Relationship Lending and Monetary Policy Pass-Through

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Abstract

This paper investigates the link between bank-firm lending relationships and monetary policy pass-through, focusing on episodes of low interest rates. Using administrative tax and bank supervisory data ranging from 1997 to 2019, we track the entirety of bank-firm relationships in Norway. Our analysis shows that when the central bank's policy rate is relatively low, firms that have maintained a long-term relationship with their bank experience a lower pass-through of further policy rate cuts. Specifically, we find that when the policy rate is around 1%, each additional year of relationship decreases the pass-through of a rate cut by 2.7 percentage points. We propose a theoretical model to rationalize our empirical findings, where state-dependent differential pass-through results from the presence of firms' switching costs and banks' leverage constraint. The model highlights that the composition of relationship lengths in the economy matters for aggregate monetary policy pass-through. The proportion of long-term relationships in the Norwegian economy significantly increased after the global financial crisis. Using the model, we calculate a counterfactual aggregate pass-through for 2017, a period of monetary easing in a low-rate environment, assuming this proportion had remained at its pre-crisis level.

JEL Classification: G21, E58, E52, E43, E50

Keywords: Relationship lending; Monetary policy pass-through; Low interest rates; Policy rate; Switching costs

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1 Introduction

In the aftermath of the 2007-2008 financial crisis, the central banks of advanced economies set their policy rates to unprecedentedly low levels. Amid a sluggish recovery, low inflation and further financial distress in Europe following the 2010-2012 sovereign debt crisis, the low interest rate environment persisted throughout the 2010s. For example, the ECB's main refinancing operations rate never rose above 1.5% during this period, even reaching 0% between 2016 and 2019. In the US, the federal funds target rate always remained below 0.5% between 2010 and 2016. In such a setting, monetary policy transmission recently received increased attention due to potentially being impaired at low interest rates.[1] At the beginning of 2020, Janet Yellen, former chairwoman of the Federal Reserve, said: "I worry about low interest rates [. . .] it has put central banks in a position where they don't have a lot of ammunition. If we have a serious recession, [. . .] we're probably not going to be able to count on central banks to offer up a significant response."[2]

The empirical and theoretical research aimed at understanding the diminished efficacy of monetary policy under low interest rates has predominantly focused on bank-level channels, particularly the implications of an effective lower bound on deposit rates. Considering that monetary policy is, in part, transmitted to the real economy at the bank-firm level — specifically through the loan conditions secured by firms with their banks — surprisingly little attention has been given to the interplay between financial intermediaries and their borrowers, despite its importance. For instance, in Norway, nearly half of corporate loans are issued by small and medium-sized banks, mostly regional Norwegian banks. Furthermore, approximately 90% of firms exclusively borrow from one bank, thus making lending relationships a dominant feature of the banking landscape. The literature on relationship lending has shown that the lending terms banks offer their customers typically depend on the duration of the existing relationship.[3] It is therefore natural to consider the possibility that the pass-through given by banks to their customers after a monetary policy change also depends on relationship

[1] See, e.g., Wang, Whited, Wu, and Xiao (2022); Heider, Saidi, and Schepens (2019); Ulate (2021); Eggertsson, Juelsrud, Summers, and Wold (2019); Brunnermeier and Koby (2018).

[2] A conversation with David Malpass and Janet Yellen at event hosted by Bipartisan Policy Center, George Washington University, February 4, 2020. (https://www.worldbank.org/en/news/speech/2020/02/04/transcript-a-conversation-with-david-malpass-and-janet-yellen-at-the-bipartisan-policy-center)

[3] While relationships can benefit firms and banks by reducing information asymmetries between them (Diamond, 1984), they also create information asymmetries among banks (Sharpe, 1990; Rajan, 1992; von Thadden, 2004; Dell'ariccia and Marquez, 2004) which lead to informational switching costs. Depending on the circumstances, banks can exploit switching costs by holding up their borrowers and extracting rents from them (see, e.g., Petersen and Rajan, 1994; Berlin and Mester, 1999; Schenone, 2010; Bolton, Freixas, Gambacorta, and Mistrulli, 2016; Kysucky and Norden, 2016; Beck, Degryse, De Haas, and van Horen, 2018; Botsch and Vanasco, 2019; Li, Lu, and Srinivasan, 2019; Liaudinskas, 2023).

length, making it instrumental in monetary policy transmission. In this paper, we contribute to the intersection of the literature on monetary policy pass-through and relationship lending by empirically estimating the heterogeneous pass-through from policy rates to loan rates for firms with different relationship lengths and for environments with different interest rate levels. Our results suggest a relationship-based explanation, which we formalize with a model, for the impairment of monetary policy transmission in low-interest rate environments.

Our empirical analyses are based on an advanced economy, Norway, which provides an almost ideal setting for our study because it collects detailed yearly balance sheets and income statements from every firm and bank operating in the country. Moreover, our data include the amounts of paid interest and outstanding loans at yearly frequency between borrowing firms and banks, which allows us to track lending relationships over time and estimate firm-bank specific average loan interest rates.

We start by presenting empirical evidence supporting the existence of state-dependent and asymmetric average within-relationship pass-through. Specifically, we find that when the Norwegian Interbank Offered Rate (NIBOR), the central bank's target rate, is one standard deviation below its mean, at approximately 1.1%, banks pass only 9% of a further policy rate cut on to firms' loan rates. In contrast, when the NIBOR is one standard deviation above its mean, at approximately 5.4%, the within-relationship pass-through rate increases to 61% of a rate cut. Our findings are consistent with recent research that has documented a lower monetary policy pass-through at low interest rates. Furthermore, our estimates reveal a significant degree of asymmetry in the pass-through rates. Specifically, banks demonstrate a much greater willingness to pass policy rate increases on to firms' loan rates.

Having evidence of impaired within-relationship monetary policy pass-through at low rates, we investigate the impact of bank-firm relationship length on the pass-through to individual firms. In a linear specification that allows for initial policy rate dependence and asymmetry, we find that when the NIBOR is low, additional years of relationship are associated with reduced pass-through of a policy rate cut. Specifically, at a NIBOR rate of 1.1%, an additional year of relationship is associated with a 2.7 percentage point decrease in the average pass-through of a further rate cut. Conversely, longer relationships are associated with greater pass-through in the event of a monetary policy tightening, with each additional year of relationship being linked to an 8 percentage point increase in pass-through of a rate hike.

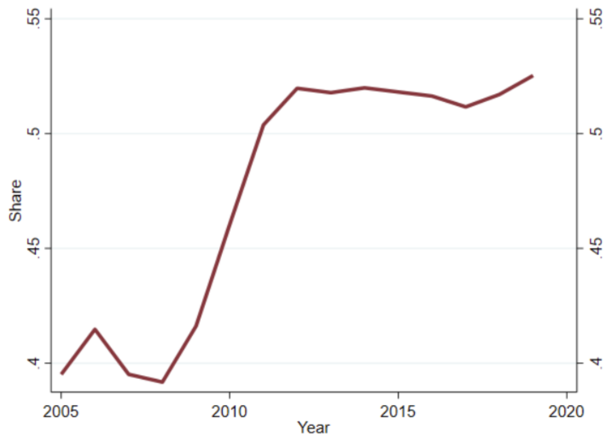
We test the robustness of these results by allowing for non-linearity in both the initial level of the policy rate and relationship length. Kernel regressions reveal a threshold effect, with relationship length having no impact on monetary policy pass-through when the NIBOR is above 1.5%. Below this threshold, we observe significant differences in pass-through based on relationship lengths, with the first years of a relationship appearing to create the greatest heterogeneity in pass-through. These findings suggest that, in a low interest rate environment, the length of bank-firm relationships is an important determinant of within-relationship monetary policy pass-through.

To understand whether differential credit supply shifts driven by banks rather than a higher increase in long-relationship firms' demand account for the lower pass-through, we examine the marginal effects of relationship length on loan growth rates following a policy rate cut. Our results suggest that long-relationship firms experience a relatively lower increase in loan volumes, supporting the view that the lower pass-through is driven by the supply side. We further explore the real effects of lower pass-through to long-relationship firms by analyzing changes in firms' tangible capital as a proxy for investment. Our findings indicate that, following a policy rate cut, an investment wedge emerges between long- and short-relationship firms. Specifically, we find that each year of relationship at the time of the shock reduces cumulated tangible capital growth rates by 0.25 percentage points over the four years that follow the shock.

The existing literature has predominantly relied on two primary mechanisms to rationalize the fact that firms become locked in relationships with their bank: information asymmetry between inside and outside banks, and firms' switching costs. We discuss how each of them can also lead to the observed lower pass-through of policy rate cuts to long-relationship borrowers, and we deduce the different consequences we should observe on the connection between the rates obtained by switchers at outside banks and the length of their previous relationship. We then investigate the extensive margin of relationship lending to discriminate between the two mechanisms. We match firms that switched banks with comparable non-switching firms to estimate the interest rate discounts that the switchers receive when transitioning to a different bank. Subsequently, we conduct a regression analysis on these discounts, considering the switchers' previous relationship durations. Our findings indicate that switchers who had maintained longer relationships with their former bank receive higher discounts at their new bank. This suggests that firms engaged in longer relationships face higher switching costs and only choose to switch when offered relatively substantial discounts. Furthermore, when we match previous short-relationship switchers with previous long-relationship switchers who both arrive at the same outside bank, we find no significant difference in interest rates, suggesting the length of the previous relationship is not informative of a borrower's quality. Hence we develop a model that rationalizes our differential pass-through results based on switching costs.

Our findings regarding differential within-relationship pass-through suggest that the distribution of relationship durations in the economy can impact the aggregate transmission of monetary policy. To elaborate, when longer-lasting relationships exhibit lower pass-through, an increase in their prevalence within the economy can reduce aggregate pass-through due to a compositional effect. This is an important consideration

Figure 1: Share of Long Bank-Firm Relationships



Notes: This chart shows the evolution of the proportion of relationships longer than 4 years in the Norwegian economy (excluding entering firms from the calculations). Calculations from our dataset.

as the proportion of long-term relationships in the Norwegian economy has significantly increased since the financial crisis. For instance, Figure 1 traces out the evolution of the share of relationships longer than 4 years. It shows it rose from an average of 40% before 2008 to an average of 52% after 2012. This raises the question: how much higher would aggregate pass-through have been post-crisis if the share of long relationships had remained stable at its pre-crisis level?

It is crucial to note that the pass-through rates specific to different relationship durations might themselves depend on the equilibrium distribution of relationship lengths in the economy. Therefore, in principle, we cannot solely rely on our empirical estimates of within-relationship pass-through to determine how much aggregate pass-through would change with an alternative composition of relationship lengths in the economy. This is because such an approach would neglect potential equilibrium effects resulting from the distribution of relationship lengths on within-relationship pass-through rates. To address this, we introduce a banking model that provides a framework rationalizing our empirical findings. We use this model to calculate counterfactual aggregate pass-through rates under a different relationship length distribution, while considering the equilibrium effects on within-relationship pass-through rates.

The model rests on three key assumptions. First, firms have heterogeneous private switching costs for changing banks. Banks cannot observe their customers' individual switching costs, but they do know the distributions of these costs by relationship duration. Second, banks face a leverage constraint that limits the amount of loans they can hold on their balance sheets. Third, banks price compete for switching firms. That is, the competitive rate that banks offer to capture switchers is the lowest rate such that no individual bank can undercut it, thereby attracting all switchers, and still satisfy its leverage constraint.

In this setting, the policy rate is crucial for banks' net worth. This gives rise to two distinct regimes: in the first regime, when the policy rate is high, banks are far from their leverage constraint, and the competitive rate is driven down to the level of the policy rate. Banks charge a constant markup above the policy rate and monetary policy pass-through is the same for all firms and equal to one. In the second regime, when the policy rate falls below a cutoff, banks become so close to their leverage constraint that the competitive rate exceeds the policy rate. Banks then charge an increasing markup above the policy rate as the latter decreases. Consequently, the pass-through of monetary policy is impaired and falls below unity. Furthermore, it varies across firms with different relationship lengths and depends on the distribution of switching costs.

We lay out the equilibrium condition under which the pass-through to long-relationship firms is lower, and show it can easily be solved analytically when switching costs are generalized Pareto distributed. Intuitively, banks provide a reduced pass-through of rate cuts to their long-term customers when the distribution of switching costs for these firms exhibits a relatively large mass in the right tail. In this case, a larger proportion of long-term relationships compared to short-term ones are bound to their respective banks because of the substantial switching costs involved. This situation enables banks to maintain a lower pass-through rate for these long-term relationships.

We use the model to conduct a counterfactual exercise under some distributional assumption and estimate that aggregate pass-through would have been up to 23% higher in 2017 if the composition of relationship length in the economy had remained as in 2006. The entire change can be attributed to a compositional effect. The within-relationship pass-through rates remain largely unaffected by the shift in the composition of relationship lengths. This has significant implications for policymakers, as it means that any alterations in the distribution of relationship lengths in the economy will have an impact on the aggregate pass-through of monetary policy. Furthermore, it suggests that reduced-form empirical estimates of within-relationship pass-through rates are largely sufficient for predicting changes in aggregate monetary policy pass-through following shifts in the composition of relationship lengths within the economy.

Related Literature Our empirical study contributes to an ongoing research agenda focused on understanding the factors contributing to the reduced efficacy of the monetary policy transmission mechanism in environments with low policy rates. The predominant focus in existing empirical studies has been on addressing the potential zero lower bound on deposit rates and its implications for bank profitability, as well as the subsequent impact on lending behavior when central banks set negative policy rates. Consequently, the majority of these studies have examined the pass-through of monetary policy to deposit rates. While Altavilla, Burlon, Giannetti, and Holton

(2022) find that conventional monetary policy can remain effective below the zero lower bound by means of the pass-through to corporate deposit rates, leading to increased investment by firms, Heider, Saidi, and Schepens (2019), Ulate (2021), and Balloch and Koby (2022) find that the pass-through to household deposit rates is seriously impaired at low policy rate levels, affecting the lending activities of banks that heavily rely on deposit funding. In this paper, we focus instead on the sensitivity of *lending* rates to monetary policy. There is less agreement in the literature on the level of pass-through to lending rates when policy rates are low. For example, Eisenschmidt and Smets (2019) and Ulate (2021) report positive pass-through, while Amzallag, Calza, Georgarakos, and Sousa (2019) and Eggertsson, Juelsrud, Summers, and Wold (2019) observe near-zero pass-through. Our research adds to this ongoing debate by showing that the pass-through to corporate lending rates is significantly diminished in a low-rate environment. To the best of our knowledge, we provide the first empirical evidence on the links between monetary policy pass-through to bank loan rates and bank-firm relationships. Our primary contribution is the evidence of relationship-induced heterogeneity in pass-through to lending rates when policy rates are low. In particular, our results suggest that in such a context, long relationships weaken the bank lending channel.

There is a small theoretical literature on the links between monetary policy and relationship lending. Hachem (2011) builds a credit-based model of production, where banks learn about the private productivity of their borrowers along the relationship. The author analyses how monetary policy affects incentives to engage in relationship lending, and how the latter matters for the response of aggregate output to shocks. The main finding is that relationships smooth the economy's output profile since banks offer policy-invariant credit terms to some of their borrowers. In contrast, we build a banking model connecting monetary policy pass-through and relationship lending based on firms' switching costs. We focus on low-rate environments and our conclusions are not in contradiction with Hachem (2011), who considers intermediate ranges of the policy rate. Bethune, Rocheteau, Wong, and Zhang (2021) build a monetary model of corporate finance with endogenous lending relationships. Entrepreneurs who match with a bank have access to external finance and better investment opportunities. The authors use their model to study optimal monetary policy after an unanticipated destruction of relationships in the economy. Our approach and objective differ in that we investigate the importance of relationship length (and not only the extensive margin of relationship lending) for monetary policy pass-through to bank loan rates, and its consequences for aggregate transmission in the low-rate environment. Araujo, Minetti, and Murro (2021) study the implications of lending relationships for monetary policy in a model where lenders provide both liquidity and expertise to firms in distress. Their focus is on banks' incentives to assist firms depending on their relationships. All these papers, in one way or another, build on the concept that lending relationships foster privileged information exchange between banks and firms. In contrast, driven by our empirical findings, we contribute to the literature by formulating a model centered on firms' switching costs and aim to comprehend their implications for monetary policy pass-through in low-rate environments.

The banking model we propose contributes to a theoretical literature that seeks to explain the impact of low and negative policy rates on the monetary transmission mechanism. Brunnermeier and Koby (2018) introduce a model that includes a reversal rate, which is the policy rate below which further monetary policy easing becomes contractionary. At low policy rates, banks' margins become thinner, negatively affecting equity and eventually leading to binding capital constraints. As equity issuance is costly, any further reduction in the policy rate must result in decreased lending. Ulate (2021) develops a banking model to investigate the effectiveness of monetary policy in negative territories, mainly through the presence of a zero lower bound on deposit rates that negatively affects bank profitability at low rates. Similarly, Eggertsson, Juelsrud, Summers, and Wold (2019) examine the impact of low policy rates in the presence of a lower bound on deposit rates, which disrupts the transmission of rate cuts to the primary source of financing for banks. Wang (2018) also studies the effects of low interest rates on monetary policy transmission, with a focus on the differences between short-term and long-term effects. Similarly to most of these models, state-dependency in our framework also relies on bank net worth being negatively affected by low policy rates, and the presence of a capital constraint limiting the amount of loans banks can take on their balance sheets. Our main contribution is to show that the interaction between the low-rate environment and the presence of heterogeneous switching costs results in differential monetary policy pass-through, contingent upon relationship length. Our model has novel implications. To the best of our knowledge, we are the first to show that the composition of relationship lengths in the economy matters for aggregate pass-through.

The rest of the paper proceeds as follows: Section 2 presents the details of the data that are used in our analyses, and the measurements of our key variables. Section 3 contains our empirical analyses. In Section 4, we present a theoretical framework to rationalize our empirical findings and conduct a counterfactual exercise. Section 5 concludes.

2 Data

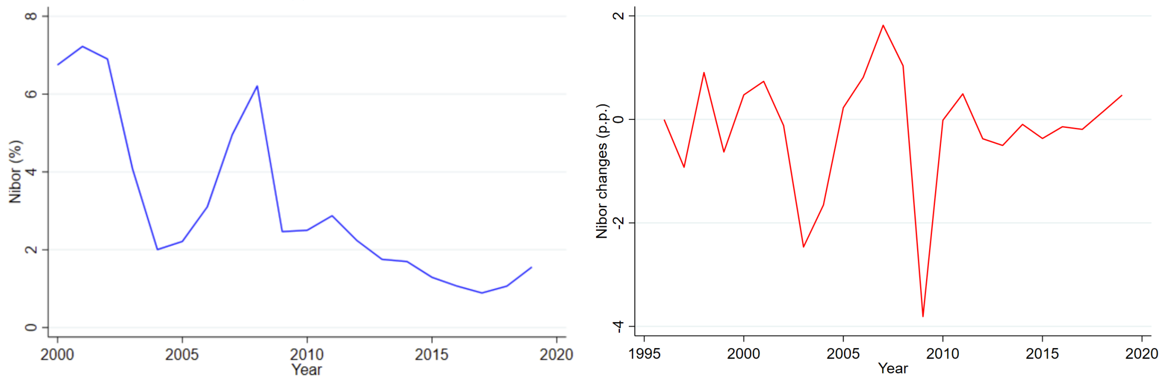
We draw data mainly from three different sources. The first source is provided by the Norwegian Tax Administration (*Skatteetaten*). By the end of each year, all banks report all outstanding loan accounts (stock) as well as interest paid on each loan account (flow) to the tax administration for tax purposes. This dataset links each loan account to a unique firm identifier, which allows us to track all bank-firm relationships from 1997 to 2019 at a yearly frequency. We define a firm and a bank to be in a relationship in a given year if either the outstanding loan amount or the interest paid is larger than zero. To construct an unbounded measure of bank-firm relationship length, we drop the existing relationships in the first year of our sample since we do not have information on their starting date. In tracking relationships through time, we account for approximately 50 bank mergers and acquisitions that took place over the 23 years that the dataset covers. Specifically, if bank A absorbs bank B, bank A typically lays hold of the information set on bank B's clients. Moreover, bank B's clients who stay with bank A after the merger do not incur switching costs. We therefore ignore the apparent switches in the data from bank B to bank A by bank B's customers, and treat these cases as continuing relationships. To obtain the average interest rate paid by a firm to its bank in year t, we divide the interest amount paid throughout year t by the average of the stocks of loan at the end of years $t-1$ and t. To get rid of clearly erroneous and extreme values of interest rates, we trim the distribution at the 5% and 95% levels. Even though we use firm-bank-account level information in our robustness analysis, the dataset for our main analysis is at the firm-bank level. It is important to note that, although we do not observe this information at the loan level, we know from Cao, Hegna, Holm, Juelsrud, König, and Riiser (2023) that 95% of corporate loans in Norway have floating rates. This means that commercial banks can freely adjust the rates they charge their corporate borrowers after a monetary policy change, including on existing loans. This allows us to estimate monetary policy pass-through using changes in interest rates estimated from total interest payments on all the existing loans of a borrower.

