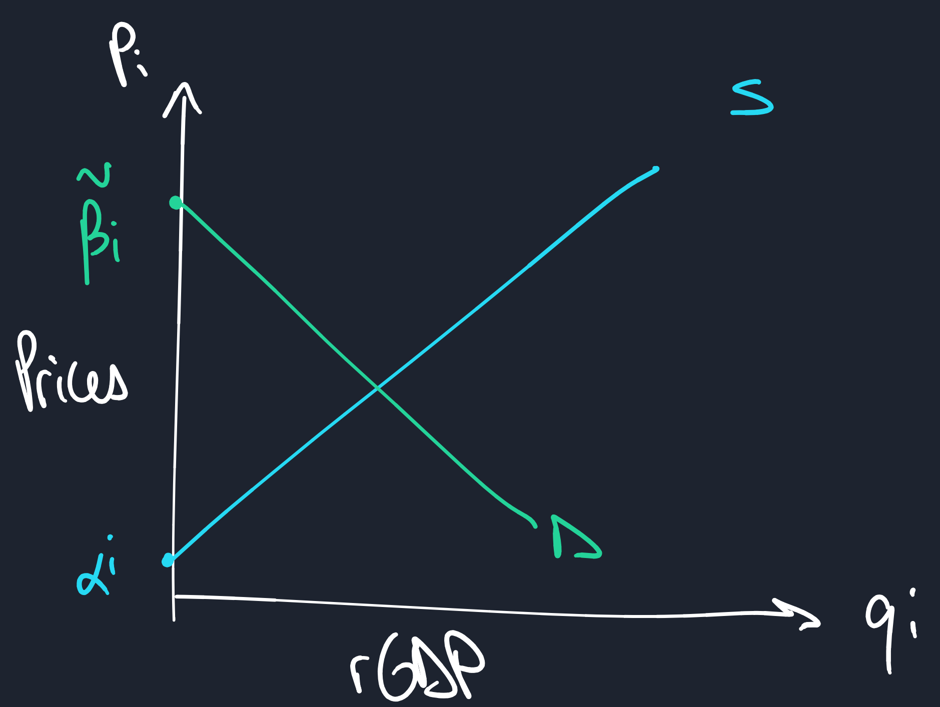
Adaptation from Jump - Kohler (2022) → Shapiro (2022) :



☆ Theory ☆

- Supply : $q_i = \sigma^i . p_i + \alpha_i$
- Demand : $q_i = -\tilde{\delta}^i . p_i + \tilde{\beta}_i \longrightarrow p_i = -\delta^i q_i + \beta_i$

with $\sigma^i, \delta^i > 0$

↳ nous : q_i et p_i = log(rGDP) et log(HICP).

⟼ Shocks :

$$\begin{cases} \varepsilon_t^s = \Delta\alpha^i = (q_{i,t} - \sigma^i p_{i,t}) - (q_{i,t-1} - \sigma^i p_{i,t-1}) & : \text{supply shock} \\ \varepsilon_t^s = \Delta q_{i,t} - \sigma^i \Delta p_{i,t} \end{cases}$$

$$\begin{cases} \varepsilon_t^d = \Delta\beta_i = (p_{i,t} + \delta^i q_{i,t}) - (p_{i,t-1} + \delta^i q_{i,t-1}) & : \text{demand shock} \\ \varepsilon_t^d = \Delta p_{i,t} + \delta^i \Delta q_{i,t} \end{cases}$$

$$\Delta q_{i,t} = \varepsilon_t^s + \sigma^i (\varepsilon_t^d - \delta^i \Delta q_{i,t})$$

$$\Delta q_{i,t} = \overbrace{\frac{1}{1+\delta^i}}^{>0} (\varepsilon_t^s + \overbrace{\sigma^i}^{>0} \varepsilon_t^d) \quad (1)$$

$$\Delta p_{i,t} = \frac{1}{\sigma^i}(\Delta q_{i,t} - \varepsilon_t^s) = \frac{1}{\sigma^i}\left(\frac{\sigma^i}{1+\delta^i}\varepsilon_t^d + \varepsilon_t^s\left(\frac{1}{1+\delta^i} - 1\right)\right)$$

$$\Delta p_{i,t} = \underbrace{\frac{1}{\sigma^i(1+\delta^i)}}_{>0} \left(\underbrace{\sigma^i}_{>0} . \varepsilon_t^d - \underbrace{\delta^i}_{>0} . \varepsilon_t^s\right) \quad (2)$$

In theory

		(1)	(2)
• Supply shock	$\Delta^+ \varepsilon^s$:	$\Delta^+ q$;	$\Delta^- p$
• Demand shock	$\Delta^+ \varepsilon^d$:	$\Delta^+ q$;	$\Delta^+ p$

