# MTMW14

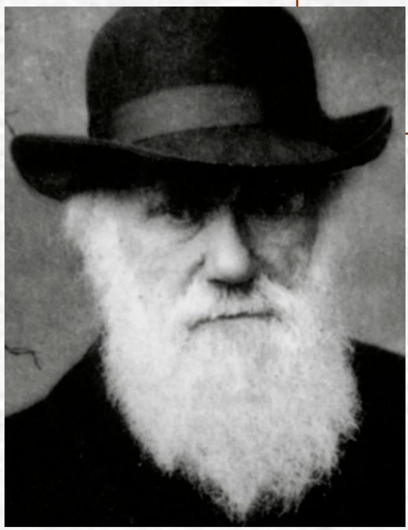
The University of Reading

# Modelling the Atmosphere and Oceans

## Lecture 3

## **Chaos and Predictability**,

## or "Natura Facit Saltus"

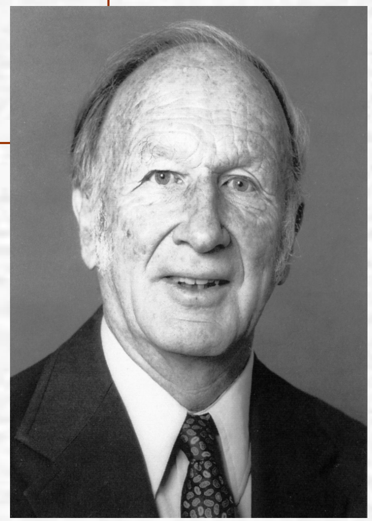


Charles Darwin (1809–1882 )


PL Vidale


*Based in part on John Methven's 2013 lecture*

Spring term 2023



Ed Lorenz (1917–2008 )

# Module summary

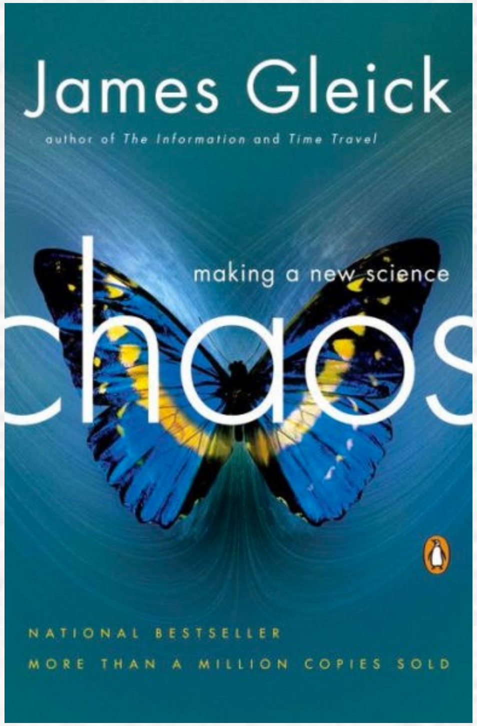
1. Introduction
2. Finite difference methods: time schemes
3. Chaos and Predictability (using complex models)
4. Fluids on rotating Earth and 2-D finite difference schemes
5. Wave dispersion in finite difference models

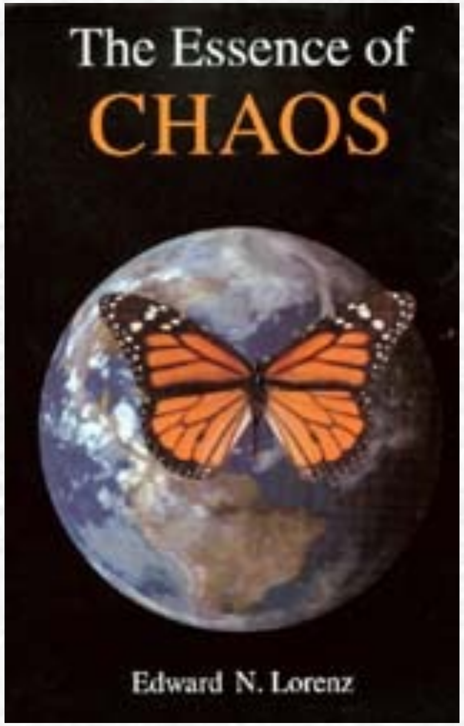
6. Numerical methods for transport by the flow
7. More methods - discrete representation of spatial distributions
8. Parameterisation of sub-grid scale turbulence and convection

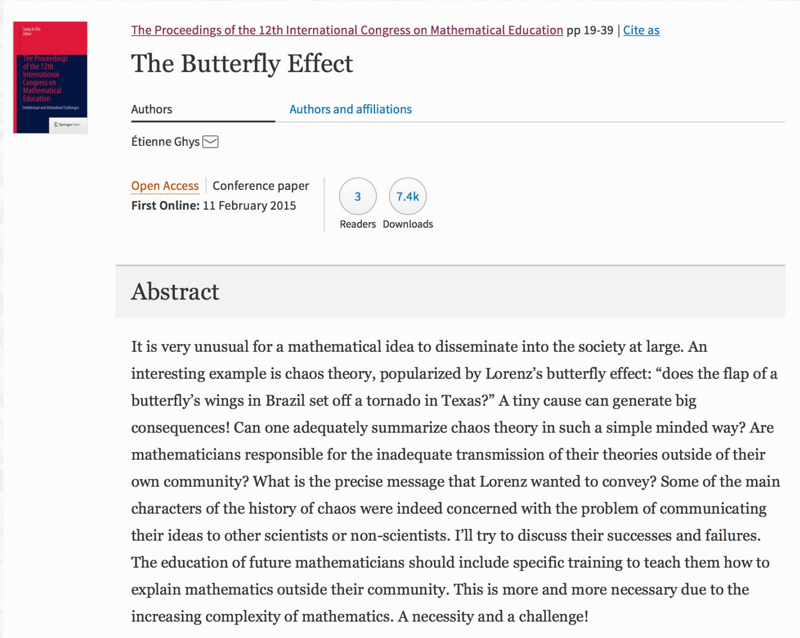
# Summary

- Uncertainty in model simulations
- Predictability and chaos
- Ensemble prediction
  - Initialisation of models and analyses
  - Ensemble definition
- Evaluation of models (with data)
  - Forecast verification
  - Improving models

# Suggested readings





The Proceedings of the 12th International Congress on Mathematical Education pp 19-39 | Cite as

## The Butterfly Effect

Authors | Authors and affiliations

Étienne Ghys

Open Access | Conference paper
First Online: 11 February 2015

3 Readers 7.4k Downloads

### Abstract

It is very unusual for a mathematical idea to disseminate into the society at large. An interesting example is chaos theory, popularized by Lorenz's butterfly effect: "does the flap of a butterfly's wings in Brazil set off a tornado in Texas?" A tiny cause can generate big consequences! Can one adequately summarize chaos theory in such a simple minded way? Are mathematicians responsible for the inadequate transmission of their theories outside of their own community? What is the precise message that Lorenz wanted to convey? Some of the main characters of the history of chaos were indeed concerned with the problem of communicating their ideas to other scientists or non-scientists. I'll try to discuss their successes and failures. The education of future mathematicians should include specific training to teach them how to explain mathematics outside their community. This is more and more necessary due to the increasing complexity of mathematics. A necessity and a challenge!

Suggested videos: https://www.youtube.com/watch?v=CeCePH_HL0g
https://www.youtube.com/watch?v=w-IHJbzRVVU

# We are culturally pre-conditioned to reject chaos theory and its implications

## *Natura non facit saltus*

Latin for »Nature does not make jumps»

Originallly in Gottfried Leibniz (*New Essays*, IV, 16:[2] *"la nature ne fait jamais des sauts"*, "nature does not make jumps")

Charles Darwin, 1879, writing about the “abominable mystery” of the sudden emergence of angiosperms on Earth.

- What worried Darwin was that the very earliest samples in the fossil record all dated back to the middle of the Cretaceous period, around 100 million years ago, and they came in a bewilderingly wide variety of shapes and sizes. This suggested **flowering plants (angiosperms) had experienced an explosive burst of diversity very shortly after their origins** – which, if true, threatened to undermine Darwin's entire model of gradual evolution through natural selection.
- In fact recently published research has revealed that angiosperms evolved relatively gradually after all. Yet this still leaves a number of key questions. The roughly 350,000 known species of flowering plants make up about 90% of all living plant species. Without them, we would have none of our major crops including those used to feed livestock, and one of the most important carbon sinks that mop up our carbon dioxide emissions would be missing.

# From determinism to chaos

We saw in previous lectures that, following Newtonian determinism, and given a theory, we could use a differential equation of this kind:

$$\frac{dq}{dt} = f\left[q(t),\ t\right].$$

This is all we need to predict the future… **subject to:**

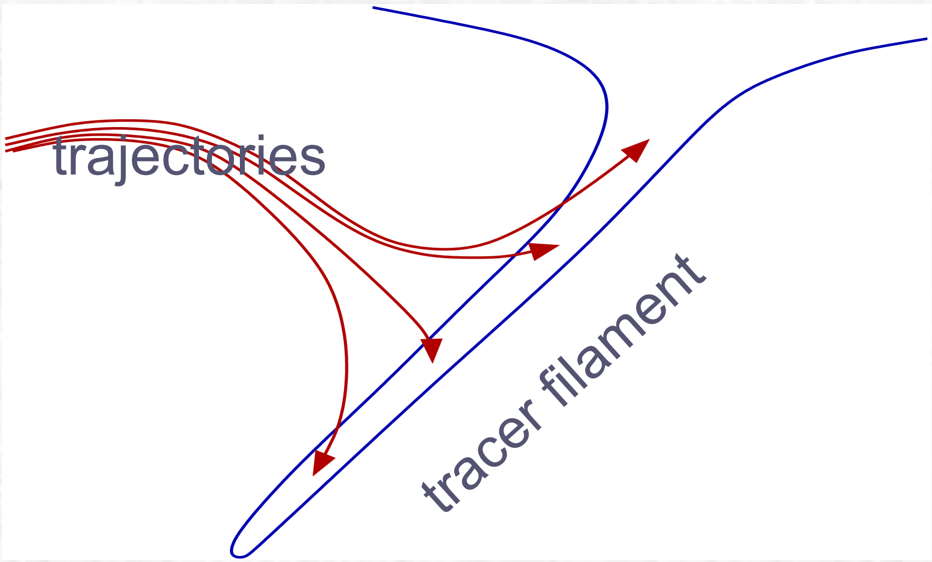
1. **boundary conditions** and
2. **initial conditions**.

(A modeller's) life is good and easy… leaving aside poor numerical methods.

# Chaos in fluid flows

Smoothly varying flow results in *apparently random* trajectories.

Chaotic systems exhibit **sensitivity to initial conditions –** *neighbouring air parcels separate at exponential rate on average.*



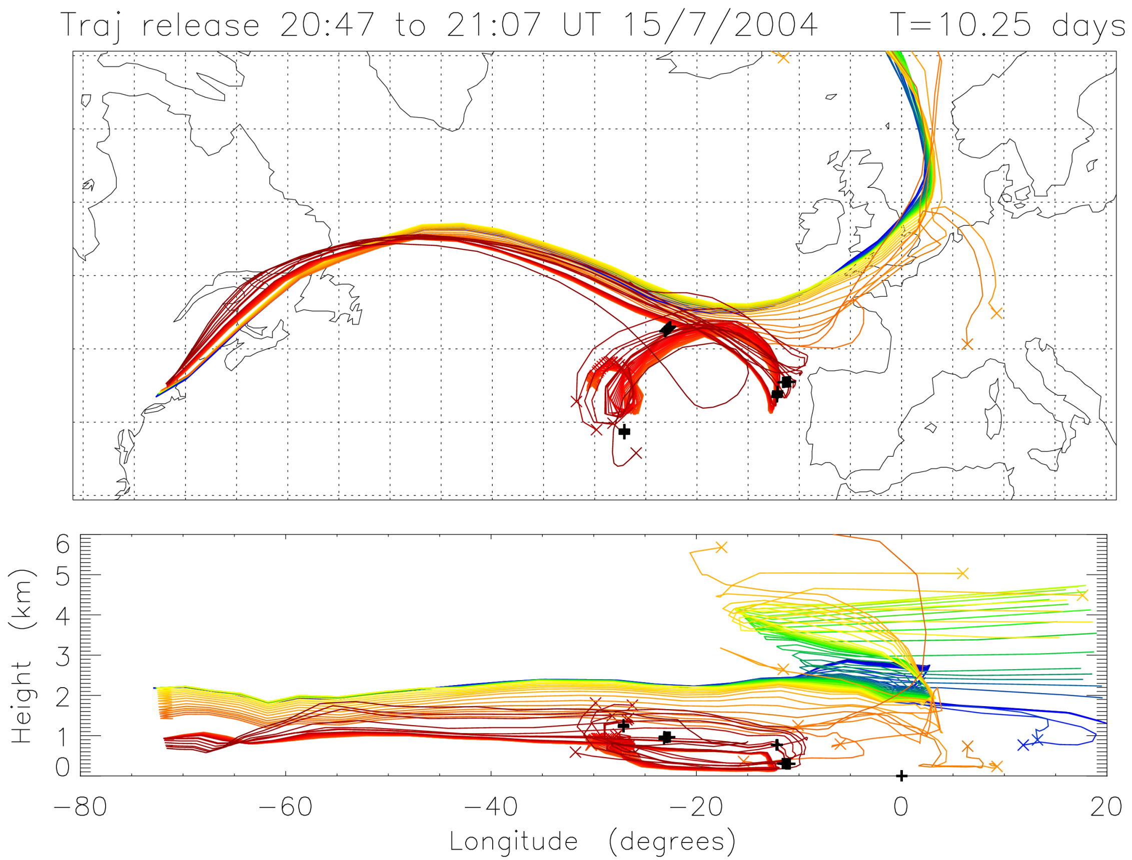

Trajectories separate rapidly forming tracer filaments

**Chaotic advection** $\Rightarrow$ tracer "cascades" to small scales.

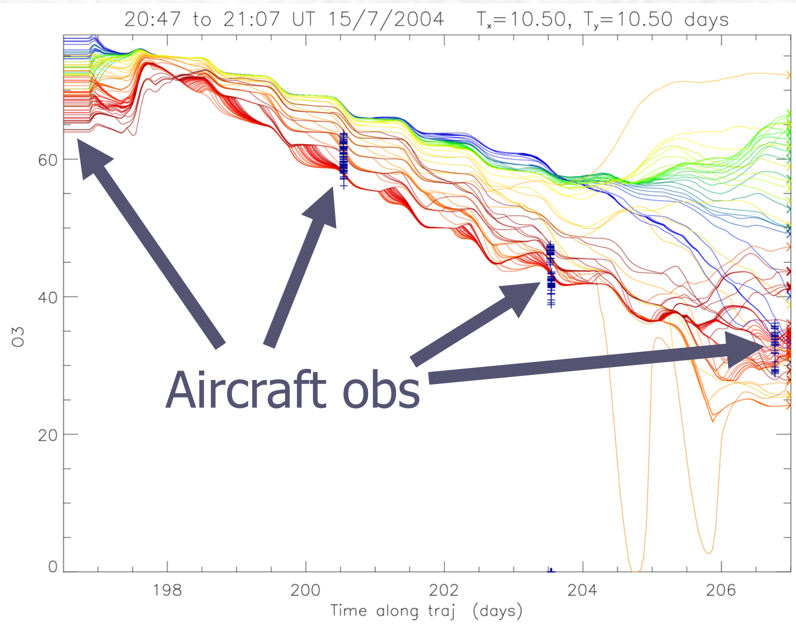
Occurs even though winds are dominated by large scales.

# Illustration: an ensemble of air-mass trajectories

Simulation of photochemical evolution of a polluted air mass.

