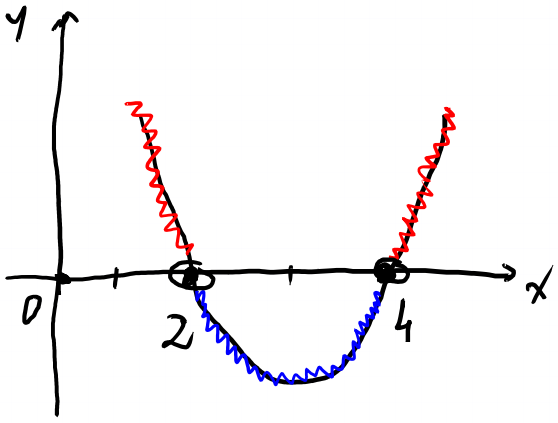
Étude de signe d'une parabole

Exemple :



=>

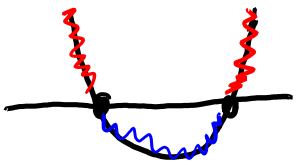
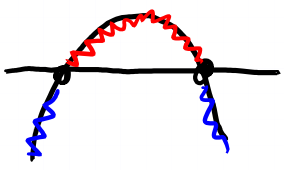
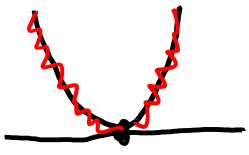
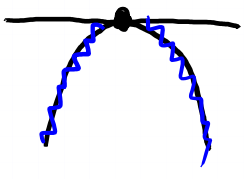
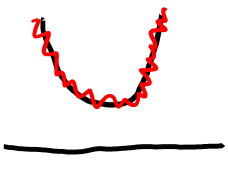
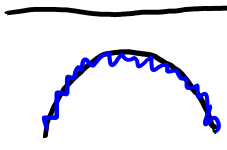
x	$-\infty$		2		4		$+\infty$
	$+$	0	$-$	0	$+$		

$$ax^2 + bx + c$$

1) Si $a > 0$ => ∪

Si $a < 0$ => ∩

2) Calcul le $\Delta = b^2 - 4ac$

	$a > 0$	$a < 0$
$\Delta > 0$		
$\Delta = 0$		
$\Delta < 0$		

3) Tableau de signe

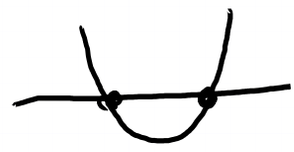
	$a > 0$	$a < 0$
$\Delta > 0$	x | x_1 | x_2 : $+$ 0 $-$ 0 $+$	x | x_1 | x_2 : $-$ 0 $+$ 0 $-$
$\Delta = 0$	x | x_1 : $+$ 0 $+$	x | x_1 : $-$ 0 $-$
$\Delta < 0$	x : $+$	x : $-$

Exemple : Étudier le signe de $2x^2+3x-2$

1) $a = 2 > 0 \Rightarrow$ ∪

2) $\Delta = ?$ $\quad a = 2 \quad b = 3 \quad c = -2$

$$\Delta = b^2 - 4ac = 3^2 - 4 \times 2 \times (-2) = 9 + 16 = 25 > 0$$



3) $x_1 = \dfrac{-b-\sqrt{\Delta}}{2a} = \dfrac{-3-5}{4} = -2 \qquad x_2 = \dfrac{-b+\sqrt{\Delta}}{2a} = \dfrac{-3+5}{4} = \dfrac{1}{2}$

Tableau de signe :

x	$-\infty$	-2	$1/2$	$+\infty$
	$+$	0	$-$ 0	$+$

Résoudre un' équation du 2ème degré

Exemple : $x^2 - 4x + 4 > 0$

$a = 1 \quad b = -4 \quad c = 4$

$a > 0 \Rightarrow$ 

$\Delta = b^2 - 4ac = (-4)^2 - 4 \times 1 \times 4 = 16 - 16 = 0$

$$x_1 = -\frac{b}{2a} = -\frac{-4}{2} = \frac{4}{2} = 2$$

Tableau de signe

x	$-\infty$		2		$+\infty$
		$+$	0	$+$	

$$S =]-\infty ; 2[\cup]2 ; +\infty[= \mathbb{R} \setminus \{2\}$$

Exercices :

1) $4x^2 - 4x + 1 < 0$

2) $x^2 + x + 3 > 0$

3) $3x^2 + 5x - 2 \leq 0$

4) $4x^2 - 9 > 0$

5) $-x^2 + 8x - 12 > 0$

6) $6x^2 - 5x + 1 \geqslant 0$

7) $4 - x^2 \leq 0$

8) $4(x^2 - 1) < 4x - 1$

9) $(x+5)(x^2 - 5x + 25) < 0$

10) $(2x-1)(4x^2 + 2x + 1) \leq 0$