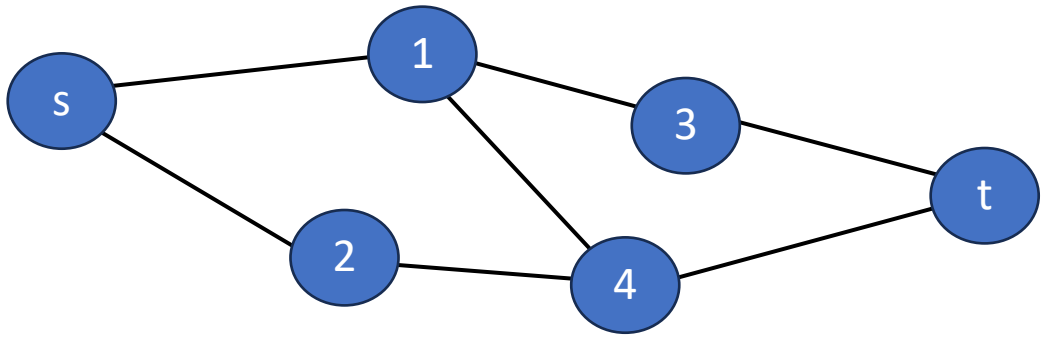
Algorithmic Game Theory

Project Assignment A.Y. 2023/2024

Consider a setting with an *undirected* graph **G=(N,E)**, where **N={1,2,…,n} U {s,t}** is the set of nodes and **s** and **t** are two distinguished *source* and *target* nodes, respectively. The objective is to define a path connecting **s** and **t**, via the nodes in {1,2,…,n} with each of them being controlled by an agent. Hereinafter, such nodes are therefore transparently viewed as the corresponding agents.

For each of the following questions, implement in Python a method that can provide results for any possible graph **G**. Report then the results obtained over the specific graph instance depicted below.



Provide arguments and explanations on the various design choices.

1. Assume that forming a path connecting **s** and **t** leads to a reward of **100$**. Then, compute the Shapley value associated with the agents in **{1,2,…,n}** as a fair way to distribute that reward among the agents in **{1,2,…,n}**, which in particular encourages cooperation.
2. Assume that each agent in **{1,2…,n}** might freely decide whether to provide her/his contribution to connect **s** and **t**. Assume, in particular, that each agent is willing to contribute only if at most two of her(his neighbors do so. Then, check whether the resulting setting admits a pure Nash equilibrium and compute one, if any.
3. Assume that **G** has treewidth bounded by some constant and provide again answer to point 2, by exploiting this additional information.
4. Assume that agent **i** in **{1,2,…,n}** has some internal utility – say **i x 10$** – for being selected in a path connecting **s** and **t**, and that s/he might cheat in declaring a different utility. Assume moreover that the goal is to form a path with the maximum overall possible utility, and compute a payment scheme that provides incentives to truthfully report such utility values.