Ex 17

$f(x) = 3x^2 - 4x + 1$

$\lim_{x \to +\infty} f(x) = +\infty - \infty = ?$ $\Rightarrow$ $3x^2 - 4x + 1 = x^2\left(3 - \frac{4x}{x^2} + \frac{1}{x^2}\right)$

$= x^2\left(3 - \frac{4}{x} + \frac{1}{x^2}\right)$

$\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} x^2\left(3 - \frac{4}{x} + \frac{1}{x^2}\right) = +\infty$

$\lim_{x \to -\infty} f(x) = +\infty - (-\infty) = +\infty + \infty = +\infty$

Ex 18

$f(x) = x^3 - 2x^2 + 5$

$\lim_{x \to +\infty} f(x) = +\infty - \infty = ?$ $\Rightarrow$ $x^3 - 2x^2 + 5 = x^3\left(1 - \frac{2x^2}{x^3} + \frac{5}{x^3}\right) =$

$= x^3\left(1 - \frac{2}{x} + \frac{5}{x^3}\right)$

$\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} x^3\left(1 - \frac{2}{x} + \frac{5}{x^3}\right) = +\infty$

$\lim_{x \to -\infty} f(x) = -\infty - \infty = -\infty$

Ex 19

$f(x) = -\frac{4}{3}x^4 - 3x^2 + \frac{1}{3}$

$\lim_{x \to +\infty} f(x) = -\infty - \infty = -\infty$

$\lim_{x \to -\infty} f(x) = -\infty - \infty = -\infty$

Ex 20

$f(x) = 6x^3 - 4x$

$$\lim_{x \to +\infty} f(x) = +\infty - \infty = ? \Rightarrow 6x^3 - 4x = x^3\left(6 - \frac{4x}{x^3}\right) = x^3\left(6 - \frac{4}{x^2}\right)$$

$$\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} x^3\left(6 - \frac{4}{x^2}\right) = +\infty$$

$$\lim_{x \to -\infty} f(x) = -\infty + \infty = ?$$

$$\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} x^3\left(6 - \frac{4}{x^2}\right) = -\infty$$

Ex 21

$$f(x) = \frac{2x+3}{x^2+1}$$

$$\lim_{x \to +\infty} f(x) = \frac{+\infty}{+\infty} = ? \Rightarrow \frac{2x+3}{x^2+1} = \frac{x\left(2 + \frac{3}{x}\right)}{x^2\left(1 + \frac{1}{x^2}\right)} = \frac{\left(2 + \frac{3}{x}\right)}{x\left(1 + \frac{1}{x^2}\right)}$$

$$\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} \frac{\left(2 + \frac{3}{x}\right)}{x\left(1 + \frac{1}{x^2}\right)} = \lim_{x \to +\infty} \frac{2}{x} = 0$$

$$\lim_{x \to -\infty} f(x) = \frac{-\infty}{+\infty} = ?$$

$$\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} \frac{2}{x} = 0$$

Ex 22

$$f(x) = \frac{x^3+1}{x^2+x+1}$$

$$\lim_{x\to+\infty} f(x) = \frac{+\infty}{+\infty} = ? \Rightarrow \frac{x^3+1}{x^2+x+1} = \frac{x^3\left(1+\frac{1}{x^3}\right)}{x^2\left(1+\frac{x}{x^2}+\frac{1}{x^2}\right)} = \frac{x\left(1+\frac{1}{x^3}\right)}{\left(1+\frac{1}{x}+\frac{1}{x^2}\right)}$$

$$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} \frac{x\left(1+\frac{1}{x^3}\right)}{\left(1+\frac{1}{x}+\frac{1}{x^2}\right)} = \lim_{x\to+\infty} x = +\infty$$