$\longmapsto z_t = \begin{pmatrix} \Delta p_t \\ \Delta q_t \end{pmatrix}$

⚠ expected comovements from shocks

Consider structural VAR :

$$A . \begin{pmatrix} \Delta q_t \\ \Delta p_t \end{pmatrix} = \mu + \sum_{i=1}^{p} A_i . z_{t-i} + \underbrace{\begin{pmatrix} \varepsilon^s \\ \varepsilon^d \end{pmatrix}}_{\text{structural shocks}}$$

Reduced - VAR : $\boxed{v_t = A^{-1} . \varepsilon_t} \longleftrightarrow A v_t = \varepsilon_t$

$\begin{pmatrix} v^s \\ v^d \end{pmatrix}$

|| Condition on A ensuring we can infer the correct effects from reduced form errors ? ⟶ determine ε_t from v_t .

$A := \begin{pmatrix} 1 & -\alpha \\ \beta & 1 \end{pmatrix}$ $\alpha, \beta > 0$ $\qquad A^{-1} = \underbrace{\frac{1}{1+\alpha\beta}}_{=1/d>0} \begin{pmatrix} 1 & \alpha \\ -\beta & 1 \end{pmatrix}$

We have from the reduced form model (omit t.)

$$\begin{pmatrix} v^s \\ v^d \end{pmatrix} = A^{-1} \begin{pmatrix} \varepsilon^s \\ \varepsilon^d \end{pmatrix} \longrightarrow \begin{pmatrix} v^s \\ v^d \end{pmatrix} = \frac{1}{d} \begin{pmatrix} \varepsilon^s + \alpha\, \varepsilon^d \\ -\beta\, \varepsilon^s + \varepsilon^d \end{pmatrix}$$

$$\Longrightarrow \boxed{\begin{matrix} v^s \sim \varepsilon^s + \alpha . \varepsilon^d \\ v^d \sim -\beta . \varepsilon^s + \varepsilon^d \end{matrix}} \quad (*)$$

From $(*)$ we have:

1/ $\varepsilon^s > 0$; $\varepsilon^d > 0 \longrightarrow v^s > 0$ & v^d ?

2/ $\varepsilon^s < 0$; $\varepsilon^d < 0 \longrightarrow v^s < 0$

3/ $\varepsilon^s < 0$; $\varepsilon^d > 0 \longrightarrow v^d > 0$ & v^s ?

4/ $\varepsilon^s > 0$; $\varepsilon^d < 0 \longrightarrow v^d < 0$

Back Tracking:

(1) $v^s > 0$, $v^d > 0 \longleftrightarrow \varepsilon^d > 0$

(2) $v^s < 0$, $v^d < 0 \longleftrightarrow \varepsilon^d < 0$

reduced-form errors of the same sign $\longmapsto$ comovements same direct° (p, q)

$\longrightarrow$ demand shock.

consistent with theory.

$\&$

(3) $v^s > 0$, $v^d < 0 \longleftrightarrow \varepsilon^s > 0$

(4) $v^s < 0$, $v^d > 0 \longleftrightarrow \varepsilon^s < 0$

reduced-form errors of opposite sign $\longmapsto$ comovements opposit° direct° (p, q)

$\longrightarrow$ supply-shock

consistent with theory

Assuming $A = \begin{pmatrix} \overset{>0}{a_{11}} & \overset{<0}{a_{12}} \\ \underset{>0}{a_{21}} & \underset{>0}{a_{22}} \end{pmatrix}$: expected structural shocks' effects on the covariates are ensured.

$\Longrightarrow$ We can thus infer aforementioned effects from the reduced-form residuals.

JUMP - KOHLER 2022

(IS) $y_t = E_t[y_{t+1}] - \sigma(i_t - E_t[\pi_{t+1}]) + g_t$

(PC) $\pi_t = \beta E_t[\pi_{t+1}] + \kappa y_t + \eta_t$ aggregate (demand / supply / monetary policy) shocks

(MP) $i_t = \phi . \pi_t + \varepsilon_t$

↳ 'strict inflation targeting'

→ Determinate solution = fc° of pre-determined variables and shocks (none here)

$$\Rightarrow \underbrace{\begin{pmatrix} y_t \\ \pi_t \\ i_t \end{pmatrix}}_{z_t} = B . \begin{pmatrix} g_t \\ \eta_t \\ \varepsilon_t \end{pmatrix}$$

↳ if W. noise $E_t[g_{t+1}] = E_t[\eta_{t+1}] = E_t[\varepsilon_{t+1}] = 0$

$\Rightarrow E_t[z_{t+1}] = 0$

$$\begin{cases} y_t = -\sigma . i_t + g_t & \text{AD} \\ \pi_t = \kappa y_t + \eta_t & \text{AS} \\ i_t = \phi . \pi_t + \varepsilon_t & \text{MP} \end{cases} \Rightarrow$$

AD - AS:

$$\begin{cases} y_t = -\sigma\phi\pi_t + \overbrace{g_t - \sigma\varepsilon_t}^{\text{AD. shock } d_t} \\ \pi_t = \kappa y_t + \underbrace{\eta_t}_{\text{AS. shock}} \end{cases}$$

↓

$$\begin{cases} y_t = K_1 . (d_t - \sigma\phi . \eta_t) \\ \pi_t = K_2 (\kappa d_t + \eta_t) \end{cases}$$

Theory:

D. shock : $\Delta d_t \to \Delta y > 0$, $\to \Delta\pi > 0$

S. shock : $\Delta\eta_t \to \Delta y < 0$, $\to \Delta\pi > 0$

instead of output y_t : $u \,\&\, -y$!

