a) $2e^{2x} - e^{x} = 1$

Changement de variable

$$X = e^{x}$$

⚠ X est strictement positif

$$e^{2x} = (e^{x})^{2} = X^{2}$$

$$2e^{2x} - e^{x} = 1$$

$$2X^{2} - X = 1$$

$$2X^{2} - X - 1 = 0 \qquad a = 2 \quad b = -1 \quad c = -1$$

$$\Delta = (-1)^{2} - 4 \times 2 \times (-1) = 1 + 8 = 9$$

$$X_1 = \frac{-(-1) - 3}{4} = \frac{1-3}{4} = -\frac{2}{4} = -\frac{1}{2} \rightarrow \text{impossible}$$

$$X_2 = \frac{-(-1) + 3}{4} = \frac{1+3}{4} = \frac{4}{4} = 1$$

$X = 1 \Rightarrow e^x = 1$

$e^x = e^0$

$x = 0 \Rightarrow S = \{0\}$

b) $e^{2x} + 2e^x - 3 \leq 0$

$X = e^x \qquad X^2 = e^{2x}$

X est strictement positif

$X^2 + 2X - 3 \leq 0 \quad a = 1 \quad b = 2 \quad c = -3$

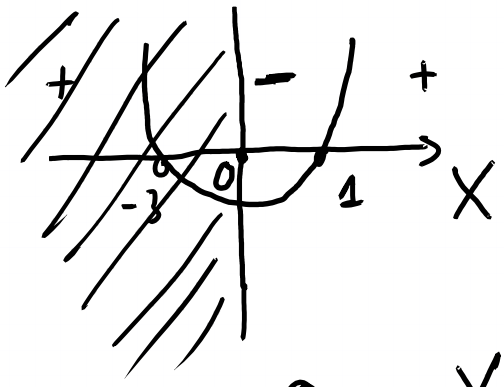
$\Delta = 2^2 - 4 \times 1 \times (-3) = 4 + 12 = 16$

$X_1 = \frac{-2-4}{2} = \frac{-6}{2} = -3$

$X_2 = \frac{-2+4}{2} = \frac{2}{2} = 1$

$a = 1 > 0$

$\Delta = 16 > 0$



$0 < X \leq 1$

$X = e^x \Rightarrow 0 < e^x \leq 1$

$\Rightarrow e^x > 0$ et $e^x \leq 1$

Toujours positif

$e^x \leq e^0$

$\boxed{x \leq 0}$

$S =]-\infty ; 0]$