Ex 7 : $\lim\limits_{\substack{x\to 1\\ x>1}} \dfrac{x^2-2x+1}{x^2-1} = \dfrac{1-2+1}{1-1} = \dfrac{0}{0} = ?$

Num : $x^2-2x+1 = (x-1)^2$

Den : $x^2-1 = (x-1)(x+1)$

$$\lim_{\substack{x\to 1\\ x>1}} \frac{(x-1)^2}{(x-1)(x+1)} = \lim_{\substack{x\to 1\\ x>1}} \frac{x-1}{x+1} = \frac{1-1}{1+1} = \frac{0}{2} = 0$$

Ex 8 : $\lim\limits_{\substack{x\to 2\\ x>2}} \dfrac{x^2-3x+2}{x^2-x-2} = \dfrac{4-6+2}{4-2-2} = \dfrac{0}{0} = ?$

Num : $x^2-3x+2 = (x-2)(ax+b) =$

$= ax^2+bx-2ax-2b$

$= ax^2+(b-2a)x-2b$

$a = 1 \qquad -2b = 2 \Rightarrow b = -1$

$b-2a = -1-2 = -3 \Rightarrow$ ok

Alors $x^2-3x+2 = (x-2)(x-1)$

Den : $x^2-x-2 = (x-2)(ax+b) =$

$= ax^2+(b-2a)x-2b$

$a = 1 \qquad -2b = -2 \Rightarrow b = 1$

Alors $x^2 - x - 2 = (x-2)(x+1)$

$$\lim_{\substack{x \to 2 \\ x > 2}} \frac{(x-2)(x-1)}{(x-2)(x+1)} = \frac{2-1}{2+1} = \frac{1}{3}$$

Ex 9 : 1. $\lim_{x \to -\infty} f(x) = \frac{e^{-\infty} - 1}{2e^{-\infty} + 1} = \frac{-1}{1} = -1$

2. $\lim_{x \to +\infty} f(x) = \frac{e^{+\infty} - 1}{2e^{+\infty} + 1} = \frac{+\infty}{+\infty} = ?$

Num : $e^x - 1 = e^x \left(1 - \frac{1}{e^x}\right)$

Den : $2e^x + 1 = e^x \left(2 + \frac{1}{e^x}\right)$

$$\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} \frac{e^x \left(1 - 1/e^x\right)}{e^x \left(2 + 1/e^x\right)} = \frac{1-0}{2+0} = \frac{1}{2}$$