNN parameters learn tasks related to specific inputs, a CNN parameter learns across the full image*
  - *Sharing weights has computational advantages, e.g., less required memory*



Why do CNN work?

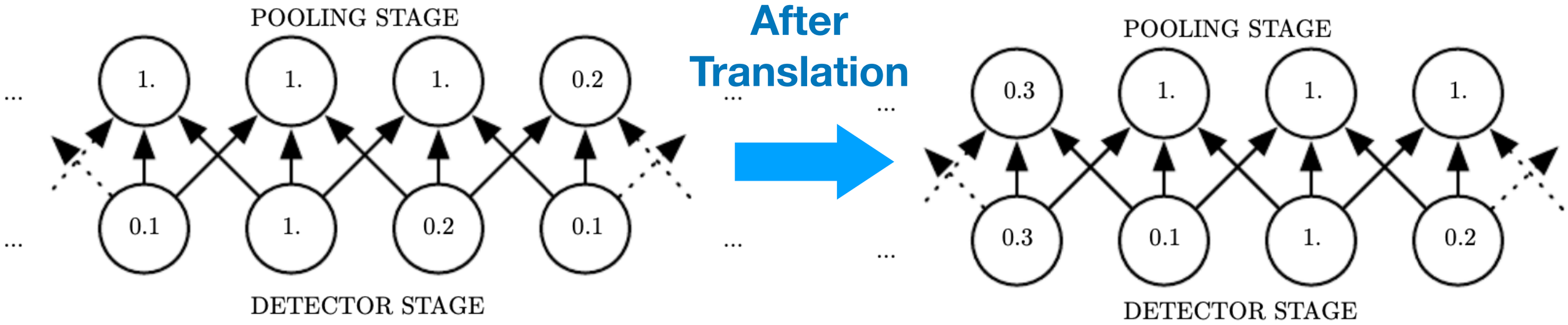
- *Equivariant representation: thanks to parameter sharing, CNN are equivariant to translation: if the input is shifted, so is the output*
- *A function* $f(x)$ *is equivariant to a function* $g(x)$ *if* $f(g(x)) = g(f(x))$
- *For CNN*
  - *Consider the operation* $S(i) = i - 1$ *that shifts the each pixel to the right (it moves the pixel* $i - 1$ *to position* i*)*
  - *(Modulo border effects), for an image I it is true that* $S(f(I)) = f(S(I))$
- *Having architectures equivariant to the problem symmetries make the learning much faster*
  - *If a DNN has to learn that to give the same answer for* I *and* $f(I)$*, it needs a much larger dataset (a dataset augmented adding all the* $S(i)$ *images), which implies a longer training process*
  - *This is built in the CNN architecture, which then arrives faster to the loss minimum*





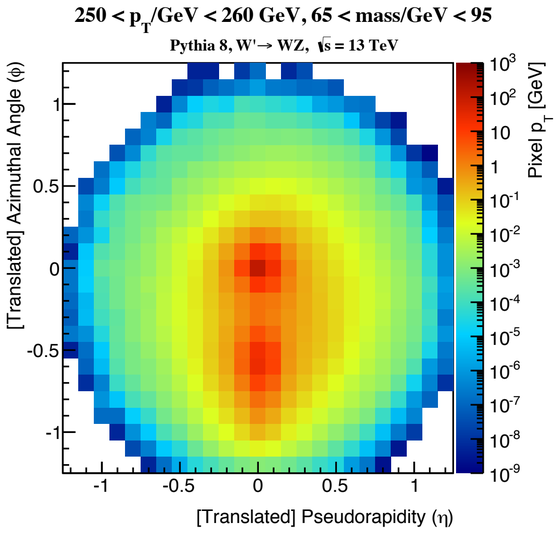
Approximate Invariance

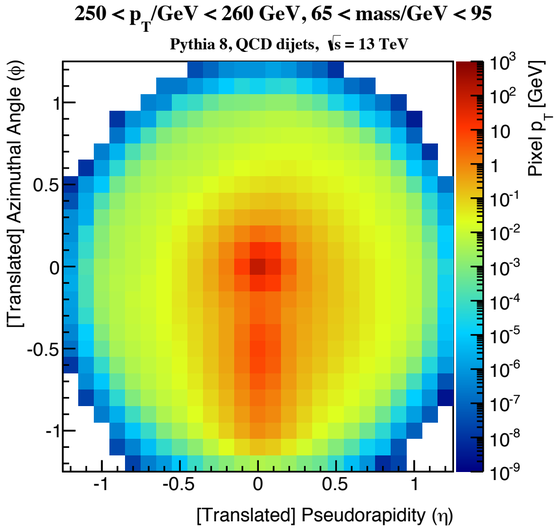
- *With pooling, network equivariant is promoted to approximate invariance: the output is about the same for small translations*
  - *See the example of MaxPooling*
- *This is imposing a strong prior via architecture choice to the learning process*



Pooling Choice

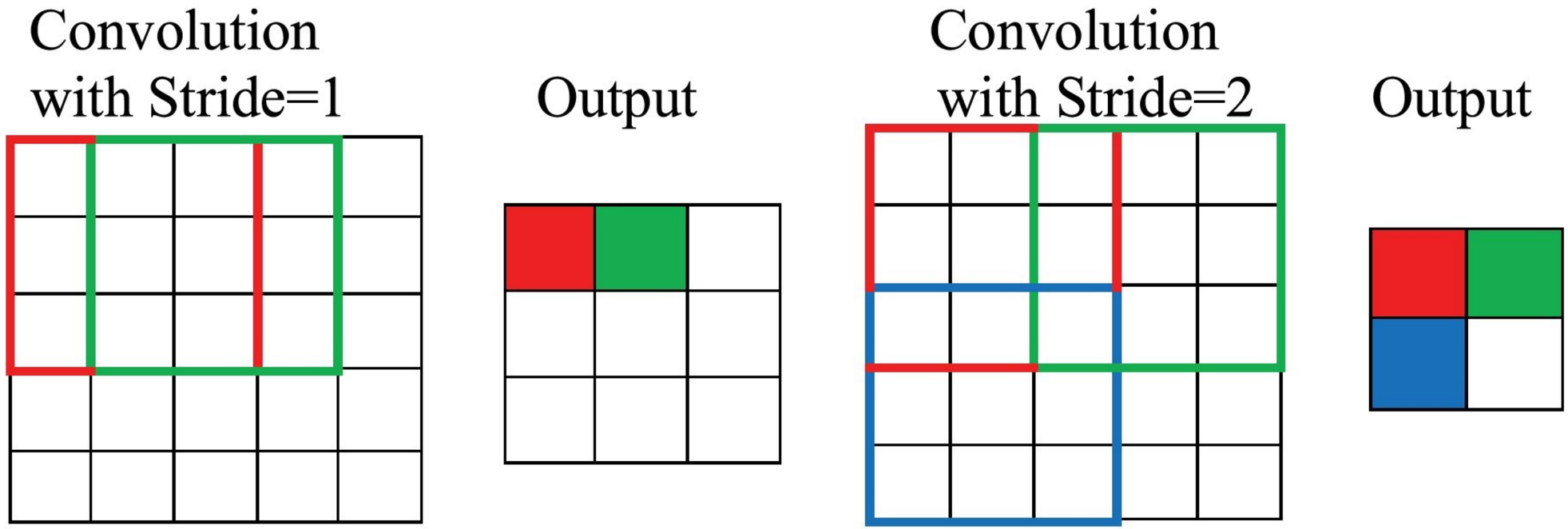
- *A strong prior could be bad if it goes against the problem nature*
  - *Example 1: try to estimate the energy of a jet from it's image*
    - *Max pooling implies informations loss*
    - *Average pooling makes more sense*
  - *Example 2: try to count how many sub-jets in the jet*
    - *Max pooling might work better than average pooling*





Other parameters: Stride

- *When applying convolution or padding, one can specify a stride*
  - *by how many pixels the kernel moves*



Other parameters: Padding

- *When the filter arrived at the edge, it might exceeds it (if n/k is not an integer)*
- *In this case, a padding rule needs to be specified*
  - *Valid: stop at the image boundary*
  - *Same: repeat the values at the boundary*
  - *Zero: fill the extra columns with zeros*

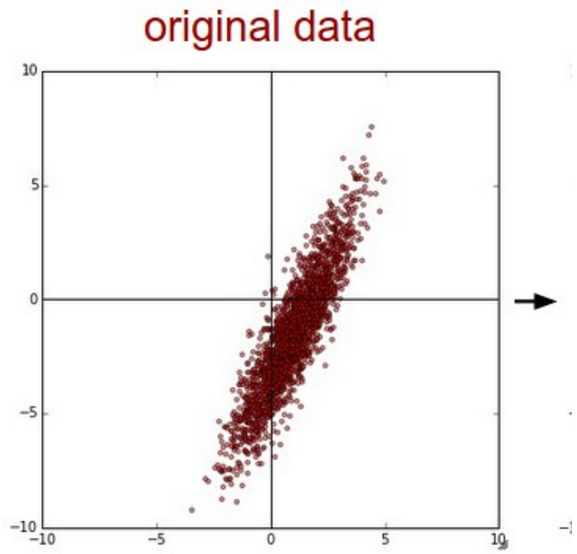
0	3	5	6	2	4	4
7	4	7	3	6	3	3
9	1	2	1	9	6	6
9	2	1	1	7	3	3
8	0	4	7	6	8	8
8	3	4	5	5	3	3
8	3	4	5	5	3	3

