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TACTICAL ASSET ALLOCATION AND MACHINE LEARNING

Empirical findings on weights' Portfolio optimization with Elastic Net Regularization

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Abstract

This paper studies how a machine learning algorithm can generate tactical allocation which outperforms returns for a pre-defined benchmark. We use three distinct and diverse data sets to implement the model which tries to forecast the next month's a selected equity index price. The algorithm used to accomplish this task is Elastic Net. Once the predictions are generated from an out-of-sample subset, we elaborate a tactical portfolio allocation aiming to maximize the return of a different combination of classical allocation between bonds and equity, and a risk parity strategy. Finally, we evaluate those returns by comparing them to the benchmark.

***Keywords*:** machine learning, elastic net, portfolio optimization, tactical allocation, investment strategy

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Introduction

Since the early 2000's portfolio managers asked themselves if a structural revolution, run by statistical and mathematical researchers, could replace their role in determining asset allocations or even forecast future returns for different asset classes. Indeed, as mentioned in a recent study of Coleman, Braiden, et al. (2020) the supervised learning methods showed at the beginning of this decade had potential to replace humankind decisions - often subject to biased behavior - by beating human decision makers or predefined benchmarks. An algorithm can read words exponentially faster than any individual. Nowadays, the job of a machine learning algorithm is further simplified by the availability of financial data collected in databases such as, Bloomberg Terminal, Eikon by Thomson Reuters, or any other government clouds (exp. USA.gov). The aim of this paper - based on the previous research of Routledge (2019) – is to prove that throughout the supervised learning approach on three different datasets an investor can maximize its portfolio's return and beat the common strategy allocation "Buy & Hold". The mentioned datasets are the following: (a) Earning yield & Price to Dividend ratio (b) an array of macroeconomic series, and (c) investors sentiment analysis. The core goal of such an algorithm is to forecast the price of the selected equity index in the upcoming month. Once the predictions are computed, we will construct a tactical asset allocation based on the magnitude of the return of those forecasts. The word "tactical" specifies a broad area of active management investment strategies aiming to take advantage of inefficiencies or temporary imbalances among asset classes – more specifically, in this work project we study the returns of US Equities. Finally, the portfolio designed by the quantitative analysis will be evaluated in comparison to a pre-defined benchmark.

I. Literature Review

The aim of Routledge's (2019) research bases its root on a simple economical objective: how to allocate wealth between two different asset classes, equity and bonds. The author set as the core question of its project, the predictions of the rate of return of a stock in excess of a risk-free asset. Although, the equity premium has an observed historical return of around 6% - its expectations are somehow oscillating between 0 and 12% (Cochrane, 2011) - it can still be forecasted to some extent. In order to create a model with such goal, the author selected the Elastic Net Regression, which contains a trade-off of regularization over the model's parameters. Throughout this research paper, we use the same powerful machine learning algorithm for two main reasons. Firstly, to maintain coherence with the literature on which this work is based on and secondly because the model is a decision maker over the features – since it uses shrinkage penalties to the estimators. The ability of the algorithm to handle and tackle down lots of non-relevant regressors that equity returns are not depending on, is undeniably useful to exploit patterns that perform well out of sample. Moreover, the writer interprets the work of Gilboa and Schmeidler (2010) on the preference parameters of the Elastic Net, and he defines the concept of simplicity as "*people like models that explain the data (e.g., likelihood) and people like models that are simple (e.g., a small number of parameters)"*. So, based on this definition, he identifies two *a* and *b* parameters that later will be used in the Elastic Net theory (or $v(\theta)$) to impose preferences over the features. Finally, he solves a utility function which aims to maximize the investor's wealth for every combination of the parameters *a* and *b* (*Appendix A*). However, *a* and *b* are not different from the more generally defined λ and $\boldsymbol{\alpha}$ hyperparameters of a typical Elastic Net regression (Aurélien, 2019).

So, the objective of this paper will shift substantially from this point on. The goal of our work project as described in the introduction will not aim to find different wealth by swinging the "simplicity" parameters, but it will focus its attention on finding the optimal λ and $\boldsymbol{\alpha}$ throughout a tuning process to predict the equity asset price in the most efficient way. From those accurate predictions we will construct a tactical portfolio allocation aiming to outperform a selected benchmark. Moreover, the author explores the model on three different datasets. The first is price-dividend data as in Cochrane (2011), and we decided to add the earning yield based on the findings of Sun (2019). The second is an array of monthly macroeconomic series from St. Louis Fed's data (FRED), in line with the finding of Ludvigson and Ng (2009). Finally, the author created a text dataset extracted from the Beige Book of the US Federal Reserve which returns information from informal surveys. The last two datasets will be additionally modified. After a careful evaluation of the original set of variables from the "hard" data we selected a cluster of them, and we added other important indicators that were missing. Based on the finding of Zouaoui, Mohamed, et al. (2010) and Chung, San-Lin, et al. (2010), we substitute the text mining dataset with an assemble of different features on investors sentiment indicators, volatility, and oil/commodity price.

We choose the Dow Jones Industrial Average Index (INDU:IND) as equity asset, because it is one of the most quoted financial barometers in the world and it is increasingly considered a valid option to trace financial markets trend. The Index is a price-weighted measure of 30 U.S. blue-chip companies – which covers all industries except transportation and utilities. We choose the 10 years US government note as bond because of its characteristic of being a benchmark that guides other interest rates. More importantly, this bond's yield is used to calculate the Bond Equity Earnings Yield Ratio (BEER), which helps to understand the value created by investing one US dollar in bonds versus investing that same amount in stocks. If the earnings yield falls below the yield of the 10-year US Treasury, the market is considered "bearish"– meaning it assumed stock prices will decline. Despite that, the US Federal Reserve considers the 10-year Treasury yield curve before making its decision to change the federal funds rate, because it serves as confidence indicator of the investors' view on the economic growth.

II. Methodology

As mentioned above the statistical regression used to generate the predictions is the Elastic Net. This algorithm combines the penalties of the Lasso (ℓ_1) and Ridge (ℓ_2). According to the literature proposed by Zou & Hastie (2005), the Ridge's βs are estimated by minimizing the following function with the partial derivative:

$$\sum_{i=1}^{n}(y_i - \beta_0 - \sum_{j=1}^{p}\beta_j x_{ij})^2 + \lambda\sum_{j=1}^{p}\beta_j^2 \;=\; RSS + \lambda\sum_{j=1}^{p}\beta_j^2, \tag{1}$$

$for\ i \in [0, n]\ and\ j \in [0, p], p\ is\ the\ number\ of\ features\ and\ n\ is\ the\ length\ of\ the\ array.$

The "tuning parameter" $\lambda \geq 0$ controls the penalty scale over the coefficients. When it is exactly equal to zero, the equation (1) will be reduced to a simple Least Squares Estimate. On the other hand, if $\lambda \rightarrow \infty$, the impact of the second term of the equation, called "shrinking penalty" increases and it will automatically drive the ℓ_2's βs closer to zero. Hence, selecting a good value of λ is critical. The main characteristic of such regularization is that the penalty will possibly shrink all the coefficients towards zero, but they will not be exactly 0. The second regularization is the Lasso, which overcomes the previously described issue of many features with very small coefficients by minimizing the following function and taking the partial derivative:

$$\sum_{i=1}^{n}(y_i - \beta_0 - \sum_{j=1}^{p}\beta_j x_{ij})^2 + \lambda\sum_{j=1}^{p}|\beta_j| \;=\; RSS + \lambda\sum_{j=1}^{p}|\beta_j|, \tag{2}$$

In the equation (2), the penalty of $\ell 1$ will set the coefficients to be exactly equal to zero, when the λ is sufficiently large. This means that those zero-coefficient features will not be used to predict the target variable.

Multicollinearity can cause serious problems for an estimation. It generally leads to large variance's and covariance's that cause the prediction to be less precise and to lead to wrong inferences. However, because the Elastic Net uses regularization techniques that attempt to reduce the variance by using penalty hyperparameters, we can conclude that by some means it performs a variable selection, that helps mitigate multicollinearity and model complexity (Tamura, Ryuta, et al., 2017).

Based on the findings of Routledge (2019), the most effective machine learning model is the Elastic Net, which incorporates penalties from both ℓ1 and ℓ2 regularization:

$$\frac{1}{2n}\sum_{i=1}^{n}(y_i - \beta x_{ji})^2 + \lambda\,(\frac{1-\alpha}{2}\sum_{j=1}^{p}\beta_j^2 + \alpha\sum_{j=1}^{p}|\beta_j|) \tag{3}$$

In addition to setting and choosing a λ value, the elastic net also allows to tune the alpha parameter where $\boldsymbol{\alpha}$ = 0 corresponds to ridge and $\boldsymbol{\alpha}$ = 1 to lasso. Hence, it is possible to choose a $\boldsymbol{\alpha}$ value between 0 and 1 to optimize the algorithm. Effectively this will shrink some coefficients and set some to 0 for sparse selection.

Before fitting the model, the input variable x will be scaled as, $\frac{x_i - Min(x)}{Max(x) - Min(x)}$ for i ∈[0,n], where i is the length of the variable's vector. This scaling methodology essentially shrinks the range such that it will be bound between 0 and 1 (or -1 to 1 if there are negative values). The transformation will be pursued to avoid that large magnitude features will influence the algorithm's prediction during the fitting process.

The next step to implement the Elastic Net algorithm will require a method to tune the parameters λ and $\boldsymbol{\alpha}$. As Aurélien (2019) highlights, the k-folds cross-validation is the most common method used by statisticians to handle such matter. The first stage is splitting each dataset into Training and Out-of-sample observations (OOS). Afterwards the training set is additionally fragmented into k-folds and one of them is recursively left out of the training process in order to evaluate the cross-validation error (or scoring error) on the test set (or "in sample fold"). The Mean Squared Error shall be minimized during the hyperparameters tuning. It is calculated as follows, $MSE = \frac{1}{n}\sum_{i=1}^{n}(y_i - \widetilde{y_i})^2$ for $j \in$ [0, n], where n is the length of the array. Such metrics measure how close the fitted data is to the actual values. To obtain a more interpretable number for the Mean Squared Error, the Dow Jones Industrial Average Index Price (or independent variable) will be transformed into a natural logarithm base before fitting the machine learning algorithm. The magnitude of the returns derived from those predictions will not be compromised, thus seems to be a valid solution without altering the outcome. Finally, the OOS is used to test the efficiency of the algorithm.

The grid search stage of the parameter tuning λ and $\boldsymbol{\alpha}$ values works as follows: each of them is assigned a list of values ($\boldsymbol{\alpha}$∈[0,1] and λ∈[0.0001,100]), and the cross-validation error is computed for each of the in fold predictions with different combinations of those parameters. Among those combinations it will select the tuning parameters values for which the cross-validation error is minimized. Finally, the model is re-fitted using all the available observations and the selected value of the optimal tuning parameter.

Once the forecasts on the OOS are generated, they will be evaluated with the same error metric, the mean square error (MSE). However, such error rate does not fully explain if the monthly price movement was successfully predicted. Hence, we used an additional approach to further explore the predicted return. The objective is to fully understand if the algorithm can predict the correct sign of the change in price. To do so, those returns will be evaluated by using a "pseudo"-confusion matrix. To clarify the usage of the word "pseudo" it is necessary to emphasize that, although such evaluation method is a technique for summarizing the performance of a classification algorithm, for the sake of this research paper the goal of such matrix will be to understand the correct prediction between negative and positive returns. The number of correct and incorrect predictions is summarized with count values and broken down into each class. It gives insight not only into the errors made by the algorithm but more importantly the types of errors that are being made.

	Positive return (Forecast)	***Negative return (Forecast)***
Positive return	*TRUE POSITIVE (TP)*	*FALSE NEGATIVE Type II Error (FN)*
Negative return	*FALSE POSITIVE Type I Error (FP)*	*TRUE NEGATIVE (TN)*

From the confusion matrix we can derive two important insights. First the precision rate, which means the proportion of the correctly predicted positive returns amongst the total positive. The economic importance of such metric is to understand weather Elastic Net regression has predicted a positive return. The second most relevant measure is the recall rate which states if the algorithm did not confuse months with positive returns for negative returns. Both rates explain the reason of over/or under-performing of the portfolio returns, which will be introduced later. Additionally, the accuracy rate is added to have a broader overview on the proportion of negative and positive returns predicted. The rates described will be calculated as follows:

$$Accuracy\ rate = \frac{TP+TN}{TP+TN+FP+FN};\ Recall\ rate = \frac{TP}{TP+FN};\ \ Precision\ rate = \frac{TP}{TP+FP} \qquad (4)\ (5)\ (6)$$

Since the accuracy rate works well with symmetric datasets – which is not the case for stock market returns distribution –, it does not solely explain the success of the algorithm. Thus, a higher percentage number on such a rate does not necessarily mean an excellent result. Perhaps the result might be mediocre or even appealing depending upon the problem. Therefore, the recall rate is computed to indicate at which percentage the class is correctly recognized. A high recall rate states that the number of false negatives (the predicted positive return is instead negative) is small, which is of fundamental importance for the later stage on the asset allocation. Indeed, an incorrect prediction of negative returns will most likely generate a poor decision on the weights to assign on each asset class. Additionally, a high precision indicates that a labelled positive return is indeed positive, or in other words a small number of false positive. To shed light on the two concepts, when there is a high recall and low precision it means that most likely the portfolio will underperform the benchmark especially in the positive economic cycle but in the negative downturn the portfolio allocation will be less likely expose on the risky asset. Low recall and high precision imply that the portfolio will probably outperform the benchmark since is able to predict one month in advance in which direction the stock market is heading. But whenever there will be a sudden sell-off, the strategy will fail, which means the allocation will not be defensive enough to protect investors from those negative returns.

