Coupling

Given two probability distribution, p and q, we are interested in constructing a joint distribution (coupling) for (X, Y), such that $X \sim p$, $Y \sim q$, and $P(X = Y)$ with high probability.

The following paper: https://arxiv.org/abs/2201.09585 proposes such a construction, and shows it may be applied to two multivariate Gaussian distributions (with different covariance matrices).

This project consists in implementing the proposed approach, and compare it to the simpler approach known as Thoriston's algoritm (Appendix I in the paper). Explain why the latter approach is valid, and is related to rejection sampling. You can consider different problems, of increasing dimension. Notice that the running time of these algorithms is random, comment on the corresponding distributions.

Bonus: If time permits, you may try to understand how you may use such coupling inside a random walk Metropolis, and replicate the experiments in Section 5.3.