. $z_t = \begin{pmatrix} u_t \\ \pi_t \end{pmatrix}$ $u_t \to$ D, $\pi_t \to$ S

↓ S.VAR(p)

$$A . z_t = a + \sum_{i=1}^{p} A_i z_{t-i} + \varepsilon_t$$

ε_t: W. noise → structural shock

↳ $z_t = A^{-1} a + A^{-1} \sum_{i=1}^{p} A_i . z_{t-i} + A^{-1} . \varepsilon_t$

$$E[z_t | z_{t-1} .. z_{t-p}] = A^{-1} . a + A^{-1} \sum_{i=1}^{p} A_i . z_{t-i}$$

$$z_t - E(z_t | z_{t-1} .. z_{t-p}) = \underline{A^{-1} . \varepsilon_t = v_t}$$

v_t → reduced form innovations (errors) → forecast error

$A . \begin{pmatrix} u_t \\ \pi_t \end{pmatrix} \longrightarrow$ v_t^u, v_t^π: $v_t = A^{-1} . \varepsilon_t$ (ε_t^d, ε_t^s)

. imposing : $A = \begin{pmatrix} 1 & -\alpha \\ \beta & 1 \end{pmatrix}$ $(\alpha, \beta > 0)$

we have : $A^{-1} = \dfrac{1}{1+\alpha\beta} \begin{pmatrix} 1 & \alpha \\ -\beta & 1 \end{pmatrix} \to \begin{pmatrix} v_t^u \\ v_t^\pi \end{pmatrix} = \dfrac{1}{1+\alpha\beta} \begin{pmatrix} 1 & \alpha \\ -\beta & 1 \end{pmatrix} \begin{pmatrix} \varepsilon_t^d \\ \varepsilon_t^s \end{pmatrix}$

$$\boxed{\begin{aligned} v_t^u &\sim \varepsilon_t^d + \alpha \varepsilon_t^s \\ v_t^\pi &\sim -\beta \varepsilon_t^d + \varepsilon_t^s \end{aligned}}$$

(hyp: $u = f(y)$ $f \downarrow$)

general form:

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

$a_{11} > 0$, $a_{12} < 0$, $a_{21} > 0$, $a_{22} > 0$

① $\varepsilon_r^d < 0$; $\varepsilon_r^s < 0$ $\Rightarrow$ $v_t^u < 0$; v_t^π ?

② $\varepsilon_r^d > 0$; $\varepsilon_r^s > 0$ $\Rightarrow$ $v_t^u > 0$; v_t^π ?

③ $\varepsilon_r^d < 0$; $\varepsilon_r^s > 0$ $\Rightarrow$ v_t^u ? ; $v_t^\pi > 0$

④ $\varepsilon_r^d > 0$; $\varepsilon_r^s < 0$ $\Rightarrow$ v_t^u ? ; $v_t^\pi < 0$

$\left.\begin{matrix} v_t^u < 0 \\ v_t^\pi > 0 \end{matrix}\right| \to \varepsilon_r^d < 0$

$\left.\begin{matrix} v_t^u > 0 \\ v_t^\pi < 0 \end{matrix}\right| \to \varepsilon_r^d > 0$

$\to$ forecast errors of opposite signs imply $\to$ demand shock = consistent w/ theory

$\left.\begin{matrix} v_t^u < 0 \\ v_t^\pi < 0 \end{matrix}\right| \to \varepsilon_r^s < 0$

$\left.\begin{matrix} v_t^u > 0 \\ v_t^\pi > 0 \end{matrix}\right| \to \varepsilon_r^s > 0$

$\to$ forecast errors of same signs imply $\to$ supply shock (consistent w/ theory)

☆ Theory gives that: demand shock should induce comovements opposite direct° for covariates; supply shock ——— same direct° for covariates

(&) imposing $A = \begin{pmatrix} a_{11} > 0 & a_{12} < 0 \\ a_{21} > 0 & a_{22} > 0 \end{pmatrix}$ sign restrictions on A

ensures that we can infer correct effects from reduced-form errors.

$\Rightarrow$ with these sign assumptions on A: we can estimate a reduced form residuals from a VAR to infer the signs of supply/demand shock.