Exercises, Class 1

2.1. $p_A(x) = \frac{1}{4} \rightarrow p_A(1) = \frac{1}{4},\ p_A(2) = \frac{1}{4},\ p_A(3) = \frac{1}{4},\ p_A(4) = \frac{1}{4}$

No bias, all outcomes are equally likely.

$p_B(x) = \frac{x}{10} \rightarrow p_B(1) = \frac{1}{10},\ p_B(2) = \frac{2}{10},\ p_B(3) = \frac{3}{10},\ p_B(4) = \frac{4}{10}$

We believe that higher values are more likely.

$p_C(x) = \frac{12}{25x} \rightarrow p_C(1) = \frac{12}{25},\ p_C(2) = \frac{12}{50},\ p_C(3) = \frac{12}{75},\ p_C(4) = \frac{12}{100}$

We believe that lower values are more likely.

2.2. $p(M=A) = p(M=B) = p(M=C) = 1/3$

Let $x = (x_1, x_2, x_3, x_4)$ be the vector counting occurrences of 1's, 2's, 3's, 4's.

$n = x_1 + x_2 + x_3 + x_4$ number of trials

$$p(M|x) = \frac{p(x|M)\,p(M)}{p(x)} \propto p(x|M)\,p(M)$$

$p(x|M)$ is a multinomial distribution, then

$$p(M|x) \propto \frac{n!}{x_1!\,x_2!\,x_3!\,x_4!}\, p_M(1)^{x_1}\, p_M(2)^{x_2}\, p_M(3)^{x_3}\, p_M(4)^{x_4} \cdot p(M)$$

We get: $p(A|x) \propto 3.34 \cdot 10^{-4}$, $p(B|x) \propto 1.72 \cdot 10^{-5}$, $p(C|x) \propto 2.23 \cdot 10^{-10}$

when $x = (25, 25, 25, 25)$. Hence, model A is the most likely.

We get: $p(A|x) \propto 2.51 \cdot 10^{-10}$, $p(B|x) \propto 4.88 \cdot 10^{-28}$, $p(C|x) \propto 4.43 \cdot 10^{-4}$

when $x = (48, 24, 16, 12)$. Model C is the most likely.

4.4. $p(x) = 6x(1-x)$ A) See IntegralOfDensity.R

B) $\int_0^1 dx\, p(x) = 6\int_0^1 dx\,(x-x^2) = 6\left[\frac{x^2}{2}-\frac{x^3}{3}\right]_0^1 = 6\left(\frac{1}{2}-\frac{1}{3}\right) = 6\,\frac{3-2}{6} = 1$

c) $p(x)$ is correctly normalized

D) From inspecting the graph, the maximal value of $p(x)$ is 1.5

4.6. 20% 1st grade, 20% 6th, 60% 11th.

From the table of conditional probabilities, we have to reconstruct that of joint probabilities.

$p(\text{food}, \text{grade}) = p(\text{food} \mid \text{grade})\, p(\text{grade})$

Then the joint probability table reads:

	ICE CREAM	FRUIT	FRENCH FRIES
1st	0.3/0.2	0.6/0.2	0.1/0.2
~~6th~~	0.6/0.2	0.3/0.2	0.1/0.2
11th	0.3/0.6	0.1/0.6	0.6/0.6

Grade and favourite food are not independent, since $p(\text{food} \mid \text{grade}) \neq p(\text{food})$

BDA HM1

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Coding exercises

4.1

```
# conditional probability of hair colors given Brown eyes
show(EyeHairProp["Brown",] / EyeProp["Brown"])
# conditional probability of eye colors given Brown hair
show(EyeHairProp[,"Brown"] / HairProp["Brown"])
```

