Ex 54

$f(x) = xe^{x} - e^{x}$ $\qquad D_f = [-1 ; 1]$

1. $f(x) = uv - e^{x}$

avec $u = x \qquad v = e^{x}$

$\qquad u' = 1 \qquad v' = e^{x}$

$f'(x) = u'v + uv' - e^{x} =$

$= 1 \times e^{x} + xe^{x} - e^{x} =$

$= \cancel{e^{x}} + xe^{x} - \cancel{e^{x}} = xe^{x}$

Étude de signe de f' :

$xe^{x} > 0$

$x > 0$ | $e^{x} > 0$ Toujours positif

Tableau de variations

x	-1		0		1
f'		$-$	0	$+$	
f	$-2/e$	$\searrow$	-1	$\nearrow$	0

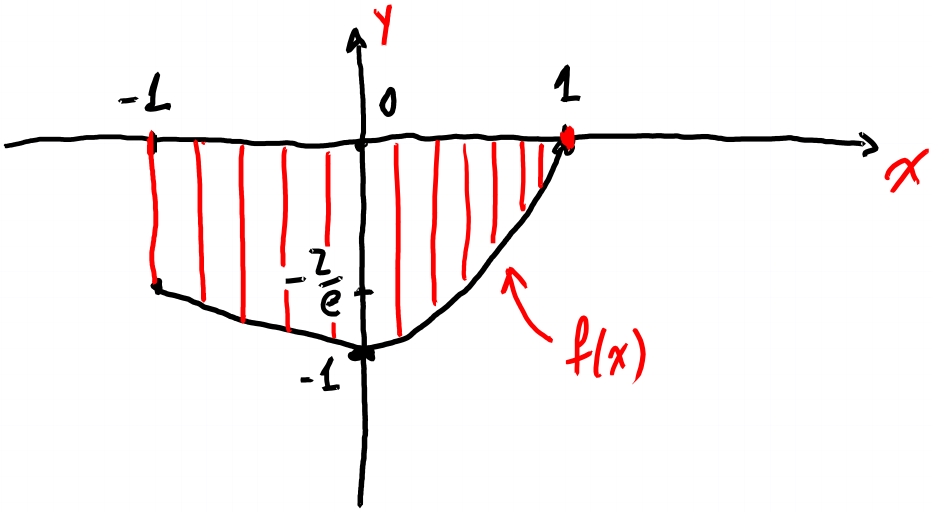
$f(x) = xe^{x} - e^{x}$

$f(-1) = -e^{-1} - e^{-1}$

$= -\frac{2}{e}$

$f(1) = e^{1} - e^{1} = 0$

$f(0) = 0 - e^{0} = -1$



2. $$A = -\int_{-1}^{1} f(x)\,dx$$

$$\int f(x)\,dx = \int (xe^x - e^x)\,dx =$$

$$= \int xe^x\,dx - \int e^x\,dx =$$

$$= \int xe^x\,dx - e^x$$

$$\int xe^x\,dx = \qquad \int uv' = uv - \int u'v$$

$u = x \qquad u' = 1$

$v' = e^x \qquad v = e^x$

$$= xe^x - \int e^x\,dx = xe^x - e^x$$

$$\int f(x)\,dx = xe^x - e^x - e^x = xe^x - 2e^x$$

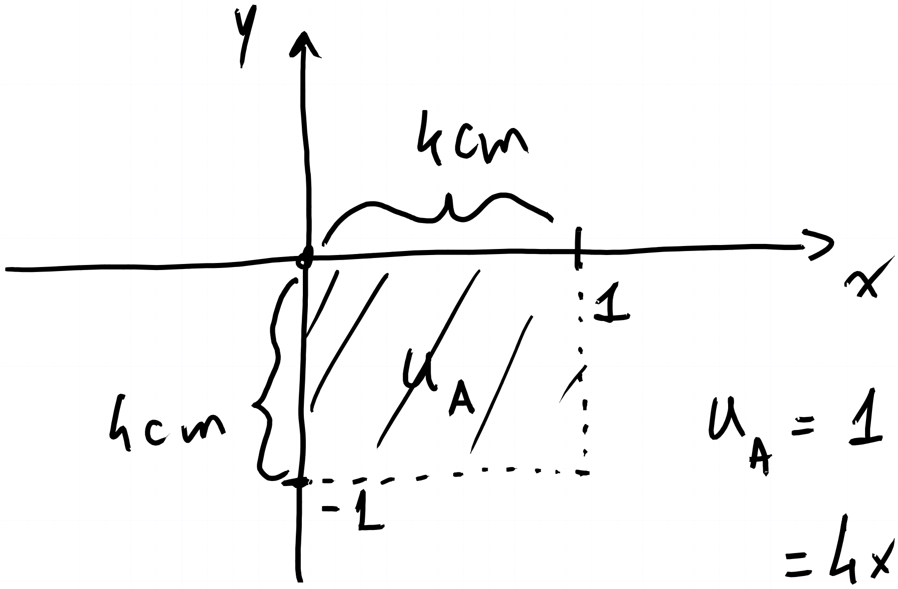
$$\int_{-1}^{1} f(x)\,dx = e^1 - 2e^1 - \left(-e^{-1} - 2e^{-1}\right) =$$

