

## Collective dynamics in binary superfluids: From dissipationless flow to dispersive shock waves

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anr<sup>®</sup>

# 1. Introduction

## Binary superfluids...

- BEC of interacting atoms in 2 hyperfine spin states · polaritons excitonic · ... Proc. Internat. School Phys. Enrico Fermi **211** (2025)  
Rev. Mod. Phys. **85**, 299 (2013)
- Elliptically polarized laser in a birefringent nonlinear medium · ... Adv. At. Mol. Opt. Phys. **74**, 157 (2025)
- $\mathcal{H} = \frac{1}{2}(|\nabla\psi_+|^2 + |\nabla\psi_-|^2) + \frac{1}{2}(|\psi_+|^4 + |\psi_-|^4) + \alpha|\psi_+|^2|\psi_-|^2$        $\alpha = g_{+-}/g_{\pm\pm}$   
 $\psi_{\pm} = \sqrt{\rho_{\pm}} e^{i \int d\mathbf{r} \cdot \mathbf{v}_{\pm}}$       Density mode  $\begin{cases} \rho = \rho_+ + \rho_- \\ \mathbf{V} \text{ fluct.} \end{cases}$       Spin mode  $\begin{cases} \sigma = \rho_+ - \rho_- \\ \mathbf{v} \text{ fluct.} \end{cases}$
- Miscibility problems · Collective & topological excitations · ...

## ...of light

Phys. Rev. Lett. **134**, 223403 (2025)

Elliptically polarized laser in a warm vapor of  $^{87}\text{Rb}$  @ LKB, Paris



Quentin Glorieux



Claire Michel



Nicolas Cheroret

## ...of matter

Phys. Rev. Lett. **128**, 083401 (2022)

BEC of  $^{39}\text{K}$  in 2 coherently coupled Zeeman states @ LCF, Palaiseau



Thomas Bourdel



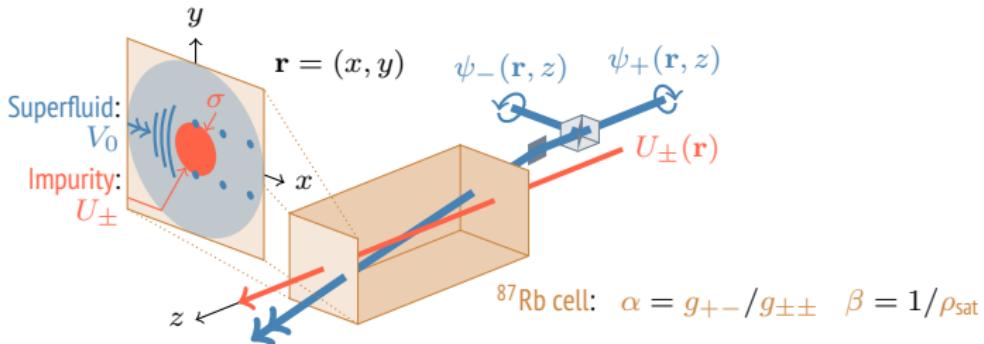
Thibault Congy



Patrick Sprenger

## 2. Critical speed of a binary superfluid of light

arXiv:2601.16005 (2026)



### Model equations

- 2-component inhomogeneous nonlinear Schrödinger-type equation, analogous to the Gross-Pitaevskii equation of Bose-Bose superfluid mixtures:

$$i\partial_z \psi_\pm = \left[ -\frac{1}{2} \nabla^2 + U_\pm(\mathbf{r}) + \frac{|\psi_\pm|^2 + \alpha |\psi_\mp|^2}{1 + \beta (|\psi_+|^2 + |\psi_-|^2)} \right] \psi_\pm$$

- What is the condition on  $V_0$  for the following incident flow to be superfluid/dissipationless?

$$\psi_\pm^{(\text{in})} = \sqrt{\frac{1}{2}} e^{i(V_0 x - \mu z)}$$

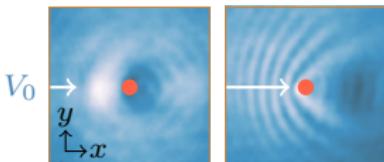
Fully balanced

$$\frac{U_\pm > 0}{\text{Obstacle}} \quad \frac{g_{\pm\pm} > g_{+-} > 0}{\text{Miscible, repulsive}} \implies 0 < \alpha < 1$$

