Ex 2 :

1) $\lim\limits_{x \to +\infty} \left(x^2 + \frac{2}{x}\right) = +\infty + 0 = +\infty$

$\lim\limits_{x \to +\infty} (2x + \ln x) = +\infty + \infty = +\infty$

2) $\lim\limits_{x \to +\infty} (2x + e^x) = +\infty + \infty = +\infty$

$\lim\limits_{\substack{x \to 0 \\ x > 0}} \frac{e^x}{x}$ $\quad u = e^x \quad v = x$

$\lim\limits_{\substack{x \to 0 \\ x > 0}} u = e^0 = 1 \quad \lim\limits_{\substack{x \to 0 \\ x > 0}} v = 0$

$\lim\limits_{\substack{x \to 0 \\ x > 0}} \frac{e^x}{x} = \frac{1}{0} = +\infty \quad (\text{car } x > 0)$

3) $\lim\limits_{x \to +\infty} \frac{1}{e^x + 1} = \frac{1}{+\infty} = 0 \qquad \lim\limits_{x \to +\infty} 3e^{-2x} = 0$

4) $\lim\limits_{x \to 1} x^2 e^x = 1 \times e^1 = e$

$\lim\limits_{x \to 1} 2x^3 \ln x = 2 \times 1^3 \times \ln(1) = 2 \times 1 \times 0 = 0$

5) $\lim\limits_{x \to +\infty} \ln(x-2) \qquad u = x - 2$

$\lim\limits_{x \to +\infty} u = +\infty$

$\lim\limits_{x \to +\infty} \ln(u) = \ln\left(\lim\limits_{x \to +\infty} u\right) = \ln(+\infty) = +\infty$

$\lim\limits_{\substack{x\to 2\\ x>2}} \ln(x-2)$ $\quad u = x-2$

$\lim\limits_{\substack{x\to 2\\ x>2}} u = 0$ avec $x-2>0$

$$\lim_{\substack{x\to 2\\ x>2}} \ln(x-2) = \ln(0) = -\infty$$

6) $\lim\limits_{x\to+\infty} 2e^{x+1}$ $\quad u = x+1$

$\lim\limits_{x\to+\infty} u = +\infty$

$$\lim_{x\to+\infty} 2e^{x+1} = 2e^{+\infty} = +\infty$$

$\lim\limits_{x\to-\infty} e^{1-x}$ $\quad u = 1-x$

$\lim\limits_{x\to-\infty} (1-x) = \lim\limits_{x\to-\infty} (-x) = +\infty$

$$\lim_{x\to-\infty} e^{1-x} = e^{+\infty} = +\infty$$

7) $$\lim_{x\to+\infty} x^2 \ln x = (+\infty)\times(+\infty) = +\infty$$

$$\lim_{x\to-\infty} (x+1)\, e^{-x} = (-\infty)\times(+\infty) = -\infty$$

8) $\lim_{x \to 0} \left(e^x + e^{-x} \right) = e^0 + e^0 = 1 + 1 = 2$

$\lim_{x \to -\infty} e^{1/x}$ $\quad u = \frac{1}{x}$ $\quad \lim_{x \to -\infty} u = 0$

$\lim_{x \to -\infty} e^{1/x} = e^0 = 1$

9) $\lim_{x \to +\infty} \left(2x + \frac{\ln x}{x} \right) = +\infty + \frac{+\infty}{+\infty} = +\infty + ?$

⚠ $\lim_{x \to +\infty} \frac{\ln x}{x} = 0$ Comparaison des fonctions.

$\lim_{x \to +\infty} \left(2x + \frac{\ln x}{x} \right) = \lim_{x \to +\infty} 2x = +\infty$

$\lim_{x \to +\infty} \left(1 + \frac{e^x}{x^2} \right) = 1 + \frac{+\infty}{+\infty} = 1 + ?$

⚠ $\lim_{x \to +\infty} \frac{e^x}{x^2} = +\infty$

$\lim_{x \to +\infty} \left(1 + \frac{e^x}{x} \right) = 1 + \infty = +\infty$