Ex 1 :

1. $f(x) = 3x^2 - 4x + 1$

$$\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} 3x^2 = 3 \times \lim_{x \to +\infty} x^2 = +\infty$$

$$\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} 3x^2 = 3 \times \lim_{x \to -\infty} x^2 = +\infty$$

$$f(x) = x^3 - 2x^2 + 5$$

$$\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} x^3 = +\infty$$

$$\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} x^3 = -\infty$$

2. $f(x) = -\dfrac{4}{3}x^4 - 3x^2 + \dfrac{1}{3}$

$$\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} \left(-\frac{4}{3}x^4\right) = -\frac{4}{3} \times \lim_{x \to +\infty} x^4 =$$

$$= -\frac{4}{3} \times (+\infty) = -\infty$$

$$\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} \left(-\frac{4}{3}x^4\right) = -\frac{4}{3} \times \lim_{x \to -\infty} x^4 =$$

$$= -\frac{4}{3} \times (+\infty) = -\infty$$

$$f(x) = 6x^3 - 4x$$

$$\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} 6x^3 = +\infty$$

$$\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} 6x^3 = -\infty$$

3) $f(x) = \dfrac{2x+3}{x^2+1}$

$$\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} \frac{2x}{x^2} = \lim_{x \to +\infty} \frac{2}{x} = 0$$

$$\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} \frac{2x}{x^2} = \lim_{x \to -\infty} \frac{2}{x} = 0$$

$$f(x) = \dfrac{x^3+1}{x^2+x+1}$$

$$\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} \frac{x^3}{x^2} = \lim_{x \to +\infty} x = +\infty$$

$$\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} \frac{x^3}{x^2} = \lim_{x \to -\infty} x = -\infty$$

$$f(x) = \frac{2x^2 - 1}{4x^2 + 5}$$

$$\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} \frac{2x^2}{4x^2} = \lim_{x \to +\infty} \frac{2}{4} = \frac{2}{4} = \frac{1}{2}$$

$$\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} \frac{2x^2}{4x^2} = \lim_{x \to -\infty} \frac{2}{4} = \frac{2}{4} = \frac{1}{2}$$