Intégration par parties.

$$(uv)' = u'v + uv'$$

$$\int (uv)' = \int (u'v + uv')$$

$$uv = \int u'v + \int \underline{uv'}$$

$$\boxed{\int uv' = uv - \int u'v}$$

⚠ : $\int v' = v \Rightarrow v$ est une primitive de v'

Exemple : $\displaystyle\int_1^2 x \ln(x)\, dx = \int uv'$

1er : $u = x$, $v' = \ln(x)$ $\Rightarrow$ $u' = 1$, $v = \int \ln(x)\, dx = \, ?$ $\Rightarrow$ <u>NON</u>

2ème : $u = \ln(x)$, $v' = x$ $\Rightarrow$ $u' = \frac{1}{x}$, $v = \int x\, dx = \frac{x^2}{2}$ $\Rightarrow$ <u>OK</u>

$$\int uv' = uv - \int u'v$$

$$\int_1^2 \ln(x)\, x\, dx = \left[\ln(x) \frac{x^2}{2}\right]_1^2 - \int_1^2 \frac{1}{x} \frac{x^2}{2}\, dx =$$

$$= \underbrace{2\ln(2)} - \int_1^2 \frac{x}{2}\,dx = 2\ln(2) - \left[\frac{x^2}{4}\right]_1^2 =$$

$$= 2\ln(2) - \left[1 - \frac{1}{4}\right] = 2\ln(2) - \frac{3}{4}$$

$$\left(\left[\ln(x)\,\frac{x^2}{2}\right]_1^2 = \ln(2)\,\frac{2^2}{2} - \ln(1)\,\frac{1^2}{2} = \right.$$

$$\left. = 2\ln(2) - 0 \times \frac{1}{2} = 2\ln(2) \right)$$

Exemple : $\int_0^1 x e^x \, dx = \int u v'$

1^{er} : $u = x$ $\quad u' = 1$

$v' = e^x \quad \Rightarrow \quad v = \int e^x \, dx = e^x$

$$\int u v' = uv - \int u' v$$

$$\int_0^1 x e^x \, dx = \left[x e^x\right]_0^1 - \int_0^1 1 \times e^x \, dx =$$

$$= 1 \times e^1 - 0 \times e^0 - \int_0^1 e^x \, dx =$$

$$= e - \left[e^x\right]_0^1 = e - [e - 1] = 1$$

$2^{ème}$: $u = e^x$ $\quad u' = e^x$

$v' = x \quad \Rightarrow \quad v = \int x \, dx = \frac{x^2}{2}$

$$\int_0^1 x e^x \, dx = \left[e^x \frac{x^2}{2}\right]_0^1 - \underbrace{\int_0^1 e^x \frac{x^2}{2} \, dx} \rightarrow ?$$

$\Rightarrow$ NON

Ex 46

$$\int_1^e (x^2+1)\ln(x)\,dx = \int uv' = uv - \int u'v$$

$$u = \ln(x) \qquad u' = \frac{1}{x}$$
$$\Rightarrow$$
$$v' = x^2+1 \qquad v = \int (x^2+1)\,dx = \frac{x^3}{3} + x$$

$$= \left[\ln(x)\left(\frac{x^3}{3}+x\right)\right]_1^e - \int_1^e \frac{1}{x}\left(\frac{x^3}{3}+x\right)dx =$$

$$= \ln(e)\left(\frac{e^3}{3}+e\right) - \ln(1)\left(\frac{1^3}{3}+1\right) - \int_1^e \left(\frac{x^2}{3}+1\right)dx =$$

$$= \frac{e^3}{3} + e - \left[\frac{x^3}{9} + x\right]_1^e =$$

$$= \frac{e^3}{3} + e - \left[\frac{e^3}{9} + e - \left(\frac{1}{9}+1\right)\right] =$$

$$= \frac{e^3}{3} + e - \left[\frac{e^3}{9} + e - \frac{10}{9}\right] =$$

$$= \frac{e^3}{3} \cancel{+e} - \frac{e^3}{9} \cancel{-e} + \frac{10}{9} = \frac{2}{9}e^3 + \frac{10}{9}$$

$$\int_0^1 x e^{2x}\,dx = \int u v' = uv - \int u' v$$

$$u = x \qquad\qquad u' = 1$$

$$\Rightarrow$$

$$v' = e^{2x} \qquad\qquad v = \int e^{2x}\,dx = \frac{e^{2x}}{2}$$

$$= \left[x \frac{e^{2x}}{2} \right]_0^1 - \int_0^1 \frac{e^{2x}}{2}\,dx =$$

$$= \frac{e^2}{2} - \left[\frac{e^{2x}}{4} \right]_0^1 = \frac{e^2}{2} - \left[\frac{e^2}{4} - \frac{1}{4} \right] =$$

$$= \frac{e^2}{2} - \frac{e^2}{4} + \frac{1}{4} = \frac{e^2+1}{4}$$

Ex 47

$$\int_0^2 x e^{-x}\,dx = \int u v' = uv - \int u' v$$

$$u = x \qquad\qquad u' = 1$$

$$\Rightarrow$$

$$v' = e^{-x} \qquad\qquad v = \int e^{-x}\,dx = -e^{-x}$$

$$= \left[x(-e^{-x}) \right]_0^2 - \int_0^2 (-e^{-x})\,dx =$$

$$= 2\left(-e^{-2}\right) + \int_0^2 e^{-x}\, dx =$$

$$= -\frac{2}{e^2} + \left[-e^{-x}\right]_0^2 = -\frac{2}{e^2} + \left[-\frac{1}{e^2} - (-1)\right] =$$

$$= -\frac{2}{e^2} - \frac{1}{e^2} + 1 = -\frac{3}{e^2} + 1$$

$$\int_1^e \ln(x)\, dx = \int_1^e 1 \times \ln(x)\, dx = \int u v' = uv - \int u' v$$

$$u = \ln(x) \qquad\qquad u' = \frac{1}{x}$$
$$\Rightarrow$$
$$v' = 1 \qquad\qquad v = \int 1\, dx = x$$

$$= \left[\ln(x)\, x\right]_1^e - \int_1^e \frac{1}{x}\, x\, dx =$$

$$= e - \left[x\right]_1^e = e - \left[e - 1\right] = 1$$

<u>Ex 49</u>

$$\int_0^1 (x+2)e^{-x}\, dx = 3 - \frac{4}{e}$$

$$\int_1^4 (x-1)\ln(x)\, dx = 4\ln(4) - \frac{3}{4}$$