Ex 10 : $f(x) = e^x - x$

1. $\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} e^x - \lim_{x \to -\infty} x$

$= 0 - (-\infty) = +\infty$

2. $\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} e^x - \lim_{x \to +\infty} x =$

$= +\infty - (+\infty) =$

$= +\infty - \infty = ?$

$f(x) = e^x - x = e^x \left(1 - \frac{x}{e^x}\right)$

$\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} e^x \left(1 - \frac{x}{e^x}\right) =$

$= \left(\lim_{x \to +\infty} e^x\right) \left[\lim_{x \to +\infty} \left(1 - \frac{x}{e^x}\right)\right] =$

$= +\infty \left[1 - 0\right] = +\infty$

Ex 11 : $\lim_{x \to +\infty} (x - \ln x) = +\infty - (+\infty)$

$= +\infty - \infty = ?$

$$\lim_{x \to +\infty} x\left(1 - \frac{\ln x}{x}\right) = \left(\lim_{x \to +\infty} x\right)\left[\lim_{x \to +\infty} \left(1 - \frac{\ln x}{x}\right)\right] =$$

$$= +\infty \left[1 - 0\right] = +\infty$$

Ex 12 : $\lim_{x \to +\infty} \frac{e^x + 1}{x^2 + 1} = \frac{+\infty}{+\infty} = ?$

$$\frac{e^x + 1}{x^2 + 1} = \frac{e^x\left(1 + \frac{1}{e^x}\right)}{x^2\left(1 + \frac{1}{x^2}\right)} = \frac{e^x}{x^2} \frac{1 + \frac{1}{e^x}}{1 + \frac{1}{x^2}}$$

$$\left(\lim_{x \to +\infty} \frac{e^x}{x^2}\right)\left(\lim_{x \to +\infty} \frac{1 + 1/e^x}{1 + 1/x^2}\right) = +\infty \; \frac{1+0}{1+0} = +\infty$$