Ex 9: $f(x) = \dfrac{e^x - 1}{2e^x + 1}$

1. $\lim\limits_{x \to -\infty} f(x) = \dfrac{0 - 1}{0 + 1} = -1$

2. $\lim\limits_{x \to +\infty} f(x) = \dfrac{+\infty}{+\infty} = ?$

$$f(x) = \frac{e^x\left(1 - \frac{1}{e^x}\right)}{e^x\left(2 + \frac{1}{e^x}\right)} = \frac{1 - \frac{1}{e^x}}{2 + \frac{1}{e^x}}$$

$$\lim_{x \to +\infty} f(x) = \frac{1 - 0}{2 + 0} = \frac{1}{2}$$

Ex 10: $f(x) = e^x - x$

1. $\lim\limits_{x \to -\infty} f(x) = 0 - (-\infty) = +\infty$

2. $\lim\limits_{x \to +\infty} f(x) = +\infty - (+\infty) = +\infty - \infty = ?$

$$f(x) = e^x\left(1 - \frac{x}{e^x}\right)$$

$$\lim_{x \to +\infty} f(x) = +\infty\,(1 - 0) = +\infty$$

Ex 11 : $\lim_{x \to +\infty} (x - \ln x) = +\infty - (+\infty) = +\infty - \infty = ?$

$$\lim_{x \to +\infty} x \left(1 - \frac{\ln x}{x}\right) = +\infty \left(1 - 0\right) = +\infty$$

Ex 12 : $\lim_{x \to +\infty} \frac{e^x + 1}{x^2 + 1} = \frac{+\infty}{+\infty} = ?$

$$\frac{e^x + 1}{x^2 + 1} = \frac{e^x \left(1 + \frac{1}{e^x}\right)}{x^2 \left(1 + \frac{1}{x^2}\right)} = \frac{e^x}{x^2} \cdot \frac{\left(1 + \frac{1}{e^x}\right)}{\left(1 + \frac{1}{x^2}\right)}$$

$$\lim_{x \to +\infty} \frac{e^x}{x^2} \cdot \frac{1 + \frac{1}{e^x}}{1 + \frac{1}{x^2}} = +\infty \; \frac{1+0}{1+0} = +\infty$$