
2018-06
E.T.
Efficient Tools for Research
Julia: My New Friend for Computing & Optimization?

« Julia, my new friend for computing and optimization? »

- **Intro to the Julia programming language, for MATLAB users**
- *Date:* 14th of June 2018
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Agenda for today [25 min]

1. What is Julia [3 min]
2. Comparison with MATLAB [3 min]
3. Examples of problems solved Julia [5 min]
4. Longer example on optimization with JuMP [10min]
5. Links for more information ? [2 min]

1. What is Julia ?

- Developed and popular from the last 7 years
- Open-source and free programming language (MIT license)
- Interpreted *and* compiled, very efficient
- But easy syntax, dynamic typing, inline documentation etc
- Multi-platform, imperative
- MATLAB-like syntax for linear algebra etc
- Designed and acknowledged as *simple to learn and use*
- Easy to run your code in parallel (multi-core & cluster)
- Used worldwide: research, data science, finance etc…

Ressources

- **Website**: JuliaLang.org for the language & Pkg.JuliaLang.org for packages
- Documentation : docs.JuliaLang.org

Comparison with MATLAB

	Julia 😃	**MATLAB** 😢
Cost	Free ✌️	Hundreds of euros / year
License	Open-source	1 year user license (no longer after your PhD!)
Comes from	A non-profit foundation, and the community	MathWorks company
Scope	Mainly numeric	Numeric only
Performances	Very good performance	Faster than Python, slower than Julia

Comparison with MATLAB

	Julia	MATLAB
Packaging	`Pkg` manager included. Based on `git` + GitHub, very easy to use	Toolboxes already included but 💰 have to pay if you wat more!
Editor/IDE	*Jupyter* is recommended (*Juno* is also good)	Good IDE already included
Parallel computations	Very easy, low overhead cost	Possible, high overhead

Comparison with MATLAB

	Julia	MATLAB
Usage	Generic, worldwide 🌎	Research in academia and industry
Fame	Young but starts to be known	Old and known, in decline
Support?	Community[1] (StackOverflow, mailing lists etc).	By MathWorks
Documentation	OK and growing, inline/online	OK, inline/online

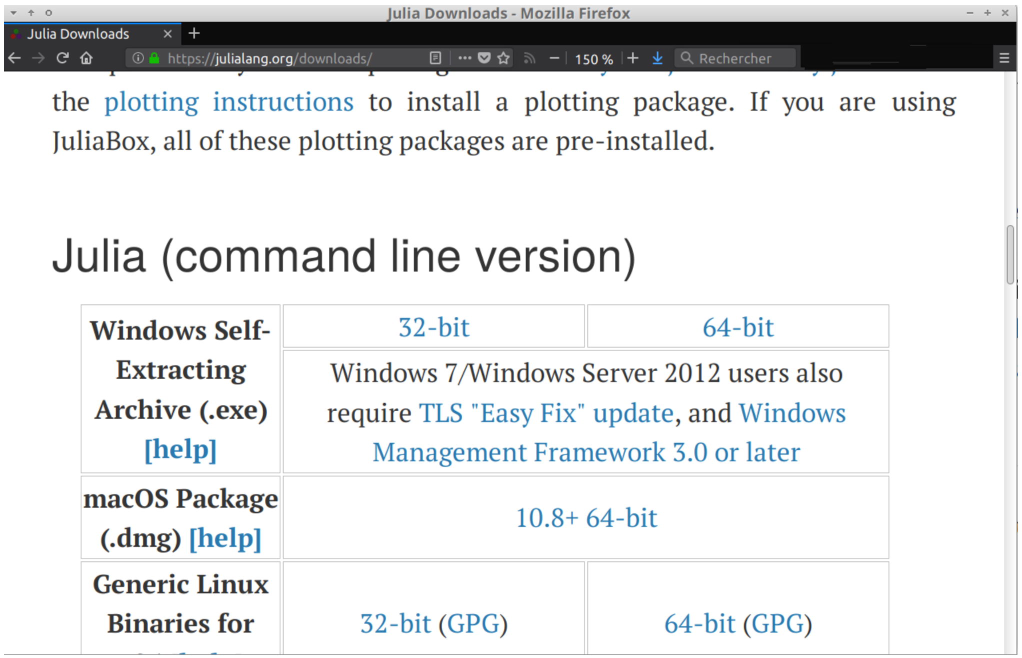
> Note[1]: **JuliaPro** offer paid licenses, if professional support is needed.

How to install Julia ⬇️ (1/2)

- You can try online *for free* on JuliaBox.com
- On Linux, Mac OS or Windows:
  - You can use the default installer 📦
    from the website julialang.org/downloads
- Takes about 4 minutes... and it's free !

> You also need Python 3 to use Jupyter ✨, I suggest to use
> Anaconda.com/download if you don't have Python yet.

