$$(uv)' = u'v + uv'$$

$$\int (uv)' = \int (u'v + uv')$$

$$uv = \int u'v + \int uv'$$

$$\boxed{\int uv' = uv - \int u'v}$$

Integrale par partie

$$\int x \ln(x)\, dx =$$

$$x = v' \qquad \rightsquigarrow \qquad v = x^2/2$$

$$\ln(x) = u \qquad \qquad u' = 1/x$$

$$= \ln(x) \frac{x^2}{2} - \int \frac{1}{x} \frac{x^2}{2}\, dx =$$

$$= \frac{x^2}{2} \ln(x) - \int \frac{x}{2}\, dx = \frac{x^2}{2} \ln(x) - \frac{x^2}{4} + C$$

$$\int x e^x \, dx = \qquad \int u v' = uv - \int u' v$$

$$x = u \qquad\qquad u' = 1$$

$$e^x = v' \quad \rightsquigarrow \quad v = e^x$$

$$= x e^x - \int 1 e^x \, dx = x e^x - \int e^x \, dx =$$

$$= x e^x - e^x + c$$

Ex 46

$$\int_1^e (x^2+1)\ln(x)\, dx =$$

$$v' = x^2+1 \quad \rightsquigarrow \quad v = \frac{x^3}{3} + x$$

$$u = \ln(x) \qquad\qquad u' = 1/x$$

$$= \left[\left(\frac{x^3}{3}+x\right)\ln(x)\right]_1^e - \int_1^e \frac{1}{x}\left(\frac{x^3}{3}+x\right) dx =$$

$$= \left[\left(\frac{x^3}{3}+x\right)\ln(x)\right]_1^e - \int_1^e \left(\frac{x^2}{3}+1\right) dx =$$

$$= \left[\left(\frac{x^3}{3}+x\right)\ln(x)\right]_1^e - \left[\frac{x^3}{9}+x\right]_1^e =$$

$$= \left(\frac{e^3}{3}+e\right)\ln(e) - \left(\frac{e^3}{9}+e-\frac{1}{9}-1\right) =$$

$$= \frac{e^3}{3} + \cancel{e} - \frac{e^3}{9} - \cancel{e} + \frac{1}{9} + 1 =$$

$$= \frac{3e^3 - e^3}{9} + \frac{1+9}{9} = \frac{2}{9}e^3 + \frac{10}{9}$$

$$\int_0^1 x e^{2x}\, dx = \qquad \int uv' = uv - \int u'v$$

$$x = u \qquad\qquad u' = 1$$

$$v' = e^{2x} \quad \rightsquigarrow \quad v = \frac{e^{2x}}{2}$$

$$= \left[x \frac{e^{2x}}{2} \right]_0^1 - \int_0^1 1 \frac{e^{2x}}{2}\, dx = \left[x \frac{e^{2x}}{2} \right]_0^1 - \frac{1}{2}\int_0^1 e^{2x}\, dx =$$

$$= \left[\frac{x e^{2x}}{2} \right]_0^1 - \frac{1}{2}\left[\frac{e^{2x}}{2} \right]_0^1 =$$

$$= \frac{e^2}{2} - 0 - \frac{1}{2}\left(\frac{e^2}{2} - \frac{1}{2} \right) = \frac{e^2}{2} - \frac{e^2}{4} + \frac{1}{4} = \frac{e^2}{4} + \frac{1}{4}$$