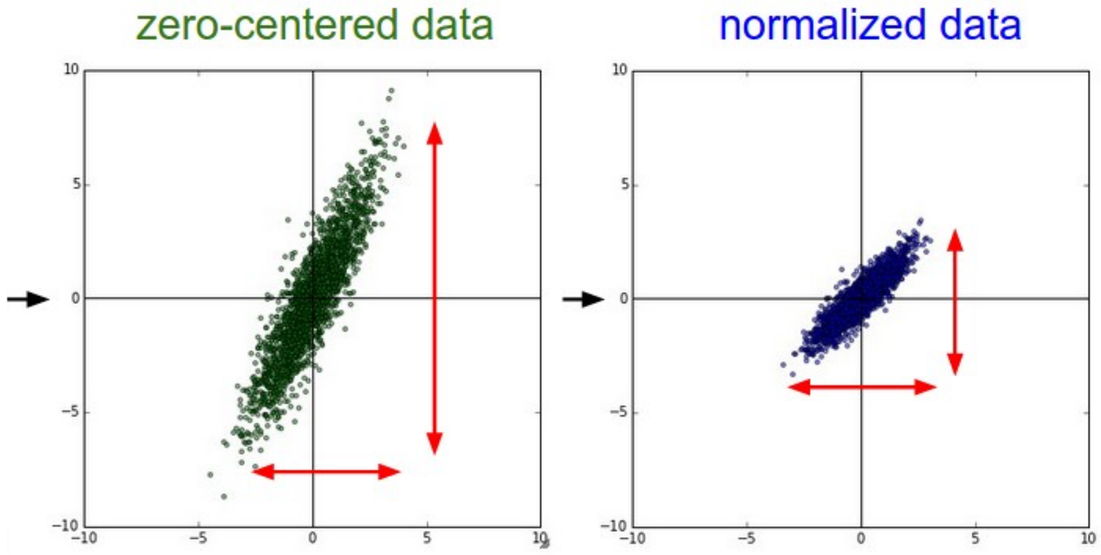
0	3	5	6	2	4
7	4	7	3	6	3
9	1	2	1	9	6
9	2	1	1	7	3
8	0	4	7	6	8
8	3	4	5	5	3

0	3	5	6	2	4	0
7	4	7	3	6	3	0
9	1	2	1	9	6	0
9	2	1	1	7	3	0
8	0	4	7	6	8	0
8	3	4	5	5	3	0
0	0	0	0	0	0	0

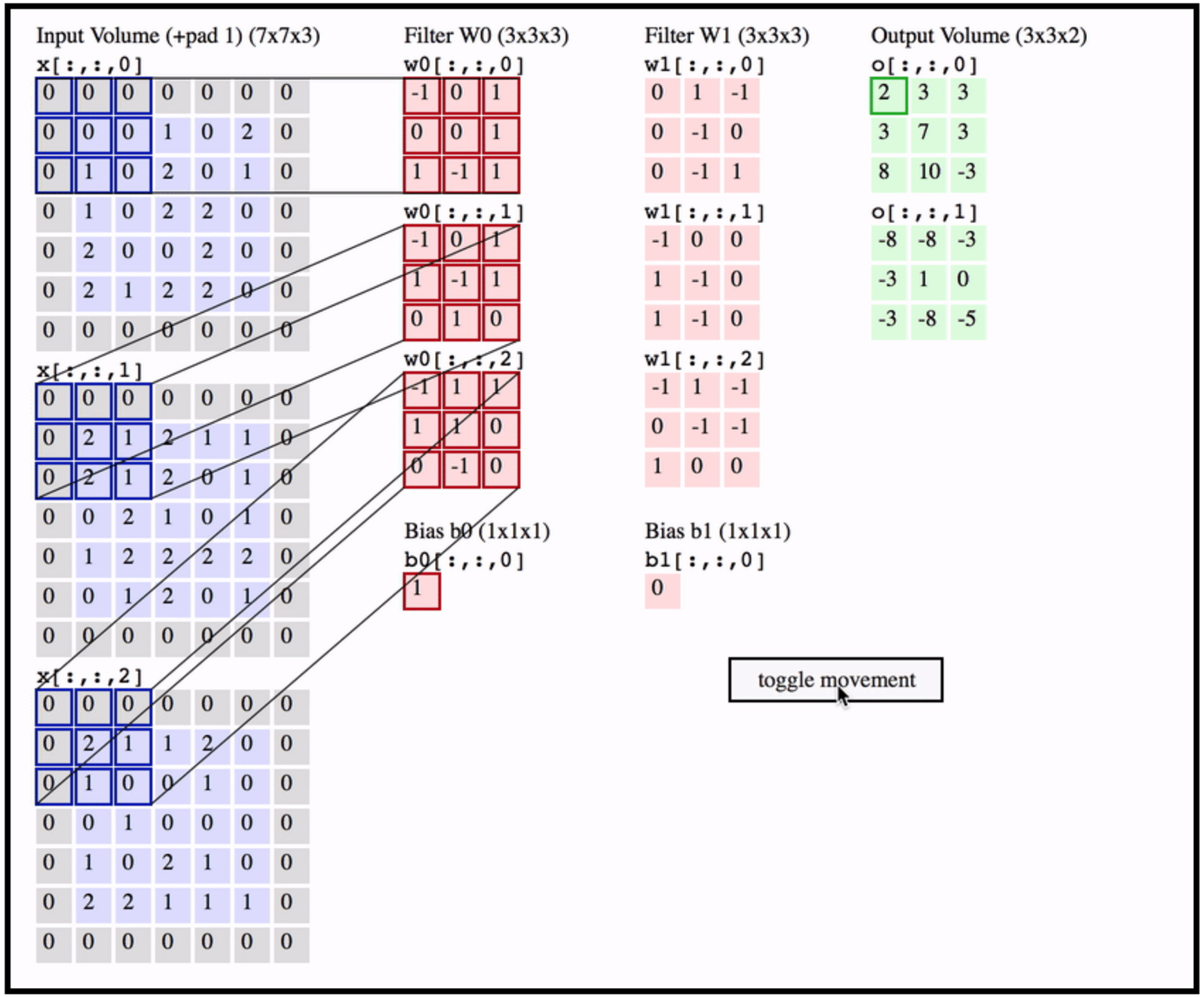
Batch Normalization

- *It is good practice to give normalized inputs to a layer*
  - *With all inputs having the same order of magnitude, all weights are equal important in the gradient*
  - *Prevents explosion of the loss function*
- *This can be done automatically with BatchNormalization*
  - *non-learnable shift and scale parameters, adjusted batch by batch*



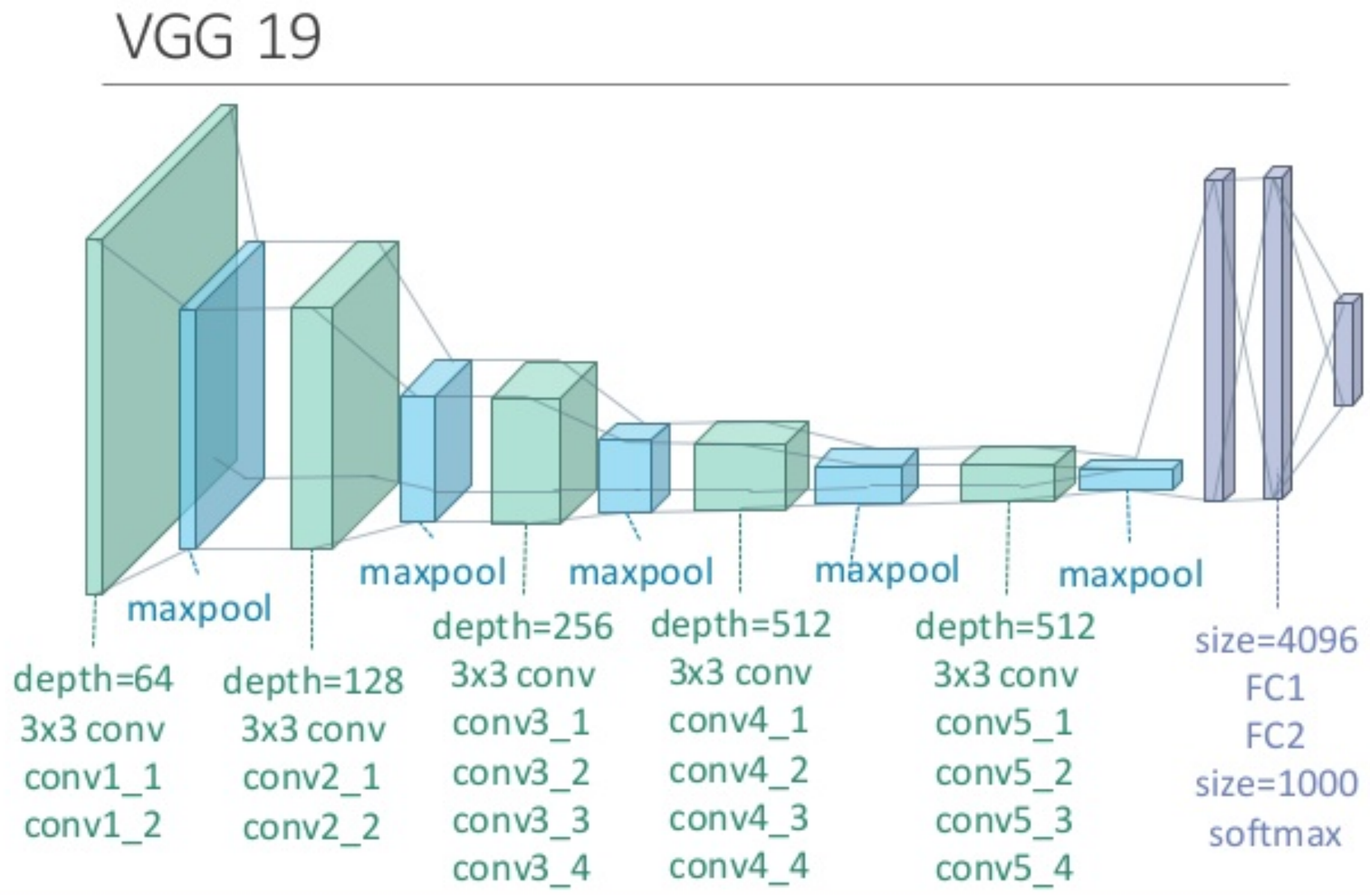


What it looks like...



