

Ex 89

$$f(x) = \ln x - x - 1 \quad I =]0; +\infty[$$

$$f'(x) = \frac{1}{x} - 1 \quad \text{Signe de } f': \frac{1}{x} - 1 > 0 \Rightarrow \frac{1-x}{x} > 0$$

$$\begin{array}{l|l} 1-x > 0 & x > 0 \\ x < 1 & \end{array}$$

$$\Rightarrow$$

x	0	1	$+\infty$
$1-x$	+	0	-
x	+	+	+

Tableau de variations :

x	0	1	$+\infty$
f'	+	0	-
f		-2	

$$f(1) = \ln(1) - 1 - 1 = -2$$

Ex 90

$$f(x) = \frac{1}{3}x^3 - x \quad I = \mathbb{R}$$

$$f'(x) = x^2 - 1 \quad \text{Signe de } f': x^2 - 1 > 0$$

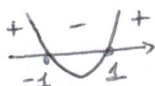


Tableau de variations :

x	$-\infty$	-1	1	$+\infty$
f'	+	0	0	+
f		$\frac{2}{3}$	$-\frac{2}{3}$	

$$f(-1) = -\frac{1}{3} + 1 = \frac{-1+3}{3} = \frac{2}{3}$$

$$f(1) = \frac{1}{3} - 1 = \frac{1-3}{3} = -\frac{2}{3}$$

Ex 91

$$f(x) = 3x^2 - 3x^3 \quad I = \mathbb{R}$$

$$f'(x) = 6x - 9x^2 \quad \text{Signe de } f': -9x^2 + 6x > 0$$

$$3x(-3x+2) > 0$$

$$\begin{array}{l|l} 3x > 0 & -3x+2 > 0 \\ x > 0 & -3x > -2 \\ & x < \frac{2}{3} \end{array}$$

x	$-\infty$	0	$\frac{2}{3}$	$+\infty$
$3x$	-	0	+	+
$-3x+2$	+	+	0	-

Tableau de variations :

x	$-\infty$	0	$\frac{2}{3}$	$+\infty$
f'	-	0	+	-
f		0	$\frac{4}{9}$	

$$f(0) = 0$$

$$f\left(\frac{2}{3}\right) = 3 \cdot \frac{4}{9} - 3 \cdot \frac{8}{27} =$$

$$= \frac{12}{9} - \frac{24}{27} = \frac{12}{9} - \frac{8}{9} =$$

$$= \frac{12-8}{9} = \frac{4}{9}$$