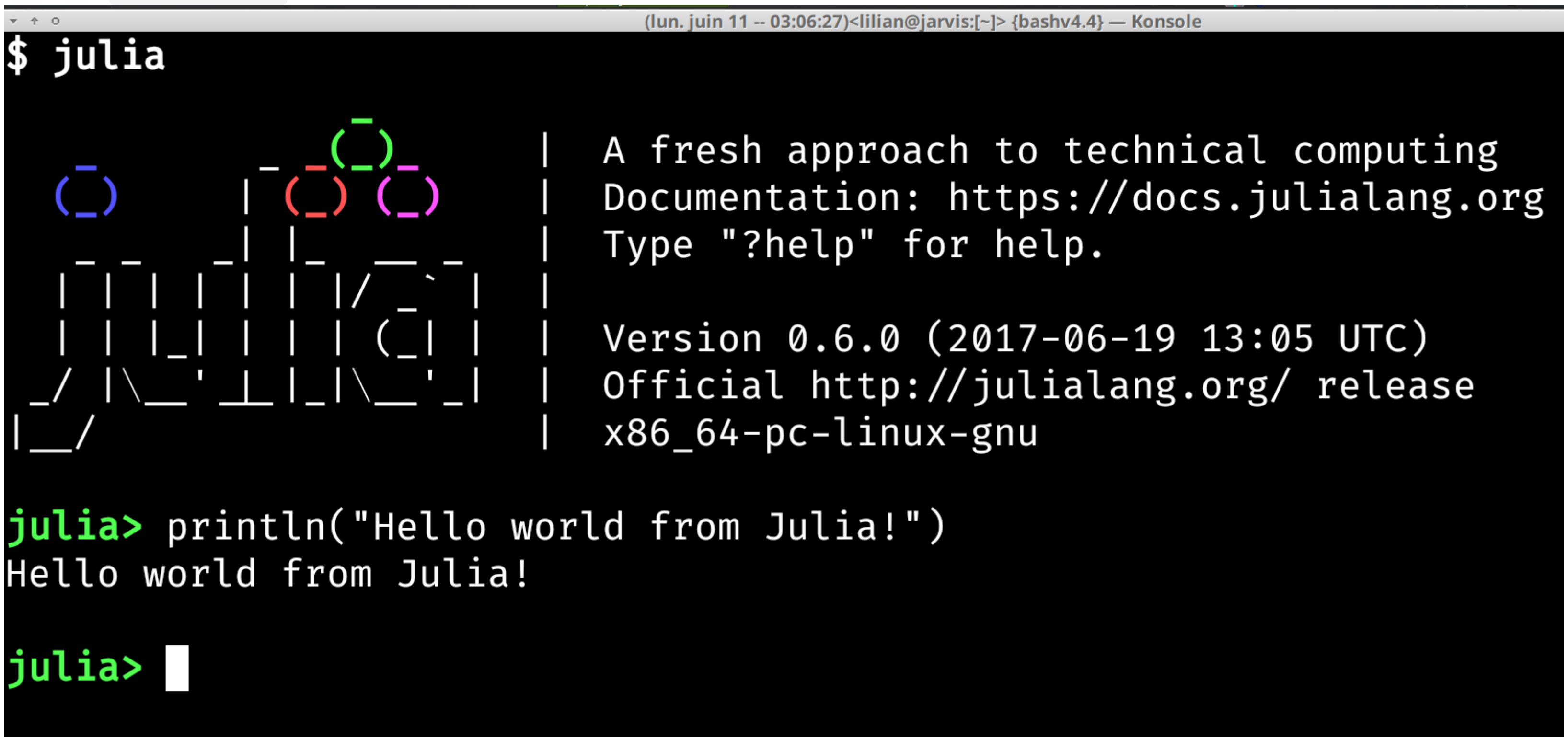
How to install Julia ⬇️ (2/2)



