Ex 25

$$\int_{-1}^{1} (x^2+1)\,dx = \left[\frac{x^3}{3} + x\right]_{-1}^{1} =$$

$$= \left(\frac{1}{3} + 1\right) - \left(\frac{(-1)^3}{3} + (-1)\right) =$$

$$= \frac{1+3}{3} - \left(-\frac{1}{3} - 1\right) =$$

$$= \frac{4}{3} - \left(\frac{-1-3}{3}\right) = \frac{4}{3} - \left(-\frac{4}{3}\right) = \frac{8}{3}$$

$$\int_{-1}^{1} (x^2+3x+5)\,dx = \left[\frac{x^3}{3} + \frac{3x^2}{2} + 5x\right]_{-1}^{1} =$$

$$= \left(\frac{1}{3} + \frac{3}{2} + 5\right) - \left(\frac{(-1)^3}{3} + \frac{3(-1)^2}{2} + 5(-1)\right) =$$

$$= \frac{1\times 2 + 3\times 3 + 5\times 6}{6} - \left(-\frac{1}{3} + \frac{3}{2} - 5\right) =$$

$$= \frac{2+9+30}{6} - \left(\frac{-1\times 2 + 3\times 3 - 5\times 6}{6}\right) =$$

$$= \frac{41}{6} - \left(\frac{-2+9-30}{6}\right) = \frac{41}{6} - \left(-\frac{23}{6}\right) = \frac{64}{6} = \frac{32}{3}$$

Ex 26

$$\int_1^4 \frac{3}{x}\,dx = \left[3\ln(x)\right]_1^4 = 3\ln(4) - 3\ln(1) = 3\ln(4)$$

$$\int_1^4 \left(x - \frac{2}{x}\right)dx = \left[\frac{x^2}{2} - 2\ln(x)\right]_1^4 =$$

$$= \left(\frac{16}{2} - 2\ln(4)\right) - \left(\frac{1}{2} - 2\ln(1)\right) =$$

$$= 8 - 2\ln(4) - \frac{1}{2} = \frac{16-1}{2} - 2\ln(4) =$$

$$= \frac{15}{2} - 2\ln(4)$$

Ex 27

$$\int_0^1 \left(x + 2 + \frac{1}{x+2}\right)dx = \left[\frac{x^2}{2} + 2x + \ln(x+2)\right]_0^1 =$$

$$\downarrow$$

$$\frac{u'}{u} = \frac{1}{x+2} \qquad \int \frac{u'}{u} = \ln(u)$$

$$= \left(\frac{1}{2} + 2 + \ln(3)\right) - \left(0 + 0 + \ln(2)\right) =$$

$$= \frac{1+4}{2} + \ln(3) - \ln(2) = \frac{5}{2} + \ln\left(\frac{3}{2}\right)$$

Ex 28

$$\int_0^1 \frac{t}{t^2+1}\,dt = \qquad \left(\frac{u'}{u} \qquad u = t^2+1 \qquad u' = 2t \right)$$

$$= \frac{1}{2}\int_0^1 \frac{2t}{t^2+1}\,dt = \left[\frac{1}{2}\ln(t^2+1) \right]_0^1 =$$

$$= \frac{1}{2}\ln(2) - \frac{1}{2}\ln(1) = \frac{1}{2}\ln(2)$$

$$\int_0^{\ln(2)} \left(e^t + e^{2t}\right) dt =$$

$$\hookrightarrow \ldots e^u \rightsquigarrow u'e^u \qquad u = 2t \quad u' = 2$$

$$= \int_0^{\ln(2)} \left(e^t + \frac{2e^{2t}}{2} \right) dt = \int_0^{\ln(2)} \left(e^t + \frac{1}{2}\,2e^{2t} \right) dt =$$

$$= \left[e^t + \frac{1}{2}e^{2t} \right]_0^{\ln(2)} = \left(e^{\ln(2)} + \frac{1}{2}e^{2\ln(2)} \right) - \left(1 + \frac{1}{2} \right) =$$

$$= 2 + \frac{1}{2}e^{\ln(2^2)} - \frac{3}{2} = 2 + \frac{1}{2}e^{\ln(4)} - \frac{3}{2} =$$

$$= 2 + \frac{4}{2} - \frac{3}{2} = 2 + 2 - \frac{3}{2} = 4 - \frac{3}{2} = \frac{5}{2}$$

Ex 29

$$\int_{-1}^{2} x(x^2+4)\,dx = \int_{-1}^{2} (x^3+4x)\,dx = \left[\frac{x^4}{4} + 4\frac{x^2}{2}\right]_{-1}^{2} =$$

$$= \left(\frac{2^4}{4} + 2\times 2^2\right) - \left(\frac{(-1)^4}{4} + 2(-1)^2\right) =$$

$$= 4 + 8 - \left(\frac{1}{4} + 2\right) = 12 - \frac{9}{4} =$$

$$= \frac{48-9}{4} = \frac{39}{4}$$

$$\int_{-1}^{2} x(x^2+4)\,dx = \rightsquigarrow \quad u'u \qquad u = x^2+4$$

$$u' = 2x$$

$$= \frac{1}{2}\int_{-1}^{2} 2x(x^2+4)\,dx = \qquad \int u'u^n = \frac{u^{n+1}}{n+1}$$

$$= \frac{1}{2}\left[\frac{(x^2+4)^2}{2}\right]_{-1}^{2} = \frac{1}{2}\left(\frac{(2^2+4)^2}{2}\right) - \frac{1}{2}\left(\frac{(1+4)^2}{2}\right) =$$