III. Data

As already mentioned above, three datasets are chosen BOB, FRED, and ALLAN, $h_n = \{BOB, FRED, ALLAN\}$, with the necessary apologies for the informality and so to preserve the adherence with the research paper of Routledge (2019).

BOB database- Price to Dividend & Earnings Yield

Following Routledge (2019) and Campbell and Shiller (1988a, 1988b), the price to dividend ratio is the default feature selected to predict price (see Figure 1), together with an additional regressor such as earning yield (see Figure 2), Sun (2019), both datasets refer to the Dow Jones Average Industrial Index. In order to obtain a realistic forecast, the P/DVD ratio is backward lagged by six months (*P/DVD6M*) and three months (*P/DVD3M*), as well the earnings yield ratio

(*Earning_yield6M* and *Earning_yield3M*). The dataset begins from 2006 July 31st. This is the shortest dataset among the others as a longer reliable data series was not available.

FRED database – Broad Array of Macro data

From the original list of variables of Ludvigson and Ng (2009) we extract the following macroeconomic indicators:

- US unemployment rate % – *EHUPUS1M*, the rate tracks the number of unemployed persons as a percentage of the labor force (the total number of employed plus unemployed). These figures generally come from a household labor force survey, and it is released quarterly.
- US real Gross Domestic Product annual change YoY – *EHGDUSY1Y*, it is the inflation adjusted value of the goods and services produced by labor and property located in the United States, it is released yearly.
- Adjusted Retail & Food Services Sales Seasonal Adj Total Monthly % Change – *RSTAMOM1M*, it measures the resale of new and used goods to the public, for personal or household consumption.
- US Trade Balance of Goods and Services Seasonal Adj – *USTBTOT2M*, it measures the difference between the movement of merchandize export and imports. The data is released every 2 months.
- FOF Federal Reserve US Households & Non-Profit Organization Net Worth Change – *NWORCHNG1M*, it tracks a breakdown of assets, liabilities and net worth for households and nonprofit organizations. It includes domestic hedge funds, private equity funds and personal trusts.

- US Personal Consumption Expenditure Core Price Index MoM Seasonal Adj – *PCE CMOM1M*, it measures the overall price changes for goods and services purchased by consumers. Also, it is the preferred measure for Fed to track inflation.
- US Industrial Production MOM Seasonal Adj– *IP CHNG1M*, it measures the output of industrial establishments in the mining and quarrying, manufacturing, and public utilities sectors.
- Chicago Purchasing Managers Index – *CHPMINDX*1M, it summarizes the current business activity at the end of each month.
- US Industrial Production Industry Groups Manufacturing MoM SA – *IPMGCHNG1M*,
- US Treasury Federal Budget Debt, US Personal Income MoM SA – *FDDSSD1M*, it is the difference between government revenues and government expenditures.
- US New One Family Houses Sold Annual Total MoM SA – *NHSLCHNG1M*, it tracks sales of newly constructed homes during the reference period. The Implicit US index is computed by taking the number of houses sold in the US and dividing it by the seasonally adjusted number of houses sold in the US.

In most of the cases, those features are announced with a certain lagged period, usually on monthly basis – the variables with different time lags are underlined in the description above. Hence, it will be necessary to resample the data so that information will be lagged to the correct announcement period. This methodology will predict the Dow Jones Industrial Average Index Price with the current information, so to avoid forward looking bias. Finally, the features are scaled to a given range as described above. The dataset begins from 31st March 1992, because we decided to cut off the longer history variables and start all of them from the same month and year to generate valid predictions and to best handle the missing values.

ALLAN database- Business Sentiment Analysis

The last dataset contains an array of selected features collected by investors and business surveys, together with commodity prices, FED rates and volatility index. The list contains:

- ISM Non-Manufacturing PMI – *NAPMNMI1M*, this PMI surveys track sentiment among purchasing managers exclusively at service sector firms. The release frequency is monthly.
- ISM Manufacturing PMI SA – *NAPMPMI1M*, differently from the PMI survey above, this accounts for all manufacturing and construction sectors. The release happens on the first half of the following months of reference.
- Merchant Wholesalers Inventories Total Monthly % Change – *MWINCHNG2M*, it tracks the level of inventories held by wholesalers. The release date is every 2 months.
- US New One Family Houses Sold (Bull & Bear) – *AAIIBULL1M* and *AAIIBEAR1M*, both AAII Investor Sentiment Surveys have become a widely followed measure of the mood of individual investors.
- US Employees on Nonfarm Payrolls Total MoM Net Change SA – *NFP TCH1M*, it measures the number of employees on business payrolls. Release on monthly basis.
- Conference Board US Leading Index MoM – *LEI CHNG1M*, it includes economic variables that tend to move before changes in the overall economy, which gives a sense of the future state of US economy. Release on monthly basis.
- Change CBOE Volatility Index – *VIX CHNG*, it estimates the expected volatility of the S&P 500® Index, so by computing the change it would capture the increase of fear of investors. For example, a positive spike would probably result in a sell-off in the market.
- Monthly change of US Initial Jobless Claims SA – *INJCJC CHNG*, since this indicator is released on weekly basis to avoid losing precious information, we compute the change in

jobless claims between months. So, a negative change represents a potential improvement in the US employment status/economy and vice versa.

- Change in yield of the US Generic Govt 10 Year – *USGG10YR CHNG*.
- Change in price of the Generic 1st 'CL' Future – *CL1 CHNG*, which is the change in the future contract in the front month of the Oil price.
- Change in price of the Generic 1st 'GC' Future – *GOLD CHNG*, which is the change in the future contract in the front month of the Gold price.

Like for the previous dataset some of the features are necessarily lagged to the correct reporting period, to avoid forward looking bias and they are resampled to monthly basis. Furthermore, the features are scaled to a given range as described above. The data starts from 31st August 1997.

The benchmark is composed by a traditional allocation of 40% in the Dow Jones Industrial Average Index and 60% in the 10-Year U.S. Treasury Note (CBT) Front Month (TY1:COM).

IV. Results and Findings

Before looking at the actual predictions we found it would be interesting to look at the correlation matrix, especially with the Dow Jones Industrial Average Index and the selected features in the datasets. Also, one of the main benefits of building such metrics is that it serves as a diagnostic for multicollinearity – which happens when there are very high intercorrelations or inter-associations amongst the independent variables. Thus, if any high correlations appears, it suggests most likely that the regression estimators will be unreliable (see Table 1). No apparent multicollinearity issue has been discovered since none of the correlations are near to -1 or 1, thus it is possible to continue with the fitting process. Once the tuning process (iterated one thousand times) is completed, an optimal model will be generated. The optimal alpha chosen by the algorithm is $\boldsymbol{\alpha} = 1.0$ and the lambda $\lambda = 0.001$. This means that the grid search results in using the Lasso only. Hence, this model will shrink the parameters to zero, whether it finds them irrelevant to add any additional information on the predictions. The optimal coefficients β* are collected in Table 2, and all of them are used to forecast the Index price. By looking at the features' importance in Figure 3, or the proportioned absolute value of the optimal coefficients, most of the information will be extrapolated from the feature Price to dividend ratio lagged by 6 months. Observing the optimal model prediction on the training set, it is not surprising that the error MSE on the unseen observation (0.39) is higher than the one on the training set (0.019). However, it is very common to obtain such results (see Figure 4). Once the price of the risky asset (INDU:IND) is predicted, the returns are calculated so that it is possible to generate the "pseudo" confusion matrix. The accuracy rate, the precision rate and the recall rate obtained are 41.18%, 58.33% and 41.18% respectively. Although, the model performs relatively well on the out-of-sample observations, the number of labelled positive returns which are instead negative (False Positive) is high. The predicted negative returns which are positive are substantially higher than the True Negatives. Hence, most likely there are already signals that it will not be possible to generate a portfolio that will overperform the target allocation when using the BOB dataset. To sum up, in the positive economic cycle the algorithm will underperform the benchmark while in the negative downturn certainly the allocation selected will be too risky and the portfolio will suffer important losses.

The next dataset is FRED - which is a collection of the macroeconomics time series. We found no evidence for the presence of multicollinearity issues since none of the correlations are closer to the unit (see Table 4). This confirms that most likely the lagged methodology has been correctly executed. Once the tuning process is finished, the optimal alpha and the lambda for FRED chosen by the algorithm are respectively, $\boldsymbol{\alpha} = 1$ and $\lambda = 0.001$. Once again, the Elastic Net regression is transformed on the Lasso, and as the optimal coefficients β* in Table 5 show, some of the features have dropped to zero. It most likely means that the linear combination in any of the subset of those zero coefficient regressors may be useful for predicting the outcomes.

The features' importance (see Figure 5) reduces the number of regressors to only seven of them (or – 41.67%) will be highly recommended. Among those left, the US Trade Balance of Goods and Services Seasonal Adj lagged by two months has the highest impact on the prediction followed by the Chicago Purchasing Managers Index lagged by one month and the US Personal Consumption Expenditure Core Price Index MoM Seasonal Adj lagged one month. The error MSE on the OOS set (0.36) is slightly lower than the one recorded in the BOB dataset, suggesting that most likely the predictions generated by the "hard" data are closer to the true values than the ones with the Earnings yield and Price to Dividend ratio (see Figure 6). Such findings are confirmed by the "pseudo" confusion matrix, where the accuracy rate, the precision rate and the recall rate are respectively 47.06%, 65.22% and 44.12%. Although the recall rate in FRED is higher than the one for BOB data, the rate is still not high enough to assume that the portfolio strategy would overperform the benchmark in the market downturn. On the other hand, the precision rate has got a substantial improvement by about seven basis point, which means a higher number of returns labelled as positive that were indeed positive (lower False Positives). Overall, the accuracy rate increased, thus we can conclude that such model is more efficient in the long-term positive cycle, but it is still not able to adjust to sudden market drops or bearish movements quickly enough.

Finally, the ALLAN dataset, which is perhaps the most fascinating one among the others. It contains mostly features that connect with business sentiment, volatility index and other perceptions of the economy from the perspective of investors, exclusively. Although differently from Routledge (2019), where the author is performing a text analysis by mining the US Federal Reserve Beige books, we decided to approach the sentiment analysis topic by using specific indexes and surveys that incorporate already the market view of the investors regarding the forward state of the market. Looking at the correlation matrix, this dataset does not contain any strong correlations (see Table 7). Once the tuning process is finished, the optimal alpha and the lambda for ALLAN chosen by the algorithm are respectively, $\boldsymbol{\alpha}$ = 1 and λ = 0.0001. This means that the Elastic Net regularization is transformed to a Lasso regression, ℓ_1. Differently from the previous two regression, the shrinkage parameter λ is much smaller, which means the penalization of the coefficients is not that high and the regression is somehow closer to an OLS. The Table 8 shows the optimal coefficients, β^*.

According to the features' importance (see Figure 7) all of the twelve variables contribute to the prediction of the Dow Jones Industrial Average index price. Amongst them, the most important variable by far in absolute value is US Employees on Nonfarm Payrolls Total MoM Net Change lagged by one month. The MSE on the OOS set (0.49) is worse than the one discovered in BOB and FRED – for the reason explained above about the low shrinkage penalty, λ – which implies a poor performance of the quasi-OLS model to predict on unseen observations (see Figure 8). Indeed, the monthly forecasts are far from the true value of the Dow Jones Industrial Average price. However, by looking at the single coefficients, those values are much smaller than the ones in BOB and FRED, so if we would have to stop to do further investigation we would not be able to explain if the model was truly capable of capturing the right magnitude of the index price. Hence, after computing the "pseudo" confusion matrix, the accuracy rate, the recall rate, and the precision rate are 41.18%, 41.18% and 58.83% respectively, very similar to the one already found with BOB dataset. Once again, the number of False Negative (Type II error) seems to be high, thus an outperformance in the relatively longer negative economic cycle is less likely to be created. However, the clear difference lays in the distribution of the returns from the forecasted price.

Indeed, the BOB predictions are negatively skewed while the ones of ALLAN are more normally distributed (see Figure 11 and Figure 12). This explains the presence of higher False Negative numbers for ALLAN, because generally it is uncommon to assume a normal distribution of returns in the market (except for Perfect Market Theory) – more often stock returns have a skewed distribution.

V. Tactical allocation strategy

Once the price of the Dow Jones Industrial Average Index is predicted within all the three datasets, the goal is to build a Portfolio which uses active management strategies ("*Tactical Asset Allocation*") that shift the weight's percentage of the two assets held – Dow Jones Industrial Average Index and 10-Year U.S. Treasury Note - to take advantage of monthly forecasted stock market prices and trends. The strategy is rebalanced at the end of the month when the data is collected. We determined the 75^{th} and 25^{th} percentile of the forecasted returns on the OOS observations, which will be later used to design the tactical allocation.

The portfolio algorithm will have three constraints: any practice of Short Selling is not allowed, the sum of weights must be one, and the deviation from the benchmark cannot be more than 20%. The last constraints try to maintain a balanced portfolio allocation as objective for the investors that seek to reduce potential volatility of the risky assets by including income-generating investments. Most likely this type of investors will accept a short-term fluctuation in value of the portfolio, but they will aim to hold such allocation for a mid- to long-range investment time horizon, so to offset the loss by going farther than the economic cycle. For convenience, from this point on, we will identify the risky asset as Equity and the risk-free investment as Bonds (with 10 years maturity).

