Ex 18

$f(x) = x^2 - x + 1$

$G(x) = \frac{x^3}{3} - \frac{x^2}{2} + x + C$

$G(1) = 0 \qquad G(1) = \frac{1^3}{3} - \frac{1^2}{2} + 1 + C = 0$

$C = -\frac{1}{3} + \frac{1}{2} - 1 = \frac{-2+3-6}{6} = -\frac{5}{6}$

Donc $G(x) = \frac{x^3}{3} - \frac{x^2}{2} + x - \frac{5}{6}$

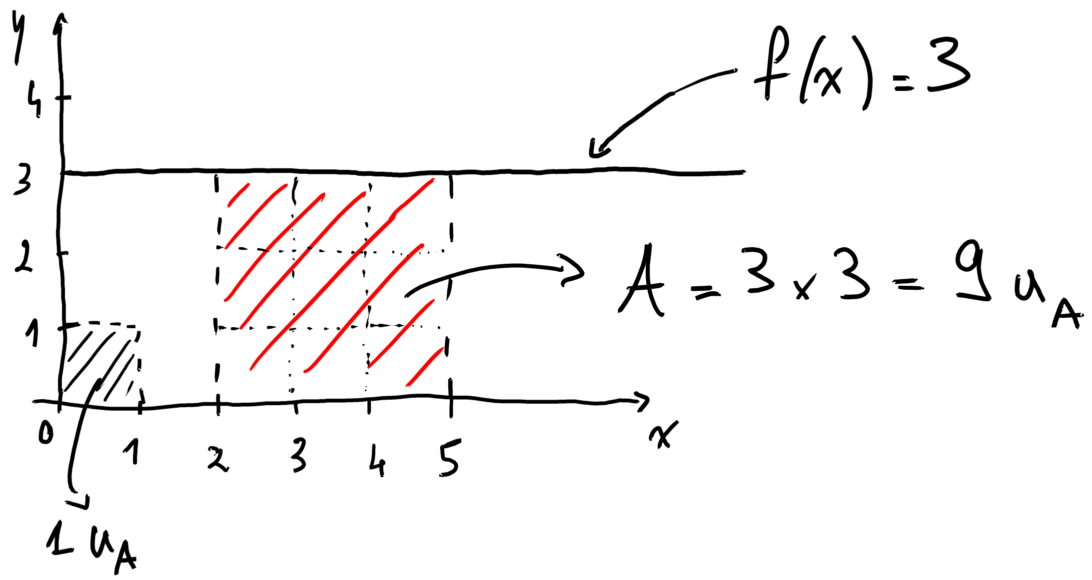
Ex 19

$f(x) = x - \frac{2}{x} \qquad G(x) = \frac{x^2}{2} - 2\ln(x) + c$

$G(1) = \frac{1^2}{2} - 2\ln(1) + C = 0$

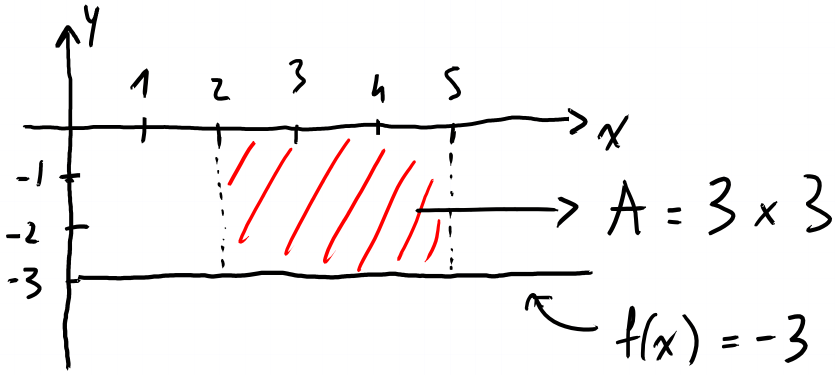
$C = -\frac{1}{2} + 2\ln(1) = -\frac{1}{2}$