1. Select the binary of your platform 📦
2. Run the binary 🏃 !
3. Wait 🕐…
4. Done 👌 ! Test with `julia` in a terminal

Different tools to use Julia

- Use `julia` for the command line for short experiments

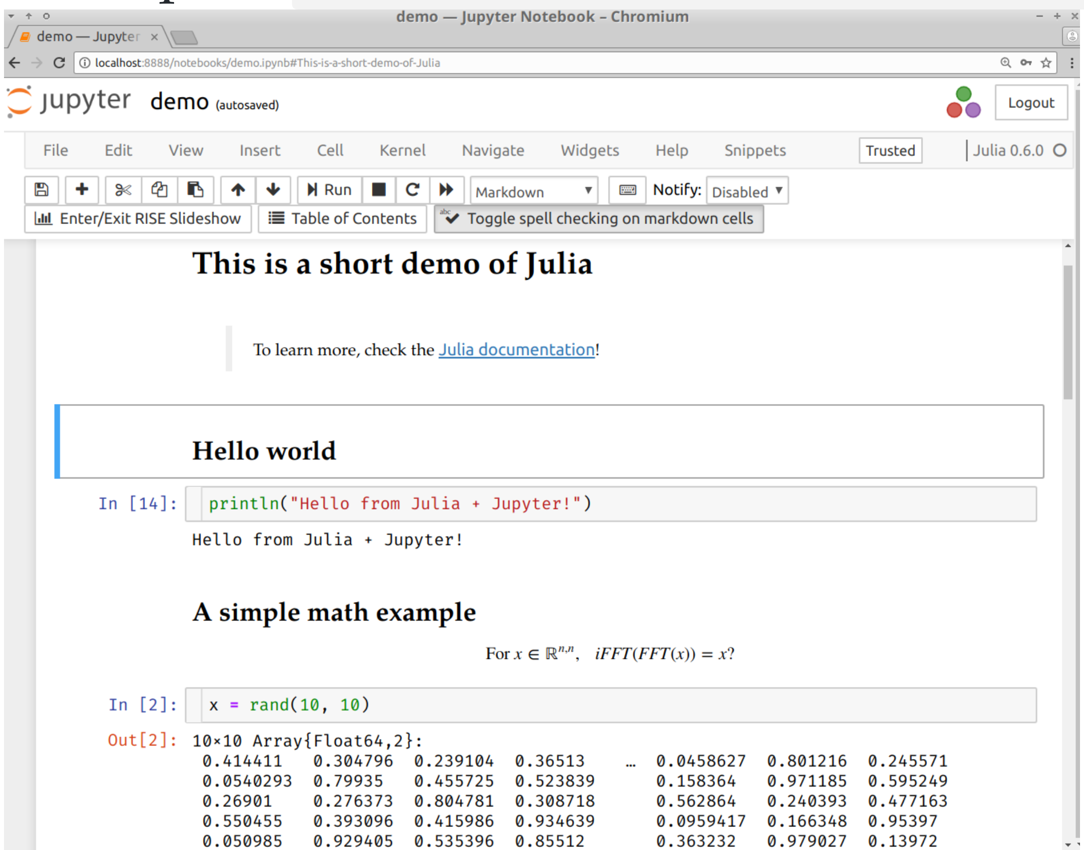


- Use the *Juno* IDE to edit large projects

> Demo time ⌚ !

Different tools to use Julia

- Use **Ju**_pyter_ notebooks to write or share your experiments (examples: `github.com/Naereen/notebooks`)



📦 How to install modules in Julia ?

- Installing is **easy** !

```
julia> Pkd.add("IJulia")  # installs IJulia
```

- Updating also!

```
julia> Pkg.update()
```

🔍 How to find the module you need ?

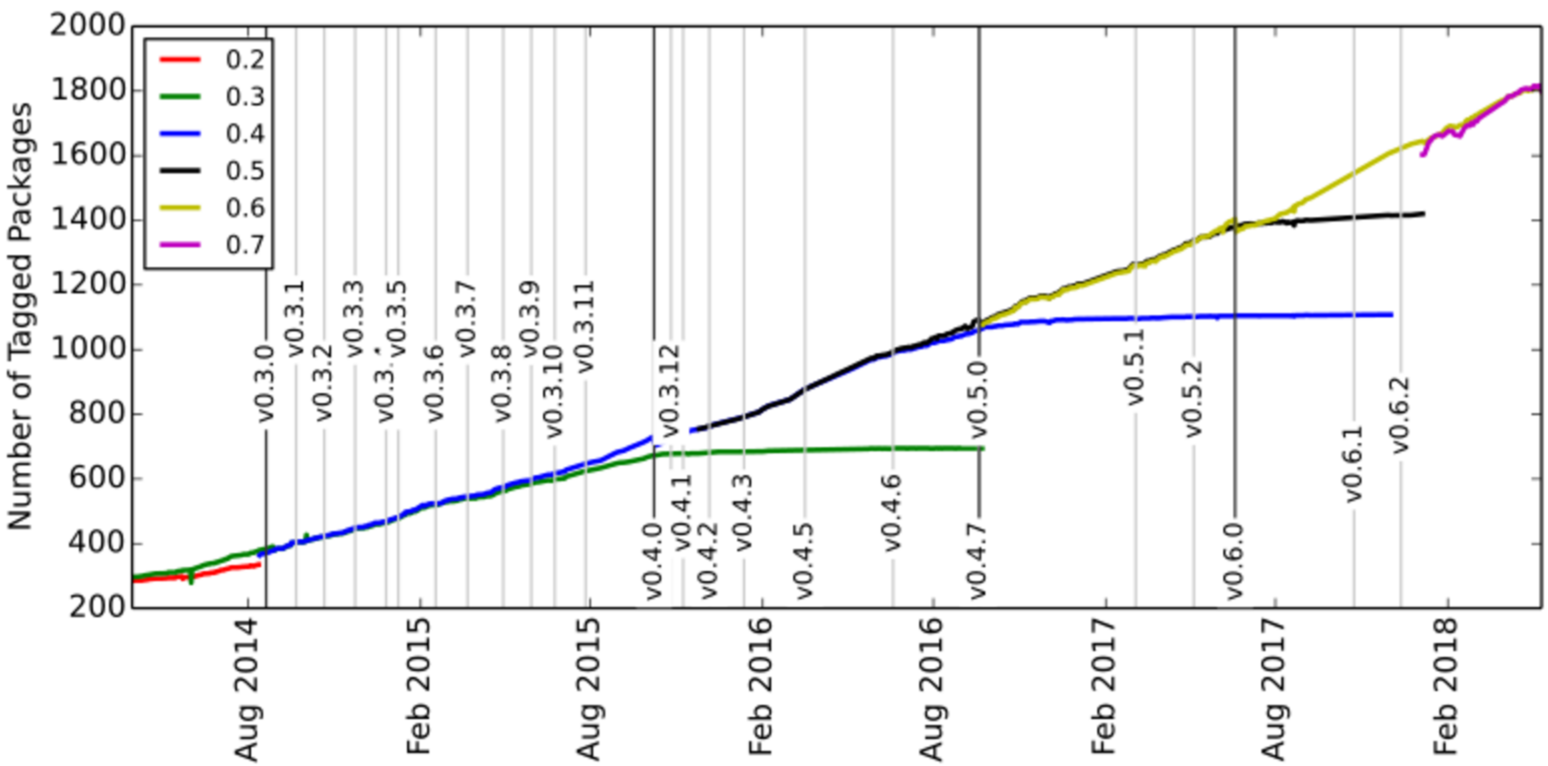
- First… ask your colleagues 😄 !
- Complete list on pkg.JuliaLang.org

Overview of famous Julia modules

- Plotting:
  - `Winston.jl` for easy plotting like MATLAB
  - `PyPlot` : interface to Matplotlib (Python)
- The JuliaDiffEq collection for **differential equations**
- The JuliaOpt collection for **optimization**
- The JuliaStats collection for **statistics**
- And many more!