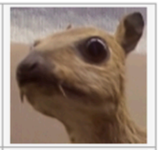
The full network

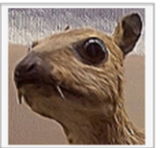
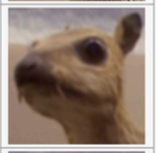
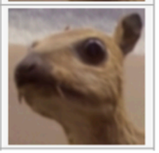
- *A full ConvNN is a sequence of Con2D+Pooling (+BatchNormalization+Dropout) layers*
- *The Conv+Pooling layer reduces the 2D image representation*
- *The use of multiple filters on the image make the output grow on a third dimension*
- *Eventually, flattening occurs and the result is given to a dense layer*



What does a ConvNN learn?

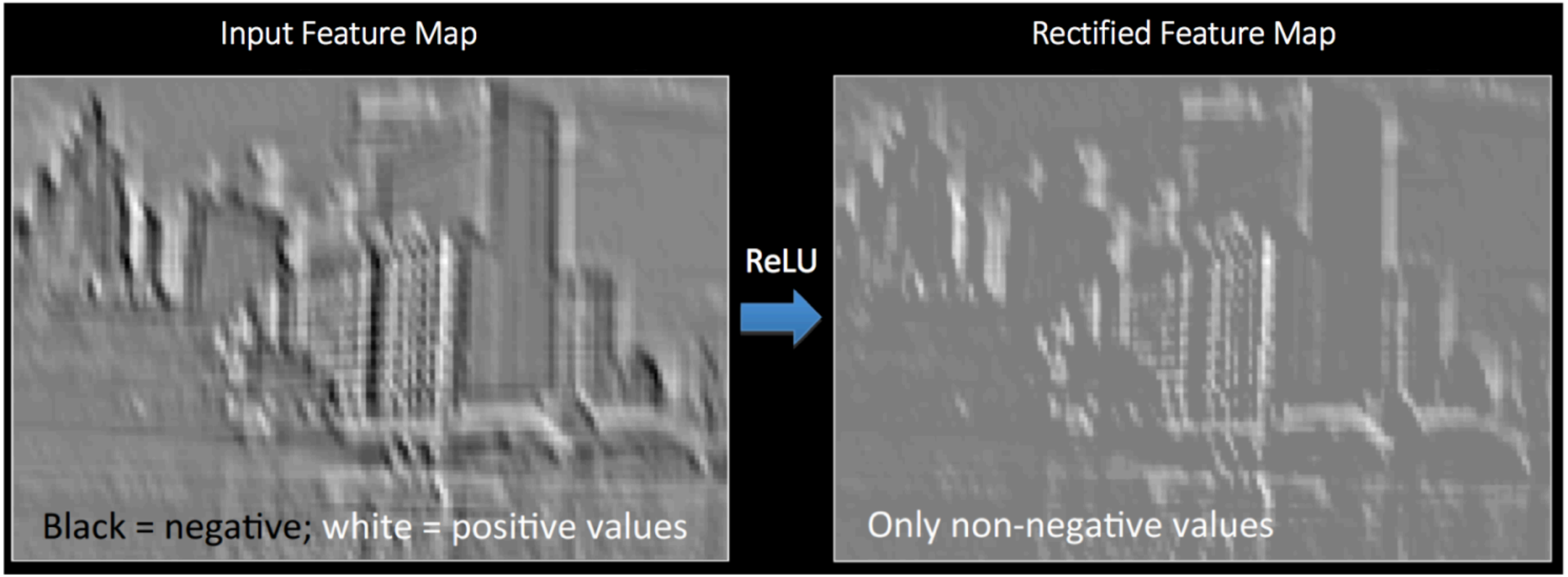
- *Each filter alters the image in a different way, picking up different aspects of the image*
  - *edges oriented in various ways*
  - *enhancing / blur of certain features*
- *It is interesting to check what each filter is doing (and to produce DeepDreams)*

Operation	Filter	Convolved Image
Identity	$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	
Edge detection	$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{bmatrix}$	
	$\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$	
	$\begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$	

Operation	Filter	Convolved Image
Sharpen	$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix}$	
Box blur (normalized)	$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$	
Gaussian blur (approximation)	$\frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$	

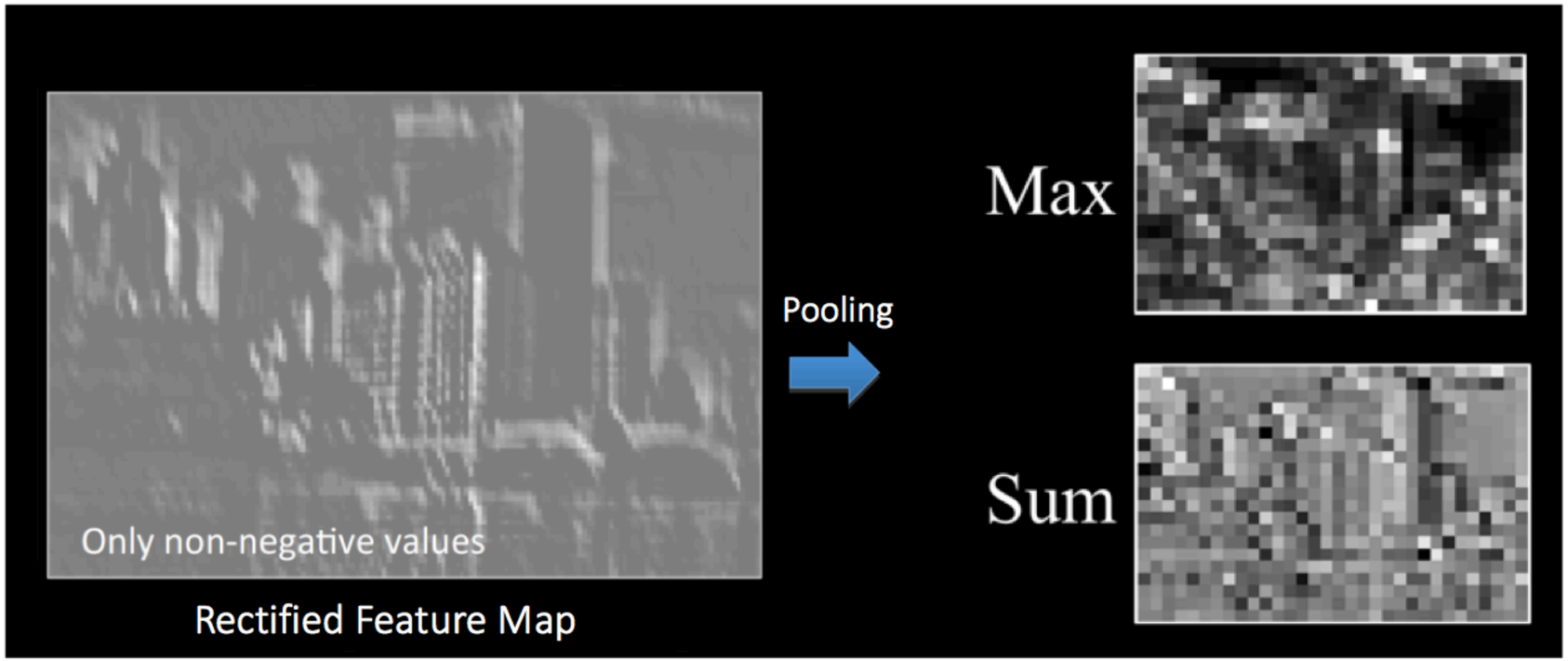
Check this blog entry on ConvNNs

What does a ConvNN learn?



