Ex 46

$$\int_0^1 x e^{2x} \, dx = \qquad \int uv' = uv - \int u'v$$

$u = x \qquad u' = 1$

$v' = e^{2x} \qquad v = \frac{e^{2x}}{2}$

$$= \left[x \frac{e^{2x}}{2} \right]_0^1 - \int_0^1 \frac{e^{2x}}{2} \, dx =$$

$$= \frac{e^2}{2} - \left[\frac{e^{2x}}{4} \right]_0^1 = \frac{e^2}{2} - \left(\frac{e^2}{4} - \frac{e^0}{4} \right) =$$

$$= \frac{e^2}{2} - \frac{e^2}{4} + \frac{1}{4} = \frac{e^2}{4} + \frac{1}{4}$$

Ex 47

$$\int_0^2 x e^{-x} \, dx = \qquad \int uv' = uv - \int u'v$$

$u = x \qquad u' = 1$

$v' = e^{-x} \qquad v = -e^{-x}$

$$= \left[x(-e^{-x}) \right]_0^2 - \int_0^2 (-e^{-x}) \, dx =$$

$$= \left[-x e^{-x} \right]_0^2 + \int_0^2 e^{-x} \, dx =$$

$$= -2e^{-2} + \left[-e^{-x} \right]_0^2 = -2e^{-2} + \left(-e^{-2} - (-e^0) \right) =$$

$$= -2e^{-2} - e^{-2} + 1 = -3e^{-2} + 1$$

$$\int_1^e \ln(x)\,dx = \int_1^e 1 \times \ln(x)\,dx =$$

$u = \ln(x) \qquad u' = \frac{1}{x}$

$v' = 1 \qquad v = x$

$$= \left[x \ln(x)\right]_1^e - \int_1^e \frac{1}{x}\, x\,dx =$$

$$= e\ln(e) - 1\ln(1) - \int_1^e 1\,dx =$$

$$= e - \left[x\right]_1^e = e - (e-1) = 1$$

Ex 48

$$\int_0^1 3x\,e^{-2x}\,dx =$$

$u = 3x \qquad u' = 3$

$v' = e^{-2x} \qquad v = \frac{e^{-2x}}{-2}$

$$= \left[3x\,\frac{e^{-2x}}{-2}\right]_0^1 - \int_0^1 3\left(\frac{e^{-2x}}{-2}\right)dx =$$

$$= 3\,\frac{e^{-2}}{-2} + \frac{3}{2}\int_0^1 e^{-2x}\,dx =$$

$$= -\frac{3}{2}e^{-2} + \frac{3}{2}\left[\frac{e^{-2x}}{-2}\right]_0^1 =$$

$$= -\frac{3}{2}e^{-2} + \frac{3}{2}\left(\frac{e^{-2}}{-2} - \frac{e^{0}}{-2}\right) =$$

$$= -\frac{3}{2}e^{-2} + \frac{3}{2}\left(-\frac{e^{-2}}{2} + \frac{1}{2}\right) =$$

