LPTMC seminar, Paris

February 3, 2026

Collective dynamics in binary superfluids: From dissipationless flow to dispersive shock waves

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LPTMS, Orsay

▷ Quantum-SOPHA
▷ STLight
▷ UniQ-RingS

1. Introduction

Binary superfluids…

- BEC of interacting atoms in 2 hyperfine spin states · polaritons excitonic
  Elliptically polarized laser in a birefringent nonlinear medium · …

  Proc. Internat. School Phys. Enrico Fermi **211** (2025)
  Rev. Mod. Phys. **85**, 299 (2013)
  Adv. At. Mol. Opt. Phys. **74**, 157 (2025)

- $$\mathcal{H} = \frac{1}{2}(|\nabla\psi_+|^2 + |\nabla\psi_-|^2) + \underbrace{\frac{1}{2}(|\psi_+|^4 + |\psi_-|^4)}_{\text{Intra-component int.}} + \underbrace{\alpha|\psi_+|^2|\psi_-|^2}_{\text{Inter-component int.}} \qquad \alpha = g_{+-}/g_{\pm\pm}$$

- $\psi_\pm = \sqrt{\rho_\pm}\,\mathrm{e}^{\mathrm{i}\int \mathrm{d}\mathbf{r}\cdot\mathbf{v}_\pm}$

  Density mode ρ, $\mathbf{V}$ fluct. $\begin{cases} \rho = \rho_+ + \rho_- \\ \mathbf{V} = \frac{1}{2}(\mathbf{v}_+ + \mathbf{v}_-) \end{cases}$ Spin mode σ, $\mathbf{v}$ fluct. $\begin{cases} \sigma = \rho_+ - \rho_- \\ \mathbf{v} = \mathbf{v}_+ - \mathbf{v}_- \end{cases}$

- Miscibility problems · Collective & topological excitations · …

…of light

Phys. Rev. Lett. **134**, 223403 (2025)

Elliptically polarized laser in a warm vapor of ^{87}Rb
@ LKB, Paris

Quentin Glorieux

Claire Michel

Nicolas Cherroret

…of matter

Phys. Rev. Lett. **128**, 083401 (2022)

BEC of ^{39}K in 2 coherently coupled Zeeman states
@ LCF, Palaiseau

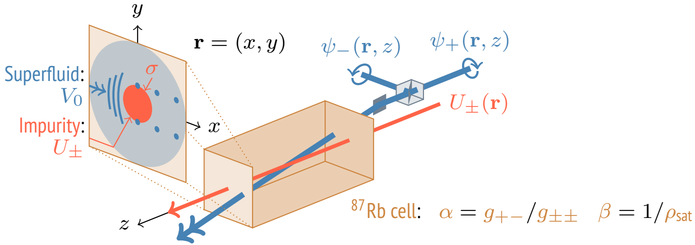
Thomas Bourdel

Thibault Congy

Patrick Sprenger

2. Critical speed of a binary superfluid of light

arXiv:2601.16005 (2026)



Model equations

- 2-component inhomogeneous nonlinear Schrödinger-type equation, analogous to the Gross-Pitaevskii equation of Bose-Bose superfluid mixtures:

$$\mathrm{i}\partial_z \psi_\pm = \left[-\frac{1}{2}\nabla^2 + U_\pm(\mathbf{r}) + \frac{|\psi_\pm|^2 + \alpha|\psi_\mp|^2}{1 + \beta(|\psi_+|^2 + |\psi_-|^2)} \right] \psi_\pm$$

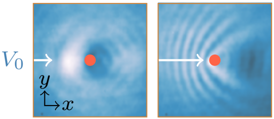
- What is the condition on V_0 for the following incident flow to be superfluid/dissipationless?

$$\psi_\pm^{(\text{in})} = \underbrace{\sqrt{\tfrac{1}{2}}\, \mathrm{e}^{\mathrm{i}(V_0 x - \mu z)}}_{\text{Fully balanced}} \qquad \underbrace{U_\pm > 0}_{\text{Obstacle}} \qquad \underbrace{g_{\pm\pm} > g_{+-} > 0}_{\text{Miscible, repulsive}} \implies 0 < \alpha < 1$$

Linear response

- Impurity $\ll$ Interactions
- Impurity-induced excitations:
  Density & spin Bogoliubov waves

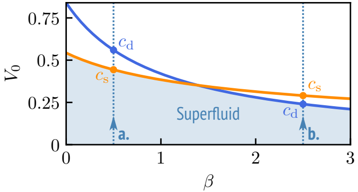
Single-component superfluid of light
in a photorefractive crystal



Nat. Commun. **9**, 2108 (2018)