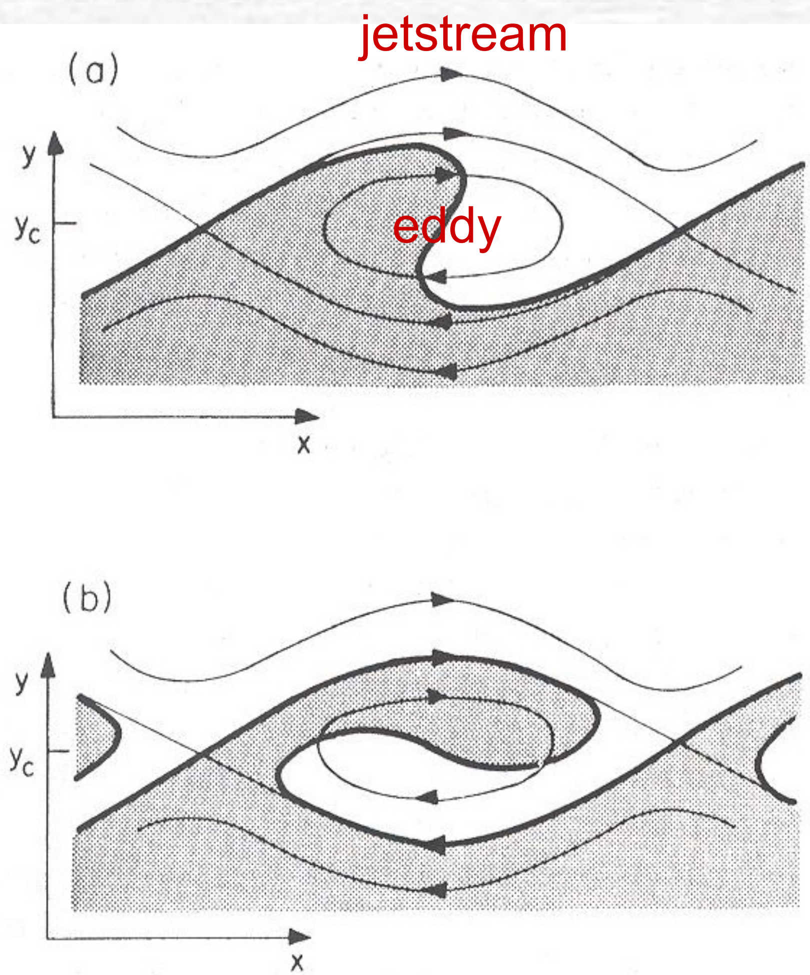



Ozone concentration (ppbv) following trajectory ensemble




Air parcel trajectories, starting from nearby locations, calculated using analysed winds

# A road to chaos - Rossby wave breaking




Streamfunction has *cat's eye* pattern.

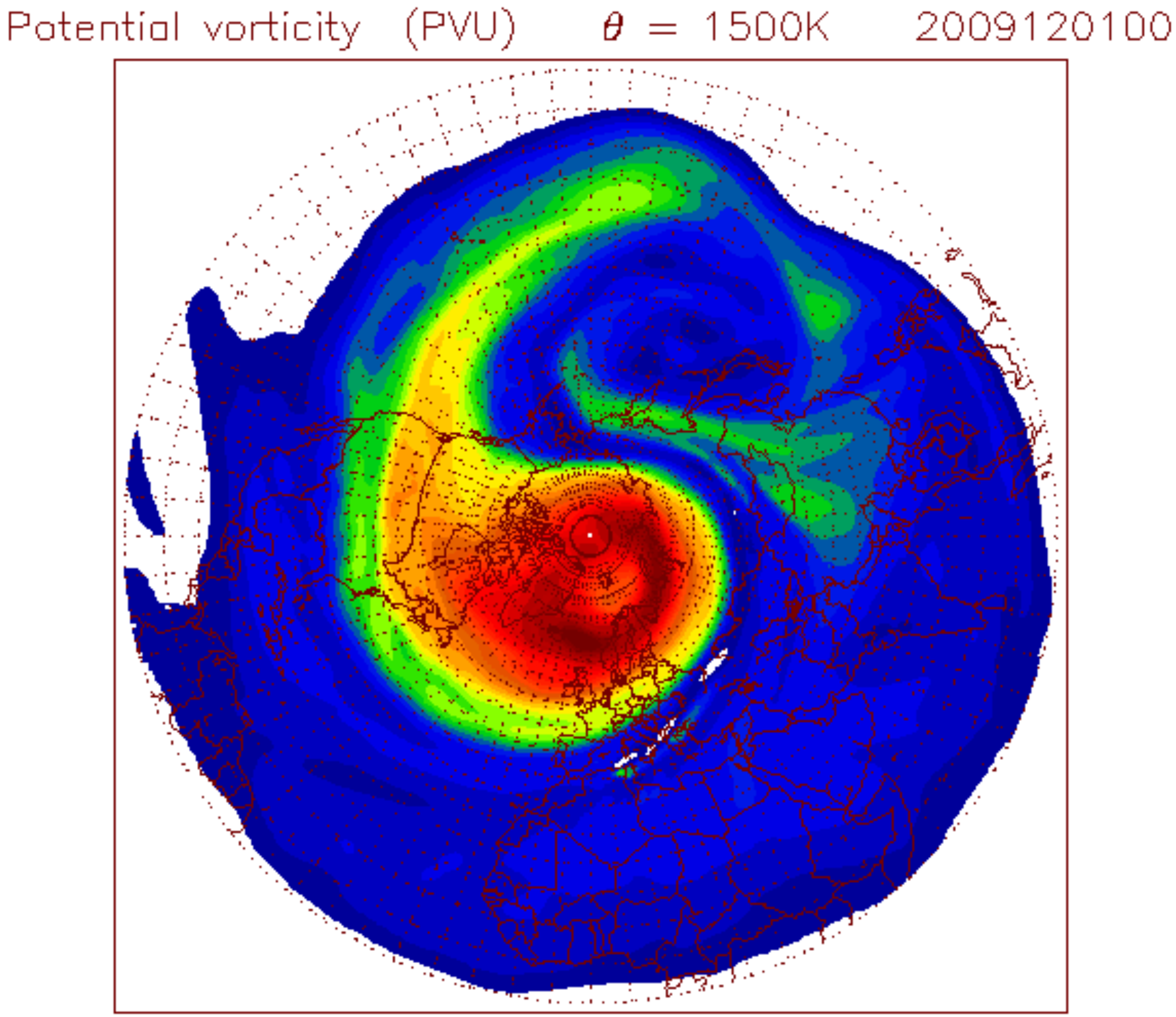
If exactly steady, air would be trapped inside eddies or jetstreams.

But, winds unsteady so trajectories can cross between regions.

Separation of trajectories $\Rightarrow$ **chaos**

From Andrews, Holton and Leovy [1987]
*Middle Atmosphere Dynamics*

# Potential vorticity – a dynamical tracer




PV stretching and folding is associated with Rossby wave breaking in the "surf zone" surrounding the stratospheric polar vortex.

Coherent structures in flow are clearly large-scale.

Evolution in PV is the "slow balanced component"

# Types of uncertainty in simulations

Initial condition uncertainty

Sensitivity to initial conditions
⇒ **chaos**

(e.g., air parcel position and initial composition)

Boundary condition uncertainty

Partly determined by factors **external** to model system, including forcing

(e.g., variation in photon flux affecting ozone photolysis)

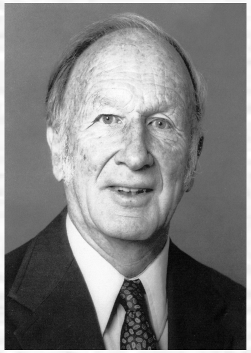
Model uncertainty

Structural

(e.g., number of species and reactions to include – here 90 species used)

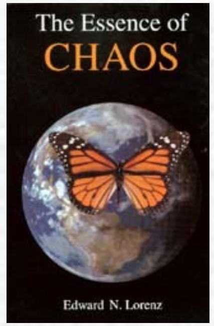
Parametric

(e.g., deposition rate coefficient for loss of ozone to surface)



From Wikipedia

# Ed Lorenz tells his tale




In 1961, Lorenz was using a simple digital computer, a Royal McBee LGP-30, to simulate weather patterns by modeling 12 variables, representing things like temperature and wind speed.

He wanted to see a sequence of data again, and, to save time, he started the simulation in the middle of its course. He did this by **entering a printout of the data that corresponded to conditions in the middle of the original simulation**. To his surprise, the weather that the machine began to predict was completely different from the previous calculation.

The culprit: **a rounded decimal number** on the computer printout. The computer worked with 6-digit precision, but the printout rounded variables off to a 3-digit number, so a value like **0.506127 printed as 0.506. This difference is tiny**, and the consensus at the time would have been that it should have no practical effect.

He states in his 1963 paper "Deterministic Nonperiodic Flow" in JAS:

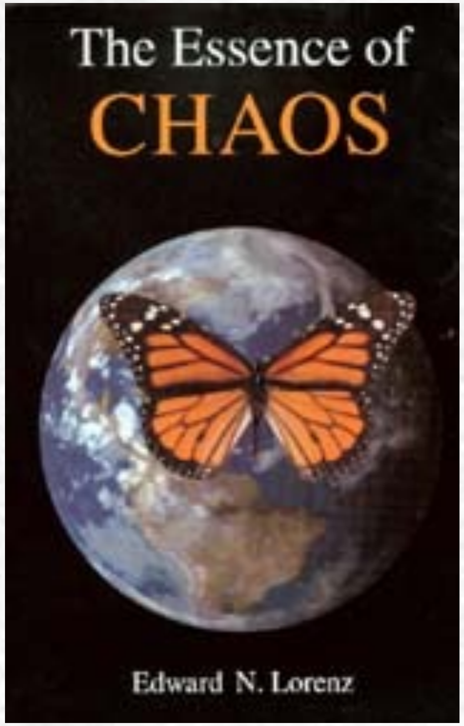
"Two states differing by imperceptible amounts may eventually evolve into two considerably different states ... If, then, there is **any error whatever in observing the present state**—and in any real system such errors seem inevitable—an acceptable prediction of an instantaneous state in the distant future may well be impossible....In view of the inevitable inaccuracy and incompleteness of weather observations, **precise very-long-range forecasting would seem to be nonexistent.**"

His description of the butterfly effect, the idea that small changes can have large consequences, followed in 1969.

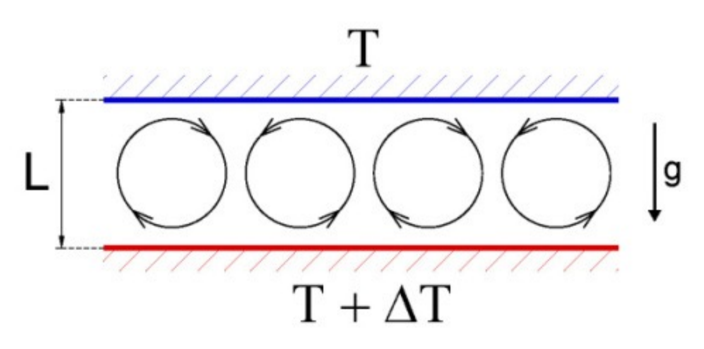
# From Lorenz's own narrative

I started the computer again and went out for a cup of coffee. When I returned about an hour later, after the computer had generated about two months of data, I found that the new solution did not agree with the original one. [...]

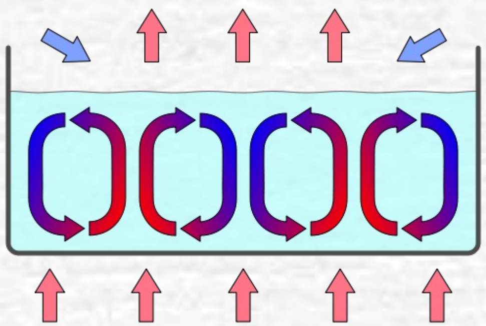
I realized that if the real atmosphere behaved in the same manner as the model, **long-range weather prediction would be impossible**, since most real weather elements were certainly not measured accurately to three decimal places.



# A simplified Lorenz model




Warm low density fluid rises
Cool high density fluid sinks



Fluid cools by loosing
heat from the surface

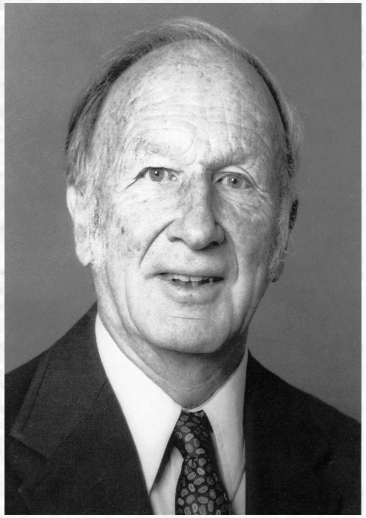
"… one flap of a sea-gull's wing may forever change the future course of the weather" (Lorenz, 1963)

System with 3 variables (*X*, *Y*, *Z*).

Evolution described by 3 simple ODEs.

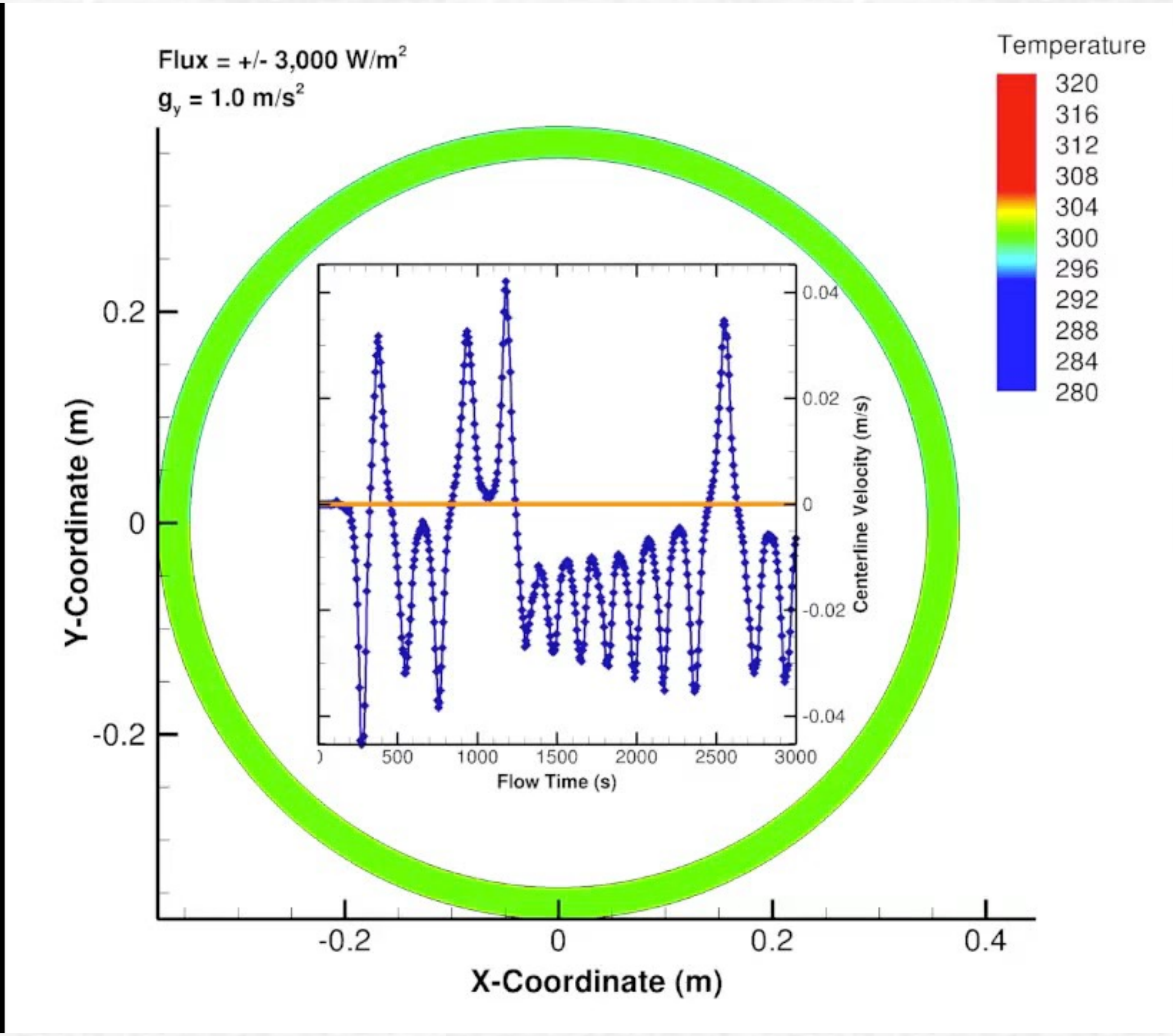
Has **nonlinear** terms.

$$\frac{dX}{dt} = -\sigma(X - Y)$$
$$\frac{dY}{dt} = \rho X - Y - XZ$$
$$\frac{dZ}{dt} = XY - \beta Z$$



Derived from highly simplified convective dynamics.