$$= -\frac{3}{2}e^{-2} - \frac{3}{4}e^{-2} + \frac{3}{4} =$$

$$= \frac{-6-3}{4}e^{-2} + \frac{3}{4} = -\frac{9}{4}e^{-2} + \frac{3}{4}$$

$$\int_1^e \ln(2x)\,dx = \int_1^e 1 \times \ln(2x)\,dx =$$

$$u = \ln(2x) \qquad u' = \frac{1}{2x}2 = \frac{1}{x}$$

$$v' = 1 \qquad v = x$$

$$= \left[x\ln(2x)\right]_1^e - \int_1^e \frac{1}{x}x\,dx =$$

$$= e\ln(2e) - \ln(2) - \left[x\right]_1^e =$$

$$= e\ln(2e) - \ln(2) - (e-1) =$$

$$= e(\ln(2) + \ln(e)) - \ln(2) - e + 1 =$$

$$= e\left(\ln(2)+1\right) - \ln(2) - e + 1 =$$

$$= e\ln(2) + \cancel{e} - \ln(2) - \cancel{e} + 1 =$$

$$= (e-1)\ln(2) + 1$$

Ex 49

$$\int_0^1 (x+2)\,e^{-x}\,dx =$$

$$u = x+2 \qquad u' = 1$$

$$v' = e^{-x} \qquad v = -e^{-x}$$

$$= \left[(x+2)(-e^{-x})\right]_0^1 - \int_0^1 (-e^{-x})\,dx =$$

$$= (1+2)(-e^{-1}) - (0+2)(-e^{0}) + \int_0^1 e^{-x}\,dx =$$

$$= -3e^{-1} + 2 + \left[-e^{-x}\right]_0^1 =$$

$$= -3e^{-1} + 2 + \left(-e^{-1} - (-e^{0})\right) =$$

$$= -3e^{-1} + 2 - e^{-1} + 1 =$$

$$= -4e^{-1} + 3$$

$$\int_1^4 (x-1)\ln(x)\,dx =$$

$$u = \ln(x) \qquad u' = \frac{1}{x}$$

$$v' = x-1 \qquad v = \frac{x^2}{2} - x$$

$$= \left[\left(\frac{x^2}{2} - x\right)\ln(x)\right]_1^4 - \int_1^4 \frac{1}{x}\left(\frac{x^2}{2} - x\right)dx =$$

$$= \left(\frac{16}{2} - 4\right)\ln(4) - \int_1^4 \left(\frac{x}{2} - 1\right)dx =$$

$$= 4\ln(4) - \left[\frac{x^2}{4} - x\right]_1^4 =$$

$$= 4\ln(4) - \left(\frac{16}{4} - 4 - \left(\frac{1}{4} - 1\right)\right) =$$

$$= 4\ln(4) - \left(\cancel{4} - \cancel{4} - \frac{1}{4} + 1\right) =$$

$$= 4\ln(4) + \frac{1}{4} - 1 = 4\ln(4) - \frac{3}{4}$$

Ex 50

$$\int_1^2 (t+1)\ln(3t)\,dt =$$

$$u = \ln(3t) \qquad u' = \frac{1}{t}$$

$$v' = t+1 \qquad v = \frac{t^2}{2} + t$$

$$= \left[\left(\frac{t^2}{2}+t\right)\ln(3t)\right]_1^2 - \int_1^2 \left(\frac{t}{2}+1\right)dt =$$

$$= \left(\frac{4}{2}+2\right)\ln(6) - \left(\frac{1}{2}+1\right)\ln(3) - \left[\frac{t^2}{4}+t\right]_1^2 =$$

$$= 4\ln(6) - \frac{3}{2}\ln(3) - \left(\frac{4}{4}+2-\left(\frac{1}{4}+1\right)\right) =$$

$$= 4\ln(6) - \frac{3}{2}\ln(3) - \left(3 - \frac{5}{4}\right) =$$

$$= 4\ln(6) - \frac{3}{2}\ln(3) - \left(\frac{12-5}{4}\right) =$$

$$= 4\ln(6) - \frac{3}{2}\ln(3) - \frac{7}{4} =$$

$$= \ln(6^4) - \frac{1}{2}\ln(3^3) - \frac{7}{4} =$$

$$= \frac{2\ln(6^4) - \ln(3^3)}{2} - \frac{7}{4} =$$

$$= \frac{\ln(6^8) - \ln(3^3)}{2} - \frac{7}{4} =$$

$$= \frac{1}{2} \ln\left(\frac{6^8}{3^3}\right) - \frac{7}{4} =$$

$$= \frac{1}{2} \ln(62208) - \frac{7}{4}$$

$$\int_{-1}^{0} (2t+1) e^{3t} \, dt =$$

$$u = 2t+1 \qquad u' = 2$$

$$v' = e^{3t} \qquad v = \frac{e^{3t}}{3}$$

$$= \left[(2t+1) \frac{e^{3t}}{3} \right]_{-1}^{0} - \int_{-1}^{0} 2 \, \frac{e^{3t}}{3} \, dt =$$

$$= \frac{1}{3} - (-2+1) \frac{e^{-3}}{3} - \frac{2}{3} \left[\frac{e^{3t}}{3} \right]_{-1}^{0} =$$

$$= \frac{1}{3} + \frac{e^{-3}}{3} - \frac{2}{3} \left(\frac{1}{3} - \frac{e^{-3}}{3} \right) =$$

