Ex 1

(E) $\quad y' - 0{,}3y = 0$

1. $y_0(x) = K e^{-\frac{(-0{,}3)}{1}x} = K e^{0{,}3x}$

2. f est solution de (E) avec $f(0) = 20$

$f(x) = K e^{0{,}3x} \Rightarrow f(0) = K e^{0} = K$

Donc $K = 20 \Rightarrow f(x) = 20 e^{0{,}3x}$

Ex 2

(E) $\quad y' - 2y = 2x + 1 \qquad [ay' + by = c(x)]$

(H) $\quad y' - 2y = 0$

1. $y_0(x) = K e^{-\frac{(-2)}{1}x} = K e^{2x}$

2. f_0 est une fonction affine $\Rightarrow f_0(x) = mx + q$

f_0 est solution de (E) $\Rightarrow f_0' - 2f_0 = 2x + 1$

$f_0' = m \Rightarrow m - 2(mx + q) = 2x + 1$

$m - 2mx - 2q = 2x + 1$

$-2mx + m - 2q = 2x + 1$

$\Rightarrow -2m = 2$ et $m - 2q = 1$

$m = -1 \qquad -1 - 2q = 1$

$q = -1$

Donc $f_0(x) = -x - 1$

3. $y_E(x) = y_\alpha(x) + f_o(x) =$

$= Ke^{2x} - x - 1$

4. $f(0) = Ke^0 - 0 - 1 = K - 1$

$f(0) = 1 \Rightarrow K - 1 = 1 \Rightarrow K = 2$

Donc $f(x) = 2e^{2x} - x - 1$

<u>Ex 3</u>

(E) $y' - 2y = -2x^2 - 2x$

(H) $y' - 2y = 0$

1. $y_0(x) = Ke^{-\frac{(-2)}{1}x} = Ke^{2x}$

2. Si $h(x)$ est solution de (E) alors

$$h' - 2h = -2x^2 - 2x$$

$h(x) = (x+1)^2 \qquad h'(x) = 2(x+1)$

$h' - 2h = 2(x+1) - 2(x+1)^2 =$

$= 2x + 2 - 2(x^2 + 2x + 1) =$

$= 2x \cancel{+2} - 2x^2 - 4x \cancel{-2} =$

$= -2x^2 - 2x \Rightarrow$ <u>Vrai</u>

Donc $h(x)$ est bien solution de (E).

3. $y_E(x) = y_0(x) + h(x) =$

$$= Ke^{2x} + (x+1)^2$$

4. f est solution de $(E) \Rightarrow f(x) = Ke^{2x} + (x+1)^2$

$f(1) = 1 \Rightarrow Ke^{2\times 1} + (1+1)^2 = 1$

$$Ke^2 + 4 = 1$$

$$Ke^2 = -3 \Rightarrow K = -\frac{3}{e^2}$$

Donc $f(x) = -\frac{3}{e^2} e^{2x} + (x+1)^2 =$

$$= -3e^{-2} e^{2x} + (x+1)^2 =$$

$$= -3e^{2(x-1)} + (x+1)^2$$

Ex 4

(E) $y' + 2y = -\frac{5}{3} e^{-3x}$

1. $y_0(x) = Ke^{-2x}$

2. Si $g(x)$ est solution de (E) alors

$$g' + 2g = -\frac{5}{3} e^{-3x}$$

$$g(x) = \frac{5}{3} e^{-3x} \Rightarrow g'(x) = \frac{5}{3} \times (-3)\, e^{-3x}$$

$$g' + 2g = -5\, e^{-3x} + \frac{10}{3} e^{-3x} =$$

$$= \frac{-15+10}{3} e^{-3x} = -\frac{5}{3} e^{-3x} \quad \underline{\text{Vrai}}$$

Donc $g(x)$ est bien solution de (E).

3. $$y_E(x) = y_0(x) + g(x) = K e^{-2x} + \frac{5}{3} e^{-3x}$$

4. $f(x)$ est solution de $(E) \Rightarrow f(x) = K e^{-2x} + \frac{5}{3} e^{-3x}$

$$f(0) = -\frac{5}{6} \Rightarrow K e^{0} + \frac{5}{3} e^{0} = -\frac{5}{6}$$

$$K = -\frac{5}{6} - \frac{5}{3} = \frac{-5-10}{6} = -\frac{15}{6}$$

Donc $f(x) = -\frac{15}{6} e^{-2x} + \frac{5}{3} e^{-3x}$

Ex 5

$$(E) \quad y' + y = 2x e^{-x}$$

1. Si $g(x)$ est solution de (E) alors

$$g' + g = 2x e^{-x} \qquad (*)$$

$$g(x) = a x^2 e^{-x} = uv \qquad u = ax^2 \quad v = e^{-x}$$

$$g'(x) = u'v + uv' = 2ax e^{-x} + ax^2(-e^{-x}) =$$
$$= 2ax e^{-x} - ax^2 e^{-x}$$

$$g' + g = 2ax e^{-x} - ax^2 e^{-x} + ax^2 e^{-x} =$$
$$= 2ax e^{-x}$$

$$(*) \Rightarrow \quad 2ax e^{-x} = 2x e^{-x}$$

Donc $a = 1$

2. $y_0(x) = K e^{-x}$

3. $y_E(x) = y_0(x) + g(x) = K e^{-x} + x^2 e^{-x}$

4. $f(x)$ est solution de $(E) \Rightarrow f(x) = K e^{-x} + x^2 e^{-x}$

$f(-1) = 2e \Rightarrow \quad Ke + e = 2e \Rightarrow K = 1$

Donc $f(x) = e^{-x} + x^2 e^{-x} = e^{-x}(x^2 + 1)$