$$= e - 2e + \frac{1}{e} + \frac{2}{e} =$$

$$= -e + \frac{3}{e} \simeq -1,615$$

$$A = 1,615\ u_A = 1,615 \times 4 \times 4\ cm^2 =$$

$$= 25,84\ cm^2 = 2584\ mm^2$$



$u_A = 1\ u_A$

$= 4 \times 4\ cm^2$

Ex 55

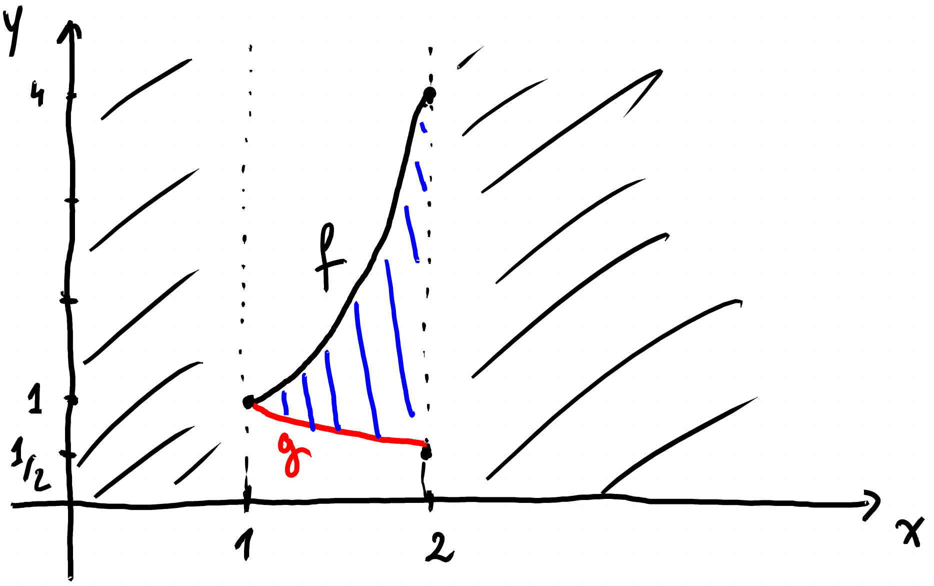
1. $f(x) = x^2$ $\qquad g(x) = 1/x$ $\qquad D_f = [1; 2]$

$f(1) = 1$

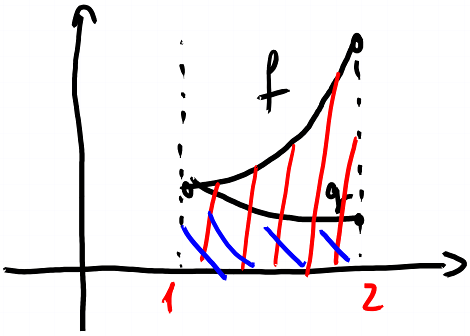
$f(2) = 4$

$g(1) = 1$

$g(2) = \frac{1}{2}$



2. a)



$\int_1^2 f$

$\int_1^2 g$

$$A = \int_1^2 (f - g)\, dx = \int_1^2 \left(x^2 - \frac{1}{x}\right) dx =$$

$$= \left[\frac{x^3}{3} - \ln(x)\right]_1^2 =$$

$$= \frac{2^3}{3} - \ln(2) - \left(\frac{1^3}{3} - \ln(1)\right) =$$

$$= \frac{8}{3} - \ln(2) - \frac{1}{3} = \frac{7}{3} - \ln(2)$$

$$A \simeq 1{,}64\ u_A$$

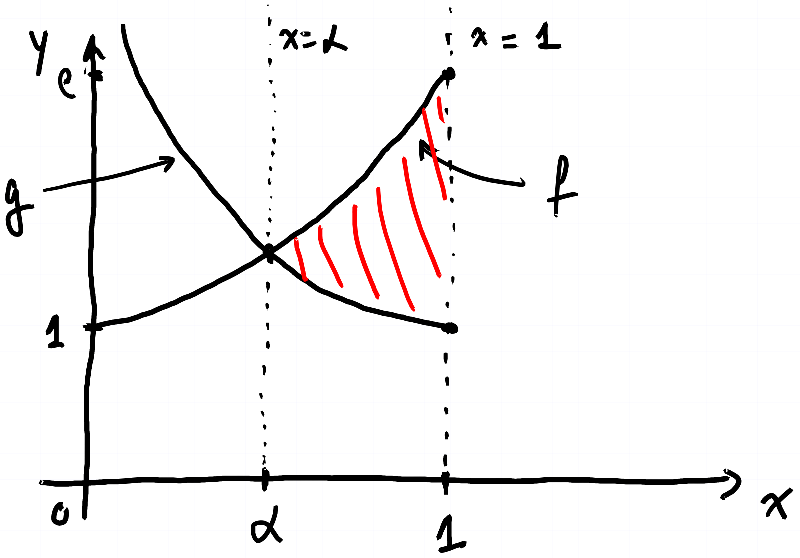
b) $u_A = 2 \times 2\,cm^2 = 4\,cm^2$

$$A = 1{,}64 \times 4\,cm^2 = 6{,}56\,cm^2$$

Ex 56

1. $f(x) = e^x \qquad D_f = [0 ; 1]$

$g(x) = \frac{1}{x} \qquad D_g =]0 ; 1]$



2. a) $$\int_\alpha^1 (f-g)\,dx = \int_\alpha^1 \left(e^x - \frac{1}{x}\right) dx =$$

$$= \left[e^x - \ln(x)\right]_\alpha^1 =$$

$$= e - \ln(1) - \left(e^\alpha - \ln(\alpha)\right) =$$

$$= e - e^\alpha + \ln(\alpha)$$

$$A = \left(e - e^\alpha + \ln(\alpha)\right) u_A =$$

$$= \left(e - e^\alpha + \ln(\alpha)\right) \times 16 \text{ cm}^2$$

b) $A = \left(e - e^{0,57} + \ln(0,57)\right) \times 16 = 6,21 \text{ cm}^2$