The theoretical description of the strategy is the following:

If the predicted return of the forward month is positive but without exceeding the 75th percentile of estimated returns up until that prediction, the Portfolio Allocation will output the signal of "Overweight Equity by 10% and Underweight Bond by 10%". If instead the forecasted return in the upcoming month is above the 75th percentile, the algorithm will generate a different message "Overweight Equity by 20% and Underweight Bond by 20%". On the other hand, if the prediction on the return is below zero but not lower than the 25th percentile, the signals generated will be "Underweight Equity by 10% and Overweight Bond by 10%". Finally, if the INDU Index's return for the forward month is falling below the 25th percentile the investors must "Underweight Equity by 20% and Overweight Bond by 20%". This last portfolio is the most defensive portfolio allocation with respect to the volatility, and as a matter of fact the type of strategy will shift from "balanced-portfolio" to "income-portfolio". Which means that the main goal for such asset distribution would be aimed to further minimize the risk – it often offers a modest long-term growth of the principal.

VI. Evaluation of the Portfolios risk-adjusted returns

Evaluating the portfolios' performance compared to a benchmark is not less important for an investor or a portfolio manager. Therefore, in this section the tactical portfolios based on the three data sets will be tested, in order to confirm the applicability of the model and to the check whether the algorithm was able to generate valid investments strategies.

The first evaluation metric is the Sharpe Ratio, which measures the risk-adjusted returns. Intuitively, the metric represents the additional amount of return that an investor would receive per unit of risk. Though, this does not necessarily translate to a lower-volatility portfolio but indicates a better historical risk-adjusted performance. In other words, a higher Sharpe ratio connotates that the portfolio's risk-return relationship is more proportional or optimal. However, one of the main limitations is that the standard deviation used to calculate it, it assumes that returns are normally distributed, while in most of the cases they are skewed.

$$Sharpe\ Ratio = \frac{R_p - risk\ free}{\sigma_p} \tag{7}$$

The risk free used in the equation (7) is the US 3-months T-Bill, because it represents the opportunity cost, or the missed-out benefits of an individual when choosing one alternative investment over another. Often the Sharpe ratio does not clearly underline if the investors' risk taken is being compensated. Nevertheless, such ratio can be manipulated by the portfolio managers that seek to boost the historical risk-adjusted returns, perhaps by lengthening the frequency interval. Hence, to avoid a misleading presentation, a valid solution is the Sortino ratio that differs from the previous one in that it only considers the standard deviation of the downside risks.

$$Sortino\ Ratio = \frac{R_p - risk\ free}{\sigma_{downside}} \tag{8}$$

Because having a positive volatility in the portfolio is most likely a benefit, the Sortino ratio provides a better view on portfolio's risk-adjusted performance. The risk free used in the equation (8) remains the US 3-months T-Bill.

By looking at these first two metrics between the three portfolios $\{p_{BOB}, p_{FRED}, p_{ALLAN}\}$ there is a clear winner. The p_{BOB} returns a discouraging risk-adjustment metric of 0.31, while considering only the negative scenarios to compute the standard deviation the value is the lowest among the cluster, 0.32. The p_{FRED}, which collect the macroeconomic data has a Sharpe ratio of 0.73 and the highest Sortino value of 0.86. The third portfolio p_{ALLAN} has a Sharpe of 0.65, yet its Sortino ratio is only 0.63. If we would have built a ranking based on the ability to generate valid risk-adjustment returns for the investor, the FRED will most likely the most preferred. So data suggests that the ranking would be as in the following order, $p_{FRED} > p_{ALLAN} > p_{BOB}$. But the real question is: are they any better than the predefined Benchmark (40% INDU:IND + 60% TY1:COM)? No, none of them beat the benchmark with a Sharpe ratio of 0.78 and a Sortino ratio of 0.95. Thus, the investor would most likely choose the benchmark over the machine learning algorithm generated portfolio (see Figure 9).

Next, it is interesting to further analyze the single portfolios since January 2020, the total number of positive and negative monthly returns (starting from the beginning of the 2016), the maximum drawdown – meaning the maximum historical negative return the portfolio has generated –, the annualized alpha (excess return versus the benchmark) and the beta (the adjusted magnitude of portfolio's return compare to the benchmark). To calculate the last two systems of measurement, the portfolio's returns are regressed versus the benchmark's returns, with a simple linear regression (OLS), as following:

$$y_{it} = \alpha + \beta x_{it} + \varepsilon \tag{6}$$

Where in the equation (6) x is the $r_{BENCHMARK}$ and y is the $r_{PORTFOLIOS}$. The Table 10 collects the metrics' list. According to the previous assessment the Benchmark is the most preferred investment for an individual investor that seek for a balanced strategy, the data in Table 10 once again offer the confirmation of that. Without any doubt the BOB Portfolio is unable to compete with the other strategies. Its annualized alpha is negative, the return since January 2020 is by about 350-basis points (-9.59%) worse than the benchmark, and looking at the maximum drawdown as well the BOB portfolio does not seem to be a valid option. Among the strategies left the FRED portfolio underperformed the benchmark by 30-basis points since the beginning of 2020 but it provides a lower maximum monthly historical loss of about -3.9%, since January 2016. However, the main limitation is that its beta is closer to 1 and we can observe 2 more months of negative returns than the benchmark. Finally, the ALLAN portfolio registered a poor performance since January 2020, -6.94%, means a 90 basis points worse than the benchmark, and it has negative annualized alpha as well. The total number of negative and positive monthly returns is the same as the 40/60 allocation, however, it does not mean that the time when those occur are the same as the benchmark. Overall, the beta of each of the three portfolios is closer to 1, meaning that most likely the magnitude of changes is about the same as the "Buy & Hold" strategy. By analyzing Figure 9, the ALLAN and BOB portfolios underperform constantly compared to the FRED Portfolio. Although from 2016 the US economy had a great performance, still the machine learning algorithm was successful at predicting the upcoming positive cycle with macro-indicator and the business sentiment databases – so, the tactical allocation selected during those periods captured the risk advantage of being more exposed to the risky asset. On the other hand, the cumulative returns are slightly underperforming the benchmark and the machine learning algorithm was not able to predict the damage of worldwide coronavirus spread and the consequently sell off in February and March 2020. This last crisis is useful to understand the limitations of the algorithm. Indeed, the information gathered neither from the business surveys (ALLAN) nor the macro economical FED decision (FRED) seems to quickly respond. Does this mean that the market is too often "bullish"? Does it not understand the short-term implications of upcoming viruses or/ structural crisis? We personally believe that those doubts might be true, but to be sure of it, further investigation must be conducted.

VII. New portfolio tactical allocation strategy adding risk parity

Once, we understood the pitfalls of the model we can adjust the choices of asset allocation to overcome sudden drawdowns of the economy. To do so, we decided to introduce the risk parity with the goal of minimizing the losses on sell-off events. Indeed, this volatility strategy allows the investors to often overweight the Bonds allocation, which historically are considered safe heaven assets during negative stock market returns. Assuming the benchmark will remain the same and the Short Selling constraint previously set remaining valid, the new strategy's description is the following:

If the predicted return of the forward month is positive but without exceeding the *75th percentile*, the Portfolio Allocation will output the signal of "Overweight Equity by 10% and Underweight Bond by 10%". If instead the forecasted return in the upcoming month is above the *75th percentile*, the algorithm will generate a different message "Overweight Equity by 20% and Underweight Bond by 20%". On the other hand, if the prediction on the return is below zero but not lower than the *25th percentile,* the signals generated will be "Risk Parity, Leverage 1.5". Finally, if the INDU Index's return for the forward month is falling below the *25th percentile* the investors must shift towards "Risk Parity, Leverage 2". The risk parity allocation, known as well as mean-variance optimization, consists of allocating the optimal weights on each asset class by equalizing the risk contributions of each of them. The leverage is usually added to scale the allocation to the desired volatility (risk). The target volatility selected in this research paper is 7.0%, with a leverage ratio of 1.5 and 2. The strategy is built as follows:

Consider a portfolio of N assets and the respective asset weight (w_i), which form a vector w. The covariance matrix of the assets is denoted as $\sum$. The standard deviation will be computed based on the previous six-monthly returns' observation. The volatility of the portfolio is $\sigma(w) = \sqrt{w'\sum w}$ and the total risk contribution of asset is computed as follows $\sigma_i(w) = \frac{w_i(\sum w)_i}{\sqrt{w'\sum w}}$. Given the homogeneity of degree one characteristic of the portfolio's volatility, then $\sigma(w) = \sum_{i=1}^{N} \sigma_i(w)$.

Equal risk contribution means $\sigma_i(w) = \sigma(w)/N$. Then we solve for, $\underset{w}{\operatorname{argmin}} \sum_{i=1}^{N} [\, w - \frac{\sigma(w)^2}{(\sum w)_i N}]^2$.

The minimization problem above will define the weight portfolio of each asset class respecting a target volatility. To find the final portfolio allocation with risk parity, the weights must be weighted based on the two leverage of ratios, 1.5 and 2. Once again, what is left is to evaluate how the strategy performs compared to the benchmark.

VIII. Final evaluation of the Portfolio based on risk-adjusted returns

Analyzing the Sharpe and Sortino between the three portfolios $\{p'_{BOB}, p'_{FRED}, p'_{ALLAN}\}$, we can observe an improvement of the metrics, with a clear winner. The p'_{FRED} has the highest Sharpe (1.0968), as well its Sortino ratio is 1.5215. The p'_{BOB} returns a strong upturn of risk-adjustment metric from 0.31 to 0.62, while its Sortino ratio is now 0.72. The third portfolio p'_{ALLAN}, which collects the sentiment analyses data has the second-best Sharpe ratio 0.9725 and a Sortino value of 0.9481. We can assume that the previous ranking remains invariant, as following $p'_{FRED} > p'_{ALLAN} > p'_{BOB}$. Knowing the portfolios have performed better in relation to their previous versions, it remains to understand if they do exceed the Benchmark (40% INDU:IND + 60% TY1:COM) as well.

All of them beat the benchmark (see Figure 13). Moreover, we need to assess if the described outperformance come exclusively from the risk parity. The p'_{FRED}, is the portfolio allocation which among the others beat the benchmark and provide investors a valid alternative to the risk parity strategies. So, we can arrange a preliminary conclusion by looking at Figure 13: the risk parity allocation provides a benefit to the portfolios in terms of performance, but the main reason why this happened cannot be related to the machine learning model. Instead, the consistence decline of the yield on the 10 Year US Treasury Note in the past 3 years implied a positive cycle performance of the Bonds that lead to excellent returns for our portfolios (see Figure 15) – when yield decreases, the present value of the bond increases. Furthermore, since the risk parity is chosen about 26 times on average in each portfolio, the outperformance in the downturn periods is only random. The Table 11 confirms the previous findings. The FRED portfolio outperforms the 40/60 strategy by realizing an annualized alpha of 2.827 as well as providing a lower maximum monthly historical loss of about -3.93%, since January 2016 – which means 50-basis points lower than the benchmark. Furthermore, its beta remains approximately the same, around 1, thus the magnitude of price activity is very similar to the benchmark. It has got the highest cumulative return since January 2020 (-5.81 %), meaning 20 basis points more than the benchmark. Finally, if the cumulative chart implied an outperformance of the three portfolios over the target allocation, the comparison with the Risk Parity Leverage 1.5 and 2 shows a different outcome. In fact, from the Table 11 the p'_{FRED} beats the Risk parity x1.5 only 50% of the total months from January 2016 and no more than 36% as against the Risk parity x2.

In conclusion, although the adoption of risk parity allocation among the portfolios clearly improves the performance of those compared to the benchmark (40/60), without a scrupulous investigation we might have overestimated the accomplishment of the portfolios $\{p'_{BOB}, p'_{FRED}, p'_{ALLAN}\}$. Moreover, looking at the last 3 months observation, all the datasets do not predict in time the market downturn due to the COVID-19 crisis. But the recent FED decision on cutting interest rate to 0.25-0, benefits our portfolio indirectly. Indeed, the late adoption of risk parity in the month of March for BOB portfolio, implied an overweight on the bond’s asset class and creates and advantage in terms of performance. We can assess that the macroeconomic indicators are yet far from explaining the market variability.

IX. Conclusions

The goal of the research paper was to demonstrate that applying Elastic Net regularization to forecast the Dow Jones Industrial Average Index price would have generated a superior tactical asset allocation beat a predefined benchmark due to a superior asset allocation strategy. Although the results are interesting, in general the algorithm does not offer a great prediction of the stock index and fails to avoid a negative downturn of the economy/stock prices. Or in other words, this explains the inability to often provide positive annualized excess returns on adverse scenarios. For example, by looking at the recent Coronavirus crisis (COVID-19) which damages the United States economy – 22 millions of American lost their job in 6 weeks (see Figure 16) – the returns since January 2020 (without risk parity allocations) are in at least two cases (ALLAN and BOB) lower than the benchmark. Which also implies that most likely the allocation was too exposed to the equity side in this period. The "Benchmark" strategy was the best choice for the investor compared to the returns generated by the two portfolios p_{ALLAN} and p_{BOB}. The machine learning algorithm that uses the macroeconomic variables to predict the equity asset prices generates a portfolio that offers the investors a valid alternative to the benchmark. Indeed, by applying the diversification benefits of risk parity portfolios as well – which include balanced correlations to underlying asset classes and stronger downside protection against severe losses – the tactical asset allocation strategy is able to overperform the benchmark in more than 50% of the cases.

