$f(x)$, $F(x)$ est primitive de $f(x)$

si $F'(x) = f(x)$

<u>Exemple</u>: $f(x) = 1$

$F(x) = ?$ $\quad F'(x) = f(x) = 1$

$F(x) = x \Rightarrow F'(x) = 1$

Donc $F(x)$ est primitive de $f(x)$

Mais : $G(x) = x + 3$

→ Vérifier que $G(x)$ est aussi primitive

$G'(x) = 1 + 0 = 1 \Rightarrow G'(x) = f(x)$

Donc $G(x)$ est primitive de $f(x)$

Si $F(x)$ est primitive de $f(x)$

Alors $G(x) = F(x) + C$ est aussi primitive de $f(x)$ car $G'(x) = F'(x)$

Ex 1

a) $F(x) = \frac{x^3}{3} + 2x^2 + 4$ $\qquad f(x) = x^2 + 4x$

Je dois montrer que $F' = f$

$$F'(x) = \frac{\cancel{3}x^2}{\cancel{3}} + 2 \times 2x + 0 = x^2 + 4x = f(x)$$

Donc $F(x)$ est primitive de $f(x)$

c) $F(x) = e^{-2x} + 3e^x + 5$ $\qquad f(x) = 3e^x - 2e^{-2x}$

$F' = f$ $\qquad F(x) = e^u + 3e^x + 5$

$(e^u)' = e^u \times u'$ avec $u = -2x$ $\quad u' = -2$

$$F'(x) = -2e^{-2x} + 3e^x + 0 = 3e^x - 2e^{-2x} = f(x)$$

Donc F est primitive de f.

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f	F
a	$ax + C$
$\frac{1}{x}$	$\ln x + C$
x^n	$\frac{x^{n+1}}{n+1} + C$ $\rightarrow$ si $\boxed{n \neq -1}$

car si $n = -1 \Rightarrow x^{-1} = \frac{1}{x}$

f	F
x^n $n \neq -1$	$\frac{x^{n+1}}{n+1} + C$

$$f(x) = x = x^1 \Rightarrow F(x) = \frac{x^{1+1}}{1+1} + C = \frac{x^2}{2} + C$$

$$f(x) = x^2 \Rightarrow F(x) = \frac{x^{2+1}}{2+1} + C = \frac{x^3}{3} + C$$

$$f(x) = 2x^3 \Rightarrow F(x) = 2\frac{x^{3+1}}{3+1} = 2\frac{x^4}{4} = \frac{1}{2}x^4$$

$$f(x) = 2x^{-4} \Rightarrow F(x) = 2\frac{x^{-4+1}}{-4+1} = 2\frac{x^{-3}}{-3} = -\frac{2}{3}x^{-3}$$

$$= -\frac{2}{3x^3}$$

$$f(x) = \sqrt{x} = x^{1/2}$$

$$\hookrightarrow F(x) = \frac{x^{1/2+1}}{1/2+1} = \frac{x^{3/2}}{3/2} = \frac{2}{3}x^{3/2} = \frac{2}{3}\sqrt{x^3}$$

$$f(x) = \frac{1}{\sqrt{x}} = \frac{1}{x^{1/2}} = x^{-1/2}$$

$$\hookrightarrow F(x) = \frac{x^{-1/2+1}}{-1/2+1} = \frac{x^{1/2}}{1/2} = 2x^{1/2} = 2\sqrt{x}$$

f	F
e^x	e^x
e^u	?
$e^u \times u'$	e^u

Example :

1) $f(x) = 3e^{3x}$

? $f(x) = u' \times e^u$

$u = 3x \quad u' = 3$

Donc $f(x) = u' e^u \Rightarrow$ Oui

Alor : $F(x) = e^{3x} + C$

2) $f(x) = e^{3x} = e^u$ avec $u = 3x$

$F(x) = ?$ dors $\underline{u' = 3}$

[Obj : $f(x) = u' \times e^u$] $\leftarrow$

$$f(x) = \frac{3 \times e^{3x}}{3} = \frac{1}{3}\left(3e^{3x}\right) = \frac{1}{3}\left(e^u \times u'\right)$$

$$F(x) = \frac{1}{3} e^{3x} + C$$

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f	F
$u^n \times u'$ $n \neq -1$	$\frac{u^{n+1}}{n+1} + c$

$f(x) = (3x+1)^2 = u^2$

avec $u = 3x+1 \quad u' = 3$

$$f(x) = \frac{(3x+1)^2 \times 3}{3} = \frac{1}{3}\left(3(3x+1)^2\right)$$

$$\Rightarrow F(x) = \frac{1}{3} \frac{(3x+1)^3}{3} = \frac{1}{9}(3x+1)^3$$

Ex 3

$f(x) = x^2 - 3x \qquad F(x) = \frac{x^3}{3} - 3\frac{x^2}{2} + C$

$g(x) = -2x^3 + 4x - 5 \qquad G(x) = -2\frac{x^4}{4} + 4\frac{x^2}{2} - 5x + C$

$= -\frac{x^4}{2} + 2x^2 - 5x + C$

Ex 4

$f(x) = 4e^x - 2x \qquad F(x) = 4e^x - 2\frac{x^2}{2} + C$

$= 4e^x - x^2 + C$

Ex 5

$f(x) = \frac{1}{x} + 3x \qquad F(x) = \ln(x) + \frac{3x^2}{2} + C$

$g(x) = x^2 - \frac{2}{x^2} = x^2 - 2x^{-2}$

$G(x) = \frac{x^3}{3} - 2\frac{x^{-1}}{-1} = \frac{x^3}{3} + 2x^{-1} = \frac{x^3}{3} + \frac{2}{x} + C$