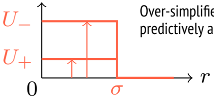
Landau criterion

$$V_0 < V_c = \min\left\{ c_d = \sqrt{\frac{1+\alpha}{2}}\,\frac{1}{1+\beta},\; c_s = \sqrt{\frac{1-\alpha}{2}}\,\frac{1}{\sqrt{1+\beta}} \right\}$$

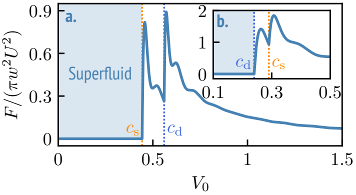


Drag force

$$F = \int d^2\mathbf{r}\; \begin{bmatrix}\psi_+^* & \psi_-^*\end{bmatrix} \begin{bmatrix}\partial_x U_+(\mathbf{r}) & 0 \\ 0 & \partial_x U_-(\mathbf{r})\end{bmatrix} \begin{bmatrix}\psi_+ \\ \psi_-\end{bmatrix}$$



Over-simplified but
predictively accurate



Superfluid hydrodynamics

- Stationary Bernoulli equations: $\cos\theta \equiv \sigma/\rho \qquad U, u \equiv U_+ \pm U_-$

$$\begin{cases} \dfrac{(1+\alpha)\rho}{1+\beta\rho} = \dfrac{1+\alpha}{1+\beta} - U\mathbf{1}_{r<\sigma} - (V^2 - V_0^2) - \dfrac{v^2}{4} + \dfrac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}} + \dfrac{\nabla\cdot(\rho\nabla\theta)}{2\rho\tan\theta} - \dfrac{(\nabla\theta)^2}{4} \\ \dfrac{(1-\alpha)\sigma}{1+\beta\rho} = -u\mathbf{1}_{r<\sigma} - \mathbf{V}\cdot\mathbf{v} - \dfrac{\nabla\cdot(\rho\nabla\theta)}{2\rho\sin\theta} \end{cases}$$

- Stationary continuity equations:

$$\begin{cases} \nabla\cdot\left(\rho\mathbf{V} + \dfrac{\sigma\mathbf{v}}{2}\right) = 0 \\ \nabla\cdot\left(\sigma\mathbf{V} + \dfrac{\rho\mathbf{v}}{2}\right) = 0 \end{cases}$$

Critical velocity

The flow monotonically flattens at infinity $\Longleftrightarrow$ The stationary continuity equations are strongly elliptic:

$$\text{SE}\Big[\rho = \underbrace{\text{fn.}(\mathbf{V},\mathbf{v})}_{\text{Bernoulli}},\ \ \mathbf{V},\ \ \sigma = \underbrace{\text{fn.}(\mathbf{V},\mathbf{v})}_{\text{Bernoulli}},\ \ \mathbf{v}\Big] > 0$$

$$\text{SE}(\mathbf{r}; V_0, \alpha, \beta, U, u) > 0$$

$$V_0 < V_c(\alpha, \beta, U, u)$$

Local generalization of the Landau criterion for an imbalanced mixture $\sigma \gg 1$: LDA ✓

Chaplygin method

Sci. Mem. Moscow Univ. Math. Phys. **21**, 1 (1902)
Phys. Rev. Lett. **69**, 1644 (1992)
Phys. Rev. A **109**, 013317 (2024)

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- Stationary continuity equations: Compressibility $\ll 1$

$$\begin{cases} \nabla\cdot\left(\rho\mathbf{V} + \dfrac{\sigma\mathbf{v}}{2}\right) = 0 \\ \nabla\cdot\left(\sigma\mathbf{V} + \dfrac{\rho\mathbf{v}}{2}\right) = 0 \end{cases} \qquad \begin{cases} \nabla\cdot\mathbf{V} = \nabla\cdot\mathbf{v} = 0 \\ \text{Appropriate boundary conditions @ } r = \sigma \end{cases}$$

Critical velocity

The flow monotonically flattens at infinity $\Longleftrightarrow$ The stationary continuity equations are strongly elliptic:

$$\mathrm{SE}\Big[\rho = \underbrace{\mathrm{fn.}(\mathbf{V},\mathbf{v})}_{\text{Bernoulli}},\ \mathbf{V},\ \sigma = \underbrace{\mathrm{fn.}(\mathbf{V},\mathbf{v})}_{\text{Bernoulli}},\ \mathbf{v}\Big] > 0$$

$$\mathrm{SE}(\mathbf{r}; V_0, \alpha, \beta, U, u) > 0$$

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Local generalization of the Landau criterion for an imbalanced mixture $\quad \sigma \gg 1$: LDA ✓

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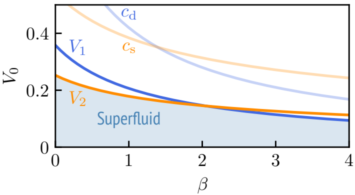
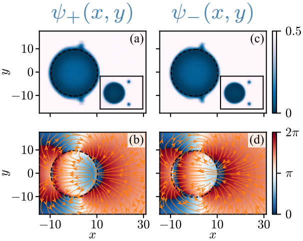
Phys. Rev. Lett. **69**, 1644 (1992)
Phys. Rev. A **109**, 013317 (2024)

