

Exercice: Dresser le tableau de variations

$$1) f(x) = \frac{3x}{x+2} \quad I = \mathbb{R} \setminus \{-2\}$$

$$2) f(x) = -\frac{3}{x^2-5x} \quad I = \mathbb{R} \setminus \{0; 5\}$$

$$3) f(x) = \frac{x^2+7}{x+3} \quad I = [-1; 5]$$

$$1) f(x) = \frac{u}{v} \quad \begin{array}{ll} u = 3x & v = x+2 \\ u' = 3 & v' = 1 \end{array}$$

$$\begin{aligned} f'(x) &= \frac{u'v - uv'}{v^2} = \frac{3(x+2) - (3x)(1)}{(x+2)^2} = \\ &= \frac{\cancel{3x} + 6 - \cancel{3x}}{(x+2)^2} = \frac{6}{(x+2)^2} \end{aligned}$$


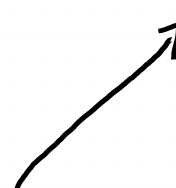
( $\triangle$   $x+2=0$  impossible  $\Rightarrow x=-2$  v.I.)

Étude de signe de  $f'$ :

6 est positif ;  $(x+2)^2$  est positif sauf pour  $x=-2$

$x$	$-\infty$	$-2$	$+\infty$
$6$	+		
$(x+2)^2$	+		+
$f'$	+		+

Tableau de variations:

$x$	$-\infty$	$-2$	$+\infty$
$f'$	+		+
$f$			

$$2) \quad f(x) = \frac{u}{v} \quad \begin{array}{ll} u = -3 & v = x^2 - 5x \\ u' = 0 & v' = 2x - 5 \end{array}$$

$$f'(x) = \frac{u'v - uv'}{v^2} = \frac{0(x^2 - 5x) - (-3)(2x - 5)}{(x^2 - 5x)^2} =$$

$$= \frac{6x - 15}{(x^2 - 5x)^2}$$




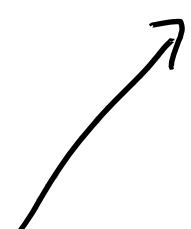
$$\left( \triangle! \quad \begin{array}{l} x^2 - 5x = 0 \text{ impossible} \\ x(x - 5) = 0 \Rightarrow x = 0 \text{ et } x = 5 \text{ V.I.} \end{array} \right)$$

Étude de signe de  $f'$ :

$$6x - 15 > 0 \Leftrightarrow x > \frac{15}{6} ; \quad (x^2 - 5x)^2 \text{ est positif sauf pour } x = 0 \text{ et } x = 5.$$

$x$	$-\infty$	$0$	$15/6$	$5$	$+\infty$
$6x - 15$		-	$\emptyset$	+	
$(x^2 - 5x)^2$	+		+		+
$f'$	-		- $\emptyset$ +		+

Tableau de variations:

x	$-\infty$	0	$15/6$	5	$+\infty$
f'	-		- 0 +		+
f			 $f(15/6)$		

$$f\left(\frac{15}{6}\right) = 0,48$$

$$3) f(x) = \frac{x^2+7}{x+3} \quad I = [-1; 5]$$

$$f(x) = \frac{u}{v} \quad u = x^2+7 \quad v = x+3$$

$$u' = 2x \quad v' = 1$$

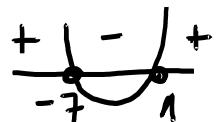
$$f'(x) = \frac{u'v - uv'}{v^2} = \frac{(2x)(x+3) - (x^2+7)(1)}{(x+3)^2} =$$

$$= \frac{2x^2 + 6x - x^2 - 7}{(x+3)^2} = \frac{x^2 + 6x - 7}{(x+3)^2}$$

signe de  $f'$ :

$$x^2 + 6x - 7 \quad a=1 \quad \cup \quad b=6 \quad c=-7$$

$$\Delta = 36 - 4 \times 1 \times (-7) = 36 + 28 = 64$$



$$x_1 = \frac{-6-8}{2} = -7 \quad x_2 = \frac{-6+8}{2} = 1$$

$(x+3)^2$  est positif sauf pour  $x=-3$  V.I.

$x$	-1	1	5
$f'$	-	0	+
$f$	$f(-1)$	$f(1)$	$f(5)$

Diagram showing the function values at the critical points and endpoints. Arrows indicate the path from  $f(-1)$  to  $f(1)$  and from  $f(1)$  to  $f(5)$ .

$$f(-1) = \frac{(-1)^2 + 7}{-1 + 3} = \frac{8}{2} = 4$$

$$f(1) = \frac{1^2 + 7}{1 + 3} = \frac{8}{4} = 2$$

$$f(5) = \frac{5^2 + 7}{5 + 3} = \frac{32}{8} = 4$$