Ex 1 :

$$\ln 3 + \ln \frac{1}{3} = \cancel{\ln 3} + \ln 1 - \cancel{\ln 3} = 0$$

$$\ln e^3 + \ln e = 3 + 1 = 4$$

$$e^{-\ln 2} = \frac{1}{e^{\ln 2}} = \frac{1}{2}$$

$$\ln \sqrt{e^5} = \ln (e^5)^{1/2} = \ln e^{5/2} = \frac{5}{2}$$

$$e^{\ln 5 - \ln 3} = \frac{e^{\ln 5}}{e^{\ln 3}} = \frac{5}{3}$$

$$\ln e^3 + e^{\ln 3} = 3 + 3 = 6$$

Ex 2 :

1. $\ln x + 2 = 0$

Ensemble de définition :

$x > 0 \qquad D =]0 ; +\infty[$

Solution de l'équation :

$\ln x = -2 \iff x = e^{-2} \qquad S = \{e^{-2}\}$

$\ln(x+1) - 3 = 0$

Ensemble de définition:

$x+1 > 0 \iff x > -1 \qquad D =]-1 ; +\infty[$

Solution de l'éq:

$\ln(x+1) = 3$

$x+1 = e^3 \iff x = e^3 - 1 \Rightarrow S = \{e^3 - 1\}$

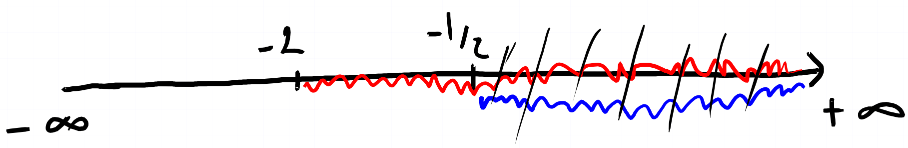
2. $\ln(x+2) = \ln(2x+1)$

Ensemble de définition

$x+2 > 0$ **et** $2x+1 > 0$

$x > -2 \qquad 2x > -1$

$x > -\frac{1}{2}$



$D =]-1/2 ; +\infty[$

Solution de l'éq:

$x+2 = 2x+1$

$-x = -1$

$x = 1 \qquad S = \{1\}$

$2\ln x + \ln 3 = 0$

Ensemble de définition:

$x > 0 \qquad D =]0;+\infty[$

Solution de l'équation:

$2\ln x = -\ln 3$

$\ln x = -\frac{\ln 3}{2}$

$x = e^{-\frac{\ln 3}{2}} \qquad S = \left\{ \frac{1}{\sqrt{3}} \right\}$

$$e^{-\frac{\ln 3}{2}} = \frac{1}{e^{\frac{\ln 3}{2}}} = \frac{1}{\sqrt{e^{\ln 3}}} = \frac{1}{\sqrt{3}}$$

II Méthode:

$2\ln x + \ln 3 = 0$

$D =]0;+\infty[$

$\ln(x^2) + \ln 3 = 0$

$\ln(3x^2) = 0$

$3x^2 = e^0 \iff 3x^2 = 1 \iff x^2 = \frac{1}{3}$

$\Leftrightarrow \quad x_1 = -\frac{1}{\sqrt{3}} \qquad x_2 = \frac{1}{\sqrt{3}}$

NON car

$x_1 \notin D \Rightarrow S = \left\{ \frac{1}{\sqrt{3}} \right\}$

3. $\ln(x+2) = 0$

Ensemble de définition

$x+2 > 0 \Leftrightarrow x > -2 \Rightarrow D =]-2 ; +\infty[$

Solution de l'équation :

$x+2 = e^0 \Leftrightarrow x = -1 \qquad S = \{-1\}$

4. $e^{2x} - 3 = 0 \qquad D = \mathbb{R}$

$e^{2x} = 3$

$2x = \ln 3 \Leftrightarrow x = \frac{\ln 3}{2} \qquad S = \left\{ \frac{\ln 3}{2} \right\}$

$e^{2x} = e^{x+1} \qquad D = \mathbb{R}$

$2x = x+1 \Leftrightarrow x = 1 \qquad S = \{1\}$

5. $e^{4x} - 2e^{3x} = 0$

$e^{3x}(e^x - 2) = 0$

$e^{3x} = 0$ ou $e^{x} - 2 = 0$

impossible $\quad e^{x} = 2$

$x = \ln 2 \qquad S = \{\ln 2\}$

$e^{0,2x} = 2e^{-0,2x} \qquad D = \mathbb{R}$

$e^{0,2x} - 2e^{-0,2x} = 0$

$e^{-0,2x}\left(e^{0,4x} - 2\right) = 0$

$e^{-0,2x} = 0$ ou $e^{0,4x} - 2 = 0$

impossible $\quad e^{0,4x} = 2$

$0,4x = \ln 2$

$x = \dfrac{\ln 2}{0,4} \qquad S = \left\{\dfrac{\ln 2}{0,4}\right\}$

$\ln(e^{0,2x}) = \ln(2e^{-0,2x})$

$0,2x = \ln 2 - 0,2x$

$0,4x = \ln 2$

$x = \dfrac{\ln 2}{0,4}$

6. $e^{2x} - 2e^{x} - 3 = 0 \qquad D = \mathbb{R}$

Changement de variable $e^{x} = X$; $X > 0$

$\Rightarrow e^{2x} = X^2$

$\Rightarrow X^2 - 2X - 3 = 0$

$\Delta = (-2)^2 - 4 \times (1) \times (-3) = 4 + 12 = 16$

$X_1 = \frac{2+4}{2} = 3$ $\qquad$ $X_2 = \frac{2-4}{2} = -1 \rightarrow$ NON <0

$\hookrightarrow e^x = 3 \Rightarrow x = \ln 3$ $\qquad$ $S = \{\ln 3\}$

$e^{2x} - 2e^x + 2 = 0$ $\qquad$ $D = \mathbb{R}$

Changement de variable : $e^x = X$; $X > 0$

$\Rightarrow e^{2x} = X^2$

$X^2 - 2X + 2 = 0$

$\Delta = (-2)^2 - 4 \times 1 \times 2 = 4 - 8 = -4 < 0$

Pas de solution $\Rightarrow S = \emptyset$