$$= \frac{1}{3} + \frac{e^{-3}}{3} - \frac{2}{9} + \frac{2}{9} e^{-3} =$$

$$= \frac{3-2}{9} + \frac{3+2}{9} e^{-3} =$$

$$= \frac{1}{9} + \frac{5}{9} e^{-3}$$

Ex 53

$$f(x) = \frac{\ln(x)}{x^2} \qquad D_f = [1 ; +\infty[$$

1. $f(x) = \frac{u}{v}$ $\qquad u = \ln(x) \qquad v = x^2$

$$u' = \frac{1}{x} \qquad v' = 2x$$

$$f'(x) = \frac{u'v - uv'}{v^2} = \frac{\frac{1}{x} x^2 - \ln(x)\, 2x}{x^4} =$$

$$= \frac{x - 2x \ln(x)}{x^4} = \frac{1 - 2\ln(x)}{x^3}$$

Étude de signe de f' sur $D_f = [1 ; +\infty[$

$$1 - 2\ln(x) > 0$$

$$-2\ln(x) > -1$$

$$\ln(x) < 1/2 \Rightarrow x < e^{1/2}$$

$$x < \sqrt{e}$$

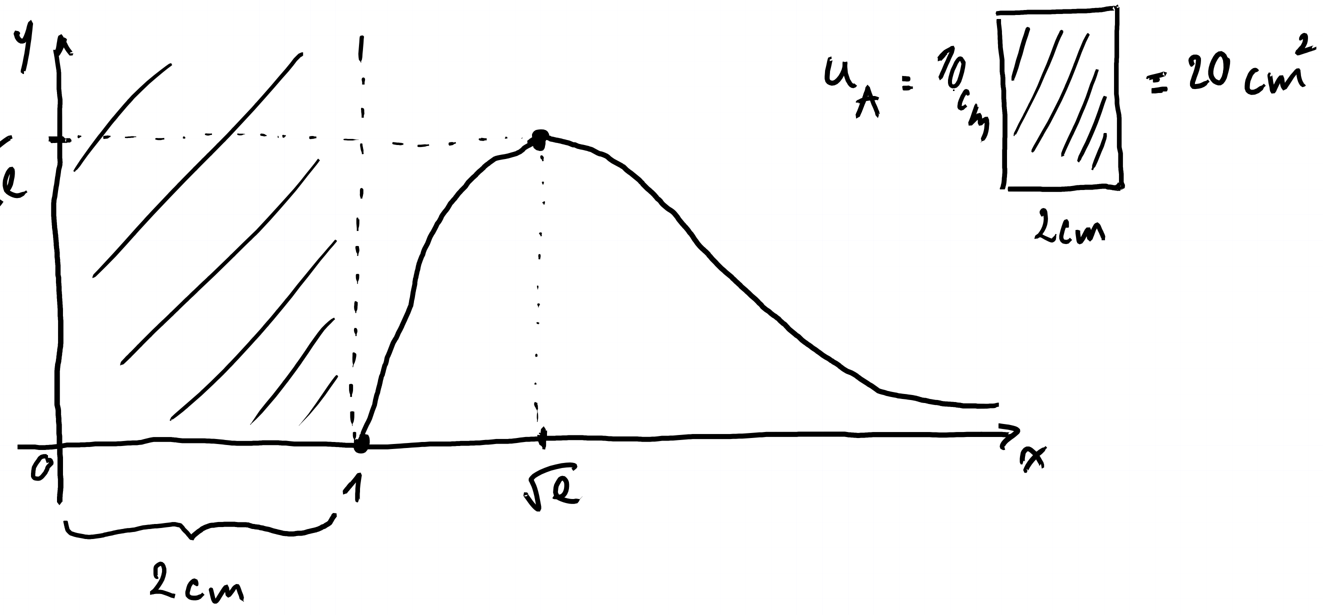
Tableau de variations

x	1		$\sqrt{e}$		$+\infty$
f'		$+$	0	$-$	
f	0	↗	$1/2e$	↘	0

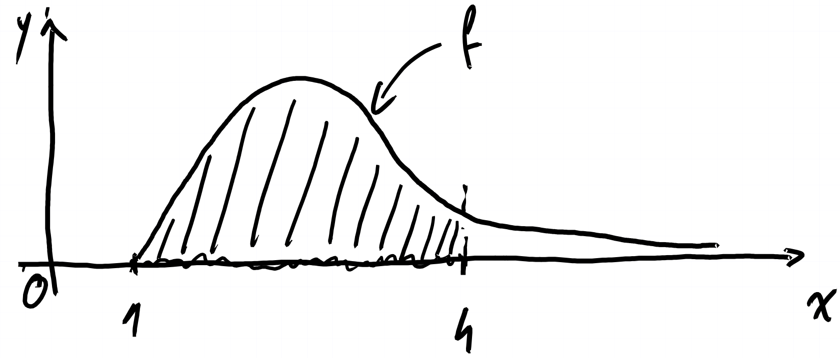
$$f(1) = 0$$

$$f(\sqrt{e}) = \frac{\ln(\sqrt{e})}{e} =$$

$$= \frac{1}{2e}$$



2.