We match this dataset with data from our second source, the firm register data provided by Brønnøysund Register Centre (*Brønnøysundregistrene*). By the end of each year, all firms operating in Norway are required to register their balance sheets and financial statements at the Register Centre. In our analysis, we drop financial firms and government-owned firms. The third source is the yearly balance sheet reports of all banks operating in Norway, including subsidiaries and branches of foreign-owned banks (mostly Swedish and Danish), between 2000 and 2019 from the financial market statistics (*Offentlig Regnskapsrapportering fra Banker og Finansieringsforetak*: financial reports from banks and financial undertakings). Our final dataset comprises 205 banks and $289,086$ firms for a total of $460,722$ unique bank-firm relationships. A particularity of the Norwegian setting is the very low share of firms that simultaneously maintain multiple bank relationships. Roughly 90% of firms only borrow from one bank at a given point in time.

To analyze the pass-through level of monetary policy, we primarily use average annual changes in the Norwegian Interbank Offered Rate (NIBOR). The latter is the target interest rate used by Norges Bank to conduct monetary policy, and we therefore interchangeably refer to it as the policy rate throughout the rest of the paper. We have daily data on the level of the NIBOR. To construct average annual changes, we follow Gertler and Karadi (2015). For each day of the year, we cumulate the daily changes of the NIBOR on any day during the previous 365 days. We then average these yearly changes across each day of the year. This procedure ensures that a change in the NIBOR on December 31^{st} of year t will mostly be reflected in year $t+1$ in the series of average annual changes. Figure 2a shows the evolution of the NIBOR over our sample period and Figure 2b shows the average annual changes. Changes in the policy rate obviously are endogenous: the central bank adapts its policy to respond to changes in the state of the economy, which may themselves have confounding effects on the change in loan rates charged by banks to firms. Yet, it is still common in the literature to use changes in the policy rate and attempt to address endogeneity concerns by including different sets of fixed effects, see, for example Greenwald (2019). The use of industry-time fixed effects, for example, controls for the macroeconomic shocks that might have caused the policy rate change, even in the case they have different effects on different sectors. As robustness checks, we also re-run our entire analyses using identified monetary policy shocks. We draw these series of exogenous shocks from Brubakk, ter Ellen, Robstad, and Xu (2022), who follow Jarociński and Karadi (2020) and extract monetary policy surprises from forward rates' movements in a 30-minute window around the central bank's monetary policy announcements. We then build annual monetary policy shock series following Gertler and Karadi (2015).

Table 1 provides some summary statistics of the main variables used in our analyses. Figure 3 shows the five-year moving average of the NIBOR and the evolution of bank switching intensity, defined as the number of firms switching banks divided by the total number of firms in the economy in a given year. The chart shows a positive correlation between the two measures, indicating fewer firms switch when the policy rate is low. Figure 4 shows the evolution of the composition of relationships in the economy by duration. It is clear that, consistently with decreasing switching rates, the share of relatively long relationships in the economy drastically increases in the years following the global financial crisis and the introduction of low policy rates. For example, the share of relationships longer than 4 years increased from an average of 40% (pre-crisis), to an average of 52% (post-crisis). The sharp increase in the share of long relationships following 2008 is partly due to a mechanical effect: lower firm entry (and therefore lower relationship creation) during the crisis.

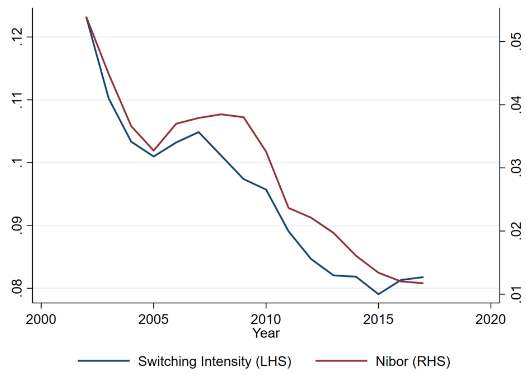
Figure 2



(a) NIBOR: Levels

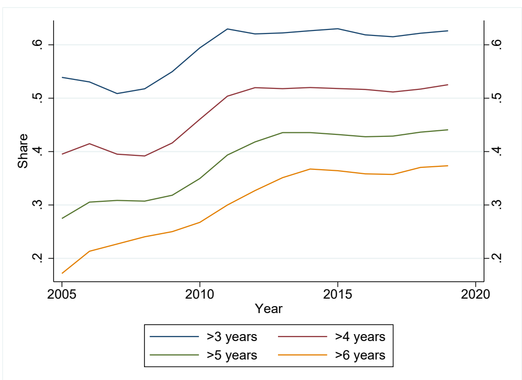
(b) NIBOR: Annual Changes

Figure 3: NIBOR (Five-Year Moving Average) and Bank Switching Intensity



Notes: Switching intensity in a given year is defined as the number of firms starting a new relationship (i.e. borrowing from a new bank) divided by the total number of operating firms in the economy.

Figure 4: Composition of Relationship Lengths



Notes: Stacked chart showing the evolution of the composition of relationships lengths in the Norwegian economy, excluding entering firms from the calculations.

Table 1: Descriptive Statistics

Variable	Observations	Mean	Std. Dev.	Min.	p25	p50	p75	Max.
Macro Variables								
3-month Nibor (%)	20	3.14	2.12	.89	1.63	2.35	4.53	7.23
ΔNibor (p.p.)	20	-.18	1.25	-3.81	-.37	-.06	.48	1.82
Exogenous MP Shock (p.p.)	20	-.02	.19	-.41	-.07	-.01	.06	.51
Relationships								
Length (years)	1,316,117	4.83	3.78	1	2	4	7	22
Loans								
Interest Received (1000 NOK)	1,316,117	404,077	5,135,723	1	14,308	57,637	198,460	5.23e+09
Loan Amount Outstanding (1000 NOK)	1,316,117	9,636,235	1.47e+08	0	107,574	849,160	3,791,451	1.52e+11
Interest Rate (%)	1,316,117	7.01	4.45	.12	4.09	6.02	8.71	26.85
ΔInterest Rate (p.p.)	998,702	-.26	2.84	-9.76	-1.40	0	.96	8.51
Firms								
Total Assets (1000 NOK)	692,363	81,844	1,738,939	0	2414	5977	17140	8.78e+08
Total Debt/Total Assets	692,363	.71	.22	0	.59	.76	.88	1
Age (years)	692,363	12.66	12.01	1	5	9	16	167
Credit Rating (1-5 ordinal)	682,549	3.36	.88	1	3	3	4	5
No. of Creditor Banks	692,363	1.19	.51	1	1	1	1	71
Banks								
Total Assets (1000 NOK)	2.113	2.94e+07	1.47e+08	86,096	1,529,436	2,908,598	8,960,664	2.10e+09
Interbank Borrowing/Total Liabilities	2.113	.05	.12	0	0	.01	.04	.98
Deposits/Total Liabilities	2.113	.72	.16	0	.65	.74	.82	.99
Loans/Deposits	2.113	1.46	3.30	0	1.13	1.27	1.41	114.11
Equity/Total Assets	2.113	.10	.03	0	.08	.10	.12	.30
Liquid Assets/Total Assets	2.113	.06	.08	0	.03	.05	.07	.99
Financal Securities/Total Assets	2.113	.11	.07	0	.07	.09	.13	.83

Notes: This table reports summary statistics of key macro, bank, and firm variables. The summary statistics for the relationships and loans characteristics are calculated at the bank-firm-year level.

3 Empirical Analysis

3.1 Relationship Duration and Monetary Policy Pass-Through

We investigate the connection between relationship duration and monetary policy pass-through, and how it depends on the initial level of the policy rate. We start by looking at the pass-through to lending rates. We show that long-relationship borrowers' rates decrease relatively less following a policy rate cut at initial low rate. Next, in an attempt to distinguish between bank loan supply and firm loan demand to explain this pass-through result, we investigate the changes in loan volumes following monetary policy changes. Finally, we show that the identified channel has real effects through investment in tangible capital.

3.1.1 Lending Rates

To evaluate the magnitude and economic significance of any effect that relationship duration may have on monetary policy pass-through, it is useful to first estimate average within-relationship pass-through. We do so by estimating the following regression, in which γ_1 is the coefficient of interest:

$$\Delta r_{ibt} = \alpha_{ib} + \epsilon^m_{t-1}\gamma_1 + \mathbf{Z}_{i,t-1}\delta_{\mathbf{1}} + \mathbf{W}_{b,t-1}\delta_{\mathbf{2}} + \mathbf{V}_{t-1}\delta_{\mathbf{3}} + \epsilon_{ibt}, \tag{1}$$

where Δr_{ibt} is the change in interest rate paid by firm i to bank b between $t-1$ and t. α_{ib} are firm-bank (i.e. relationship) fixed effects. ϵ^m_{t-1} is the monetary policy change in $t-1$. $\mathbf{Z}_{it}$ are firm-level controls including age, size, leverage, and credit rating. $\mathbf{W}_{bt}$ are bank-level controls including size measured by the logarithm of total assets, interbank borrowing to total liabilities ratio, deposits to total liabilities ratio, loans to deposits ratio, equity to total assets ratio, liquid assets to total assets ratio, and financial securities to total assets ratio. $\mathbf{V}_t$ are macroeconomic controls including GDP growth, inflation, a measure of market volatility (VIX index), oil prices, the NOK/USD exchange rate, and the slope of the yield curve measured as the difference between the yields on 10y-NIBOR and 3m-NIBOR. Column 2 of Table 2 shows that a 1 p.p. increase in the NIBOR is associated with a 0.483 p.p. increase in loan rates on average.

We then allow for asymmetric pass-through, depending on whether monetary policy is tightening or loosening.

$$\Delta r_{ibt} = \alpha_{ib} + \epsilon^m_{t-1}[\gamma_1 + tight_{t-1}\gamma_2] + tight_{t-1}\gamma_3 + \mathbf{Z}_{i,t-1}\delta_{\mathbf{1}} + \mathbf{W}_{b,t-1}\delta_{\mathbf{2}} + \mathbf{V}_{t-1}\delta_{\mathbf{3}} + \epsilon_{ibt}, \tag{2}$$

where the dummy variable $tight_t$ equals one when the policy rate increases. Column 4 of Table 2 shows that pass-through displays strong asymmetry with only 26% of a policy rate cut transmitted to firms, while banks increase rates by 143% of a policy rate hike on average. It is worth noting here that we estimate *within-relationship* pass-through, that is, conditional on the relationship already existing at the moment of the monetary policy change and surviving it. In other words, we estimate pass-through to those firms that choose to stay in the relationship after and despite the change in the interest rate offered by their banks. In particular, these pass-through estimates do not take changes in interest rates offered to new customers (switchers) into account. This can help explain the relatively large (small) size of the estimate in case of policy rate hike (cut).

Finally, we add an interaction term with the initial level of the NIBOR, allowing pass-through to depend on the initial stance of monetary policy.

$$\begin{aligned}\Delta r_{ibt} = \alpha_{ib} + \epsilon^m_{t-1}[\gamma_1 + tight_{t-1}\gamma_2 + i_{t-1}(\gamma_3 + tight_{t-1}\gamma_4)] + tight_{t-1}\gamma_5 + i_{t-1}\gamma_6 \\ + tight_{t-1} \times i_{t-1}\gamma_7 + \mathbf{Z}_{i,t-1}\delta_{\mathbf{1}} + \mathbf{W}_{b,t-1}\delta_{\mathbf{2}} + \mathbf{V}_{t-1}\delta_{\mathbf{3}} + \epsilon_{ibt},\end{aligned} \tag{3}$$

where i_{t-1} is the level of the NIBOR at time $t-1$. Column 6 of Table 2 shows that as the policy rate decreases, any further cut is transmitted to a lesser extent. Symmetrically, as the policy rate increases, smaller shares of further hikes are passed on to firms. Table 3 summarizes this by calculating average within-relationship monetary policy pass-through in the case of a monetary expansion/contraction in a low/high initial policy rate environment. It is constructed using the coefficient estimates of regression (3). When the NIBOR is one standard deviation below its mean, *only* 9% *of a further rate cut is passed on to firms.* This contrasts with the 61% that are transmitted when the initial policy rate is one standard deviation above the mean.

Having shown that average within-relationship pass-through following a policy rate cut is decreasing in the initial level of said policy rate, we next investigate the role of bank-firm relationship duration. We run similar regressions to those estimating the average pass-through, but now include interaction terms between relationship duration and monetary policy shocks. Since we are interested in the marginal effect of a bank-firm-time level variable (i.e. relationship length) on within-relationship pass-through, we can now include time fixed effects, which was not possible for the estimation of *average* within-relationship monetary policy pass-through. We run the following set of regressions and report the estimated coefficients of interest in Table 4.

Table 2: Within-Relationship Pass-Through

	(1)	(2)	(3)	(4)	(5)	(6)
ϵ^m_{t-1}	0.143***	0.483***	0.022	0.263***	-0.640***	-0.042
	(0.012)	(0.031)	(0.011)	(0.017)	(0.031)	(0.065)
$tight_{t-1} \times \epsilon^m_{t-1}$			0.286***	1.167***	4.324***	3.160***
			(0.045)	(0.045)	(0.135)	(0.155)
$\epsilon^m_{t-1} \times i_{t-1}$					0.252***	0.121***
					(0.012)	(0.021)
$tight_{t-1} \times \epsilon^m_{t-1} \times i_{t-1}$					-0.761***	-0.388***
					(0.037)	(0.025)
N	937476	763122	937476	763122	937476	763122
Macro Controls	No	Yes	No	Yes	No	Yes
Firm Controls	No	Yes	No	Yes	No	Yes
Bank Controls	No	Yes	No	Yes	No	Yes
Industry-Time FE	No	No	No	No	No	No
ILS-Time FE	No	No	No	No	No	No
Bank-Time FE	No	No	No	No	No	No
Bank-Firm FE	Yes	Yes	Yes	Yes	Yes	Yes

Dually clustered (bank and firm levels) standard errors in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Notes: Columns (1)-(2) report the estimated coefficients of interest from regression (1). Columns (3)-(4) report the estimated coefficients of interest from regression (2). Columns (5)-(6) report the estimated coefficients of interest from regression (3). Macroeconomic controls **V** include GDP growth, inflation, market volatility (VIX index), oil prices, the NOK/USD exchange rate, and the slope of the yield curve (difference between the yields on 10y-NIBOR and 3m-NIBOR). Firm controls **Z** include age, size, leverage, and credit rating. Bank controls **W** include size measured by the logarithm of total assets, interbank borrowing to total liabilities ratio, deposits to total liabilities ratio, loans to deposits ratio, equity to total assets ratio, liquid assets to total assets ratio, and financial securities to total assets ratio.

$$\Delta r_{ibt} = \alpha_{ib} + length_{ib,t-1}\mathbf{X}_{t-1}\beta + \epsilon_{ibt} \tag{4}$$

$$\Delta r_{ibt} = \alpha_{ib} + \alpha_{bt} + length_{ib,t-1}\mathbf{X}_{t-1}\beta + \epsilon_{ibt} \tag{5}$$

$$\Delta r_{ibt} = \alpha_{ib} + \alpha_{bt} + \alpha_{jlst} + length_{ib,t-1}\mathbf{X}_{t-1}\beta + \epsilon_{ibt} \tag{6}$$

Table 3: Within-Relationship Pass-Through: Marginal Effects

	Easing ($tight_{t-1} = 0$)	Tightening ($tight_{t-1} = 1$)
Low Nibor	0.090*	2.827***
($i_{t-1} = 1.09\%$)	(0.044)	(0.129)
High Nibor	0.610***	1.685***
($i_{t-1} = 5.37\%$)	(0.056)	(0.099)

Dually clustered (bank and firm levels) standard errors in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Notes: This table shows the marginal effects of MP (change in NIBOR) on Δr (change in loan rates) when monetary policy is loosening/tightening and when the initial level of the NIBOR is one standard deviation below/above its mean. Marginal effects are calculated from specification (3) and the estimated coefficients of column (6) in Table 2.

$$\Delta r_{ibt} = \alpha_{ib} + \alpha_{jt} + \alpha_{bt} + length_{ib,t-1}\mathbf{X}_{t-1}\beta + \mathbf{Z}_{i,t-1}\mathbf{U}_{i,t-1}\delta + \epsilon_{ibt}, \tag{7}$$

where $length_{ib,t-1}$ is the relationship length between firm i and bank b at time $t-1$,

$$\mathbf{X}'_{t-1} = \begin{pmatrix} 1 \\ \epsilon_t^m \\ i_t \\ tight_t \\ \epsilon_t^m \times i_t \\ \epsilon_t^m \times tight_t \\ tight_t \times i_t \\ \epsilon_t^m \times tight_t \times i_t \end{pmatrix},$$

and

$$\mathbf{U}'_{t-1} = \begin{pmatrix} \mathbf{X}'_{t-1} & \ldots & \mathbf{X}'_{t-1} \end{pmatrix},$$

with the column dimension of $\mathbf{U}'_{t-1}$ being equal to the number of firm-level controls in $\mathbf{Z}_{i,t-1}$.

In regression (4), we do not control for anything beyond bank-firm fixed effects (α_{ib}). However, relationship length between a given bank and firm is not randomly assigned, and might be correlated with bank and firm characteristics that influence loan interest rates. In regression (5), we add bank-time fixed effects (α_{bt}). Identification within bank-time addresses the concern that relationship duration may be correlated with bank balance sheet items (such as deposit ratios), which also affect pass-through. With bank-time fixed effects, we estimate the marginal effect of relationship length on pass-through by comparing pass-through to firms borrowing from the same bank, but with different relationship lengths. To control for the demand side and identify supply shocks, the literature typically relies on the inclusion of firm-time fixed effects. Under the assumption that firms have the same demand for credit across banks at one point in time, identification within firm-time ensures that any estimated effect does not come from changes in firm demand for credit. However, the structure of bank-firm relationships in Norway prevents us from relying on firm-time fixed effects. Indeed, approximately 90% of the firms in our dataset only borrow from a single bank at any given point in time. In regression (6), we try to circumvent this issue by following Degryse, De Jonghe, Jakovljević, Mulier, and Schepens (2019) and add firm industry-location-size-time fixed effects (α_{jlst}). Under the assumption that firms within the same cell have the same demand for credit across banks, these fixed effects allow to identify supply shocks. In regression (7), we only include firm industry-time fixed effects (α_{jt}) and control for a set of firm characteristics (age, size, leverage, credit rating). We also interact these firm characteristics with the monetary policy shocks, allowing loan rates of firms with different observables to react differently to the same policy rate change. Insofar as selection into a given relationship length is correlated with these characteristics, controlling for them should reduce the endogeneity bias. In addition to firm characteristics, vector **Z** in regression (7) also contains a time-varying HHI index at the county level to capture the level of local bank competition and market power. This addresses the concern that localities with higher bank concentration may precisely be the ones where firms tie longer relationships.