Finally, there are additional works that are worth pursuing. The ALLAN dataset can be improved by combining forum/journal API and scraping official statements/or Google Trends, to create a real sentiment text analysis. The risk-free asset can also be predicted, to build a more theoretical centered portfolio strategy. The algorithm can be further developed by perhaps using a Deep Neural Network with Elastic Net regularization which at the same time implies a bigger dataset for the variables or decreasing the frequency from monthly to weekly. Perhaps, by handling the missing values due to the historical discrepancy of features' history, we can use boosting regressions. Moreover, it would be interesting to apply this model to different markets and geographical areas. Smaller marketplaces are probably more intrinsically connected with change on investors sentiment behavior or local central bank strategies.

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APPENDIX A

According to Gilboa, Schmeidler et al. (2003, 2010, 2012) to better understand the decision-making model at the foundation of this work project, it is necessary to define some statistical concepts. The data is described in the following format (x_n, y_n), where $x_n \in \mathbb{X}$ represents the group of signals or regressors, and $y_n \in \mathbb{Y}$ is the independent variable, or the state that will be predicted. $\mathcal{P}(\mathbb{X}, \mathbb{Y})$ is the set of joint probability distribution over $\mathbb{X} \times \mathbb{Y}$. Hence, the datasets of length *n* is defined as $h_n = ((x_0, y_1), \dots, (x_n, y_{n+1}))$, because the goal of predicting the state *y* one period head with the signals *x* of the previous time period. A "theory", which is defined as a probabilistic distribution, is the prediction generating process that is demarcated as, $\theta(y, x)$. The empirical regression to predict the price, $\mathbf{P}_{t+h}$, give the information at time t, $\mathbf{X}_t$, is the following, $\ln(\mathbf{P}_{t+h}) = \theta' \boldsymbol{X}_t + \boldsymbol{\varepsilon}$, where **h** is the time variation. The optimal θ* is obtain by solving for

$$\theta^*(h_n, a, b) = \underset{\theta}{\text{argmax}} \sum_{(x,y)\in h_n} \ell(y, x, \theta) + b\,(\frac{1-a}{2}\sum_{j=1}^{n}\beta_j^2 + a\sum_{j=1}^{n}|\beta_j|\,),$$

$$for\ j\ \in [0, n]\ where\ n\ is\ the\ length\ of\ the\ matrix\ array$$

It is possible to recursive change the "simplicity" parameters of *a* and *b* across to obtain the different solutions.

APPENDIX B

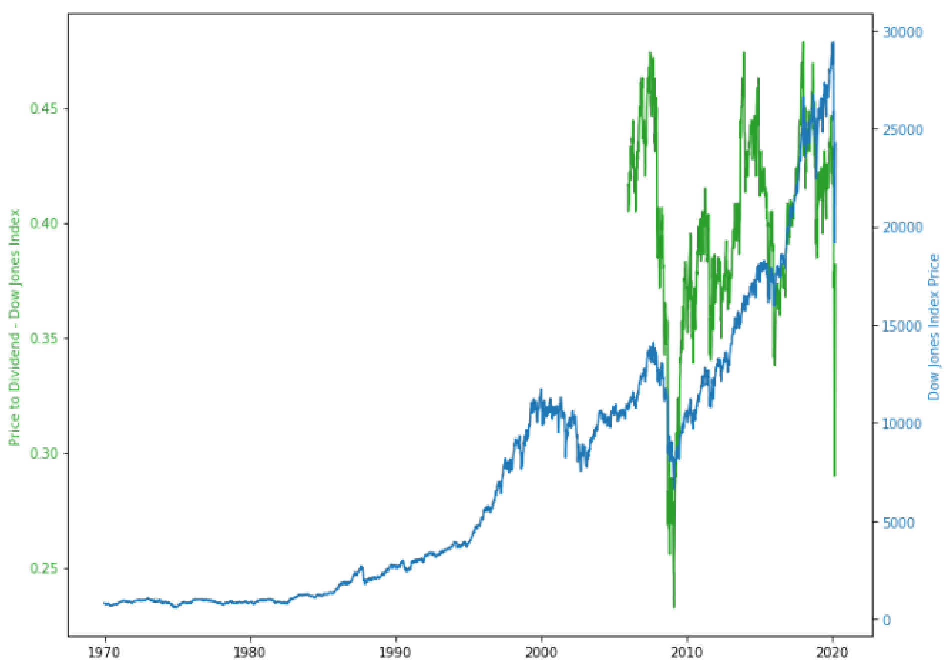


Figure 1 The Dow Jones Industrial Average Index price vs. its Price to Dividend ratio

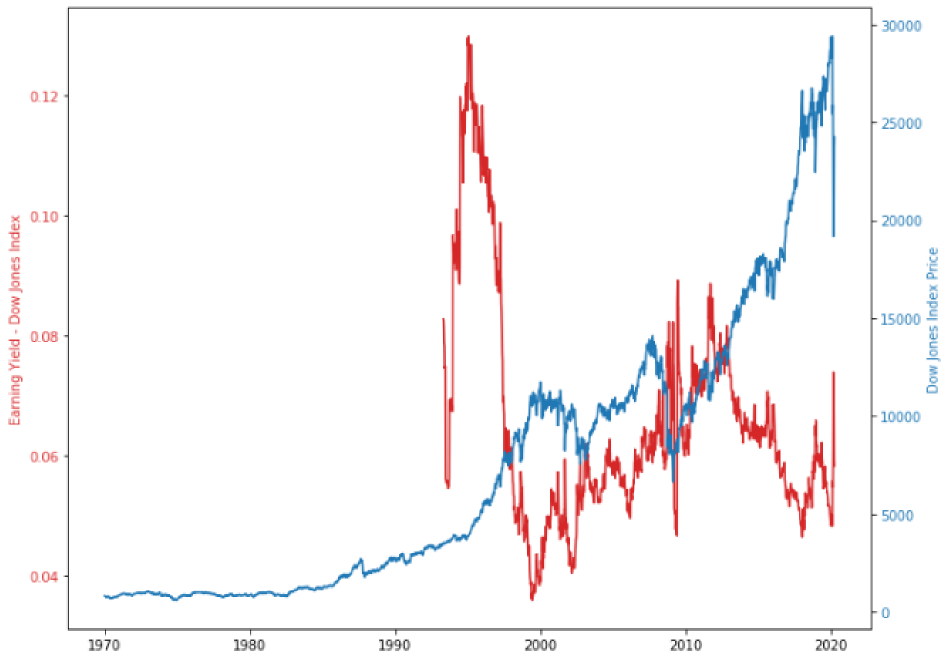


Figure 2 The Dow Jones Industrial Average Index price vs. its Earning yield

```
                 DWJ_price   P/DVD3M   P/DVD6M  Earning_yield3M
DWJ_price         1.000000  0.498343  0.462972        -0.417893
P/DVD3M           0.498343  1.000000  0.803852        -0.523825
P/DVD6M           0.462972  0.803852  1.000000        -0.376617
Earning_yield3M  -0.417893 -0.523825 -0.376617         1.000000
Earning_yield6M  -0.396091 -0.441502 -0.516851         0.921386

                 Earning_yield6M
DWJ_price              -0.396091
P/DVD3M                -0.441502
P/DVD6M                -0.516851
Earning_yield3M         0.921386
Earning_yield6M         1.000000

<matplotlib.axes._subplots.AxesSubplot at 0x29de88d79e8>
```

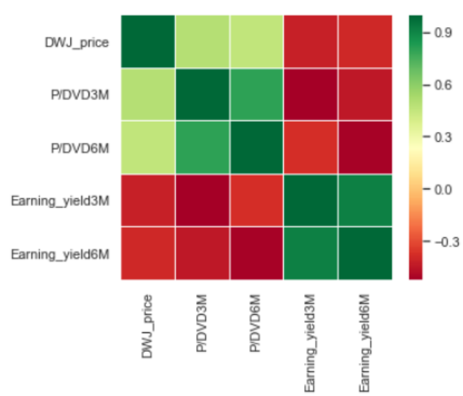


Table 1 Correlation Matrix - BOB Dataset. The P/DVD and Earning_yield ratios have got moderate type correlation between 0.4 and 0.5 in absolute terms. The Price to Dividend ratio lagged six (0.46) and three

months (0.50) has got a positive magnitude relationship, while the Earning yields lagged three months (-0.42) and six months (-0.40) have got a negative sign.

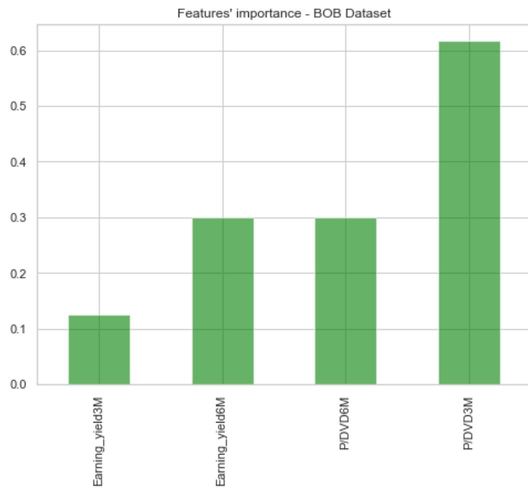


Figure 3 Features' importance - BOB Dataset

	β^*_{BOB}
P/DVD6M	*0.616*
P/DVD3M	*0.2996*
Earning yield3M	*0.1245*
Earning yield6M	*0.2987*

Table 2 Optimal coefficient from Elastic Net Regression - BOB Dataset

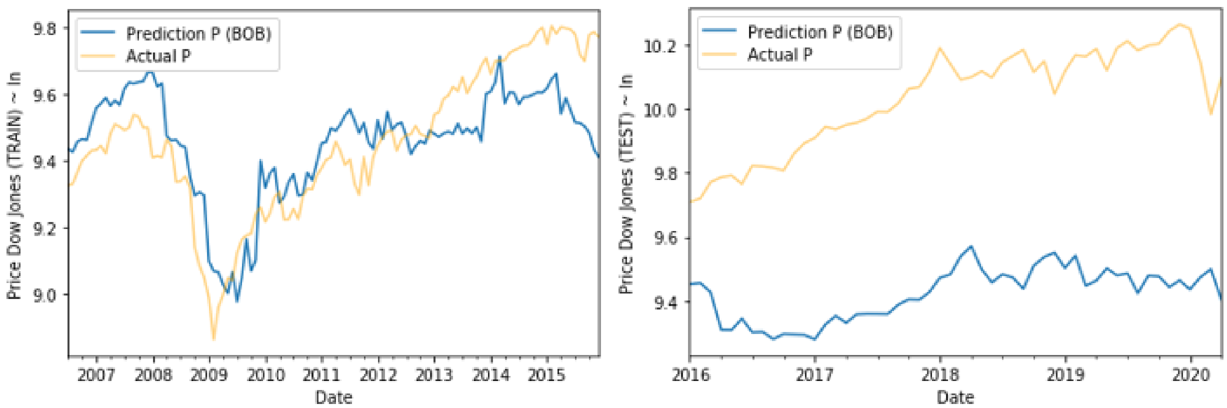
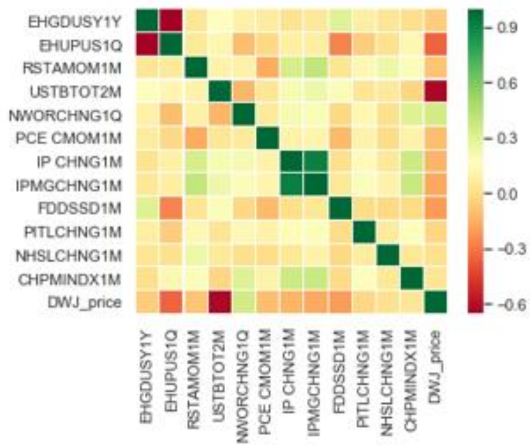


Figure 4 Training and OOS predictions on BOB with Elastic Net and grid search cross validation for parameters tuning. The k-fold of CV contains 2 years of observed monthly data and the optimal parameters which reduces the MSE are α = 1 and λ = 0.001 using 1000 iterations. In this occasion the training collection identify to effectively perform the first fitting step with Elastic Net algorithm is bounded between the 31st July 2006 and the 31st December 2016. The correspondent OOS set is therefore between 1st January 2016 and the 30th April 2020. We opt for this timeseries split because in the recent years the stock market saw a brilliant return, so to better evaluate the portfolio it is necessary to have a more volatile out of sample set. At the same time, to not compromise the training process and avoid worst Mean Square Error, the cut off was made in 2016. Respectively they contain hundred twenty-five months and fifty months. Each of the five folds of the cross-validation is composed by approximately two years of observed ratios.