# ANIMATION of a physical chaotic pendulum




NATURA FACIT SALTUS

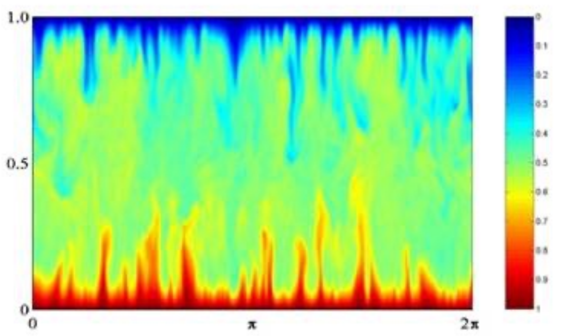
# Terms in Lorenz's equations

$$\frac{dX}{dt} = -\sigma(X - Y)$$
$$\frac{dY}{dt} = \rho X - Y - XZ$$
$$\frac{dZ}{dt} = XY - \beta Z$$

**X** is proportional to **convective intensity**

**Y** is proportional to **temperature difference** between ascending and descending currents

**Z** is the **difference** in **vertical temperature profile** from linearity



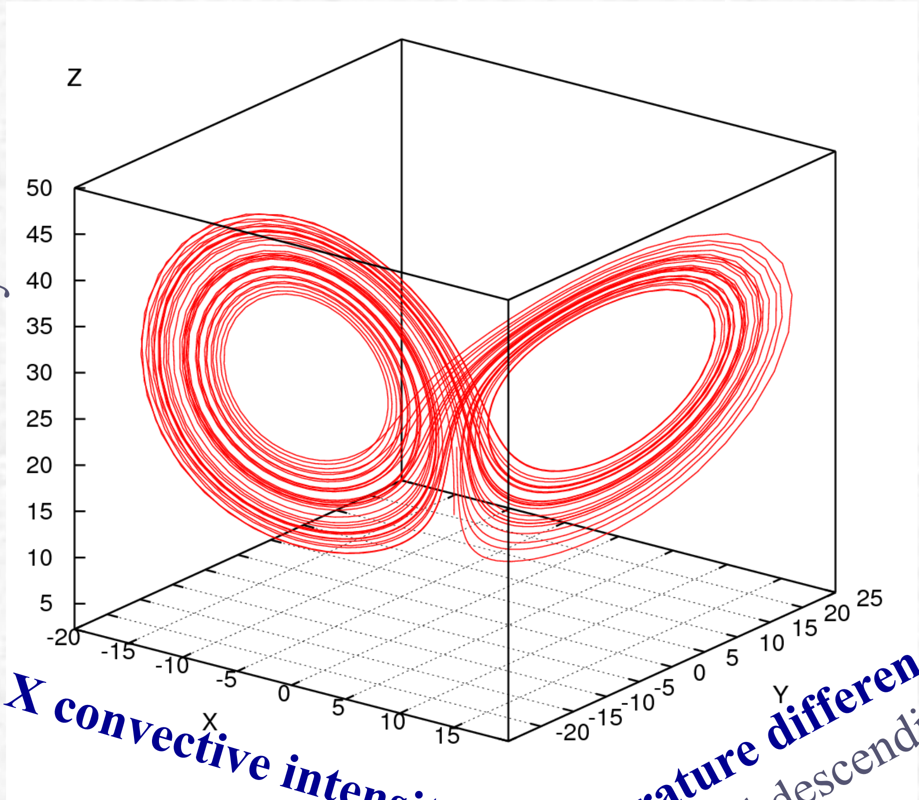

$\sigma$ is the **Prandtl number**: ratio of momentum diffusivity (Kinematic viscosity) and thermal diffusivity.

$\rho$ is the **Rayleigh number**: determines whether the heat transfer is primarily in the form of conduction or convection.

$\beta$ is a geometric factor

# Chaos discovered using Lorenz (1963) model

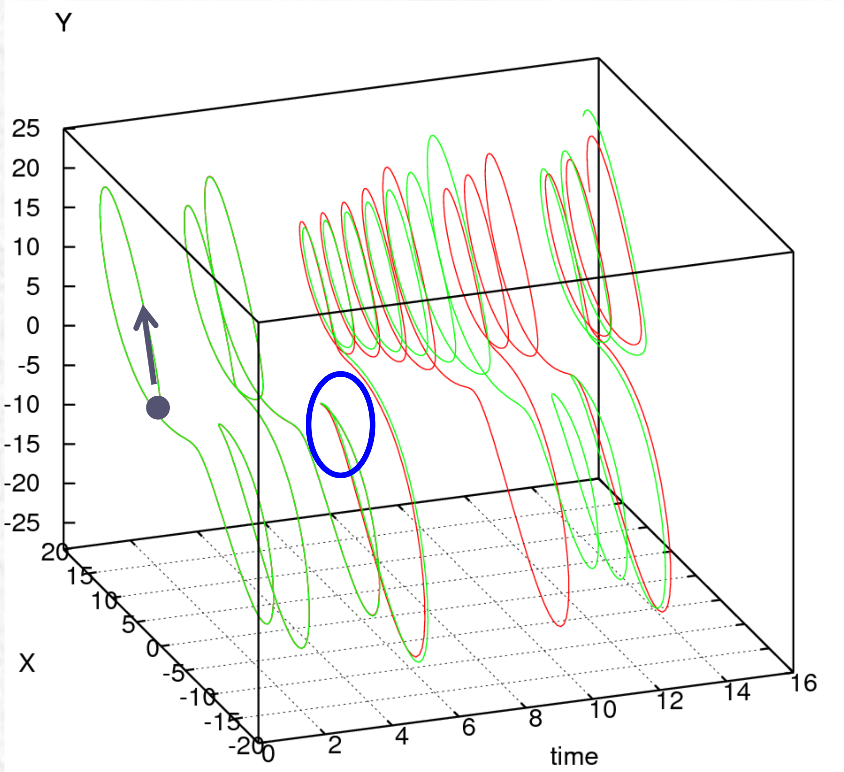
Trajectories in "phase space" are similar forming a *strange attractor* with 2 wings – the Lorenz "butterfly"




**Z difference** in **vertical temperature profile** from linearity

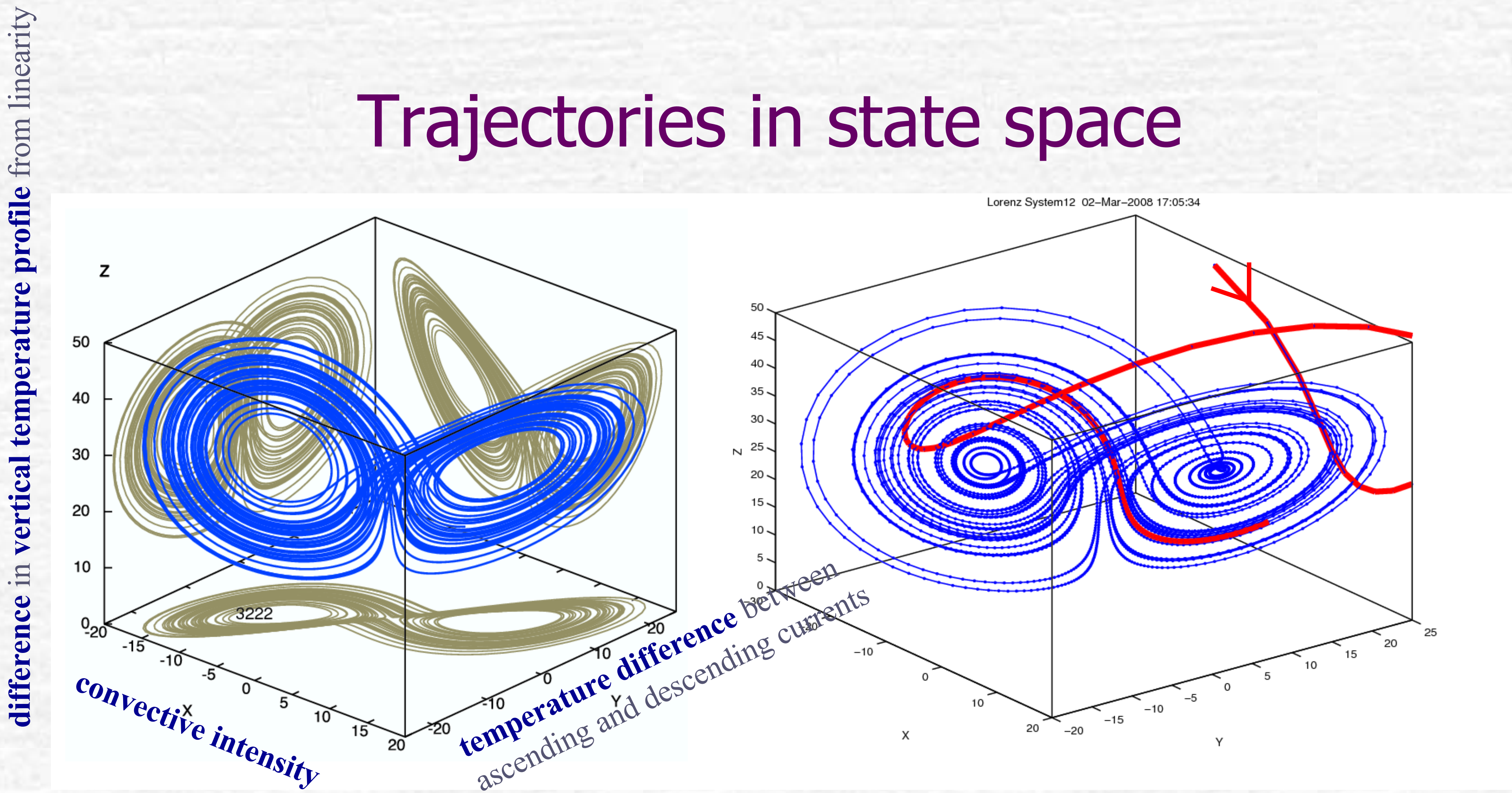
**X convective intensity**

**Y temperature difference** between ascending and descending currents




Individual trajectories cross between the wings. Two neighbours separate until they cross at different times.

# Trajectories in state space

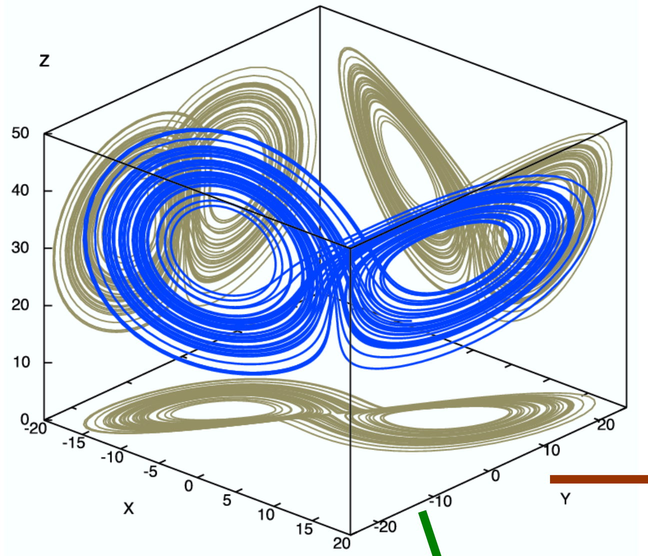


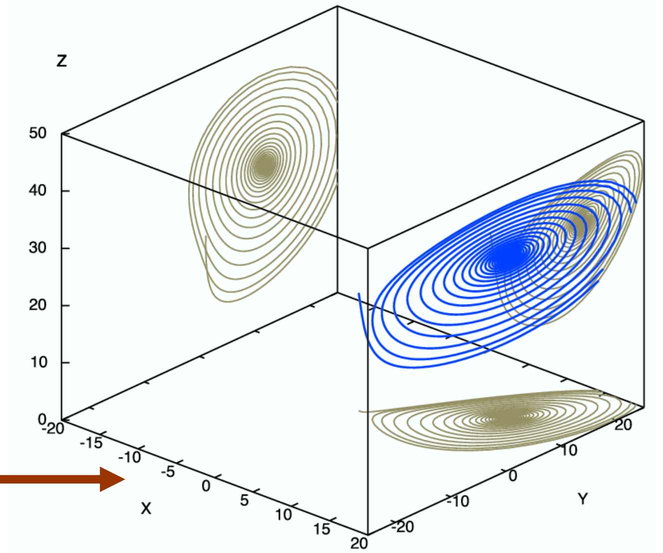

Solution for σ=10, ρ=29 and β=8/3 shown in blue.

Grey shows *projections* onto 2D coordinate planes.

Red trajectory starts far from the *strange attractor* and rapidly converges into it.

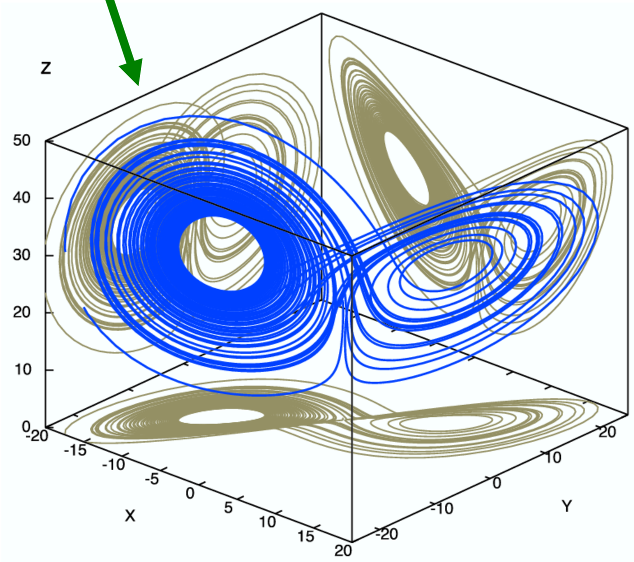
# Forcing nonlinear systems



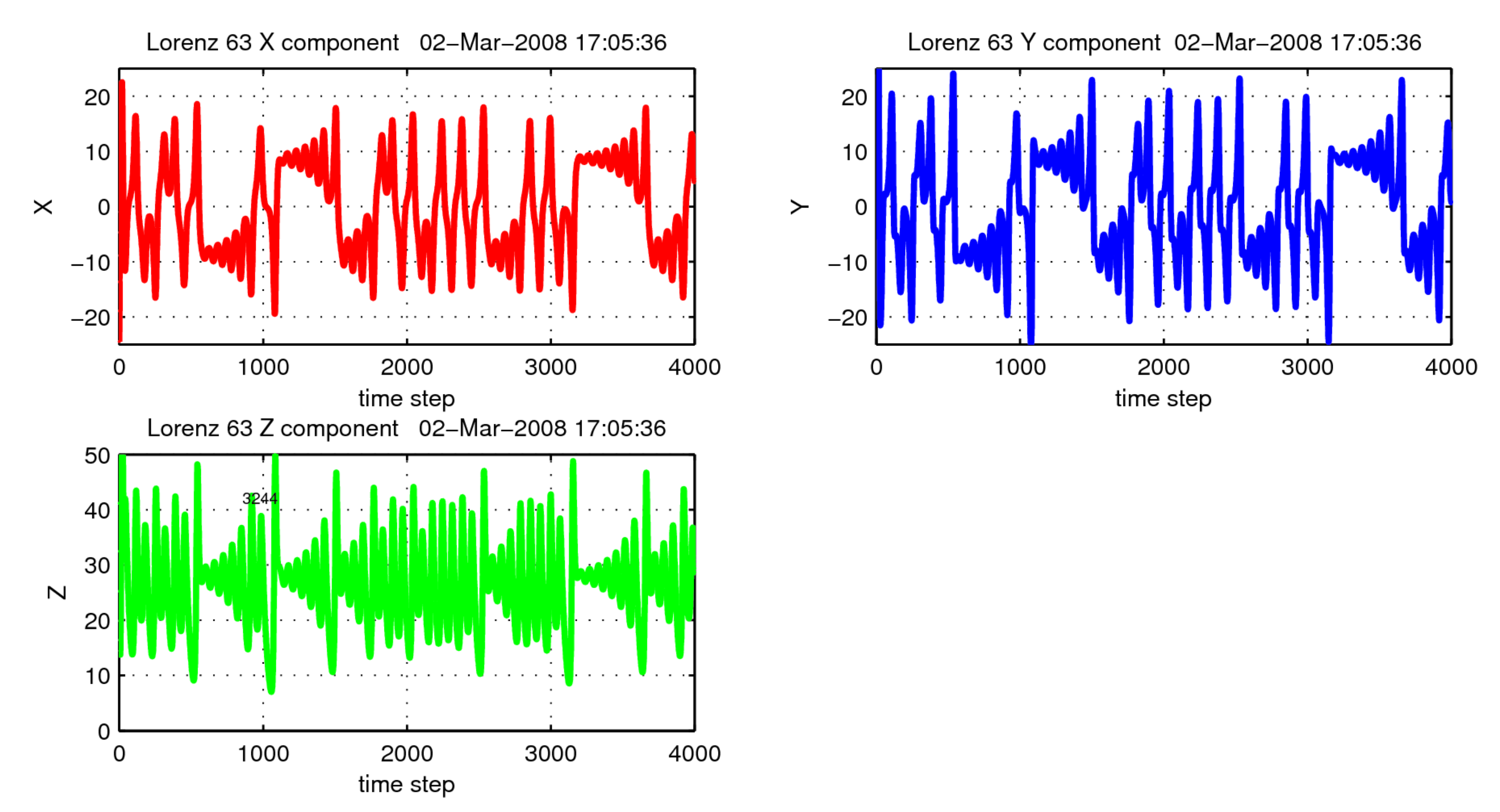

Apply constant forcing in –*X*, -*Y* direction

⇒ Attractor does not shift but left wing becomes more densely populated

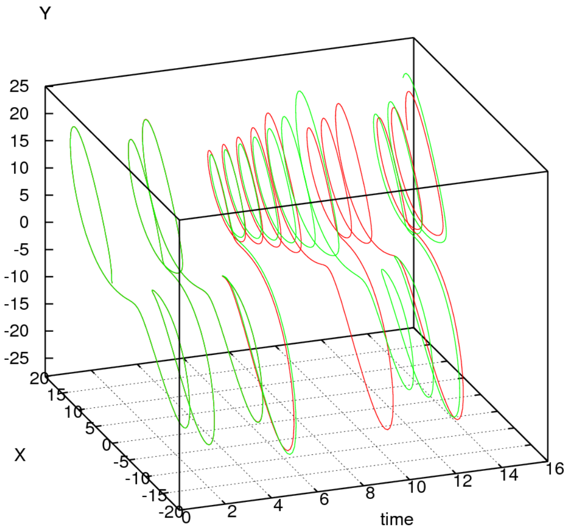



As parameter is varied, solution *bifurcates* to entirely different behaviour (decaying orbit). Here β=5.