> Find more specific packages on GitHub.com/svaksha/Julia.jl/

Many packages, and a quickly growing community



> Julia is still in development, in version v0.6 but version 1.0 is planned soon!

2. Main differences in syntax between Julia and MATLAB

> Ref: cheatsheets.quantecon.org

	Julia	MATLAB
File ext.	`.jl`	`.m`
Comment	`# blabla...`	`% blabla...`
Indexing	`a[1]` to `a[end]`	`a(1)` to `a(end)`
Slicing	`a[1:100]` (view)	`a(1:100)` (⚠️ copy)
Operations	Linear algebra by default	Linear algebra by default
Block	Use `end` to close all blocks	Use `endif` `endfor` etc

	Julia	**MATLAB**
Help	`?func`	`help func`
And	`a & b`	`a && b`
Or	`a | b`	`a || b`
Datatype	`Array` of *any* type	multi-dim doubles array
Array	`[1 2; 3 4]`	`[1 2; 3 4]`
Size	`size(a)`	`size(a)`
Nb Dim	`ndims(a)`	`ndims(a)`
Last	`a[end]`	`a(end)`

	Julia	**MATLAB**
Tranpose	`a.'`	`a.'`
Conj. transpose	`a'`	`a'`
Matrix x	`a * b`	`a * b`
Element-wise x	`a .* b`	`a .* b`
Element-wise /	`a ./ b`	`a ./ b`
Element-wise ^	`a ^ 3`	`a .^ 3`
Zeros	`zeros(2, 3, 5)`	`zeros(2, 3, 5)`
Ones	`ones(2, 3, 5)`	`ones(2, 3, 5)`
Identity	`eye(10)`	`eye(10)`
Range	`range(0, 100, 2)` or `1:2:100`	`1:2:100`

	Julia	MATLAB
Maximum	`max(a)`	`max(max(a))` ?
Random matrix	`rand(3, 4)`	`rand(3, 4)`
L2 Norm	`norm(v)`	`norm(v)`
Inverse	`inv(a)`	`inv(a)`
Solve syst.	`a \ b`	`a \ b`
Eigen vals	`V, D = eig(a)`	`[V,D]=eig(a)`
FFT/IFFT	`fft(a)`, `ifft(a)`	`fft(a)`,`ifft(a)`

> Very close to MATLAB for linear algebra!

3. Scientific problems solved with Julia

> Just to give examples of syntax and modules

1. 1D numerical integration and plot
2. Solving a 2^{nd} order Ordinary Differential Equation

3.1. 1D numerical integration and plot

> Exercise : evaluate and plot this function on [-1, 1] :
>
> $$\mathrm{Ei}(x) := \int_{-x}^{\infty} \frac{\mathrm{e}^u}{u} \,\mathrm{d}u$$

How to?

Use packages and everything is easy!

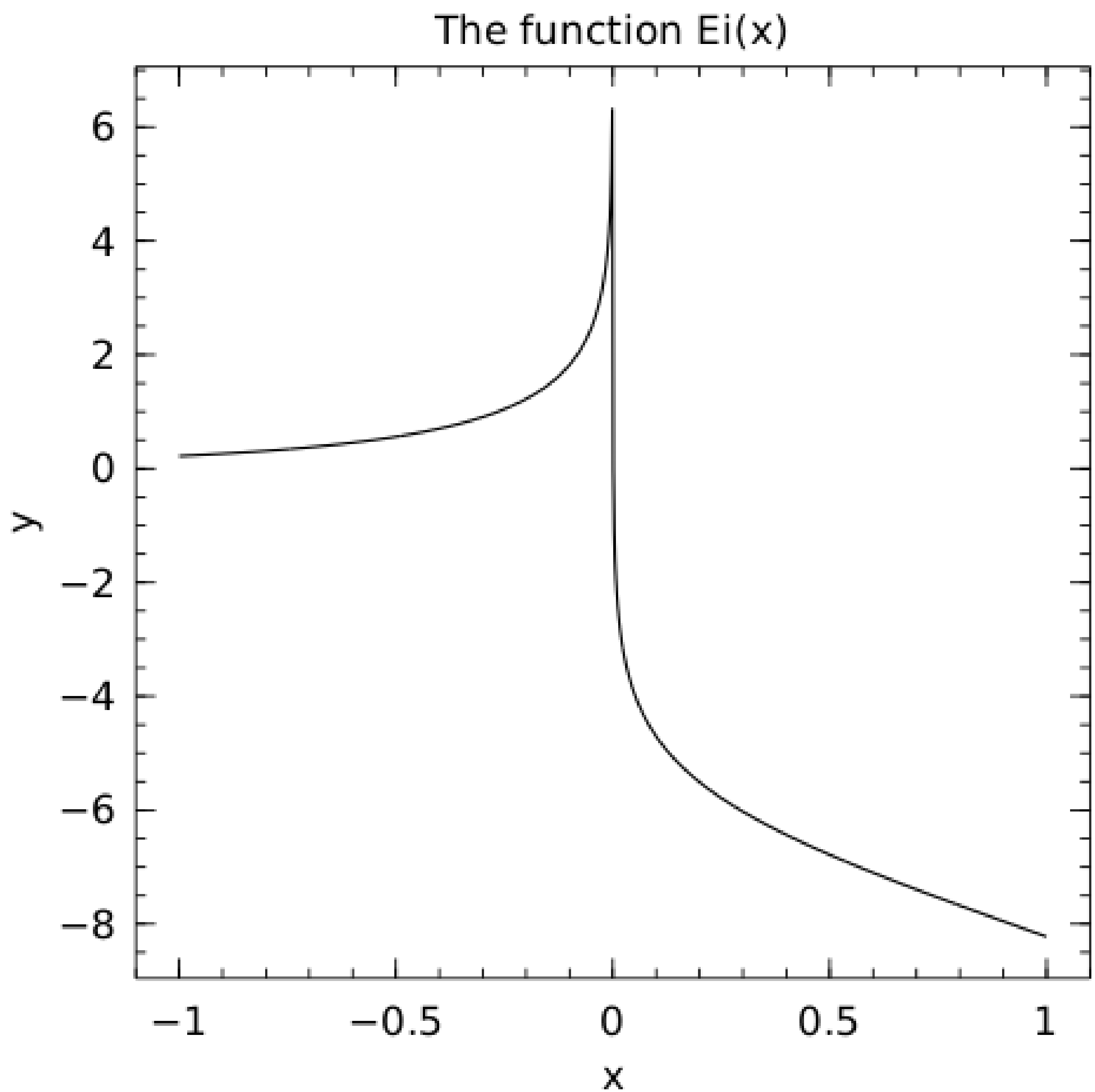
- `QuadGK.jl` for integration
- `Winston.jl` for 2D plotting

```julia
using QuadGK

function Ei(x, minfloat=1e-3, maxfloat=100)
    f = t -> exp(-t) / t  # inline function, with '- >'
    if x > 0
        return quadgk(f, -x, -minfloat)[1]
             + quadgk(f, minfloat, maxfloat)[1]
    else
        return quadgk(f, -x, maxfloat)[1]
    end
end

X = linspace(-1, 1, 1000)   # 1000 points
Y = [ Ei(x) for x in X ]

using Winston
plot(X, Y)
title("The function Ei(x)")
xlabel("x"); ylabel("y")
savefig("figures/Ei_integral.png")
```