$$= \frac{1}{4}64 - \frac{1}{4}25 = \frac{64-25}{4} = \frac{39}{4}$$

$$\int_{1}^{2} \frac{xe^x+1}{x}\,dx = \int_{1}^{2}\left(\frac{xe^x}{x} + \frac{1}{x}\right)dx = \int_{1}^{2}\left(e^x + \frac{1}{x}\right)dx =$$

$$= \left[e^x + \ln(x)\right]_{1}^{2} = \left(e^2 + \ln(2)\right) - \left(e^1 + \ln(1)\right) =$$

$$= e^2 + \ln(2) - e$$

Ex 30

$$\int_0^{\ln(2)} (e^x - e^{-x})\,dx = \int_0^{\ln(2)} (e^x + \underbrace{(-1)e^{-x}}_{u'e^u})\,dx =$$

avec $u = -x$
$u' = -1$

$$= \left[e^x + e^{-x} \right]_0^{\ln(2)} =$$

$$= \left(e^{\ln(2)} + e^{-\ln(2)} \right) - (1 + 1) =$$

$$= \left(2 + \frac{1}{e^{\ln(2)}} \right) - 2 = \cancel{2} + \frac{1}{2} - \cancel{2} = \frac{1}{2}$$

$$\int_e^{e^2} \frac{1}{x \ln(x)}\,dx = \int_e^{e^2} \underset{\substack{\downarrow \\ u'}}{\frac{1}{x}}\ \underset{\substack{\downarrow \\ 1/u}}{\frac{1}{\ln(x)}}\,dx =$$

avec $u = \ln(x)$

$$\int \frac{u'}{u} = \ln(u)$$

$$= \int_e^{e^2} \frac{1/x}{\ln(x)}\,dx = \left[\ln(\ln(x)) \right]_e^{e^2} =$$

$$= \ln(\ln(e^2)) - \ln(\ln(e)) =$$

$$= \ln(2) - \ln(1) = \ln(2)$$

Ex 32

$$\int \left(x^2 + 2e^{-2x}\right) dx = \int \left(x^2 + 2\frac{(-2)}{-2}e^{-2x}\right) dx =$$

e^u $\quad u = -2x \rightsquigarrow u'e^u$

$u' = -2$

$$= \int \left(x^2 - (-2)e^{-2x}\right) dx = \frac{x^3}{3} - e^{-2x} + C$$

Ex 33

$$\int \frac{x^2}{x+1} dx = \int \frac{x^2 - 1 + 1}{x+1} dx = \int \left(\frac{x^2-1}{x+1} + \frac{1}{x+1}\right) dx =$$

$$= \int \left(\frac{\cancel{(x+1)}(x-1)}{\cancel{(x+1)}} + \frac{1}{x+1}\right) dx =$$

$$= \int \left(x - 1 + \frac{1}{x+1}\right) dx = \frac{x^2}{2} - x + \ln(x+1) + C$$

Ex 34

$$\int x e^{x^2+1} dx = \qquad e^u \text{ avec } u = x^2+1 \quad u' = 2x$$

$$= \frac{1}{2}\int 2x\, e^{x^2+1} dx = \frac{1}{2} e^{x^2+1} + C$$

Ex 35

$$\int \frac{x^2}{2x-1}\,dx = \qquad \left[(2x-1)(2x+1) = 4x^2-1\right]$$

$$= \frac{1}{4}\int \frac{4x^2}{2x-1}\,dx = \frac{1}{4}\int \frac{4x^2-1+1}{2x-1}\,dx =$$

$$= \frac{1}{4}\int \left(\frac{4x^2-1}{2x-1} + \frac{1}{2x-1}\right)dx =$$

$$= \frac{1}{4}\int \left(\frac{(2x+1)\cancel{(2x-1)}}{\cancel{2x-1}} + \frac{1}{2x-1}\right)dx =$$

$$= \frac{1}{4}\int \left(2x+1 + \frac{1}{2x-1}\right)dx =$$

$\rightarrow \frac{1}{u}$ avec $u = 2x-1 \quad u' = 2$

$\rightarrow u'/u$

$$= \frac{1}{4}\int \left(2x+1 + \frac{1}{2}\,\frac{2}{2x-1}\right)dx =$$

$$= \frac{1}{4}\left(2\,\frac{x^2}{2} + x + \frac{1}{2}\ln(2x-1)\right) + C$$

$$= \frac{x^2}{4} + \frac{x}{4} + \frac{1}{8}\ln(2x-1) + C$$

Ex 36

$$\int\left(x+3-\frac{4}{x^3}\right)dx = \int\left(x+3-4x^{-3}\right)dx =$$

$$= \frac{x^2}{2}+3x-4\,\frac{x^{-3+1}}{-3+1}+C =$$

$$= \frac{x^2}{2}+3x-4\,\frac{x^{-2}}{-2}+C = \frac{x^2}{2}+3x+\frac{2}{x^2}+C$$

Ex 37

$$\int x(x^2+1)^2\,dx = \qquad u'u^2 \quad u=x^2+1 \quad u'=2x$$

$$= \frac{1}{2}\int 2x\,(x^2+1)^2\,dx = \frac{1}{2}\,\frac{(x^2+1)^3}{3}+C = \frac{(x^2+1)^3}{6}+C$$

Ex 38

$$\int \frac{2e^x}{e^x+1}\,dx = \qquad \frac{u'}{u} \quad u=e^x+1 \quad u'=e^x$$

$$= 2\int\frac{e^x}{e^x+1}\,dx = 2\ln\left(e^x+1\right)+C$$