Impenetrable impurity

$U > (1+\alpha)/(1+\beta)$

$$V_c = \min\left\{ V_1\colon c_d \sqrt{\frac{(1+\beta)\left[\sqrt{(1+\beta)(121+25\beta)} - (11+5\beta)\right]}{9\beta}},\ V_2\colon c_s \sqrt{\frac{2(1+\alpha)}{11+5\alpha}} \right\}$$





Densities

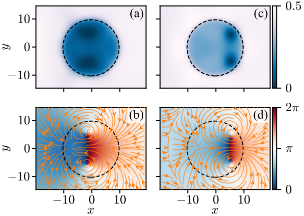
Phases / Velocities

Vortex-antivortex pairs at the impurity's poles

Penetrable impurity

Cumbersome $V_c(\alpha, \beta, U, u)$

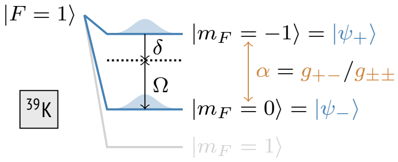
Jones-Roberts solitons inside the impurity



3. Nonlinear periodic waves in a binary BEC

In progress (2026)

BEC in two Rabi-coupled Zeeman states



$$\mathcal{H} = \frac{1}{2}(|\psi_+|^4 + |\psi_-|^4) + \alpha|\psi_+|^2|\psi_-|^2 - \frac{\Omega}{2}(\psi_+^*\psi_- + \psi_-^*\psi_+) + \frac{\delta}{2}(|\psi_+|^2 - |\psi_-|^2)$$

Reminder: $\rho = |\psi_+|^2 + |\psi_-|^2$

Cubic-quintic nonlinear Schrödinger equation

Ground state for $\frac{|1-\alpha|\rho}{\Omega} \ll 1$:

D. Petrov, Orsay   T. Bourdel, Palaiseau   L. Tarruell, Barcelona

$$\mathcal{H}_{\text{gs}} \simeq -\frac{\sqrt{\Omega^2+\delta^2}}{2}\rho + g_2\frac{\rho^2}{2} + \boxed{g_3\frac{\rho^3}{3}}$$

$$\begin{cases} g_2 = 1 - \frac{1}{2}\dfrac{1-\alpha}{1+\delta^2/\Omega^2} \\ g_3 = -\frac{3}{4}\dfrac{(1-\alpha)^2\delta^2/\Omega^2}{\Omega(1+\delta^2/\Omega^2)^{5/2}} \boxed{<0} \end{cases}$$

$\Longrightarrow$ Effective cubic-quintic nonlinear Schrödinger dynamics for a scalar field $\varphi(\mathbf{r},t)$ with density $|\varphi|^2 = \rho$:

$$\mathrm{i}\partial_t\varphi = (-\tfrac{1}{2}\nabla^2 + g_2|\varphi|^2 + g_3|\varphi|^4)\varphi$$

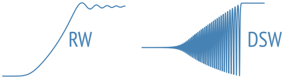
Contact dispersive shock waves

- 1D configuration (BEC trapped in the $y\!-\!z$ plane): $\quad i\partial_t\varphi = (-\frac{1}{2}\partial_{xx} + \tilde{g}_2|\varphi|^2 + \tilde{g}_3|\varphi|^4)\varphi$
- Riemann initial-value problem: $\quad \varphi = \sqrt{\rho}\,\mathrm{e}^{\mathrm{i}\int \mathrm{d}x\, v}$

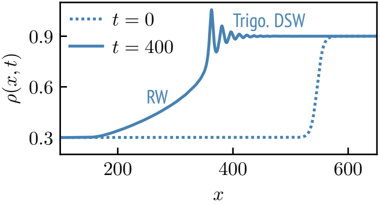
$\rho(x, t=0)$ $\quad v(x, t=0)$

$$\xrightarrow[\text{Nonlinear evolution}]{t > 0}$$

Typical $\rho(x, t)$ when $\tilde{g}_3 = 0$



- Contact dispersive shock waves emerge when $\tilde{g}_3 \neq 0$:



Whitham modulation theory

Nonlinear Periodic Waves and Their Modulations (2000)

Physica D **333**, 11 (2016)

Within experimental reach @ LCF:

$$(\Delta x, \Delta t)_{\text{Contact DSW}} \simeq (14.4\,\mu\mathrm{m}, 318.4\,\mathrm{ms})$$

$\omega_\perp/(2\pi) = 300\,\mathrm{Hz} \quad \Omega/(2\pi) = 25.4\,\mathrm{kHz}$

