Ex 98

$f(x) = 3 + 2\ln x - (\ln x)^2 \qquad I =]0;+\infty[$

$f'(x) = \frac{2}{x} - \frac{2}{x}\ln x = \frac{2}{x}(1-\ln x)$ Signe de f' : $\frac{2}{x}(1-\ln x) > 0 \Rightarrow \frac{1-\ln x}{x} > 0$

$1 - \ln x > 0$
$-\ln x > -1$
$\ln x < 1$
$x < e$

$x > 0$

x	0		e		$+\infty$
$1-\ln x$	‖	$+$	0	$-$	
x	‖	$+$		$+$	

Tableau de variations :

x	0		e		$+\infty$
f'	‖	$+$	0	$-$	
f	‖	$\nearrow$	$f(e)$	$\searrow$	

$f(e) = 3 + 2\ln(e) - (\ln(e))^2 =$
$= 3 + 2 - 1 = 4$

Ex 99

$f(x) = x + \frac{1}{2(e^x+1)} \qquad D_f = \mathbb{R}$

1. $\lim\limits_{x\to-\infty} f(x) = -\infty + \frac{1}{2(e^{-\infty}+1)} = -\infty + \frac{1}{2} = -\infty$

$\lim\limits_{x\to+\infty} f(x) = +\infty + \frac{1}{2(e^{+\infty}+1)} = +\infty + \frac{1}{+\infty} = +\infty + 0 = +\infty$

2. $f'(x) = 1 - \frac{2e^x}{4(e^x+1)^2} = 1 - \frac{e^x}{2(e^x+1)^2} = \frac{2(e^x+1)^2 - e^x}{2(e^x+1)^2} = \frac{2(e^{2x}+2e^x+1) - e^x}{2(e^x+1)^2} =$

$= \frac{2e^{2x}+4e^x+2-e^x}{2(e^x+1)^2} = \frac{2e^{2x}+3e^x+2}{2(e^x+1)^2}$

3. Signe de f' : $2e^{2x}+3e^x+2 > 0$
$X = e^x \Rightarrow 2X^2+3X+2 > 0$
$\Delta = 9 - 4\times2\times2 = -7 < 0$
$+ \; (+) \; +$

$2(e^x+1)^2 > 0$
Toujours positif.

Tableau de variations :

x	$-\infty$		$+\infty$
f'		$+$	
f	$-\infty$	$\nearrow$	$+\infty$