Infinite-state strategies for infinite-state quantitative systems
Internship proposal, ENS Lyon

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1 Context

Games on graphs. We consider *zero-sum games on graphs* [FBB+23]. In computer science, such games are often used to model the interaction between a (controllable) computer system and its (uncontrollable) environment [BCJ18, Chapter 27]. The system and the environment are seen as the two players. We confer an objective to the system, and the goal is to decide whether we can control the system such that the objective is guaranteed, no matter what uncontrollable events occur in the environment. In game-theoretic vocabulary, this corresponds to finding a *winning strategy* (the controller) for the system against the antagonistic environment. This problem is called *reactive synthesis*. Two classes of objectives are often distinguished: *qualitative* objectives (e.g., ω-regular objectives [GTW02]) are those where a player either wins or loses, while *quantitative* objectives are those where the goal is to optimize some numerical value (e.g., total payoff, mean payoff, discounted payoff).

Strategy complexity. We consider the following question: when winning is possible in a game, *how much information* must be remembered to make optimal decisions? This information is usually encoded in automata-like structures. A research topic that started decades ago, but is still a frequent topic in the recent literature, seeks to better understand these *memory requirements* of objectives and games. Given some objective and some class of games, how *simple* can a winning strategy be?

2 Research questions

Finite-state vs. infinite-state games Many results focus on *finite-state* games, where the state space of the game is finite. In this context, the strategy complexity of the players is often well understood [DJW97,GZ05]. For instance, all aforementioned classical **quantitative objectives** are known to be *memoryless-determined* in finite-state games, which is roughly the lowest possible strategy complexity. This property is at the core of many algorithms to *solve* related reactive synthesis problems [FBB+23].

Unfortunately, the situation is much less clear when we play over *infinite-state* games: quantitative objectives are in general not memoryless-determined anymore (and actually, not even *finite-memory-determined*); see Figure 1 for an example of a quantitative game illustrating the need for infinite memory to implement a winning strategy. Our goal in this internship is to explore the strategy complexity of quantitative objectives over infinite-state games.

Research questions. Even if finite-memory strategies are insufficient, one can still wonder what kind of *infinite-memory* strategies are sufficient. Open questions include:

– characterizing the shape of the memory structures sufficient to implement winning strategies;
– determining for which objectives using some natural infinite memory structures suffices;
– studying the strategy complexity in some classes of infinite-state, but finitely representable games (such as games generated by *pushdown processes*).

Recent work by a subset of the supervisors provide answers to the second question for the *step-counter* memory structure and for objectives with low *Borel complexity* [BIP+24]. This work is a starting point. A close line of work is also accessible for *stochastic* games (see, e.g., [KMS+20]); we intend to focus here on *deterministic* games.

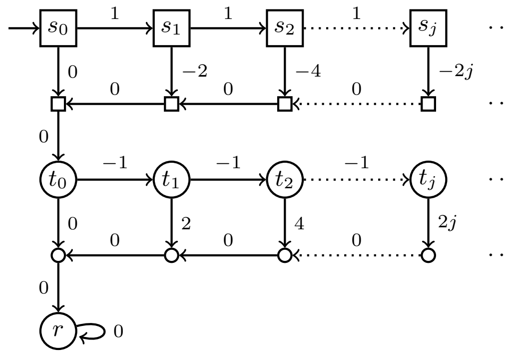

Fig. 1. Infinite-state game in which Player 1 (controlling circles) seeks to obtain a total sum of weights above 0 infinitely often, whereas Player 2 (controlling squares) aims for the opposite. Here, Player 1 can win by "outcounting" Player 2 before moving to r, in order for the sum of weights to go over 0. To do so, Player 1 needs infinite memory, but a simple infinite structure counting the number of steps played is sufficient.

3 Practical matters

The student will be hosted in the *Formal Methods Group* at the University of Mons in Belgium, which is part of the **Mathematics** and the **Computer Science Department**. The student will be supervised by Sougata Bose, Mickael Randour, and Pierre Vandenhove.

If you are interested, please contact all three supervisors by email. Though internships are usually not paid in Belgium, financial support can be discussed.

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