Donc $G(x) = \frac{x^2}{2} - 2\ln(x) - \frac{1}{2}$



$$\int_2^5 f(x)\,dx = \int_2^5 3\,dx = \Big[3x\Big]_2^5 =$$

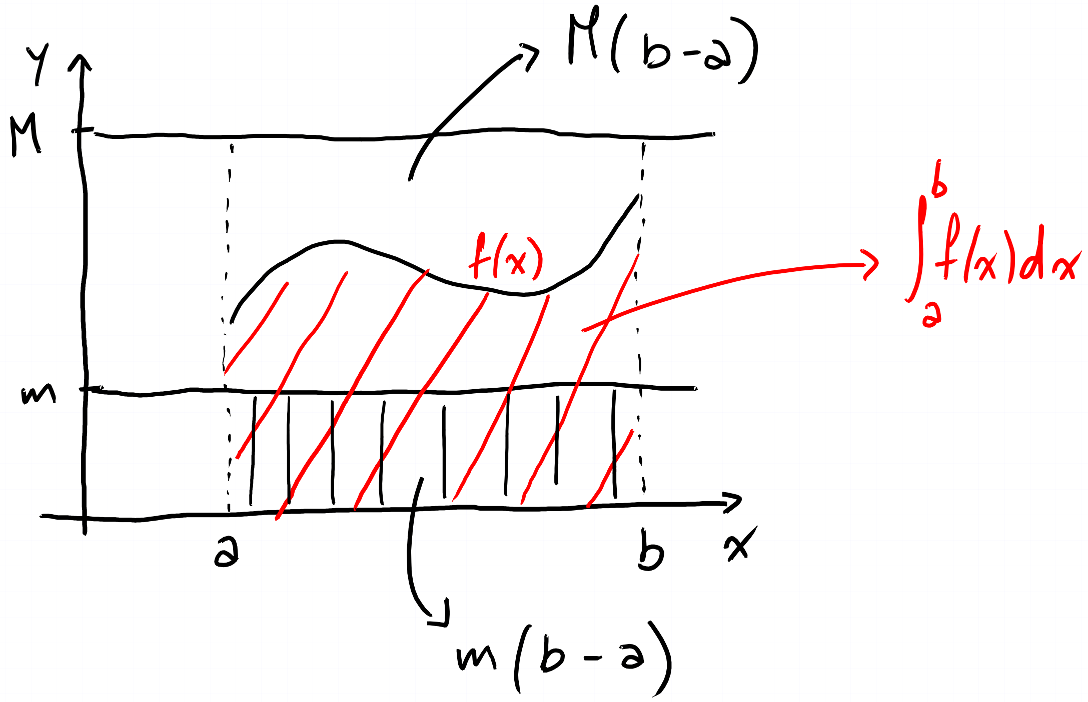
$$= (3 \times 5) - (3 \times 2) = 9$$



$$\int_2^5 f(x)\,dx = \int_2^5 (-3)\,dx = \Big[-3x\Big]_2^5 =$$

$$= (-3 \times 5) - (-3 \times 2) = -15 + 6 = -9$$

$$\Rightarrow \quad A = -\int_2^5 f(x)\,dx = 9\ u_A$$



Donc $\quad m(b-a) \leq \int_a^b f(x)\,dx \leq M(b-a)$

Valeur moyenne d'une fonction f(x) sur l'intervalle [a ; b]

$$\boxed{\frac{1}{b-a}\int_a^b f(x)\,dx}$$

Ex 25

$$\int_{-1}^{1}(x^2+1)\,dx = \left[\frac{x^3}{3}+x\right]_{-1}^{1} =$$

$$=\left(\frac{1^3}{3}+1\right)-\left(\frac{(-1)^3}{3}+(-1)\right)=$$

$$=\frac{4}{3}-\left(-\frac{1}{3}-1\right)=\frac{4}{3}-\left(-\frac{4}{3}\right)=\frac{8}{3}$$

$$\int_{-1}^{1}(x^2+3x+5)\,dx=\left[\frac{x^3}{3}+3\,\frac{x^2}{2}+5x\right]_{-1}^{1}=$$

$$=\left(\frac{1}{3}+\frac{3}{2}+5\right)-\left(-\frac{1}{3}+\frac{3}{2}-5\right)=$$

$$=\frac{2+9+30}{6}-\left(\frac{-2+9-30}{6}\right)=$$

$$=\frac{41}{6}-\left(-\frac{23}{6}\right)=\frac{41+23}{6}=\frac{64}{6}=\frac{32}{3}$$

Ex 26

$$\int_{1}^{4}\frac{3}{x}\,dx=\left[3\ln(x)\right]_{1}^{4}=3\ln(4)-3\ln(1)=3\ln(4)$$

$$\int_{1}^{4}\left(x-\frac{2}{x}\right)dx=\left[\frac{x^2}{2}-2\ln(x)\right]_{1}^{4}=8-2\ln(4)-\frac{1}{2}=$$

$$=\frac{16-1}{2}-2\ln(4)=\frac{15}{2}-2\ln(4)$$

Ex 27

$$\int_0^1 \left(x+2+\frac{1}{x+2}\right) dx = \left[\frac{x^2}{2}+2x+\ln(x+2)\right]_0^1 =$$

$$= \left(\frac{1}{2}+2+\ln(3)\right) - \left(\ln(2)\right) =$$

$$= \frac{5}{2} + \ln(3) - \ln(2) = \frac{5}{2} + \ln\left(\frac{3}{2}\right)$$

Ex 28

$$\int_0^1 \frac{t}{t^2+1} dt = \qquad \left(\int \frac{u'}{u} = \ln(u)\right)$$

$$u = t^2+1 \quad u' = 2t$$

$$= \frac{1}{2}\int_0^1 \frac{2t}{t^2+1} dt = \frac{1}{2}\left[\ln(t^2+1)\right]_0^1 =$$