## Linear response

- Impurity  $\ll$  Interactions
- Impurity-induced excitations:  
Density & spin Bogoliubov waves

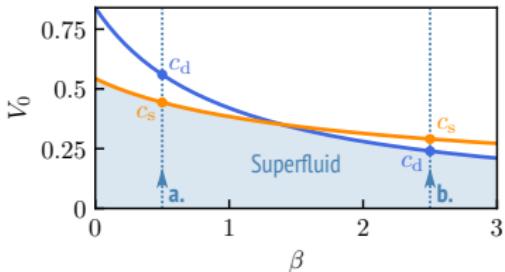
Single-component superfluid of light  
in a photorefractive crystal



Nat. Commun. 9, 2108 (2018)

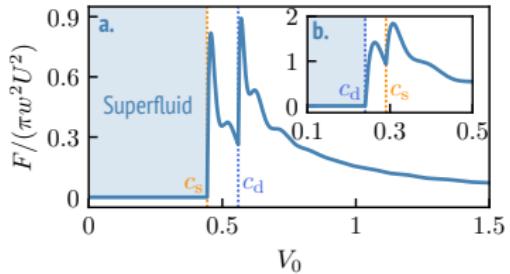
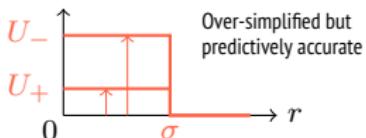
## Landau criterion

$$V_0 < V_c = \min \left\{ c_d = \sqrt{\frac{1+\alpha}{2}} \frac{1}{1+\beta}, c_s = \sqrt{\frac{1-\alpha}{2}} \frac{1}{\sqrt{1+\beta}} \right\}$$



## Drag force

$$F = \int d^2r [\psi_+^* \psi_-^*] \begin{bmatrix} \partial_x U_+(r) & 0 \\ 0 & \partial_x U_-(r) \end{bmatrix} \begin{bmatrix} \psi_+ \\ \psi_- \end{bmatrix}$$



## Superfluid hydrodynamics

- Stationary Bernoulli equations:

$$\cos \theta \equiv \sigma / \rho \quad U, u \equiv U_+ \pm U_-$$

$$\begin{cases} \frac{(1+\alpha)\rho}{1+\beta\rho} = \frac{1+\alpha}{1+\beta} - U \mathbf{1}_{r<\sigma} - (V^2 - V_0^2) - \frac{v^2}{4} + \frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}} + \frac{\nabla \cdot (\rho \nabla \theta)}{2\rho \tan \theta} - \frac{(\nabla \theta)^2}{4} \\ \frac{(1-\alpha)\sigma}{1+\beta\rho} = -u \mathbf{1}_{r<\sigma} - \mathbf{V} \cdot \mathbf{v} - \frac{\nabla \cdot (\rho \nabla \theta)}{2\rho \sin \theta} \end{cases}$$

- Stationary continuity equations:

$$\begin{cases} \nabla \cdot \left( \rho \mathbf{V} + \frac{\sigma \mathbf{v}}{2} \right) = 0 \\ \nabla \cdot \left( \sigma \mathbf{V} + \frac{\rho \mathbf{v}}{2} \right) = 0 \end{cases}$$

## Critical velocity

The flow monotonically flattens at infinity  $\iff$  The stationary continuity equations are strongly elliptic:

$$SE\left[\rho = \frac{fn.(\mathbf{V}, \mathbf{v})}{Bernoulli}, \mathbf{V}, \sigma = \frac{fn.(\mathbf{V}, \mathbf{v})}{Bernoulli}, \mathbf{v}\right] > 0$$

$$SE(\mathbf{r}; V_0, \alpha, \beta, U, u) > 0$$

$$V_0 < V_c(\alpha, \beta, U, u)$$

Local generalization of the Landau criterion  
for an imbalanced mixture  $\sigma \gg 1$ : LDA ✓

### Chaplygin method

Sci. Mem. Moscow Univ. Math. Phys. 21, 1 (1902)

Phys. Rev. Lett. 69, 1644 (1992)

Phys. Rev. A 109, 013317 (2024)