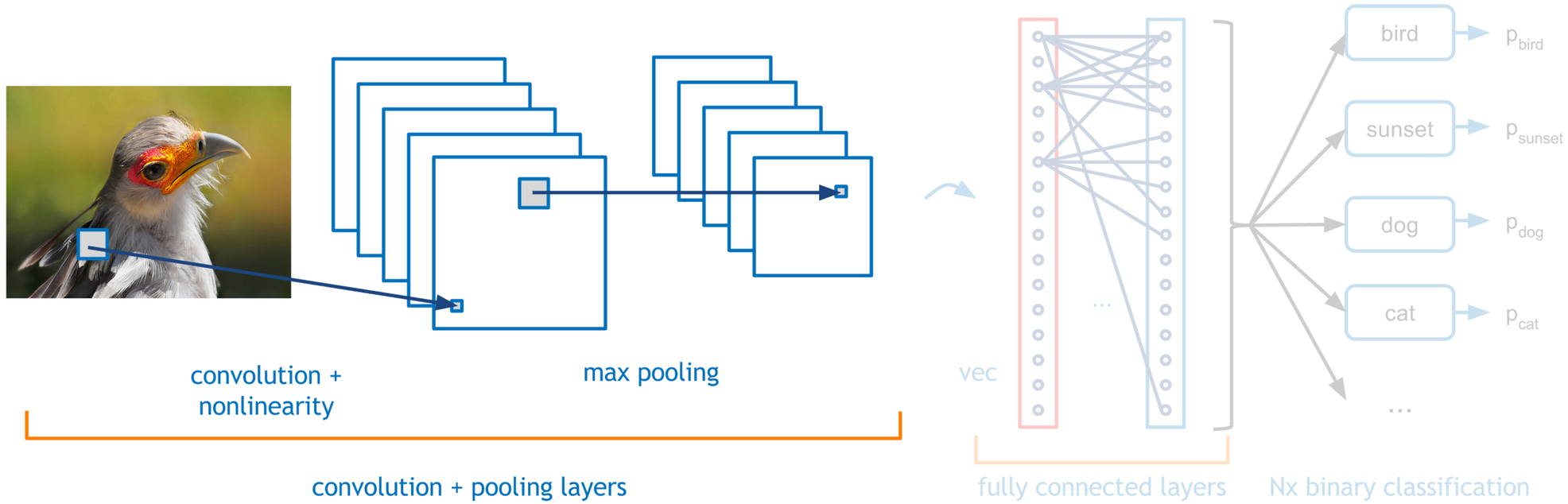
What does a ConvNN learn?

- *Pooling is important in smoothing out the image*
  - *reduce parameters downstream (and prevent overfitting)*
  - *makes processing independent to local features (distortion, translation)*
  - *yields a scale invariant representation of the image (which could be good or bad)*



Two tasks in one network

- ***The Conv layer stars from RAW data and defines interesting quantities (high level features)***
- *The HLFs replace the physics-motivated inputs of a DNN*
- *The DNN at the end exploits the engineered features to accomplish the task*



Two tasks in one network

- *The Conv layer stars from RAW data and defines interesting quantities (high level features)*
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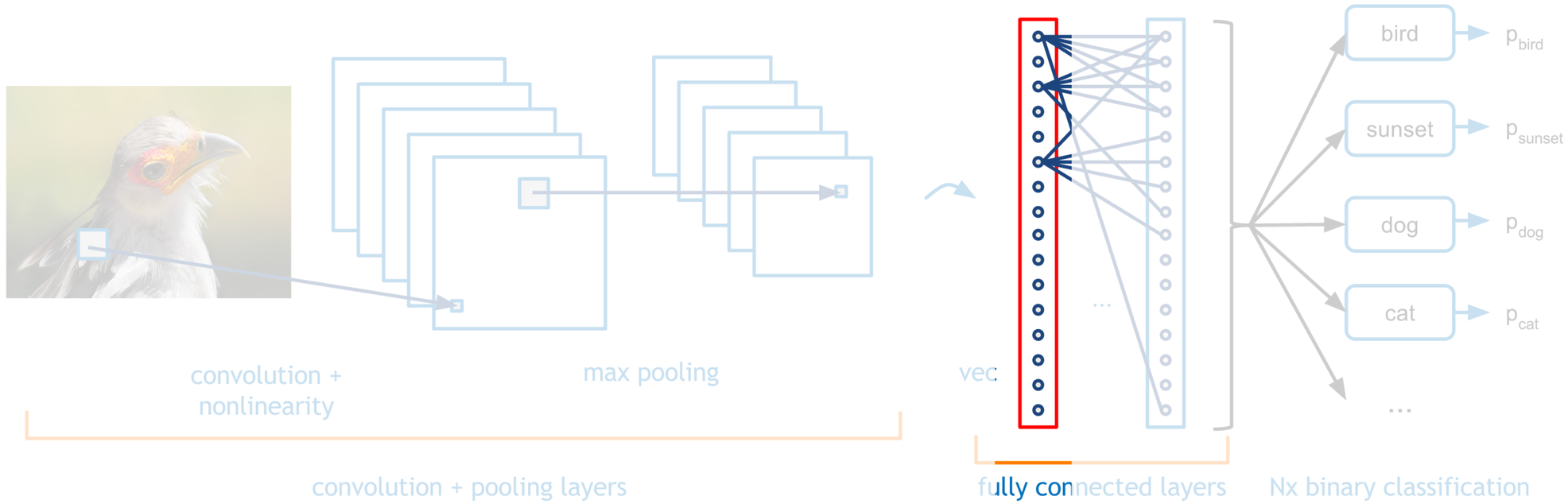

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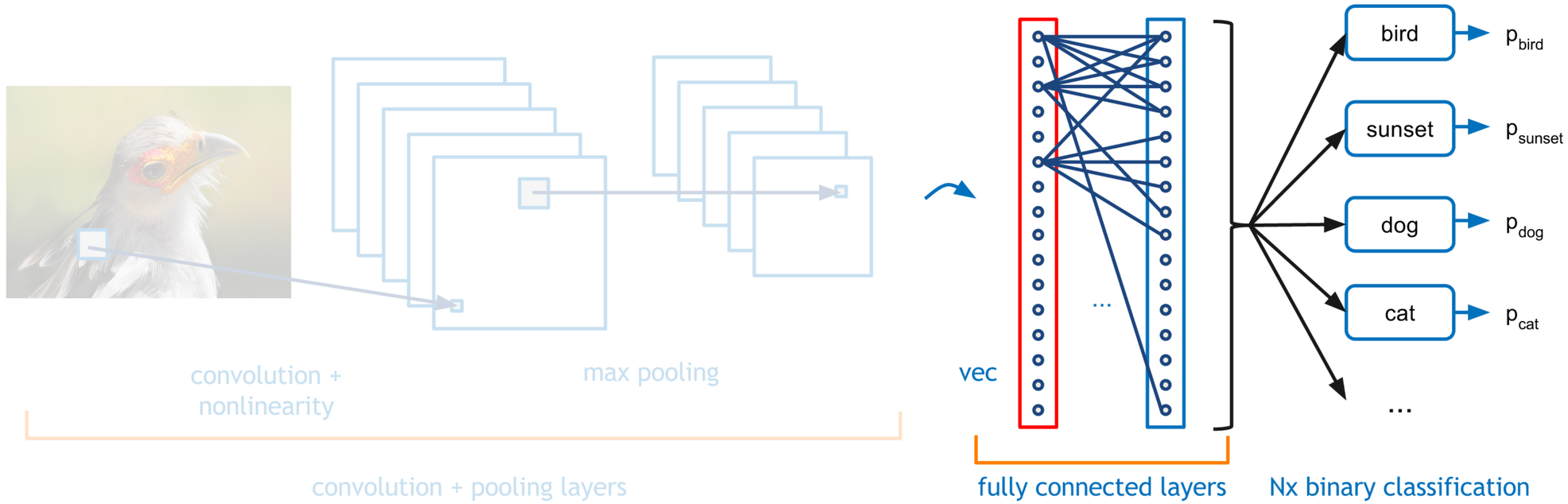
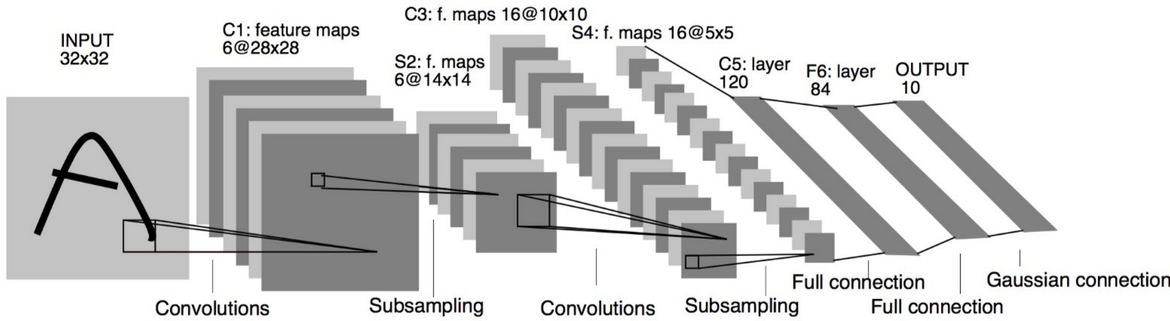
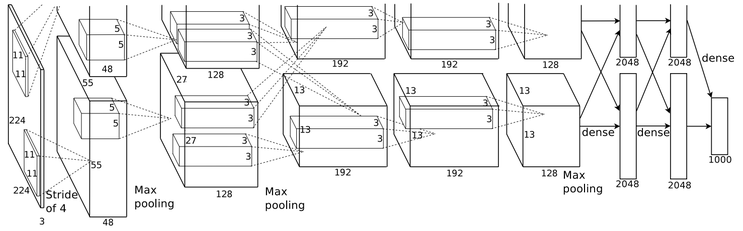
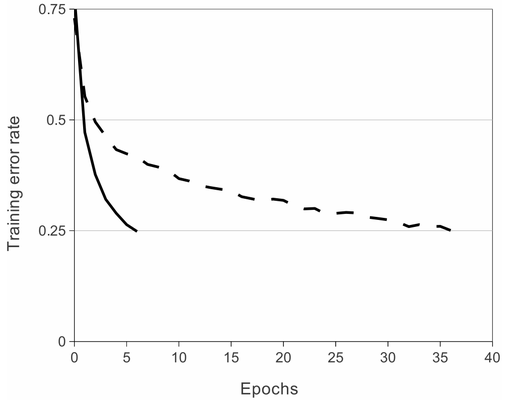


