Ex 26

$$\int_1^4 \frac{3}{x}\,dx = \Big[3\ln(x)\Big]_1^4 = 3\ln(4) - 3\ln(1) = 3\ln(4)$$

$$\int_1^4 \left(x - \frac{2}{x}\right)dx = \left[\frac{x^2}{2} - 2\ln(x)\right]_1^4 =$$

$$= \frac{4^2}{2} - 2\ln(4) - \left(\frac{1^2}{2} - 2\ln(1)\right) =$$

$$= 8 - 2\ln(4) - \frac{1}{2} = \frac{16-1}{2} - 2\ln(4) =$$

$$= \frac{15}{2} - 2\ln(4)$$

Ex 27

$$\int_0^1 \left(x + 2 + \frac{1}{x+2}\right)dx = \left[\frac{x^2}{2} + 2x + \ln(x+2)\right]_0^1 =$$

$$\frac{1}{x+2} \downarrow \frac{1}{u} \quad \text{avec } u = x+2 \Rightarrow u' = 1 \Rightarrow \frac{1}{x+2} = \frac{u'}{u} \rightarrow \ln(u)$$

$$f \rightarrow F$$

$$= \frac{1}{2} + 2 + \ln(3) - \big(0 + 0 + \ln(2)\big) =$$

$$= \frac{1+4}{2} + \ln(3) - \ln(2) = \frac{5}{2} + \ln\left(\frac{3}{2}\right)$$

Ex 28

$$\int_0^1 \frac{t}{t^2+1}\,dt = \qquad u = t^2 + 1 \Rightarrow u' = 2t \leadsto \frac{u'}{u}$$

$$= \frac{1}{2}\int_0^1 \frac{2t}{t^2+1}\,dt = \frac{1}{2}\int_0^1 \frac{u'}{u}\,dt = \left[\frac{1}{2}\ln(u)\right]_0^1 =$$

$$= \left[\frac{1}{2}\ln(t^2+1)\right]_0^1 = \frac{1}{2}\ln(2) - \frac{1}{2}\ln(1) = \frac{1}{2}\ln(2)$$

$$\int_0^{\ln(2)} (e^t + e^{2t})\, dt$$

$$\int e^t\, dt = e^t + c$$

$$\int e^{2t}\, dt = \int e^u\, dt \quad \text{avec} \quad u = 2t \Rightarrow u' = 2$$

$$= \frac{1}{2}\int 2e^{2t}\, dt = \frac{1}{2} e^{2t} + c$$

$$\int_0^{\ln(2)} (e^t + e^{2t})dt = \left[e^t + \frac{1}{2} e^{2t} \right]_0^{\ln(2)} =$$

$$= e^{\ln(2)} + \frac{1}{2} e^{2\ln(2)} - \left(e^0 + \frac{1}{2} e^0 \right) =$$

$$= 2 + \frac{1}{2} e^{\ln(2^2)} - \left(1 + \frac{1}{2}\right) =$$

$$= 2 + \frac{1}{2} \times 4 - \frac{3}{2} = 2 + 2 - \frac{3}{2} =$$

$$= 4 - \frac{3}{2} = \frac{8-3}{2} = \frac{5}{2}$$

<u>Ex 29</u>

$$\int_{-1}^{2} x(x^2+4)\, dx = \qquad \text{" } \underset{\displaystyle f}{u'u} \text{ "} \rightsquigarrow \underset{\displaystyle F}{\frac{u^2}{2}}$$

$$u = x^2 + 4 \Rightarrow u' = 2x$$

$$= \frac{1}{2}\int_{-1}^{2} 2x(x^2+4)\, dx = \left[\frac{1}{2} \frac{(x^2+4)^2}{2} \right]_{-1}^{2} =$$

$$= \frac{1}{4}(4+4)^2 - \frac{1}{4}(1+4)^2 = \frac{64}{4} - \frac{25}{4} = \frac{39}{4}$$

$$\left(\int x(x^2+4)\,dx = \int (x^3+4x)\,dx = \frac{x^4}{4} + 4\frac{x^2}{2} \right)$$

$$\int_1^2 \frac{xe^x+1}{x}\,dx = \int_1^2 \left(\frac{\cancel{x}e^x}{\cancel{x}} + \frac{1}{x}\right)dx = \int_1^2 \left(e^x + \frac{1}{x}\right)dx =$$

$$= \left[e^x + \ln(x) \right]_1^2 = e^2 + \ln(2) - (e^1 + \ln(1)) =$$

$$= e^2 + \ln(2) - e$$

Ex 30

$$\int_0^{\ln(2)} (e^x - e^{-x})\,dx$$

$$\int e^x\,dx = e^x$$

$$\int e^{-x}\,dx = \int e^u\,dx \qquad u = -x \Rightarrow u' = -1$$

$$= -\int (-1)\,e^{-x}\,dx = -e^{-x}$$

$$\int_0^{\ln(2)} (e^x - e^{-x})\,dx = \left[e^x - (-e^{-x}) \right]_0^{\ln(2)} =$$