The function Ei(x)
6
4
2
0
−2
−4
−6
−8
y
−1
−0.5
0
0.5
1
x

3.2. Solving a 2^{nd} order ODE

> Goal : solve and plot the differential equation of a pendulum:
>
> $$\theta''(t) + b\,\theta'(t) + c\,\sin(\theta(t)) = 0$$
>
> For $b = 1/4, c = 5, \theta(0) = \pi - 0.1, \theta'(0) = 0, t \in [0, 10]$

How to?

Use packages!

- `DifferentialEquations.jl` function for ODE integration
- `Winston.jl` for 2D plotting

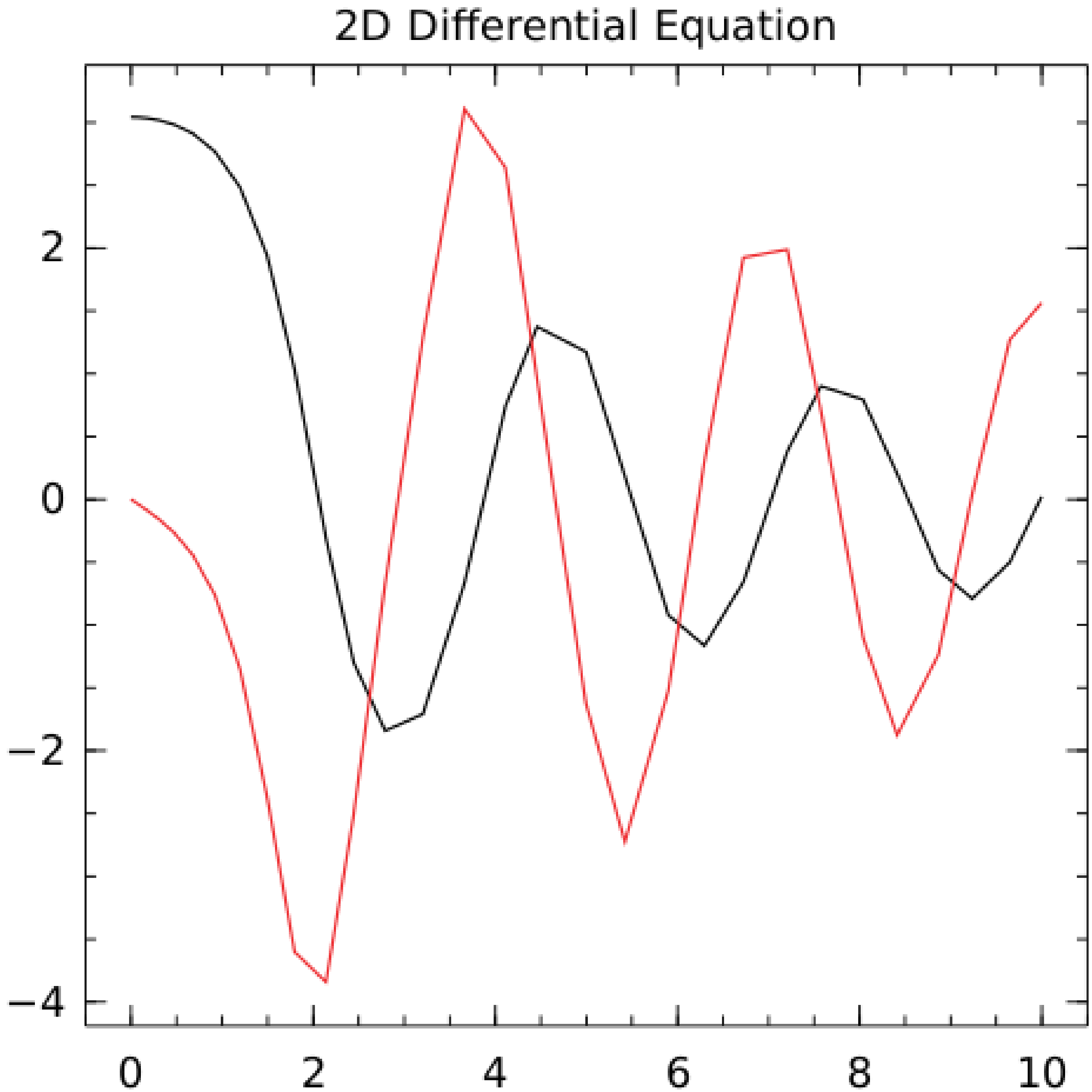
```julia
using DifferentialEquations

b, c = 0.25, 5.0

# macro magic!
pend2 = @ode_def Pendulum begin
  dθ = ω  # <-- yes, this is UTF8
  dω = (-b * ω) - (c * sin(θ))
end

prob = ODEProblem(pend, y0, (0.0, 10.0))
sol = solve(prob)        # ↑ solve on interval [0,10]
t, y = sol.t, hcat(sol.u...)'

using Winston
plot(t, y[:, 1], t, y[:, 2])
title("2D Differential Equation")
savefig("figures/Pendulum_solution.png")
```