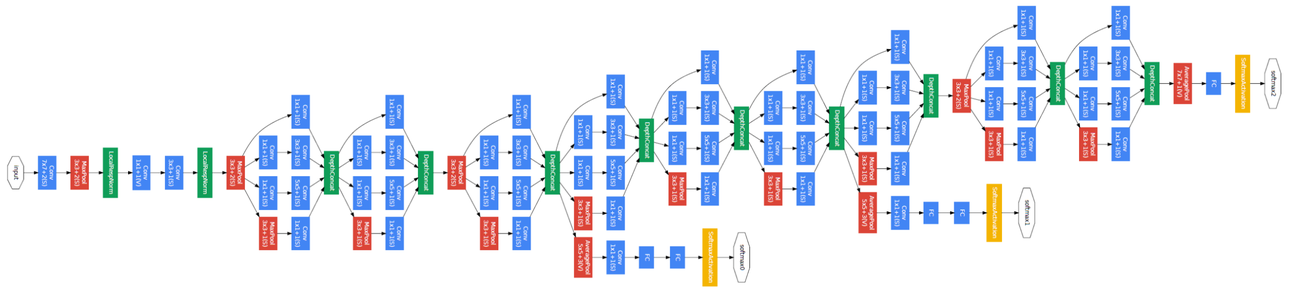
A history of ConvNNs

- *Neocognitron (1980): translation-invariant image processing with NNs*
- *LeNet (1989): considered the very first ConvNN, designer for digit recognition (ZIP codes)*
- *AlexNet (2012): the first big ConvNN (60M parameters, 650K neurtons), setting the state of the art: trained on GPUs, using ReLU and Dropout*
- *GoogleNet (2014): built on AlexNet, introduced an inception model to reduce e the number of parameters*



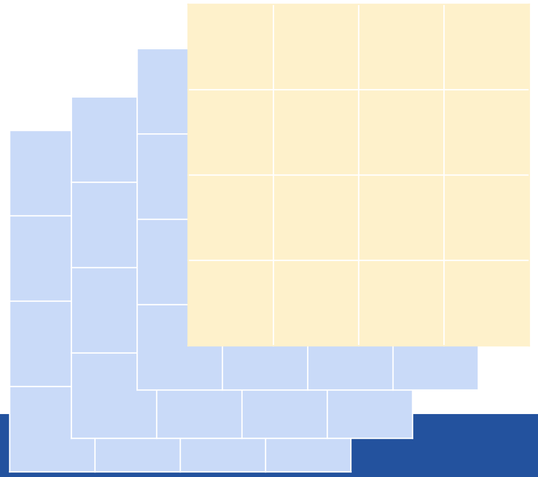


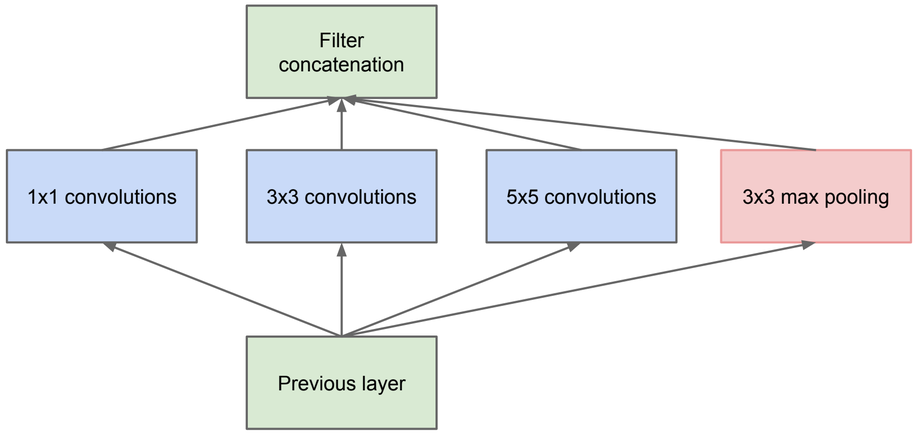




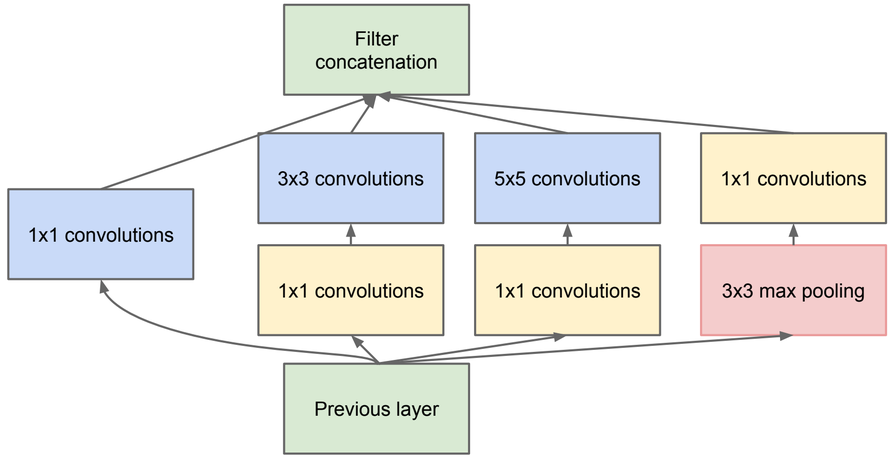
Inception Module

- *Rather than going deeper and deeper, inception architecture go wider*
- *Several conv layers, with different filter size, process the same inputs*
- *This way, more features can be detected from the same image*
- *The outcome of this parallel processing is then recombined through a concatenation step ass channels of an image*





(a) Inception module, naïve version

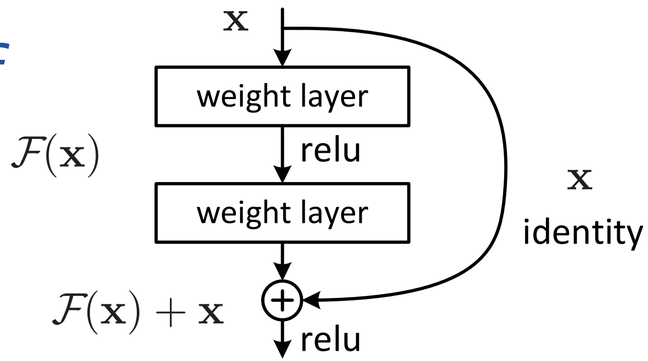


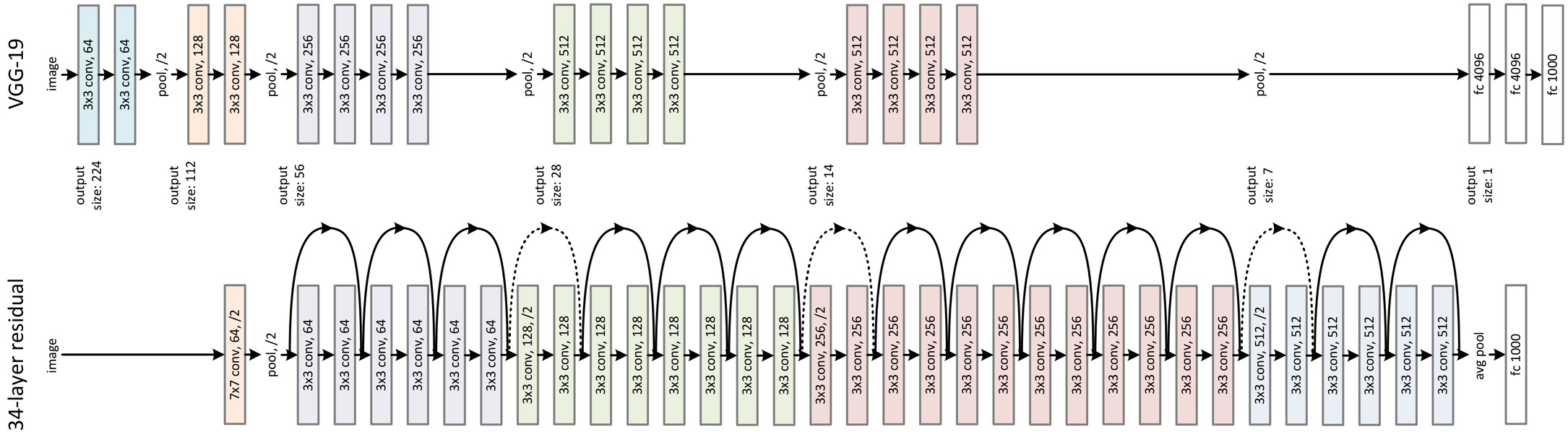
(b) Inception module with dimension reductions