## Superfluid hydrodynamics

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- Stationary continuity equations:

Compressibility  $\ll 1$

$$\begin{cases} \nabla \cdot \left( \rho \mathbf{V} + \frac{\sigma \mathbf{v}}{2} \right) = 0 \\ \nabla \cdot \left( \sigma \mathbf{V} + \frac{\rho \mathbf{v}}{2} \right) = 0 \end{cases} \quad \begin{cases} \nabla \cdot \mathbf{V} = \nabla \cdot \mathbf{v} = 0 \\ \text{Appropriate boundary conditions @ } r = \sigma \end{cases}$$

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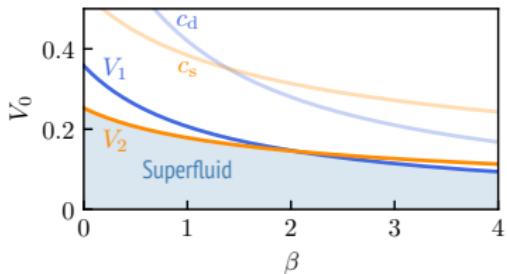
Phys. Rev. A 109, 013317 (2024)

$$V_0 < V_c(\alpha, \beta, U, u)$$

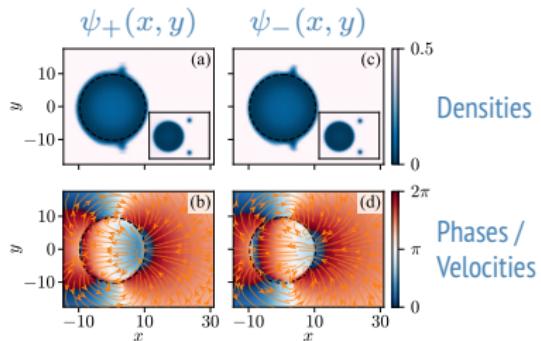
## Impenetrable impurity

$$U > (1 + \alpha)/(1 + \beta)$$

$$V_c = \min \left\{ V_1: c_d \sqrt{\frac{(1 + \beta)[\sqrt{(1 + \beta)(121 + 25\beta)} - (11 + 5\beta)]}{9\beta}}, V_2: c_s \sqrt{\frac{2(1 + \alpha)}{11 + 5\alpha}} \right\}$$



Vortex-antivortex pairs at the impurity's poles

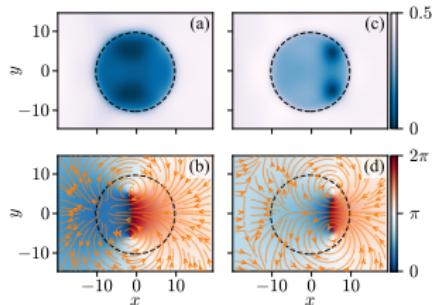


## Penetrable impurity

Cumbersome yet analytical

$$V_c(\alpha, \beta, U, u)$$

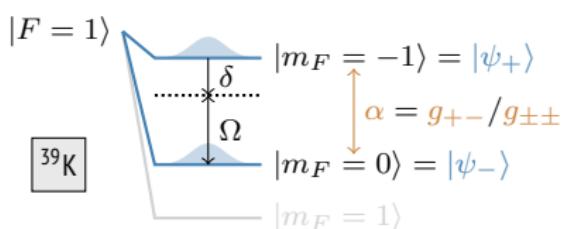
Jones-Roberts solitons inside the impurity



### 3. Nonlinear periodic waves in a binary BEC

In progress (2026)

#### BEC in two Rabi-coupled Zeeman states



$$\begin{aligned}\mathcal{H} = & \frac{1}{2}(|\psi_+|^4 + |\psi_-|^4) + \alpha|\psi_+|^2|\psi_-|^2 \\ & - \frac{\Omega}{2}(\psi_+^*\psi_- + \psi_-^*\psi_+) \\ & + \frac{\delta}{2}(|\psi_+|^2 - |\psi_-|^2)\end{aligned}$$

Reminder:  $\rho = |\psi_+|^2 + |\psi_-|^2$

#### Cubic-quintic nonlinear Schrödinger equation

Ground state for  $\frac{|1-\alpha|\rho}{\Omega} \ll 1$ :

