

BDA HM11

Piero Birello

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17.1

Qualitative discrepancy of the model with data can be already seen from the posterior predictive check regression lines fitted on the whole dataset. Variations between counties shift vertically separate subsets of data, and we can account for this introducing a hierarchical model, as done indeed. However, what such hierarchical linear model cannot account for are consistently higher income values for intermediate family size, as well as consistently lower income values for small and large family size. Similar considerations apply to posterior predictive checks for the hierarchical linear model introducing variations between counties, which we do not show here for brevity.

A possible approach to reject a model without comparing it to another one (via posterior odds or BF) is to compute the so called *Bayesian p-value* as follows: sample predictions of the data according to the obtained posterior distribution, and meanwhile compute a measure of discrepancy of such predictions with respect to ground truth values. One should treat this value as a normal NHT p-value, and consequently reject or not the model. This approach presents similar limitations to p-values, among which the fact that any set threshold brings to rejection of the true hypothesis with some finite probability irrespective of the sample size. More generally, such methodology is conceptually opposite to the one of Bayesian linear regression, which is explicitly designed to properly treat the noise distribution for the dependent value around its mean.

17.2

We run a linear regression on weight and height data using standardized variables first and non standardized variables after. MCMC chains have different behaviour as visible from Figure 2. When using non standardized data, autocorrelation decays at longer distances resulting in significantly smaller effective sample size. However, despite impacting algorithmic efficiency, standardization does not impact posterior estimates of the parameters, which are consistent among the two linear regression models although showing different uncertainty (Figure 3).

