1) $f(x) = e^x + 2x - e^3$

$f'(x) = e^x + 2$

2) $f(x) = 2x e^x = u\,v$

$u = 2x \qquad v = e^x$

$u' = 2 \qquad v' = e^x$

$f'(x) = u'v + uv' = 2e^x + 2xe^x =$

$= 2e^x(1 + x)$

3) $f(x) = (5x^2 - 2x)e^x = u\,v$

$u = 5x^2 - 2x \qquad v = e^x$

$u' = 10x - 2 \qquad v' = e^x$

$f'(x) = u'v + uv' = (10x-2)e^x + (5x^2 - 2x)e^x =$

$= e^x\left((10x-2) + (5x^2 - 2x)\right) =$

$= e^x(10x - 2 + 5x^2 - 2x) =$

$= e^x(5x^2 + 8x - 2)$

4) $f(x) = (e^x + 2)(e^x - e) = uv$

$u = e^x + 2 \qquad v = e^x - e$

$u' = e^x \qquad v' = e^x$

$f'(x) = u'v + uv' = e^x(e^x - e) + (e^x + 2)e^x =$

$= e^x\left((e^x - e) + (e^x + 2)\right) =$

$= e^x\left(e^x - e + e^x + 2\right) =$

$= e^x(2e^x - e + 2)$

5) $f(x) = \dfrac{2e^x - 1}{e^x + 3} = \dfrac{u}{v}$

$u = 2e^x - 1 \qquad v = e^x + 3$

$u' = 2e^x \qquad v' = e^x$

$f'(x) = \dfrac{u'v - uv'}{v^2} = \dfrac{2e^x(e^x + 3) - (2e^x - 1)e^x}{(e^x + 3)^2} =$

$= \dfrac{e^x\left(2(e^x + 3) - (2e^x - 1)\right)}{(e^x + 3)^2} =$

$= \dfrac{e^x\left(\cancel{2e^x} + 6 - \cancel{2e^x} + 1\right)}{(e^x + 3)^2} = \dfrac{7e^x}{(e^x + 3)^2}$

6) $f(x) = e^{x^3 + \frac{2}{5}x^2 - 1} = e^u$

$u = x^3 + \frac{2}{5}x^2 - 1 \qquad u' = 3x^2 + \frac{4}{5}x$

$f'(x) = u' e^u = \left(3x^2 + \frac{4}{5}x\right) e^{x^3 + \frac{2}{5}x^2 - 1}$

7) $f(x) = e^{\frac{x+1}{x^2+1}} = e^u$

$u = \frac{x+1}{x^2+1} = \frac{w}{v} \qquad w = x+1 \quad w' = 1$

$v = x^2+1 \quad v' = 2x$

$u' = \frac{w'v - wv'}{v^2} = \frac{x^2+1 - (x+1)2x}{(x^2+1)^2} =$

$= \frac{x^2+1-2x^2-2x}{(x^2+1)^2} = \frac{-x^2-2x+1}{(x^2+1)^2}$

$f'(x) = u' e^u = \frac{-x^2-2x+1}{(x^2+1)^2} e^{\frac{x+1}{x^2+1}}$