Table 4 shows that the estimates are similar across specifications. In particular, regressions (6) and (7), which both attempt to control for demand, yield quantitatively close estimates in the case of a monetary policy expansion. The negative coefficient on the interaction between length and monetary policy shock, along with the positive coefficient on the interaction between length, monetary policy shock, and NIBOR indicate that, after a policy rate cut, the pass-through is decreasing in relationship length when the NIBOR is low. Table 5 shows the marginal effects (calculated from the estimates of regression (7)) of relationship length on pass-through in the case of a monetary policy expansion/contraction in a low/high initial policy rate environment. It shows that when the NIBOR is one standard deviation below its mean, each additional year of relationship at the moment of the shock *reduces* the pass-through of a policy rate cut by 2.7 percentage points. This effect represents roughly one-third of the average within-relationship pass-through from Table 3. On the other hand, each additional year of relationship *increases* the pass-through of a policy rate hike by 8 percentage points. It appears that at low policy rates, banks take advantage of their long-relationship customers in both directions of a policy rate change. In contrast, when the NIBOR is one standard deviation above its mean, the pass-through following a policy rate cut is larger for firms in long relationships.

Table 4: Pass-Through and Relationship Length

	(1)	(2)	(3)	(4)
$length_{ibt} \times \epsilon^m_{t-1}$	-0.062***	-0.053***	-0.057***	-0.055***
	(0.010)	(0.012)	(0.012)	(0.012)
$length_{ibt} \times \epsilon^m_{t-1} \times i_{t-1}$	0.030***	0.024***	0.026***	0.026***
	(0.004)	(0.005)	(0.005)	(0.005)
$tight_{t-1} \times length_{ibt} \times \epsilon^m_{t-1}$	0.056*	0.085*	0.073**	0.149***
	(0.028)	(0.034)	(0.028)	(0.034)
$tight_{t-1} \times length_{ibt} \times \epsilon^m_{t-1} \times i_{t-1}$	0.004	-0.027**	-0.025***	-0.040***
	(0.007)	(0.009)	(0.007)	(0.008)
N	937476	937449	703029	865407
Macro Controls	No	No	No	No
Firm Controls	No	No	No	Yes
Bank Controls	No	No	No	No
Industry-Time FE	No	No	No	Yes
ILS-Time FE	No	No	Yes	No
Bank-Time FE	No	Yes	Yes	Yes
Bank-Firm FE	Yes	Yes	Yes	Yes

Dually clustered (bank and firm levels) standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Notes: Columns (1)-(4) report the estimated coefficients of interest from regressions (4)-(7) respectively. Firm controls **Z** include age, size, leverage, and credit rating.

Regressions (4) - (7) assume that marginal effects are linear in the initial level of the NIBOR. To address potential non-linearity concerns, especially when interest rates are low, we run Kernel regressions (8) at different initial levels of the policy rate. We weight data points using Epanechnikov's kernel centered at 0.1 increments of the NIBOR and a Silverman bandwidth. Since the regressions are now centered around a specific initial NIBOR level, we remove the dummy $tight_{t-1}$ from the specifications. However, note that the observations of low NIBOR are associated with policy rate cuts. The results we get in this region are therefore to be interpreted in the context of a monetary policy expansion.

$$(8) \qquad \Delta r_{ibt} = \alpha_{ib} + \alpha_{jt} + \alpha_{bt} + length_{ib,t-1}(\beta_0 + \beta_1 \epsilon^m_{t-1}) + \mathbf{Z}_{i,t-1}(\gamma_{\mathbf{1}} + \gamma_{\mathbf{2}} \epsilon^m_{t-1}) + \epsilon_{ibt}$$

Table 5: Pass-Through and Relationship Length: Marginal Effects

	Easing ($tight_{t-1} = 0$)	Tightening ($tight_{t-1} = 1$)
Low Nibor	-0.027***	0.079*
($i_{t-1} = 1.09\%$)	(0.007)	(0.033)
High Nibor	0.083***	0.020
($i_{t-1} = 5.37\%$)	(0.014)	(0.185)

Dually clustered (bank and firm levels) standard errors in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Notes: This table shows the marginal effects of *length* (relationship length) on the marginal effect of MP (change in NIBOR) on Δr (change in loan rates) when monetary policy is loosening/tightening and when the initial level of the NIBOR is one standard deviation below/above its mean. In other words, the table shows $\frac{\partial^2 \Delta r_t}{\partial \epsilon^m_{t-1} \partial length_{t-1}}$. Marginal effects are calculated from specification (7) and the estimated coefficients of column (4) in Table 4.

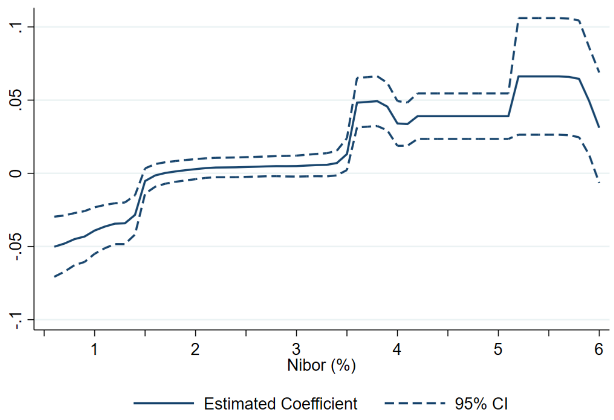
Figure 5 plots the estimated coefficients β_1 against the initial level of the NIBOR. It shows the marginal effect of relationship length on pass-through displays non-linearity with three apparent regimes. When the NIBOR is above 3.5%, firms in longer relationships seem to get more pass-through. However, confidence intervals are relatively large as we have few observations of high policy rates. When the NIBOR is in an intermediate range, between 1.5% and 3.5%, relationship length is irrelevant for monetary policy pass-through. Finally, when the NIBOR is below 1.5%, firms in long relationships get relatively less pass-through. Quantitatively, the estimated effects for this last regime are even larger than those estimated in Table 5. When the initial level of the NIBOR is 1.1%, each additional year of relationship reduces pass-through by 3.7 percentage points.

Next, we also allow for non-linearity in relationship length by using dummies instead of the continuous variable $length_{ib,t-1}$. We run the following kernel regressions, using the same kernel and bandwidth as in (8):

$$(9) \quad \Delta r_{ibt} = \alpha_{ib} + \alpha_{It} + \alpha_{bt} + \sum_{s=1}^{2} \mathcal{I}_{sib,t-1}(\beta_{0,s} + \beta_{1,s}\epsilon^m_{t-1}) + \mathbf{Z}_{i,t-1}(\gamma_{\mathbf{1}} + \gamma_{\mathbf{2}}\epsilon^m_{t-1}) + \epsilon_{ibt},$$

where $\mathcal{I}_{1ib,t-1}$ is a dummy variable equal to 1 if relationship length between bank b and firm i at time $t-1$ is between 5 and 8 years. $\mathcal{I}_{2ib,t-1}$ equals 1 if relationship length is longer than 8 years. The left-hand side of Figure 6 plots the estimated coefficients $\beta_{1,1}$ against the initial level of the NIBOR, and the right-hand side adds the estimated coefficients $\beta_{1,2}$. These regressions reveal substantial non-linearity in relationship length. When the NIBOR is at 1.1%, firms whose relationships are between

Figure 5: Pass-Through and Relationship Length: Allowing for Non-linearity in NIBOR



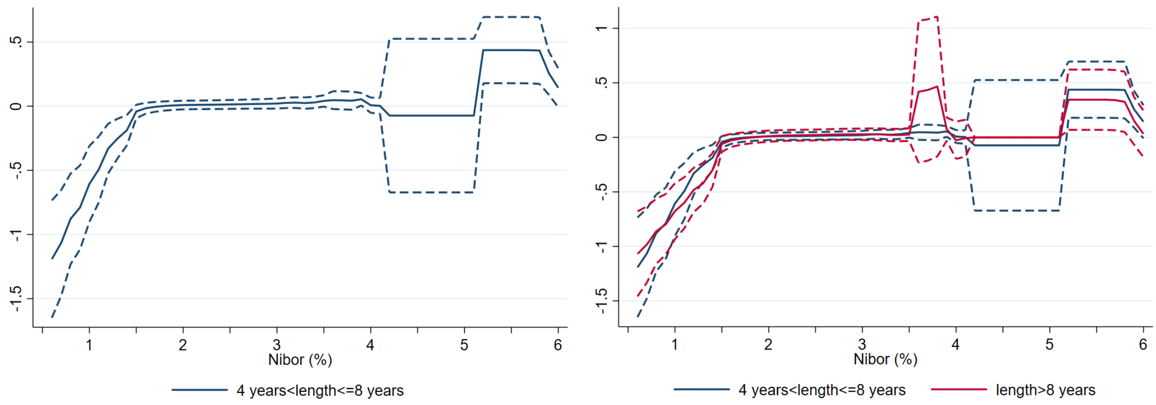
Notes: The figure plots coefficient β_1 from kernel regressions (8), where points are weighted using Epanechnikov's kernel centered at 0.1 increments on the x-axis and a Silverman bandwidth (here = 1.07). Coefficient β_1 is the marginal effect of *length* (relationship length) on the marginal effect of MP (change in NIBOR) on Δr (change in loan rates). I.e. $\beta_1 = \frac{\partial^2 \Delta r_t}{\partial \epsilon^m_{t-1} \partial length_{t-1}}$.

5 and 8 years get approximately 50 percentage points less pass-through than firms with a relationship shorter than 5 years. The difference in pass-through between firms in a relationship shorter than 5 years and firms with a relationship longer than 8 years is of similar magnitude. This suggests it is the variation in relationship duration during the first 8 years of relationship that matters for differential pass-through. In other words, the marginal effect of relationship length on pass-through is itself decreasing in relationship length. This explains why its estimate was of much smaller magnitude in our previous fully linear specifications (4) to (7). Finally, allowing for non-linearity in relationship length seems to indicate the presence of only two regimes with a cutoff around 1.5% for the NIBOR. I.e., we do not get the previous results of larger pass-through to long-relationship firms at high NIBOR levels anymore.

3.1.2 Lending Volumes

In our previous analysis, we regress a change in equilibrium prices (loan rates) on the change in banks' marginal cost of funds (the policy rate) in an attempt to uncover the relevance of relationship length for monetary policy pass-through. One concern for identification is that firms' loan demand may be shifting at the same time than monetary policy and correlated with relationship length. For example, we may worry that long-relationship firms increase their loan demand relatively more after a monetary policy expansion. The resulting higher increase in equilibrium loan volumes could therefore explain why these firms get relatively less pass-through. From a monetary policy standpoint though, we really are interested in understanding whether our dif-

Figure 6: Pass-Through and Relationship Length: Allowing Non-Linearity in NIBOR and Relationship Length



Notes: The left-hand side figure plots coefficient $\beta_{1,1}$ from kernel regressions (9), where points are weighted using Epanechnikov's kernel centered at 0.1 increments on the x-axis and a Silverman bandwidth (here = 1.07). Coefficient $\beta_{1,1}$ is the additional effect of having a relationship aged between 5 and 8 years compared to the reference group (relationships shorter than 5 years) on the marginal effect of MP (change in NIBOR) on Δr (change in loan rates). The right-hand side figure adds coefficient $\beta_{1,2}$ from kernel regressions (9). Coefficient $\beta_{1,2}$ is the additional effect of having a relationship longer than 8 years compared to the reference group (relationships shorter than 5 years) on the marginal effect of MP (change in NIBOR) on Δr (change in loan rates).

ferential pass-through results stem from bank credit supply.

Our previous analysis already attempts to control for firm credit demand in two ways. In a first specification, we include industry-location-size-time fixed effects. In a second specification, we include industry-time fixed effects and control for the most important firm-level characteristics. To further rule out that our results could be entirely driven by firms' demand side, we run the same regressions as in (4)-(7), but using loan growth rates from $t-1$ to t as the outcome variable. Table 6 reports the results. The estimated coefficients on the interaction between relationship length and monetary policy shocks (first row) are either insignificant or positive. This means that when the NIBOR is at 0%, additional years of relationship have a non-negative effect on the marginal effect of a policy rate cut on loan growth rates. In other words, long-relationship firms *do not* have a higher loan growth rate than short-relationship firms following the policy change. If anything, their loan growth rate is lower. This rules out the possibility that our differential pass-through results from Section 3.1.1 may be driven by firms' demand side only. Banks' credit supply must have increased relatively more for short-relationship firms.

3.1.3 Real Effects

We investigate whether lower pass-through to long-relationship firms after a policy rate cut at initial low rate has real effects. To do so, we regress tangible capital growth

Table 6: Credit Growth and Relationship Length

	(1)	(2)	(3)	(4)
$length_{ibt} \times \epsilon^m_{t-1}$	0.000	0.004*	0.007***	0.001
	(0.002)	(0.002)	(0.002)	(0.002)
$length_{ibt} \times \epsilon^m_{t-1} \times i_{t-1}$	0.001	-0.001	-0.002**	-0.001
	(0.001)	(0.001)	(0.001)	(0.001)
$tight_{t-1} \times length_{ibt} \times \epsilon^m_{t-1}$	-0.003	-0.044***	-0.051***	-0.011*
	(0.007)	(0.004)	(0.004)	(0.004)
$tight_{t-1} \times length_{ibt} \times \epsilon^m_{t-1} \times i_{t-1}$	0.003*	0.011***	0.013***	0.005***
	(0.001)	(0.001)	(0.001)	(0.001)
N	968586	968564	703932	873884
Macro Controls	No	No	No	No
Firm Controls	No	No	No	Yes
Bank Controls	No	No	No	No
Industry-Time FE	No	No	No	Yes
ILS-Time FE	No	No	Yes	No
Bank-Time FE	No	Yes	Yes	Yes
Bank-Firm FE	Yes	Yes	Yes	Yes

Dually clustered (bank and firm levels) standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Notes: Columns (1)-(4) report the estimated coefficients of interest from regressions (4)-(7), but using loan growth rates $\frac{L_{ibt}-L_{ib,t-1}}{L_{ib,t-1}}$ as the outcome variable, where L_{ibt} is the loan volume between bank b and firm i at time t. Firm controls $\mathbf{Z}$ include age, size, leverage, and credit rating.

rates – a proxy for firm investment – at yearly horizons $h \in \{0, \ldots, 8\}$ on the same variables as in regression (7). We use the coefficients on the interaction terms between relationship length and monetary policy shocks to calculate the wedges in cumulated tangible capital growth h years after the monetary policy shock, which are due to the length of the relationship at the moment of the shock.

We run the following regressions for $h \in \{0, \ldots, 8\}$:

$$\frac{k_{i,t+h} - k_{i,t-1}}{k_{i,t-1}} = \alpha_{ib} + \alpha_{jt} + \alpha_{bt} + length_{ib,t-1}\mathbf{X}_{t-1}\beta_h + \mathbf{Z}_{i,t-1}\mathbf{U}_{i,t-1}\delta_h + \epsilon_{ibt}, \tag{10}$$

where k_{it} is the tangible capital of firm i at time t,

$$\mathbf{X}'_{t-1} = \begin{pmatrix} 1 \\ \epsilon_t^m \\ i_t \\ tight_t \\ \epsilon_t^m \times i_t \\ \epsilon_t^m \times tight_t \\ tight_t \times i_t \\ \epsilon_t^m \times tight_t \times i_t \end{pmatrix},$$

and

$$\mathbf{U}'_{t-1} = \begin{pmatrix} \mathbf{X'}_{t-1} & \dots & \mathbf{X'}_{t-1} \end{pmatrix}.$$

Figure 7 plots the marginal effects of relationship length on monetary policy pass-through to capital growth rates at horizons $h \in \{0, \dots, 8\}$ for a monetary expansion when the NIBOR is at 1.1%. The positive estimates mean that additional years of relationship at the moment of the shock reduce tangible capital growth following a policy rate cut. The figure shows an inverted u-shaped wedge path. Initially, investment in tangible capital reacts independently from the length of the relationship firms maintain with their banks. A wedge then builds over time and peaks four years after the monetary policy shock before disappearing. Each year of existing relationship at the moment of the policy rate cut reduces cumulated tangible capital growth over the next four years by 0.25 percentage points.

These results suggest that the lower pass-through of policy rate cuts to long-relationship firms' lending rates, associated with lower loan growth rates, translate into real effects through investment in tangible capital.

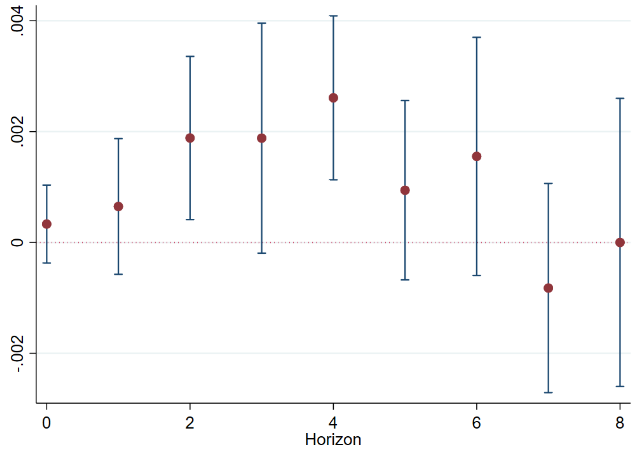
3.2 Robustness Analysis

In this section, we conduct further robustness checks to ensure that our results are robust to different measurements of monetary policy shocks as well as more detailed, account-level information.

3.2.1 Exogenous Monetary Policy Shocks

To address the endogeneity of movements in the NIBOR beyond using different sets of fixed effects, we re-run our entire analyses using identified monetary policy shocks from Brubakk, ter Ellen, Robstad, and Xu (2022). These shocks are identified in a 30-minute window around monetary policy announcements and can be thought of as the unexpected component of a change in policy rate. They are therefore arguably uncorrelated with any unobservables potentially affecting bank-firm loan interest rates.

Figure 7: Relationship Length and Capital Growth



Notes: The figure shows the marginal effects of relationship length on the marginal effect of MP (change in NIBOR) on tangible capital growth rates $\frac{\partial^2 g_{k,t+h}}{\partial \epsilon^m_{t-1} \partial lengtht-1}$ at horizons $h \in \{0, \ldots, 8\}$ for a monetary policy easing when the initial NIBOR is 1.1%. Marginal effects are calculated from the coefficient estimates of regression (10).

Section 6.1 of the Appendix contains the tables showing our results using these shocks. Our conclusions remain qualitatively unchanged.

3.2.2 Account-Level Information

Our analysis thus far has used bank-firm level data. Here, we leverage the high granularity of our dataset, specifically its inclusion of bank-firm-account level information. This granular detail enables us to investigate the mechanisms by which banks adjust loan interest rates following changes in the Nibor. Specifically, we examine whether interest rate pass-through occurs predominantly via new loans or existing ones. To explore this, we re-estimate regressions (4)-(7) in two ways: (i) measuring interest rates based only on new accounts, and (ii) conducting the regressions at the bank-firm-account level.

Although our dataset does not provide loan-level information to directly identify the age composition of bank loan portfolios, new accounts are typically associated with new loans. Thus, restricting interest rate measurements to newly opened accounts approximates an analysis of new loans. Additionally, conducting regressions at the bank-firm-account level rather than the bank-firm level isolates pass-through for accounts that existed in the previous period. To the extent that new loans are not added to existing accounts (though this cannot be entirely ruled out), this approach identifies interest rate pass-through for existing loans, highlighting the role of floating-rate adjustments.