$$A = \int_1^4 f(x)\,dx = \int_1^4 \frac{\ln(x)}{x^2}\,dx =$$

$$u = \ln(x) \qquad u' = \frac{1}{x}$$

$$v' = \frac{1}{x^2} \qquad v = -\frac{1}{x}$$

$$= \left[-\frac{1}{x}\ln(x)\right]_1^4 - \int_1^4 \frac{1}{x}\left(-\frac{1}{x}\right)dx =$$

$$= -\frac{1}{4}\ln(4) + \int_1^4 \frac{1}{x^2}\,dx =$$

$$= -\frac{1}{4}\ln(4) + \left[-\frac{1}{x}\right]_1^4 =$$

$$= -\frac{1}{4}\ln(4) + \left(-\frac{1}{4} - (-1)\right) =$$

$$= -\frac{1}{4}\ln(4) + \left(\frac{3}{4}\right) = \frac{3}{4} - \frac{1}{4}\ln(4)$$

$$A = \left(\frac{3}{4} - \frac{1}{4}\ln(4)\right) u_A = \left(\frac{3}{4} - \frac{1}{4}\ln(4)\right) 20 \text{ cm}^2 =$$

$$= 8{,}07 \text{ cm}^2 = 807 \text{ mm}^2$$

Valeur moyenne de f entre 1 et 4 :

$$\frac{1}{4-1}\int_1^4 f(x)\,dx = \frac{807}{3} \text{ mm}^2$$

Déf : Valeur moyenne de f entre a et b :

$$\frac{1}{b-a}\int_a^b f(x)\,dx$$

Ex 54

$f(x) = xe^x - e^x \qquad D_f = [-1; 1]$

1. $f(x) = e^x(x-1) = uv$

$u = e^x \qquad v = x - 1$

$u' = e^x \qquad v' = 1$

$f'(x) = u'v + uv' = e^x(x-1) + e^x =$

$= e^x(x-1+1) = xe^x$

Étude de signe de f' :

$x > 0$	$e^x > 0$ Toujours

Tableau de variations :

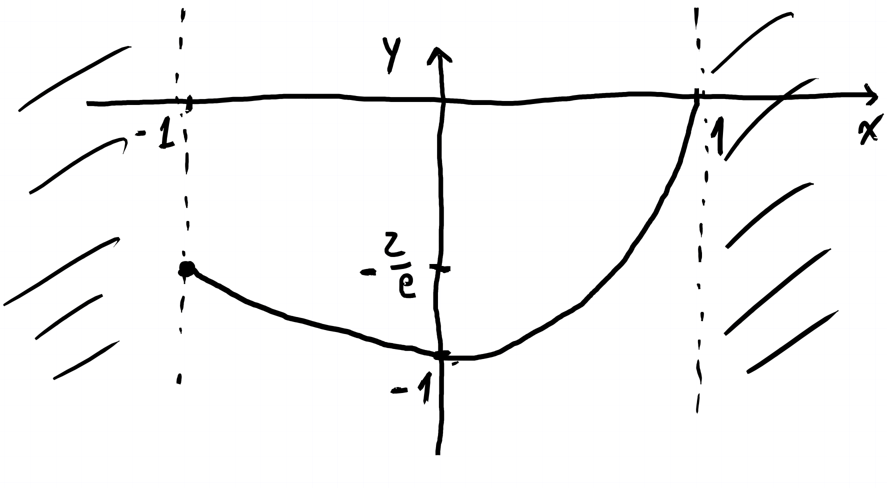
x	-1		0		1
f'		$-$	0	$+$	
f	$-2/e$	$\searrow$	-1	$\nearrow$	0

$f(x) = e^x(x-1)$

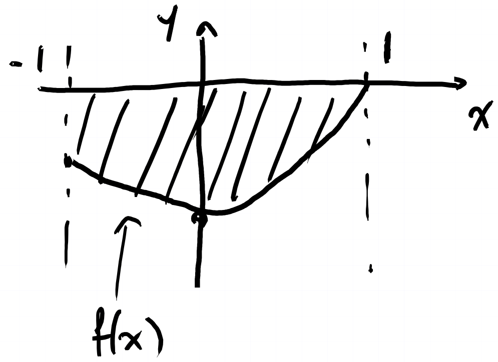
$f(-1) = e^{-1}(-2)$
$= -\frac{2}{e}$

$f(0) = e^0(-1)$
$= -1$

$f(1) = 0$



2.



