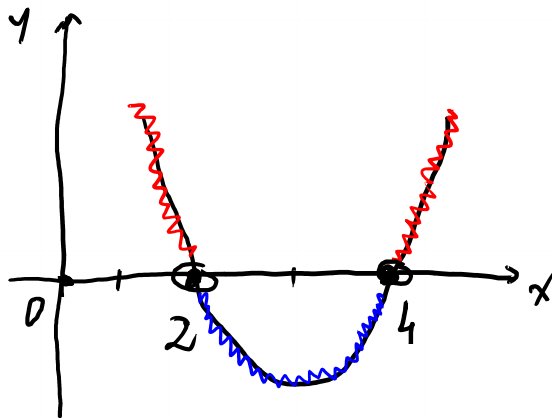


Étude de signe d'une parabole

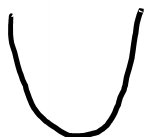
Exemple :




\Rightarrow

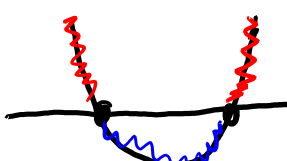
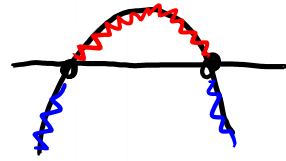
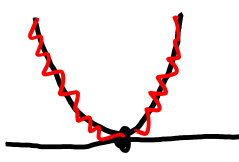
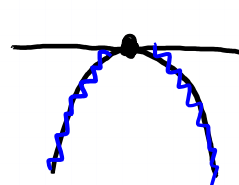
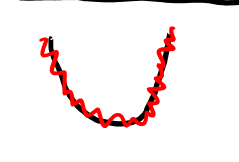
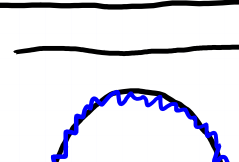
x	$-\infty$	2	4	$+\infty$	
	+	0	-	0	+

$$ax^2 + bx + c$$

1) Si $a > 0 \Rightarrow$ 

Si $a < 0 \Rightarrow$ 

2) Calcul le $\Delta = b^2 - 4ac$

	$a > 0$	$a < 0$
$\Delta > 0$		
$\Delta = 0$		
$\Delta < 0$		

3) Tableau de signe

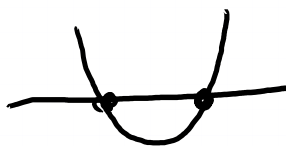
	$a > 0$			$a < 0$		
$\Delta > 0$	x	x_1	x_2	x	x_1	x_2
		+	-		-	+
$\Delta = 0$	x	x_1		x	x_1	
		+			-	
$\Delta < 0$	x			x		
		+			-	

Exemple : Étudier le signe de $2x^2 + 3x - 2$

1) $a = 2 > 0 \Rightarrow \cup$

2) $\Delta = ?$ $a = 2$ $b = 3$ $c = -2$

$$\Delta = b^2 - 4ac = 3^2 - 4 \times 2 \times (-2) = 9 + 16 = 25 > 0$$



3) $x_1 = \frac{-b - \sqrt{\Delta}}{2a} = \frac{-3 - 5}{4} = -2$ $x_2 = \frac{-b + \sqrt{\Delta}}{2a} = \frac{-3 + 5}{4} = \frac{1}{2}$

Tableau de signe:

x	$-\infty$	-2	$1/2$	$+\infty$
	+	-	+	+

Résoudre une équation du 2^{ème} degré

Exemple: $x^2 - 4x + 4 > 0$

$$a = 1 \quad b = -4 \quad c = 4$$

$$a > 0 \Rightarrow \cup$$

$$\Delta = b^2 - 4ac = (-4)^2 - 4 \times 1 \times 4 = 16 - 16 = 0$$



$$x_1 = -\frac{b}{2a} = -\frac{-4}{2} = \frac{4}{2} = 2$$

Tableau de signe

x	$-\infty$	2	$+\infty$
	+	\emptyset	+

$$S =]-\infty; 2[\cup]2; +\infty[= \mathbb{R} \setminus \{2\}$$

Exercices:

1) $4x^2 - 4x + 1 < 0$

6) $6x^2 - 5x + 1 \geq 0$

2) $x^2 + x + 3 > 0$

7) $4 - x^2 \leq 0$

3) $3x^2 + 5x - 2 \leq 0$

8) $4(x^2 - 1) < 4x - 1$

4) $4x^2 - 9 > 0$

9) $(x+5)(x^2 - 5x + 25) < 0$


5) $-x^2 + 8x - 12 > 0$

10) $(2x-1)(4x^2 + 2x + 1) \leq 0$

Correction

1) $4x^2 - 4x + 1 < 0$

$a = 4 > 0$  $b = -4$ $c = 1$

$\Delta = (-4)^2 - 4 \times 4 \times 1 = 16 - 16 = 0$ 


$x_1 = -\frac{-4}{2 \times 4} = \frac{1}{2} \Rightarrow$

x	$-\infty$	$1/2$	$+\infty$
	+	\emptyset	+

$S = \emptyset$

2) $x^2 + x + 3 > 0$

$a = 1 > 0$  $b = 1$ $c = 3$

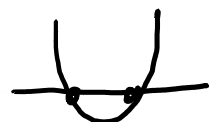
$\Delta = 1^2 - 4 \times 1 \times 3 = 1 - 12 = -11 < 0 \Rightarrow$ 