Ex 6

$f(x) = 3x^2 - \frac{4}{x^2} = 3x^2 - 4x^{-2}$

$F(x) = \cancel{3}\frac{x^3}{\cancel{3}} - 4\frac{x^{-1}}{-1} = x^3 + \frac{4}{x}$

$$g(x) = 1 + \frac{2}{x^2} - \frac{1}{x^4} = 1 + 2x^{-2} - x^{-4}$$

$$G(x) = x + 2\frac{x^{-1}}{-1} - \frac{x^{-3}}{-3} + C = x - \frac{2}{x} + \frac{1}{3x^3}$$

<u>Ex 9</u>

$$f(x) = 2e^{2x} = 2e^{u} \qquad u = 2x \quad u' = 2$$

Donc $f(x) = u'e^{u} \Rightarrow F(x) = e^{2x} + C$

<u>Ex 10</u>

$$f(x) = e^{-x} = e^{u} \qquad u = -x \quad u' = -1$$

$$f(x) = \frac{(-1)\,e^{-x}}{-1} = \frac{1}{-1}\,u'\,e^{u} = (-1)\,u'e^{u} = -u'e^{u}$$

$$F(x) = -e^{u} + C = -e^{-x} + C$$

<u>Ex 11</u>

$$f(x) = 2e^{3x+1} = 2e^{u} \qquad u = 3x+1 \quad u' = 3$$

$$f(x) = 2\times\frac{3\times e^{3x+1}}{3} = \frac{2}{3}\left(3e^{3x+1}\right) = \frac{2}{3}u'e^{u}$$

$$F(x) = \frac{2}{3}e^{u} + C = \frac{2}{3}e^{3x+1} + C$$

Ex 12

$f(x) = x + 4e^{-3x} = x + 4e^{u}$ $\quad u = -3x \quad u' = -3$

$f(x) = x + 4 \times \frac{(-3)e^{-3x}}{-3} = x - \frac{4}{3}\left(-3e^{-3x}\right) = x - \frac{4}{3}u'e^{u}$

$F(x) = \frac{x^2}{2} - \frac{4}{3}e^{-3x} + C$

Ex 13

$f(x) = 2x\left(e^{x^2}\right) = 2x\, e^{u}$ $\quad u = x^2 \quad u' = 2x$

$= u'\, e^{u}$

$F(x) = e^{u} + C = e^{x^2} + C$

Ex 14

$f(x) = \frac{1}{(x-2)^2} = \frac{1}{u^2} = u^{-2}$ $\quad u = x - 2 \quad u' = 1$

$f(x) = u' \times u^{-2}$ $\quad F(x) = \frac{u^{-2+1}}{-2+1} + C = -u^{-1} + C$

$= \frac{-1}{u} + C = -\frac{1}{x-2} + C$

Ex 16

$$f(x) = x(x^2+1)^3 = x\,u^3 \qquad u = x^2+1 \qquad u' = 2x$$

$$f(x) = \frac{2x\,u^3}{2} = \frac{1}{2}(2x\,u^3) = \frac{1}{2}(u'u^3)$$

$$F(x) = \frac{1}{2}\,\frac{u^4}{4} + c = \frac{1}{8}(x^2+1)^4 + C$$

Ex 17

$$f(x) = \frac{1}{x-2} = \frac{1}{u} \qquad u = x-2 \qquad u' = 1$$

$$f(x) = \frac{u'}{u} \qquad F(x) = \ln(u) + c = \ln(x-2) + C$$

Ex 18

$$f(x) = x^2 - x + 1 \quad ; \quad \underline{G(1) = 0}$$

$$G(x) = \frac{x^3}{3} - \frac{x^2}{2} + x + C$$

$$G(1) = \frac{1^3}{3} - \frac{1^2}{2} + 1 + C = 0$$

$$C = -\frac{1}{3} + \frac{1}{2} - 1 = \frac{-2+3-6}{6} = -\frac{5}{6}$$

$$G(x) = \frac{x^3}{3} - \frac{x^2}{2} + x - \frac{5}{6}$$

Ex 19

$f(x) = x - \frac{2}{x}$; $G(1) = 0$

$$G(x) = \frac{x^2}{2} - 2\ln(x) + c$$

$$G(1) = \frac{1}{2} - 2\ln(1) + c = 0$$

$$c = -\frac{1}{2} + 2\overset{\to 0}{\ln(1)} = -\frac{1}{2}$$

$$G(x) = \frac{x^2}{2} - 2\ln(x) - \frac{1}{2}$$

Ex 20

$f(x) = 2x + \frac{1}{x}$; $G(2) = 0$

$$G(x) = \frac{\cancel{2}x^2}{\cancel{2}} + \ln(x) + c = x^2 + \ln(x) + c$$

$$G(2) = 4 + \ln(2) + c = 0$$

$$c = -4 - \ln(2)$$

$$G(x) = x^2 + \ln(x) - 4 - \ln(2)$$