2D Differential Equation
2
0
−2
−4
0
2
4
6
8
10

Conclusion (1/2)

Sum-up

- I hope you got a good introduction to Julia
- It's not hard to migrate from MATLAB to Julia
- Good start:
  `docs.JuliaLang.org/en/stable/manual/getting-started`

Conclusion (2/2)

> *Thanks for joining 👏 !*

Your mission, if you accept it… 💥

1. 👶 *Padawan level:* Train yourself a little bit on Julia
   ↪ JuliaBox.com ? Or install it on your laptop!
   And ead introduction in the Julia manual!
2. 👩‍🎓 *Jedi level:* Try to solve a numerical system, from your research or teaching, **in Julia instead of MATLAB**
3. ⚔️ *Master level:* From now on, try to use open-source & free tools for your research (Julia, Python and others)… 🤑

Examples

1. **Iterative computation**: signal filtering
2. **Optimization**: robust regression on RADAR data

Ex. 1: Iterative computation

Objective:

- show the efficiency of Julia's Just-in-Time (JIT) compilation
- but also its fragility...

Iterative computation: signal filtering

The classical saying:

> *"Vectorized code often runs much faster than the corresponding code containing loops."* (cf. MATLAB doc)

does not hold for Julia, because of its **just-in-time compiler**.

Example of a computation that cannot be vectorized

Smoothing of a signal $\{u_k\}_{k\in\mathbb{N}}$:

$$y_k = a y_{k-1} + (1-a) u_k, \quad k \in \mathbb{N}^+$$

Parameter a tunes the smoothing (none: $a = 0$, strong $a \to 1^-$).

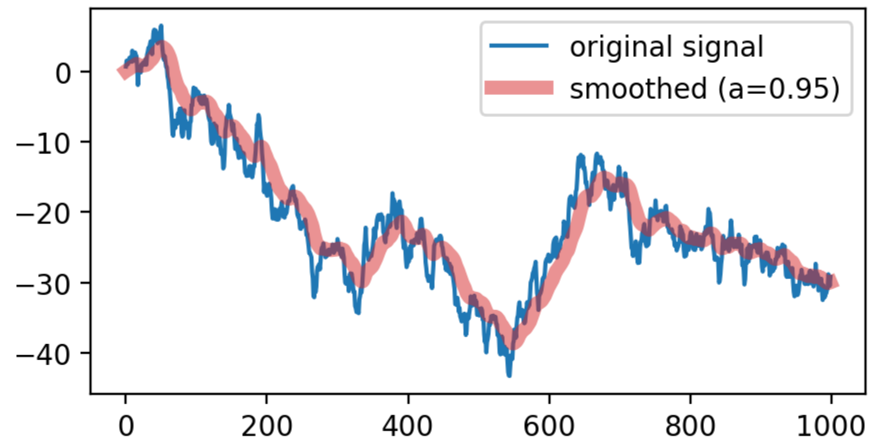
Iteration (`for` loop) cannot be avoided.

Signal filtering in Julia

```
function smooth(u, a)
    y = zeros(u)

    y[1] = (1-a)*u[1]
    for k=2:length(u)
        y[k] = a*y[k-1] + (1-a)*u[k]
    end

    return y
end
```



Performance of the signal filter

Implementation	Time for 10 Mpts	notes
Julia	50-70 ms	
Python native	4400 ms	
SciPy's `lfilter`	70 ms	many lines of C
Python + `@numba.jit`	50 ms	since 2012

```python
@numba.jit # ← factor x100 speed-up!
def smooth_jit(u, a):
    y = np.zeros_like(u)

    y[0] = (1-a)*u[0]
    for k in range(1,len(u)):
        y[k] = a*y[k-1] + (1-a)*u[k]
    return y
```

Conclusion on the performance

For this simple iterative computation:

- Julia performs very well, much better than native Python
- but it's possible to get the same with fresh Python tools (Numba)
- more realistic example needed