The results of these analyses are presented in Tables 11 and 12 in Section 6.2 of the Appendix. Table 11 reports regressions (4)-(7) results using only new account data, while Table 12 shows results based on account-level interest rate changes. Table 11 indicates that the lower pass-through of rate reductions to long-relationship firms in a low-rate environment, as observed in our main analysis, does not hold for new accounts. This finding suggests, and is corroborated by Table 12, that the lower pass-through to long-relationship firms occurs primarily at the intensive margin—that is, through existing loans and the floating-rate mechanism. By contrast, the higher pass-through of rate increases to long-relationship firms in the low-rate environment is evident for both new and existing accounts.

It is worth noting that, in Table 11, the coefficients used to compute the marginal effect of relationship length on pass-through under policy easing have the same signs as in the main analysis. Their statistical insignificance could reflect reduced sample size when focusing solely on new accounts. Nevertheless, our findings suggest that floating interest rates are the key mechanism through which banks differentiate pass-through to firms of varying relationship lengths following changes in Nibor, with differential adjustments occurring primarily in the context of existing loans.

3.3 Extensive Margin Analysis

Our empirical analysis shows that firms that have maintained a relatively long relationship with their bank get less pass-through of a monetary policy rate cut at initial low rate. In this section, we explore the extensive margin of relationship lending, namely the conditions that switching firms obtain at their new bank and how they relate to the length of their previous relationship, to shed light on the potential mechanism driving our within-relationship pass-through results.

The literature on relationship lending has identified two main mechanisms explaining why firms typically get locked in relationships with their bank: information asymmetry and switching costs. First, inside banks typically accumulate private information on the quality of their borrowers over time and can exploit this informational advantage over outside banks by charging higher rates than would prevail under perfect information. If inside banks can extract rent from a relationship, they will price loans to ensure its continuity. Second, costs incurred by firms when switching banks can allow their inside bank to charge relatively high rates without inducing a switch. Each one of these mechanisms can potentially explain why firms in relatively long relationships get less pass-through of a rate cut at initial low rate.

To see how information asymmetry can potentially explain our empirical findings on pass-through, consider a loan market where borrowers' quality (e.g. credit risk) is private information and revealed to inside banks through repeated interactions ("learning by lending"). In such a market, outside banks can infer a borrower's expected quality from the observed relationship length it currently has with its inside bank. In other words, relationship length sends a signal about the quality of a borrower. Everything else equal, the spread between the rates long- and short-relationship firms obtain when switching to an outside bank reflects their difference in average quality. In this framework, if a policy rate cut correlates with a relative decline in long-relationship borrowers' quality, resulting in the rate they can obtain at an outside bank falling by less than that of short-relationship borrowers, inside banks can pass less of this cut to their long-relationship customers without inducing a switch.

Alternatively, if following a policy rate cut, long-relationship firms' switching costs increase relative to those of short relationships, inside banks can pass less of the cut to their long-relationship borrowers. A negative correlation between long-relationship borrowers' relative switching costs and the policy rate is however not needed to induce less pass-through to long relationships. As detailed in the following theoretical section of the paper, the mere presence of heterogeneous switching costs that are uncorrelated with the policy rate but whose levels depend on the length of the relationship, coupled with bank capital regulation, results in less pass-through to long-relationship borrowers at initial low rate.

The information asymmetry and switching costs mechanisms can both explain our within-relationship pass-through results, but they have different implications for the switchers' discounts and rates we should observe, and how they relate to relationship length. In the case of information asymmetry, we should observe a significant spread between the rates obtained by long- and short-relationship firms at the outside bank, reflecting their difference in expected quality. Moreover, this spread should be decreasing in the level of the policy rate in the low-rate environment, reflecting the relative worsening of long-relationship borrowers' quality. In the case of switching costs, we should observe a significant correlation between switchers' discounts (to the extent they provide an approximation of switching costs) and the length of their previous relationship.

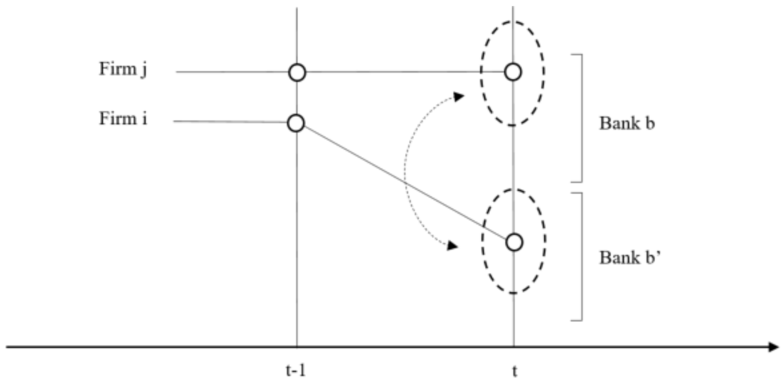
We follow the matching methodology from Ioannidou and Ongena (2010) to estimate the discounts that firms obtain when switching banks. We then relate these discounts (and rates obtained at the outside banks) to the length of their previous relationship in order to determine which of information asymmetry or switching costs is the privileged explanation of our within-relationship pass-through results.

As explained in Section 2, the dataset we use in our within-relationship pass-through analysis does not provide information on contractual interest rates. Instead, we obtain interest rate estimates by dividing annual interest payments by the annual average stock of loans. Since we don't know on which day of the year loans are issued, this results in a particularly noisy estimate for the first year of a relationship. However, to calculate switching discounts, having precise measures of loan interest rates for the first year is key. In the following analysis, we therefore use the relatively new credit register of Norges Bank (ENGA), which only covers the years 2015-2019, but reports contractual loan interest rates.

We start by matching switching firms with non-switching firms. Figure (8) illustrates the procedure. Suppose firm i borrows from bank b until year $t-1$ and switches to bank b' in year t, where it pays the interest rate $r_{ib't}$. We find a comparable non-switching firm j, which borrows from bank b in both $t-1$ and t. We interpret the rate r_{jbt} paid by firm j to bank b in t as the counterfactual rate firm i would have paid in t if it had stayed with bank b. The discount $s_{ibb't}$ firm i obtains by switching from bank b to bank b' in t is therefore estimated to be:

$$s_{ibb't} = r_{jbt} - r_{ib't} \tag{11}$$

Figure 8: Matching switching with non-switching firms



We match switching firms with non-switching firms on year, inside (i.e. origin) bank, industry, size, age, leverage, credit rating, and whether the length of relationship with the inside bank is short or long. We use a baseline cutoff of 4 years for the definition of short/long relationships and run the analysis using different cutoffs in the appendix. Matches are exact on year, bank, industry, credit rating, and previous short or long relationship. For size, age, and leverage, we use a $\pm 30\%$ window around the switcher's value. Our dataset contains $47'948$ matched pairs.

Next, we regress the estimated discounts on a dummy indicating whether the switcher's previous relationship was long or short, controlling for the variables we used for matching:

$$s_{ibb't} = \sum_{k=2015}^{2019} \mathcal{I}_k(\beta_k d_{ibb't} + \mathbf{Z}_{ibb't}\gamma_k) + \alpha_{ind} + \alpha_{b'} + \epsilon_{ibb't}, \tag{12}$$

where $s_{ibb't}$ is the discount obtained by firm i when switching from b to b' in t, $\mathbf{Z}_{ibb't}$ are the firm controls used for matching (size, age, leverage, credit rating), α_{ind} are industry fixed effects, $\alpha_{b'}$ are outside bank fixed effects, and:

$$\begin{aligned} \mathcal{I}_k &= 1 \quad \text{if} \quad k = t \\ d_{ibb't} &= 1 \quad \text{if previous relationship of switching firm } i \text{ was long } (\geq 4 \text{ years}). \end{aligned}$$

The matching procedure described above typically assigns more than one matching non-switching firm to each switching firm (29 on average). To account for this multiplicity, we weight the regression by the inverse number of matches for each switching firm and cluster the standard errors at the switching firm level.

Figure 9(b) plots the estimated coefficients β_k. They show the difference in discounts obtained by switching firms, which is due to having a previous long relationship. Our results indicate that firms with a previous long relationship consistently obtain a larger discount than firms with a previous short relationship when switching banks, with this difference ranging from 20 to 40 basis points for most years. Section 6.3 of the appendix contains robustness checks with unweighted regressions and different length cutoffs for the definition of long relationship.

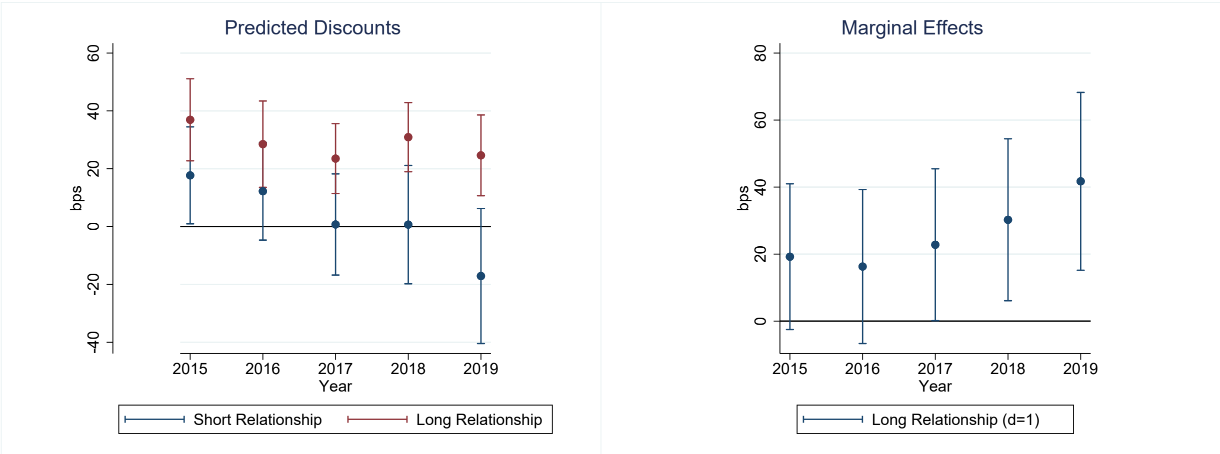
Bank characteristics most likely influence the offers made by the inside and outside banks, and therefore affect the calculated discounts. Following Ioannidou and Ongena (2010), we run regression (12) again, but where switching discounts are now calculated from matching switching firms on the *outside* (i.e. destination) bank (yielding $52'754$ matched pairs):

$$s_{ib't} = r_{jb't} - r_{ib't} \tag{13}$$

That is, we calculate the discount firm i obtains when switching from bank b in $t-1$ to bank b' in t by comparing its new rate $r_{ib't}$ at bank b' in t with the rate $r_{jb't}$ of a comparable existing customer j of bank b' in t. By calculating discounts as a difference in rates made by the same (outside) bank, we ensure that they do not simply reflect unobserved heterogeneity between inside and outside banks.

Figure 10(b) plots the estimated coefficients β_k. The results are comparable to those displayed in Figure (9). Switchers with a previous long relationship obtain larger

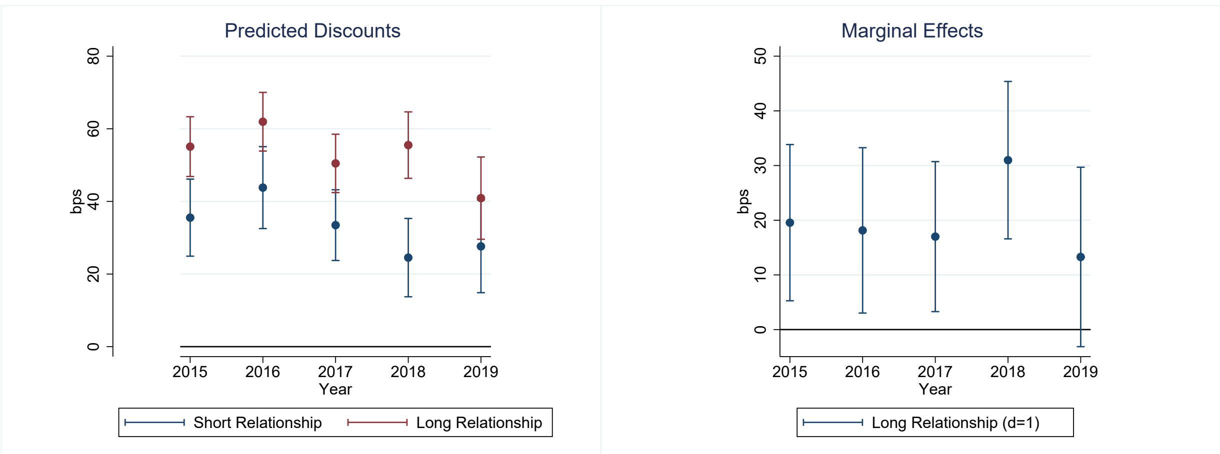
Figure 9: Previous Relationship Length and Switchers' Discounts: Matching on Inside Banks



(a) Predicted discounts **(b)** Relationship length marginal effect

Notes: Figure (a) shows the predicted discounts for switchers with previous short and previous long relationships. Figure (b) shows the estimated coefficients β_k from regression (12). For each year, they show the additional discount obtained by switchers whose previous relationship length is longer than or equal to 4 years.

Figure 10: Previous Relationship Length and Switchers' Discounts: Matching on Outside Banks

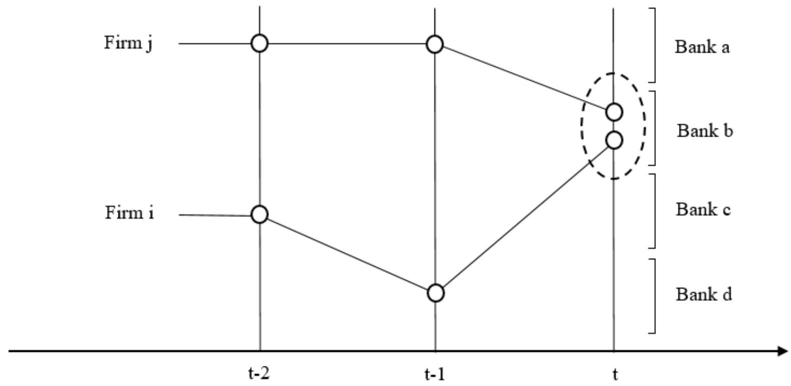


(a) Predicted discounts **(b)** Relationship length marginal effect

Notes: Figure (a) shows the predicted discounts for switchers with previous short and previous long relationships. Figure (b) shows the estimated coefficients β_k from regression (12). For each year, they show the additional discount obtained by switchers whose previous relationship length is longer than or equal to 4 years.

discounts than switchers with a previous short relationship. The difference in discounts is on the same order of magnitude as before, i.e. around 20 basis points. Section 6.3 of the appendix contains robustness checks with unweighted regressions and alternative cutoffs for the definition of long relationships.

Figure 11: Matching among Switching Firms



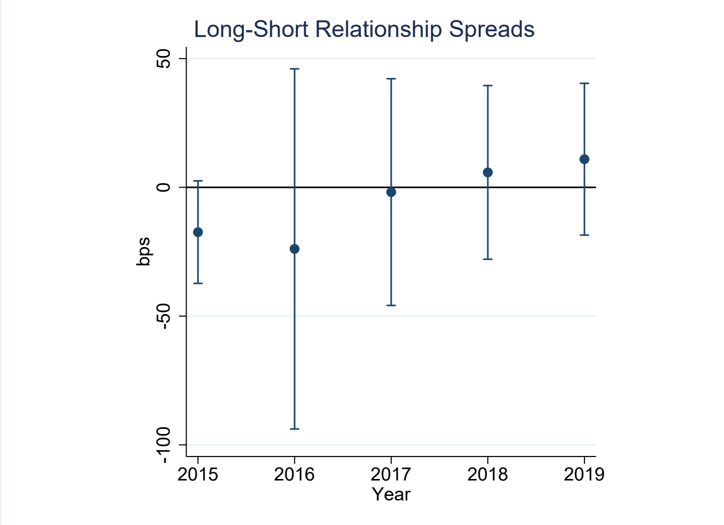
Our matching analysis thus far shows that switchers with a previous long relationship obtain relatively large discounts. To the extent that discounts can be seen as estimates of switching costs, our results suggest that switching costs are increasing in the length of relationship. Next, to investigate the spread between the rates obtained by long- and short-relationship firms after switching to the same outside bank, we match switching firms among themselves. Figure 11 illustrates the procedure. Using our baseline cutoff of 4 years, we split switchers into two groups: firms with a previous long relationship and firms with a previous short relationship. We then create pairs of switching firms matching on year, outside bank, and firm characteristics (industry, size, age, leverage, credit rating). Our dataset contains 683 matched pairs. We calculate the spread s_{ijt} between the loan rates r_{it} and r_{jt} of two similar firms i and j arriving at the same bank b in year t, where the main difference between the firms is that firm i has a previous short relationship with bank d whereas firm j has a previous long relationship with bank a. Next, we regress the spreads on year dummies:

$$s_{ijt} = \sum_{k=2015}^{2019} \mathcal{I}_k \beta_k + \epsilon_{ijt}, \tag{14}$$

where $s_{ijt} = r_{it} - r_{jt}$ and $\mathcal{I}_k = 1$ if $k = t$. Figure 12 plots the estimated coefficients β_k. It shows there is no significant spread between the rates obtained by switchers with a previous short relationship and switchers with a previous long relationship at an outside bank. This suggests that for all three years covered by our credit register data, relationship length does not carry any signal about a borrower's quality. Banks do not seem to price loans differently depending on a switcher's previous relationship length. Section 6.3 of the appendix contains robustness checks with unweighted regressions and different cutoffs for the definition of long relationship.

Overall, our findings indicate that switchers who maintained longer relationships with their former bank receive higher discounts at their new bank. This suggests the

Figure 12: Previous Relationship Length and Switchers' New Rates



Notes: The figure shows the estimated coefficients β_k from regression (14). For each year, they show the spread between the rates secured by switchers with previous short relationships and the rates secured by switchers with previous long relationships is insignificant.

presence of heterogeneous switching costs with firms engaged in longer relationships facing higher switching costs and only choosing to switch when offered relatively substantial discounts. When we match previous short-relationship switchers with previous long-relationship switchers that arrive at the same outside bank, we find no significant difference in loan rates, suggesting the absence of correlation between a borrower's expected quality and the length of its relationship. Based on this evidence, we lean towards the explanation of switching costs in developing a model that rationalizes our differential pass-through results.