$$= \frac{1}{2}\ln(2) - \frac{1}{2}\ln(1) = \frac{1}{2}\ln(2)$$

$$\int_0^{\ln 2} (e^t + e^{2t})\, dt = \qquad \left(\int u'e^u = e^u\right)$$

$$u = 2t \quad u' = 2$$

$$= \int_0^{\ln 2} e^t\, dt + \frac{1}{2}\int_0^{\ln 2} 2e^{2t}\, dt =$$

$$= \left[e^t \right]_0^{\ln 2} + \frac{1}{2}\left[e^{2t} \right]_0^{\ln 2} =$$

$$= e^{\ln 2} - e^0 + \frac{1}{2} e^{2\ln 2} - \frac{1}{2} e^0 =$$

$$= 2 - 1 + \frac{1}{2} e^{\ln 2^2} - \frac{1}{2} =$$

$$= \frac{1}{2} + \frac{1}{2} \times 4 = \frac{1}{2} + 2 = \frac{5}{2}$$

<u>Ex 29</u>

$$\int_{-1}^{2} x(x^2+4)\,dx = \int_{-1}^{2} (x^3+4x)\,dx =$$

$$= \left[\frac{x^4}{4} + 4\,\frac{x^2}{2} \right]_{-1}^{2} = \frac{2^4}{4} + 2 \times 2^2 - \left(\frac{1}{4} + 2 \right) =$$

$$= 4 + 8 - \frac{9}{4} = 12 - \frac{9}{4} = \frac{48-9}{4} = \frac{39}{4}$$

$$\int_1^2 \frac{xe^x+1}{x}\,dx = \int_1^2 \left(e^x + \frac{1}{x} \right) dx = \left[e^x + \ln(x) \right]_1^2 =$$

$$= e^2 + \ln(2) - e$$

Ex 30

$$\int_0^{\ln 2} (e^x - e^{-x})\, dx = \qquad \left(\int u' e^u = e^u\right)$$

$u = -x \quad u' = -1$

$$= \int_0^{\ln 2} e^x\, dx + \int_0^{\ln 2} (-1)\, e^{-x}\, dx =$$

$$= \left[e^x\right]_0^{\ln 2} + \left[e^{-x}\right]_0^{\ln 2} =$$

$$= e^{\ln 2} - e^0 + \left(e^{-\ln 2} - e^0\right) =$$

$$= 2 - 1 + \left(\frac{1}{2} - 1\right) = 1 - \frac{1}{2} = \frac{1}{2}$$

$$\int_e^{e^2} \frac{1}{x \ln(x)}\, dx = \int_e^{e^2} \frac{1}{x} \frac{1}{\ln(x)}\, dx = \int_e^{e^2} \frac{\frac{1}{x}}{\ln(x)}\, dx =$$

$u = \ln(x) \quad u' = \frac{1}{x} \qquad \left(\int \frac{u'}{u} = \ln(u)\right)$

$$= \left[\ln(\ln(x))\right]_e^{e^2} = \ln(\ln e^2) - \ln(\ln e) =$$

$$= \ln(2) - \ln(1) = \ln(2)$$

Ex 32

$f(x) = x^2 + 2e^{-2x} =$ $\left(\int u'e^u = e^u \right)$ $u = -2x$ $u' = -2$

$= x^2 + 2 \frac{1}{(-2)} (-2) e^{-2x} =$

$= x^2 - (-2) e^{-2x}$

Donc $F(x) = \frac{x^3}{3} - e^{-2x} + C$

Ex 33

$f(x) = \frac{x^2}{x+1} =$ " $x^2 - 1 = (x+1)(x-1)$ "

$= \frac{x^2 - 1 + 1}{x+1} = \frac{x^2-1}{x+1} + \frac{1}{x+1} =$

$= \frac{(x+1)(x-1)}{x+1} + \frac{1}{x+1} = x - 1 + \frac{1}{x+1}$

Donc $F(x) = \frac{x^2}{2} - x + \ln(x+1) + C$

Ex 34

$f(x) = x e^{x^2+1} =$ $\int u'e^u = e^u$ $u = x^2+1$ $u' = 2x$

$= \frac{1}{2} 2x e^{x^2+1}$

Donc $F(x) = \frac{1}{2} e^{x^2+1} + C$

Ex 35

$$f(x) = \frac{x^2}{2x-1} = \qquad \text{"}(2x-1)(2x+1) = 4x^2-1\text{"}$$

$$= \frac{1}{4}\ \frac{4x^2}{2x-1} = \frac{1}{4}\ \frac{4x^2-1+1}{2x-1} =$$

$$= \frac{1}{4}\ \frac{4x^2-1}{2x-1} + \frac{1}{4}\ \frac{1}{2x-1} = \frac{1}{4}\ \frac{(2x-1)(2x+1)}{2x-1} + \frac{1}{4}\ \frac{1}{2x-1} =$$

$$= \frac{1}{4}(2x+1) + \frac{1}{4}\ \frac{1}{2x-1} =$$

$$u = 2x+1 \qquad \int \frac{u'}{u} = \ln(u)$$

$$u' = 2$$

$$= \frac{1}{4}(2x+1) + \frac{1}{4} \times \frac{1}{2}\ \frac{2}{2x-1}$$

Donc $F(x) = \frac{1}{4}\left(2\frac{x^2}{2} + x\right) + \frac{1}{4} \times \frac{1}{2} \ln(2x-1) + C$

$$= \frac{x^2}{4} + \frac{x}{4} + \frac{1}{8} \ln(2x-1) + C$$