Fragility of Julia's JIT Compilation

The efficiency of the compiled code relies on **type inference**.

```julia
function smooth1(u, a)
    y = 0
    for k=1:length(u)
        y = a*y + (1-a)*u[k]
    end
    return y
end
```

```julia
function smooth2(u, a)
    y = 0.
    for k=1:length(u)
        y = a*y + (1-a)*u[k]
    end
    return y
end
```

An order of magnitude difference

```
julia> @time smooth1(u, 0.9);
  0.212018 seconds (30.00 M allocations: 457.764 MiB ...)
```

```
julia> @time smooth2(u, 0.9);
  0.024883 seconds (5 allocations: 176 bytes)
```

Fortunately, Julia gives a good diagnosis tool

```
julia> @code_warntype smooth1(u, 0.9);
...
y::Union{Float64, Int64}
...
```

`y` is either `Float64` or `Int64` when it should be just `Float64`.

Cause: initialization `y=0` vs. `y=0.`!

Ex. 2: Optimization in Julia

Objective: demonstrate **JuMP**, a Modeling Language for Optimization in Julia.

Some research groups migrate to Julia just for this package!

Optimization problem

Example problem: Identifying the sea clutter in Weather Radar data.

- is a **robust regression** problem
  - is an optimization problem

An “IETR-colored” example, inspired by:

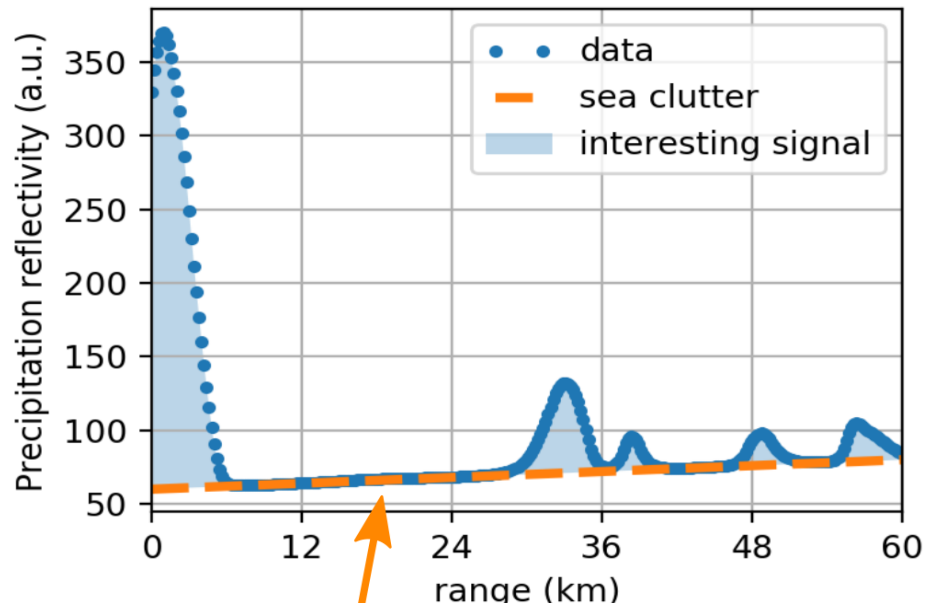
- Radar data+photo: P.-J. Trombe *et al.*, “Weather radars – the new eyes for offshore wind farms?,” *Wind Energy*, 2014.
- Regression methods: S. Boyd and L. Vandenberghe, Convex Optimization. Cambridge University Press, 2004. (Example 6.2)

Weather radar: the problem of sea clutter

Offshore wind farm (Horns Rev, Denmark)

Weather radar: X-band (9.41 GHz), 60 km range

Reflection data, for one azimuth



Objective: **fit this trend** (to substract it)

Given n data points (x_i, y_i), fit a linear trend:

$$\hat{y} = a.x + b$$

An optimization problem with two parameters: a (slope), b (intercept)

Regression as an optimization problem

The parameters for the trend (a, b) should minimize a criterion J which penalizes the residuals $r_i = y_i - \hat{y} = y_i - a.x + b$:

$$J(a, b) = \sum_i \phi(r_i)$$

where ϕ is the *penaly function*, to be chosen:

- $\phi(r) = r^2$: quadratic deviation $\rightarrow$ least squares regression
- $\phi(r) = |r|$: absolute value deviation
- $\phi(r) = h(r)$: Huber loss
- ...

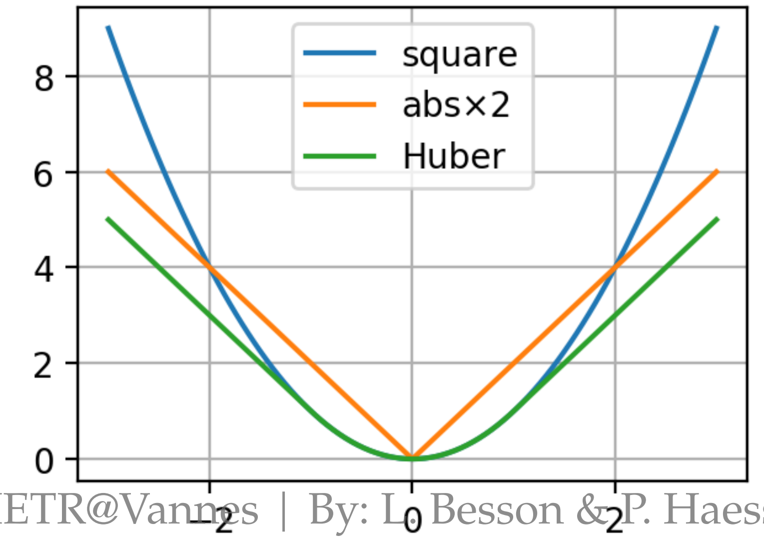
Choice of penalty function

The choice of the loss function influences:

