21/07/20

$\begin{pmatrix} a & b \\ b & d \end{pmatrix}$ $\quad P(\lambda) = \begin{vmatrix} a-\lambda & b \\ b & d-\lambda \end{vmatrix} = (a-\lambda)(d-\lambda) - b^2$ 1/2

$P(\lambda) = \lambda^2 - (a+d)\lambda + ad - b^2$

$\Delta = (a+d)^2 - 4(ad - b^2) = (a-d)^2 + 4b^2$

$\lambda_{1,2} = \dfrac{a+d \pm \sqrt{(a-d)^2 + 4b^2}}{2}$

$\lambda_1, \lambda_2 \geq 0$
$a+d \geq 0$

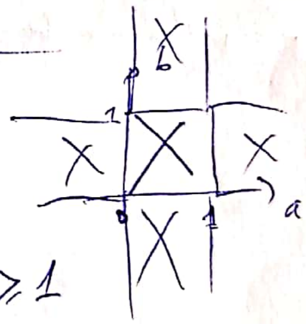
$\lambda_1 \lambda_2 \geq 0$
↳ $ad - b^2 \geq 0$
↳ $\boxed{b^2 \leq ad}$

$\lambda_2 \geq 0 \to a+d \geq \sqrt{(a-d)^2 + 4b^2}$

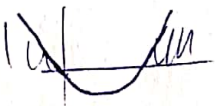
↳ $(a+d)^2 \geq (a-d)^2 + 4b^2$

$\underbrace{(a+d-a+d)}_{2d}\underbrace{(a+d+a-d)}_{2a} \geq 4b^2$

$f = x^a y^b$ sur $]0,1[^2$



$\dfrac{\partial^2 f}{\partial x^2} = a(a-1)x^{a-2}y^b \geq 0$

↳  $a \leq 0$ ou $a \geq 1$

↓
ex $\dfrac{1}{x^{0.1}}$ ou $x, x^{1.5}, x^2 \ldots$

$\dfrac{\partial^2 f}{\partial x \partial y} = a x^{a-1} b y^{b-1}$

$x^{2a-2} y^{2b-2}$

"$ad - b^2 \geq 0$" $\to a(a-1)b(b-1)x^{a-2}y^{b-2}x^a y^b - (a x^{a-1} b y^{b-1})^2 \geq 0$

↳ $(a^2-a)(b^2-b) - a^2b^2 \geq 0 \to (ab)(1-b-a) \geq 0$

$= \cancel{a^2b^2} + ab - ab^2 - a^2b \cancel{-a^2b^2} \geq 0$

$ab(ab - a - b + 1 - ab) \geq 0$

$a \leq 0$, $b \geq 1$	$a, b \geq 1$ (1)
$ab \leq 0$ (2)	(3) $a \geq 1$, $b \leq 0$

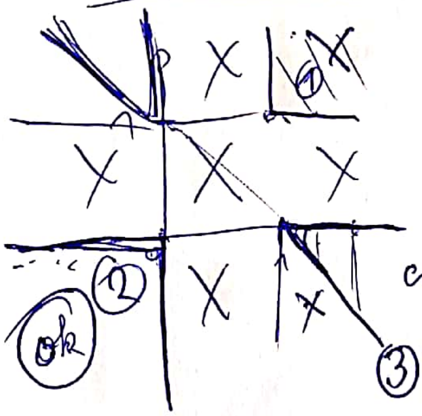
$\rightarrow a$

(1) $ab \geq 0 \rightarrow 1 - b - a \geq 0$

$\hookrightarrow a + b \leq 1 \rightarrow$ impossible!

(2) $ab \geq 0 \rightarrow a+b \leq 1 \rightarrow$ tjs vrai!

(3) $ab \leq 0 \rightarrow \cancel{a+b} \geq 1$



ex : $a = 1,5$
$0 \geq b \geq 1 - 1,5 = -0,5$

$a = 1$
$b = 0$

$a = 2$
$0 \geq b \geq -1$