

$$f(x) = x e^{1/x} = x e^u$$

$$D_f = \mathbb{R} \setminus \{0\}$$

$$\text{avec } u = \frac{1}{x} \Rightarrow u' = -\frac{1}{x^2}$$

$$f(x) = x e^{1/x} = w v$$

$$\text{avec } w = x \quad \text{et} \quad v = e^{1/x} = e^u$$

$$w' = 1$$

$$v' = u' e^u = -\frac{1}{x^2} e^{1/x}$$

$$f'(x) = w' v + w v' = e^{1/x} + x \left(-\frac{1}{x^2} e^{1/x} \right) =$$

$$= e^{1/x} - \frac{1}{x} e^{1/x} = e^{1/x} \left(1 - \frac{1}{x} \right) =$$

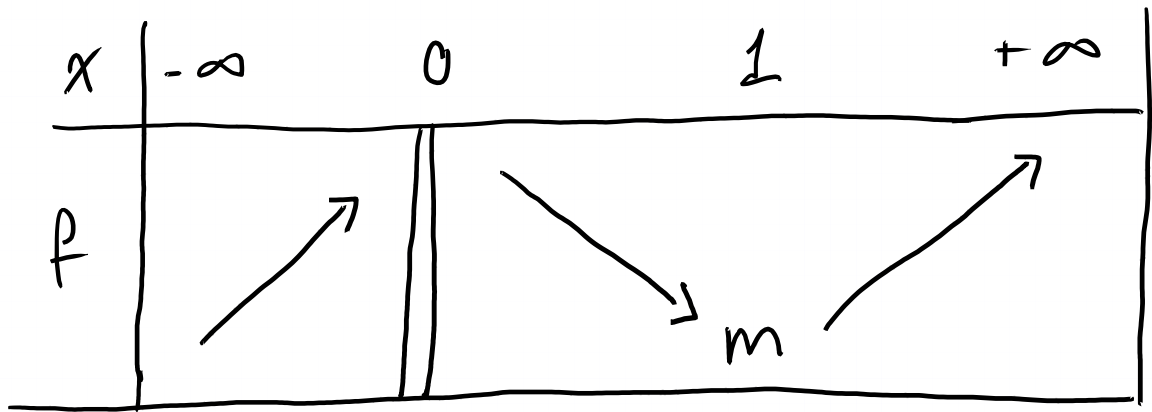
$$= e^{1/x} \left(\frac{x-1}{x} \right)$$

Étude de signe de f' :

$$\begin{array}{c|c|c} e^{1/x} > 0 & x-1 > 0 & x > 0 \\ \text{Toujours} & x > 1 & \end{array}$$

| x | $-\infty$ | 0 | 1 | $+\infty$ |
|-----------|-----------|-----|-----|-----------|
| $e^{1/x}$ | + | | + | + |
| $x-1$ | - | | - 0 | + |
| x | - | | + | + |
| f' | + | | - 0 | + |

Tableau de variations

| x | $-\infty$ | 0 | 1 | $+\infty$ |
|-----|--|-----|-----|-----------|
| f |  | | | |

$$m = f(1) = 1 \times e^{\frac{1}{1}} = e$$