4 Theoretical Framework

Our empirical results on differential pass-through imply that the composition of relationship lengths in the economy may play a role in aggregate monetary policy transmission. If we define aggregate monetary policy pass-through as the weighted sum of length-specific within-relationship pass-throughs:

$$\underbrace{\Pi}_{\text{Aggregate Pass-Through}} = \sum_{l=1}^{L} \underbrace{p_l}_{\text{Share}} * \underbrace{\pi_l}_{\text{Within-Relationship Pass-Through}}, \tag{15}$$

where the longest relationship length in the economy is L years, $\{\pi_1, \ldots, \pi_L\}$ are the length-specific within-relationship pass-throughs, and $\{p_1, \ldots, p_L\}$ are the shares of relationship lengths with $\sum_{l=1}^{L} p_l = 1$, it follows that the change in aggregate pass-through resulting from a change in relationship length composition $\{\Delta p_1, \ldots, \Delta p_L\}$ in the economy is:

$$\underbrace{\Delta\Pi}_{\text{Change in Aggregate Pass-Through}} = \underbrace{\sum_{l=1}^{L} \Delta p_l * \pi_l}_{\text{Composition Effect}} \tag{16}$$

Specifically, if pass-through to long-relationship firms is relatively small, an increasing share of long relationships impairs aggregate pass-through via a composition effect. This is an important consideration in view of the evolution of the composition of relationship lengths over the past 20 years in the Norwegian economy. As depicted in Figure 4, the share of long-relationship firms has dramatically increased after the financial crisis, in a period where the central bank considerably lowered its policy rate. The question arises whether monetary easing would have been better transmitted to loan rates, if the relationship length profile of the economy had remained stable. To answer this question, we cannot simply rely, à priori, on our empirical estimates. The reason is that the estimated difference in pass-through between short and long relationships, as well as the pass-through levels themselves, may depend on the observed composition of relationship length in the economy. In other words, the within-relationship pass-throughs in equation (15) may be functions of the shares $\{p_1, \ldots, p_L\}$ such that aggregate pass-through should actually be written:

$$\underbrace{\Pi}_{\text{Aggregate Pass-Through}} = \sum_{l=1}^{L} \underbrace{p_l}_{\text{Share}} * \underbrace{\pi_l(p_1, \ldots, p_L)}_{\text{Within-Relationship Pass-Through}} \tag{17}$$

Our empirical estimates of within-relationship pass-through might therefore not be enough to calculate a counterfactual aggregate pass-through under an alternative distribution of relationship lengths in the economy. Indeed, if aggregate pass-through is defined by equation (17), its change following a change in the composition of relationship length $\{\Delta p_1, \ldots, \Delta p_L\}$ becomes:

$$\underbrace{\Delta\Pi}_{\text{Change in Aggregate Pass-Through}} = \underbrace{\sum_{l=1}^{L} \Delta p_l * \pi_l}_{\text{Composition Effect}} + \underbrace{\sum_{l=1}^{L} p_l * \Delta\pi_l + \sum_{l=1}^{L} \Delta p_l * \Delta\pi_l}_{\text{GE Effects}} \tag{18}$$

If within-relationship pass-throughs depend on the entire distribution of relationship lengths in the economy, the change in aggregate pass-through following a change in this distribution does not only comprise a composition effect but also general equilibrium effects. To provide some intuition, imagine an economy with a $50\% - 50\%$ share of short and long relationships where banks are able to give relatively more pass-through to their short-term customers because they have a pool of long-term customers to whom they can pass relatively less of a policy rate cut. In an economy with short relationships only, and for the same profitability target, banks would need to give their short-term customers less pass-through than in the initial economy, possibly leaving the aggregate pass-through unchanged. To answer questions about counterfactual aggregate pass-through, we therefore need a model allowing the entire composition of relationship length to affect length-specific within-relationship pass-through.

We construct a static (i.e. one-period) banking model, which rationalizes our empirical findings, and allows us to answer counterfactual questions. Agents (firms and banks) make decisions at the beginning of the period, taking the policy rate i as given, to maximize end-of-period payoffs. In what follows, we outline the model setup and characterize the equilibrium conditions. The comparative statics exercise shows how equilibrium prices (i.e. interest rates) would react to an unexpected shock to i at the beginning of the period. We provide a condition under which lower pass-through of a policy rate cut to long-relationship firms obtains when the initial policy rate is low, and study how it relates to the composition of relationship lengths in the economy.

4.1 Firms

There is a continuum of firms of mass 1. Each firm inelastically demands 1 unit of lending to be paid back with interest at the end of the period. Note this means aggregate lending $\mathbf{L}$ is fixed and equal to one. Each firm has a non-negative private cost of switching bank $c_j \geq 0$. Banks cannot observe their clients' individual switching costs.

At the beginning of the period, all firms are exogenously matched with a bank and either are in a *short* or *long* relationship. This can be seen as there being two types of firms in the economy. Although this is a static model, a short relationship between firm j and bank b at the beginning of the period can be thought of as firm j having switched to bank b in the previous (not modeled) period. A long relationship at the beginning of the period can be thought of as the relationship being short or long in the previous (not modeled) period. In this static setting, the initial shares of short vs. long relationships are exogenous. The two types of firms differ in that they draw their switching costs from two different distributions. The switching cost distribution of firms in short relationships is characterized by a density function $f_s(c)$. That of long relationships by $f_l(c)$.

At the beginning of the period, a firm is offered a rate r_s or r_l (depending on its type) by the bank it is currently matched with, and an outside option r_{out} by competing banks. A firm decides to switch to an outside bank if the discount it gets covers its private switching costs: $r_k - r_{out} > c_j$, where $k \in \{s, l\}$.

4.2 Banks

There is a continuum of banks of mass 1. We look for a symmetric equilibrium and therefore consider a representative bank. The bank takes the policy rate i and the rate offered by other competing banks r_{out} as given. The bank doesn't know its clients' private switching costs, but it knows the distributions $f_s(c)$ and $f_l(c)$.

The asset side of the bank balance sheet is made of loans given out to short and long relationship clients, as well as to firms that switch away from their current bank. Given the firms' switching behavior, the loan demands by short-relationship firms $L_s(r_s)$ and long-relationship firms $L_l(r_l)$ faced by the bank are:

$$L_s(r_s) = [1 - F_s(r_s - r_{out})]p_s\mathbf{L}, \tag{19}$$

$$L_l(r_l) = [1 - F_l(r_l - r_{out})]p_l\mathbf{L}, \tag{20}$$

where F_s and F_l respectively are the cumulative density functions of the short- and long-relationship firms' switching costs, p_s and p_l are the shares of short and long relationships in the economy at the beginning of the period with $p_s + p_l = 1$, and $\mathbf{L}$ is the amount of aggregate lending.

The total number of switching firms in the economy and the outside rate r_{out} are taken as given by the bank. If the bank sets $r_{sw} > r_{out}$, it does not attract any switching firm. On the other hand, if the bank sets $r_{sw} < r_{out}$, it can extend as many loans to switchers as its leverage constraint allows. If the bank sets $r_{sw} = r_{out}$, switchers are evenly split across all banks in the economy. In this latter case, denote $\bar{L}_{sw}$ the amount of switchers each bank gets. Note that in our representative bank setting, $\bar{L}_{sw}$ is actually the total number of switchers in the economy. That is, the bank absorbs all switchers by setting $r_{sw} = r_{out}$. The loan demand from switchers the representative bank faces can therefore be written as:

$$L_{sw}(r_{sw}) = \begin{cases} > \bar{L}_{sw} & \text{if } r_{sw} < r_{out} \\ \bar{L}_{sw} & \text{if } r_{sw} = r_{out} \\ 0 & \text{if } r_{sw} > r_{out} \end{cases} \tag{21}$$

In a symmetric equilibrium, the representative bank sets $r_{sw} = r_{out}$ and attracts $\bar{L}_{sw}$, which is the total number of switchers in the economy.

The bank can also invest in some financial assets S at the policy rate i. Since financial assets will always be positive, the policy rate represents the marginal cost of issuing an extra unit of loan. The liability side of the balance sheet consists of equity E and deposits D. Equity E is exogenous and deposits D stem from a constant elasticity deposit supply function, which we take from the literature without explicitly modeling a household side.

$$D(r_d) = \left(\frac{1+r_d}{1+\bar{r}_d}\right)^{-\epsilon^d} \mathbf{D}, \tag{22}$$

where $\bar{r}_d$ is the average deposit rate and $\epsilon^d < -1$ means that banks that pay higher deposit rates attract more deposits.
Aggregate lending $\mathbf{L}$ and deposits $\mathbf{D}$ are fixed. In a symmetric equilibrium, all banks (i.e. the representative bank) set the same loan and deposit rates and hold the aggregate quantities $\mathbf{L}$ of loans and $\mathbf{D}$ of deposits on their balance sheet.

The bank's problem is to choose the rates it offers to short-relationship firms r_s, long-relationship firms r_l, switching firms r_{sw}, depositors r_d, and the amount of financial securities S to maximize its period-two net worth, subject to its balance sheet constraint and a net worth constraint.

The bank's problem can therefore be written:

(23)

$$\max_{r_{sw},r_s,r_l,r_d,S} \quad N = (1+r_{sw})L_{sw}(r_{sw})+(1+r_s)L_s(r_s)+(1+r_l)L_l(r_l)+(1+i)S-(1+r_d)D(r_d)$$

s.t.

$$
\begin{aligned}
L_{sw}(r_{sw}) &= \begin{cases} > \bar{L}_{sw} & \text{if } r_{sw} < r_{out} \\ \bar{L}_{sw} & \text{if } r_{sw} = r_{out} \\ 0 & \text{if } r_{sw} > r_{out} \end{cases} \\
L_s(r_s) &= [1 - F_s(r_s - r_{out})]p_s\mathbf{L} \\
L_l(r_l) &= [1 - F_l(r_l - r_{out})](1 - p_s)\mathbf{L} \\
D(r_d) &= \left(\frac{1+r_d}{1+\bar{r}_d}\right)^{-\epsilon^d}\mathbf{D} \\
L_{sw} + L_s + L_l + S &= E + D && \text{(Balance sheet constraint)} \\
\lambda(L_{sw} + L_s + L_l) &\leq N && \text{(Net worth constraint),}
\end{aligned}
$$

where $1/\lambda$ is the maximum leverage a bank can take.

The first order conditions for r_s, r_l and r_d yield the bank's optimal pricing rule.

Lemma 1. *The optimal loan and deposit rates r_s^*, r_l^*, and r_d^* are implicitly defined by:*

$$r_s^* - i = \frac{1 - F_s(r_s^* - r_{out})}{f_s(r_s^* - r_{out})} + \lambda\frac{\xi}{1+\xi}, \tag{24}$$

$$r_l^* - i = \frac{1 - F_l(r_l^* - r_{out})}{f_l(r_l^* - r_{out})} + \lambda\frac{\xi}{1+\xi}, \tag{25}$$

$$1 + r_d^* = \frac{\epsilon^d}{\epsilon^d - 1}(1+i), \tag{26}$$

where ξ is the Lagrange multiplier on the leverage constraint.

Proof. See Appendix 6.4. □

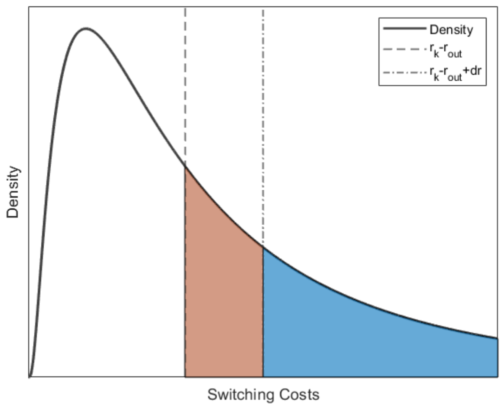
From (24) and (25), we clearly see that the switching cost distributions play a crucial role for optimal rate setting. The bank chooses a markup above the policy rate, which equals the ratio between the survival function and the density function evaluated at the markup above the outside rate. This also means that the spread between the policy rate and the outside rate matters for the optimal markups.
To provide more intuition about Lemma 1 and the bank's first order conditions, consider Figure 13, which traces out a stylized switching costs probability density function. As stated in equations (19)-(20), the corresponding cumulative distribution function represents the loan demand faced by the bank. That is, when charging firms the markup $r_k - r_{out}$ (dashed line) above the outside rate, the bank effectively serves firms with switching costs higher than $r_k - r_{out}$ (red area and blue area) while firms with smaller switching costs take the outside option (white area). If the bank increases the markup by dr to $r_k - r_{out} + dr$ (dash-dotted line), two opposite effects occur. On the one side,

the bank now earns more revenue from firms with relatively high switching costs that remain captive after the markup increase (blue area). This is the inframarginal gain, the left-hand side of equation (27). On the other side, when the markup increases, firms with switching costs higher than $r_k - r_{out}$ but lower than $r_k - r_{out} + dr$ now find it more attractive to take the outside option (red area). This is a loss for the bank, the right-hand side of equation (27). The bank chooses its markups such that the inframarginal gain equals the loss on marginal switchers:

$$\underbrace{dr[1 - F_k(r_k^* - r_{out} + dr)]}_{\text{Inframarginal Gain}} = \underbrace{(r_k^* - i)[F_k(r_k^* - r_{out} + dr) - F_k(r_k^* - r_{out})]}_{\text{Loss on Marginal Switchers}} \tag{27}$$

Equation (27) is equivalent to equations (24)-(25) when $\xi = 0$.

Figure 13: Optimal Rate Setting



Notes: The figure shows a stylized switching costs probability density function. When charging the markup $r_k - r_{out}$, the bank serves firms with switching costs higher than $r_k - r_{out}$ (red area + blue area). When charging the markup $r_k - r_{out} + dr$, the bank serves firms with switching costs higher than $r_k - r_{out} + dr$ (blue area). The bank chooses its markup such that when considering an increase of dr, the inframarginal gain equals the loss on marginal switchers.

4.3 Equilibrium and Comparative Statics

Before we can study the effects of the policy rate i on equilibrium prices, we still need to characterize the equilibrium outside rate r_{out}. We assume that banks compete for switchers à la Bertrand. That is, taking r_{out} as given, an atomistic bank will consider setting its own rate for switchers r_{sw} slightly below r_{out} to attract more switchers than competing banks. To understand until which level r_{out} will be driven down throughout this process, the bank's leverage constraint is crucial. When an atomistic bank sets $r_{sw} = r_{out} - \epsilon$, it attracts all switchers and can potentially replace all of its financial securities S paying rate i by loans to switchers paying the higher rate r_{sw}. If the bank's net worth is high enough such that it can do so without violating its constraint, all banks will proceed the same way (by symmetry), meaning that the original outside rate r_{out} we started with is too high. When banks' net worth is high enough, this process will therefore drive r_{out} down to the policy rate i, where it is not worth for any bank to go lower (i being the marginal cost of issuing an extra unit of loan). However, when banks' net worth is low, r_{out} will be competed down to a level *above* the policy rate i. The reason being that at some level $r_{out} > i$, banks will not be able to set $r_{sw} = r_{out} - \epsilon$ and take on more switchers on their balance sheet without violating the leverage constraint.

Under the assumption that $\mathbf{E} < \mathbf{L} < \mathbf{D}$, the bank's net worth N is increasing in the policy rate since the latter is the rate of return on on securities $S > 0$.[4] Therefore, there exists a threshold interest rate $\bar{i}$ above which net worth N is high enough to drive r_{out} down to i, and below which r_{out} is larger than i. In other words, the equilibrium outside rate is a function of the policy rate:

$$r_{out}(i) = \begin{cases} i & \text{if } i \geq \bar{i} \\ g(i) > i & \text{if } i < \bar{i} \end{cases}, \tag{28}$$

where $g(i)$ is a continuous function with $g(\bar{i}) = i$ and $g'(i) < 1$. $g(i)$ is the highest rate for r_{out} that no individual bank could undercut, thereby substituting additional loans to switchers L_{sw} for financial securities S, without violating its constraint. In other words, $g(i)$ is the highest rate above the policy rate such that the bank's net worth constraint binds.

We are now in a position to study how the representative bank's optimal rates r_s and r_l respond to a change in the policy rate i. That is, we provide an analytical expression for monetary policy pass-through by applying the implicit function theorem on the first order conditions (24) and (25).

Proposition 1. *The monetary policy pass-through to short- and long-relationship firms' loan rates is given by:*

$$\frac{dr_s^*}{di} = \frac{1 + \frac{\partial r_{out}}{\partial i}\left[1 + (r_s^* - i)\frac{f_s'(r_s^* - g(i))}{f_s(r_s^* - g(i))}\right]}{1 + \left[1 + (r_s^* - i)\frac{f_s'(r_s^* - g(i))}{f_s(r_s^* - g(i))}\right]}, \tag{29}$$

$$\frac{dr_l^*}{di} = \frac{1 + \frac{\partial r_{out}}{\partial i}\left[1 + (r_l^* - i)\frac{f_l'(r_l^* - g(i))}{f_l(r_l^* - g(i))}\right]}{1 + \left[1 + (r_l^* - i)\frac{f_l'(r_l^* - g(i))}{f_l(r_l^* - g(i))}\right]}. \tag{30}$$

[4] By the envelope theorem, one can easily see that $\frac{dN}{di} = S(1 + \xi) > 0$.

Proof. See Appendix 6.4. □

From Proposition 1, it follows directly:

Corollary 1. *The interest rate threshold $\bar{i}$ defines two distinct regimes.*

1. *When $i > \bar{i}$ and $\frac{\partial r_{out}}{\partial i} = 1$, it is easy to see that:*

$$\frac{dr_s^*}{di} = \frac{dr_l^*}{di} = 1. \tag{31}$$

   In words, when the policy rate is high enough, the outside rate r_{out} is driven down to i and banks charge a constant markup for both long- and short-relationship firms, implying full pass-through.

2. *When $i < \bar{i}$ and $\frac{\partial r_{out}}{\partial i} = g'(i) < 1$, we have:*

$$\frac{dr_s^*}{di} < 1 \quad , \quad \frac{dr_l^*}{di} < 1, \quad \text{and} \quad \frac{dr_s^*}{di} \neq \frac{dr_l^*}{di} \tag{32}$$

   In words, when the policy rate falls below the threshold $\bar{i}$, the pass-throughs to both short- and long-relationship firms are not complete anymore, and are not the same.

Figure 15 illustrates the two-regime result of Corollary 1. When the policy rate is above the threshold $\bar{i}$ (white area), the policy rate and the outside rate coincide. Banks charge a constant markup above the policy rate for both short-relationship and long-relationship firms, meaning pass-through is full for all customers. When the policy rate falls below the threshold $\bar{i}$ (shaded area) and banks become constrained, the outside rate lies above the policy rate. The markups optimally charged by banks above the policy rate are now decreasing functions of the policy rate, meaning pass-through is incomplete. Furthermore, the pass-through of a policy rate change differs for short- and long-relationship firms. We can derive the following proposition, which establishes the condition under which pass-through is lower for long-relationship firms.

Proposition 2. *Let $i < \bar{i}$ so that $\frac{\partial r_{out}}{\partial i} = g'(i) < 1$. The pass-through of a change in i is lower for long-relationship firms than for short-relationship firms, i.e. $\frac{dr_l^*}{di} < \frac{dr_s^*}{di}$ if:*

$$\frac{r_s^* - i}{r_s^* - g(i)} \epsilon_{f_s|_{r_s^* - g(i)}} < \frac{r_l^* - i}{r_l^* - g(i)} \epsilon_{f_l|_{r_l^* - g(i)}}, \tag{33}$$

where $\epsilon_f|_{x^*} = \frac{f'(x^*)}{f(x^*)} x^*$ is the elasticity of the density function $f(x)$ at $x = x^*$.

Proof. See Appendix 6.4. □

Proposition 2 provides a condition that ensures a lower pass-through to long-relationship firms when banks are constrained if it holds in equilibrium. This condition stipulates that the weighted elasticity of the switching cost density for long-relationship firms f_l must be larger than that of short-relationship firms f_s, at the optimal markups. The weights are defined by the ratios between the optimal markup above the policy rate and the optimal markup above the outside rate. To provide more intuition, consider the case where these elasticities are negative, i.e. $f'(x^*) < 0$. The condition for a lower pass-through to long relationships requires the weighted elasticity of their switching cost density to be smaller than that of short relationships in absolute value term at the optimum. Broadly speaking, this means that for long relationships, there must be a larger mass under the probability density function to the right of the optimal markup compared to short relationships. The reason this leads to a lower pass-through to long-relationship firms is akin to the discussion on the bank's first order conditions following Lemma 1. When this is the case and the policy rate goes down, the bank gives less pass-through (i.e. decreases the markup they charge above the policy rate by relatively less) to long relationships because a relatively high share of these customers is locked in the relationship due to high switching costs. Lowering the markup therefore decreases revenue on this high share of locked in customers (i.e. large inframarginal loss in equation (27)) and only earns a relatively small mass of firms, which decide not to switch because of the lower markup (i.e. small gain on marginal switchers in equation (27)). The reasoning for the higher pass-through to short-relationship firms is the opposite. When the mass to the right of the optimal markup is relatively small, decreasing the markup only modestly affects revenue on locked in customers (i.e. small inframarginal loss), while it can earn a large mass of firms by preventing them from switching (i.e. large gain on marginal swsitchers). In Section 4.4, we provide more intuition on the condition outlined in Proposition 2 by assuming a specific functional form for the switching cost distributions.