Date	p_BOB_ret	message
2016-01-31 00:00:00	0	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2016-02-29 00:00:00	0.979027	Underweight Dow Jones (20.00%) + Overweight US 10Y Treasury (80.00%)
2016-03-31 00:00:00	0.346196	Underweight Dow Jones (20.00%) + Overweight US 10Y Treasury (80.00%)
2016-04-30 00:00:00	0.819112	Underweight Dow Jones (30.00%) + Overweight US 10Y Treasury (70.00%)
2016-05-31 00:00:00	0.0217989	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2016-06-30 00:00:00	-0.763208	Underweight Dow Jones (20.00%) + Overweight US 10Y Treasury (80.00%)
2016-07-31 00:00:00	1.60078	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2016-08-31 00:00:00	-0.730337	Underweight Dow Jones (20.00%) + Overweight US 10Y Treasury (80.00%)
2016-09-30 00:00:00	0.342936	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2016-10-31 00:00:00	-1.00609	Underweight Dow Jones (30.00%) + Overweight US 10Y Treasury (70.00%)
2016-11-30 00:00:00	-0.702693	Underweight Dow Jones (30.00%) + Overweight US 10Y Treasury (70.00%)
2016-12-31 00:00:00	0.787766	Underweight Dow Jones (30.00%) + Overweight US 10Y Treasury (70.00%)
2017-01-31 00:00:00	0.440523	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2017-02-28 00:00:00	2.63632	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2017-03-31 00:00:00	-0.617653	Underweight Dow Jones (30.00%) + Overweight US 10Y Treasury (70.00%)
2017-04-30 00:00:00	1.05597	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2017-05-31 00:00:00	0.55044	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2017-06-30 00:00:00	0.489465	Underweight Dow Jones (30.00%) + Overweight US 10Y Treasury (70.00%)
2017-07-31 00:00:00	0.911454	Underweight Dow Jones (30.00%) + Overweight US 10Y Treasury (70.00%)
2017-08-31 00:00:00	0.482422	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2017-09-30 00:00:00	1.21651	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2017-10-31 00:00:00	2.01568	Underweight Dow Jones (30.00%) + Overweight US 10Y Treasury (70.00%)
2017-11-30 00:00:00	0.369409	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2017-12-31 00:00:00	2.1955	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2018-01-31 00:00:00	3.96135	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2018-02-28 00:00:00	-2.99832	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2018-03-31 00:00:00	-2.52481	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2018-04-30 00:00:00	-0.0257545	Underweight Dow Jones (20.00%) + Overweight US 10Y Treasury (80.00%)
2018-05-31 00:00:00	0.602746	Underweight Dow Jones (20.00%) + Overweight US 10Y Treasury (80.00%)
2018-06-30 00:00:00	0.0463275	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2018-07-31 00:00:00	2.11428	Underweight Dow Jones (30.00%) + Overweight US 10Y Treasury (70.00%)
2018-08-31 00:00:00	1.15956	Underweight Dow Jones (20.00%) + Overweight US 10Y Treasury (80.00%)
2018-09-30 00:00:00	-0.608422	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2018-10-31 00:00:00	-3.93045	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2018-11-30 00:00:00	1.94286	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2018-12-31 00:00:00	-3.93485	Underweight Dow Jones (20.00%) + Overweight US 10Y Treasury (80.00%)
2019-01-31 00:00:00	1.30999	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2019-02-28 00:00:00	3.37956	Underweight Dow Jones (20.00%) + Overweight US 10Y Treasury (80.00%)
2019-03-31 00:00:00	1.08577	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2019-04-30 00:00:00	0.966848	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2019-05-31 00:00:00	-3.01917	Underweight Dow Jones (30.00%) + Overweight US 10Y Treasury (70.00%)
2019-06-30 00:00:00	2.82748	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2019-07-31 00:00:00	0.822159	Underweight Dow Jones (20.00%) + Overweight US 10Y Treasury (80.00%)
2019-08-31 00:00:00	1.82426	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2019-09-30 00:00:00	0.54422	Underweight Dow Jones (30.00%) + Overweight US 10Y Treasury (70.00%)
2019-10-31 00:00:00	-0.307206	Underweight Dow Jones (20.00%) + Overweight US 10Y Treasury (80.00%)
2019-11-30 00:00:00	0.637446	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2019-12-31 00:00:00	0.786501	Underweight Dow Jones (20.00%) + Overweight US 10Y Treasury (80.00%)
2020-01-31 00:00:00	1.59526	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2020-02-29 00:00:00	-5.07075	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2020-03-31 00:00:00	-6.1191	Underweight Dow Jones (20.00%) + Overweight US 10Y Treasury (80.00%)

Table 3 BOB Portfolio (p $_{BOB}$) monthly return and tactical allocation choices



	EHGDUSY1Y	EHUPUS1Q	RSTAMOM1M	USTBTOT2M	NWORCHNG1Q	PCE CMOM1M
EHGDUSY1Y	1.000000	-0.654085	0.043754	0.168032	0.095992	0.082161
EHUPUS1Q	-0.654085	1.000000	0.060985	0.124079	-0.099977	-0.010954
RSTAMOM1M	0.043754	0.060985	1.000000	0.109479	0.099818	-0.159277
USTBTOT2M	0.168032	0.124079	0.109479	1.000000	-0.124274	0.045050
NWORCHNG1Q	0.095992	-0.099977	0.099818	-0.124274	1.000000	0.063131
PCE CMOM1M	0.082161	-0.010954	-0.159277	0.045050	0.063131	1.000000
IP CHNG1M	0.030031	0.091223	0.346231	0.210276	0.214943	0.112672
IPMGCHNG1M	0.051867	0.085821	0.407976	0.239166	0.192938	0.120744
FDDSSD1M	0.307704	-0.263046	0.039391	0.197582	-0.011585	-0.113623
PITLCHNG1M	0.084507	-0.053067	0.137542	0.037997	0.136637	0.104943
NHSLCHNG1M	0.043344	0.020042	0.247049	0.062298	0.014794	-0.001149
CHPMINDX1M	0.013069	0.136551	0.188687	-0.026545	0.306443	0.104311
DWJ_price	-0.052652	-0.344949	-0.082457	-0.626022	0.359723	-0.104145

	IP CHNG1M	IPMGCHNG1M	FDDSSD1M	PITLCHNG1M	NHSLCHNG1M \
EHGDUSY1Y	0.030031	0.051867	0.307704	0.084507	0.043344
EHUPUS1Q	0.091223	0.085821	-0.263046	-0.053067	0.020042
RSTAMOM1M	0.346231	0.407976	0.039391	0.137542	0.247049
USTBTOT2M	0.210276	0.239166	0.197582	0.037997	0.062298
NWORCHNG1Q	0.214943	0.192938	-0.011585	0.136637	0.014794
PCE CMOM1M	0.112672	0.120744	-0.113623	0.104943	-0.001149
IP CHNG1M	1.000000	0.919510	0.016398	0.155407	0.053136
IPMGCHNG1M	0.919510	1.000000	0.052005	0.191963	0.111471
FDDSSD1M	0.016398	0.052005	1.000000	0.000286	-0.012468
PITLCHNG1M	0.155407	0.191963	0.000286	1.000000	0.075217
NHSLCHNG1M	0.053136	0.111471	-0.012468	0.075217	1.000000
CHPMINDX1M	0.376432	0.380081	-0.006632	0.183779	0.052359
DWJ_price	-0.131326	-0.165198	-0.198893	-0.015491	0.020691

	CHPMINDX1M	DWJ_price
EHGDUSY1Y	0.013069	-0.052652
EHUPUS1Q	0.136551	-0.344949
RSTAMOM1M	0.188687	-0.082457
USTBTOT2M	-0.026545	-0.626022
NWORCHNG1Q	0.306443	0.359723
PCE CMOM1M	0.104311	-0.104145
IP CHNG1M	0.376432	-0.131326
IPMGCHNG1M	0.380081	-0.165198
FDDSSD1M	-0.006632	-0.198893
PITLCHNG1M	0.183779	-0.015491
NHSLCHNG1M	0.052359	0.020691
CHPMINDX1M	1.000000	0.044853
DWJ_price	0.044853	1.000000

Table 4 Correlation Matrix - FRED Dataset. There are strong negative moderate correlation between the Dow Jones Industrial average Index price and the US Trade Balance of Goods and Services Seasonal Adj lagged by two moths (-0.63), following by less strong numbers in US unemployment rate lagged by one quarter (-0.35), US Treasury Federal Budget Debt lagged by one month and US Industrial Production Industry Groups Manufacturing MoM SA lagged by one month (respectively -0.20 and -0.165). Instead one of the strongest positive correlations can be found in FOF Federal Reserve US Households & NPO Net Worth Change lagged by one quarter (0.36).

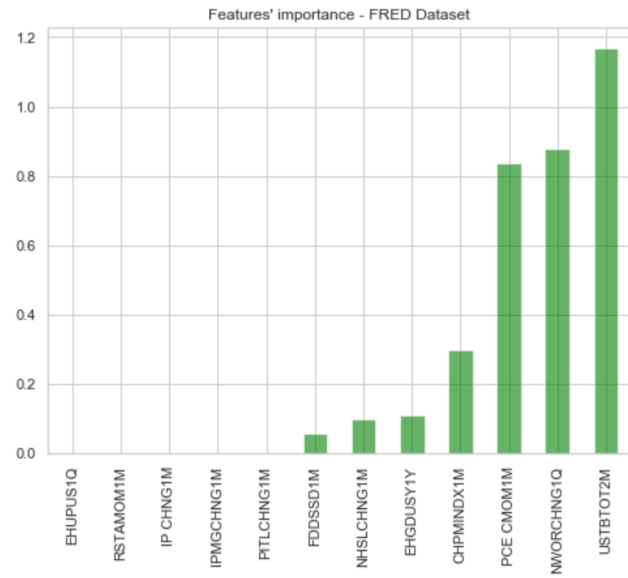


Figure 5 Features' importance FRED dataset

	β^*_{FRED}
EHGDUSY1Y	*0.1115*
EHUPUS1Q	*0*
RSTAMOM1M	*0*
USTBTOT2M	*1.16915*
NWORCHNG1Q	*0.87735*
PCE CMOM1M	*-0.83839*
IP CHNG1M	*0*
IPMGCHNG1M	*0*
FDDSSD1M	*-0.056646*
PITLCHNG1M	*0*
NHSLCHNG1M	*0.097743*
CHPMINDX1M	*-0.29894*

Table 5 Optimal coefficient from Elastic Net Regression - FRED Dataset

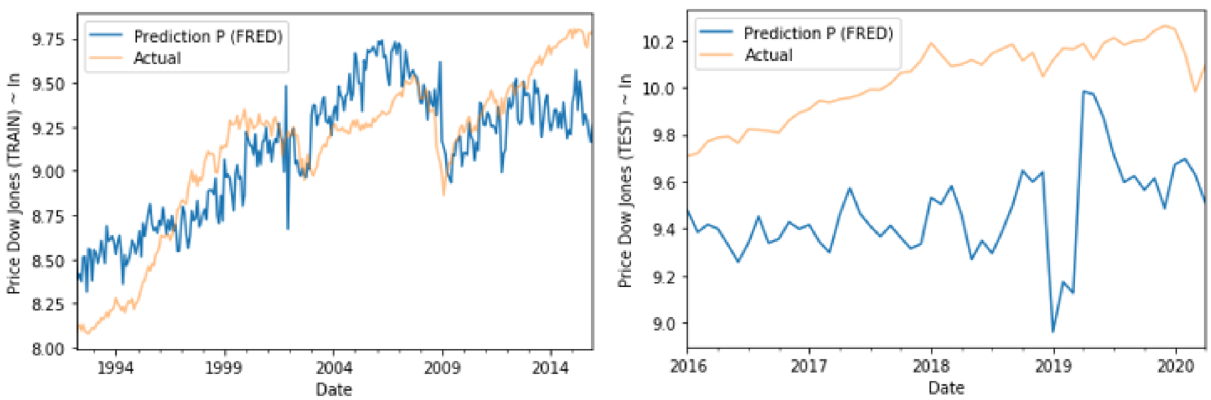
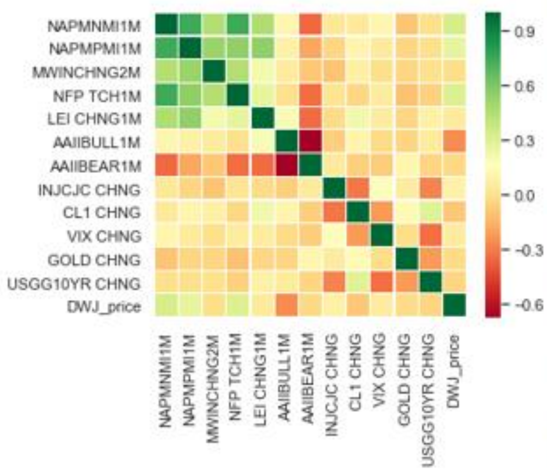


Figure 6 Training and OOS predictions on FRED with Elastic Net and grid search cross validation for parameters tuning. The k-fold of CV contains 2 years of observed monthly data and the optimal parameters which reduces the MSE are α = 1 and λ = 0.0001 using 1000 iterations. In this second dataset the training set for training Elastic Net algorithm is bounded between the 31st March 1992 and the 31st December 2016. Although, the FRED array is longer than BOB, we decide to maintain a two-year frame in each of the folds of the cross validation. This decision has been statistically tested, if the fold would have been larger than 2 years the improvement on the minimization objective of the error MSE would have been marginal, while decreasing the time length would have generate a worst error rate. Moreover, it is important to have got a long enough in sample test set to correctly evaluate the training process, thus a fold shorter than 24 months would not be preferred. The correspondent OOS set is as for the previous scenario between 1st January 2016 and the 30th April 2020.