# Irregularity in time series




*X* and *Y* variables show intermittent flips between two *regimes* – the "butterfly' s wings"




# Chaos

- Exhibits ***sensitivity to initial conditions.***
- Trajectories from nearby points diverge exponentially on average until separated by scale of attractor
- Time series initially track together but eventually become uncorrelated (*as if random*)
- *Nonlinearity* is necessary for chaos.

# Types of predictability

Initial condition uncertainty

Sensitivity to initial conditions
$\Rightarrow$ **chaos**

**Predictability of the first kind**

*"weather trajectory"*

Boundary condition uncertainty

Partly determined by factors **external** to model system, including forcing

**Predictability of the second kind**

*change in "climate regime"*

Model uncertainty

Structural

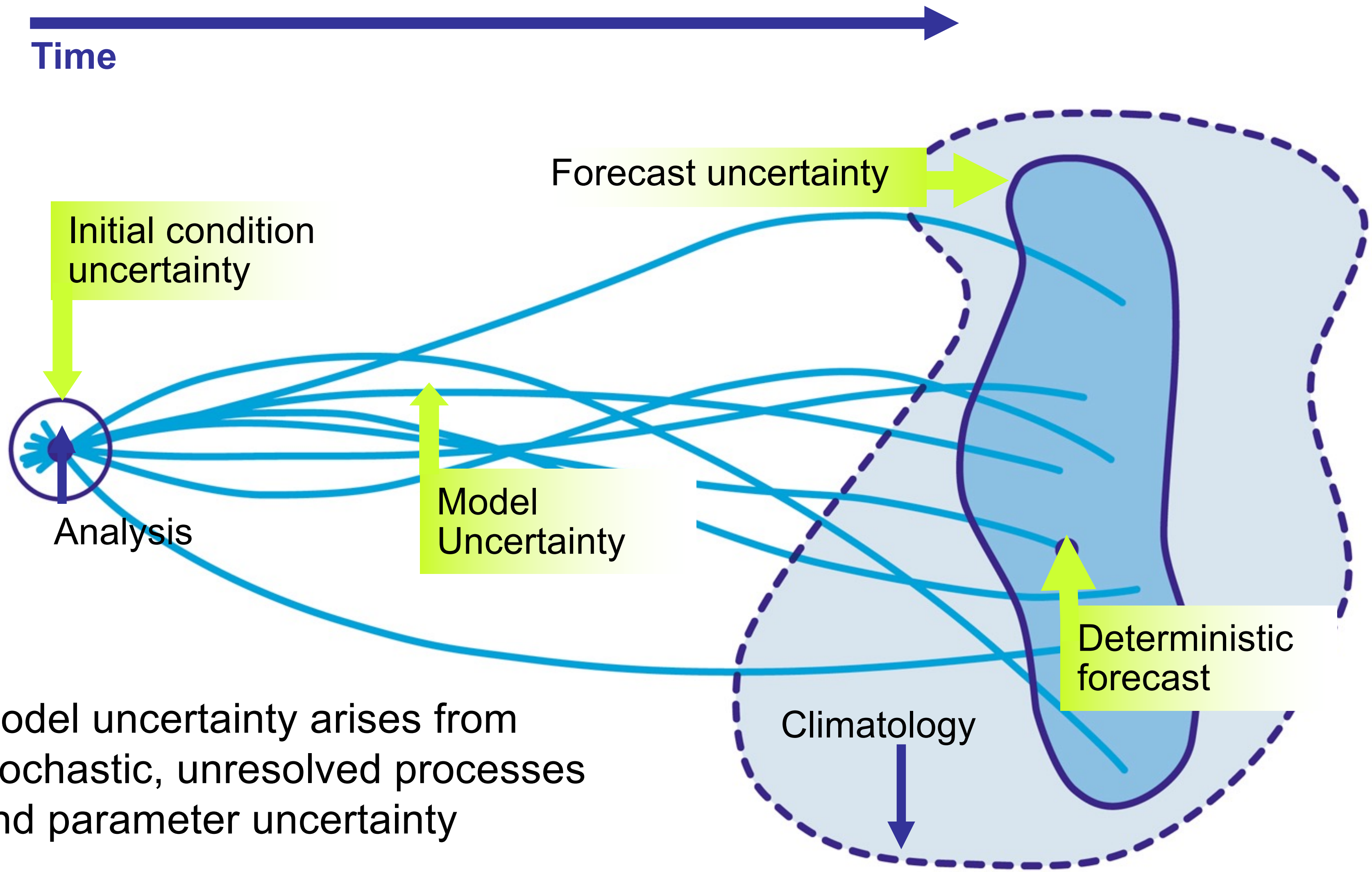
*Same model?*

Parametric

Bifurcation

*Rapid climate change*

# Ensemble Prediction Systems: Representing uncertainty

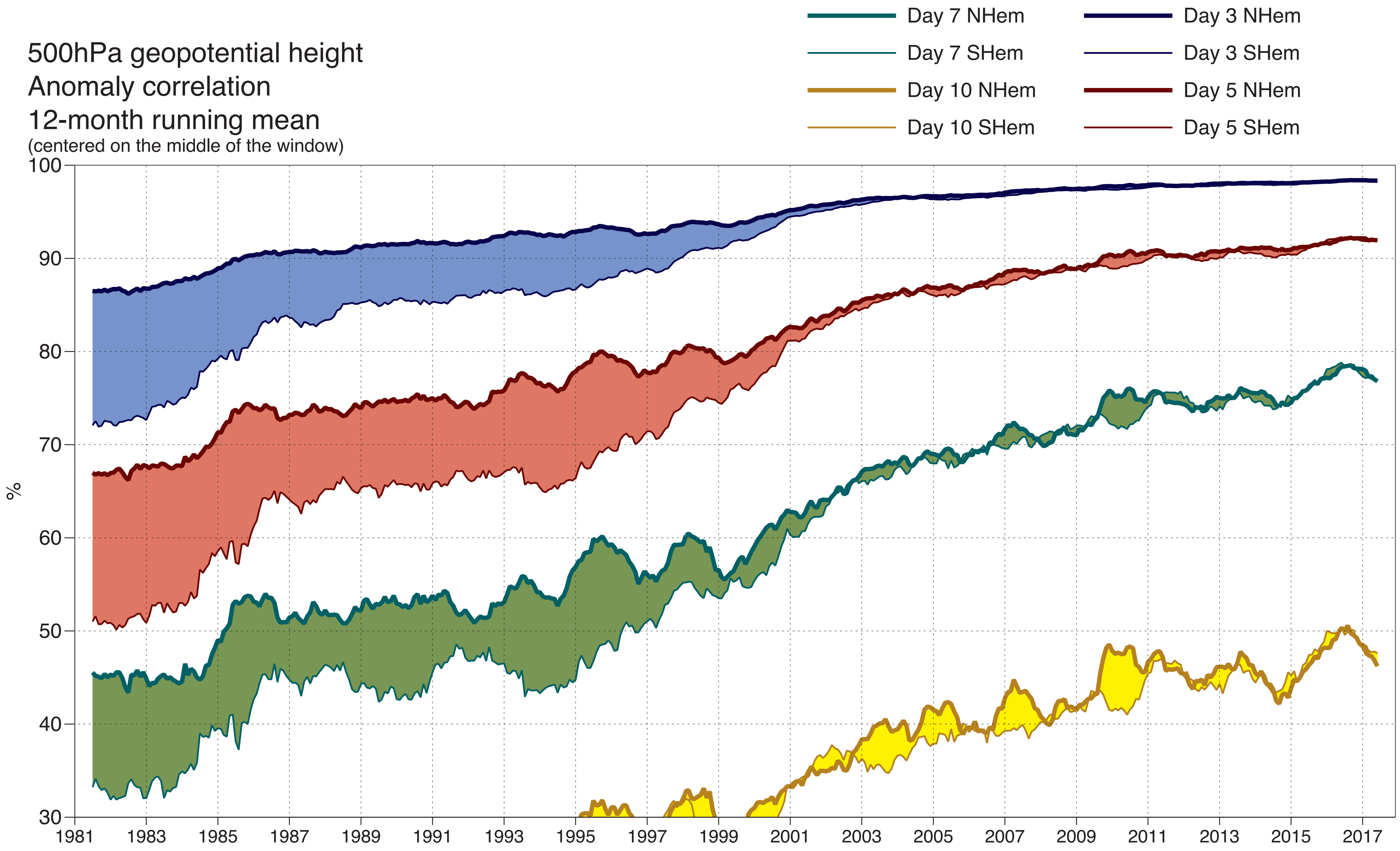



Model uncertainty arises from stochastic, unresolved processes and parameter uncertainty



J. Slingo, Met Office

# Model skill has improved in the last 30 years. But predictability of the first kind is limited



# What does *predictability* mean?

- Predictability is a property of the physical system being examined.

*Predictability of the first kind* refers to the degree to which trajectories from neighbouring initial conditions stay coherent.

Associated with the average rate of separation of trajectories.

*Limit of predictability* refers to a notion that at some point in the future trajectories that started as neighbours will eventually be uncorrelated, no matter how good the forecast.

- *Predictive skill* measures the quality of forecasts produced by a model.

It quantifies the ability of an ensemble system to predict the probabilities of events.

- Expect predictive skill to be lower when predictability is lower.
- CANNOT measure predictive skill from a single forecast or event.

# Ensembles for weather prediction

Atmospheric flow is chaotic

⇒ know that however good the estimate of ICs, forecasts (even using a *perfect model*) will diverge from reality

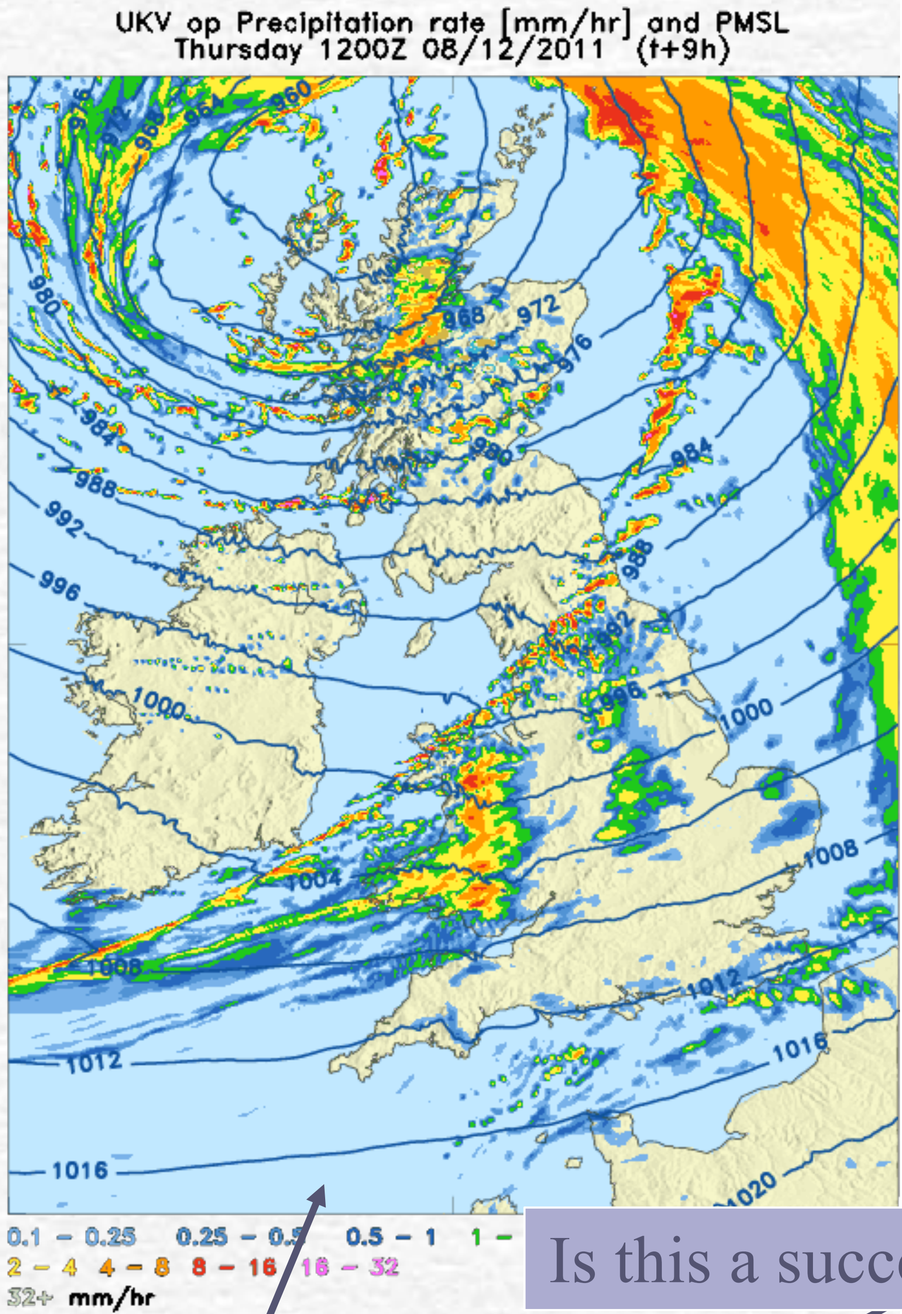
⇒ Time limit to predictability (of the first kind).

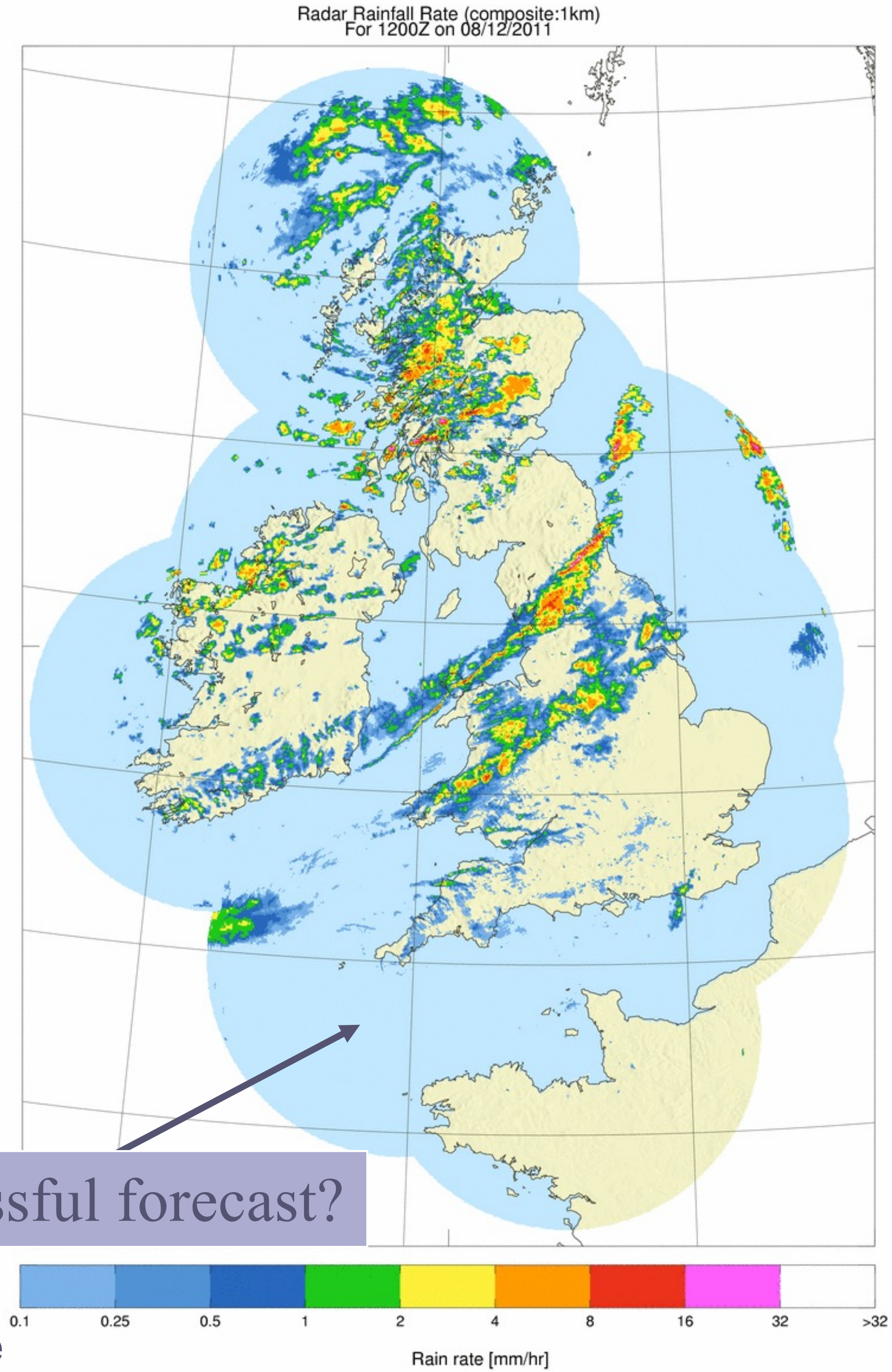
Aim of ensembles

1. Span the range of outcomes for atmosphere at a **given lead time**
2. Forecast the <u>uncertainty</u> in forecasts!

Issues with ensembles

- State space is huge, but number of ensemble members is limited
  - ⇒ Constrain initial spread to match uncertainty in initial state estimate
  - ⇒ Calculate structures growing fastest (in chosen **metric**)
    - ⇒ Hope that rapidly growing perturbations will capture range of outcomes




Is this a successful forecast?