$$= \left[e^x + e^{-x} \right]_0^{\ln(2)} =$$

$$= e^{\ln(2)} + e^{-\ln(2)} - (e^0 + e^0) =$$

$$= \cancel{2} + \frac{1}{e^{\ln(2)}} - \cancel{2} = \frac{1}{2}$$

$$\int_e^{e^2} \frac{1}{x\ln(x)}\,dx = \qquad u = \ln(x) \Rightarrow u' = \frac{1}{x}$$

$$= \int_e^{e^2} \frac{u'}{u}\,dx = \Big[\ln(u)\Big]_e^{e^2} =$$

$$= \Big[\ln(\ln(x))\Big]_e^{e^2} = \ln(\ln(e^2)) - \ln(\ln(e)) =$$

$$= \ln(2) - \ln(1) = \ln(2)$$

Ex 32

$$\int (x^2 + 2e^{-2x})\,dx$$

$$\int x^2\,dx = \frac{x^3}{3}$$

$$\int 2e^{-2x}\,dx = \int 2e^u\,dx \qquad u = -2x \Rightarrow u' = -2$$

$$= -\int -2e^{-2x}\,dx = -e^{-2x}$$

$$\int (x^2 + 2e^{-2x})\,dx = \frac{x^3}{3} - e^{-2x} + C$$

Ex 33

$$\int \frac{x^2}{x+1}\,dx = \int \frac{x^2 - 1 + 1}{x+1}\,dx$$

$$\left(⚠: \quad x^2 - 1 = (x+1)(x-1) \qquad \mathbb{R} \right)$$

$$= \int \frac{(x+1)(x-1) + 1}{x+1}\,dx =$$

$$= \int \left(\frac{\cancel{(x+1)}(x-1)}{\cancel{x+1}} + \frac{1}{x+1} \right) dx =$$

$$= \int \left(x - 1 + \frac{1}{x+1} \right) dx =$$

$$= \frac{x^2}{2} - x + \ln(x+1) + C$$

Ex 34

$$\int x e^{x^2+1} \, dx = \int x e^{u} \, dx \qquad u = x^2+1 \Rightarrow u' = 2x$$

$$= \frac{1}{2} \int 2x \, e^{x^2+1} dx = \frac{1}{2} e^{x^2+1} + C$$

Ex 35

$$\int \frac{x^2}{2x-1} \, dx = \qquad \left(\triangle \quad (2x-1)(2x+1) = 4x^2 - 1 \right)$$

$$= \frac{1}{4} \int \frac{4x^2}{2x-1} \, dx = \frac{1}{4} \int \frac{4x^2 - 1 + 1}{2x-1} \, dx =$$

$$= \frac{1}{4} \int \frac{(2x-1)(2x+1) + 1}{2x-1} \, dx =$$

$$= \frac{1}{4} \int \left(\frac{\cancel{(2x-1)}(2x+1)}{\cancel{2x-1}} + \frac{1}{2x-1} \right) dx =$$

$$= \frac{1}{4} \int \left(2x + 1 + \frac{1}{2x-1} \right) dx =$$

$\hookrightarrow \frac{1}{u}$ avec $u = 2x - 1 \Rightarrow u' = 2$

$$= \frac{1}{4}\int\left(2x+1+\frac{1}{2}\,\frac{2}{2x-1}\right)dx =$$

$$= \frac{1}{4}\left(\frac{2x^2}{2}+x+\frac{1}{2}\ln(2x-1)\right)+C =$$

$$= \frac{1}{4}\left(x^2+x+\frac{1}{2}\ln(2x-1)\right)+C$$

Ex 36

$$\int\left(x+3-\frac{4}{x^3}\right)dx$$

$$\int x\,dx = \frac{x^2}{2} \qquad \int 3\,dx = 3x$$

$$\int\frac{1}{x^3}\,dx = \int x^{-3}\,dx = \frac{x^{-3+1}}{-3+1} = \frac{x^{-2}}{-2} = -\frac{1}{2x^2}$$

$$\int\left(x+3-\frac{4}{x^3}\right)dx = \frac{x^2}{2}+3x-4\left(-\frac{1}{2x^2}\right) =$$

$$= \frac{x^2}{2}+3x+\frac{2}{x^2}+C$$

Ex 37

$$\int x(x^2+1)^2\,dx = \qquad u = x^2+1 \Rightarrow u' = 2x$$

$$= \frac{1}{2}\int 2x(x^2+1)^2\,dx = \frac{1}{2}\int u'u^2\,dx =$$

$$= \frac{1}{2}\,\frac{u^3}{3}+C = \frac{1}{6}(x^2+1)^3+C$$