Date	p_FRED_ret	message
2016-01-31 00:00:00	0	Underweight Dow Jones (20.00%) + Overweight US 10Y Treasury (80.00%)
2016-02-29 00:00:00	0.933623	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2016-03-31 00:00:00	2.18999	Underweight Dow Jones (30.00%) + Overweight US 10Y Treasury (70.00%)
2016-04-30 00:00:00	0.900779	Underweight Dow Jones (30.00%) + Overweight US 10Y Treasury (70.00%)
2016-05-31 00:00:00	0.0217989	Underweight Dow Jones (20.00%) + Overweight US 10Y Treasury (80.00%)
2016-06-30 00:00:00	1.11704	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2016-07-31 00:00:00	3.76431	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2016-08-31 00:00:00	-0.624243	Underweight Dow Jones (20.00%) + Overweight US 10Y Treasury (80.00%)
2016-09-30 00:00:00	0.342936	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2016-10-31 00:00:00	-1.00609	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2016-11-30 00:00:00	1.05705	Underweight Dow Jones (30.00%) + Overweight US 10Y Treasury (70.00%)
2016-12-31 00:00:00	0.787766	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2017-01-31 00:00:00	0.793457	Underweight Dow Jones (20.00%) + Overweight US 10Y Treasury (80.00%)
2017-02-28 00:00:00	1.64973	Underweight Dow Jones (30.00%) + Overweight US 10Y Treasury (70.00%)
2017-03-31 00:00:00	-0.509832	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2017-04-30 00:00:00	1.17853	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2017-05-31 00:00:00	0.55044	Underweight Dow Jones (20.00%) + Overweight US 10Y Treasury (80.00%)
2017-06-30 00:00:00	0.0165031	Underweight Dow Jones (30.00%) + Overweight US 10Y Treasury (70.00%)
2017-07-31 00:00:00	0.911454	Underweight Dow Jones (30.00%) + Overweight US 10Y Treasury (70.00%)
2017-08-31 00:00:00	0.482422	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2017-09-30 00:00:00	0.842829	Underweight Dow Jones (30.00%) + Overweight US 10Y Treasury (70.00%)
2017-10-31 00:00:00	0.984687	Underweight Dow Jones (30.00%) + Overweight US 10Y Treasury (70.00%)
2017-11-30 00:00:00	0.369409	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2017-12-31 00:00:00	2.1955	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2018-01-31 00:00:00	3.96135	Underweight Dow Jones (30.00%) + Overweight US 10Y Treasury (70.00%)
2018-02-28 00:00:00	-2.23388	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2018-03-31 00:00:00	-2.52481	Underweight Dow Jones (20.00%) + Overweight US 10Y Treasury (80.00%)
2018-04-30 00:00:00	-0.9148	Underweight Dow Jones (20.00%) + Overweight US 10Y Treasury (80.00%)
2018-05-31 00:00:00	0.602746	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2018-06-30 00:00:00	-0.949806	Underweight Dow Jones (30.00%) + Overweight US 10Y Treasury (70.00%)
2018-07-31 00:00:00	1.01589	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2018-08-31 00:00:00	1.52773	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2018-09-30 00:00:00	0.646094	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2018-10-31 00:00:00	-3.93045	Underweight Dow Jones (30.00%) + Overweight US 10Y Treasury (70.00%)
2018-11-30 00:00:00	1.34259	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2018-12-31 00:00:00	-3.93485	Underweight Dow Jones (20.00%) + Overweight US 10Y Treasury (80.00%)
2019-01-31 00:00:00	1.30999	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2019-02-28 00:00:00	3.37956	Underweight Dow Jones (30.00%) + Overweight US 10Y Treasury (70.00%)
2019-03-31 00:00:00	0.900524	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2019-04-30 00:00:00	1.24759	Underweight Dow Jones (30.00%) + Overweight US 10Y Treasury (70.00%)
2019-05-31 00:00:00	-0.400145	Underweight Dow Jones (20.00%) + Overweight US 10Y Treasury (80.00%)
2019-06-30 00:00:00	2.20385	Underweight Dow Jones (20.00%) + Overweight US 10Y Treasury (80.00%)
2019-07-31 00:00:00	-0.0209904	Underweight Dow Jones (20.00%) + Overweight US 10Y Treasury (80.00%)
2019-08-31 00:00:00	1.82426	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2019-09-30 00:00:00	0.285466	Underweight Dow Jones (30.00%) + Overweight US 10Y Treasury (70.00%)
2019-10-31 00:00:00	-0.307206	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2019-11-30 00:00:00	1.91931	Underweight Dow Jones (20.00%) + Overweight US 10Y Treasury (80.00%)
2019-12-31 00:00:00	-0.0118019	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2020-01-31 00:00:00	0.118235	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2020-02-29 00:00:00	-3.81979	Underweight Dow Jones (30.00%) + Overweight US 10Y Treasury (70.00%)
2020-03-31 00:00:00	-2.62786	Underweight Dow Jones (20.00%) + Overweight US 10Y Treasury (80.00%)

Table 6 Portfolio FRED (p_{FRED}) monthly return and tactical allocation choices



	NAPMNMI1M	NAPMPMI1M	MWINCHNG2M	NFP TCH1M	LEI CHNG1M
NAPMNMI1M	1.000000	0.748133	0.445388	0.748654	0.457699
NAPMPMI1M	0.748133	1.000000	0.522018	0.553339	0.550137
MWINCHNG2M	0.445388	0.522018	1.000000	0.443305	0.208512
NFP TCH1M	0.748654	0.553339	0.443305	1.000000	0.264191
LEI CHNG1M	0.457699	0.550137	0.208512	0.264191	1.000000
AAIIBULL1M	0.115669	0.089365	0.061689	0.006639	0.193903
AAIIBEAR1M	-0.350172	-0.186597	-0.082475	-0.337845	-0.348169
INJCJC CHNG	0.041647	-0.035599	-0.096456	0.027000	-0.016479
CL1 CHNG	0.044829	0.102970	0.076526	-0.033225	0.222933
VIX CHNG	0.111643	0.052138	0.000430	0.064338	0.067330
GOLD CHNG	-0.095938	-0.035513	-0.037059	-0.104034	-0.039277
USGG10YR CHNG	-0.001281	0.000410	0.006251	-0.052181	0.083882
DWJ_price	0.338245	0.263009	-0.007574	0.305785	0.055092

	AAIIBULL1M	AAIIBEAR1M	INJCJC CHNG	CL1 CHNG	VIX CHNG
NAPMNMI1M	0.115669	-0.350172	0.041647	0.044829	0.111643
NAPMPMI1M	0.089365	-0.186597	-0.035599	0.102970	0.052138
MWINCHNG2M	0.061689	-0.082475	-0.096456	0.076526	0.000430
NFP TCH1M	0.006639	-0.337845	0.027000	-0.033225	0.064338
LEI CHNG1M	0.193903	-0.348169	-0.016479	0.222933	0.067330
AAIIBULL1M	1.000000	-0.676913	-0.057206	0.105781	-0.015891
AAIIBEAR1M	-0.676913	1.000000	0.052714	-0.060319	-0.066329
INJCJC CHNG	-0.057206	0.052714	1.000000	-0.314646	0.153015
CL1 CHNG	0.105781	-0.060319	-0.314646	1.000000	-0.222046
VIX CHNG	-0.015891	-0.066329	0.153015	-0.222046	1.000000
GOLD CHNG	-0.040855	0.110940	0.066602	0.129103	-0.006167
USGG10YR CHNG	0.059047	-0.040933	-0.283383	0.290204	-0.335844
DWJ_price	-0.259183	-0.024390	0.087214	-0.086256	0.066520

	GOLD CHNG	USGG10YR CHNG	DWJ_price
NAPMNMI1M	-0.095938	-0.001281	0.338245
NAPMPMI1M	-0.035513	0.000410	0.263009
MWINCHNG2M	-0.037059	0.006251	-0.007574
NFP TCH1M	-0.104034	-0.052181	0.305785
LEI CHNG1M	-0.039277	0.083882	0.055092
AAIIBULL1M	-0.040855	0.059047	-0.259183
AAIIBEAR1M	0.110940	-0.040933	-0.024390
INJCJC CHNG	0.066602	-0.283383	0.087214
CL1 CHNG	0.129103	0.290204	-0.086256
VIX CHNG	-0.006167	-0.335844	0.066520
GOLD CHNG	1.000000	-0.222876	-0.018102
USGG10YR CHNG	-0.222876	1.000000	-0.039488
DWJ_price	-0.018102	-0.039488	1.000000

Table 7 Correlation Matrix - ALLAN Dataset. The ISM Non-Manufacturing PMI has got a positive correlation with the Dow Jones Industrial Average Index Price of about 0.34, following by US Employees on Nonfarm Payrolls Total MoM Net Change with 0.31 and ISM Manufacturing PMI at 0.26, all of them respectively lag by one month. While the bullish survey on US New One Family Houses Sold lagged by one month with an opposite sign of -0.26. All the other variables have got correlations closer to zero.

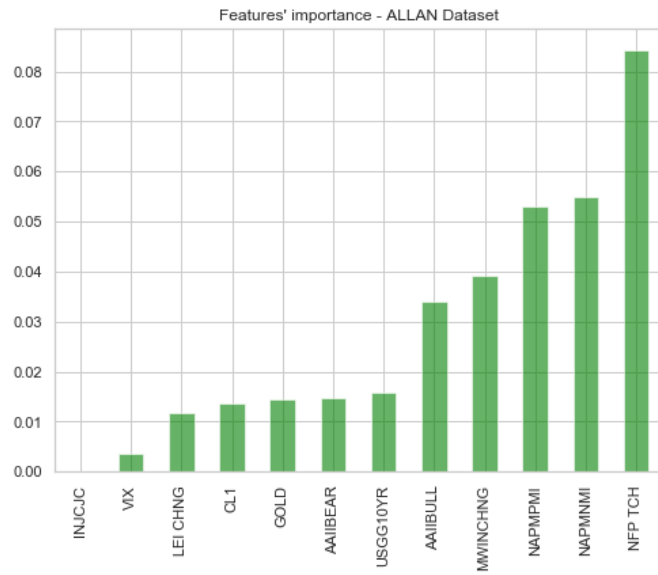


Figure 7 Features' importance - ALLAN Dataset

	β^*_{ALLAN}
NAPMNMI1M	*0.05495*
NAPMPMI1M	*0.05302*
MWINCHNG2M	*0.039195*
NFP TCH1M	*0.084338*
LEI CHNG1M	*0.0184*
AAIIBULL1M	*-0.03396*
AAIIBEAR1M	*-0.01471*
INJCJC CHNG	*-0.00018*
CL1 CHNG	*-0.013161*
VIX CHNG	*-0.00344*
GOLD CHNG	*-0.014432*
USGG10YR CHNG	*0.015707*

Table 8 Optimal coefficient from Elastic Net Regression - ALLAN Dataset

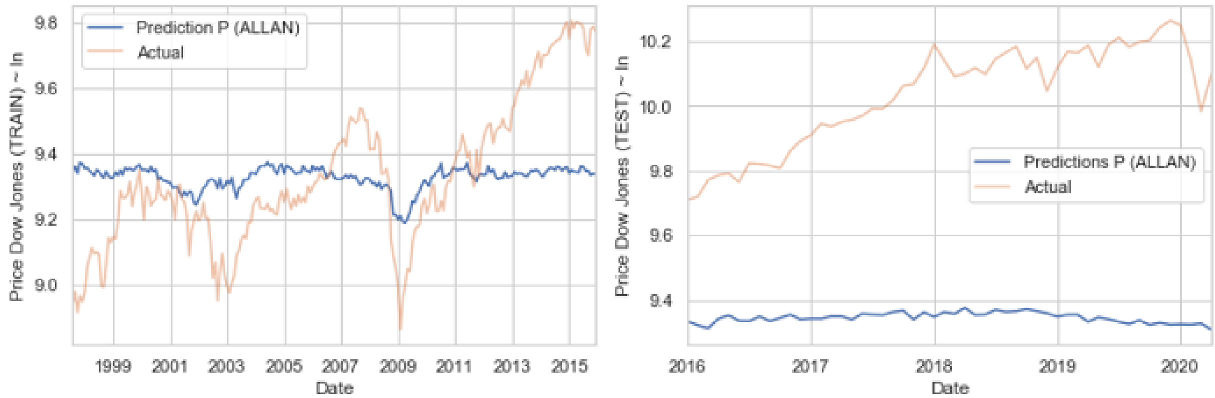


Figure 8 Training and OOS predictions on ALLAN with Elastic Net and grid search cross validation for parameters tuning. The k-fold of CV contains 2 years of observed monthly data and the optimal parameters which reduces the MSE are α = 1 and λ = 0.001 using 1000 iterations. In this third dataset the training set for training Elastic Net algorithm is bounded between the 31st August 1997 and the 31st December 2016. The correspondent OOS set is as for the previous scenario between 1st January 2016 and the 30th April 2020. Respectively they contain two-hundred twenty-one and fifty months. Each of the nine folds of the cross-validation is composed by two years of observed indexes.

