

$$1. \quad f(x) = \frac{-x^2 + 2x - 1}{x} = \frac{-x^2}{x} + \frac{2x}{x} - \frac{1}{x} =$$

$$= -x + 2 - \frac{1}{x}$$

Donc $a = -1$ $b = 2$ $c = -1$

2ème méthode:

$$f(x) = ax + b + \frac{c}{x} = \frac{ax \cdot x + b \cdot x + c}{x} =$$

$$= \frac{ax^2 + bx + c}{x}$$

Donc $a = -1$ $b = 2$ $c = -1$

$$2. \quad f(x) = -x + 2 - \frac{1}{x}$$

$$f'(x) = -1 + 0 - \left(-\frac{1}{x^2}\right) = -1 + \frac{1}{x^2} = \frac{-x^2 + 1}{x^2}$$


2ème méthode:

$$f(x) = \frac{-x^2 + 2x - 1}{x} = \frac{u}{v} \quad \begin{array}{ll} u = -x^2 + 2x - 1 & v = x \\ u' = -2x + 2 & v' = 1 \end{array}$$

$$f'(x) = \frac{u'v - uv'}{v^2} = \frac{(-2x + 2)x - (-x^2 + 2x - 1)1}{x^2} =$$

$$= \frac{-2x^2 + 2x + x^2 - 2x + 1}{x^2} = \frac{-x^2 + 1}{x^2}$$

3. Étude de signe de f' sur $\mathbb{R} \setminus \{0\}$

Num: $-x^2 + 1$ $a = -1$  $b = 0$ $c = 1$


$$\Delta = 0^2 - 4 \times (-1) \times 1 = 4 > 0$$



$$x_1 = \frac{0 - 2}{-2} = 1$$

$$x_2 = \frac{0 + 2}{-2} = -1$$

x	$-\infty$	-1	1	$+\infty$	
$-x^2 + 1$	$-$	\emptyset	$+$	\emptyset	$-$

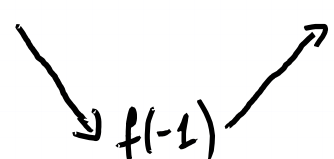
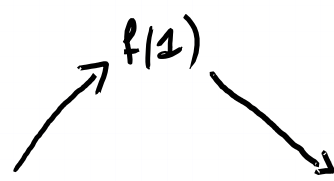
Den: x^2 $a = 1$  $b = 0$ $c = 0$

$$\Delta = 0^2 - 4 \times 1 \times 0 = 0$$



$$x_1 = -\frac{0}{2} = 0 \quad \underline{\text{V.I.}}$$

x	$-\infty$	0	$+\infty$
x^2	$+$	\parallel	$+$

x	$-\infty$	-1	0	1	$+\infty$	
$-x^2+1$	$-$	\emptyset	$+$	\emptyset	$-$	
x^2		$+$		$+$		
f'	$-$	\emptyset	$+$	$+$	\emptyset	$-$
f						

$$f(-1) = 4$$

$$f(1) = 0$$

4. a. tangente horizontale \rightarrow coefficient directeur est zéro
 \downarrow

$$f'(x) = 0$$

$$\text{Donc } \frac{-x^2+1}{x^2} = 0 \Leftrightarrow -x^2+1 = 0 \quad \boxed{x=0 \text{ v.l.}}$$
$$S = \{-1; 1\}$$

b. $f'(x) = 3$

$$\frac{-x^2+1}{x^2} = 3 \Leftrightarrow -x^2+1 = 3x^2 \quad \boxed{x=0 \text{ v.l.}}$$
$$-4x^2+1 = 0$$

$$a = -4 \quad b = 0 \quad c = 1$$

$$\Delta = 0^2 - 4 \times (-4) \times 1 = 16$$

$$x_1 = \frac{0-4}{-8} = \frac{1}{2} \quad x_2 = \frac{0+4}{-8} = -\frac{1}{2}$$

$$S = \left\{-\frac{1}{2}; \frac{1}{2}\right\}$$

5. T: $y = f'(-2)(x - (-2)) + f(-2)$

$$f'(-2) = \frac{-(-2)^2+1}{(-2)^2} = \frac{-4+1}{4} = -\frac{3}{4}$$

$$f(-2) = -(-2) + 2 - \frac{1}{(-2)} = 2 + 2 + \frac{1}{2} = \frac{8+1}{2} = \frac{9}{2}$$

$$T: y = -\frac{3}{4}(x+2) + \frac{9}{2} = -\frac{3}{4}x - \frac{3}{2} + \frac{9}{2} = -\frac{3}{4}x + \frac{6}{2}$$

$$\text{Donc } T: y = -\frac{3}{4}x + 3$$

b. Les axes du repère:

$$\text{axe des } x: y = 0$$

$$\text{axe des } y: x = 0$$

$$f(x) = \frac{-x^2 + 2x - 1}{x^2}$$

Intersection avec l'axe des x :

$$y = 0 \Rightarrow \text{image nulle} \Rightarrow f(x) = 0$$

$$\frac{-x^2 + 2x - 1}{x^2} = 0 \Leftrightarrow -x^2 + 2x - 1 = 0 \quad \boxed{x=0 \text{ v. I.}}$$

$$-x^2 + 2x - 1 = 0 \quad a = -1 \quad b = 2 \quad c = -1$$

$$\Delta = 2^2 - 4 \times (-1) \times (-1) = 4 - 4 = 0$$

$$x_1 = \frac{-2}{-2} = 1$$

Le point d'intersection entre \mathcal{C} et l'axe des x

$$\text{est : } \begin{cases} x = 1 \\ y = 0 \end{cases}$$

Intersection avec l'axe des y :

$$x=0 \Rightarrow f(0) = ? \quad \text{car } x=0 \text{ v.I.}$$

Je ne peux pas calculer l'image de $x=0$,
donc \mathcal{C} n'a pas d'intersection avec
l'axe des y .