__Exercice :__ Dresser le tableau de variations

1) $f(x) = \dfrac{3x}{x+2}$ $\qquad I = \mathbb{R} \setminus \{-2\}$

2) $f(x) = -\dfrac{3}{x^2-5x}$ $\qquad I = \mathbb{R} \setminus \{0 ; 5\}$

3) $f(x) = \dfrac{x^2+7}{x+3}$ $\qquad I = [-1 ; 5]$

1) $f(x) = \dfrac{u}{v}$ $\qquad u = 3x \qquad v = x+2$

$\qquad u' = 3 \qquad v' = 1$

$$f'(x) = \frac{u'v - uv'}{v^2} = \frac{3(x+2) - (3x)(1)}{(x+2)^2} =$$

$$= \frac{\cancel{3x} + 6 - \cancel{3x}}{(x+2)^2} = \frac{6}{(x+2)^2}$$

(⚠ $x+2 = 0$ impossible $\Rightarrow$ $x = -2$ V.I.)

Étude de signe de f' :

6 est positif ; $(x+2)^2$ est positif sauf pour $x = -2$

x	$-\infty$	-2	$+\infty$
6	+	+	+
$(x+2)^2$	+	‖	+
f'	+	‖	+

Tableau de variations :

x	$-\infty$		-2		$+\infty$
f'		$+$	$\|$	$+$	
f		$\nearrow$	$\|$	$\nearrow$	

2) $f(x) = \dfrac{u}{v}$ $\qquad u = -3 \qquad v = x^2 - 5x$

$\qquad\qquad u' = 0 \qquad v' = 2x - 5$

$$f'(x) = \frac{u'v - uv'}{v^2} = \frac{0\,(x^2-5x) - (-3)(2x-5)}{(x^2-5x)^2} =$$

$$= \frac{6x - 15}{(x^2-5x)^2}$$

(⚠ $x^2 - 5x = 0$ impossible

$x(x-5) = 0 \Rightarrow x = 0$ et $x = 5$ V.I.)

Étude de signe de f' :

$6x - 15 > 0 \iff x > \frac{15}{6}$; $(x^2-5x)^2$ est positif sauf pour $x = 0$ et $x = 5$.

x	$-\infty$		0		$15/6$		5		$+\infty$
$6x-15$		$-$		$-$	0	$+$		$+$	
$(x^2-5x)^2$		$+$	$\|$	$+$		$+$	$\|$	$+$	
f'		$-$	$\|$	$-$	0	$+$	$\|$	$+$	

Tableau de variations:

x	$-\infty$		0		$15/6$		5		$+\infty$
f'		$-$	$\|\|$	$-$	0	$+$	$\|\|$	$+$	
f		$\searrow$	$\|\|$	$\searrow$	$f\left(\frac{15}{6}\right)$	$\nearrow$	$\|\|$	$\nearrow$	

$$f\left(\frac{15}{6}\right) = 0{,}48$$

3) $f(x) = \dfrac{x^2+7}{x+3}$ $\quad I = [-1\,;\,5]$

$f(x) = \dfrac{u}{v}$ $\quad u = x^2+7 \quad v = x+3$

$\quad u' = 2x \quad v' = 1$

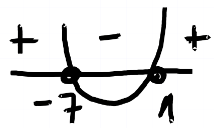
$$f'(x) = \frac{u'v - uv'}{v^2} = \frac{(2x)(x+3) - (x^2+7)(1)}{(x+3)^2} =$$

$$= \frac{2x^2+6x-x^2-7}{(x+3)^2} = \frac{x^2+6x-7}{(x+3)^2}$$

signe de f':

$x^2+6x-7 \qquad a = 1 \qquad b = 6 \qquad c = -7$

$\Delta = 36 - 4\times1\times(-7) = 36+28 = 64$



$x_1 = \dfrac{-6-8}{2} = -7 \qquad x_2 = \dfrac{-6+8}{2} = 1$

$(x+3)^2$ est positif sauf pour $x=-3$ V.I.

x	-1		1		5
f'		$-$	0	$+$	
f	$f(-1)$	↘	$f(1)$	↗	$f(5)$

$$f(-1) = \frac{(-1)^2+7}{-1+3} = \frac{8}{2} = 4$$

$$f(1) = \frac{1^2+7}{1+3} = \frac{8}{4} = 2$$

$$f(5) = \frac{5^2+7}{5+3} = \frac{32}{8} = 4$$