$$f(x) = (0{,}25x)\, e^{-0{,}125x^2} = u\,v$$

$$u = 0{,}25x \quad \rightarrow \quad u' = 0{,}25$$

$$v = e^{-0{,}125x^2} \quad \Rightarrow \quad v' = -0{,}125 \cdot 2x \cdot e^{-0{,}125x^2}$$

$$= -0{,}25x\, e^{-0{,}125x^2}$$

$$f'(x) = u'v + uv' =$$

$$= 0{,}25\, e^{-0{,}125x^2} + 0{,}25x\left(-0{,}25x\, e^{-0{,}125x^2}\right) =$$

$$= 0{,}25\, e^{-0{,}125x^2} - 0{,}0625x^2\, e^{-0{,}125x^2} =$$

$$= e^{-0{,}125x^2}\left(0{,}25 - 0{,}0625x^2\right) =$$

$$= 0{,}0625\, e^{-0{,}125x^2}\left(\frac{0{,}25}{0{,}0625} - x^2\right) =$$

$$= 0{,}0625\, e^{-0{,}125x^2}\left(4 - x^2\right) =$$

$$= 0{,}0625\, e^{-0{,}125x^2}\left(2^2 - x^2\right) =$$

$$= 0{,}0625\, e^{-0{,}125x^2}(2+x)(2-x)$$

Étude de signe de f' :

$0{,}0625\,e^{-0{,}125x^2}$	$2+x>0$	$2-x>0$
Toujours positif	$x>-2$	$-x>-2$
		$x<2$

x	$-\infty$		-2		2		$+\infty$
$0{,}0625\,e^{-0{,}125x^2}$		$+$	|	$+$	|	$+$	
$2+x$		$-$	0	$+$	|	$+$	
$2-x$		$+$	|	$+$	0	$-$	
f'		$-$	0	$+$	0	$-$	

Sur $]-2;2[$ $f'>0$

Tableau de variations :

x	$-\infty$		-2		2		$+\infty$
f'		$-$	0	$+$	0	$-$	
f		↘		↗		↘	

Sur $]2;+\infty[$ f est décroissante.

Équation de la tangente au point d'absaisse 0

$$T: \quad y = f'(0)(x-0) + f(0)$$

$$f'(0) = 0{,}0625\,(2+0)(2-0)\,e^0 =$$

$$= 0{,}0625 \times 4 \times 1 = 0{,}25$$

$$f(0) = 0{,}25 \times 0 \times e^0 = 0$$

Donc $T: \quad y = 0{,}25\,x$

$$\frac{x+5}{x-1} \leq \frac{x-3}{x+2}$$

$$\left[\begin{array}{l} x-1=0 \Rightarrow x=1 \quad \text{V.I.} \\ x+2=0 \Rightarrow x=-2 \quad \text{V.I.} \end{array}\right]$$

$$\frac{x+5}{x-1} - \frac{x-3}{x+2} \leq 0$$

$$\frac{(x+5)(x+2)-(x-3)(x-1)}{(x-1)(x+2)} \leq 0$$

$$\frac{x^2+2x+5x+10-(x^2-x-3x+3)}{(x-1)(x+2)} \leq 0$$

$$\frac{\cancel{x^2}+7x+10-\cancel{x^2}+4x-3}{(x-1)(x+2)} \leq 0$$

$$\frac{11x+7}{(x-1)(x+2)} \leq 0$$

$11x+7>0$	$x-1>0$	$x+2>0$
$x>-\frac{7}{11}$	$x>1$	$x>-2$
	V.I.	V.I.

x	$-\infty$	-2		$-\frac{7}{11}$		1	$+\infty$
$11x+7$	$-$	‖	$-$	0	$+$	‖	$+$
$x-1$	$-$	‖	$-$		$-$	‖	$+$
$x+2$	$-$	‖	$+$		$+$	‖	$+$
Pr	$-$	‖	$+$	0	$-$	‖	$+$

$$S = \left]-\infty ; -2\right[\cup \left[-\frac{7}{11} ; 1\right[$$

$$f(x) = \frac{2x^2 - x - 6}{x-1}$$

$$f(0) = \frac{2\times 0 - 0 - 6}{0-1} = \frac{-6}{-1} = 6$$

$$f(-2) = \frac{2\times(-2)^2 - (-2) - 6}{-2-1} = \frac{2\times 4 + 2 - 6}{-3} = -\frac{4}{3}$$

Étude de signe de $f(x) = \dfrac{2x^2 - x - 6}{x - 1}$ $\quad (x = 1 \text{ V.I.})$

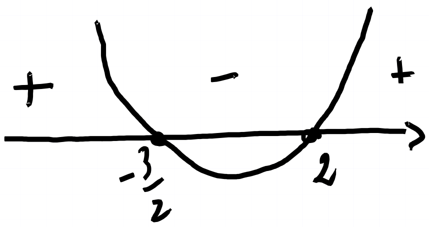
$2x^2 - x - 6 > 0$

$\Delta = (-1)^2 - 4 \times 2 \times (-6) =$

$= 1 + 48 = 49$

$x_1 = \dfrac{-(-1) - \sqrt{49}}{2 \times 2} = \dfrac{1-7}{4} = -\dfrac{6}{4} = -\dfrac{3}{2}$

$x_2 = \dfrac{-(-1) + \sqrt{49}}{2 \times 2} = \dfrac{1+7}{4} = \dfrac{8}{4} = 2$