Single UKV T+9 forecast

Radar image

High resolution (1.5km) Met Office ensemble forecast

Verifying at 16 UTC. Forecast from 03 UTC 8/12/11

MOGREPS-UK members (4 out of 12)

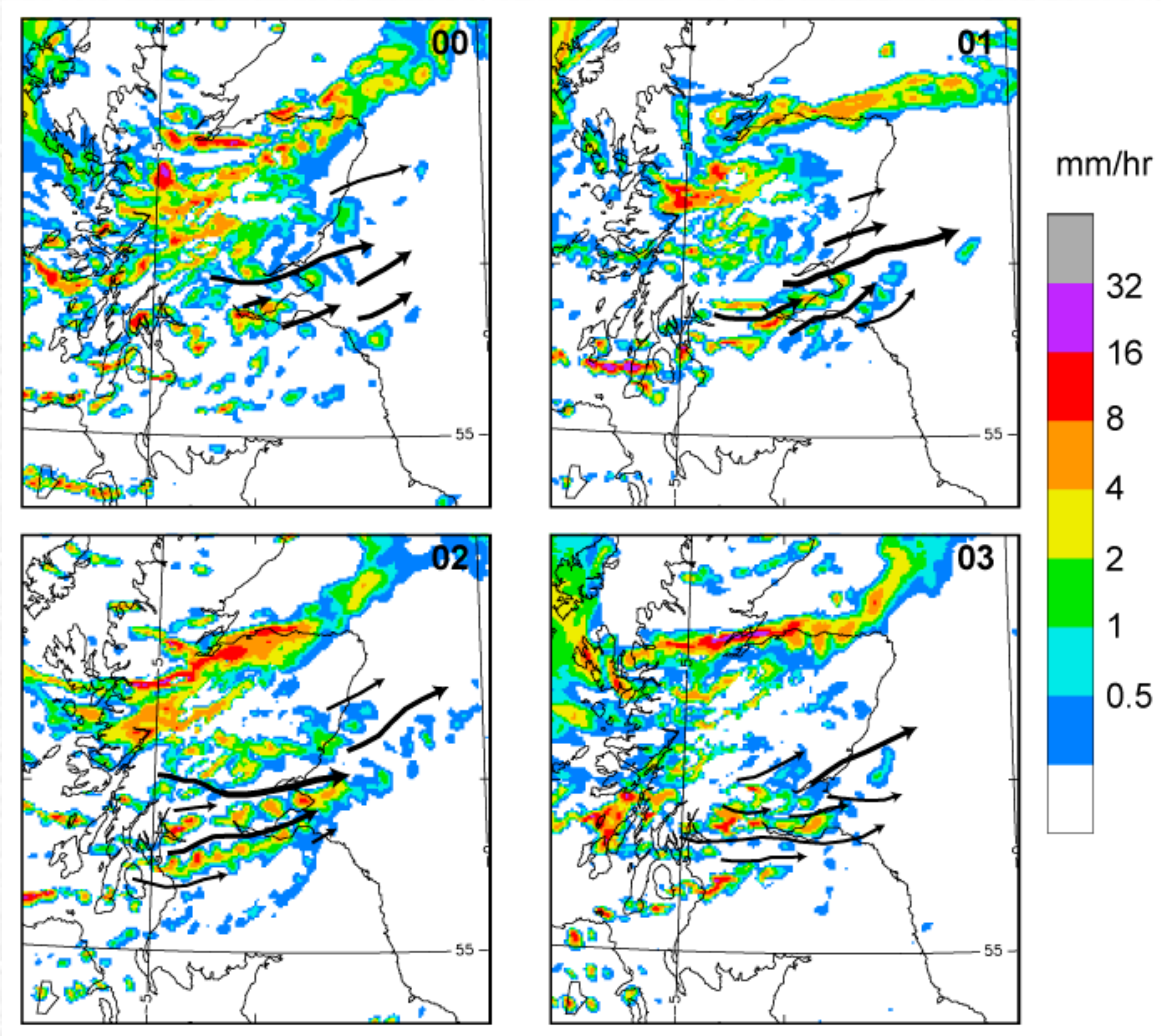
Precipitation intensity

Arrows along high wind cores

and

Dry slots between rain

Assume that members are equally likely






Nigel Roberts, MetOffice@reading

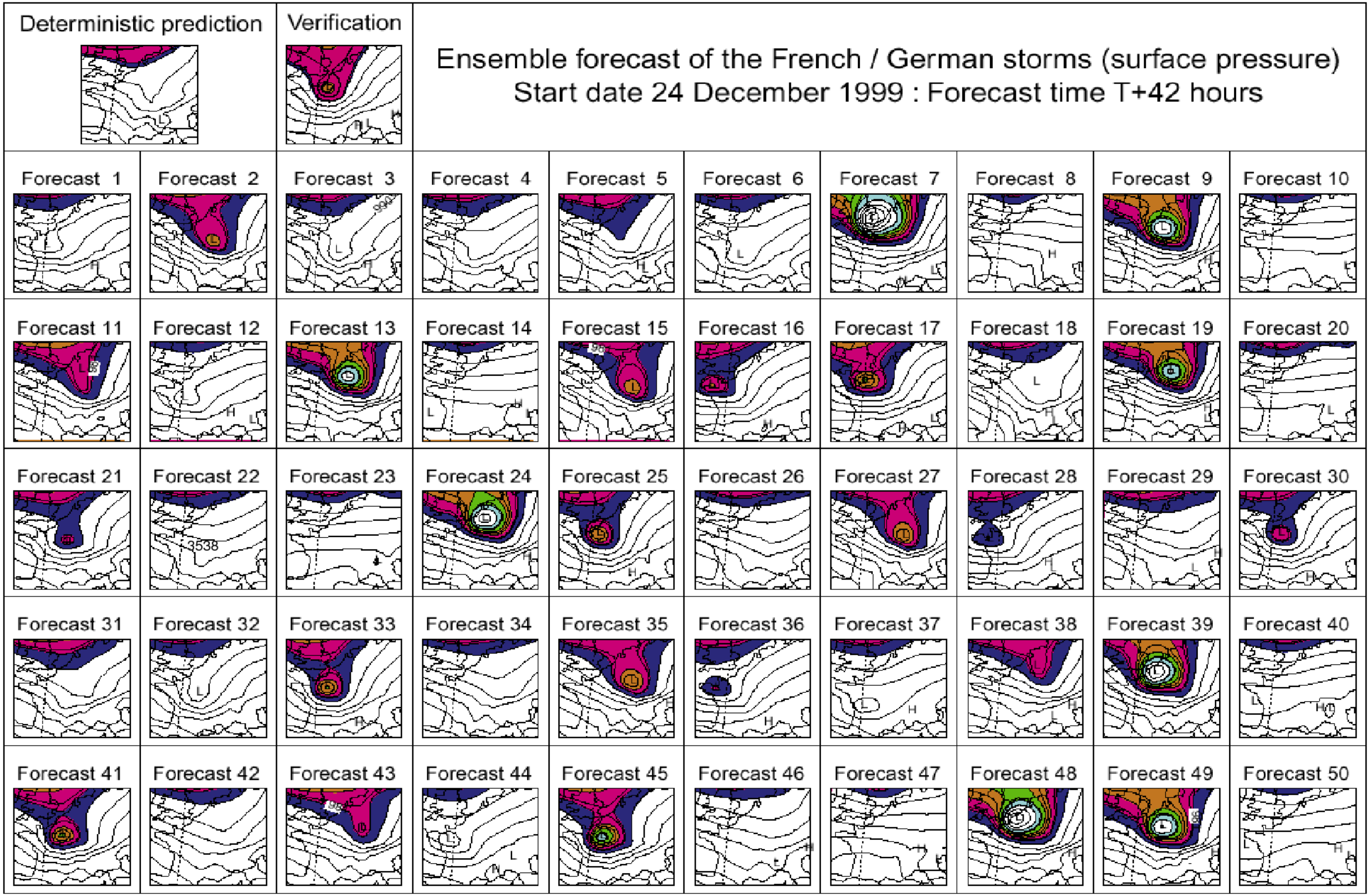
# Assimilating data into forecasts

- Control forecasts start from an **analysis** – the best estimate of the current atmospheric state.

- Analyses are obtained through **data assimilation**
  - most recent model forecast is compared with latest observations and pulled towards them.
  - forecast-obs (least squares) is minimised, weighting by uncertainties.

- Other members of the ensemble are created by:
  - modifying the **initial conditions** with fast growing perturbations
  - perturbing model **parameters**
  - adding **stochastic noise** to the outputs of model processes

- Crucially, perturbation magnitude is scaled so the average ensemble spread matches the *statistics of forecast error* (at an optimisation lead time, usually 2 days)


Deterministic prediction
Verification
Ensemble forecast of the French / German storms (surface pressure)
Start date 24 December 1999 : Forecast time T+42 hours
Forecast 1
Forecast 2
Forecast 3
Forecast 4
Forecast 5
Forecast 6
Forecast 7
Forecast 8
Forecast 9
Forecast 10
Forecast 11
Forecast 12
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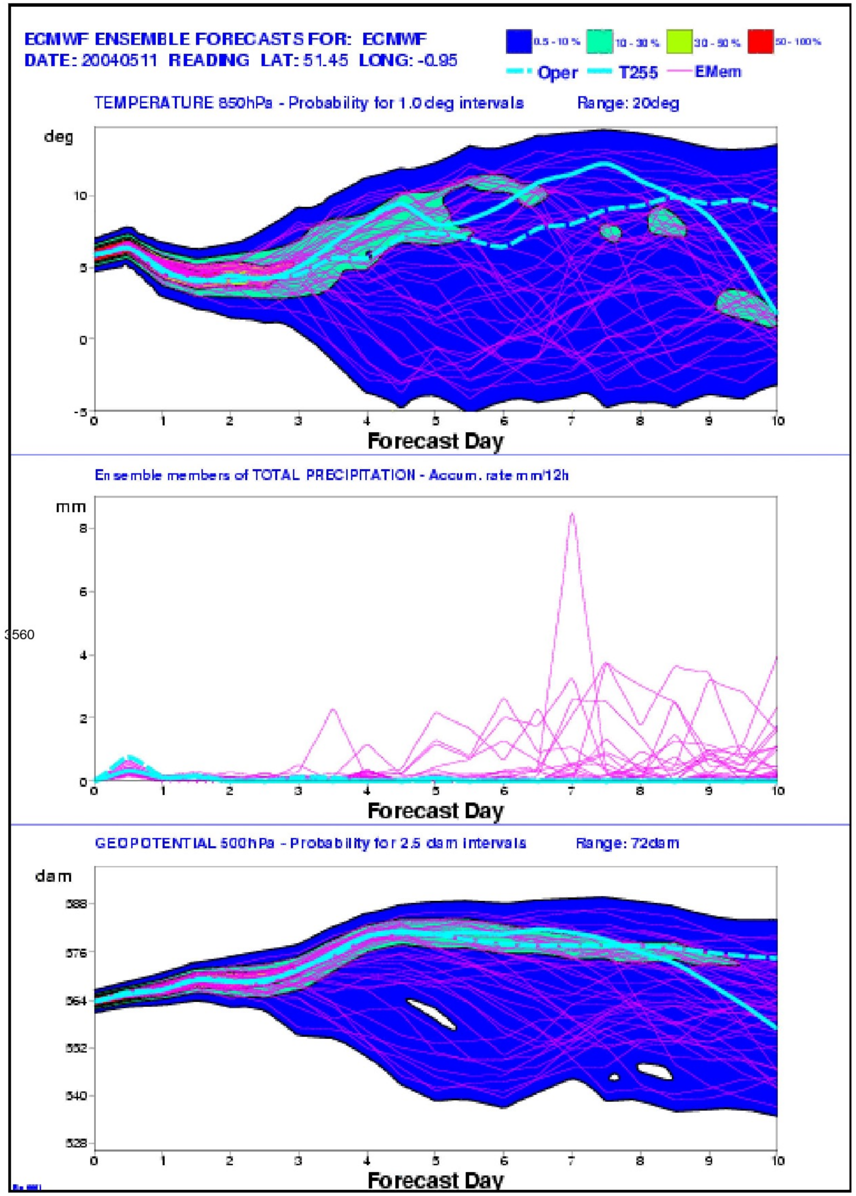
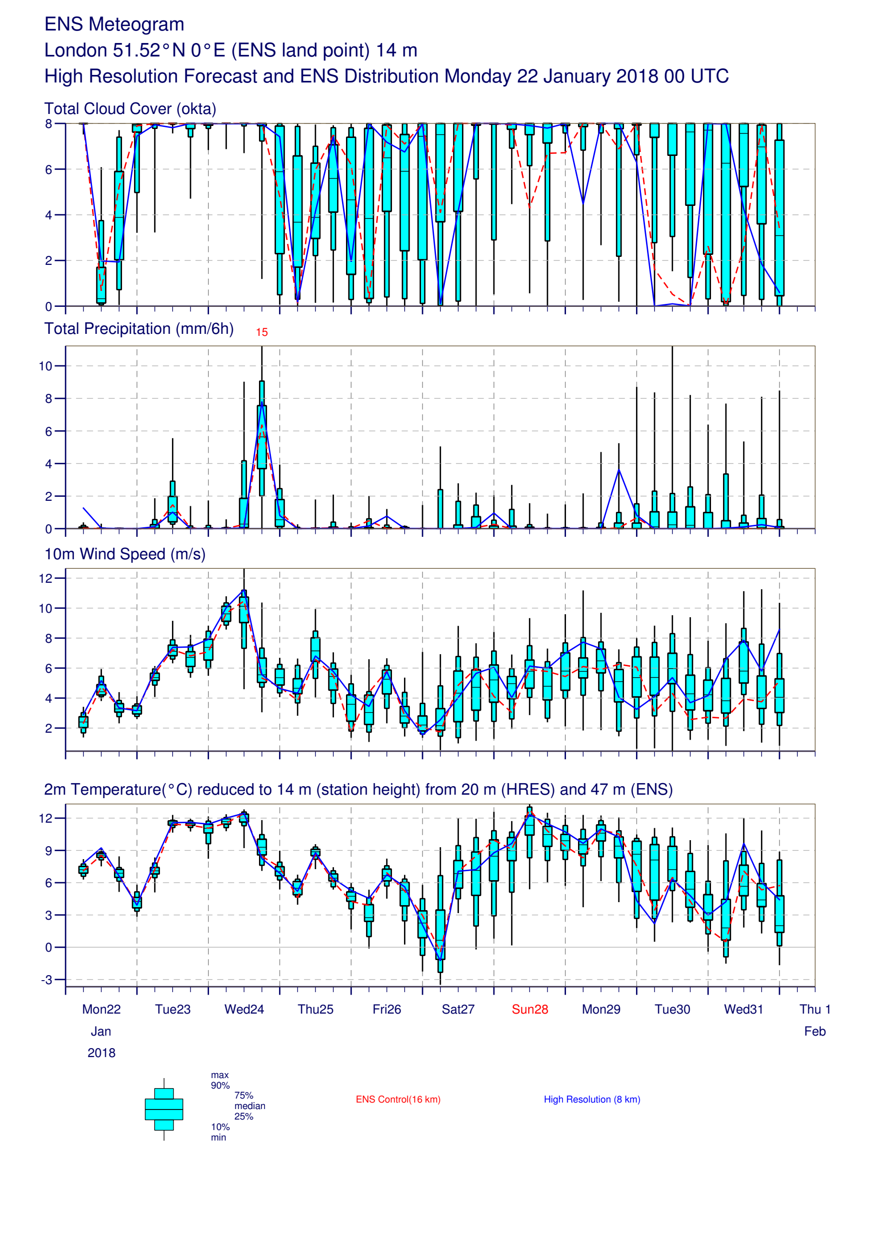
# Forecast ensemble properties