D. Petrov, Orsay    T. Bourdel, Palaiseau    L. Tarruell, Barcelona

$$\mathcal{H}_{\text{gs}} \simeq -\frac{\sqrt{\Omega^2 + \delta^2}}{2}\rho + g_2 \frac{\rho^2}{2} + \boxed{g_3 \frac{\rho^3}{3}}$$

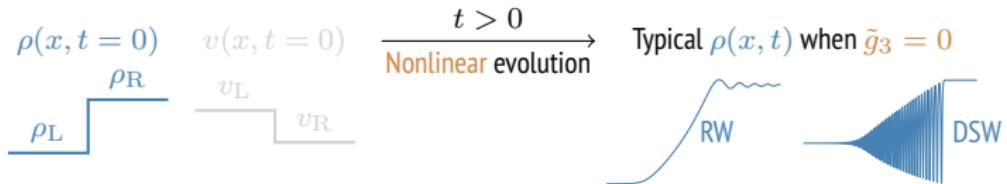
$$\begin{cases} g_2 = 1 - \frac{1}{2} \frac{1-\alpha}{1+\delta^2/\Omega^2} \\ g_3 = -\frac{3}{4} \frac{(1-\alpha)^2 \delta^2 / \Omega^2}{\Omega(1+\delta^2/\Omega^2)^{5/2}} \boxed{< 0} \end{cases}$$

Effective cubic-quintic nonlinear Schrödinger dynamics for a scalar field  $\varphi(\mathbf{r}, t)$  with density  $|\varphi|^2 = \rho$ :

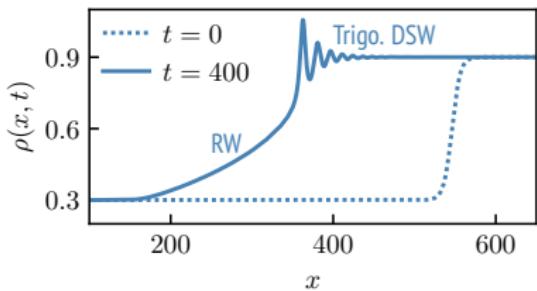
$$i\partial_t \varphi = \left(-\frac{1}{2}\nabla^2 + g_2|\varphi|^2 + g_3|\varphi|^4\right)\varphi$$

## Contact dispersive shock waves

- 1D configuration (BEC trapped in the  $y-z$  plane):  $i\partial_t \varphi = (-\frac{1}{2}\partial_{xx} + \tilde{g}_2|\varphi|^2 + \tilde{g}_3|\varphi|^4)\varphi$
- Riemann initial-value problem:  $\varphi = \sqrt{\rho} e^{i \int dx v}$



- Contact dispersive shock waves emerge when  $\tilde{g}_3 \neq 0$ :



### Whitham modulation theory

*Nonlinear Periodic Waves and Their Modulations* (2000)  
Physica D 333, 11 (2016)

Within experimental reach @ LCF:

$$(\Delta x, \Delta t)_{\text{Contact DSW}} \simeq (14.4 \mu\text{m}, 318.4 \text{ ms})$$

$$\begin{aligned} \omega_\perp / (2\pi) &= 300 \text{ Hz} & \Omega / (2\pi) &= 25.4 \text{ kHz} \\ \rho &\simeq 2.5 \times 10^9 \text{ m}^{-1} & \delta / \Omega &= 0.9 \end{aligned}$$

# 4. Vortex-induced self-propulsion against a superflow

arXiv:2512.09028 (2025) & Forthcoming (2026)



Myrann Baker-Rasooli



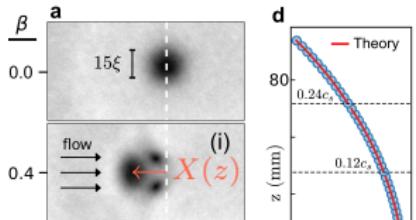
Tangui Aladjidi

@ LKB, Paris

## Upstream motion driven by downstream vortices

$$i\partial_z \psi = [-\frac{1}{2}\nabla^2 + U(\mathbf{r} - X\hat{\mathbf{x}}) + |\psi|^2]\psi$$

$$k \frac{d^2 X}{dz^2} \propto \int d^2 \mathbf{r} |\psi|^2 \partial_x U(\mathbf{r} - X\hat{\mathbf{x}})$$