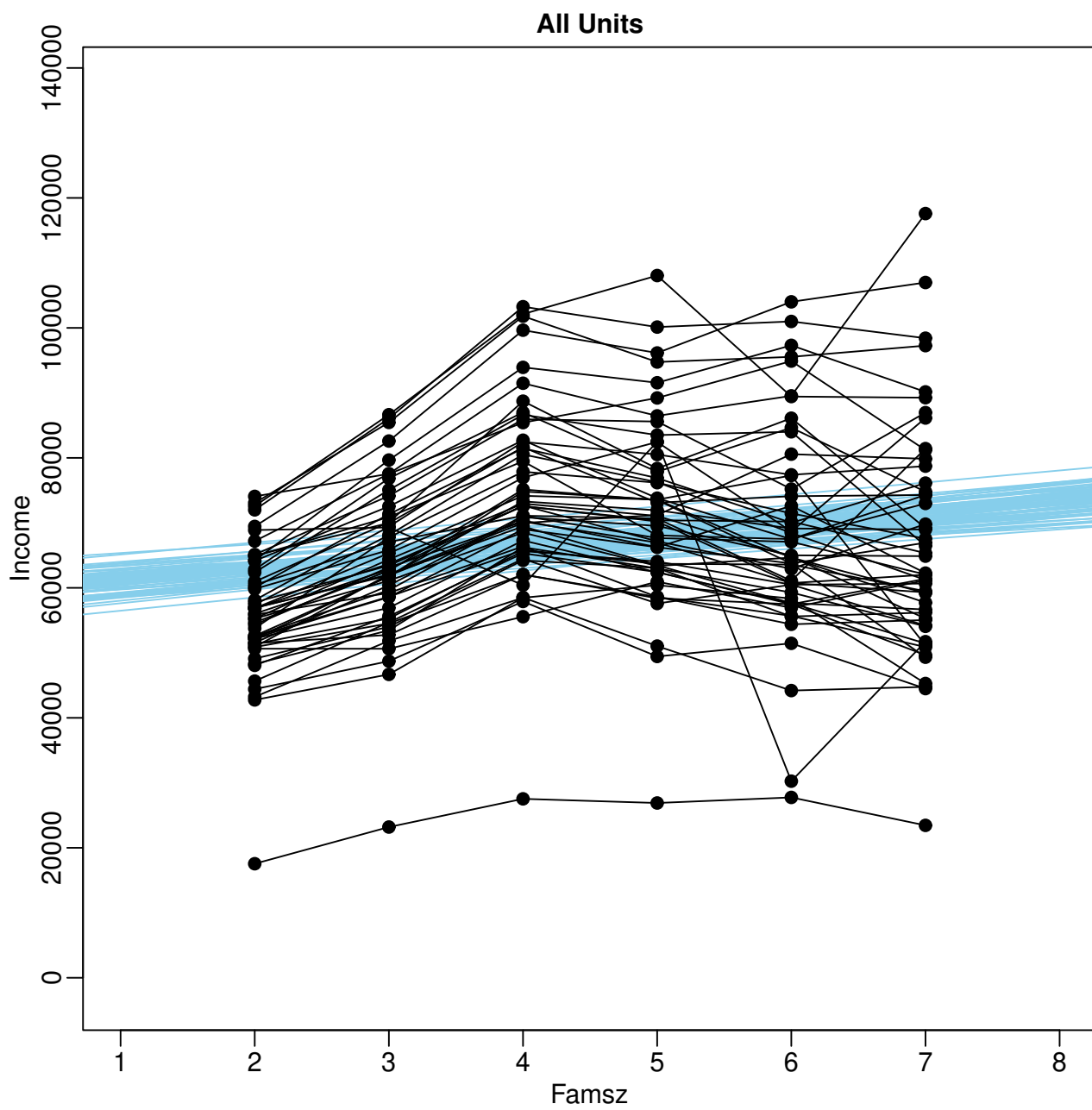


Figure 1: Caption

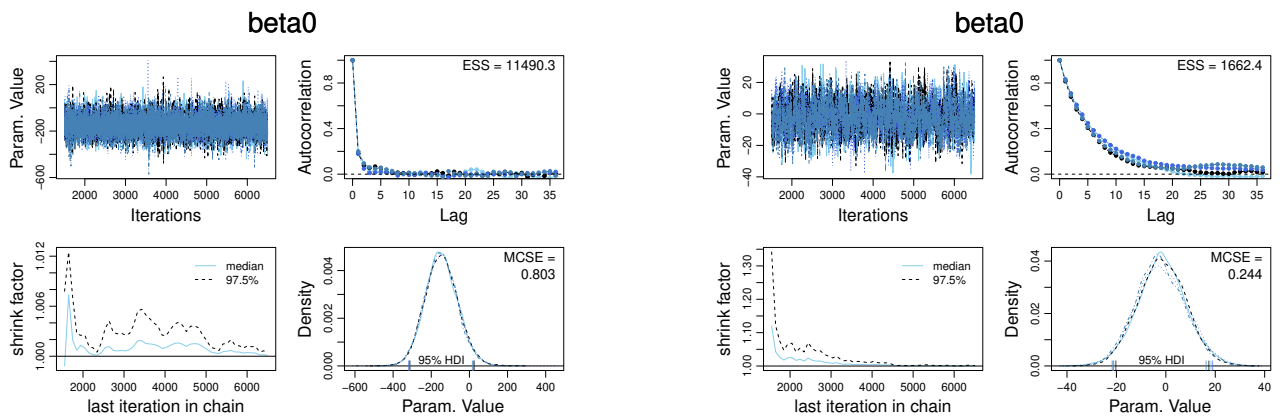


Figure 2: Chain diagnostics, exercise 17.2.

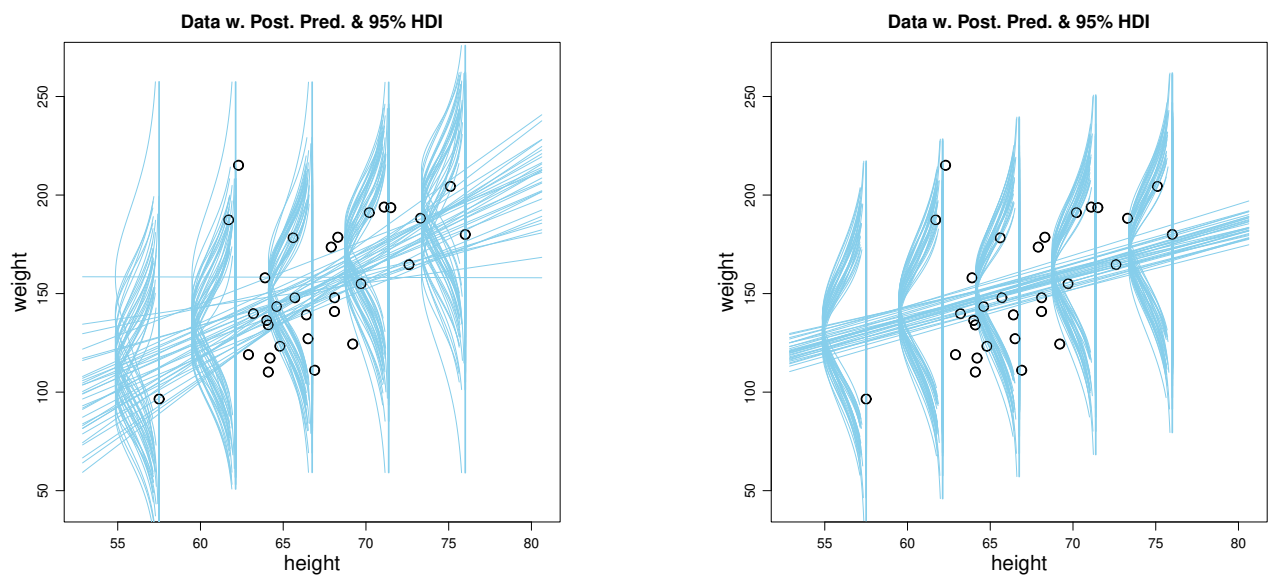


Figure 3: Posterior predictive checks, exercise 17.2.