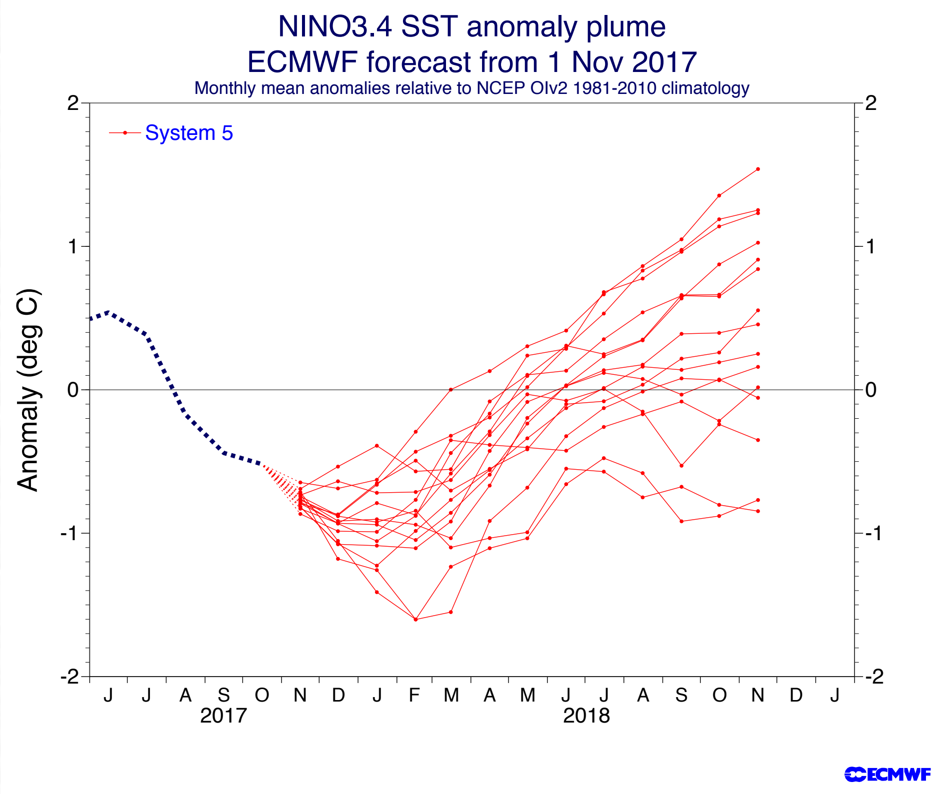

- On verification, the sequence of *analyses* usually lies within ensemble plume.
- Ensemble mean outperforms single high resolution forecast (partly smoothing)
- Ensemble can split into clusters – depends on variable
- Ensemble perturbations derived from rapidly growing subspace (using linearised dynamics)

⇒ Hope that histogram of ensemble predictions ≈ prediction for probability (PDF) of event

# A typical "meteogram" for a single location

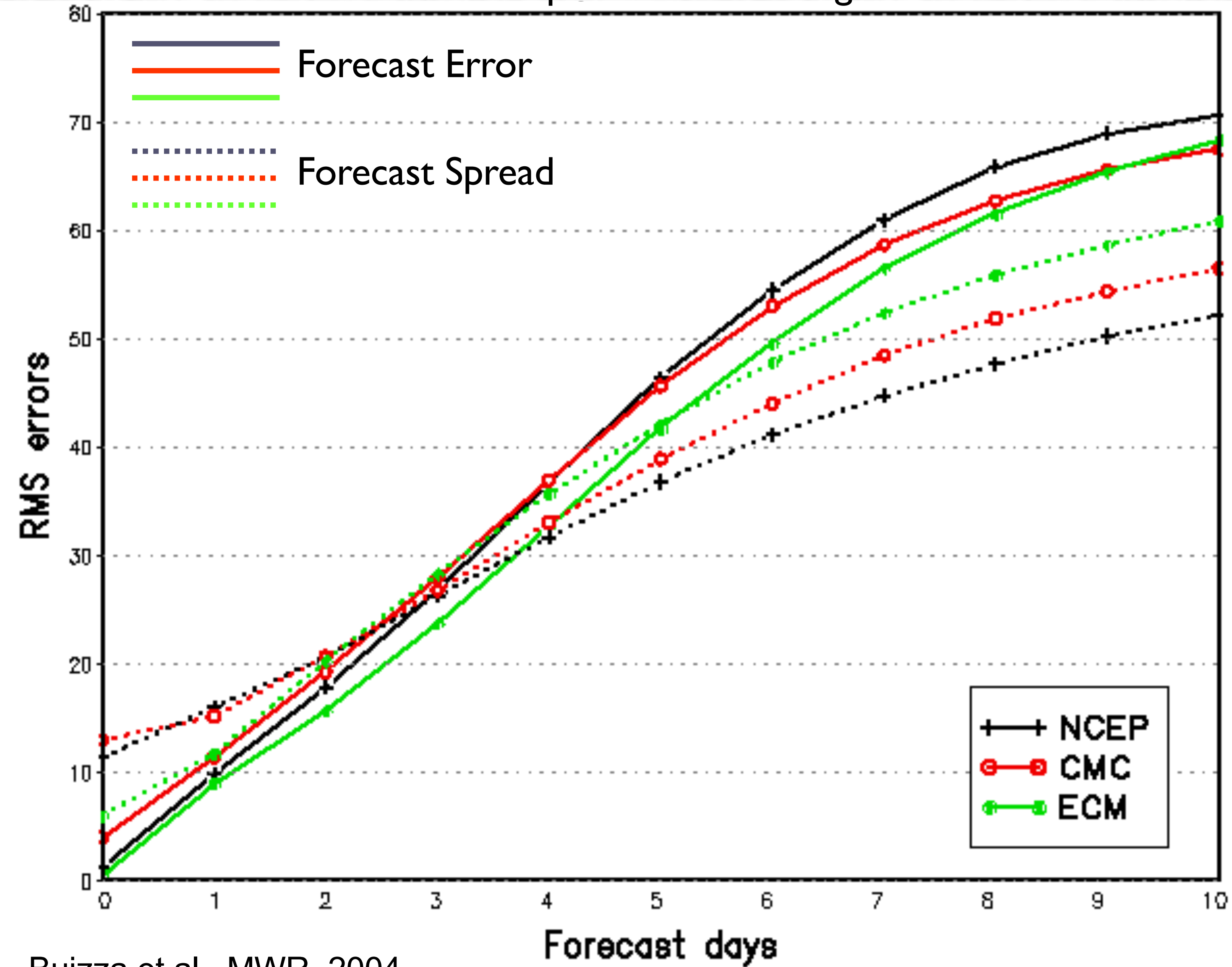



# The equivalent seasonal product



# Problem of under-dispersion of the ensemble system

Northern Hemisphere 500hPa Height Forecasts




Buizza et al., MWR, 2004

**RMS error grows faster than the spread**

Ensemble is **under-dispersive.**

Ensemble forecast is **over-confident.**

Ensemble spread increased by identifying initial perturbations (singular vectors) that give maximum error growth.

J. Slingo, Met Office

# Mitigating overconfidence

- Control forecasts start from an **analysis** – the best estimate of the current atmospheric state.

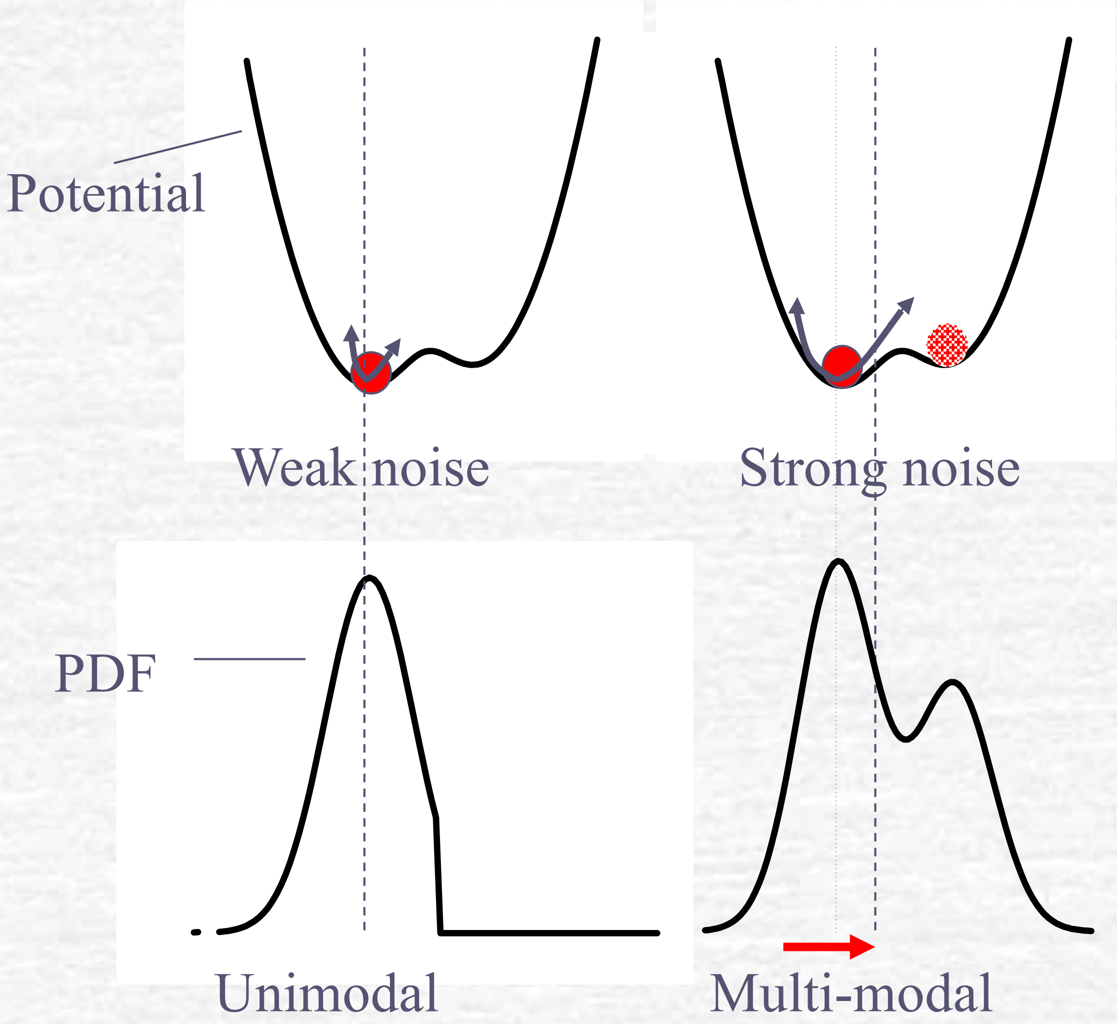
- Analyses are obtained through **data assimilation**
  - most recent model forecast is compared with latest observations and pulled towards them.
  - forecast-obs (least squares) is minimised, weighting by uncertainties.

- Other members of the ensemble are created by:
  - modifying the **initial conditions** with fast growing perturbations to obtain the maximum possible spread
  - perturbing model **parameters**
  - adding **stochastic noise** to the model solution trajectories

- Crucially, perturbation magnitude is scaled so the average ensemble spread matches the *statistics of forecast error* (at an optimisation lead time, usually 2 days)

# Stochastic parameterizations:

## reducing model error and enhancing internal variability




**Stochastic parameterizations can change mean and variance of PDF:**

- Impacts variability of model and increases ensemble spread
- Impacts **systematic error** (e.g. blocking, precipitation error)

J. Slingo, Met Office

# Stochastic physics in Ensemble Prediction Systems

Met Office employs three schemes to address different sources of model error:

- Random Parameters (RP)
  - Error due to approximations in parameterisation
- Stochastic Convective Vorticity (SCV)
  - Unresolved impact of organised convection (MCSs)
- Stochastic Kinetic Energy Backscatter (SKEB)
  - Excess dissipation of energy at small scales

J. Slingo, Met Office

# IFS: Stochastically Perturbed Parameterisation Tendencies (SPPT)

- Operational scheme in ECMWF's ensemble prediction system
- Perturbations to total parametrised tendency of physical processes with multiplicative noise
- Perturbed physical tendencies:

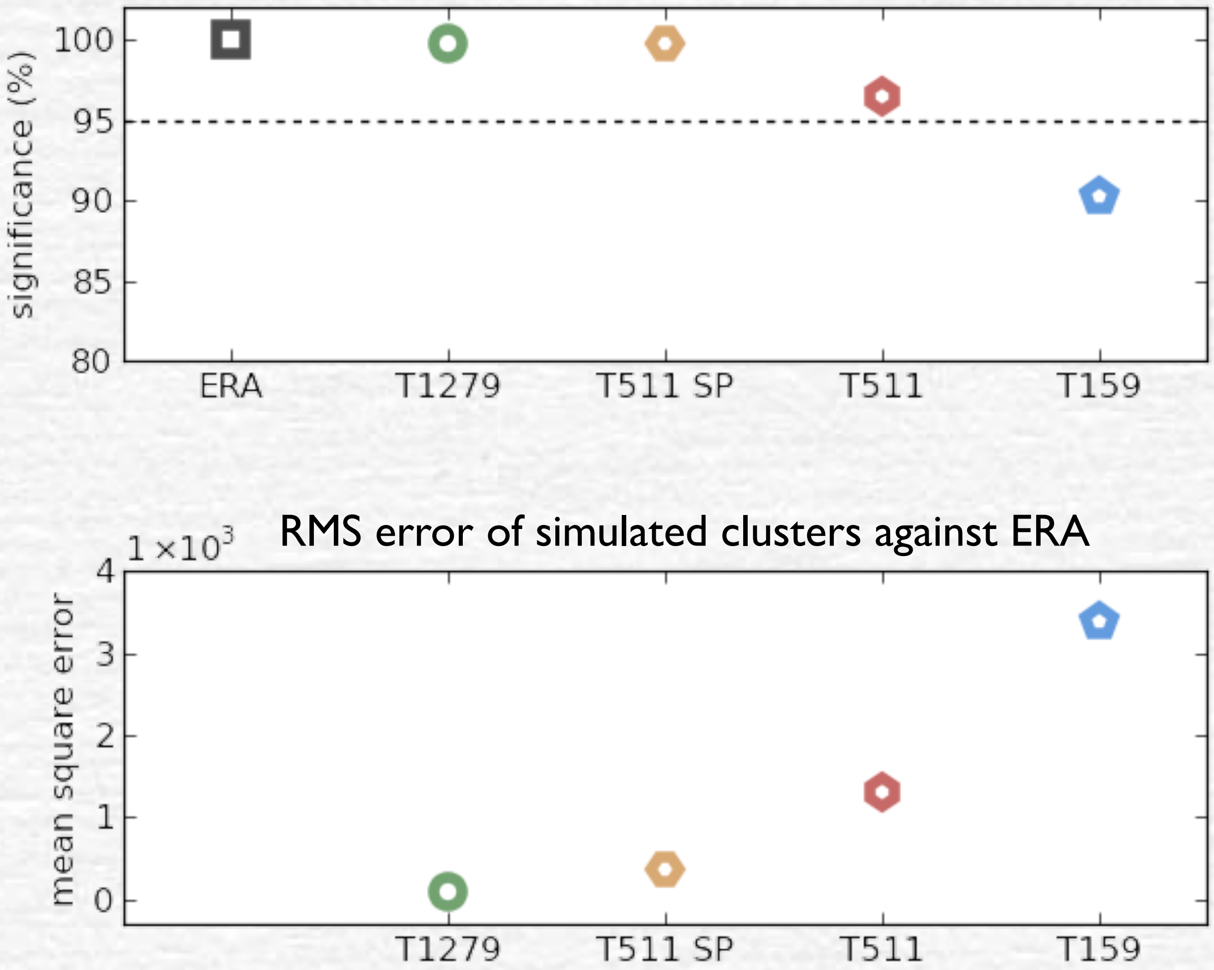
$$\boldsymbol{X_p = (1+r\mu)\, X_c} \qquad \textit{for } X=\{u,v,T,q\}$$

- *r* is a uni-variate random number described through a spectral pattern generator which is smooth in space and time
- Spectral coefficients of *r* are described with an AR(1) process
- Gaussian distribution, truncated at ±2s

# Project Athena: AMIP runs

Dawson, Corti Palmer, GRL 2012

Probability that clusters are not produced from a chance sampling of a gaussian




This suggests that adding stochastic noise is equivalent to a doubling of resolution

# How to improve model predictions?

Initial condition uncertainty

*Better state estimation via*

*⇒ Data assimilation system (inc. model)*

*⇒ Better observations (more accurate measurements, targeted obs. etc)*

Boundary condition uncertainty

*Include boundary processes into model (e.g., soil model, coupled ocean, ice sheets)*

Model uncertainty

Structural

*Cross-pollination*

Parametric

*Inference of better parameter values and range*

# This is the time to ask a climate contrarian's classic question

"How can you predict climate if you cannot predict the weather?"