A history of ConvNNs

- *VGGNet(2014):exploit small filters (3x3, 1x1), previously considered not optimal in a stack of Conv layers (rather than 1Conv+1Pooling)*

- *ResNet (2015):implemented skip connections, which were proven to boost performances*

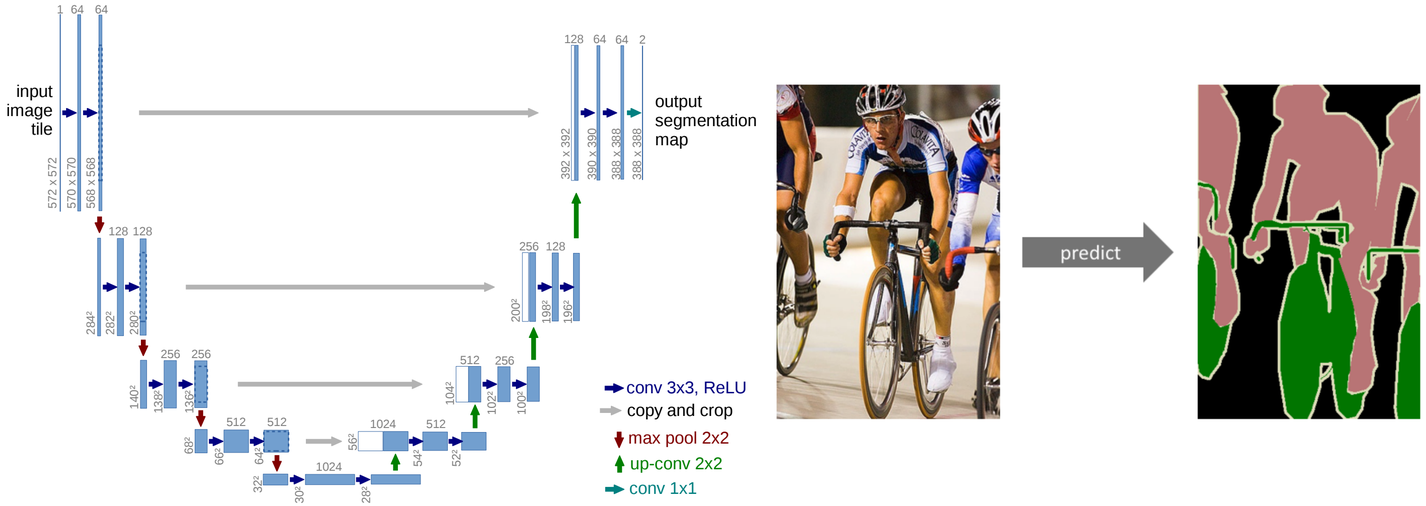




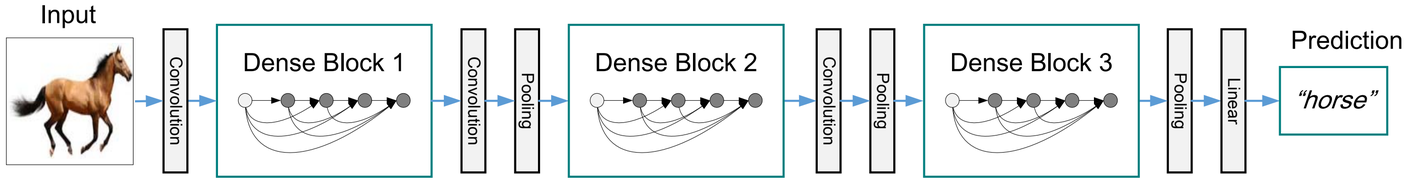
Check this blog entry on ConvNNs

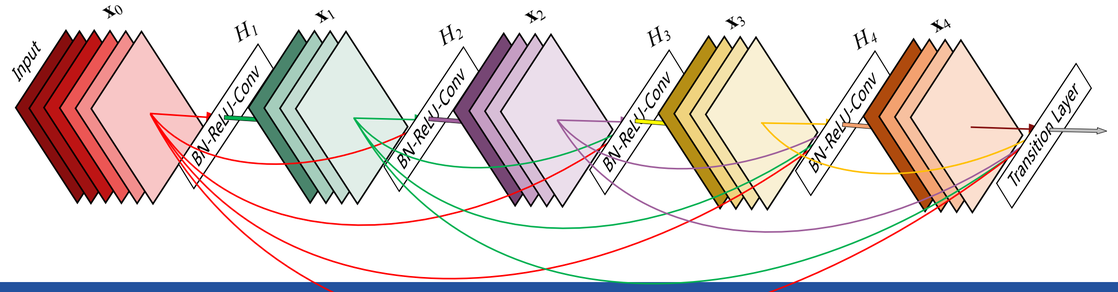
A history of ConvNNs

- *U-net (2015): conv layers with skip connections, in a downsampling+upsampling U-shaped sequence. Introduced for semantic segmentation*



- *DenseNet (2016): uses skip connections between a given layer and all the layers downstream in a dense block, with Conv layers in between*

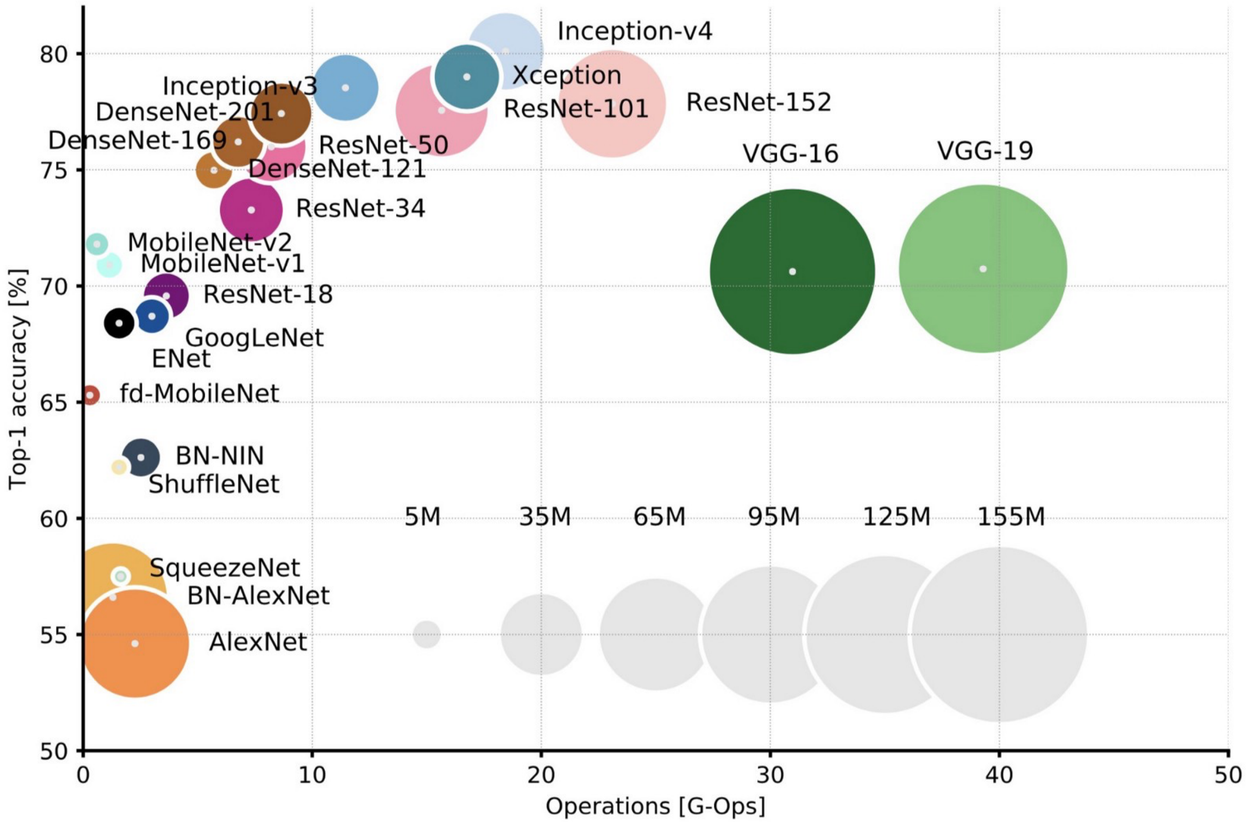




The computational cost

- *In this evolution, computing-vision networks drastically improved in performance*
- *This came at a big cost in complexity (number of parameters and operations)*
- *The inference with this network became particularly slow*
  - *big interest in optimize these networks on dedicated resources, e.g. FPGAs*
- *Many cloud providers provide optimized versions of these networks:*
  - *you can just use them (re-training if needed), rather than inventing your own one*