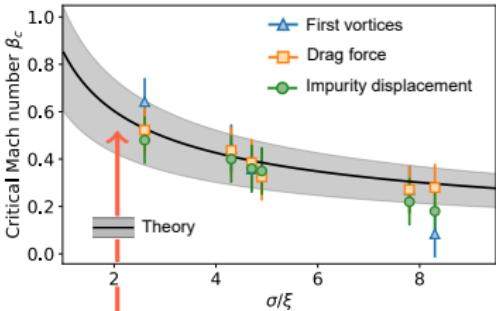


$$X(z) \simeq -C^{\text{st}} \Delta \rho z^2 + O(z)$$

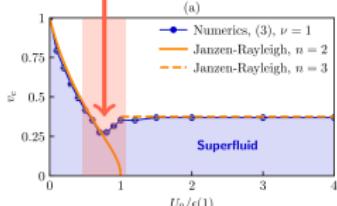
$$\Delta \rho \simeq \frac{2\sqrt{2}}{\sigma} \left( \frac{V_0}{\sigma} - \frac{\sqrt{2}}{\sigma} \right) \gtrsim 0.16$$

## Critical speed for vortex shedding

$V_c$  vs. impurity's radius  $\sigma$



$$V_c \simeq \sqrt{\frac{8}{11} \frac{\delta}{\sigma}} \quad \delta \equiv \sigma - r_{v-\text{av}} \sim 1$$



$V_c$  vs. impurity's amplitude  $U_0$  for  $\sigma \gg 1$

Phys. Rev. A  
109, 013317 (2024)

See also:  
Phys. Rev. A  
107, 023310 (2023)

## 5. Summary and outlook

### Critical speed of a binary superfluid of light

arXiv:2601.16005 (2026)

- $V_c = \min\{c_d, c_s\} \xleftarrow[\text{Landau criterion}]{0 \leftarrow U_{\pm}} V_c = \text{fn.}(U_{\pm})$   
Strongly elliptic stationary flow
- $V_0 \gtrsim V_c$ : Vortex-antivortex pairs & Jones-Roberts solitons in both components  $\psi_+$  &  $\psi_-$

### Nonlinear periodic waves in a binary BEC

In progress (2026)

- Effective cubic-quintic nonlinear Schrödinger description at large Rabi frequency:  
 $i\partial_t\varphi = (-\frac{1}{2}\nabla^2 + g_2|\varphi|^2 - |g_3||\varphi|^4)\varphi$
- Contact dispersive shock waves (rarefaction + trigonometric) in a 1D configuration

### Vortex-induced self-propulsion against a superflow

arXiv:2512.09028 (2025) & Forthcoming (2026)

- Upstream trajectory of the mobile impurity:  $X(z) \simeq -C^{\text{st}} \Delta\rho z^2 + O(z)$
- Critical speed for vortex shedding:  $V_c \simeq \sqrt{\frac{8}{11}}\sigma^{-1/2}$

## Ongoing work and future directions

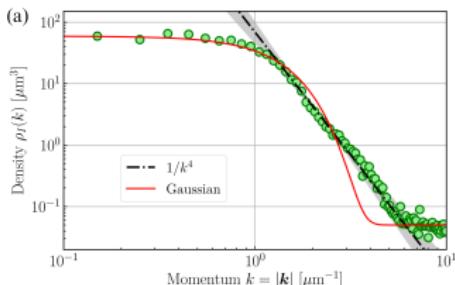
- Quantum depletion & OTOCs in a superfluid of light

Phys. Rev. A **92**, 043802 (2015)

$$[\hat{\psi}(\mathbf{r}, t; z), \hat{\psi}^\dagger(\mathbf{r}', t'; z)] = \frac{2\hbar\omega_0}{n_0\epsilon_0 c} \delta^{(2)}(\mathbf{r} - \mathbf{r}') \delta(t - t')$$

- Nonequilibrium Tan contact in a quenched binary BEC

Phys. Rev. Lett. **130**, 153401 (2023)



$$\rho(k, t) \underset{k \rightarrow \infty}{\simeq} \frac{\mathcal{C}(t)}{k^4}$$

- Quenched 2D soliton gas
- KPZ dynamics in quantum scars