4.2

```
N = 500 # Specify the total number of flips, denoted N.
pHeads = 0.8 # Specify underlying probability of heads.
# Flip a coin N times and compute the running proportion of heads at each flip.
# Generate a random sample of N flips (heads=1, tails=0):
flipSequence = sample( x=c(0,1), prob=c(1-pHeads,pHeads), size=N, replace=TRUE )
# Compute the running proportion of heads:
r = cumsum( flipSequence ) # Cumulative sum: Number of heads at each step.
n = 1:N                    # Number of flips at each step.
runProp = r / n            # Component by component division.
# Graph the running proportion:
plot( n , runProp , type="o" , log="x" , col="skyblue" ,
      xlim=c(1,N) , ylim=c(0.0,1.0) , cex.axis=1.5 ,
      xlab="Flip Number" , ylab="Proportion Heads" , cex.lab=1.5 ,
      main="Running Proportion of Heads" , cex.main=1.5 )
# Plot a dotted horizontal reference line:
abline( h=pHeads , lty="dotted" )
# Display the beginning of the flip sequence:
flipLetters = paste( c("T","H")[flipSequence[1:10]+1] , collapse="" )
displayString = paste0( "Flip Sequence = " , flipLetters , "..." )
text( N , .9 , displayString , adj=c(1,0.5) , cex=1.3 )
# Display the relative frequency at the end of the sequence.
text( N , .8 , paste("End Proportion =",runProp[N]) , adj=c(1,0.5) , cex=1.3 )
```

4.4

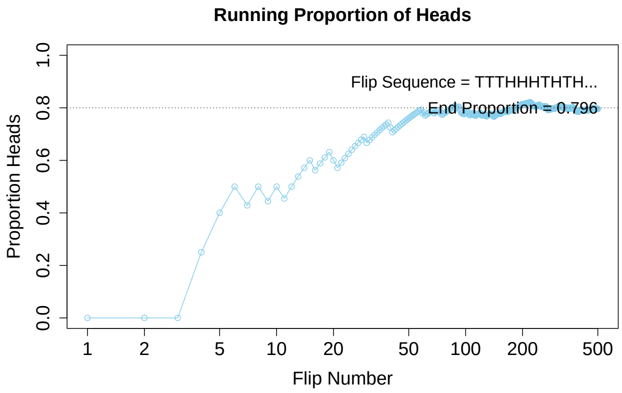

Figure 1: Figure for exercise 4.2

```
source("DBDA2E-utilities.R")

# Graph of normal probability density function, with comb of intervals.
xlow  = 0 # Specify low end of x-axis.
xhigh = 1 # Specify high end of x-axis.
dx = (xhigh-xlow)/1000                # Specify interval width on x-axis
# Specify comb of points along the x axis:
x = seq( from = xlow , to = xhigh , by = dx )
# Compute y values, i.e., probability density at each value of x:
y = 6*x*(1-x)
# Plot the function. "plot" draws the intervals. "lines" draws the bell curve.
openGraph(width=7,height=5)
plot( x , y , type="h" , lwd=1 , cex.axis=1.5
      , xlab="x" , ylab="p(x)" , cex.lab=1.5
      , main="Probability Density" , cex.main=1.5 )
lines( x , y , lwd=3 ,  col="skyblue" )
# Approximate the integral as the sum of width * height for each interval.
area = sum( dx * y )
print(area)
saveGraph( file = "Figures/IntegralOfDensity" , type="pdf" )
```

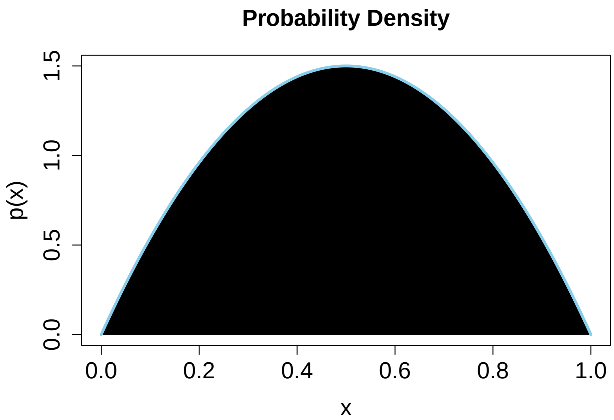

Figure 2: Figure for exercise 4.4