Exercise A

An electronics company, X Electronics, released a new monitor (the *SuperScreen*) a little more than a year ago. We know that X Electronics buys some of the monitor components such as the LCD panels from other manufacturers. At the time of the production of SuperScreen, they had two viable options for LCD panel manufacturers: company A or company B. When X Electronics had to choose between the two providers, they had to take two things into consideration: the price and the reliability of the panels within the one-year warranty period.

The most common failure of LCD panels is that some of the pixels malfunction (e.g. black dot defect: the pixel never emits light). If 3 or more defective pixels appear on a panel in the first year after purchase, X Electronics refunds the customer. In this exercise, we assume that all the customers return their monitors if at least 3 defective pixels appear on the panel.

The LCD panels produced by company A are more expensive than those from company B, as well as more reliable. Let's denote with p_A the probability that 3 or more defective pixels will appear within the first year on a panel manufactured by company A. This probability depends on the settings of the machine with which the panels are produced, leading to uncertainty in the p_A value. We know that p_A comes from a truncated normal distribution with domain [0.0,0.1] and parameters $\mu = 0.05$ and $\sigma = 0.01$. In JAGS code:

```
pA ~ dnorm(0.05,1/(0.01)^2)T(0.0,0.1)
```

If nobody changes the settings of the machine, all the panels produced by it will have a fixed $p_A^\star$ chance of failure, but we don't have any knowledge of the exact $p_A^\star$ just the distribution it comes from.

As mentioned before, the panels of company B are less reliable. Let's denote with p_B the probability that 3 or more defective pixels will appear in the first year of use of a company B panel. p_B comes from a truncated normal distribution with domain [.05, .15] and is parametrized by $\mu = 0.1$ and $\sigma = 0.02$, or in JAGS code:

```
pB ~ dnorm(0.1,1/(0.02)^2)T(0.05,0.15)
```

X Electronics has stated that it uses LCD panels bought from company A. We want to assess their claims based on consumer reports.

We have data from 5 different stores that each sold 50 SuperScreen monitors a year ago. The vector y contains the number of the monitors that were returned to the shop due of defective pixels (please remember, we assumed that every costumer returns the product if it has 3 or more defective pixels):

```
y = c(7,5,4,4,2)
```

Tasks

1. Rewrite the `BernBetaJagsFull.r` code provided to model the above task. Use the histogram created by the program to decide whether to believe X Electronics' statement that it uses the panels from company A. Explain your decision. You should submit the modified code, the resulting histogram and your explanation.

Hints:

- Use the Binomial distribution instead of Bernoulli distribution.
- Use p_A as θ.
- The prior distribution of p_A should be the domain-truncated normal distribution that was given above. Be careful: the second parameter of the `dnorm` distribution in JAGS is precision (the inverse of variance).

2. Let's assume that X Electronics, contrary to their claims, used the panels of company B. Do you believe this after seeing the data? Change the prior in task 1 from p_A's to p_B's distribution. Submit the new histogram and interpret the results.

3. Suppose that each store sold 500 SuperScreen monitors instead of 50 and that the number of returned monitors are given by the y' vector:

   ```
   y' = 10*y
   ```

   Does this change the credibility of X Electronics? Repeat task 1 and 2 using the new data, and interpret the results.

4. [Bonus task: you do not need to submit this, but if manage to do it, you can get extra points] Do a model comparison to determine if X Electronics is lying. The first model (M_1) is the honest company model which states that X Electronics' statement is true, and they indeed use company A's panels. The second model (M_2) is the dishonest company model according to which the SuperScreen has LCD panels supplied by company B. Determine the Bayes factor:

$$BF = \frac{P(D|M_1)}{P(D|M_2)}$$

Exercise B

For this exercise, we will continue our analysis of the SuperScreen monitors. At a later stage of the production of the SuperScreen, X Electronics decided to change the supplier of the LCD panels and switch to a third company, C. They produce the LCD panels for the SuperScreen monitors at five production sites, which use essentially the same machines, but which inevitably differ in some settings.

We assume that the probability p_C^i of 3 or more defective pixels appearing in the first year on the panels produced at site i (where $i \in [1,2,3,4,5]$, for the five sites) depends only on the settings of the machines at the site. We further assume that all p_C^i values come from the same distribution, since the machines used are nearly identical, but for their settings.

After one year of distribution, X Electronincs collected data from five stores. Each monitor's LCD panel has a unique product number which identifies the production site of the panel. Every store sold 40 monitors from each production site, which means that altogether the 5 stores sold 1000 monitors.

For each store and for each production site the number of faulty monitors is given by the following table:

store	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
production site	1	1	1	1	1	2	2	2	2	2	3	3	3	3	3	4	4	4	4	4	5	5	5	5	5
number of faulty monitors	1	5	5	5	2	4	2	1	3	3	3	5	3	4	6	4	2	4	5	1	1	3	2	4	1

Task

Estimate the parameters of the distribution of p_C^i based on the given data, assuming the distribution is a truncated normal distribution with unknown μ and σ values. In JAGS code:

```
pCi ~ dnorm(μ,1/σ^2)T(0.0,0.5)
```

Our prior belief is that μ is about 0.1. To formalize our beliefs we choose `beta(1.1,9.9)` as a hyper prior to μ. Our hyper prior for the σ parameter is the `gamma(shape = 2, rate = 28)` distribution, which is quite narrow with mean 0.025, to reflect our assumption about the σ parameter being small relative to the μ parameter. The `assignment2_task2_script.r` script contains everything you will need (it reads data, runs chain and plots results) except the model itself. Your only task is to write the model solving the above problem. To see how you can construct such a model, see the `BernBetaMuKappaJags.r` file.

You should submit the and the histograms of the sampled hyper-prior parameters.

Exercise C

Dr. Jenner is looking for experimental evidence that the perceived price of a placebo can affect the placebo's efficiency to suppress cold symptoms. It is known that without medication 30% of people infected with a cold virus make a full recovery within 5 days.

In an experiment conducted by Dr. Jenner 90 healthy volunteers took part. At the beginning of the experiment they were infected with a cold virus and then were either given a placebo that was advertised as a drug costing 2$/pill (30 participants), 8$/pill (30 participants) or no pill at all (30 participants). On the 6th day, participants returned to meet with a physician to check their symptomatology. Importantly, they were told not to take any active medication during the one-week period of the experiment and were oblivious with respect to the purpose of the study. At the end of the experiment, Dr. Jenner found that 8 participants from the non-placebo group had recovered, while 11 and 14 had recovered from the cheap and expensive placebo groups respectively.

Task

Determine whether the placebos had any impact on recovery and if so, whether the price modulated the placebo's effect. To that end, you will need the histograms of $\mu_{\text{noPlacebo}} - \mu_{\text{2\$Placebo}}$, $\mu_{\text{noPlacebo}} -$

$\mu_{8\$\text{Placebo}}$ and $\mu_{2\$\text{Placebo}} - \mu_{8\$\text{Placebo}}$. Please submit your code alongside the histograms. This is analogous to the case of multiple coins from multiple mints.

Hints:

- Use the Binomial distribution instead of Bernoulli distribution.
- You might want to use nested indexing. Check page 253 of the textbook for an example.

You will also find attached two functions that will help you with plotting your results (`openGraphSaveGraph.r` and `plotPost.r`). These functions are called in some of the other scripts provided so make sure you have them in the r path if you wish to use them. However, you are welcome to use your own plotting functions.