Ex 2

Tangente

Pour les fonctions suivantes déterminer une équation de la tangente à la courbe $\mathscr{C}_f$ au point d'abscisse a.

1) $f(x) = -x^2 + 2x - 8\,;\quad a = -2$

2) $f(x) = \dfrac{x+3}{1-2x}\,;\quad a = -1$

3) $f(x) = x^2 + 1 - \dfrac{1}{x^2+1}\,;\quad a = 1$

1) $f'(x) = -2x + 2$

$f'(-2) = -2\times(-2) + 2 = 6$

$f(-2) = -(-2)^2 + 2\times(-2) - 8 = -4 - 4 - 8 = -16$

Équation de la tangente:

$$y = f'(-2)\left(x - (-2)\right) + f(-2)$$

$$y = 6(x+2) - 16 = 6x + 12 - 16 = 6x - 4$$

$$\boxed{y = 6x - 4}$$

2) $f(x) = \dfrac{u}{v}$ $\quad u = x + 3 \quad u' = 1$

$v = 1 - 2x \quad v' = -2$

$$f'(x) = \frac{u'v - uv'}{v^2} = \frac{1(1-2x) - (x+3)(-2)}{(1-2x)^2} =$$

$$= \frac{1 - 2x - (-2x - 6)}{(1-2x)^2} = \frac{1 - 2x + 2x + 6}{(1-2x)^2} =$$

$$= \frac{7}{(1-2x)^2}$$

$$f'(-1) = \frac{7}{(1-2\times(-1))^2} = \frac{7}{(1+2)^2} = \frac{7}{9}$$

$$f(-1) = \frac{-1+3}{1-2\times(-1)} = \frac{2}{1+2} = \frac{2}{3}$$

Équation tangente:

$$y = f'(-1)(x-(-1)) + f(-1)$$

$$y = \frac{7}{9}(x+1) + \frac{2}{3} = \frac{7}{9}x + \frac{7}{9} + \frac{2}{3} =$$

$$= \frac{7}{9}x + \frac{7+6}{9} = \frac{7}{9}x + \frac{13}{9}$$

$$\boxed{y = \frac{7}{9}x + \frac{13}{9}}$$

3) $f(x) = x^2 + 1 - \frac{1}{u}$ $\qquad u = x^2+1 \qquad u' = 2x$

$$f'(x) = 2x - \left(-\frac{u'}{u^2}\right) = 2x + \frac{2x}{(x^2+1)^2}$$

$$f'(1) = 2 + \frac{2}{4} = 2 + \frac{1}{2} = \frac{5}{2}$$

$$f(1) = 1 + 1 - \frac{1}{2} = 2 - \frac{1}{2} = \frac{3}{2}$$

Équation tangente :

$$y = f'(1)(x-1) + f(1)$$

$$y = \frac{5}{2}(x-1) + \frac{3}{2} = \frac{5}{2}x - \frac{5}{2} + \frac{3}{2} = \frac{5}{2}x - \frac{2}{2} = \frac{5}{2}x - 1$$

$$\boxed{y = \frac{5}{2}x - 1}$$