$\rho \simeq 2.5 \times 10^9\,\mathrm{m}^{-1} \quad \delta/\Omega = 0.9$

4. Vortex-induced self-propulsion against a superflow

arXiv:2512.09028 (2025) & Forthcoming (2026)

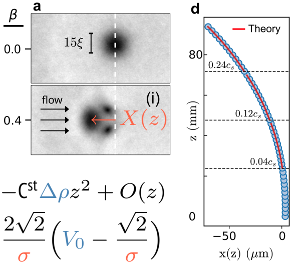
Myrann Baker-Rasooli   Tangui Aladjidi

@ LKB, Paris

Upstream motion driven by downstream vortices

$$\mathrm{i}\partial_z\psi = [-\tfrac{1}{2}\nabla^2 + U(\mathbf{r} - X\hat{\mathbf{x}}) + |\psi|^2]\psi$$

$$k\frac{\mathrm{d}^2X}{\mathrm{d}z^2} \propto \int \mathrm{d}^2\mathbf{r}\ |\psi|^2\partial_x U(\mathbf{r} - X\hat{\mathbf{x}})$$

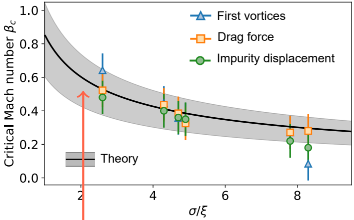


$$X(z) \simeq -\mathrm{C}^{\mathrm{st}}\Delta\rho z^2 + O(z)$$

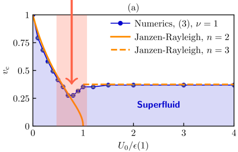
$$\Delta\rho \simeq \frac{2\sqrt{2}}{\sigma}\underbrace{\left(V_0 - \frac{\sqrt{2}}{\sigma}\right)}_{\gtrsim 0.16}$$

Critical speed for vortex shedding

V_c vs. impurity's radius σ



$$V_c \simeq \sqrt{\frac{8\delta}{11}\frac{1}{\sigma}} \qquad \delta = |r_{\text{v-av}} - \sigma| \sim 1$$



Phys. Rev. A **109**, 013317 (2024)

See also:
Phys. Rev. A **107**, 023310 (2023)

V_c vs. impurity's amplitude U_0 for $\sigma \gg 1$

5. Summary and outlook

Critical speed of a binary superfluid of light

arXiv:2601.16005 (2026)

- $\underbrace{V_\mathrm{c} = \min\{c_\mathrm{d}, c_\mathrm{s}\}}_{\text{Landau criterion}} \xleftarrow{0 \leftarrow U_\pm} \underbrace{V_\mathrm{c} = \text{fn.}(U_\pm)}_{\text{Strongly elliptic stationary flow}}$
- $V_0 \gtrsim V_\mathrm{c}$: Vortex-antivortex pairs & Jones-Roberts solitons in both components ψ_+ & ψ_-

Nonlinear periodic waves in a binary BEC

In progress (2026)

- Effective cubic-quintic nonlinear Schrödinger description at large Rabi frequency: $\mathrm{i}\partial_t \varphi = (-\frac{1}{2}\nabla^2 + g_2|\varphi|^2 - |g_3||\varphi|^4)\varphi$
- Contact dispersive shock waves (rarefaction + trigonometric) in a 1D configuration

Vortex-induced self-propulsion against a superflow

arXiv:2512.09028 (2025) & Forthcoming (2026)

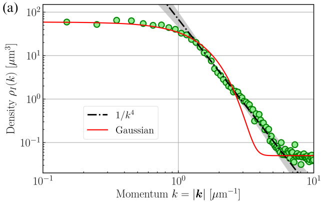
- Upstream trajectory of the mobile impurity: $X(z) \simeq -\mathrm{C^{st}}\Delta\rho z^2 + O(z)$
- Critical speed for vortex shedding: $V_\mathrm{c} \simeq \sqrt{\frac{8}{11}}\sigma^{-1/2}$

Ongoing work and future directions

- Quantum depletion & OTOCs in a superfluid of light — Phys. Rev. A **92**, 043802 (2015)

$$[\hat{\psi}(\mathbf{r},t;z),\hat{\psi}^\dagger(\mathbf{r}',t';z)] = \frac{2\hbar\omega_0}{n_0\epsilon_0 c}\delta^{(2)}(\mathbf{r}-\mathbf{r}')\delta(t-t')$$

- Nonequilibrium Tan contact in a quenched binary BEC — Phys. Rev. Lett. **130**, 153401 (2023)



$$\rho(k,t) \underset{k\to\infty}{\simeq} \frac{\mathcal{C}(t)}{k^4}$$

- Quenched 2D soliton gas
- KPZ dynamics in quantum scars

Merci !