7) $f(x) = \left(\frac{x-3}{x-2}\right)^2$ V I : $x-2=0 \Rightarrow x=2$ $D_f = \mathbb{R} \setminus \{2\}$

$\rightarrow f(x) = u^2$ avec $u = \frac{x-3}{x-2}$ $\boxed{f' = 2uu'}$

$u = \frac{w}{v}$ $w = x-3$ $v = x-2$

$w' = 1$ $v' = 1$

$u' = \frac{w'v - wv'}{v^2} = \frac{x-2-(x-3)}{(x-2)^2} = \frac{\cancel{x}-2-\cancel{x}+3}{(x-2)^2} = \frac{1}{(x-2)^2}$

$f'(x) = 2 \underbrace{\frac{x-3}{x-2}}_{u} \underbrace{\frac{1}{(x-2)^2}}_{u'} = 2\frac{x-3}{(x-2)^3}$

$(x-2)(x-2)^2 = (x-2)^3$

$A \times A^2 = A^3$

Étude de signe :

$f'(x) = \frac{2(x-3)}{(x-2)^3}$ $2 > 0$ Tous. | $x-3>0$ $x>3$ | $(x-2)^3 > 0$ $x-2>0$ $x>2$ V.I.

x	$-\infty$		2		3		$+\infty$
2	$+$		$+$		$+$		
$x-3$	$-$		$-$	0	$+$		
$(x-2)^3$	$-$		$+$		$+$		
f'	$+$		$-$	0	$+$		

8) $f(x) = x^2 + 1 - \dfrac{2x}{x+3}$ V.I : $x+3 = 0 \Rightarrow x = -3$

$D_f = \mathbb{R} \setminus \{-3\}$

$f(x) = x^2 + 1 - \dfrac{u}{v}$ $\quad u = 2x \quad v = x+3$

$u' = 2 \quad v' = 1$

$$f'(x) = 2x - \frac{u'v - uv'}{v^2} = 2x - \frac{2(x+3) - 2x}{(x+3)^2} =$$

$$= 2x - \frac{2x + 6 - 2x}{(x+3)^2} = 2x - \frac{6}{(x+3)^2} =$$

$$= \frac{2x(x+3)^2 - 6}{(x+3)^2} = \frac{2x(x^2 + 6x + 9) - 6}{(x+3)^2} =$$

$$= \frac{2x^3 + 12x^2 + 18x - 6}{(x+3)^2} = \frac{2(x^3 + 6x^2 + 9x - 3)}{(x+3)^2}$$

Avec la calculatrice : $f'(x) > 0$ si $x > 0,28$

$f'(x) = 0$ si $x \simeq 0,28$

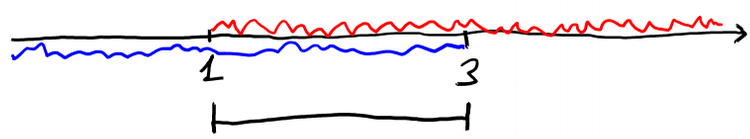
$f'(x) < 0$ si $x < 0,28$

9) $f(x) = \sqrt{x-1}\sqrt{3-x} = uv$

Ensemble de définition : $x - 1 \geq 0$ et $3 - x \geq 0$

$x \geq 1$ $\quad -x \geq -3$

$x \leq 3$



$D_f = [1 ; 3]$

$f'(x) = u'v + uv' \quad u = \sqrt{x-1} \quad v = \sqrt{3-x}$

$u' = \dfrac{1}{2\sqrt{x-1}} \quad v' = \dfrac{-1}{2\sqrt{3-x}}$

$$f'(x) = \frac{1}{2\sqrt{x-1}}\sqrt{3-x} + \sqrt{x-1}\left(-\frac{1}{2\sqrt{3-x}}\right) =$$

$$= \frac{\sqrt{3-x}}{2\sqrt{x-1}} - \frac{\sqrt{x-1}}{2\sqrt{3-x}} = \frac{\sqrt{3-x}\sqrt{3-x} - \sqrt{x-1}\sqrt{x-1}}{2\sqrt{x-1}\sqrt{3-x}} =$$

$$= \frac{3-x-(x-1)}{2\sqrt{x-1}\sqrt{3-x}} = \frac{3-x-x+1}{2\sqrt{x-1}\sqrt{3-x}} = \frac{-2x+4}{2\sqrt{x-1}\sqrt{3-x}}$$

Étude de signe : $-2x+4>0$

$-2x>-4$

$2x<4 \longrightarrow \frac{2x}{2}<\frac{4}{2} \rightarrow x<2$

$x<2$

$D_f = [1;3]$

x	1		2		3
f'		+	0	−	