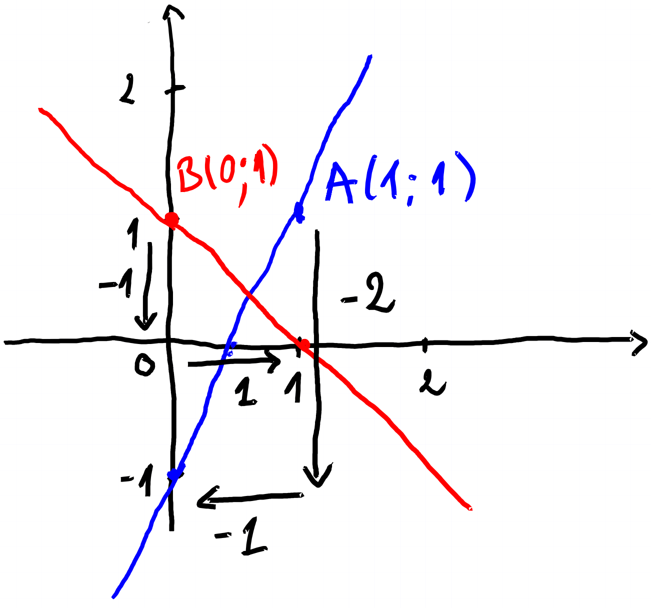
$x - 1 > 0$

$x > 1$

V.I.

x	$-\infty$		$-3/2$		1		2		$+\infty$
$2x^2 - x - 6$		$+$	0	$-$	$\|\|$	$-$	0	$+$	
$x - 1$		$-$		$-$	$\|\|$	$+$		$+$	
f		$-$	0	$+$	$\|\|$	$-$	0	$+$	

Sur $]2 ; +\infty[$ $f > 0$



$f'(0)$ est le coefficient directeur de la droite rouge $\Rightarrow f'(0) = \frac{-1}{1} = -1$

$f'(1)$ est le coefficient directeur de la droite bleue $\Rightarrow f'(1) = \frac{-2}{-1} = 2$

$$f(x) = \frac{2x^2-3}{x^2-7} = \frac{u}{v}$$

avec $u = 2x^2 - 3 \Rightarrow u' = 4x$

$v = x^2 - 7 \Rightarrow v' = 2x$

$$f'(x) = \frac{u'v - uv'}{v^2} = \frac{4x(x^2-7) - (2x^2-3)\,2x}{(x^2-7)^2} =$$

$$= \frac{\cancel{4x^3} - 28x - \cancel{4x^3} + 6x}{(x^2-7)^2} = -\frac{22x}{(x-7)^2}$$

$$A = (5-2x)^2 = 5^2 - 5 \times 2x \times 2 + 4x^2 =$$
$$= 25 - 20x + 4x^2$$

$$A = (5-2x)^2 = (5-2x)(5-2x) =$$
$$= 25 - 10x - 10x + 4x^2 = 25 - 20x + 4x^2$$

---

x	-10		-3	-2	0		2	3	5		$+\infty$
f	-4	$\searrow$	-7	$f(-2)$	1	$\nearrow$	5	$f(3)$	-1	$\searrow$	

$-1 \leq f(3) \leq 5$ $\qquad$ $-7 \leq f(-2) \leq 1$

$$1 - e^{x+4} \geq 0$$
$$-e^{x+4} \geq -1$$
$$e^{x+4} \leq 1$$
$$e^{x+4} \leq e^0$$
$$x+4 \leq 0$$
$$x \leq -4$$
$$S =]-\infty ; -4]$$

Déterminer la fonction affine f tels que :

$f(3) = 0$ et $f(5) = 6$

1er Méthode : $f(x) = ax + b$

$A(3;0)$ $B(5;6)$

$$a = \frac{y_B - y_A}{x_B - x_A} = \frac{6-0}{5-3} = \frac{6}{2} = 3$$

$f(x) = 3x + b$

$f(3) = 0 \Rightarrow 3 \times 3 + b = 0$

$9 + b = 0 \Rightarrow b = -9$

Donc $f(x) = 3x - 9$

2ème Méthode : $f(x) = ax + b$

$$\begin{aligned} f(3) = 0 &\Rightarrow \\ f(5) = 6 &\Rightarrow \end{aligned} \begin{cases} 3a + b = 0 \\ 5a + b = 6 \end{cases} \Rightarrow \begin{cases} b = -3a \\ 5a - 3a = 6 \end{cases}$$

$$\Rightarrow \begin{cases} b = -3a \\ 2a = 6 \end{cases} \Rightarrow \begin{cases} b = -3a \\ a = 3 \end{cases}$$

$$\Rightarrow \begin{cases} b = -3 \times 3 = -9 \\ a = 3 \end{cases}$$

Donc $f(x) = 3x - 9$

$$A = 2(e^x + 1)\left(e^x - \frac{1}{2}\right) =$$

$$= 2\left(e^x \times e^x - e^x \times \frac{1}{2} + 1 \times e^x - 1 \times \frac{1}{2}\right) =$$

$$= 2\left(e^{x+x} - \frac{1}{2}e^x + e^x - \frac{1}{2}\right) =$$

$$= 2\left(e^{2x} - \frac{1}{2}e^x + e^x - \frac{1}{2}\right) =$$

$$= 2e^{2x} - e^x + 2e^x - 1 =$$

$$= 2e^{2x} + e^x - 1$$