Ex 5

$f(x) = \frac{1}{x} + 3x \qquad g(x) = x^2 - \frac{2}{x^2} \qquad I =]0;+\infty[$

$$F(x) = \ln(x) + 3\frac{x^2}{2} + C$$

$$G(x) = \frac{x^{2+1}}{2+1} - 2\left(-\frac{1}{x}\right) = \frac{x^3}{3} + \frac{2}{x} + C$$

Ex 6

$f(x) = 3x^2 - \frac{4}{x^2} \qquad g(x) = 1 + \frac{2}{x^2} - \frac{1}{x^4}$

$$F(x) = \cancel{3}\frac{x^3}{\cancel{3}} - 4\left(-\frac{1}{x}\right) + C = x^3 + \frac{4}{x} + C$$

f	F
x^n	$\frac{x^{n+1}}{n+1}$

$\frac{1}{x^4} = x^{-4}$

$\to n = -4$

f	F
x^{-4}	$\frac{x^{-4+1}}{-4+1} = \frac{x^{-3}}{-3} = -\frac{1}{3x^3}$

$$G(x) = x - \frac{2}{x} - \left(-\frac{1}{3x^3}\right) + C$$

$$= x - \frac{2}{x} + \frac{1}{3x^3} + C$$

Attention!

$$\boxed{x^{-1} = \frac{1}{x} \longrightarrow F = \ln(x)}$$

$$x^2 \Rightarrow F = \frac{x^3}{3}$$

$$x^{-5} \Rightarrow F = \frac{x^{-5+1}}{-5+1} = \frac{x^{-4}}{-4} = -\frac{1}{4x^4}$$

$$f = \frac{1}{\sqrt{x}} = \frac{1}{x^{1/2}} = x^{-1/2}$$

$$\hookrightarrow F = \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} = \frac{x^{1/2}}{1/2} =$$

$$\longrightarrow F = 2\sqrt{x}$$

Ex 10

$$f(x) = e^{-x} = e^{u} \quad \text{avec} \quad u = -x$$

$$\boxed{\text{si } f(x) = e^{u} u' \longrightarrow F(x) = e^{u} + C}$$

$$u' = -1 \Rightarrow f(x) = \frac{(-1)}{-1} e^{-x} = \frac{1}{-1}(-1) e^{-x}$$

$$= -1 \underbrace{u' e^{u}}$$

$$\text{Donc } F(x) = -1 e^{-x} + C = -e^{-x} + C$$

Ex 11

$$f(x) = 2e^{3x+1} = 2e^{u} \qquad u = 3x+1 \qquad u' = 3$$

$$f(x) = 2 \times \frac{3}{3} \times e^{3x+1} = \frac{2}{3} \times \underbrace{3}_{u'} \times \underbrace{e^{3x+1}}_{e^{u}}$$

$$F(x) = \frac{2}{3} e^{3x+1} + C$$

Ex 12

$$f(x) = x + 4e^{-3x} = x + 4e^{u} \qquad u = -3x \qquad u' = -3$$

$$f(x) = x + 4 \times \frac{(-3)}{-3} \times e^{-3x} = x - \frac{4}{3} \underbrace{(-3)}_{u'} \underbrace{e^{-3x}}_{e^{u}}$$

$$F(x) = \frac{x^2}{2} - \frac{4}{3} e^{-3x} + C$$

Ex 14

$$f(x) = \frac{1}{(x-2)^2} = \frac{1}{u^2} = u^{-2}$$

$$\boxed{\begin{aligned} u &= x-2 \\ u' &= 1 \end{aligned}}$$

$$\rightarrow \left[u^{n} \times u' \longrightarrow \frac{u^{n+1}}{n+1} \right]$$

$$f(x) = \overset{u'}{1} \times u^{-2} = u' \times u^{-2}$$

$$n = -2 \Rightarrow F(x) = \frac{u^{-2+1}}{-2+1} = \frac{u^{-1}}{-1}$$

$f(x) = u^{n}$ avec $n = -2$

$$f(x) = \underline{u' \times u^{n}}$$

$$u = x-2 \Rightarrow F(x) = \frac{(x-2)^{-1}}{-1} + C =$$

$$= -\frac{1}{x-2} + C$$

Ex 15

$$f(x) = \frac{3}{(x+1)^2} = \frac{3}{u^2} = 3u^{-2}$$

$$u = x+1 \qquad u' = 1$$

$$f(x) = 3 \times \underset{\uparrow \atop u'}{1} \times u^{-2}$$

$$F(x) = 3\,\frac{u^{-2+1}}{-2+1} + C = 3\,\frac{u^{-1}}{-1} + C$$

$$= -3\,\frac{1}{u} + C = -\frac{3}{u} + C$$

$$= -\frac{3}{x+1} + C$$

Ex 17

$$f(x) = \frac{1}{x-2} = \frac{1}{u} = u^{-1} \qquad u = x-2 \quad u' = 1$$

$$f(x) = \frac{1}{u} = \frac{u'}{u} \Rightarrow F(x) = \ln(u) + C$$

$$= \ln(x-2) + C$$

$$x > 2$$

$$f(x) = \frac{1}{3x+2} = \frac{1}{u} \qquad u = 3x+2 \qquad u' = 3$$

$$f(x) = \frac{3}{u}\,\frac{1}{3} = \frac{1}{3}\,\frac{3}{u} = \frac{1}{3}\,\frac{u'}{u}$$