- the optimization result (fit quality)
    - e.g. in the presence of outliers
- the properties of optimization problem: convexity, smoothness

Properties of each function

- quadratic: convex, smooth, heavy weight for strong deviations
- absolute value: convex, not smooth
- Huber: a mix of the two



How to solve the regression problem

Option 1: a big bag of tools

a specific package for each type of regression:

- "least square toolbox" (→ MultivariateStats.jl)
- "least absolute value toolbox" (→ quantile regression)
- "Huber toolbox" (i.e. robust regression → ???)
- ...

Option 2: the "One Tool"

→ a **Modeling Language for Optimization**

+more **freedom to explore variants** of the problem

Modeling Languages for Optimization

Purpose: make it easy to ***specify*** *and* ***solve*** *optimization problems without expert knowledge.*

JuMP: optimization modeling in Julia

The JuMP package offers a domain-specific modeling language for mathematical optimization.

JuMP **interfaces with many optimization solvers**: open-source (Ipopt, GLPK, Clp, ECOS...) and commercial (CPLEX, Gurobi, MOSEK...).

Other Modeling Languages for Optimization:

- Standalone software: AMPL, GAMS
- Matlab: YALMIP (previous seminar), CVX
- Python: Pyomo, PuLP, CVXPy

> Claim: JuMP is **fast**, thanks to Julia's metaprogramming capabilities (generation of Julia code within Julia code).

Regression with JuMP — common part

Given `x` and `y` the 300 data points:

```
m = Model(solver = ECOSSolver())

@variable(m, a)
@variable(m, b)

res = a*x .- y +b
```

`res` ("residuals") is an Array of 300 elements of type `JuMP.GenericAffExpr{Float64,JuMP.Variable}`, i.e. a semi-symbolic affine expression.

Now, we need to specify the penalty on those residuals.

Regression choice: least squares

$$\min \sum_i r_i^2$$

reformulated as a Second-Order Cone Program (SOCP):

$$\min j, \quad \text{such that } \|r\|_2 \leq j$$

```
@variable(m, j)
@constraint(m, norm(res) <= j);
@objective(m, Min, j)
```

(SOCP → ECOS solver)

Regression choice: least absolute deviation

$$\min \sum_i |r_i|$$

reformulated as a Linear Program (LP)

$$\min \sum_i t_i, \quad \text{such that } -t_i \leq r_i \leq t_i$$

```
@variable(m, t[1:n] )
@constraint(m, res .<= t)
@constraint(m, res .>= -t)
@objective(m, Min, sum(t));
```

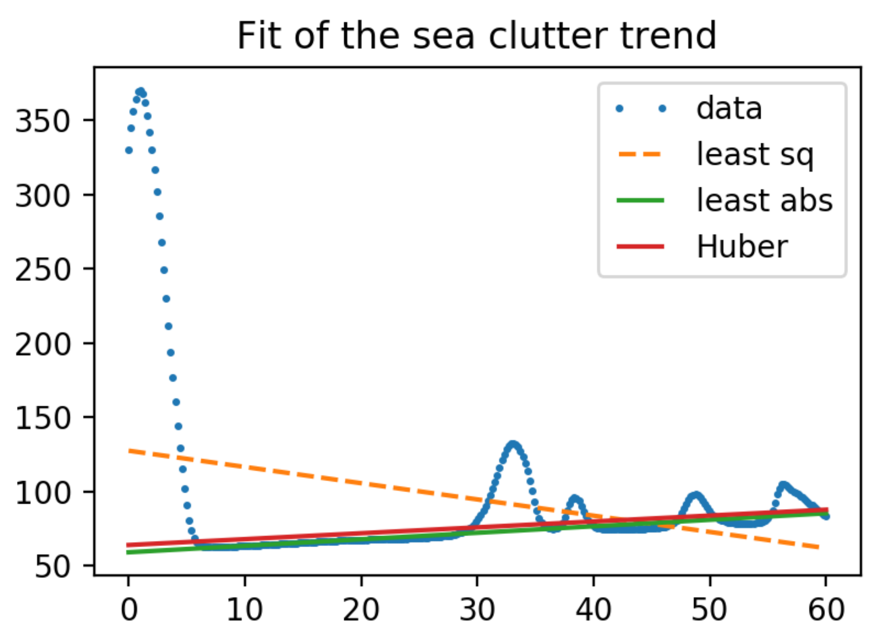
Solve! ⚙ <

```
julia> solve(m)
[solver blabla... ⌛ ]
:Optimal # hopefully
```

```
> getvalue(a), getvalue(b)
(-1.094, 127.52) # for least squares
```

Observations:

- least abs. val., Huber: OK
- least squares: NOT OK



JuMP: summary

A modeling language for optimization, withing Julia:

- gives access to all classical optimization solvers
- very fast (claim)
- gives freedom to explore many variations of an optimization problem (fast prototyping)

More on optimization with Julia:

- JuliaOpt: host organization of JuMP
- Optim.jl: implementation of classics in Julia (e.g. Nelder-Mead)
- JuliaDiff: Automatic Differentiation to compute gradients, thanks to Julia's strong capability for code introspection