Exercice 2

Dans chacun des cas, calculer $f'(x)$ en précisant l'ensemble de définition de f :

1. $f(x) = 4x^3 - 5x^2 + x - 1$
2. $f(x) = 5x^3 - \frac{1}{x}$
3. $f(x) = (x^2+1)(x^3-2x)$
4. $f(x) = \dfrac{2x^2-3}{x^2+7}$
5. $f(x) = \dfrac{2x-1}{x+1}$
6. $f(x) = -x + 2 + \dfrac{2}{3x}$
7. $f(x) = \dfrac{1}{x+x^2}$
8. $f(x) = (2x+1)^2$
9. $f(x) = x(5x-3)$

1. $D = \mathbb{R}$

$f'(x) = 12x^2 - 10x + 1$

2. $x=0$ V.I. $\Rightarrow$ $D = \mathbb{R} \setminus \{0\}$

$f(x) = 5x^3 - \frac{1}{v}$ $\qquad v = x \quad v' = 1$

$$f'(x) = 15x^2 - \left(-\frac{v'}{v^2}\right) = 15x^2 - \left(-\frac{1}{x^2}\right) = 15x^2 + \frac{1}{x^2}$$

3. $f(x) = uv \qquad u = x^2+1 \quad u' = 2x \qquad D = \mathbb{R}$

$v = x^3-2x \quad v' = 3x^2-2$

$$f'(x) = u'v+uv' = 2x(x^3-2x)+(x^2+1)(3x^2-2) =$$

$$= 2x^4-4x^2+3x^4-2x^2+3x^2-2 =$$

$$= 5x^4-3x^2-2$$

4. $x^2+7=0 \iff x^2=-7$ impossible $\Rightarrow$ pas de V.I.

$D = \mathbb{R}$

$f(x) = \dfrac{u}{v} \qquad u = 2x^2-3 \quad u' = 4x$

$v = x^2+7 \quad v' = 2x$

$$f'(x) = \frac{u'v-uv'}{v^2} = \frac{4x(x^2+7)-(2x^2-3)2x}{(x^2+7)^2} =$$

$$= \frac{\cancel{4x^3}+28x-\cancel{4x^3}+6x}{(x^2+7)^2} = \frac{34x}{(x^2+7)^2}$$

5. $x+1=0 \iff x=-1$ V.I. $\Rightarrow D = \mathbb{R}\setminus\{-1\}$

$f(x) = \dfrac{u}{v} \qquad u = 2x-1 \quad u' = 2$

$v = x+1 \quad v' = 1$

$$f'(x) = \frac{u'v-uv'}{v^2} = \frac{2(x+1)-(2x-1)1}{(x+1)^2} =$$

$$= \frac{2\cancel{x} + 2 - \cancel{2x} + 1}{(x+1)^2} = \frac{3}{(x+1)^2}$$

6. $3x = 0 \Leftarrow\Rightarrow x = 0$ V.I. $\Rightarrow D = \mathbb{R} \setminus \{0\}$

$$f(x) = -x + 2 + \frac{2}{v} \qquad v = 3x \quad v' = 3$$

$$f'(x) = -1 + 0 + 2\left(-\frac{v'}{v^2}\right) = -1 + 2\left(-\frac{3}{(3x)^2}\right) =$$

$$= -1 - \frac{6}{9x^2} = -1 - \frac{2}{3x^2}$$