Ex 53

$$f(x) = \frac{\ln(x)}{x^2} \qquad D_f = [1 ; +\infty[$$

1) $f(x) = \frac{u}{v}$ $\quad u = \ln(x) \quad u' = 1/x$

$\quad v = x^2 \quad v' = 2x$

$$f'(x) = \frac{u'v - uv'}{v^2} = \frac{\frac{1}{x}x^2 - \ln(x)\,2x}{x^4} =$$

$$= \frac{x - 2x\ln(x)}{x^4} = \frac{1 - 2\ln(x)}{x^3}$$

Étude de signe de f' sur $D_f = [1 ; +\infty[$:

$1 - 2\ln(x) > 0 \Rightarrow -2\ln(x) > -1$

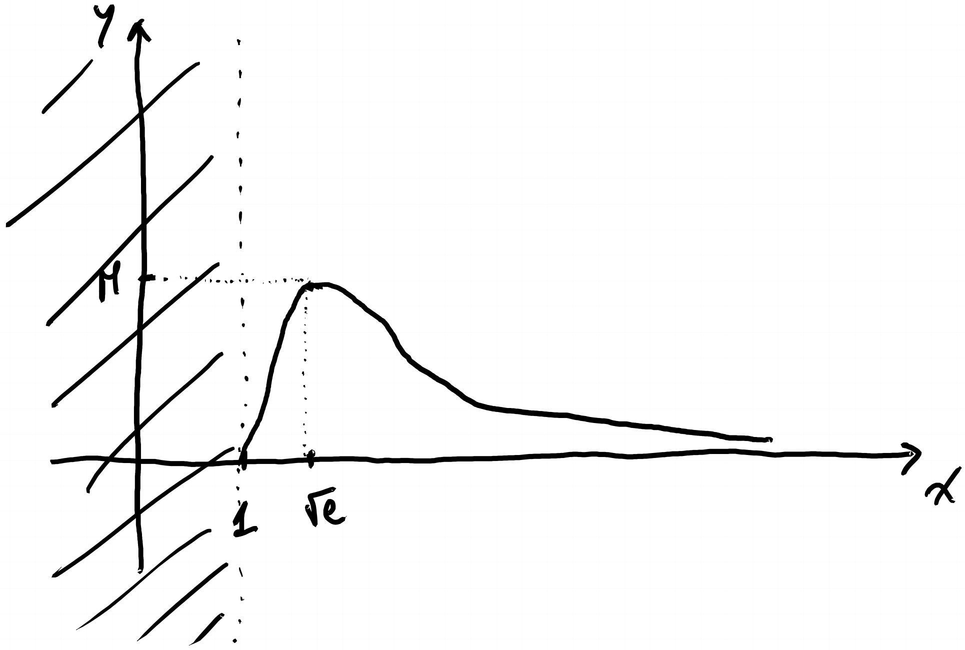
$\Rightarrow 2\ln(x) < 1 \Rightarrow \ln(x) < \frac{1}{2} \Rightarrow x < \sqrt{e}$

Tableau de variations :

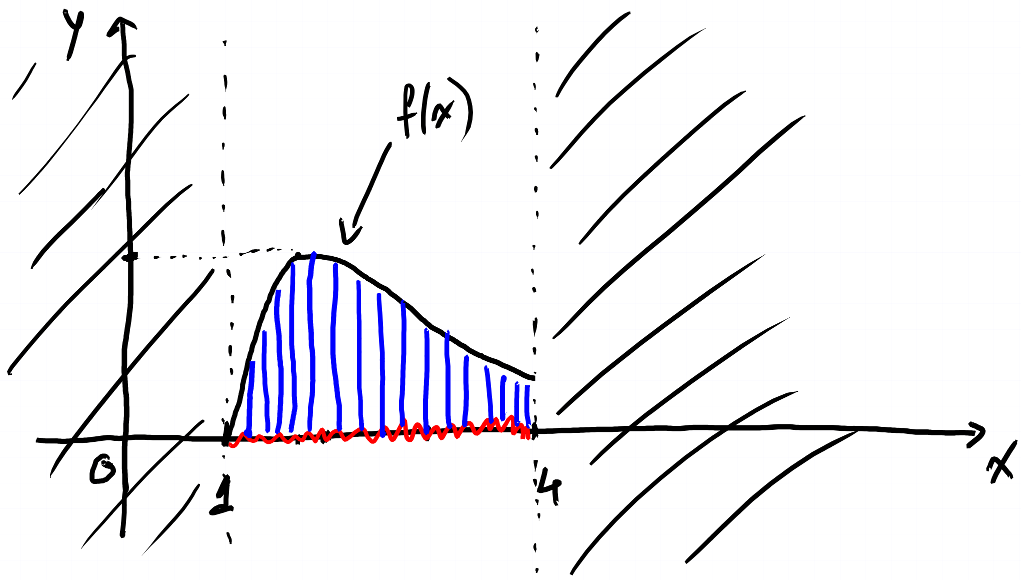
x	1		$\sqrt{e}$		$+\infty$
f'		$+$	0	$-$	
f	0	$\nearrow$	M	$\searrow$	0

$f(1) = \frac{\ln(1)}{1^2} = 0 \qquad \lim_{x \to +\infty} \frac{\ln(x)}{x^2} = 0$

$M = f(\sqrt{e}) = \frac{\ln(\sqrt{e})}{e} = \frac{1}{2e}$



2) $M(x;y)$ t.q. $1 \leq x \leq 4$ et $0 \leq y \leq f(x)$



$$A = \int_1^4 f(x)\,dx = \int_1^4 \frac{\ln(x)}{x^2}\,dx =$$

$$= \int_1^4 x^{-2} \ln(x)\,dx = \int u v'$$

$u = \ln(x)$ $\Rightarrow$ $u' = \frac{1}{x}$

$v' = x^{-2}$ $\qquad v = \int x^{-2}\,dx = \frac{x^{-2+1}}{-2+1} = \frac{x^{-1}}{-1} = -\frac{1}{x}$

$$\int uv' = uv - \int u'v =$$

$$= \left[\ln(x)\left(-\frac{1}{x}\right) \right]_1^4 - \int_1^4 \frac{1}{x}\left(-\frac{1}{x}\right) dx =$$

$$= -\frac{1}{4}\ln(4) + \int_1^4 x^{-2}\, dx =$$

$$= -\frac{1}{4}\ln(4) + \left[-\frac{1}{x}\right]_1^4 =$$

$$= -\frac{1}{4}\ln(4) + \left[-\frac{1}{4} + 1\right] =$$

$$= -\frac{1}{4}\ln(4) + \frac{3}{4}$$

Donc $\mathcal{A} = \left[\frac{3}{4} - \frac{1}{4}\ln(4)\right] u_A$

$$u_A = 2\,\text{cm} \times 10\,\text{cm} = 20\,\text{cm}^2 = 2000\,\text{mm}^2$$

$$\mathcal{A} = \left[\frac{3}{4} - \frac{1}{4}\ln(4)\right] \times 2000\,\text{mm}^2 = 807\,\text{mm}^2$$

<u>Bonus</u> : Valeur moyenne de f entre 1 et 4

$$\frac{1}{4-1}\int_1^4 f(x)\,dx = \frac{807}{3}\,\text{mm}^2$$