x	$-\infty$	$+\infty$
	+	

 $\Rightarrow S = \mathbb{R} =]-\infty; +\infty[$

3) $3x^2 + 5x - 2 < 0$

$a = 3 > 0$  $b = 5$ $c = -2$

$\Delta = 5^2 - 4 \times 3 \times (-2) = 25 + 24 = 49 > 0$ 

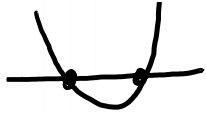
$x_1 = \frac{-5-7}{6} = -2$ $x_2 = \frac{-5+7}{6} = \frac{1}{3}$

x	$-\infty$	-2	$1/3$	$+\infty$
	+	\emptyset	-	+

$S =]-2; \frac{1}{3}[$

$$4) \quad 4x^2 - 9 > 0$$

$$a = 4 > 0 \quad \cup \quad b = 0 \quad c = -9$$

$$\Delta = 0^2 - 4 \times 4 \times (-9) = 144 > 0 \Rightarrow$$


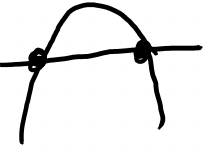
$$x_1 = \frac{0 - 12}{8} = -\frac{3}{2} \quad x_2 = \frac{0 + 12}{8} = \frac{3}{2}$$

x	$-\infty$	$-\frac{3}{2}$	$\frac{3}{2}$	$+\infty$	
	+	0	-	0	+

$$S =]-\infty; -\frac{3}{2}[\cup]\frac{3}{2}; +\infty[$$

$$5) \quad -x^2 + 8x - 12 > 0$$

$$a = -1 < 0 \quad \cap \quad b = 8 \quad c = -12$$

$$\Delta = 8^2 - 4 \times (-1) \times (-12) = 64 - 48 = 16 > 0 \Rightarrow$$



$$x_1 = \frac{-8 - 4}{-2} = 6 \quad x_2 = \frac{-8 + 4}{-2} = 2$$

x	$-\infty$	2	6	$+\infty$	
	-	0	+	0	-

$$S =]2; 6[$$

$$6) \quad 6x^2 - 5x + 1 \geq 0$$

$$a = 6 > 0 \quad \cup \quad b = -5 \quad c = 1$$

$$\Delta = (-5)^2 - 4 \times 6 \times 1 = 25 - 24 = 1 > 0 \Rightarrow$$


$$x_1 = \frac{5 - 1}{12} = \frac{1}{3} \quad x_2 = \frac{5 + 1}{12} = \frac{1}{2}$$

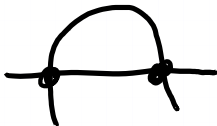
x	$-\infty$	$\frac{1}{3}$	$\frac{1}{2}$	$+\infty$	
	+	0	-	0	+

$$S =]-\infty; \frac{1}{3}] \cup [\frac{1}{2}; +\infty[$$

$$7) 4 - x^2 \leq 0$$

$$-x^2 + 4 \leq 0$$

$$a = -1 < 0 \quad \cap \quad b = 0 \quad c = 4$$

$$\Delta = 0^2 - 4 \times (-1) \times 4 = 16 > 0 \Rightarrow$$


$$x_1 = \frac{0-4}{-2} = 2 \quad x_2 = \frac{0+4}{-2} = -2$$

x	$-\infty$	-2	2	$+\infty$
	-	0	+	-

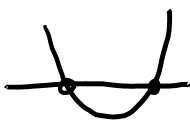
$$S =]-\infty; -2] \cup [2; +\infty[$$

$$8) 4(x^2 - 1) < 4x - 1$$

$$4x^2 - 4 < 4x - 1$$

$$4x^2 - 4x - 3 < 0$$

$$a = 4 > 0 \quad \cup \quad b = -4 \quad c = -3$$

$$\Delta = (-4)^2 - 4 \times 4 \times (-3) = 16 + 48 = 64 > 0 \Rightarrow$$


$$x_1 = \frac{4-8}{8} = -\frac{1}{2} \quad x_2 = \frac{4+8}{8} = \frac{3}{2}$$

x	$-\infty$	$-\frac{1}{2}$	$\frac{3}{2}$	$+\infty$
	+	0	-	+

$$S =]-\frac{1}{2}; \frac{3}{2}[$$

$$9) (x+5)(x^2-5x+25) \leq 0$$

$$\begin{array}{l|l} x+5 > 0 \quad \textcircled{+} & x^2-5x+25 \quad a=1 > 0 \cup b=-5 \quad c=25 \\ x > -5 \quad \downarrow & \\ x > -5 \quad \text{à droite} & \Delta = (-5)^2 - 4 \times 1 \times 25 = -75 < 0 \quad \underline{\cup} \\ \text{de } -5 & \end{array}$$

Tableau de
signe:

x	$-\infty$	-5	$+\infty$
$x+5$	-	\emptyset	+
$x^2-5x+25$	+		
Produit	-	\emptyset	+

$$S =]-\infty; -5[$$

$$10) (2x-1)(4x^2+2x+1) \leq 0$$

$$\begin{array}{l|l} 2x-1 > 0 & 4x^2+2x+1 \quad a=4 > 0 \cup b=2 \quad c=1 \\ x > \frac{1}{2} & \\ & \Delta = 2^2 - 4 \times 4 \times 1 = -12 < 0 \quad \underline{\cup} \end{array}$$

Tableau de
signe:

x	$-\infty$	$\frac{1}{2}$	$+\infty$
$2x-1$	-	\emptyset	+
$4x^2+2x+1$	+		
Produit	-	\emptyset	+

$$S =]-\infty; \frac{1}{2}]$$