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} x = -\infty$$

Ex 23

$$f(x) = \frac{2x^2-1}{4x^2+5}$$

$$\lim_{x\to+\infty} f(x) = \frac{+\infty}{+\infty} = ? \Rightarrow \frac{2x^2-1}{4x^2+5} = \frac{x^2\left(2-\frac{1}{x^2}\right)}{x^2\left(4+\frac{5}{x^2}\right)} = \frac{2-\frac{1}{x^2}}{4+\frac{5}{x^2}}$$

$$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} \frac{2-\frac{1}{x^2}}{4+\frac{5}{x^2}} = \lim_{x\to+\infty} \frac{2}{4} = \frac{2}{4} = \frac{1}{2}$$

$$\lim_{x\to-\infty} f(x) = \frac{1}{2}$$

Ex 24

$$\lim_{x\to+\infty} \left(x^2+\frac{2}{x}\right) = +\infty$$

$\lim_{x\to+\infty} (2x+\ln x) = +\infty+\infty$ [redacted] $= +\infty$

[redacted]

Ex 25

$\lim_{x\to+\infty} (2x+e^x) =$ [redacted] $+\infty+\infty = +\infty$

$$\lim_{\substack{x\to 0\\ x>0}} \frac{e^x}{x} = \frac{1}{0^+} = +\infty$$

Ex 26

$\lim_{x\to+\infty} \frac{1}{e^x+1} = \frac{1}{+\infty} = 0$ $\qquad$ $\lim_{x\to+\infty} 3e^{-2x} = e^{-\infty} = 0$

Ex 27

$\lim_{x\to 1} x^2 e^x = 1 \times e^1 = e$ $\qquad$ $\lim_{x\to 1} 2x^3 \ln x = 2 \times 1 \times \ln(1) = 0$

Ex 28

$\lim_{x\to+\infty} \ln(x-2) = +\infty$ $\qquad$ $\lim_{\substack{x\to 2\\ x>2}} \ln(x-2) = \ln(0^+) = -\infty$

Ex 29

$\lim_{x\to+\infty} 2e^{x+1} = 2e^{+\infty} = +\infty$ $\qquad$ $\lim_{x\to-\infty} e^{1-x} = e^{+\infty} = +\infty$

Ex 30

$\lim_{x\to+\infty} x^2 \ln x = +\infty$ [redacted]

$\lim_{x\to-\infty} (x+1)e^{-x} = (-\infty)e^{+\infty} = (-\infty)\times(+\infty) = -\infty$

Ex 31

$\lim_{x\to 0} (e^x + e^{-x}) = 1 + 1 = 2$ $\qquad$ $\lim_{x\to-\infty} e^{\frac{1}{x}} = e^{\frac{1}{-\infty}} = e^0 = 1$

Ex 32

$\lim_{x\to+\infty} \left(2x + \frac{\ln x}{x}\right) = +\infty + 0 = +\infty$

$\lim_{x\to+\infty} \left(1 + \frac{e^x}{x^2}\right) = 1 + \infty = +\infty$

Ex 37

$$\lim_{\substack{x \to 1 \\ x > 1}} \frac{x^2 - 2x + 1}{x^2 - 1} = \frac{1 - 2 + 1}{1 - 1} = \frac{0}{0} = ?$$

[redacted] $$\frac{x^2 - 2x + 1}{x^2 - 1} = \frac{x^2 - 2x + 1}{(x+1)(x-1)} = \frac{(x-1)(x-1)}{(x+1)(x-1)} = \frac{x-1}{x+1}$$

$$\lim_{\substack{x \to 1 \\ x > 1}} \frac{x^2 - 2x + 1}{x^2 - 1} = \lim_{\substack{x \to 1 \\ x > 1}} \frac{x-1}{x+1} = \frac{0}{2} = 0$$

Ex 38

$$\lim_{\substack{x \to 2 \\ x < 2}} \frac{x^2 - 3x + 2}{x^2 - x - 2} = \frac{4 - 6 + 2}{4 - 2 - 2} = \frac{0}{0} = ?$$

$$\frac{x^2 - 3x + 2}{x^2 - x - 2} = \frac{(x-2)(ax+b)}{(x-2)(cx+d)} = \frac{ax^2 + bx - 2ax - 2b}{cx^2 + dx - 2cx - 2d} = \frac{ax^2 + x(b-2a) - 2b}{cx^2 + x(d-2c) - 2d}$$