Date	p_ALLAN_ret	message
2016-01-31 00:00:00	0	Underweight Dow Jones (20.00%) + Overweight US 10Y Treasury (80.00%)
2016-02-29 00:00:00	0.933623	Underweight Dow Jones (20.00%) + Overweight US 10Y Treasury (80.00%)
2016-03-31 00:00:00	0.346196	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2016-04-30 00:00:00	1.14578	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2016-05-31 00:00:00	0.252571	Underweight Dow Jones (20.00%) + Overweight US 10Y Treasury (80.00%)
2016-06-30 00:00:00	1.11704	Underweight Dow Jones (30.00%) + Overweight US 10Y Treasury (70.00%)
2016-07-31 00:00:00	2.14166	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2016-08-31 00:00:00	-0.624243	Underweight Dow Jones (20.00%) + Overweight US 10Y Treasury (80.00%)
2016-09-30 00:00:00	0.342936	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2016-10-31 00:00:00	-0.965415	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2016-11-30 00:00:00	1.93692	Underweight Dow Jones (20.00%) + Overweight US 10Y Treasury (80.00%)
2016-12-31 00:00:00	0.444958	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2017-01-31 00:00:00	0.793457	Underweight Dow Jones (30.00%) + Overweight US 10Y Treasury (70.00%)
2017-02-28 00:00:00	1.89638	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2017-03-31 00:00:00	-0.581713	Underweight Dow Jones (30.00%) + Overweight US 10Y Treasury (70.00%)
2017-04-30 00:00:00	1.05597	Underweight Dow Jones (20.00%) + Overweight US 10Y Treasury (80.00%)
2017-05-31 00:00:00	0.433417	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2017-06-30 00:00:00	0.647119	Underweight Dow Jones (30.00%) + Overweight US 10Y Treasury (70.00%)
2017-07-31 00:00:00	0.911454	Underweight Dow Jones (30.00%) + Overweight US 10Y Treasury (70.00%)
2017-08-31 00:00:00	0.482422	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2017-09-30 00:00:00	1.21651	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2017-10-31 00:00:00	2.01568	Underweight Dow Jones (20.00%) + Overweight US 10Y Treasury (80.00%)
2017-11-30 00:00:00	0.346712	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2017-12-31 00:00:00	2.74225	Underweight Dow Jones (20.00%) + Overweight US 10Y Treasury (80.00%)
2018-01-31 00:00:00	0.246536	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2018-02-28 00:00:00	-3.38053	Underweight Dow Jones (30.00%) + Overweight US 10Y Treasury (70.00%)
2018-03-31 00:00:00	-0.842131	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2018-04-30 00:00:00	-0.0257545	Underweight Dow Jones (20.00%) + Overweight US 10Y Treasury (80.00%)
2018-05-31 00:00:00	0.602746	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2018-06-30 00:00:00	-0.700772	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2018-07-31 00:00:00	2.66347	Underweight Dow Jones (30.00%) + Overweight US 10Y Treasury (70.00%)
2018-08-31 00:00:00	1.15956	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2018-09-30 00:00:00	0.332465	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2018-10-31 00:00:00	-3.24062	Underweight Dow Jones (30.00%) + Overweight US 10Y Treasury (70.00%)
2018-11-30 00:00:00	1.34259	Underweight Dow Jones (30.00%) + Overweight US 10Y Treasury (70.00%)
2018-12-31 00:00:00	-1.63062	Underweight Dow Jones (20.00%) + Overweight US 10Y Treasury (80.00%)
2019-01-31 00:00:00	1.30999	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2019-02-28 00:00:00	2.91609	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2019-03-31 00:00:00	0.53004	Underweight Dow Jones (20.00%) + Overweight US 10Y Treasury (80.00%)
2019-04-30 00:00:00	0.124632	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2019-05-31 00:00:00	-3.01917	Underweight Dow Jones (30.00%) + Overweight US 10Y Treasury (70.00%)
2019-06-30 00:00:00	2.82748	Underweight Dow Jones (30.00%) + Overweight US 10Y Treasury (70.00%)
2019-07-31 00:00:00	0.26006	Underweight Dow Jones (30.00%) + Overweight US 10Y Treasury (70.00%)
2019-08-31 00:00:00	1.23346	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2019-09-30 00:00:00	0.54422	Underweight Dow Jones (20.00%) + Overweight US 10Y Treasury (80.00%)
2019-10-31 00:00:00	-0.424496	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2019-11-30 00:00:00	1.91931	Underweight Dow Jones (30.00%) + Overweight US 10Y Treasury (70.00%)
2019-12-31 00:00:00	0.254299	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2020-01-31 00:00:00	0.487493	Underweight Dow Jones (30.00%) + Overweight US 10Y Treasury (70.00%)
2020-02-29 00:00:00	-1.31789	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2020-03-31 00:00:00	-6.1191	Underweight Dow Jones (20.00%) + Overweight US 10Y Treasury (80.00%)

Table 9 ALLAN Portfolio (p_{ALLAN}) monthly return and tactical allocation choices

Figure 9. Cumulative monthly return of the portfolios, $p = \{p_{BOB}, p_{FRED}, p_{ALLAN}\}$ vs. the benchmark (40% INDU:IND + 60% TY1:COM)

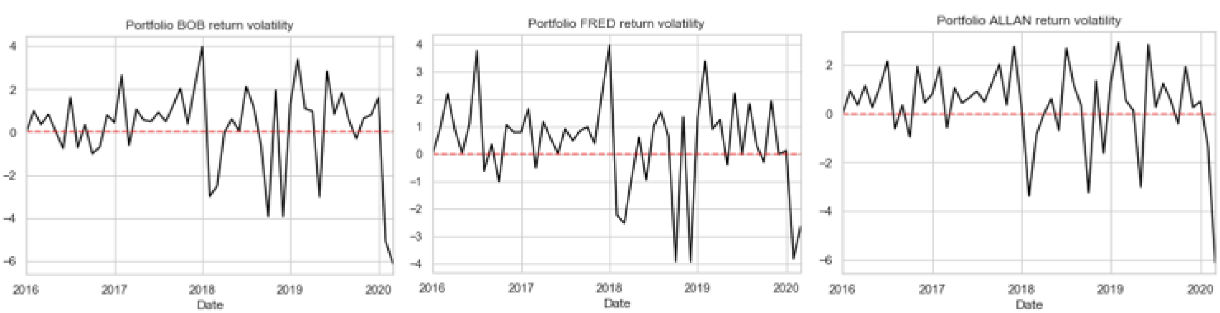


Figure 10 Volatility of Portfolios' returns, $p= \{p_{BOB}, p_{FRED}, p_{ALLAN}\}$ – out of sample time frame from January 2016 to March 2020

	p_{BOB}	p_{FRED}	p_{ALLAN}	**Benchmark**
Beta	1.2338	1.014	0.97	–
Alpha (annualized)	-2.96	0.134	-0.27	–
Max monthly down-turn	-6.119%	-3.935%	-6.119%	-4.3735%
Annualized Stdev	6.96	5.91	5.75	5.2536
Annualized return	2.47%	4.60%	4.02%	4.40%
Positive/Negative Months	35/15	35/15	37/13	37/13
Since Jan 2020	-9.5946%	-6.3294%	-6.9495	-6.0856%
Total months overperforming	24/50	26/50	23/50	–

Table 10 Key metrics of the portfolios, $p = \{p_{BOB}, p_{FRED}, p_{ALLAN}\}$

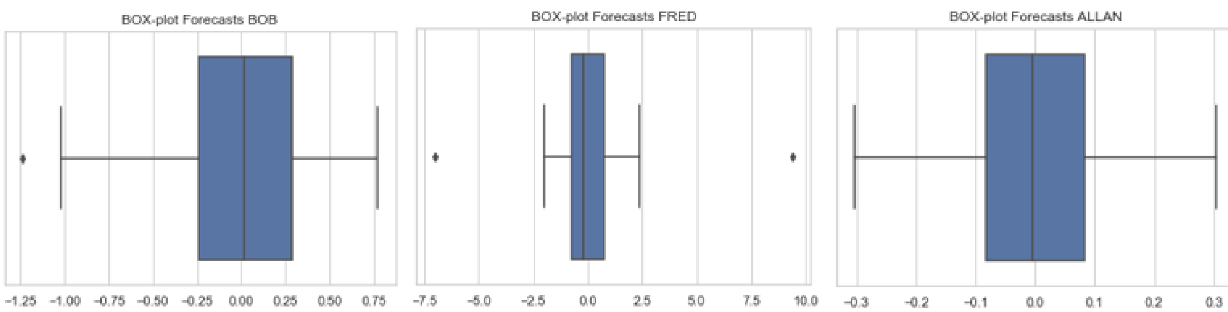


Figure 11 Box plot, distribution of forecast's returns

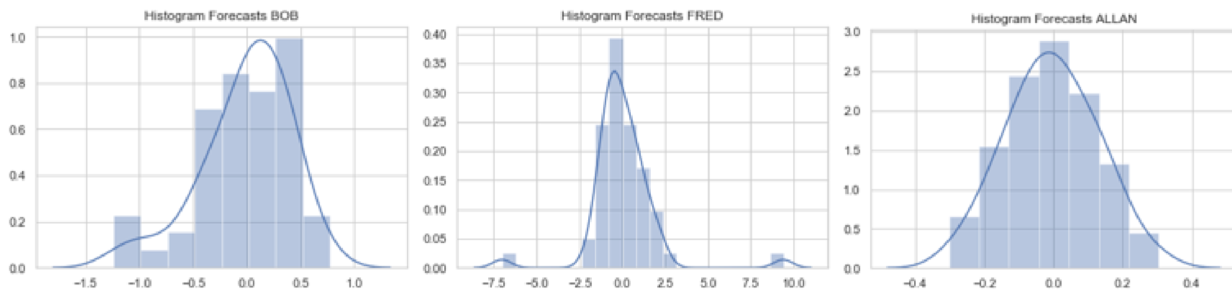


Figure 12 Histogram forecast's returns. The forecast using BOB and FRED dataset are skewed while the ones with ALLAN are more normally distributed. The skewness help to justify the performance of the algorithm on predicting correct positive and negative return.

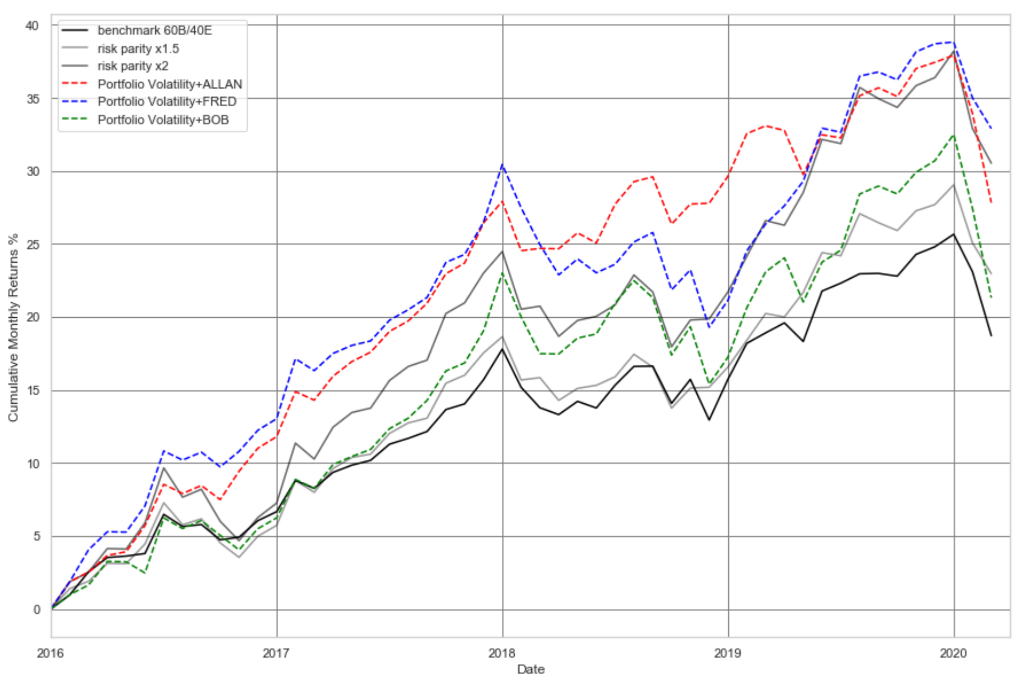


Figure 13 Cumulative monthly return of the portfolios, p' = {p' $_{BOB}$, p' $_{FRED}$, p' $_{ALLAN}$} vs. the benchmark (40% INDU:IND + 60% TY1:COM). The cumulative return of the risk parity with leverage 1.5 and 2 are plotted as well.