2021 Nobel Prize in Physics to Klaus Hasselmann:

- In the 1970s, **Klaus Hasselmann** created a model that links together weather and climate, thus answering the question of why climate models can be reliable despite weather being changeable and chaotic. The "genius contribution" of Hasselmann's was the 1970s introduction of the first 'conceptual model' for Earth's climate[1] — a simple set of equations that captures global phenomena with just a few variables. This approach has given insights complementary to those from global circulation models, which are brute-force, geographically detailed calculations. "

## Stochastic climate models

### Part I. Theory

By K. HASSELMANN, *Max-Planck-Institut für Meteorologie, Hamburg, FRG*

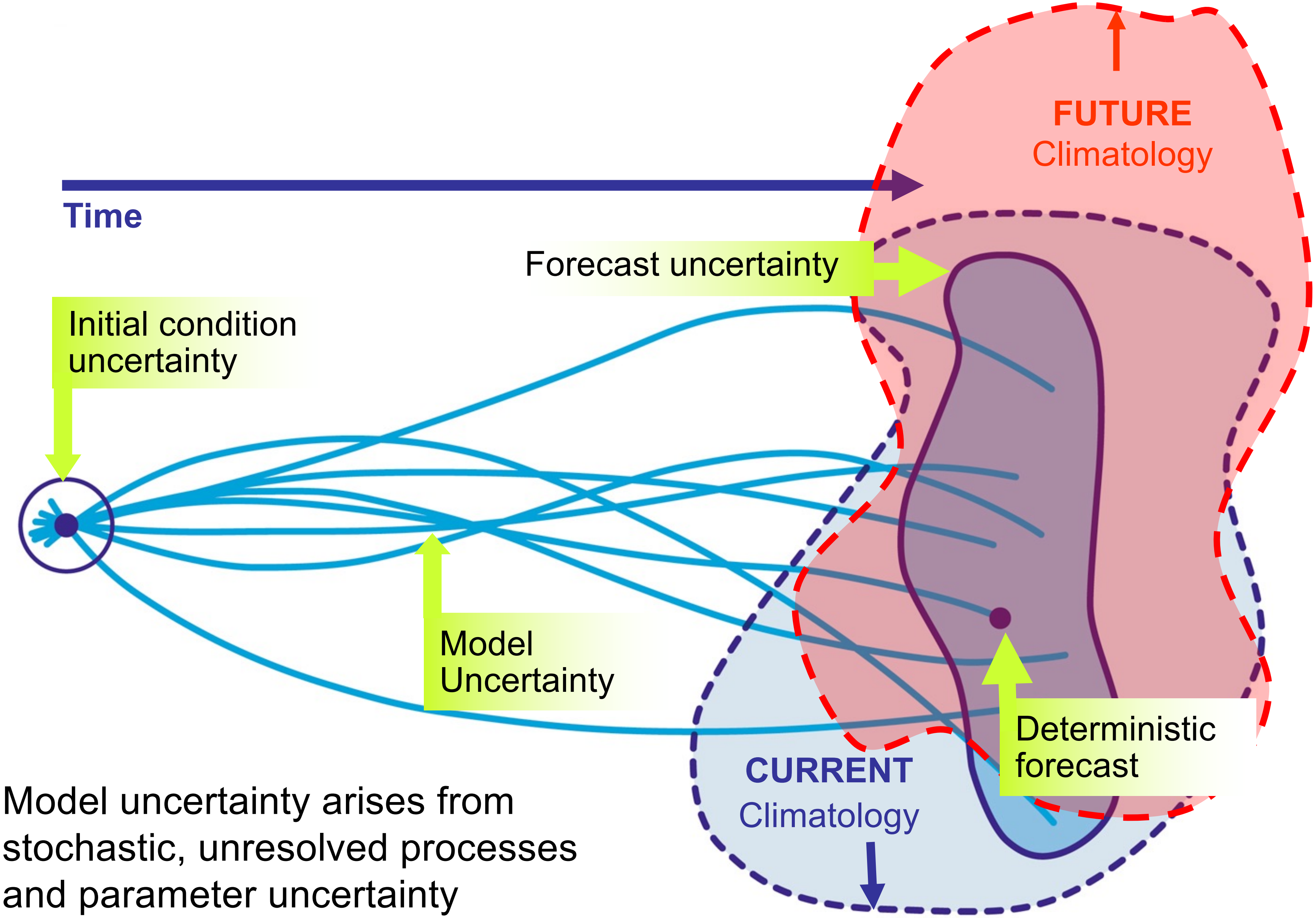
(Manuscript received January 19; in final form April 5, 1976)

ABSTRACT

A stochastic model of climate variability is considered in which slow changes of climate are explained as the integral response to continuous random excitation by short period "weather" disturbances. The coupled ocean–atmosphere–cryosphere–land system is divided into a rapidly varying "weather" system (essentially the atmosphere) and a slowly responding "climate" system (the ocean, cryosphere, land vegetation, etc.). In the usual Statistical Dynamical Model (SDM) only the average transport effects of the rapidly varying weather components are parameterised in the climate system. The resultant prognostic equations are deterministic, and climate variability can normally arise only through variable external conditions. The essential feature of stochastic climate models is that the non-averaged "weather" components are also retained. They appear formally as random forcing terms. The climate system, acting as an integrator of this short-period excitation, exhibits the same random-walk response characteristics as large particles interacting with an ensemble of much smaller particles in the analogous Brownian motion problem. The model predicts "red" variance spectra, in qualitative agreement with observations. The evolution of the climate probability distribution is described by a Fokker-Planck equation, in which the effect of the random weather excitation is represented by diffusion terms. Without stabilising feedback, the model predicts a continuous increase in climate variability, in analogy with the continuous, unbounded dispersion of particles in Brownian motion (or in a homogeneous turbulent fluid). Stabilising feedback yields a statistically stationary climate probability distribution. Feedback also results in a finite degree of climate predictability, but for a stationary climate the predictability is limited to maximal skill parameters of order 0.5.

Hasselmann also developed methods for identifying specific signals, fingerprints, that both natural phenomena and human activities imprint in the climate. His methods have been used to prove that the increased temperature in the atmosphere is due to human emissions of carbon dioxide.

# Challenge of a changing climate




Model uncertainty arises from stochastic, unresolved processes and parameter uncertainty

# Chaos and climate modelling

- We like to think that we are setting up our numerical climate experiments in a way that exploits predictability of the second kind
- There are dangers in making this assumption and we must remember that:
  - We are always dealing with internal variability
  - Sensitivity to initial conditions comprises contributions from other components of the system that have slower evolution (longer memories)
  - There are some well-known climate attractors: our models will very easily “fall” into those attractors

# What will happen to TCs in the future?

Typhoons will migrate poleward … and a NA hurricane reduction

SPECIAL U.S. CLIVAR Hurricanes COLLECTION

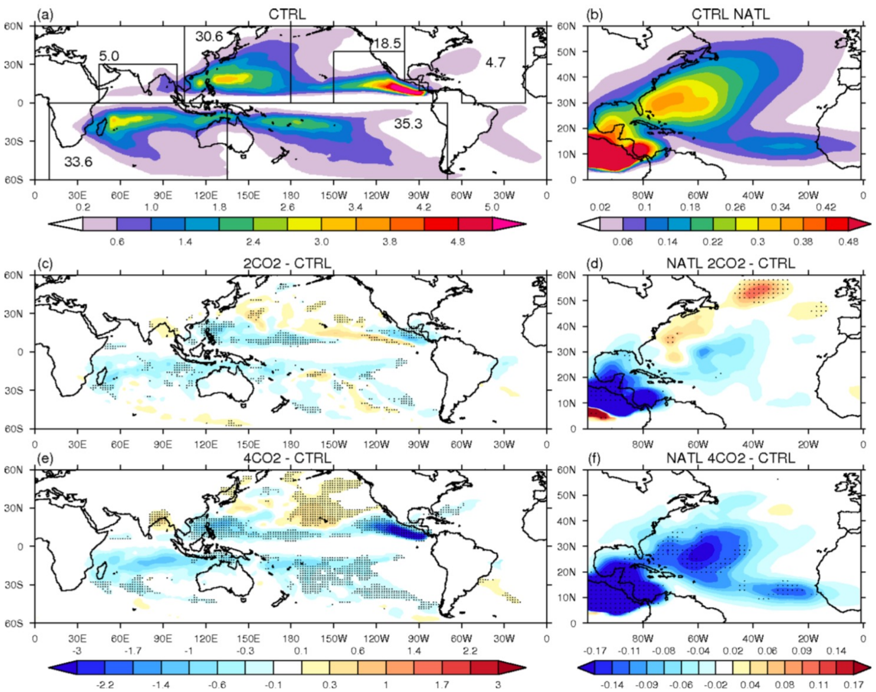
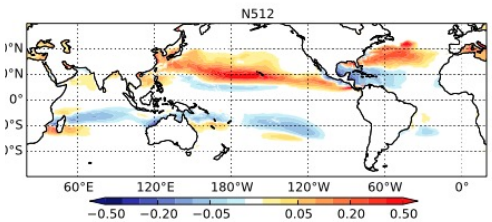



FIG. 2. Tropical cyclone track density, same as figure 1, for (a) HiGEM present-day simulation (b) The same as for (a) but North Atlantic (c) 2CO2 - present-day simulation (d) North Atlantic 2CO2 - present-day simulation (e) 4CO2 - present-day simulation and (f) North Atlantic 4CO2 - present-day simulation. Stippling shows where changes are outside the range of 5×30-year present-day simulations.

Bell et al. J. Clim. 2012, idealised HiGEM simulations

GPI-based estimates agree in the Pacific, albeit not in the Atlantic




PRACE

UPSCALE

## 2012 UPSCALE MODELLING CAMPAIGN

JOURNAL OF CLIMATE

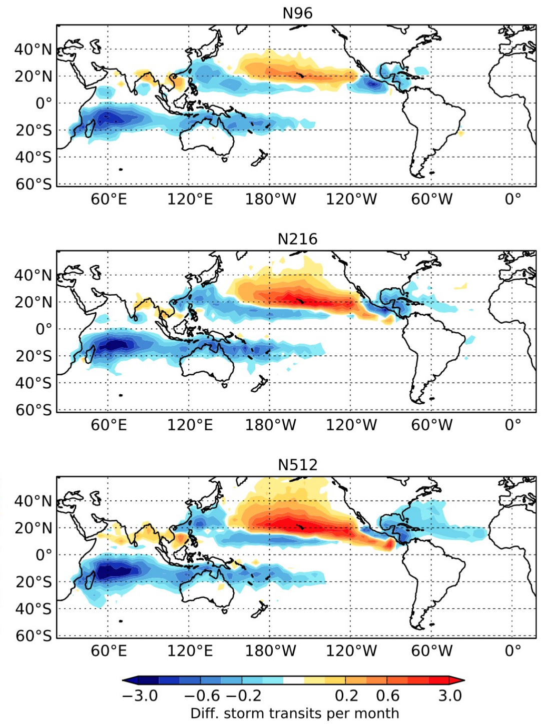
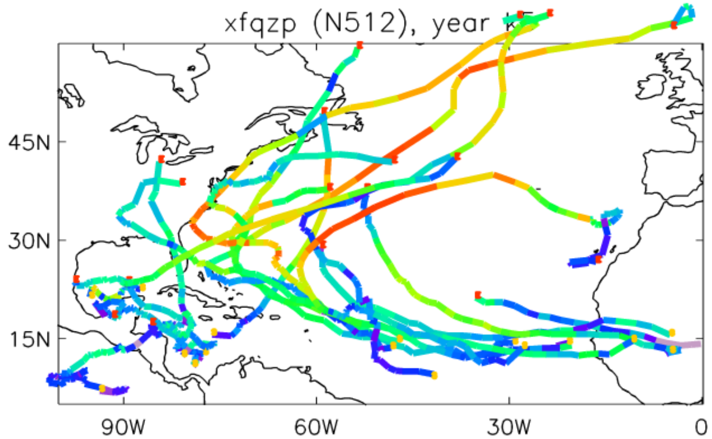
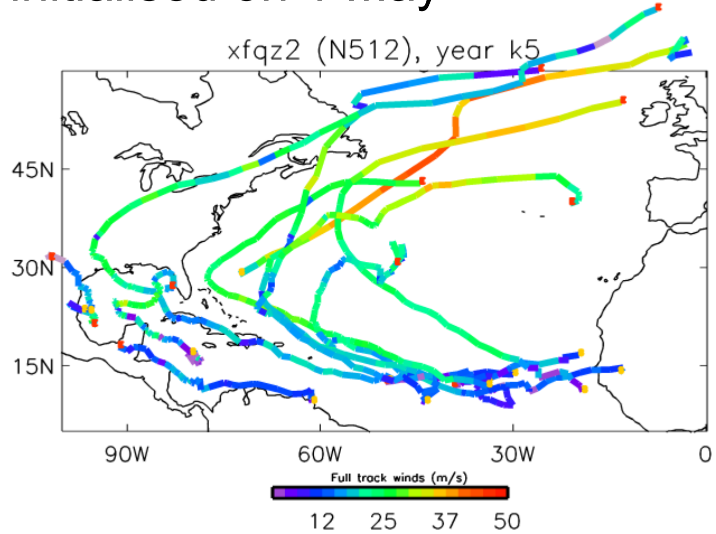


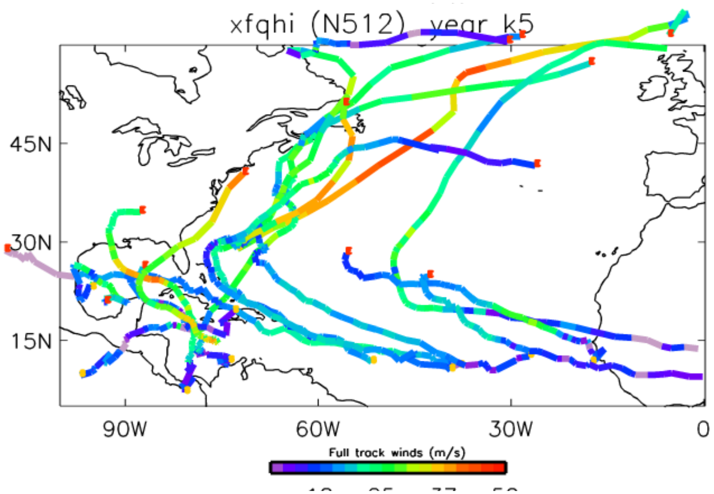

FIG. 12. Change in tropical cyclone track density (storm transits per month per unit area equivalent to a 4° spherical cap) between the future climate and present climate integrations for the whole 1986–2010 period and for the whole ensemble at each model resolution: (top)–(bottom) N96, N216, and N512.