$$\Rightarrow \quad ax^2 + x(b - 2a) - 2b = x^2 - 3x + 2$$

$$\Rightarrow \quad a = 1 \quad \text{et} \quad -2b = 2$$
$$b = -1$$

$$cx^2 + x(d - 2c) - 2d = x^2 - x - 2$$

$$\Rightarrow \quad c = 1 \quad \text{et} \quad -2d = -2$$
$$d = 1$$

$$\Rightarrow \quad \frac{x^2 - 3x + 2}{x^2 - x - 2} = \frac{(x-2)(x-1)}{(x-2)(x+1)} = \frac{x-1}{x+1}$$

$$\lim_{\substack{x \to 2 \\ x < 2}} \frac{x^2 - 3x + 2}{x^2 - x - 2} = \lim_{\substack{x \to 2 \\ x < 2}} \frac{x-1}{x+1} = \frac{2-1}{2+1} = \frac{1}{3}$$

Ex 39

1. $\lim\limits_{x \to -\infty} \dfrac{e^x - 1}{2e^x + 1} = \dfrac{e^{-\infty} - 1}{e^{-\infty} + 1} = \dfrac{-1}{+1} = -1$

2. $\lim\limits_{x \to +\infty} \dfrac{e^x - 1}{2e^x + 1} = \lim\limits_{x \to +\infty} \dfrac{e^x\left(1 - \frac{1}{e^x}\right)}{e^x\left(2 + \frac{1}{e^x}\right)} = \lim\limits_{x \to +\infty} \dfrac{1 - \frac{1}{e^x}}{2 + \frac{1}{e^x}} = \dfrac{1}{2}$

Ex 40

1. $\lim\limits_{x \to -\infty} e^x - x = e^{-\infty} + \infty = 0 + \infty = +\infty$

2. $\lim\limits_{x \to +\infty} e^x - x = \lim\limits_{x \to +\infty} e^x\left(1 - \frac{x}{e^x}\right) = \lim\limits_{x \to +\infty} e^x = +\infty$

Ex 41

$\lim\limits_{x \to +\infty} (x - \ln x) = \lim\limits_{x \to +\infty} x\left(1 - \frac{\ln x}{x}\right) = \lim\limits_{x \to +\infty} x = +\infty$

Ex 42

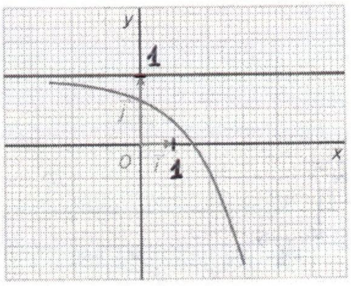
$\lim\limits_{x \to +\infty} \dfrac{e^x + 1}{x^2 + 1} = \lim\limits_{x \to +\infty} \dfrac{e^x\left(1 + \frac{1}{e^x}\right)}{x^2\left(1 + \frac{1}{x^2}\right)} = \lim\limits_{x \to +\infty} \dfrac{e^x}{x^2} = +\infty$

Lecture graphique

Fiche l'Essentiel

Pour chacun des exercices 43 à 44, *donner par lecture graphique la limite en + ∞ et en – ∞ de chaque fonction représentée.*

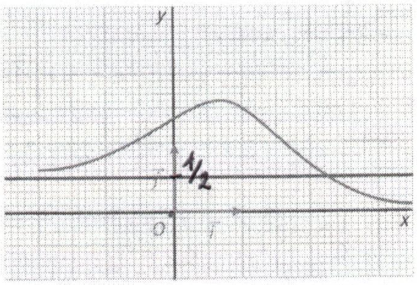
43



$\lim_{x \to -\infty} f = 1$

$\lim_{x \to +\infty} f = -\infty$

44

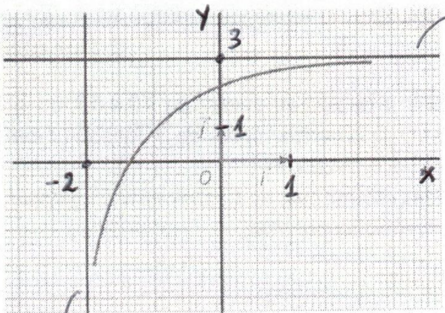


$\lim_{x \to -\infty} f = \frac{1}{2}$

$\lim_{x \to +\infty} f = 0$

Pour chacun des exercices 45 à 48, *donner pour chaque fonction les équations des asymptotes à la courbe représentative.*