Figure 14 Volatility of Portfolios' returns, p' = {p' $_{BOB}$, p' $_{FRED}$, p' $_{ALLAN}$} – out of sample time frame from January 2016 to March 2020

	p' BOB	***p' FRED***	***p' ALLAN***	***Benchmark***
Beta	1.3306	1.1154	1.071	–
Alpha (annualized)	-0.8457	2.827	1.8258	–
Max monthly down-turn	-6.1191%	-3.9349%	-6.1191%	-4.3735%
Annualized Stdev	7.557	6.791	6.4326	5.2536
Annualized return	5.014%	7.7391%	6.5459%	4.40%
Positive/Negative Months	34/16	36/14	37/13	37/13
Since Jan 2020	-9.3838%	-5.811%	-9.6102%	-6.0856%
Total month overperforming benchmark	33/50	34/50	32/50	–
Total month overperforming rp1.5	26/50	25/50	25/50	–
Total month overperforming rp2	20/50	18/50	18/50	–

Table 11 Key metrics of the portfolios, p' = {p' $_{BOB}$, p' $_{FRED}$, p' $_{ALLAN}$}

Date	p_Vol_BOB_ret	message
2016-01-31 00:00:00	0	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2016-02-29 00:00:00	0.979027	Risk Parity, leverage 2.0
2016-03-31 00:00:00	0.658943	Risk Parity, leverage 2.0
2016-04-30 00:00:00	1.6148	Risk Parity, leverage 1.5
2016-05-31 00:00:00	-0.0226078	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2016-06-30 00:00:00	-0.763208	Risk Parity, leverage 2.0
2016-07-31 00:00:00	3.76857	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2016-08-31 00:00:00	-0.730337	Risk Parity, leverage 2.0
2016-09-30 00:00:00	0.542883	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2016-10-31 00:00:00	-1.00609	Risk Parity, leverage 1.5
2016-11-30 00:00:00	-0.994636	Risk Parity, leverage 1.5
2016-12-31 00:00:00	1.44621	Risk Parity, leverage 1.5
2017-01-31 00:00:00	0.741418	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2017-02-28 00:00:00	2.63632	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2017-03-31 00:00:00	-0.617653	Risk Parity, leverage 1.5
2017-04-30 00:00:00	1.63458	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2017-05-31 00:00:00	0.55044	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2017-06-30 00:00:00	0.489465	Risk Parity, leverage 1.5
2017-07-31 00:00:00	1.41486	Risk Parity, leverage 1.5
2017-08-31 00:00:00	0.722383	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2017-09-30 00:00:00	1.21651	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2017-10-31 00:00:00	2.01568	Risk Parity, leverage 1.5
2017-11-30 00:00:00	0.550631	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2017-12-31 00:00:00	2.1955	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2018-01-31 00:00:00	3.96135	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2018-02-28 00:00:00	-2.99832	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2018-03-31 00:00:00	-2.52481	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2018-04-30 00:00:00	-0.0257545	Risk Parity, leverage 2.0
2018-05-31 00:00:00	1.09262	Risk Parity, leverage 2.0
2018-06-30 00:00:00	0.281076	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2018-07-31 00:00:00	2.11428	Risk Parity, leverage 1.5
2018-08-31 00:00:00	1.53768	Risk Parity, leverage 2.0
2018-09-30 00:00:00	-1.16625	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2018-10-31 00:00:00	-3.93045	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2018-11-30 00:00:00	1.94286	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2018-12-31 00:00:00	-3.93485	Risk Parity, leverage 2.0
2019-01-31 00:00:00	1.85619	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2019-02-28 00:00:00	3.37956	Risk Parity, leverage 2.0
2019-03-31 00:00:00	2.44697	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2019-04-30 00:00:00	0.966848	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2019-05-31 00:00:00	-3.01917	Risk Parity, leverage 1.5
2019-06-30 00:00:00	2.724	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2019-07-31 00:00:00	0.822159	Risk Parity, leverage 2.0
2019-08-31 00:00:00	3.84469	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2019-09-30 00:00:00	0.54422	Risk Parity, leverage 1.5
2019-10-31 00:00:00	-0.53954	Risk Parity, leverage 2.0
2019-11-30 00:00:00	1.4881	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2019-12-31 00:00:00	0.786501	Risk Parity, leverage 2.0
2020-01-31 00:00:00	1.80603	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2020-02-29 00:00:00	-5.07075	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2020-03-31 00:00:00	-6.1191	Risk Parity, leverage 2.0

Table 12 Portfolio BOB + Risk Parity (p' $_{BOB}$) monthly returns and tactical allocation choices

Date	p_Vol_FRED_ret	message
2016-01-31 00:00:00	0	Risk Parity, leverage 2.0
2016-02-29 00:00:00	1.86087	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2016-03-31 00:00:00	2.18999	Risk Parity, leverage 1.5
2016-04-30 00:00:00	1.23586	Risk Parity, leverage 1.5
2016-05-31 00:00:00	-0.0226078	Risk Parity, leverage 2.0
2016-06-30 00:00:00	1.7892	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2016-07-31 00:00:00	3.76431	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2016-08-31 00:00:00	-0.624243	Risk Parity, leverage 2.0
2016-09-30 00:00:00	0.542883	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2016-10-31 00:00:00	-1.00609	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2016-11-30 00:00:00	1.05705	Risk Parity, leverage 1.5
2016-12-31 00:00:00	1.44621	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2017-01-31 00:00:00	0.793457	Risk Parity, leverage 2.0
2017-02-28 00:00:00	4.11119	Risk Parity, leverage 1.5
2017-03-31 00:00:00	-0.819496	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2017-04-30 00:00:00	1.17853	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2017-05-31 00:00:00	0.55044	Risk Parity, leverage 2.0
2017-06-30 00:00:00	0.306684	Risk Parity, leverage 1.5
2017-07-31 00:00:00	1.41486	Risk Parity, leverage 1.5
2017-08-31 00:00:00	0.722383	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2017-09-30 00:00:00	0.842829	Risk Parity, leverage 1.5
2017-10-31 00:00:00	2.39411	Risk Parity, leverage 1.5
2017-11-30 00:00:00	0.550631	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2017-12-31 00:00:00	2.1955	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2018-01-31 00:00:00	3.96135	Risk Parity, leverage 1.5
2018-02-28 00:00:00	-2.96389	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2018-03-31 00:00:00	-2.52481	Risk Parity, leverage 2.0
2018-04-30 00:00:00	-2.07089	Risk Parity, leverage 2.0
2018-05-31 00:00:00	1.09262	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2018-06-30 00:00:00	-0.949806	Risk Parity, leverage 1.5
2018-07-31 00:00:00	0.586678	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2018-08-31 00:00:00	1.52773	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2018-09-30 00:00:00	0.646094	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2018-10-31 00:00:00	-3.93045	Risk Parity, leverage 1.5
2018-11-30 00:00:00	1.3759	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2018-12-31 00:00:00	-3.93485	Risk Parity, leverage 2.0
2019-01-31 00:00:00	1.85619	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2019-02-28 00:00:00	3.37956	Risk Parity, leverage 1.5
2019-03-31 00:00:00	1.83522	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2019-04-30 00:00:00	1.24759	Risk Parity, leverage 1.5
2019-05-31 00:00:00	1.68077	Risk Parity, leverage 2.0
2019-06-30 00:00:00	3.632	Risk Parity, leverage 2.0
2019-07-31 00:00:00	-0.281004	Risk Parity, leverage 2.0
2019-08-31 00:00:00	3.84469	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2019-09-30 00:00:00	0.285466	Risk Parity, leverage 1.5
2019-10-31 00:00:00	-0.53954	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2019-11-30 00:00:00	1.91931	Risk Parity, leverage 2.0
2019-12-31 00:00:00	0.551984	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2020-01-31 00:00:00	0.118235	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2020-02-29 00:00:00	-3.81979	Risk Parity, leverage 1.5
2020-03-31 00:00:00	-2.10947	Risk Parity, leverage 2.0

Table 13 Portfolio FRED + Risk Parity (p' $_{FRED}$) monthly return and tactical allocation choices

Date	p_Vol_ALLAN_ret	message
2016-01-31 00:00:00	0	Risk Parity, leverage 2.0
2016-02-29 00:00:00	1.86087	Risk Parity, leverage 2.0
2016-03-31 00:00:00	0.658943	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2016-04-30 00:00:00	1.14578	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2016-05-31 00:00:00	0.252571	Risk Parity, leverage 2.0
2016-06-30 00:00:00	1.7892	Risk Parity, leverage 1.5
2016-07-31 00:00:00	2.82643	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2016-08-31 00:00:00	-0.624243	Risk Parity, leverage 2.0
2016-09-30 00:00:00	0.542883	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2016-10-31 00:00:00	-0.965415	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2016-11-30 00:00:00	1.93692	Risk Parity, leverage 2.0
2016-12-31 00:00:00	1.58036	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2017-01-31 00:00:00	0.793457	Risk Parity, leverage 1.5
2017-02-28 00:00:00	3.08339	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2017-03-31 00:00:00	-0.581713	Risk Parity, leverage 1.5
2017-04-30 00:00:00	1.63458	Risk Parity, leverage 2.0
2017-05-31 00:00:00	1.00537	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2017-06-30 00:00:00	0.647119	Risk Parity, leverage 1.5
2017-07-31 00:00:00	1.41486	Risk Parity, leverage 1.5
2017-08-31 00:00:00	0.722383	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2017-09-30 00:00:00	1.21651	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2017-10-31 00:00:00	2.01568	Risk Parity, leverage 2.0
2017-11-30 00:00:00	0.734174	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2017-12-31 00:00:00	2.74225	Risk Parity, leverage 2.0
2018-01-31 00:00:00	1.48249	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2018-02-28 00:00:00	-3.38053	Risk Parity, leverage 1.5
2018-03-31 00:00:00	0.156815	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2018-04-30 00:00:00	-0.0257545	Risk Parity, leverage 2.0
2018-05-31 00:00:00	1.09262	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2018-06-30 00:00:00	-0.700772	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2018-07-31 00:00:00	2.66347	Risk Parity, leverage 1.5
2018-08-31 00:00:00	1.53768	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2018-09-30 00:00:00	0.332465	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2018-10-31 00:00:00	-3.24062	Risk Parity, leverage 1.5
2018-11-30 00:00:00	1.3759	Risk Parity, leverage 1.5
2018-12-31 00:00:00	0.0546227	Risk Parity, leverage 2.0
2019-01-31 00:00:00	1.85619	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2019-02-28 00:00:00	2.91609	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2019-03-31 00:00:00	0.53004	Risk Parity, leverage 2.0
2019-04-30 00:00:00	-0.317778	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2019-05-31 00:00:00	-3.01917	Risk Parity, leverage 1.5
2019-06-30 00:00:00	2.724	Risk Parity, leverage 1.5
2019-07-31 00:00:00	-0.210753	Risk Parity, leverage 1.5
2019-08-31 00:00:00	2.88352	Overweight Dow Jones (60.00%) + Underweight US 10Y Treasury (40.00%)
2019-09-30 00:00:00	0.54422	Risk Parity, leverage 2.0
2019-10-31 00:00:00	-0.598609	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2019-11-30 00:00:00	1.91931	Risk Parity, leverage 1.5
2019-12-31 00:00:00	0.422686	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2020-01-31 00:00:00	0.487493	Risk Parity, leverage 1.5
2020-02-29 00:00:00	-3.97861	Overweight Dow Jones (50.00%) + Underweight US 10Y Treasury (50.00%)
2020-03-31 00:00:00	-6.1191	Risk Parity, leverage 2.0

Table 14 Portfolio ALLAN + Risk Parity (p'_{ALLAN}) monthly return and tactical allocation choices

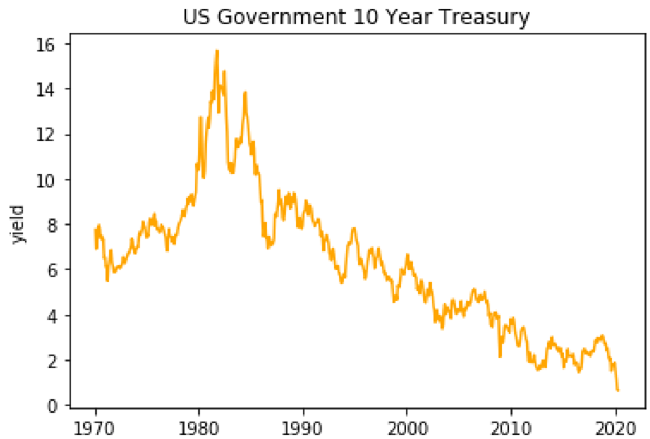


Figure 15 The 10 Year US Treasury Note yield from 1970

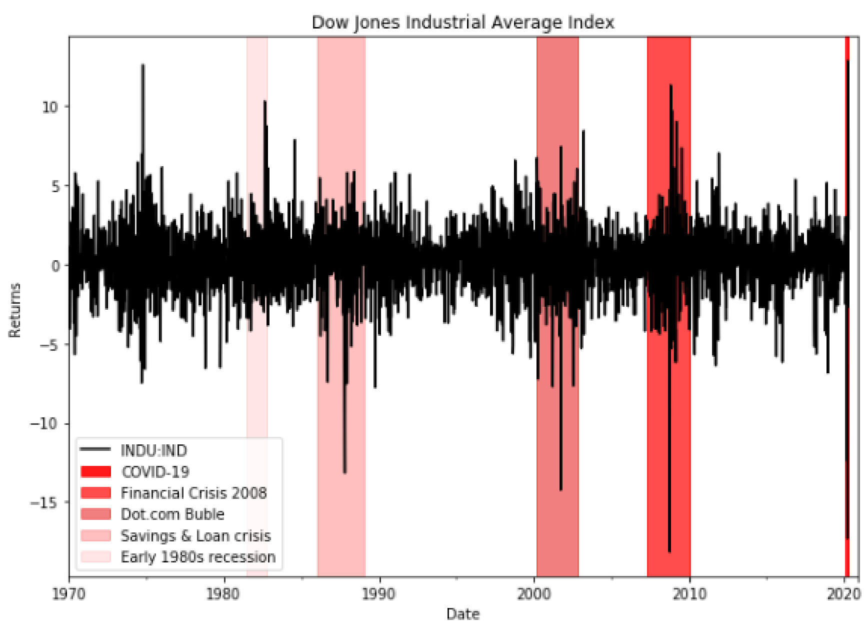


Figure 16 Dow Jones Industrial Average Index weekly returns, highlighted in red the different economic crises which affect the United States of America since 1970