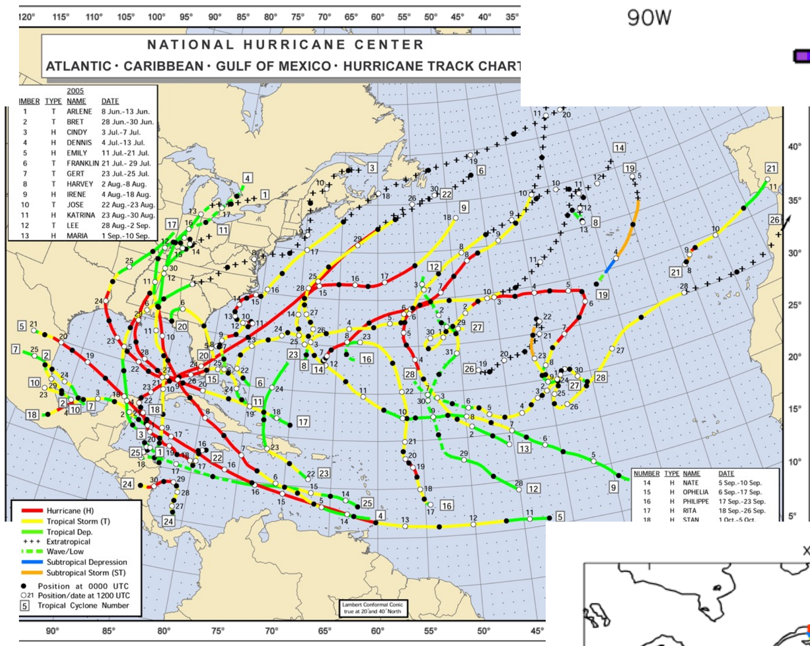
Roberts et al. 2015. Journal of Climate, RCP 8.5 scenario

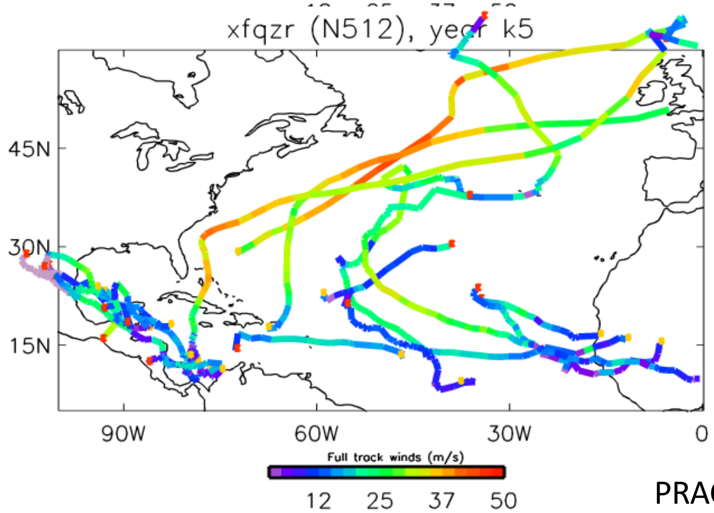
# 2005 Atlantic storm track at N512: 5 members initialised on 1 May

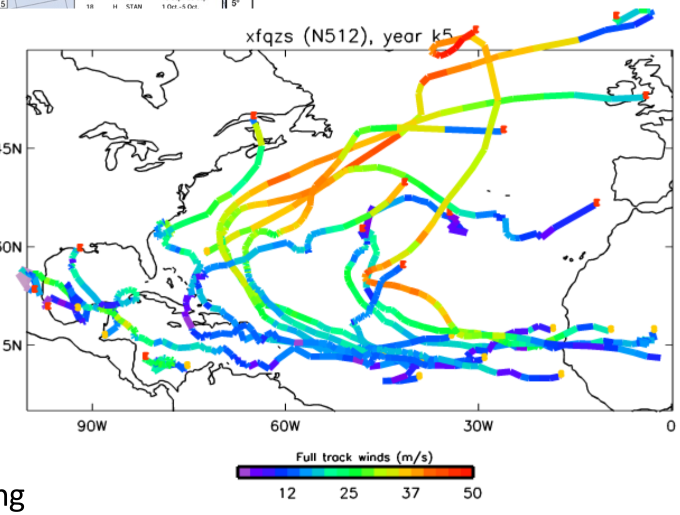


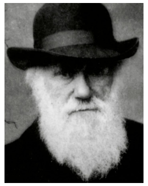




PRACE UPSCALE, Project PI: P.L. Vidale, Reading



NATURA FACIT SALTUS, or “The Abominable Mystery” of Stochastic Physics

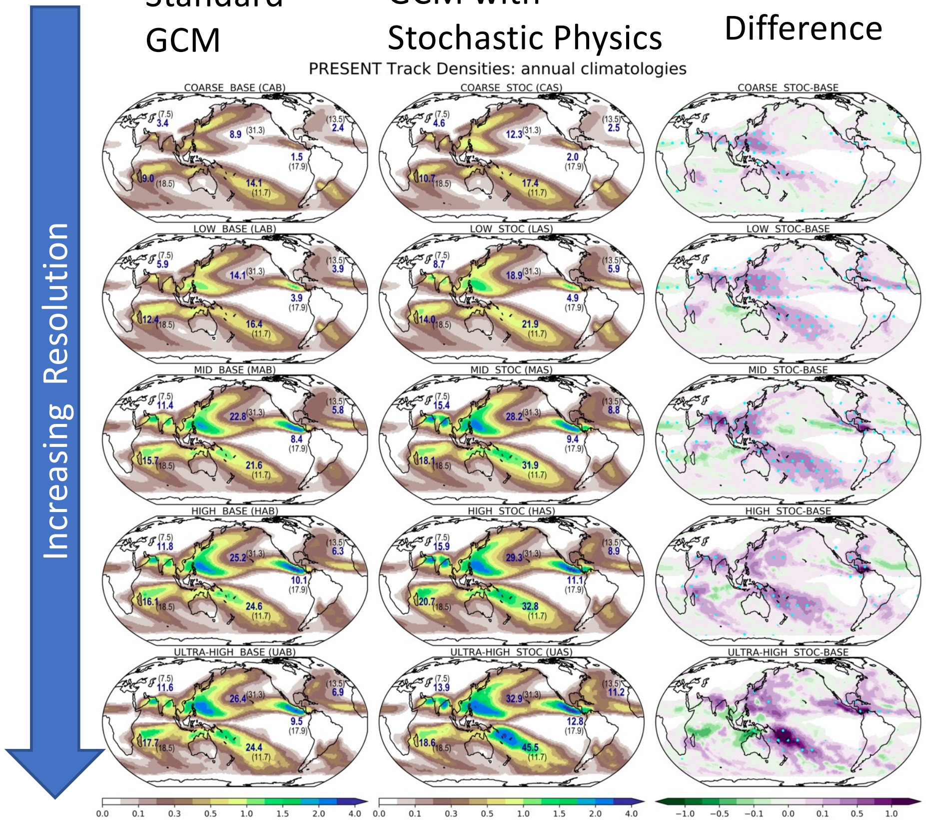
# Both **resolution** and **Stochastic Physics** improve climate means (count, distribution, intensity spectra)

Stochastic Physics used as a surrogate for resolution in the simulation of **Tropical Cyclones** in the EC-Earth 3.1 model.

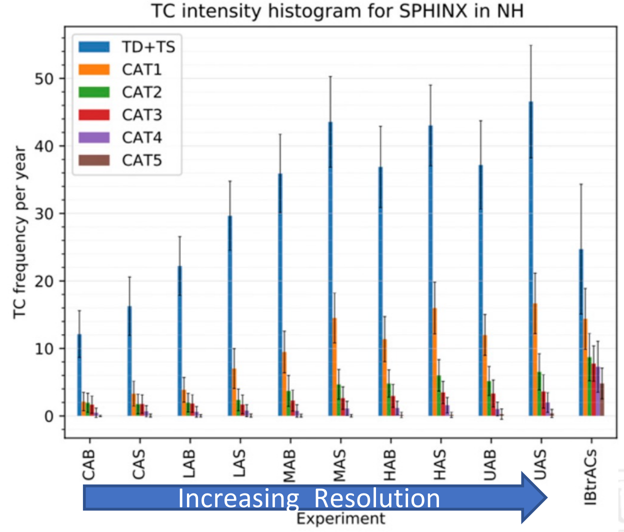
Ensemble of 30x2 simulations, 30yrs each

We can mimic **50% of the effects of resolution with just an additional 5%** increase in CPU costs, instead of a ~240% increase.

(for reference, a doubling of resolution incurs a factor of 8x in costs)




Vidale et al. *J. Clim.* 2021




Observed distribution of Tropical Cyclones

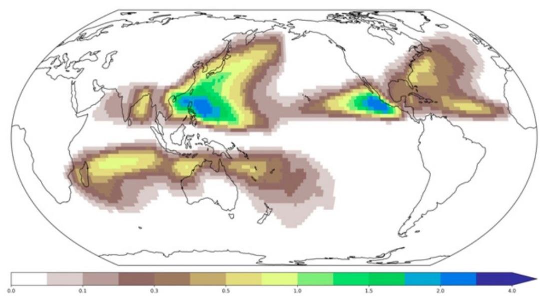
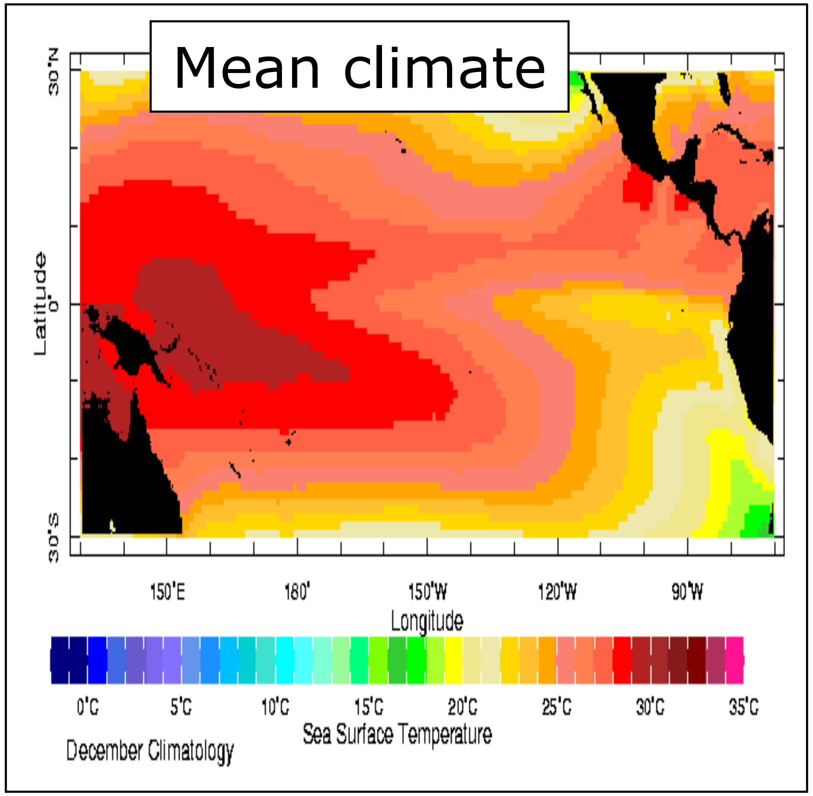
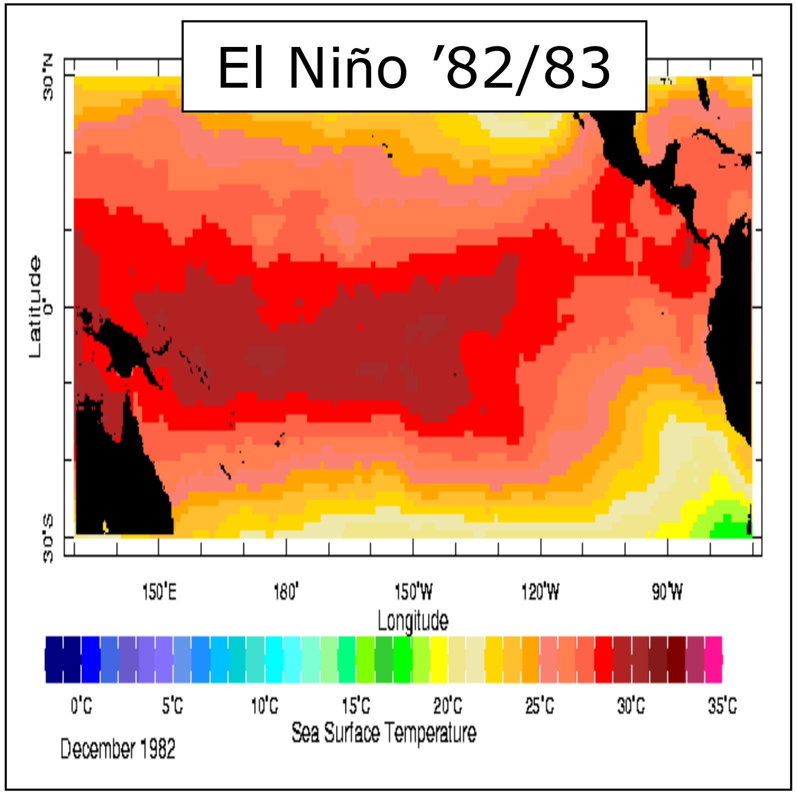
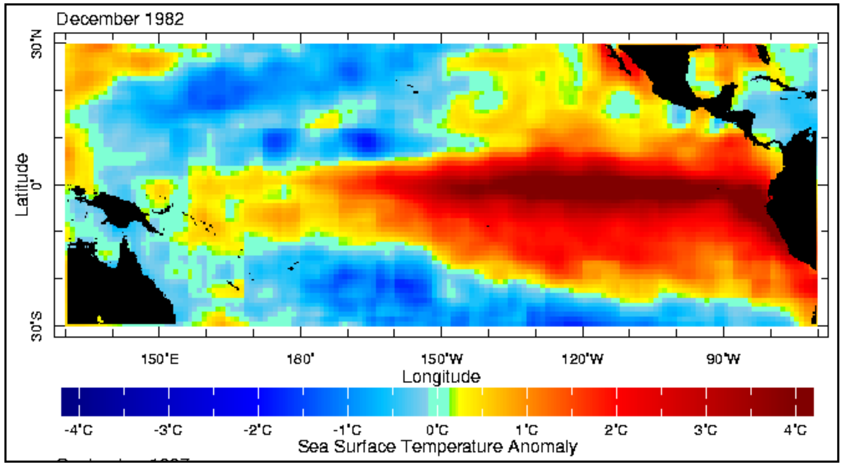


FIG. 2. TC track densities from the IBTrACS dataset for the Climate-SPHINX simulation period (1979–2008).

# A climate seesaw: El Niño

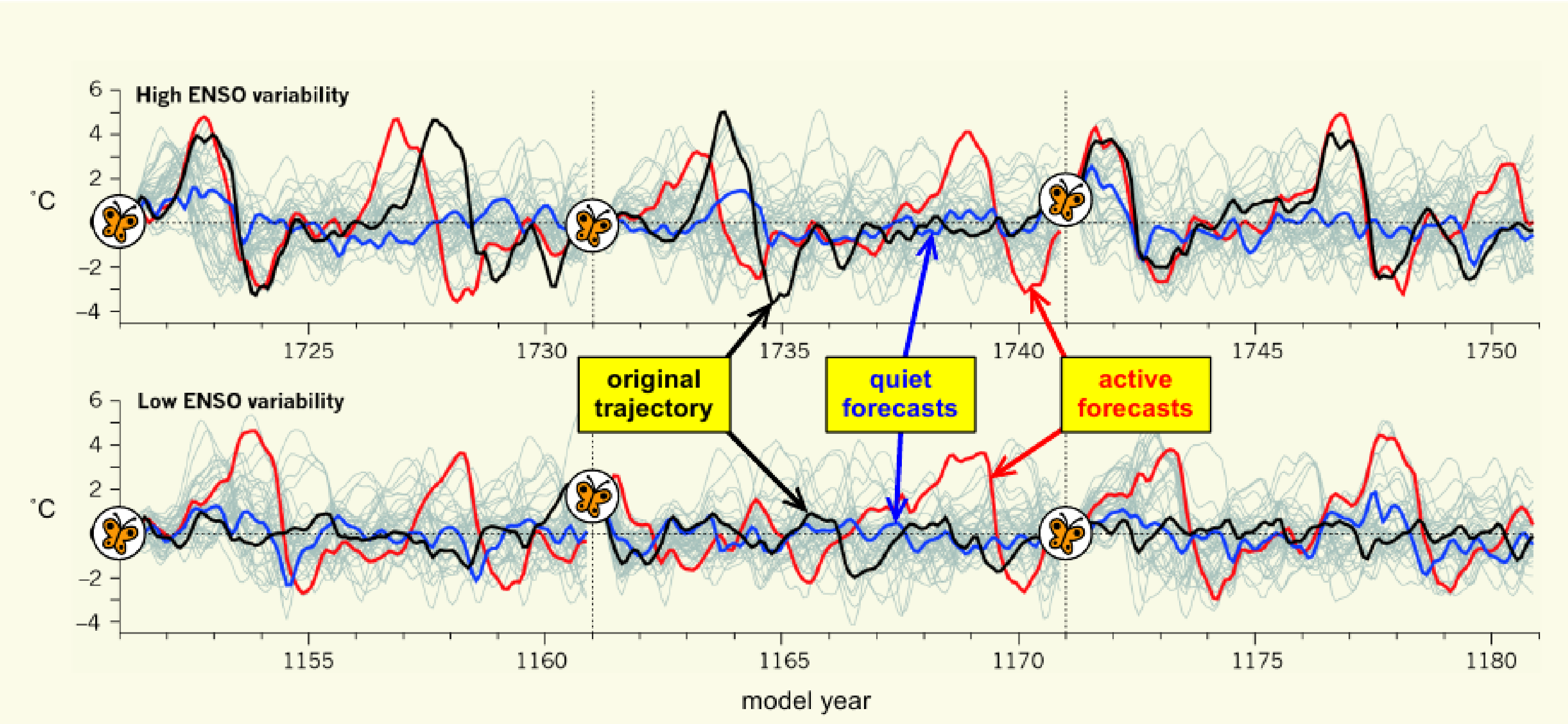


# ENSO modulation: is it decadally predictable?

**"Perfect-model" forecasts** of NINO3 SSTA, for extreme-ENSO epochs simulated by CM2.1




(External forcings **held fixed** at 1860 values.)

*Wittenberg et al. (J. Climate, 2014)*

Effects of the proverbial "flap of a butterfly's wing"...

# Summary

- Chaos is a **property of the system** we are simulating:
  - it is not just a shortcoming of the numerical approximations we employ; it is a consequence of the nonlinearity of the system, and its sensitivity to initial conditions, which are uncertain by definition
  - numerical models do spawn additional chaos, because of their design:
    - By accident: restarts, parameterisation branches
    - By design: use of stochastic parameterisations
- Key concept for chaos: sensitivity to initial conditions
  - Degrees of freedom in our system are a crucial pre-requisite
- Predictability of first → second kind: cover applications from NWP → climate prediction
- In practice: explore the growth of the ensemble spread to assess reliability of any forecast/prediction

Another “freebie” for interpreting the results you will obtain in P1:

Poincaré–Bendixson theorem