Ex 53

$$f(x) = \frac{\ln(x)}{x^2} \qquad D_f = [1 ; +\infty[$$

1) $$f = \frac{u}{v} \qquad f' = \frac{u'v - uv'}{v^2}$$

$$u = \ln(x) \qquad v = x^2$$
$$u' = 1/x \qquad v' = 2x$$

$$f'(x) = \frac{\frac{1}{x}x^2 - \ln(x)\,2x}{x^4} = \frac{x - 2x\ln(x)}{x^4} =$$
$$= \frac{x[1 - 2\ln(x)]}{x^4} = \frac{1 - 2\ln(x)}{x^3}$$

Étude signe f' :

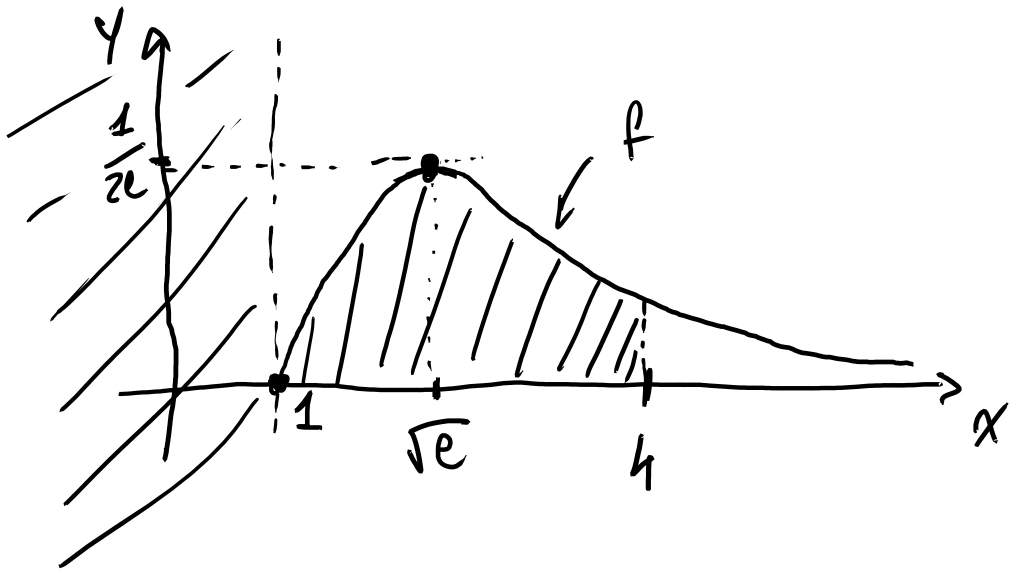
$$1 - 2\ln(x) > 0$$
$$-2\ln(x) > -1$$
$$2\ln(x) < 1$$
$$\ln(x) < 1/2 \Rightarrow x < e^{1/2}$$
$$x < \sqrt{e}$$

x	1		$\sqrt{e}$		$+\infty$
f'		$+$	0	$-$	
f	0	$\nearrow$	$1/2e$	$\searrow$	0

$$f(1) = \frac{\ln(1)}{1^2} = 0$$

$$f(\sqrt{e}) = \frac{\ln(\sqrt{e})}{e} = \frac{1/2}{e} = \frac{1}{2e}$$

$$\lim_{x \to +\infty} \frac{\ln(x)}{x^2} = 0 \rightarrow y=0 \text{ est asymptote horizontale}$$



2) $\int_1^4 \frac{\ln(x)}{x^2}\,dx$ $\qquad \int uv' = uv - \int u'v$

$u = \ln(x) \qquad u' = 1/x$

$v' = \frac{1}{x^2} \qquad v = \int \frac{1}{x^2}\,dx = \int x^{-2}\,dx = \frac{x^{-2+1}}{-2+1} = -\frac{1}{x}$

$$\int_1^4 \frac{\ln(x)}{x^2}\,dx = \left[-\frac{1}{x}\ln(x)\right]_1^4 - \int_1^4 \frac{1}{x}\left(-\frac{1}{x}\right)dx =$$

$$= -\frac{1}{4}\ln(4) - \int_1^4 \left(-\frac{1}{x^2}\right)dx$$

$$= -\frac{1}{4}\ln(4) + \left[-\frac{1}{x}\right]_1^4 =$$

$$= -\frac{1}{4}\ln(4) + \left(-\frac{1}{4} - \left(-\frac{1}{1}\right)\right) =$$

$$= -\frac{1}{4}\ln(4) - \frac{1}{4} + 1 =$$

$$= -\frac{1}{4}\ln(4) + \frac{3}{4}$$

$$\text{Aire} = \left(-\frac{1}{4}\ln(4) + \frac{3}{4}\right) u_{\text{Aire}} \qquad u_{\text{Aire}} = 2 \times 10 \text{ cm}^2$$

$$= 20 \text{ cm}^2$$

$$= (0{,}75 - 0{,}35) \times 20 \text{ cm}^2 = 8 \text{ cm}^2$$