

Ex 92

$$f(x) = \ln x - \sqrt{x} \quad I = ]0; +\infty[$$

$$f'(x) = \frac{1}{x} - \frac{1}{2\sqrt{x}} \quad \text{Signe de } f': \frac{1}{x} - \frac{1}{2\sqrt{x}} > 0 \Rightarrow \frac{2 - \sqrt{x}}{2x} > 0$$

$$\begin{array}{l|l} 2 - \sqrt{x} > 0 & 2x > 0 \\ -\sqrt{x} > -2 & x > 0 \\ \sqrt{x} < 2 & \\ x < 4 & \end{array}$$

x	0	4	+\infty
$2 - \sqrt{x}$	2	0	-
$2x$	+	+	+

Tableau de variations:

x	0	4	+\infty
$f'$	2	0	-
$f$		-0,61	

$$f(4) = \ln(4) - 2 \approx -0,61$$

Ex 93

$$f(x) = x + 10 - 5 \ln(x+2) \quad I = [0; 10]$$

$$f'(x) = 1 - \frac{5}{x+2} \quad \text{Signe de } f': 1 - \frac{5}{x+2} > 0 \Rightarrow \frac{x+2-5}{x+2} > 0 \Rightarrow \frac{x-3}{x+2} > 0$$

$$\begin{array}{l|l} x-3 > 0 & x+2 > 0 \\ x > 3 & x > -2 \end{array}$$

x	0	3	10
$x-3$	-	0	+
$x+2$	+	+	+

Tableau de variations:

x	0	3	10
$f'$	-	0	+
$f$	$f(0)$	$f(3)$	$f(10)$

$$f(0) = 10 - 5 \ln(2)$$

$$f(3) = 13 - 5 \ln(5)$$

$$f(10) = 20 - 5 \ln(12)$$

Ex 94

$$f(x) = x^2 - 18 \ln x + 18 \quad I = ]0; +\infty[$$

$$f'(x) = 2x - \frac{18}{x} \quad \text{Signe de } f': 2x - \frac{18}{x} > 0 \Rightarrow \frac{2x^2 - 18}{x} > 0 \Rightarrow \frac{2(x^2 - 9)}{x} > 0$$

$$\Rightarrow \frac{2(x+3)(x-3)}{x} > 0 \Rightarrow \frac{(x+3)(x-3)}{x} > 0$$

$$\begin{array}{l|l|l} x+3 > 0 & x-3 > 0 & x > 0 \\ x > -3 & x > 3 & \end{array}$$

x	0	3	+\infty
$x+3$	3	+	+
$x-3$	-	0	+
$x$	+	+	+

Tableau de variations:

x	0	3	+\infty
$f'$	3	0	+
$f$		$f(3)$	

$$f(3) = 9 - 18 \ln(3) + 18$$

$$= 27 - 18 \ln(3)$$