From Proposition 1 and Proposition 2, it is clear that the pass-through ($\frac{dr_s^*}{di}$ and $\frac{dr_l^*}{di}$), and whether there is less pass-through to long relationships ($\frac{dr_l^*}{di} < \frac{dr_s^*}{di}$) depend on the outside rate ($r_{out} = g(i)$) and its derivative ($g'(i)$). The latter are determined in equilibrium by the binding bank's constraint, which itself depends on the shares of short and long relationships in the economy (p_s and p_l). Thus, for a given level of the policy rate i, changing the shares p_s and p_l will alter the individual pass-throughs. That is, when modifying the shares p_s and p_l, the change in aggregate pass-through may not only come from a composition effect, but also from the fact that the individual pass-throughs themselves are changing. This consideration highlights why we cannot, à priori, just use our empirical estimates to calculate a counterfactual aggregate pass-through under an alternative composition of relationship length. In the next section, we make an assumption about the functional form of the switching cost distributions, which allows us to assess the importance of this general equilibrium effect.

4.4 Special Case

For general distributions, there is no closed-form solution for the bank's optimal choices and equilibrium outside rate, and the model must be solved numerically. In this section, we look at the special case where switching costs follow a generalized Pareto distribution (GPD). In this case, we can derive analytical solutions for the optimal rates and pass-throughs. This allows to get a better intuition for Proposition 2, and explicitly calculate counterfactual aggregate pass-through under an alternative distribution of relationship length in the economy.

Recall that the pdf and cdf of a random variable following a generalized Pareto distribution with location parameter μ, scale parameter σ, and shape parameter ξ are respectively given by:

$$f(x) = \frac{1}{\sigma}\left(1+\xi\frac{x-\mu}{\sigma}\right)^{-\left(\frac{1}{\xi}+1\right)} \tag{34}$$

$$F(x) = 1-\left(1+\xi\frac{x-\mu}{\sigma}\right)^{-\frac{1}{\xi}} \tag{35}$$

Lemma 2. *Let $f_s \sim GPD(\mu_s, \sigma_s, \xi_s)$ and $f_l \sim GPD(\mu_l, \sigma_l, \xi_l)$. The optimal loan rates r_s^* and r_l^* are given by:*

$$r_s^* = \frac{i+\sigma_s-\xi_s(r_{out}+\mu_s)}{1-\xi_s}, \tag{36}$$

$$r_l^* = \frac{i+\sigma_l-\xi_l(r_{out}+\mu_l)}{1-\xi_l}. \tag{37}$$

Proof. See Appendix 6.4. □

Next, we can explicitly derive the policy rate cutoff $\bar{i}$, below which the bank's constraint becomes binding, the outside rate is above the policy rate, and the pass-throughs to short and long relationships differ.

Proposition 3. *Let $f_s \sim GPD(\mu_s, \sigma_s, \xi_s)$ and $f_l \sim GPD(\mu_l, \sigma_l, \xi_l)$. The threshold $\bar{i}$ below which the bank's net worth constraint binds and $\frac{\partial r_{out}}{\partial i} < 1$ is given by:*

$$\bar{i} = \frac{\lambda L - E - \tau_s - \tau_l}{1+S}, \tag{38}$$

where:

$$\begin{aligned}
\tau_s &= p_s \left(\frac{\sigma_s - \xi_s \mu_s}{1-\xi_s}\right)\left(1 + \frac{\xi_s}{1-\xi_s}\frac{\sigma_s - \mu_s}{\sigma_s}\right)^{-\frac{1}{\xi_s}}, \\
\tau_l &= p_l \left(\frac{\sigma_l - \xi_l \mu_l}{1-\xi_l}\right)\left(1 + \frac{\xi_l}{1-\xi_l}\frac{\sigma_l - \mu_l}{\sigma_l}\right)^{-\frac{1}{\xi_l}}.
\end{aligned}$$

Proof. See Appendix 6.4. □

Note that the threshold $\bar{i}$ increases with the capital requirement λ and decreases with equity E and securities S. The composition of relationship length also affect the threshold through τ_s and τ_l.

The GPD assumption also allows to solve explicitly for the derivative of the outside rate with respect to the policy rate.

Proposition 4. *Let $f_s \sim GPD(\mu_s, \sigma_s, \xi_s)$ and $f_l \sim GPD(\mu_l, \sigma_l, \xi_l)$. The monetary policy pass-through to the outside rate r_{out} when $i < \bar{i}$ is given by:*

$$\frac{\partial r_{out}}{\partial i} = 1 - \frac{L+S}{L\left(1 - (r_{out} - i)\left[\frac{p_s}{\sigma_s}\kappa_s^{-\frac{1}{\xi_s}-1} + \frac{p_l}{\sigma_l}\kappa_l^{-\frac{1}{\xi_l}-1}\right]\right)}, \tag{39}$$

where:

$$\begin{aligned}
\kappa_s &= 1 + \frac{\xi_s}{\sigma_s(1-\xi_s)}(\sigma_s - \mu_s - (r_{out} - i)), \\
\kappa_l &= 1 + \frac{\xi_l}{\sigma_l(1-\xi_l)}(\sigma_l - \mu_l - (r_{out} - i)).
\end{aligned}$$

Proof. See Appendix 6.4. □

The result from Lemma 2 can then be used to obtain the pass-throughs to long and short relationships.

Proposition 5. *Let $f_s \sim GPD(\mu_s, \sigma_s, \xi_s)$ and $f_l \sim GPD(\mu_l, \sigma_l, \xi_l)$. The monetary policy pass-throughs to short- and long-relationship firms' loan rates are given by:*

$$\frac{dr_s^*}{di} = \frac{1 - \xi_s \frac{\partial r_{out}}{\partial i}}{1-\xi_s}, \tag{40}$$

$$\frac{dr_l^*}{di} = \frac{1 - \xi_l \frac{\partial r_{out}}{\partial i}}{1-\xi_l}. \tag{41}$$

Proof. See Appendix 6.4. □

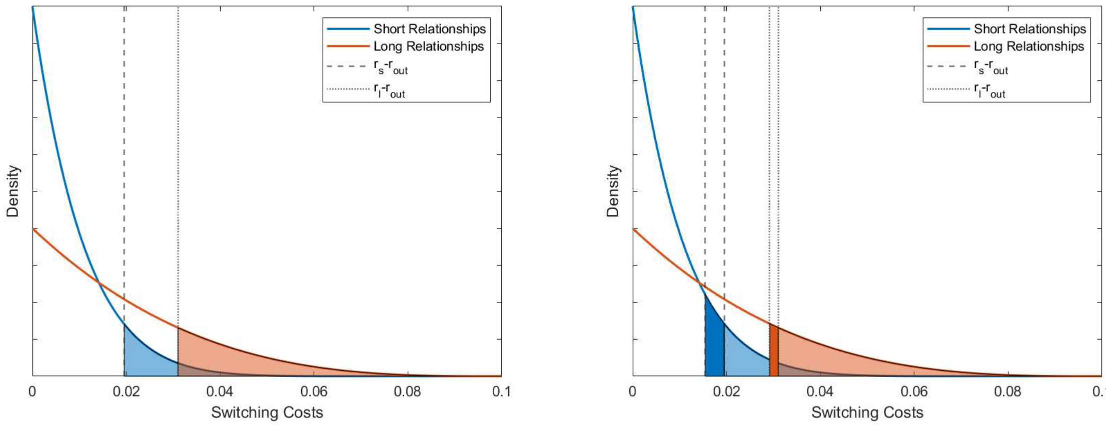
Finally, we easily obtain the equivalent of Proposition 2 in the case of generalized Pareto distributed switching costs.

Proposition 6. *Let $f_s \sim GPD(\mu_s, \sigma_s, \xi_s)$ and $f_l \sim GPD(\mu_l, \sigma_l, \xi_l)$. Let $i < \bar{i}$ so that $\frac{\partial r_{out}}{\partial i} = g'(i) < 1$. The pass-through of a change in i is lower for long-relationship firms than for short-relationship firms, i.e. $\frac{dr_l^*}{di} < \frac{dr_s^*}{di}$ if:*

$$\xi_s > \xi_l. \tag{42}$$

Proof. See Appendix 6.4. □

Figure 14: Generalized Pareto Distribution - Stylized Example



(a) Optimal rate setting given policy rate i_0

(b) Optimal rate setting given $i_1 < i_0$ with constrained bank

Notes: The figures show two probability density functions of generalized Pareto distributions with shape parameters $\xi_s = -0.1$ (short relationships) and $\xi_l = -0.25$ (long relationships). In Figure (a), the vertical dashed lines show the corresponding optimal markups above the outside rate r_{out} charged to short-relationship and long-relationship firms by the bank for a given policy rate i_0. Figure (b) shows the optimal markups for a lower policy rate $i_1 < i_0$. The lower shape parameter of the switching cost distribution for long-relationship firms implies a relatively low pass-through of a policy rate cut when the bank is constrained.

Proposition 6 states that in the case switching costs are generalized Pareto distributed, whether long relationships get less pass-through than short relationships only depends on the shape parameters of the distributions. For illustration, Figure 14 (a) shows the densities of two generalized Pareto distributed random variables with the associated optimal markups chosen by the bank for an arbitrary level of the policy rate i_0 and outside rate r_{out}. The scale parameters have been chosen so that the supports of the two densities are the same. The density with the higher shape parameter (short relationships) has a smaller mass in the tail to the right of the optimal markup than the density with the lower shape parameter (long relationships). Recall that the mass to the right of the optimal markup is locked-in with the bank since switching costs are high. The mass to the left of the optimal markup switches since the discount thus obtained more than covers the switching costs. Figure 14 (b) illustrates a policy rate cut. When the policy rate decreases and constrained banks decrease the markup they charge their customers, the two opposite effects already discussed after Lemma 1 and Proposition 2 take place. On the one hand, decreasing the markup decreases the amount of switchers. Banks thus serve a larger mass of customers, which increases revenue (gain on marginal switchers; dark shaded areas). On the other hand, decreasing the markup lowers the revenue earned on all customers (inframarginal loss). Banks will decrease markups up to the point where the two effects cancel out and the FOCs in Lemma 2 are satisfied at the new policy rate. On Figure 14 (b), it is clear that it is optimal to decrease the markup by relatively less (i.e. give less pass-through) for long relationships. Since there is more mass in the tail to the right of the initial optimal markup charged to long relationships (light shaded red area > light shaded blue area), decreasing the markup to the same extent as for the short relationships would not prevent as many customers from switching, and it would lower the revenues earned from the higher share of locked-in customers.

We end this section by noting that in this special case, Proposition 6 tells us the composition of relationship length in the economy is irrelevant to whether long relationships get less pass-through than short relationships or not. However, it is clear from Proposition 5 that this composition still matters for the levels of the pass-through through the equilibrium object $g'(i)$. Calculating a counterfactual aggregate pass-through under an alternative distribution of relationship lengths requires taking this general equilibrium effect into account. We tackle this task in the next section.

4.5 Counterfactual Exercise

In Sections 4.3 and 4.4, we highlighted how within-relationship pass-through depends on the composition of relationship length in the economy. In this section, we use the results of the special case from Section 4.4 to calculate a counterfactual aggregate pass-through, taking this equilibrium effect into account. More specifically, we ask how much higher would aggregate pass-through of a policy rate cut have been with a higher share of short relationships, at a time when the policy rate was low. Figure 4 shows the share of relationships that are longer than 6 years increased from 21% in 2006 to 36% in 2017. In 2017, the policy rate was 0.89%, which is below our estimated threshold $\bar{i}$ for differential pass-through (cf. Figure 5). We use our model to get the actual aggregate pass-through in 2017 and estimate the counterfactual aggregate pass-through that would have prevailed if the share of long relationships had remained stable at its 2006 level. We compute the aggregate pass-through as the weighted sum of the within-relationship pass-throughs for short and long relationships, and the pass-through for switchers (dr_{out}/di), where the weights are given by the respective shares of short/long relationships and switchers.

Our main assumption is that the switching cost distributions have remained constant over the entire period of our sample. In other words, we assume that the decline in the share of short relationships observed after the financial crisis is not due to any change in the switching cost distributions. It is rather explained by factors that are exogenous to our model and unrelated to switching costs, like a decline in firm entry during the crisis for example. We then use moments from our data and the equations of the model to back out the implied parameters of the switching cost distributions. With these parameters at hand, we solve the model using a counterfactual share of short relationships. We thus obtain within-relationship pass-throughs, which take equilibrium effects into account and allow us to calculate a counterfactual aggregate pass-through.

We use 7 equations for 7 unknowns. 6 of these equations involve data moments from 2017: the optimal markup equations (36)-(37), the pass-through equations (40)-(41), the binding net worth constraint from the bank's problem, and the share of switching firms at the optimum. The seventh equation targets the spread between the interest rates paid by short and long relationships when the policy rate is above the threshold $\bar{i}$. Panels A and B of Table 7 show the data input used in these equations. Panel A shows the targeted moments from the data, which are endogenous in the model. Panel B shows the exogenous parameters of the model. All the parameters come from the data, except for the deposit supply elasticity (taken from the literature) and the capital requirement parameter. The latter is set to a slightly higher level than Basel requirements to ensure the constraint binds at low policy rates. This is necessary in our framework, since with fixed aggregate quantities, the decline in the policy rate only brings the bank closer to the constraint through its earnings on securities S and not through an increase in aggregate loans L. The 7 unknowns are the 3 parameters of the generalized Pareto distribution for short and long relationships, and the outside rate in 2017. Panel C of Table 7 shows the estimates.

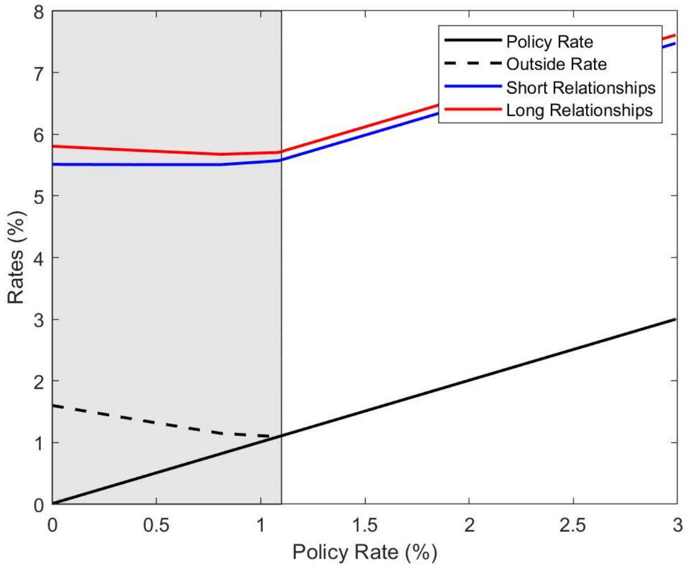
Table 7: Calibration

	A. Targeted Moments	
Short Relationship Markup (2017)	$r_s^* - i$	4.66%
Long Relationship Markup (2017)	$r_l^* - i$	4.84%
Short Relationship Pass-Through (2017)	dr_s^*/di	0.22
Long Relationship Pass-Through (2017)	dr_l^*/di	0.1
Proportion of Switchers (2017)	L_{sw}/L	0.1
Interest Rate Spread (2012-2015)	$r_l^* - r_s^*$	0.12%
	B. Exogenous Parameters	
Policy Rate (2017)	i	0.89%
Share of Short Relationships (2017)	p_s	0.64
Share of Long Relationships (2017)	p_l	0.36
Bank Equity/Total Assets (2000-2019)	$E/(L+S)$	8.22%
Bank Securities/Total Assets (2000-2019)	$S/(L+S)$	15.5%
Capital Requirements	λ	0.15
Deposit Supply Elasticity	ϵ	-10
	C. Estimates	
Short Relationship Sw. Cost Distribution - Location	μ_s	0.04
Short Relationship Sw. Cost Distribution - Scale	σ_s	0.05
Short Relationship Sw. Cost Distribution - Shape	ξ_s	-1.85
Long Relationship Sw. Cost Distribution - Location	μ_l	0.02
Long Relationship Sw. Cost Distribution - Scale	σ_l	0.13
Long Relationship Sw. Cost Distribution - Shape	ξ_l	-3
Outside rate (2017)	r_{out}	1.11%

Notes: The table shows the moments of the data we target, the exogenous parameters of the model, and the estimates.

We use these estimates to solve the model for any share of short/long relationships in the economy. Figure 15 shows the optimal rates and equilibrium outside rates for the shares that were prevailing in 2017. We calculate the aggregate pass-through to be 0.146. We then re-solve the model using the short/long relationships shares from 2006. We calculate a counterfactual pass-through of 0.179, that is 23% higher. As shown in equation (18), we can decompose the change in aggregate pass-through into a composition effect and general equilibrium effects. In the present case, we find:

Figure 15: Policy Rate, Outside Rate, Optimal Rates, and Pass-Through



Notes: The figure shows the equilibrium outside rate and optimal rates charged to short- and long-relationship firms for different levels of the policy rate and for the relationship length distribution of 2017. The figure illustrates the two regimes: equal and full pass-through when $i > \bar{i} \approx 1.1\%$ and heterogeneous, incomplete pass-through when $i < \bar{i}$.

$$\underbrace{\Delta\Pi}_{\text{Change in Aggregate Pass-Through}} = \underbrace{\sum_{l\in\{s,l,sw\}} \Delta p_l * \pi_l}_{\text{Composition Effect: } +23\%} + \underbrace{\sum_{l\in\{s,l,sw\}} p_l * \Delta\pi_l + \sum_{l\in\{s,l,sw\}} \Delta p_l * \Delta\pi_l}_{\text{GE Effects } \approx 0}, \tag{43}$$

where the 3 relationship lengths $\{s, l, sw\}$ stand for short, long, and switcher respectively.

Interestingly, all of the change is coming from the composition effect. Indeed, the equilibrium effect on the within-relationship pass-throughs going through the outside rate is negligible. The within-relationship pass-throughs remain virtually unchanged for a different composition of short and long relationships in the economy.

The fact that the change in aggregate monetary policy pass-through resulting from a shift in the composition of relationship lengths in the economy is entirely determined by a composition effect carries two important implications.

Firstly, it indicates that when the within-relationship pass-throughs differ for various relationship lengths, any alteration in the composition of relationship lengths will inevitably affect aggregate pass-through. This is because compositional effects are not counteracted by equilibrium effects. This underscores the significance of the distribution of relationship lengths as a state variable that should be considered by central bankers when designing policy, necessitating ongoing tracking of its evolution.

Secondly, as changes in the composition of relationship lengths do not impact within-relationship pass-through, empirical estimates of within-relationship pass-through for different relationship lengths are sufficient to calculate aggregate pass-through under alternative distributions of relationship lengths.

5 Conclusion

In this paper, we investigate how bank-firm lending relationships shape the monetary policy pass-through to banks' loan rates, particularly how low monetary policy rates modify such a channel. Using Norwegian administrative tax and bank supervisory data spanning over two decades, we are able to track the universe of bank-firm relationships in the economy.