https://towardsdatascience.com/neural-network-architectures-156e5bad51ba



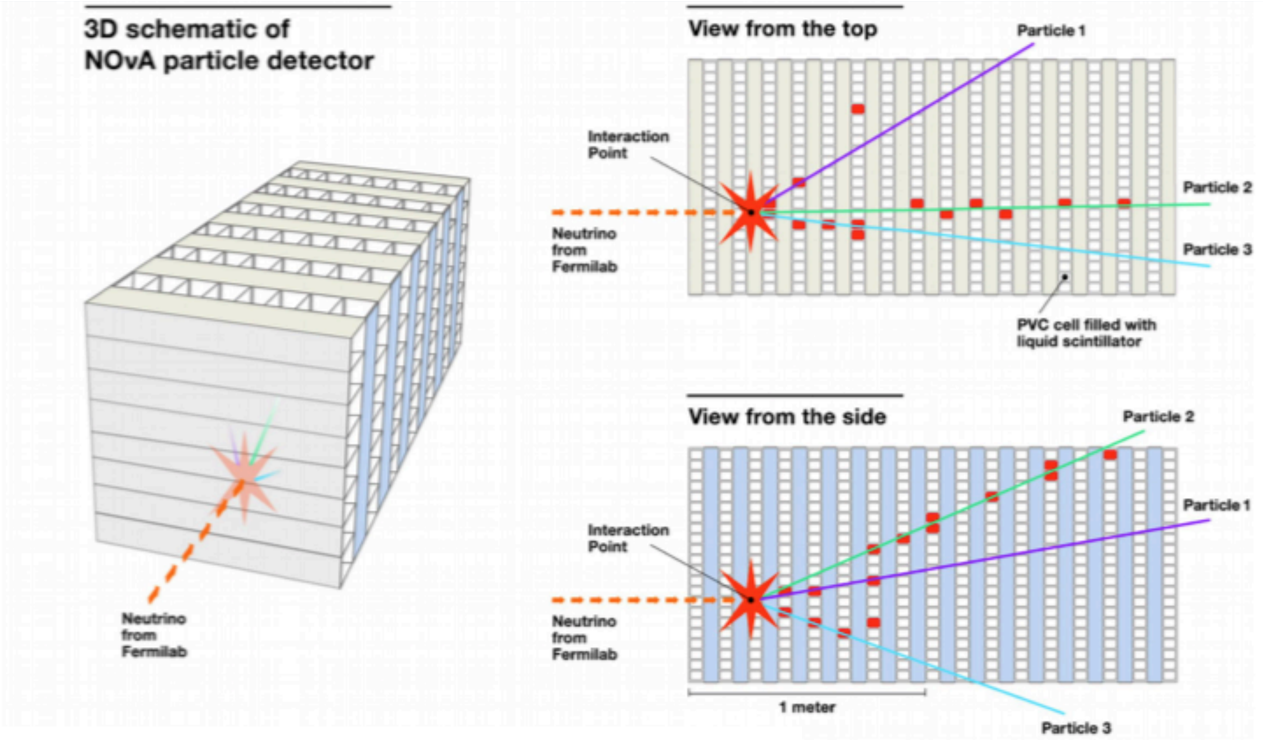
Conclusions

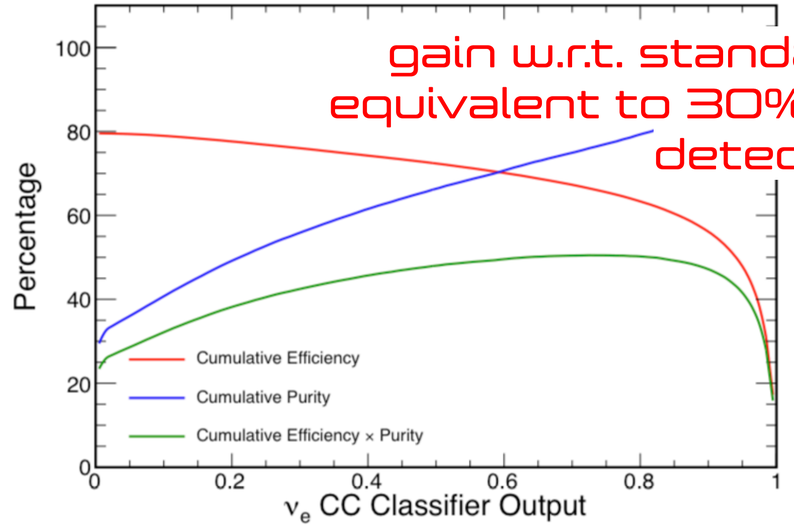
- *Convolutional Neural Networks has redefined computing vision*
- *We reviewed the basic ingredients*
- *We looked at the historical evolution*
- *We looked at performance cost*
- *We will discuss more the inference efficiency at the end of the course*

Backup

Example: PID for ν experiments

- *Many HEP detectors (particularly underground) more and more structured as regular arrays of sensors*
- *Modern computer-vision techniques work with images as arrays of pixel sensor (in 1D, 2D, and 3D)*
- *These techniques were applied by Nova on electron and muon ID*
- *Impressive gain over traditional techniques (comparable to +30% detector == $$$ saved)*

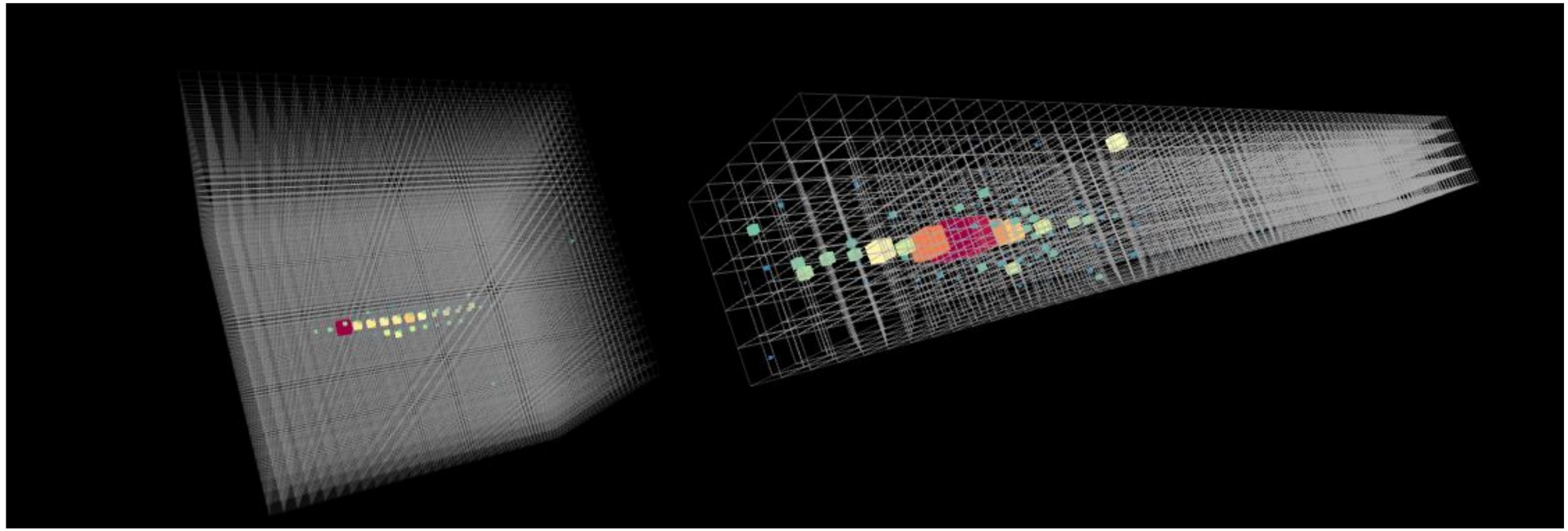




gain w.r.t. standard approach:
equivalent to 30% bigger mass of detector

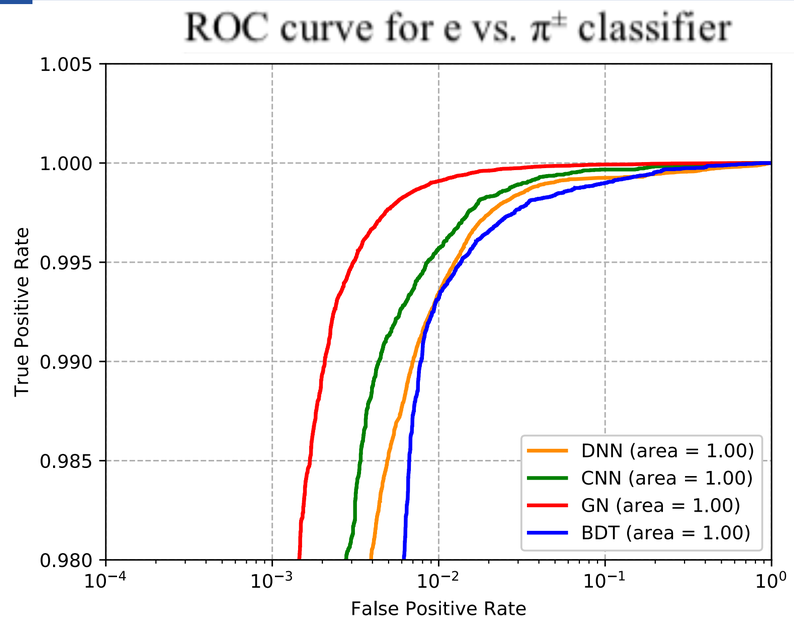
Calorimetry & Computer Vision

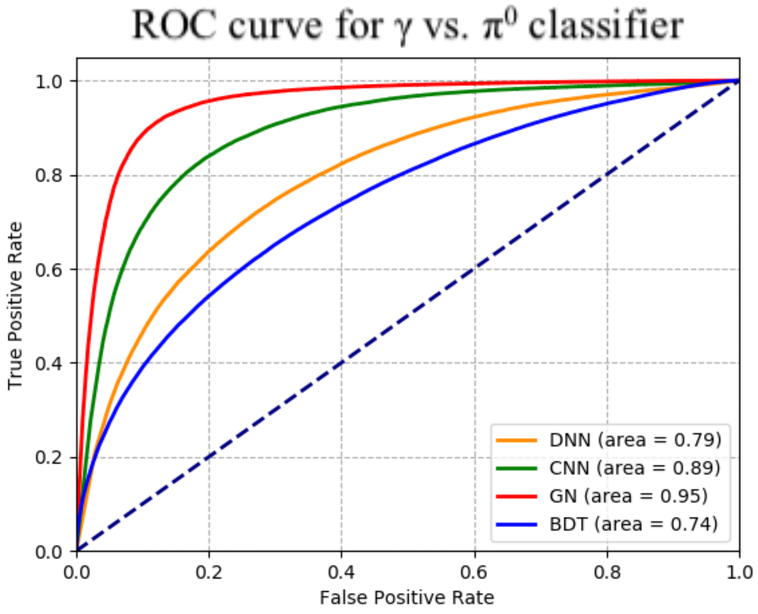
- *(next generation) digital calorimeters: 3D arrays of sensors with more regular geometry*
- *Ideal configuration to apply Convolutional Neural Network*
  - *speed up reconstruction at similar performances*
  - *and possibly improve performances*