$$F(x) = \frac{1}{3}\ln(u) + C = \frac{1}{3}\ln(3x+2) + C$$

<u>Ex 18</u>

$$f(x) = x^2 - x + 1 \qquad \underline{G(1) = 0}$$

$$G(x) = \frac{x^3}{3} - \frac{x^2}{2} + x + C$$

$$G(1) = \frac{1^3}{3} - \frac{1^2}{2} + 1 + C = 0$$

$$= \frac{1}{3} - \frac{1}{2} + 1 + C = 0$$

$$C = -1 + \frac{1}{2} - \frac{1}{3} = \frac{-6+3-2}{6} = -\frac{5}{6}$$

$$G(x) = \frac{x^3}{3} - \frac{x^2}{2} + x - \frac{5}{6} \qquad G(1) = 0$$

Ex 19

$f(x) = x - \frac{2}{x}$ $\qquad G(1) = 0$

$G(x) = \frac{x^2}{2} - 2\ln(x) + C$

$G(1) = \frac{1}{2} - 2\ln(1) + C = 0$

$\frac{1}{2} + C = 0 \Rightarrow C = -1/2$

$G(x) = \frac{x^2}{2} - 2\ln(x) - 1/2$

Ex 13

$f(x) = 2x(e^{x^2}) = 2x(e^u)$

avec $u = x^2$ $\quad u' = 2x$

$f(x) = u'e^u \Rightarrow F(x) = e^{x^2} + C$

Ex 16

$f(x) = x(x^2+1)^3 = x\,u^3$

avec $u = x^2+1$ $\qquad u' = 2x$

$f(x) = \frac{2x}{2}(x^2+1)^3 = \frac{1}{2}u'u^3$

$\Rightarrow F(x) = \frac{1}{2}\frac{u^4}{4} + C = \frac{1}{2}\frac{(x^2+1)^4}{4} = \frac{1}{8}(x^2+1)^4$

Example: $f(x) = x + \frac{1}{x} + e^{2x}$

$$F(x) = \frac{x^2}{2} + \ln|x| + \frac{e^{2x}}{2} + C$$

$$\boxed{\int_1^2 f(x)\,dx = F(2) - F(1)}$$

$$F(2) = \frac{2^2}{2} + \ln(2) + \frac{e^4}{2} + C$$

$$F(1) = \frac{1}{2} + \ln(1) + \frac{e^2}{2} + C = \frac{1}{2} + \frac{e^2}{2} + C$$

$$F(2) - F(1) = \frac{4}{2} + \ln(2) + \frac{e^4}{2} + C - \left[\frac{1}{2} + \frac{e^2}{2} + C\right] =$$

$$= 2 + \ln(2) + \frac{e^4}{2} + \cancel{C} - \frac{1}{2} - \frac{e^2}{2} - \cancel{C} =$$

$$= \frac{3}{2} + \ln(2) + \frac{e^4}{2} - \frac{e^2}{2} \approx 25,8$$

Ex 25

1) $\int_{-1}^{1} (x^2+1)\,dx =$

$$f(x) = x^2 + 1$$

$$F(x) = \frac{x^3}{3} + x$$

$$= F(1) - F(-1) = \frac{1}{3} + 1 - \left[\frac{(-1)^3}{3} - 1\right] =$$

$$= \frac{4}{3} - \left[-\frac{1}{3} - 1\right] = \frac{4}{3} - \left[-\frac{4}{3}\right] = \frac{4}{3} + \frac{4}{3} = \frac{8}{3}$$

2) $\int_{-1}^{1}(x^2+3x+5)\,dx$

$f(x)=x^2+3x+5 \qquad F(x)=\frac{x^3}{3}+3\frac{x^2}{2}+5x$

$F(1)-F(-1)=$

$=\frac{1}{3}+3\,\frac{1}{2}+5-\left[\frac{(-1)^3}{3}+3\,\frac{(-1)^2}{2}+5(-1)\right]=$

$=\frac{1}{3}+\frac{3}{2}+5-\left[-\frac{1}{3}+\frac{3}{2}-5\right]=$

$=\frac{1}{3}+\frac{3}{2}+5+\frac{1}{3}-\frac{3}{2}+5=$

$=\frac{2+\cancel{9}+30+2-\cancel{9}+30}{6}=\frac{64}{6}=\frac{32}{3}$