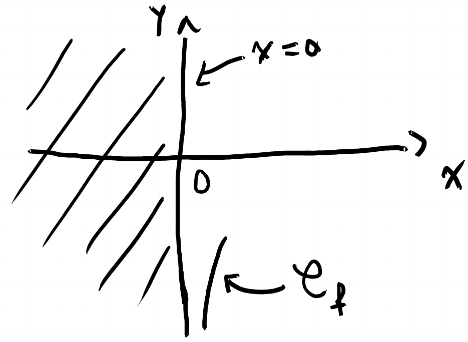
Ex 1 : $f(x) = x - 2 - \frac{1}{x}$ $\quad I =]0 ; +\infty[$

1) $\lim\limits_{\substack{x \to 0 \\ x > 0}} f(x) = 0 - 2 - \frac{1}{0} = -2 - (+\infty)$

$= -2 - \infty = -\infty$

$\lim\limits_{x \to 0} f(x) = -\infty$ donc $x = 0 \to$ asymptote verticale



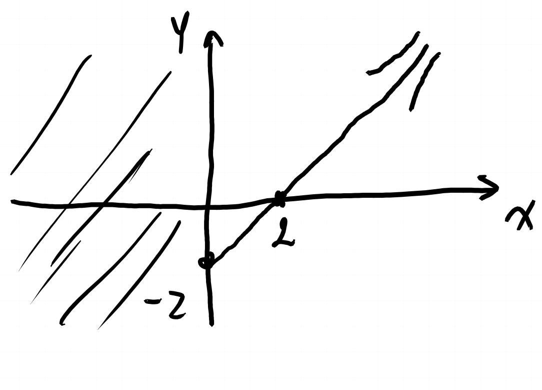
2) $\lim\limits_{x \to +\infty} [f - D] = 0$ à vérifier

$f - D = x - 2 - \frac{1}{x} - (x - 2) =$

$= \cancel{x} - \cancel{2} - \frac{1}{x} - \cancel{x} + \cancel{2} = -\frac{1}{x}$

$\lim\limits_{x \to +\infty} \left(-\frac{1}{x}\right) = -\frac{1}{+\infty} = 0$

Donc la droite D est asymptote à $\mathcal{C}_f$



3) Étude de signe de $f - D$

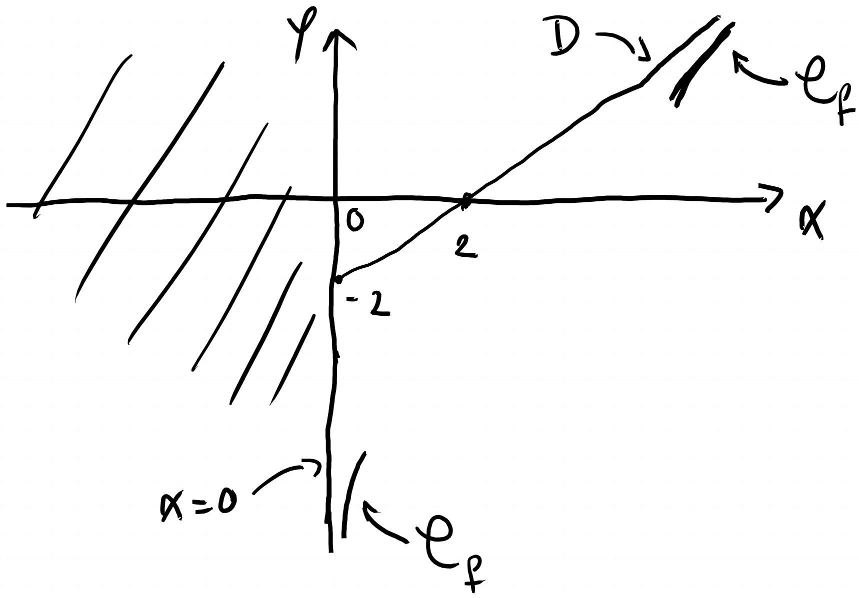
si $f - D > 0 \Rightarrow f > D \Rightarrow f$ au-dessus

si $f - D < 0 \Rightarrow f < D \Rightarrow f$ au-dessous

$$f - D = -\frac{1}{x}$$

x	0		$+\infty$
$-\frac{1}{x}$	$\|\|$	$-$	

Donc $f - D < 0 \Rightarrow f < D$

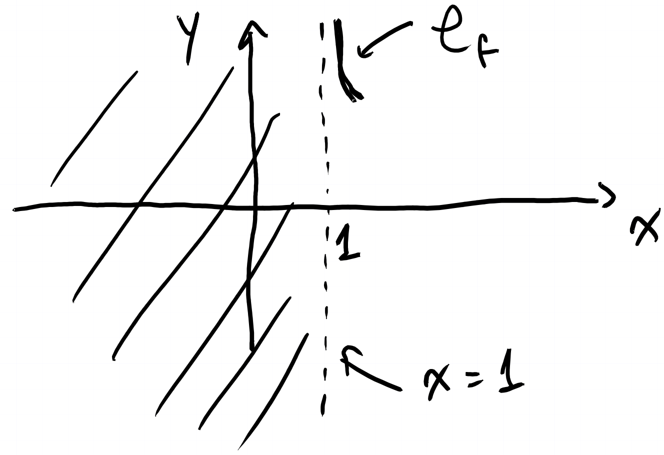


<u>Ex 2 :</u> $f(x) = \dfrac{x^2}{x-1}$ $\quad I =]1 ; +\infty[$

1) $\lim\limits_{\substack{x \to 1 \\ x > 1}} f(x) = \dfrac{1}{1-1} = \dfrac{1}{0} = +\infty$

$\hookrightarrow x - 1 > 0$

$x = 1$ est asymptote verticale.



2) $f(x) = x+1 + \dfrac{1}{x-1} = \dfrac{(x+1)(x-1)+1}{x-1} =$

$= \dfrac{x^2-1+1}{x-1} = \dfrac{x^2}{x-1} \Rightarrow$ vérifié

$f - D = x+1 + \dfrac{1}{x-1} - (x+1) = \dfrac{1}{x-1}$

$\lim\limits_{x\to+\infty} (f-D) = \dfrac{1}{+\infty} = 0$

D est asymptote à $\mathcal{C}_f$

3) Étude du signe de $f - D = \dfrac{1}{x-1}$

x	1	$+\infty$
$\dfrac{1}{x-1}$	||	+

$f - D > 0 \Rightarrow f > D$