Our analysis shows that when the monetary policy rate is relatively low, firms that have maintained a long-term relationship with their bank experience a lower pass-through of further cuts in policy rates. Specifically, when the policy rate is around 1%, each additional year of relationship length decreases within-relationship pass-through by 2.7 percentage points. This is a substantial effect, given that the average within-relationship monetary policy pass-through at that policy rate level is 9%. We show that this lower pass-through also comes with a lower loan volume increase and lower physical capital growth.

Exploring the extensive margin of relationship lending using a matching procedure, we find that switchers with a previous long relationship obtain higher discounts, consistent with relatively high switching costs.

We propose a banking model to rationalize these findings, where state-dependent differential pass-through results from the presence of firms' switching costs and banks' leverage constraint. Both our empirical results and theoretical model highlight that the composition of relationship lengths in an economy matters for aggregate monetary policy pass-through.

The share of long relationships in the Norwegian economy substantially increased after the global financial crisis. We calibrate the model to our data and use it to calculate a counterfactual aggregate pass-through that would have prevailed in 2017 if the composition of relationship lengths had remained as in 2006.

The model highlights that the entire change in aggregate pass-through following a shift in the composition of relationship lengths can be attributed to a compositional effect. The length-dependent within-relationship pass-throughs remain largely unaffected by the change in composition of relationship lengths. This has strong implications for policymakers as it suggests that the distribution of relationship lengths in the economy is a key determinant of aggregate monetary policy pass-through and should therefore be taken into account as a state variable when designing optimal policy.

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6 Appendix

6.1 Exogenous Monetary Policy Shocks

This section presents the same regression tables as our main empirical analyses from Section 3, but using identified monetary policy shocks instead of changes in the NIBOR. The notes below each table indicate the equivalent table in the main text.

Table 8: Average Pass-Through: Identified MP Shocks

	(1)	(2)	(3)	(4)	(5)	(6)
ϵ^m_{t-1}	0.111	0.889***	1.653***	4.197***	-3.707***	-0.356
	(0.098)	(0.197)	(0.124)	(0.210)	(0.212)	(0.412)
$tight_{t-1} \times \epsilon^m_{t-1}$			-3.565***	-7.568***	0.398	14.452***
			(0.175)	(0.475)	(0.374)	(2.653)
$\epsilon^m_{t-1} \times i_{t-1}$					2.222***	0.969***
					(0.096)	(0.200)
$tight_{t-1} \times \epsilon^m_{t-1} \times i_{t-1}$					-1.889***	-3.400***
					(0.065)	(0.231)
N	937476	763122	937476	763122	937476	763122
Macro Controls	No	Yes	No	Yes	No	Yes
Firm Controls	No	Yes	No	Yes	No	Yes
Bank Controls	No	Yes	No	Yes	No	Yes
Industry-Time FE	No	No	No	No	No	No
ILS-Time FE	No	No	No	No	No	No
Bank-Time FE	No	No	No	No	No	No
Bank-Firm FE	Yes	Yes	Yes	Yes	Yes	Yes

Dually clustered (bank and firm levels) standard errors in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Notes: This table is the equivalent of Table 2 in the main text.

Table 9: Pass-Through and Relationship Length: Identified MP Shocks

	(1)	(2)	(3)	(4)
$length_{ibt} \times \epsilon^m_{t-1}$	-0.963***	-0.395***	-0.426***	-0.476***
	(0.073)	(0.113)	(0.101)	(0.119)
$length_{ibt} \times \epsilon^m_{t-1} \times i_{t-1}$	0.482***	0.185***	0.195***	0.208***
	(0.030)	(0.041)	(0.036)	(0.042)
$tight_{t-1} \times length_{ibt} \times \epsilon^m_{t-1}$	0.513***	0.652***	0.596***	0.673***
	(0.141)	(0.135)	(0.117)	(0.120)
$tight_{t-1} \times length_{ibt} \times \epsilon^m_{t-1} \times i_{t-1}$	-0.383***	-0.244***	-0.231***	-0.241***
	(0.041)	(0.046)	(0.039)	(0.043)
N	937476	937449	703029	865407
Macro Controls	No	No	No	No
Firm Controls	No	No	No	Yes
Bank Controls	No	No	No	No
Industry-Time FE	No	No	No	Yes
ILS-Time FE	No	No	Yes	No
Bank-Time FE	No	Yes	Yes	Yes
Bank-Firm FE	Yes	Yes	Yes	Yes

Dually clustered (bank and firm levels) standard errors in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Notes: This table is the equivalent of Table 4 in the main text.

Table 10: Credit Growth and Relationship Length: Identified MP Shocks

	(1)	(2)	(3)	(4)
$length_{ibt} \times \epsilon^m_{t-1}$	0.057**	0.009	0.033**	-0.004
	(0.021)	(0.010)	(0.010)	(0.009)
$length_{ibt} \times \epsilon^m_{t-1} \times i_{t-1}$	-0.012	-0.000	-0.011*	0.003
	(0.011)	(0.006)	(0.005)	(0.005)
$tight_{t-1} \times length_{ibt} \times \epsilon^m_{t-1}$	-0.010	-0.130***	-0.138***	-0.080***
	(0.027)	(0.012)	(0.013)	(0.014)
$tight_{t-1} \times length_{ibt} \times \epsilon^m_{t-1} \times i_{t-1}$	0.011	0.023***	0.033***	0.013*
	(0.013)	(0.006)	(0.006)	(0.006)
N	968586	968564	703932	873884
Macro Controls	No	No	No	No
Firm Controls	No	No	No	Yes
Bank Controls	No	No	No	No
Industry-Time FE	No	No	No	Yes
ILS-Time FE	No	No	Yes	No
Bank-Time FE	No	Yes	Yes	Yes
Bank-Firm FE	Yes	Yes	Yes	Yes

Dually clustered (bank and firm levels) standard errors in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Notes: This table is the equivalent of Table 6 in the main text.

6.2 Using Account-Level Information

This section presents the same regression tables as our main empirical analyses from Section 3, but using newly issued accounts only. The title of each table indicates whether the regressions were run using changes in the NIBOR or identified monetary policy shocks. The notes below each table indicate the equivalent table in the main text.

Table 11: Pass-Through and Relationship Length: Changes in NIBOR (New accounts)

	(1)	(2)	(3)	(4)
$length_{ibt} \times \epsilon^m_{t-1}$	-0.085	-0.099	-0.032	-0.099
	(0.067)	(0.065)	(0.092)	(0.068)
$length_{ibt} \times \epsilon^m_{t-1} \times i_{t-1}$	0.031	0.033	0.023	0.032
	(0.026)	(0.026)	(0.037)	(0.027)
$tight_{t-1} \times length_{ibt} \times \epsilon^m_{t-1}$	0.525***	0.816***	0.554**	0.779***
	(0.118)	(0.138)	(0.186)	(0.171)
$tight_{t-1} \times length_{ibt} \times \epsilon^m_{t-1} \times i_{t-1}$	-0.150***	-0.220***	-0.173***	-0.212***
	(0.033)	(0.037)	(0.050)	(0.048)
N	60773	60236	24525	56547
Macro Controls	No	No	No	No
Firm Controls	No	No	No	Yes
Bank Controls	No	No	No	No
Industry-Time FE	No	No	No	Yes
ILS-Time FE	No	No	Yes	No
Bank-Time FE	No	Yes	Yes	Yes
Bank-Firm FE	Yes	Yes	Yes	Yes

Dually clustered (bank and firm levels) standard errors in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Notes: This table is the equivalent of Table 4 in the main text.

Table 12: Pass-Through and Relationship Length: Changes in NIBOR (Account level)

	(1)	(2)	(3)	(4)
$length_{ibt} \times \epsilon^m_{t-1}$	-0.069***	-0.067***	-0.075***	-0.069***
	(0.010)	(0.012)	(0.012)	(0.013)
$length_{ibt} \times \epsilon^m_{t-1} \times i_{t-1}$	0.034***	0.032***	0.035***	0.033***
	(0.004)	(0.005)	(0.004)	(0.005)
$tight_{t-1} \times length_{ibt} \times \epsilon^m_{t-1}$	0.049*	0.072	0.039	0.113**
	(0.025)	(0.043)	(0.031)	(0.036)
$tight_{t-1} \times length_{ibt} \times \epsilon^m_{t-1} \times i_{t-1}$	0.000	-0.029*	-0.024**	-0.039***
	(0.007)	(0.011)	(0.008)	(0.010)
N	1190797	1190769	975977	1101832
Macro Controls	No	No	No	No
Firm Controls	No	No	No	Yes
Bank Controls	No	No	No	No
Industry-Time FE	No	No	No	Yes
ILS-Time FE	No	No	Yes	No
Bank-Time FE	No	Yes	Yes	Yes
Bank-Firm FE	Yes	Yes	Yes	Yes

Dually clustered (bank and firm levels) standard errors in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Notes: This table is the equivalent of Table 4 in the main text.

6.3 Extensive Margin: Robustness Analysis

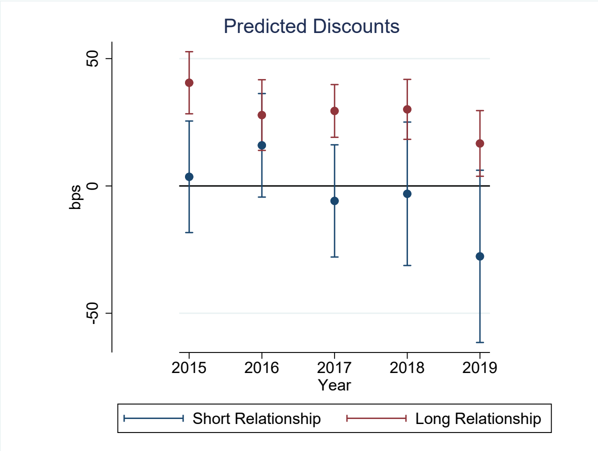
This section presents the results from Section 3.3 with alternative relationship length cutoffs for the definition of short/long relationships. It also presents results from unweighted regressions.

6.3.1 Weighted Regressions with Alternative Relationship Length Cutoffs

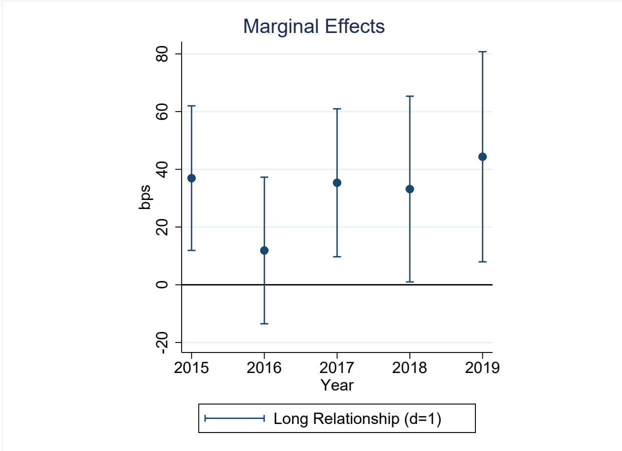
This section shows the results from Figures 9, 10, and 12 in Section 3.3 with cutoffs of 3 and 5 years for the definition of long relationships (instead of 4 in the main text). I.e. in regression (12), the dummy $d = 1$ when relationship length is longer or equal to 3 (respectively 5) years.

3-year cutoff

Figure 16: Previous Relationship Length and Switchers' Discounts: Matching on Inside Banks



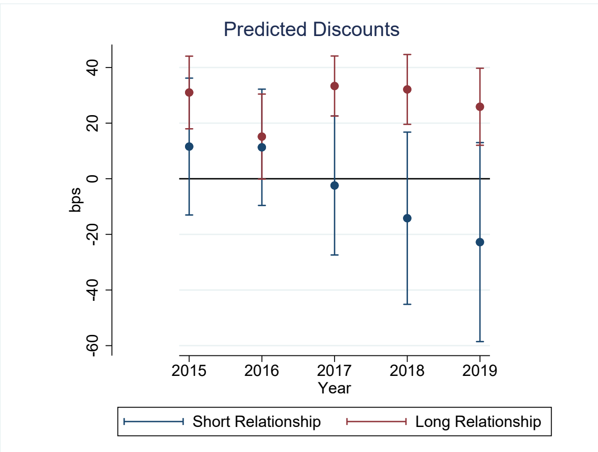
(a) Predicted discounts



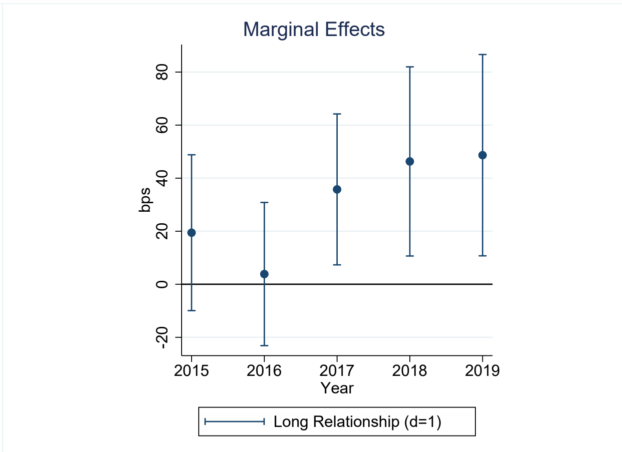
(b) Relationship length marginal effect

Notes: Figure (a) shows the predicted discounts for switchers with previous short and previous long relationships. Figure (b) shows the estimated coefficients β_k from regression (12). For each year, they show the additional discount obtained by switchers whose previous relationship length is longer than or equal to 4 years.

Figure 17: Previous Relationship Length and Switchers' Discounts: Matching on Outside Banks



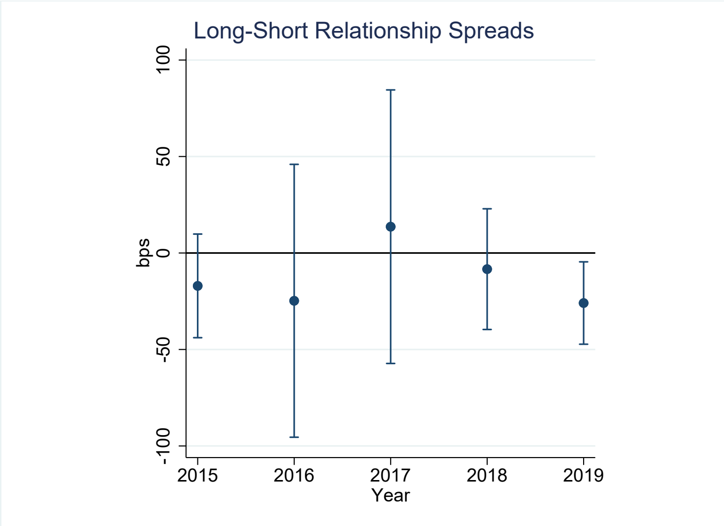
(a) Predicted discounts



(b) Relationship length marginal effect

Notes: Figure (a) shows the predicted discounts for switchers with previous short and previous long relationships. Figure (b) shows the estimated coefficients β_k from regression (12). For each year, they show the additional discount obtained by switchers whose previous relationship length is longer than or equal to 4 years.

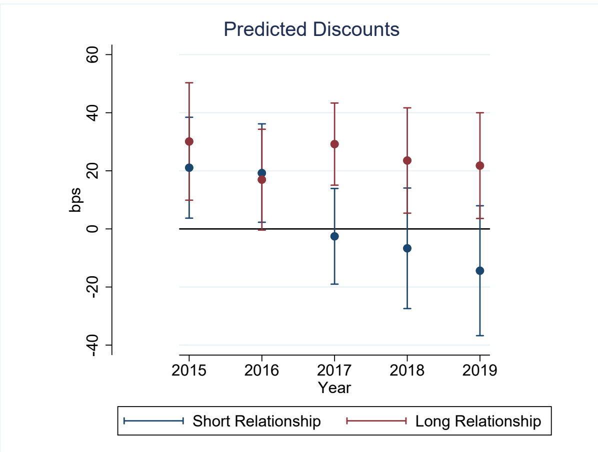
Figure 18: Previous Relationship Length and Switchers' New Rates



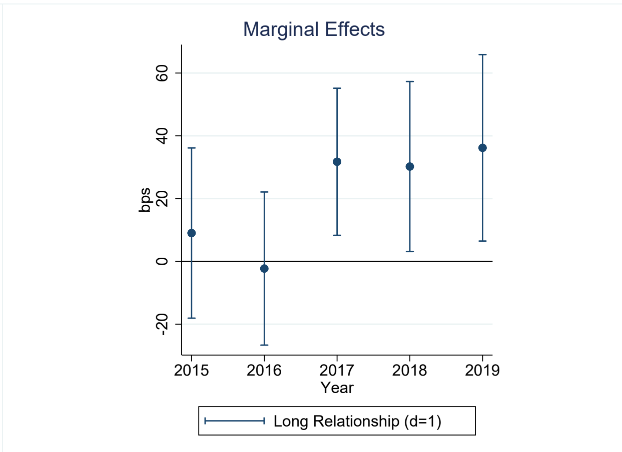
Notes: The figure shows the estimated coefficients β_k from regression (14). For each year, they show the spread between the rates secured by switchers with previous short relationships and the rates secured by switchers with previous long relationships.

5-year cutoff

Figure 19: Previous Relationship Length and Switchers' Discounts: Matching on Inside Banks



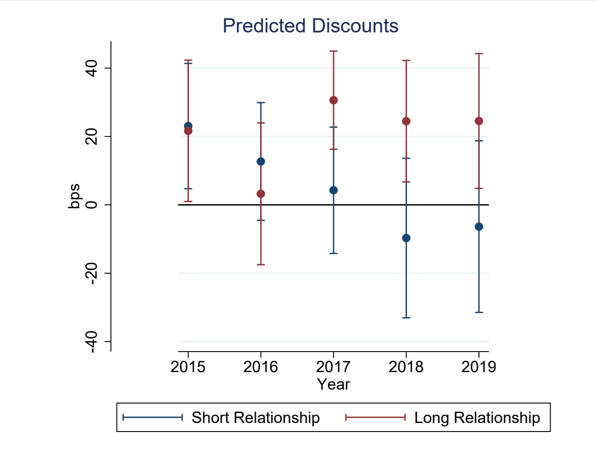
(a) Predicted discounts



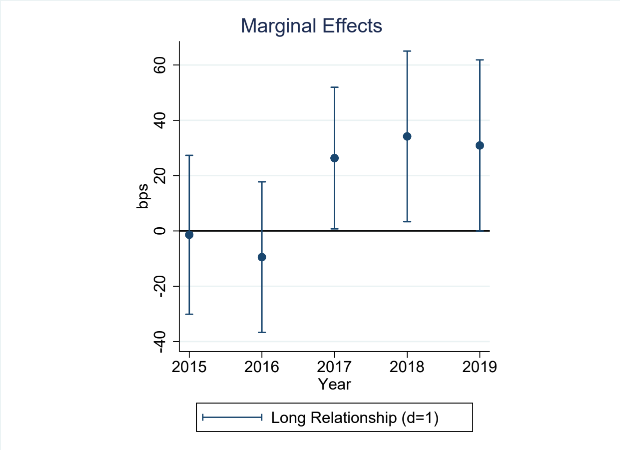
(b) Relationship length marginal effect

Notes: Figure (a) shows the predicted discounts for switchers with previous short and previous long relationships. Figure (b) shows the estimated coefficients β_k from regression (12). For each year, they show the additional discount obtained by switchers whose previous relationship length is longer than or equal to 4 years.

