F est primitive de f si $\underline{F' = f}$

Example : $f(x) = e^x$

$F(x) = e^x$ car $F' = e^x = f$

Si F est primitive alors $G = F + C$ est primitive aussi

$G' = (F+c)' = F' + c' = e^x + 0 = e^x = f$ Vrai

La primitive G t.q. $\underline{G(0) = 2}$ ←

$G(0) = e^0 + C = 1 + C$

Donc $1 + C = 2 \Rightarrow C = 1$

Alors : $\underline{G(x) = e^x + 1}$

Example : $f(x) = 2$ $F(x) = 2x + C$

car $F'(x) = 2 + 0 = 2 = f(x)$ Vrai

Difference entre primitive et dérivée :

$f(x) = x^2 + 3x$

$f'(x) = 2x + 3$

$F(x) = ?$ $\boxed{F' = f}$

$F(x) = \frac{x^3}{3} + 3\frac{x^2}{2}$

$(x^3)' = 3x^2$

$F = x^3 \Rightarrow F' = 3x^2$

$F = \frac{x^3}{3} \Rightarrow \boxed{F' = \frac{3x^2}{3} = x^2}$

$(x^2)' = 2x$

Je cherche $F(x)$ t.q. $\boxed{F' = f}$ ←

$f = x^2 \qquad F(x) = \frac{3x^2}{3} = x^2 \qquad F' = 2x \neq f$

Donc x^2 n'est pas primitive de f

$f(x) = x^2 \qquad F(x) = \frac{x^3}{3} \qquad F' = \frac{3x^2}{3} = x^2 = f$

→ Donc $\frac{x^3}{3}$ est bien primitive de f.

$$\boxed{f(x) = x^n \Rightarrow F(x) = \frac{x^{n+1}}{n+1} + C}$$

$f(x) = x^n$

$f'(x) = n x^{n-1}$

si $f(x) = x^2$ donc $n = 2$

$$F(x) = \frac{x^{2+1}}{2+1} + C = \frac{x^3}{3} + C$$

si $\underline{F(0) = 1}$ $\quad F(0) = \frac{0^3}{3} + C = 1 \Rightarrow C = 1$

$$\Rightarrow F(x) = \frac{x^3}{3} + 1$$

Example : $\underline{f(x) = u^n u'}$ $\qquad F(x) = \frac{u^{n+1}}{n+1} + C$

$f(x) = (\underbrace{x^2+3x}_{u})^2 (\underbrace{2x+3}_{u'})$ $\qquad F(x) = \frac{u^{2+1}}{2+1} + C$

$= u^2 u' \quad n = 2$ $\qquad = \frac{(x^2+3x)^3}{3} + C$

Example : $f(x) = \underbrace{e^u u'}$ $F(x) = e^u + C$

$f(x) = \underline{5e^{3x}} = 5e^u$ avec $u = 3x$

$u' = 3 \leftarrow$

$\underline{f(x) = 5e^{3x} \times \frac{3}{3}} = \underline{\frac{5}{3} e^{3x} \times 3} = \frac{5}{3} e^u u'$

$f(x) = \frac{5}{3} e^u u' \Rightarrow F(x) = \frac{5}{3} e^u + C = \frac{5}{3} e^{3x} + C$

<u>Ex 1</u>

a) $F(x) = \frac{x^3}{3} + 2x^2 + 4$ $f(x) = x^2 + 4x$

$D_f = \mathbb{R}$; Je dois montrer que $\boxed{F' = f}$

$F' = \frac{3x^2}{3} + 2 \times 2x + 0 = x^2 + 4x = f(x) \Rightarrow$ <u>Vrai</u>

b) $F(x) = 2e^x + \frac{1}{2} x^2$ $f(x) = 2e^x + x$

Je dois montrer que $\boxed{F' = f}$ -> <u>être primitive</u>

$F'(x) = 2e^x + \frac{1}{2} 2x = 2e^x + x = f(x) \Rightarrow$ <u>Vrai</u>

<u>Ex 3</u> $f(x) = x^2 - 3x$ $F(x) = ?$

f	x^2	x
F	$\frac{x^3}{3}$	$\frac{x^2}{2}$

$\Rightarrow F(x) = \frac{x^3}{3} - 3\frac{x^2}{2} = \frac{x^3}{3} - \frac{3}{2} x^2$

$G(x) = F(x) + C = \frac{x^3}{3} - \frac{3}{2} x^2 + C$

$$g(x) = -2x^3 + 4x - 5$$

f	x^3	x	a
F	$\frac{x^4}{4}$	$\frac{x^2}{2}$	ax

$$F(x) = -2\frac{x^4}{4} + 4\frac{x^2}{2} - 5x$$

$$G(x) = F(x) + C = -\frac{1}{2}x^4 + 2x^2 - 5x + C$$