Example: Particle ID

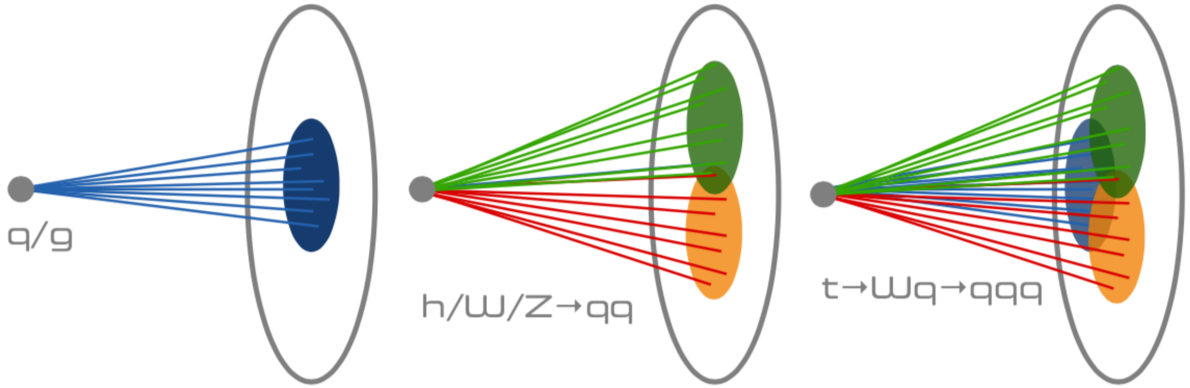
- We tried particle ID on a sample of simulated events
  - one particle/event (e, γ, π^0, π)
- Different event representations
  - high-level features related to event shape (moments of X,Y, and Z projections, etc)
  - raw data (energy recorded in each cell)
- Pre-filtered pion events to select the nasty ones and make the problem harder

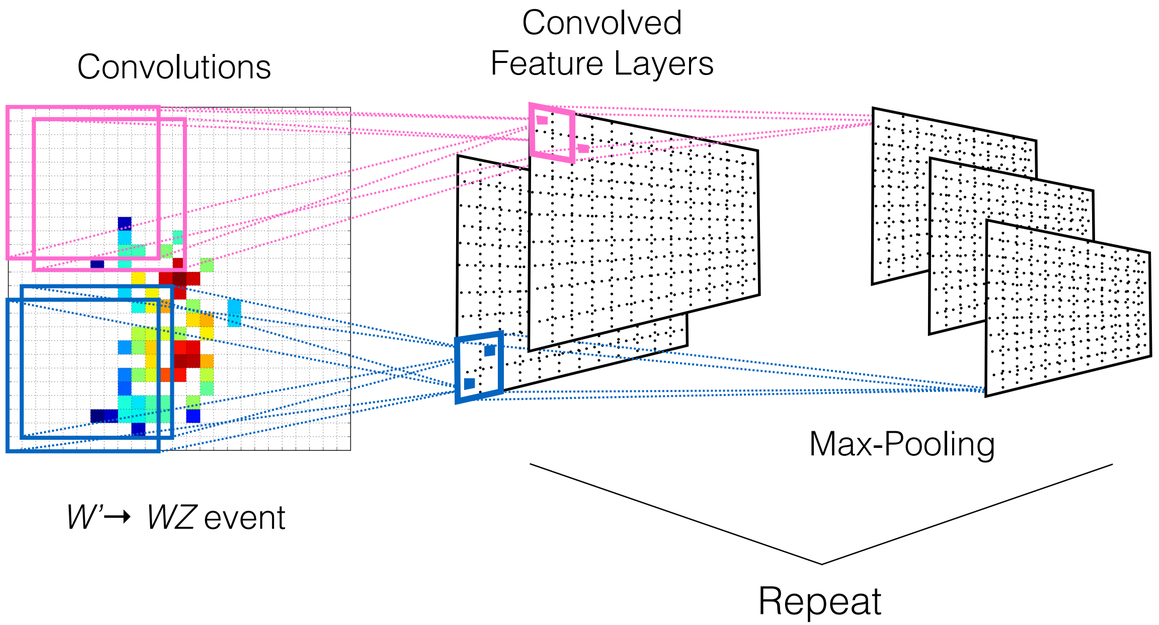




Jet as images (for ConvNN)

- *One can pixelate the surface crossed by the jet and create an image with the momentum deposited in each cell*
- *Such an image can then be processed with computing-vision techniques*
- *Pros: can benefit of the progresses made in optimizing computing vision*
- *Cons: underlying assumption on detector geometry (regular array of pixels) made sacrificing information of the actual detector*





https://arxiv.org/abs/1511.05190

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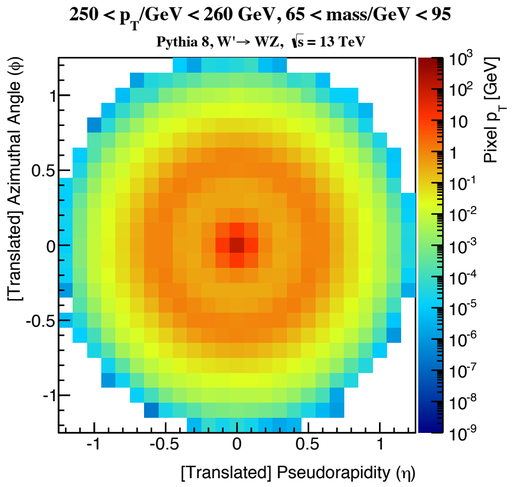

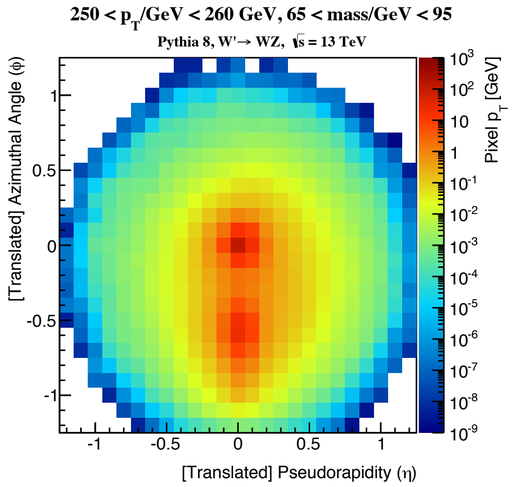

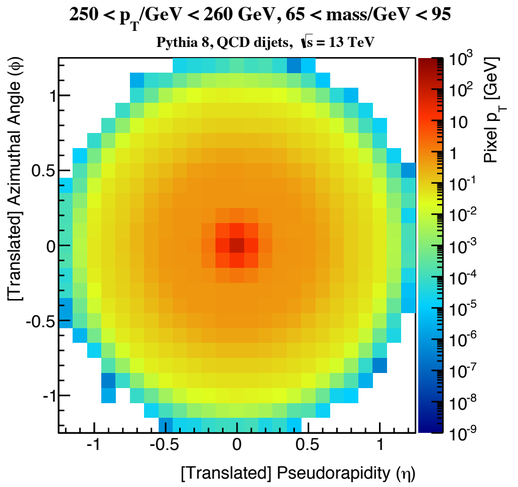

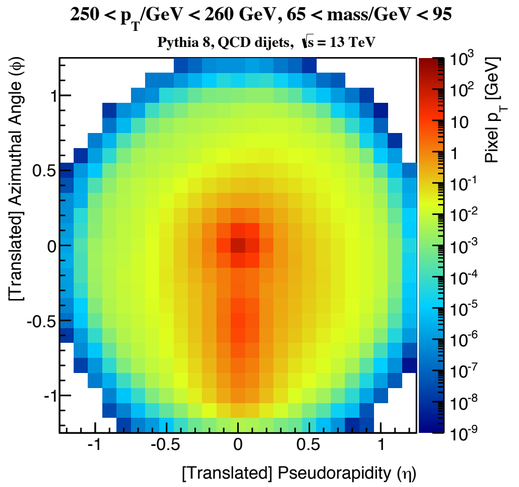

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