45

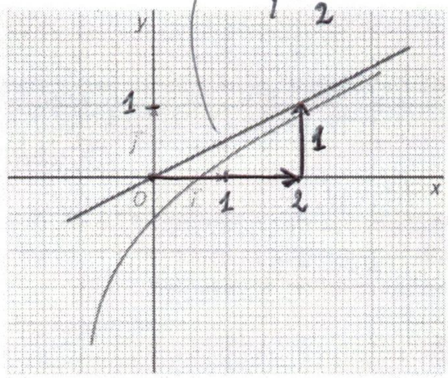


asymptote horizontale : $y = 3$

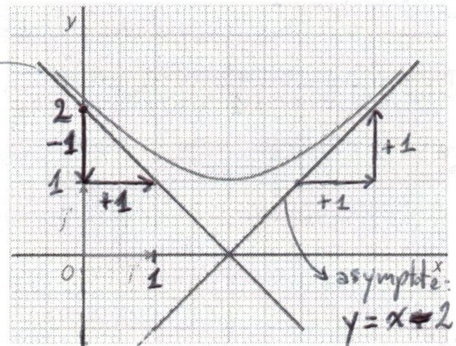
asymptote verticale : $x = -2$

46 **R**

asymptote : $y = \frac{1}{2}x$



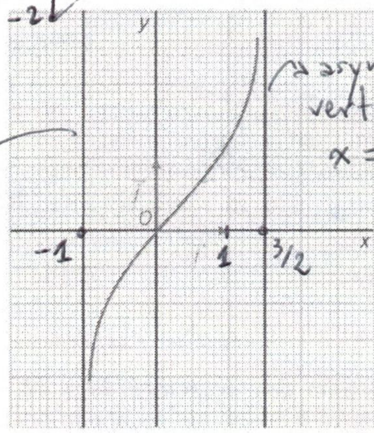
47



asymptote : $y = -x + 2$

asymptote : $y = x - 2$

48



asymptote verticale : $x = \frac{3}{2}$

asymptote verticale : $x = -1$

Ex 49

$f(x) = x - 2 - \frac{1}{x}$ $\qquad D_f =]0;+\infty[$

1. $\lim\limits_{x \to 0} f(x) = 0 - 2 - \frac{1}{0^+} = -2 - \infty = -\infty$

Donc, $x = 0$ est asymptote verticale.

2. a) $\lim\limits_{x \to +\infty} [f(x) - (x-2)]$ $\qquad f(x) - (x-2) = x - 2 - \frac{1}{x} - (x-2) = -\frac{1}{x}$

$\lim\limits_{x \to +\infty} \left(-\frac{1}{x}\right) = 0$ $\qquad$ Donc, $y = x - 2$ est asymptote à $\mathcal{C}$.

b) Étude de signe : $f(x) - (x-2)$ sur $]0;+\infty[$

$f(x) - (x-2) = -\frac{1}{x}$

Tableau de signe de $-\frac{1}{x}$:

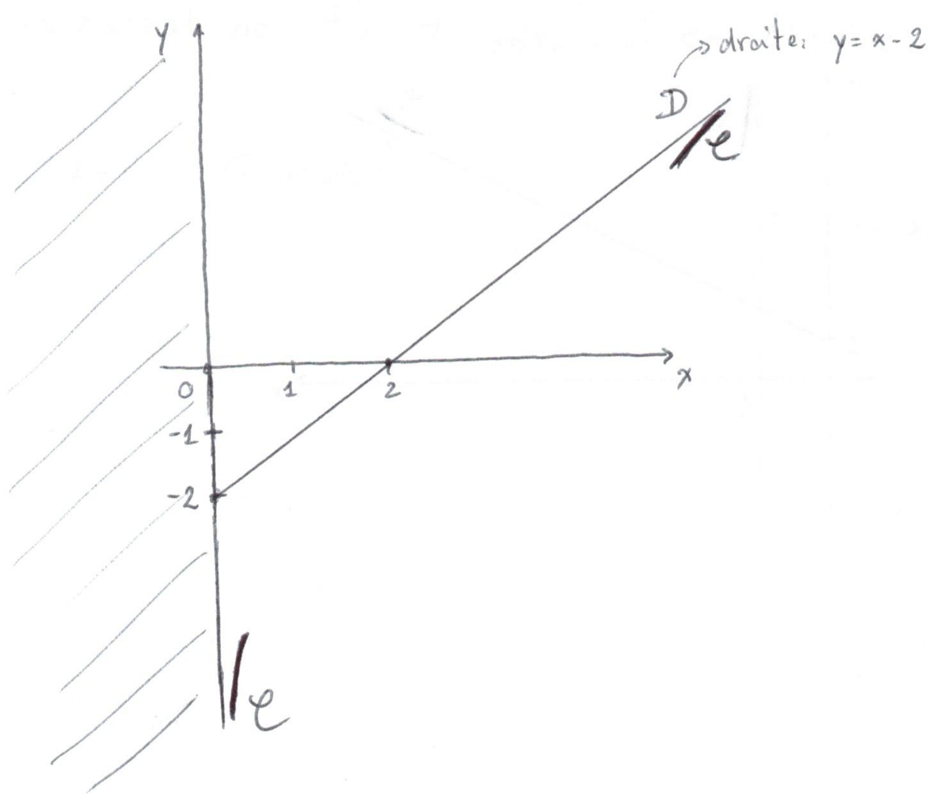
x	0	$+\infty$
$-\frac{1}{x}$	‖	$-$

Donc, $f(x) - (x-2)$ est negatif

$\Rightarrow f(x) - (x-2) < 0$

$\Rightarrow f(x) < (x-2)$ sur D_f.

Donc la courbe $\mathcal{C}$ est au-dessous de la droite D.



Ex 50

$$f(x) = \frac{x^2}{x-1} \qquad D_f =]1;+\infty[$$

1. $\lim\limits_{x \to 1} f(x) = \frac{1}{0^+} = +\infty$ Donc $x = 1$ est asymptote verticale

2. a) $x + 1 + \frac{1}{x-1} = \frac{(x+1)(x-1)+1}{x-1} = \frac{x^2-1+1}{x-1} = \frac{x^2}{x-1}$

$\lim\limits_{x \to +\infty} \left[f(x) - (x+1) \right] =$

$= \lim\limits_{x \to +\infty} \left[\left(x+1+\frac{1}{x-1} \right) - (x+1) \right] =$

$= \lim\limits_{x \to +\infty} \left[\frac{1}{x-1} \right] = \frac{1}{+\infty} = 0$

Donc la droite D d'equation $y = x + 1$ est asymptote à $\mathcal{C}$.

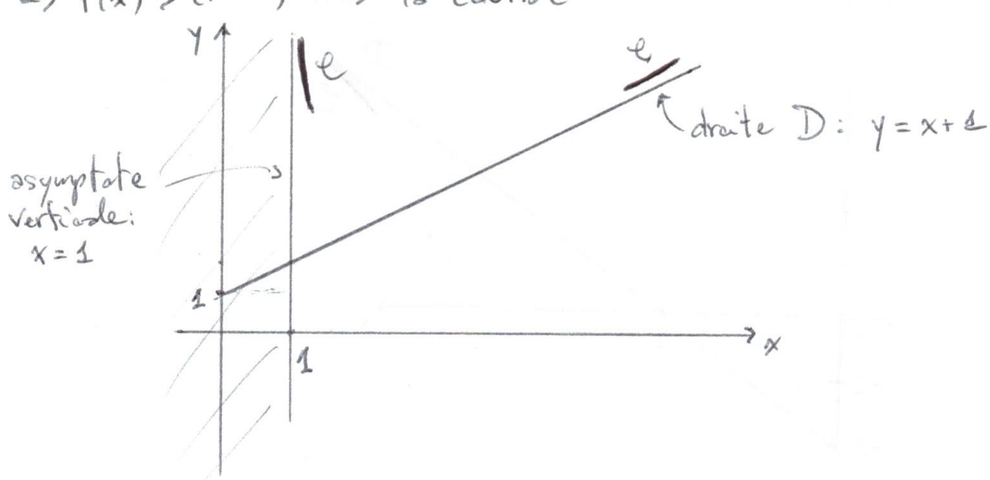
b) $f(x) - (x+1) = \frac{1}{x-1}$

Tableau de signe de $\frac{1}{x-1}$:

x	1	$+\infty$
$\frac{1}{x-1}$		$+$

Donc $f(x) - (x+1) > 0$ sur D_f.

