Assignment 1
FIE450

Before you start read the following points carefully!

1. Start your `R`-script with the line `rm(list=ls())`
2. Assign intermediate results of each sub task to the variable names given in parenthesis! Only these variables are considered for grading. If they are not there or misspelled then no points are given for this task.
3. Do not round results
4. The assignment shall be solved in one single `R`-script named `Assignment-1.R`.
5. Submit your `R`-file and two `csv`-files. You must not submit `docx`-files and `pdf`-files.
6. Comment your are `R`-code

Term structure of risk-free interest rates Assume that the term structure of risk-free interest rates is given by the prices of *risk-free* zero coupon bonds, $Z_i(t_0)$, at time t_0 and with maturity $t_i \in \{0.1, 0.2, 0.3, 0.4, 0.5\}$ years. That is the price of 1 Kronor received at t_i is worth Z_i Kronor at t_0. The following zero coupon bond prices are quoted

$$Z_1(T_0) = 0.9995 \quad Z_2(T_0) = 0.9980 \quad Z_3(T_0) = 0.9955 \quad Z_4(T_0) = 0.9920 \quad Z_5(T_0) = 0.9876$$

Compute:

1. Download the entire available time-series of daily and monthly stock price information of Norsk Hydro (Ticker: NHY.OL) denominated in Norwegian Kronor from `finance.yahoo.com`.

   For each sampling frequency calculate simple returns (`r.daily`, `r.monthly`). Then compute the annualized means (`mu.daily`, `mu.monthly`), annualized standard errors (`se.daily`, `se.monthly`) and the 95% confidence intervals (`ci.daily`, `ci.monthly`).

   Compare the results of both sampling frequencies. Which frequency do you prefer and why? Give your reasoning in not more than three lines as a comment in your `R`-code.

2. Estimate the implied volatility of a put option on Norsk Hydro (Ticker: NHY9R38) from the bid and ask quotes on January 22, 2019. These quotes are available on the webpage of the Oslo Stock Exchange. Determine the midquote (`P.market`) and the stock price on that day (`S0`). Assign the implied volatility to a variable called `sigma`.

3. Use the estimated implied volatility to price a *down-and-in barrier call option* on Norsk Hydro based on market information obtained for January 22, 2019. The barrier and strike price, respectively, shall be $b = K = 32$. The payoff function at maturity $T = 0.5$ years is given by

   $$I\{\tau(b) \leq T\}(S(T) - K)^+, \tag{1}$$

   where

   $$\tau(b) = \inf\{t_i : S(t_i) < b\} \tag{2}$$

   is the first time $t_i \in \{0.1, 0.2, 0.3, 0.4, 0.5\}$ the price of the underlying asset S drops below b and $I\{\}$ denotes the indicator of the event in braces, i.e. I is one if the expression within braces is true and zero otherwise. Thus, a down-and-in call gets "knocked in" only when the underlying asset crosses the barrier b from above.

   Incorporate the term structure of risk-free interest rates into your Monte-Carlo simulation framework using

$$S(t_{i+1}) = S(t_i)\frac{Z_i(t_0)}{Z_{i+1}(t_0)} \exp\left(-\frac{1}{2}\sigma^2(t_{i+1}-t_i) + \sqrt{t_{i+1}-t_i}\sigma W_{i+1}\right), \qquad (3)$$

where, again, $Z_i(t_0)$ denotes the price of a *risk-free* zero coupon bond at t_0 with maturity t_i and W is a standard normal distributed random variable. Simulate $n = 10000$ price paths in total.

Using regular Monte-Carlo simulation compute:

(a) Simulate paths of stock prices. (Assign this result to a variable called `S`)

(b) Compute the discounted payoffs of each scenario. (`P`)

(c) Compute the Monte Carlo estimator of the option price. (`V`)

(d) Compute the standard error of the Monte Carlo estimator. (`se`)

(e) Compute a 95% confidence interval of the Monte Carlo estimator. (`ci`)

Using Monte-Carlo simulation and antithetic variates compute:

(a) Simulate paths of stock prices. (Assign this result to a variable called `S.as`)

(b) Compute the discounted payoffs of each scenario. (`P.as`)

(c) Compute the Monte Carlo estimator of the option price. (`V.as`)

(d) Compute the standard error of the Monte Carlo estimator. (`se.as`)

(e) Compute a 95% confidence interval of the Monte Carlo estimator. (`ci.as`)

(f) Does antithetic sampling improve the accuracy of the estimate? Check!