Ex 46

$$\int_1^e (x^2+1)\ln(x)\,dx = \qquad \int uv' = uv - \int u'v$$

$u = \ln(x) \qquad u' = 1/x$

$v' = x^2+1 \qquad v = \int (x^2+1)\,dx = \frac{x^3}{3}+x$

$$= \left[\ln(x)\left(\frac{x^3}{3}+x\right)\right]_1^e - \int_1^e \frac{1}{x}\left(\frac{x^3}{3}+x\right)dx =$$

$$= \ln(e)\left(\frac{e^3}{3}+e\right) - \int_1^e\left(\frac{x^2}{3}+1\right)dx =$$

$$= \frac{e^3}{3}+e-\left[\frac{x^3}{9}+x\right]_1^e = \frac{e^3}{3}+e-\left[\frac{e^3}{9}+e-\left(\frac{1}{9}+1\right)\right] =$$

$$= \frac{e^3}{3}+\cancel{e}-\frac{e^3}{9}-\cancel{e}+\frac{1}{9}+1 = \frac{3e^3-e^3}{9}+\frac{1+9}{9} =$$

$$= \frac{2}{9}e^3+\frac{10}{9}$$

$$\int_0^1 x e^{2x}\,dx = \qquad \int uv' = uv - \int u'v$$

$u = x \qquad u' = 1$

$v' = e^{2x} \qquad v = \int e^{2x}\,dx = \frac{1}{2}\int 2e^{2x}\,dx = \frac{e^{2x}}{2}$

$$= \left[x\,\frac{e^{2x}}{2}\right]_0^1 - \int_0^1 \frac{e^{2x}}{2}\,dx =$$

$$= \frac{e^2}{2} - \frac{1}{2}\int_0^1 e^{2x}\,dx = \frac{e^2}{2} - \frac{1}{2}\left[\frac{e^{2x}}{2}\right]_0^1 =$$

$$= \frac{e^2}{2} - \frac{1}{2}\left(\frac{e^2}{2} - \frac{1}{2}\right) = \frac{e^2}{2} - \frac{e^2}{4} + \frac{1}{4} =$$

$$= \frac{e^2}{4} + \frac{1}{4}$$

Ex 47

$$\int_0^2 x e^{-x}\,dx = \qquad \int u v' = uv - \int u' v$$

$u = x \qquad u' = 1$

$v' = e^{-x} \qquad v = \int e^{-x}\,dx = -\int (-1) e^{-x}\,dx = -e^{-x}$

$$= \left[x(-e^{-x})\right]_0^2 - \int_0^2 (-e^{-x})\,dx =$$

$$= \left[-x e^{-x}\right]_0^2 + \int_0^2 e^{-x}\,dx =$$

$$= -2e^{-2} + \left[-e^{-x}\right]_0^2 = -2e^{-2} - \left[e^{-x}\right]_0^2 =$$

$$= -2e^{-2} - \left(e^{-2} - 1\right) = -2e^{-2} - e^{-2} + 1 =$$

$$= -3e^{-2} + 1$$

$$\int_1^e \ln(x)\,dx = \int_1^e 1 \times \ln(x)\,dx = \qquad \int uv' = uv - \int u'v$$

$$u = \ln(x) \qquad u' = 1/x$$

$$v' = 1 \qquad v = x$$

$$= \Big[x \ln(x)\Big]_1^e - \int_1^e \frac{x}{x}\,dx =$$

$$= e\ln(e) - \Big[x\Big]_1^e = e - (e-1) = 1$$

<u>Ex 48</u>

$$\int_0^1 3x e^{-2x}\,dx = \qquad \int uv' = uv - \int u'v$$

$$u = 3x \qquad u' = 3$$

$$v' = e^{-2x} \qquad v = -\frac{e^{-2x}}{2}$$

$$= \Big[-3x\frac{e^{-2x}}{2}\Big]_0^1 + \int_0^1 3\frac{e^{-2x}}{2}\,dx =$$

$$= -3\frac{e^{-2}}{2} + \frac{3}{2}\Big[-\frac{e^{-2x}}{2}\Big]_0^1 =$$

$$= -\frac{3}{2}e^{-2} + \frac{3}{2}\Big[-\frac{e^{-2}}{2} - \Big(-\frac{1}{2}\Big)\Big] =$$

$$= -\frac{3}{2}e^{-2} - \frac{3}{4}e^{-2} + \frac{3}{4} = \frac{-6-3}{4}e^{-2} + \frac{3}{4} = -\frac{9}{4}e^{-2} + \frac{3}{4}$$

$$\int_1^e \ln(2x)\,dx = \int_1^e 1 \times \ln(2x)\,dx = \qquad \int uv' = uv - \int u'v$$

$$u = \ln(2x) \qquad u' = \frac{2}{2x} = \frac{1}{x}$$

$$v' = 1 \qquad v = x$$

$$= \left[x \ln(2x)\right]_1^e - \int_1^e \frac{x}{x}\,dx =$$

$$= e\ln(2e) - \ln(2) - \left[\;x\;\right]_1^e =$$

$$= e\ln(2e) - \ln(2) - (\;e - 1\;) =$$

$$= e\ln(2e) - \ln(2) - e + 1$$

$$= e\left[\ln(2) + \ln(e)\right] - \ln(2) - e + 1 =$$

$$= e\ln(2) + e\ln(e) - \ln(2) - e + 1 =$$

$$= \ln(2)(e-1) + \cancel{e} - \cancel{e} + 1 =$$

$$= \ln(2)(e-1) + 1$$