$\Rightarrow f(x) > (x+1) \Rightarrow$ la courbe $\mathcal{C}$ est au-dessus de D.



Ex 51

$f(x) = x + 2\dfrac{\ln x}{x}$ $\qquad D_f =]0\,;+\infty[$

1. $\lim\limits_{x \to 0} f(x) = \lim\limits_{x \to 0} x + 2\left(\frac{1}{x}\right)(\ln x) = 0 + 2(+\infty)(-\infty) = -\infty$

Donc $x = 0$ est asymptote verticale

2. a) $\lim\limits_{x \to +\infty} \left[f(x) - (x) \right] = \lim\limits_{x \to +\infty} \left[x + 2\frac{\ln x}{x} - x \right] = \lim\limits_{x \to +\infty} \left(2\frac{\ln x}{x} \right) = 0$

Donc la droite D d'equation $y = x$ est asymptote à $\mathcal{C}$.

b) $f(x) - x = 2\dfrac{\ln x}{x}$

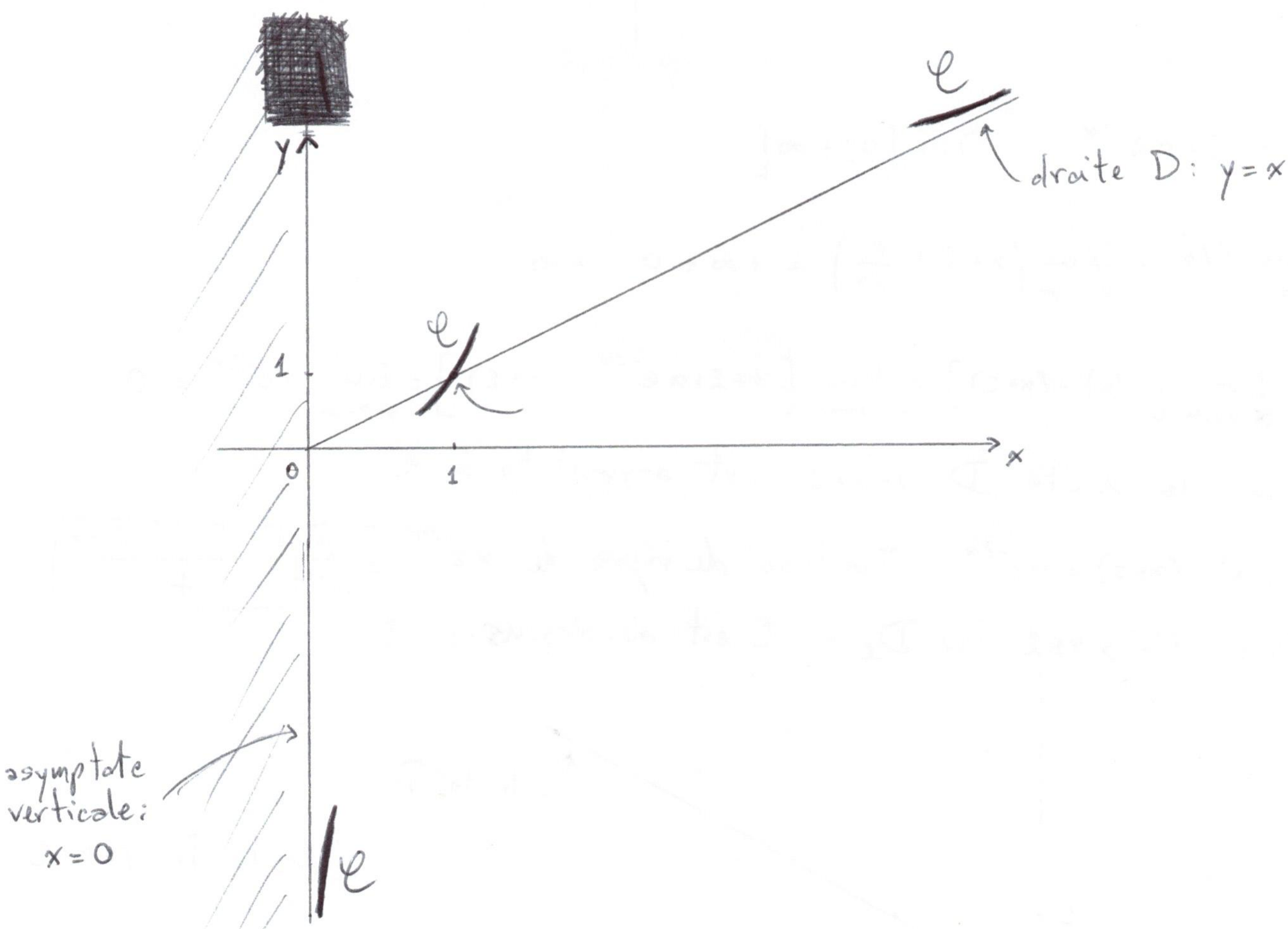
Tableau de signe de $2\dfrac{\ln x}{x}$:

x	0		1		$+\infty$
$\frac{2\ln x}{x}$		$-$	0	$+$	

Donc $f(x) - x > 0$ sur $]1\,;+\infty[$

$\Rightarrow$ Pour $x \to +\infty$ la courbe $\mathcal{C}$ est au-dessus de D.

Pour $x < 1$ la courbe $\mathcal{C}$ est au-dessous de D.



Ex 52

$f(x) = x + e^{2x}$ $\quad D_f = \mathbb{R}$

1. $\lim\limits_{x \to -\infty} f(x) = -\infty + e^{-\infty} = -\infty$

2. $\lim\limits_{x \to -\infty} \left[f(x) - (x) \right] = \lim\limits_{x \to -\infty} \left[x + e^{2x} - x \right] = \lim\limits_{x \to -\infty} e^{2x} = e^{-\infty} = 0$

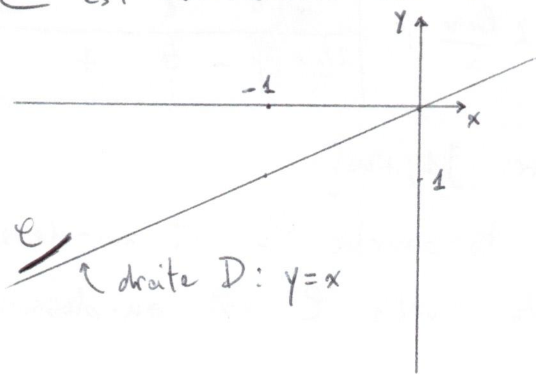
Donc la droite D $y = x$ est asymptote à $\mathcal{C}$.

$f(x) - x = e^{2x} \Rightarrow$ Tableau de signe de e^{2x} :

x	$-\infty$ $\qquad$ $+\infty$
e^{2x}	+

Donc $f(x) - x > 0$ sur $\mathbb{R} \Rightarrow f(x) > x$

$\Rightarrow$ la courbe $\mathcal{C}$ est au-dessus de D.



Ex 53

$f(x) = x + 2 + x e^{-2x}$ $\quad D_f = [0; +\infty[$

1. $\lim\limits_{x \to +\infty} f(x) = \lim\limits_{x \to +\infty} \left(x + 2 + \frac{x}{e^{2x}} \right) = +\infty + 0 = +\infty$

2. a) $\lim\limits_{x \to +\infty} \left[f(x) - (x+2) \right] = \lim\limits_{x \to +\infty} \left[x + 2 + x e^{-2x} - (x+2) \right] = \lim\limits_{x \to +\infty} x e^{-2x} = 0$

Donc la droite D $y = x + 2$ est asymptote à $\mathcal{C}$.

b) $f(x) - (x+2) = x e^{-2x}$ Tableau de signe de $x e^{-2x}$:

x	0 $\qquad$ $+\infty$
$x e^{-2x}$	+

Donc $f(x) > x + 2$ sur $D_f \Rightarrow \mathcal{C}$ est au-dessus de D.