$$A = -\int_{-1}^{1} f(x)\,dx$$

$$A = -\int_{-1}^{1} e^{x}(x-1)\,dx =$$

$u = x-1 \qquad u' = 1$

$v' = e^{x} \qquad v = e^{x}$

$$= -\left\{ \left[(x-1)e^{x}\right]_{-1}^{1} - \int_{-1}^{1} e^{x}\,dx \right\} =$$

$$= -\left\{ 0 - (-2)e^{-1} - \left[e^{x}\right]_{-1}^{1} \right\} =$$

$$= -\left\{ 2e^{-1} - \left(e^{1} - e^{-1}\right) \right\} =$$

$$= -2e^{-1} + e - e^{-1} = \left(e - 3e^{-1}\right) u_A$$

$$u_A = 4 \times 4 \text{ cm}^2 = 16 \text{ cm}^2$$

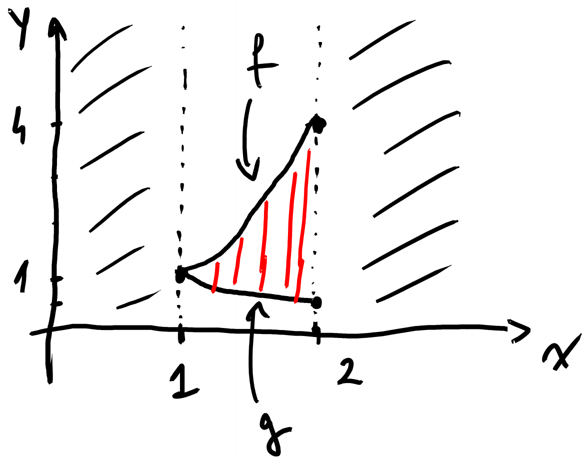
$$A = \left(e - 3e^{-1}\right) \times 16 \text{ cm}^2 = 25{,}83 \text{ cm}^2$$

$$= 2583 \text{ mm}^2$$

Ex 55

$f(x) = x^2 \quad g(x) = \frac{1}{x} \quad D_f = [1;2]$

1.



2. a) $A = \int_1^2 [f(x) - g(x)]\,dx = \int_1^2 \left(x^2 - \frac{1}{x}\right) dx =$

$= \left[\frac{x^3}{3} - \ln(x)\right]_1^2 =$

$= \frac{8}{3} - \ln(2) - \frac{1}{3} = \frac{7}{3} - \ln(2)$

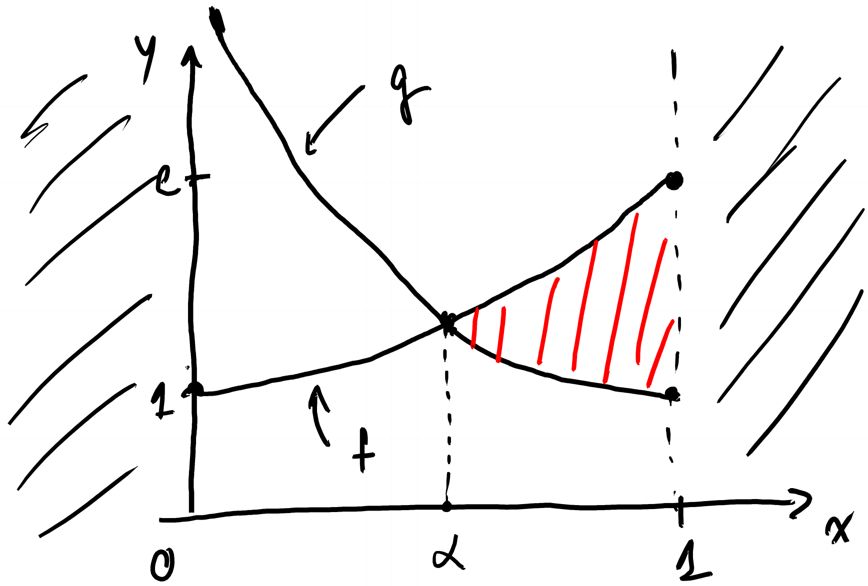
b) $A = \left(\frac{7}{3} - \ln(2)\right) \times 4 \text{ cm}^2 = 6{,}56 \text{ cm}^2$

Ex 56

$f(x) = e^x \qquad D_f = [0;1]$

$g(x) = \frac{1}{x} \qquad D_g =]0;1]$

1.



2. a) $A = \int_\alpha^1 (f-g)\,dx = \int_\alpha^1 \left(e^x - \frac{1}{x}\right) dx =$

$= \left[e^x - \ln(x) \right]_\alpha^1 = e - \left(e^\alpha - \ln(\alpha)\right) =$

$= \left(e - e^\alpha + \ln(\alpha)\right) u_A =$

$= \left(e - e^\alpha + \ln(\alpha)\right) \times 16 \text{ cm}^2$

b) $A = \left(e - e^{0,57} + \ln(0,57)\right) \times 16 \text{ cm}^2 =$

$= 6,21 \text{ cm}^2$