Figure 20: Previous Relationship Length and Switchers' Discounts: Matching on Outside Banks



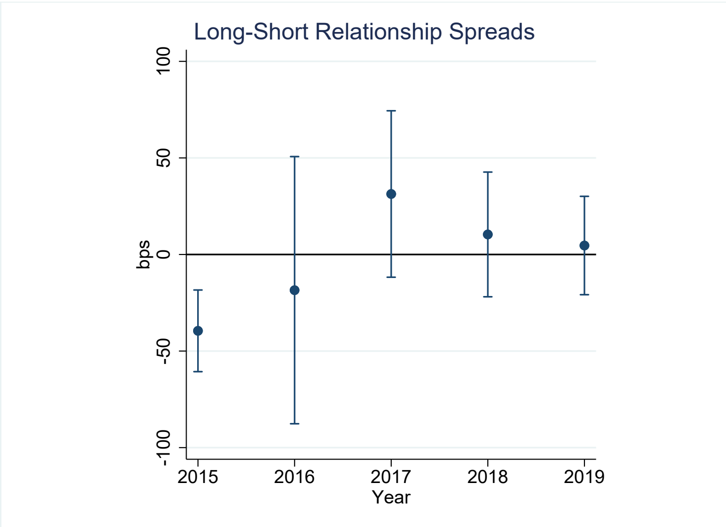
(a) Predicted discounts



(b) Relationship length marginal effect

Notes: Figure (a) shows the predicted discounts for switchers with previous short and previous long relationships. Figure (b) shows the estimated coefficients β_k from regression (12). For each year, they show the additional discount obtained by switchers whose previous relationship length is longer than or equal to 4 years.

Figure 21: Previous Relationship Length and Switchers' New Rates



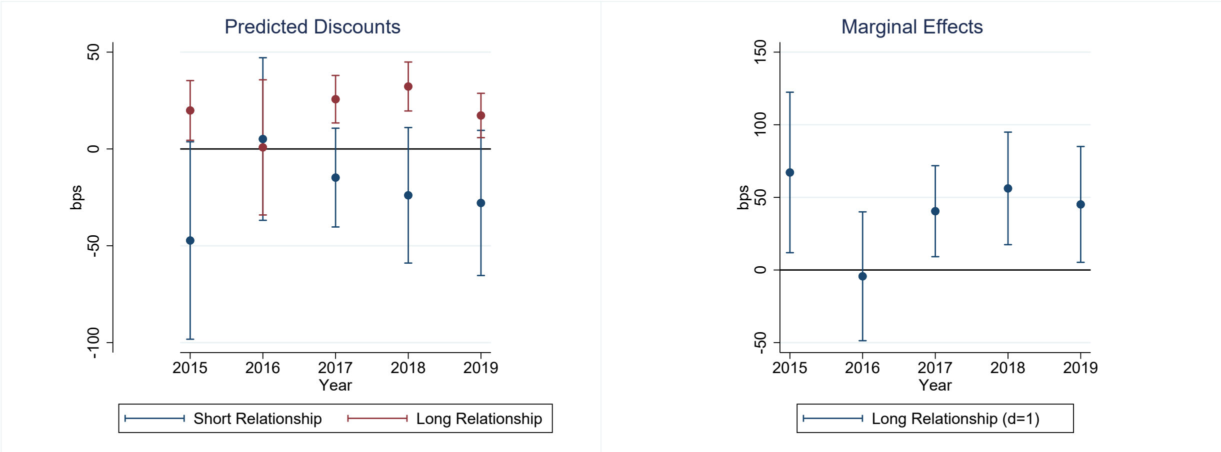
Notes: The figure shows the estimated coefficients β_k from regression (14). For each year, they show the spread between the rates secured by switchers with previous short relationships and the rates secured by switchers with previous long relationships.

6.3.2 Unweighted Regressions with Alternative Relationship Length Cut-offs

This section shows the results from Figures 9, 10, and 12 in Section 3.3 when the associated regressions are *not* weighted by the inverse number of matches. We show the results for cutoffs of 3, 4, and 5 years for the definition of long relationships.

3-year cutoff

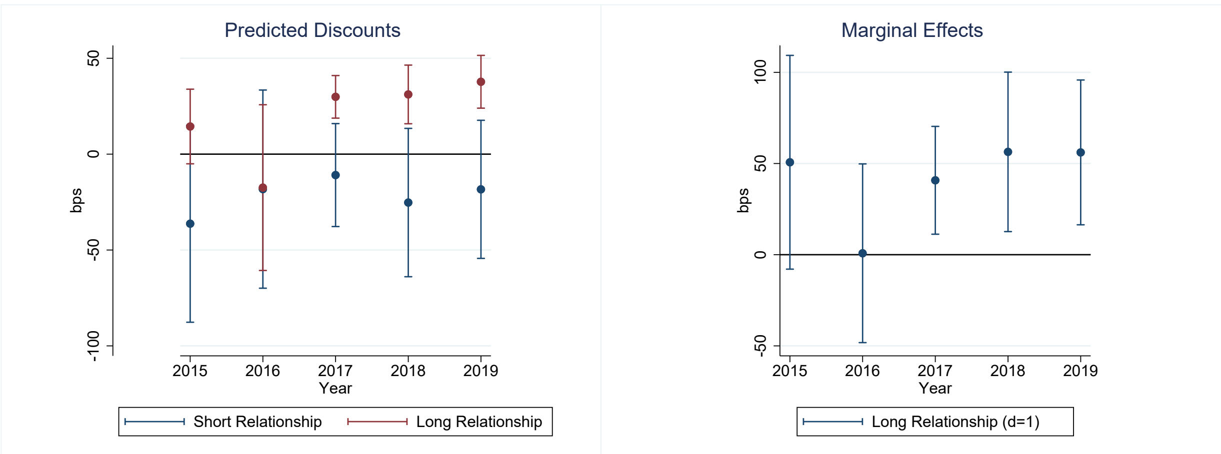
Figure 22: Previous Relationship Length and Switchers' Discounts: Matching on Inside Banks



(a) Predicted discounts **(b)** Relationship length marginal effect

Notes: Figure (a) shows the predicted discounts for switchers with previous short and previous long relationships. Figure (b) shows the estimated coefficients β_k from regression (12). For each year, they show the additional discount obtained by switchers whose previous relationship length is longer than or equal to 4 years.

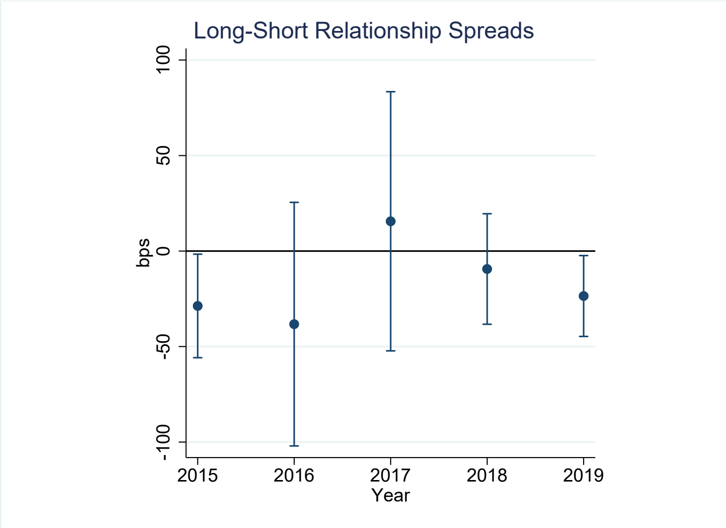
Figure 23: Previous Relationship Length and Switchers' Discounts: Matching on Outside Banks



(a) Predicted discounts **(b)** Relationship length marginal effect

Notes: Figure (a) shows the predicted discounts for switchers with previous short and previous long relationships. Figure (b) shows the estimated coefficients β_k from regression (12). For each year, they show the additional discount obtained by switchers whose previous relationship length is longer than or equal to 4 years.

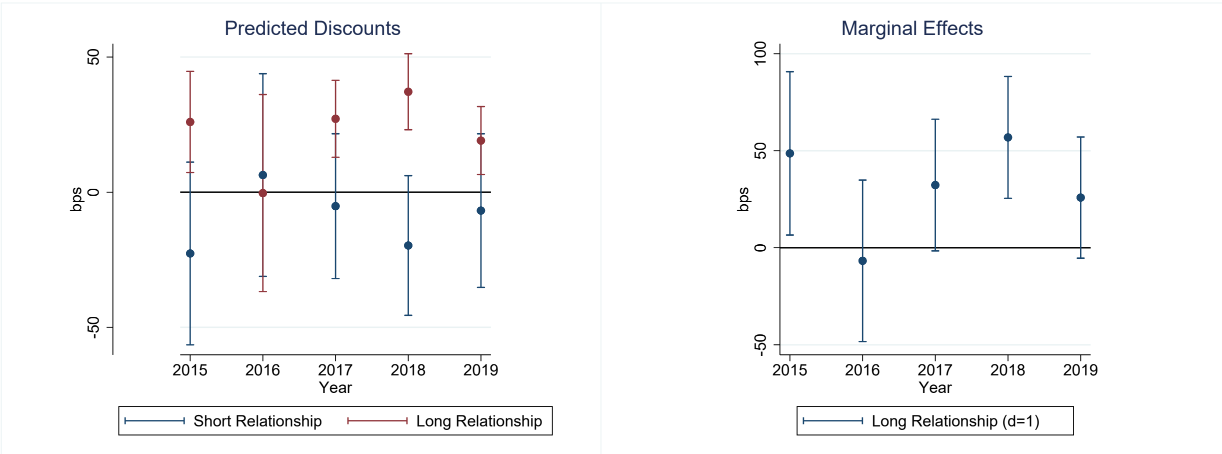
Figure 24: Previous Relationship Length and Switchers' New Rates



Notes: The figure shows the estimated coefficients β_k from regression (14). For each year, they show the spread between the rates secured by switchers with previous short relationships and the rates secured by switchers with previous long relationships.

4-year cutoff

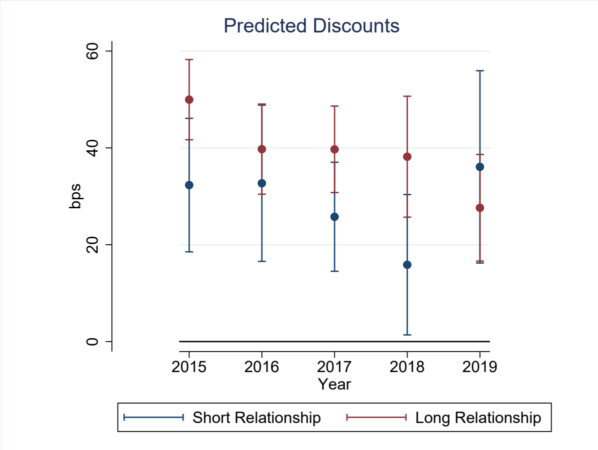
Figure 25: Previous Relationship Length and Switchers' Discounts: Matching on Inside Banks



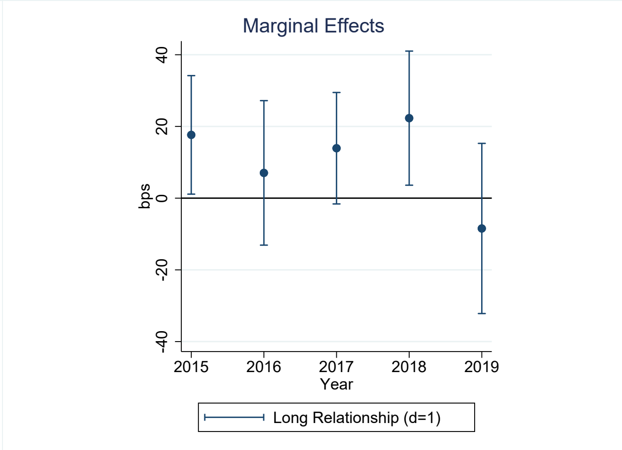
(a) Predicted discounts **(b)** Relationship length marginal effect

Notes: Figure (a) shows the predicted discounts for switchers with previous short and previous long relationships. Figure (b) shows the estimated coefficients β_k from regression (12). For each year, they show the additional discount obtained by switchers whose previous relationship length is longer than or equal to 4 years.

Figure 26: Previous Relationship Length and Switchers' Discounts: Matching on Outside Banks



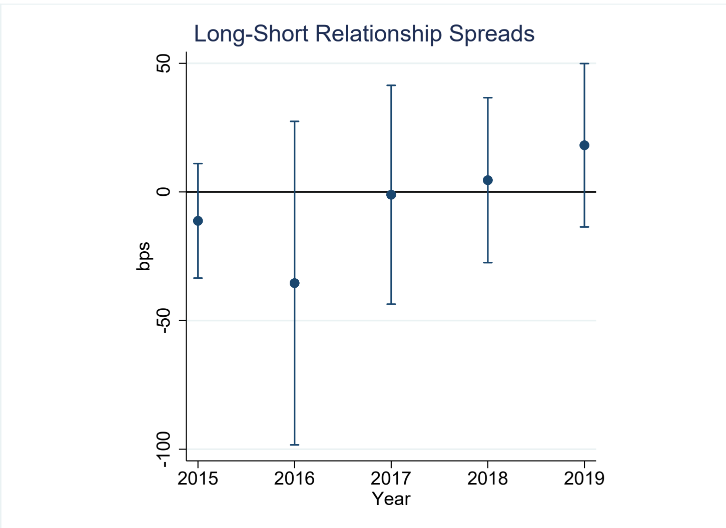
(a) Predicted discounts



(b) Relationship length marginal effect

Notes: Figure (a) shows the predicted discounts for switchers with previous short and previous long relationships. Figure (b) shows the estimated coefficients β_k from regression (12). For each year, they show the additional discount obtained by switchers whose previous relationship length is longer than or equal to 4 years.

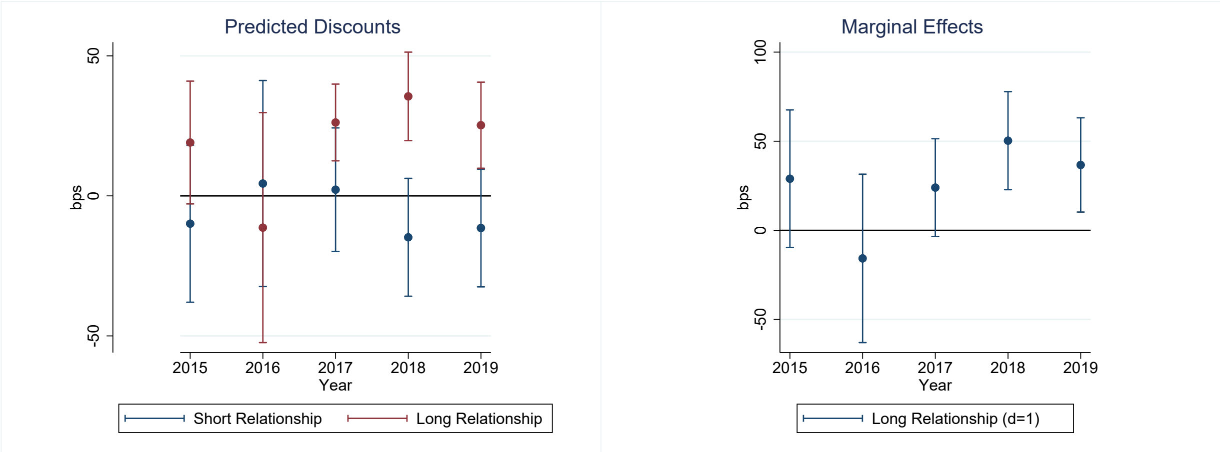
Figure 27: Previous Relationship Length and Switchers' New Rates



Notes: The figure shows the estimated coefficients β_k from regression (14). For each year, they show the spread between the rates secured by switchers with previous short relationships and the rates secured by switchers with previous long relationships.

5-year cutoff

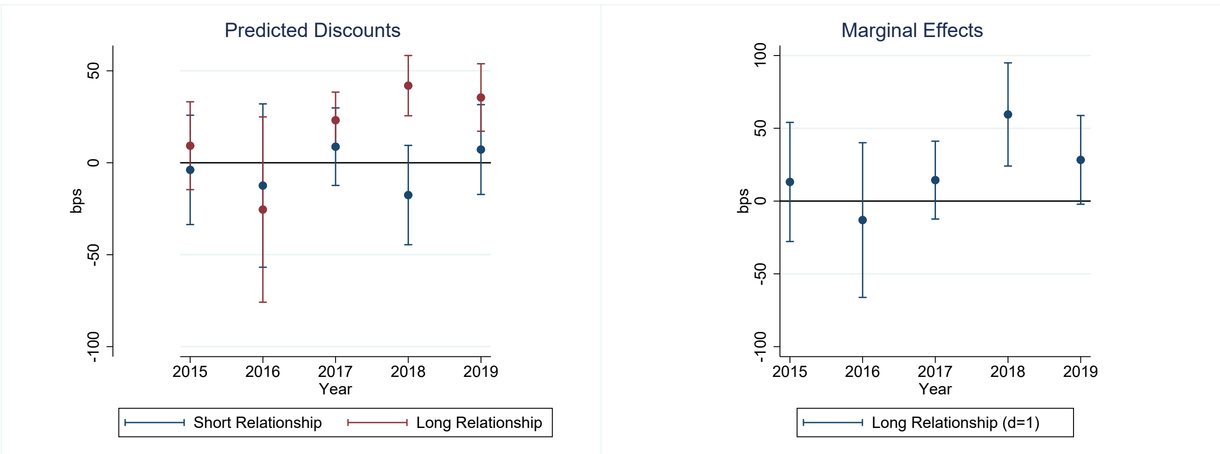
Figure 28: Previous Relationship Length and Switchers' Discounts: Matching on Inside Banks



(a) Predicted discounts **(b)** Relationship length marginal effect

Notes: Figure (a) shows the predicted discounts for switchers with previous short and previous long relationships. Figure (b) shows the estimated coefficients β_k from regression (12). For each year, they show the additional discount obtained by switchers whose previous relationship length is longer than or equal to 4 years.

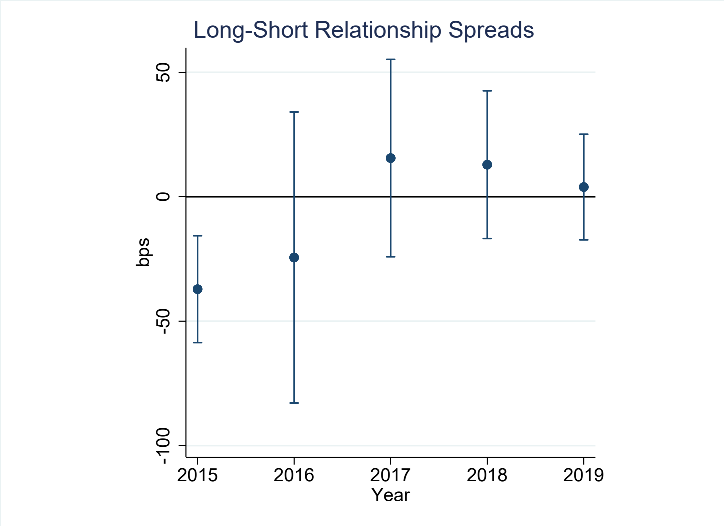
Figure 29: Previous Relationship Length and Switchers' Discounts: Matching on Outside Banks



(a) Predicted discounts **(b)** Relationship length marginal effect

Notes: Figure (a) shows the predicted discounts for switchers with previous short and previous long relationships. Figure (b) shows the estimated coefficients β_k from regression (12). For each year, they show the additional discount obtained by switchers whose previous relationship length is longer than or equal to 4 years.

Figure 30: Previous Relationship Length and Switchers' New Rates



Notes: The figure shows the estimated coefficients β_k from regression (14). For each year, they show the spread between the rates secured by switchers with previous short relationships and the rates secured by switchers with previous long relationships.

6.4 Proofs

Proof of Lemma 1

The bank's problem is:

$$\max_{r_{sw}, r_s, r_l, r_d, S} \quad N = (1+r_{sw})L_{sw}(r_{sw}) + (1+r_s)L_s(r_s) + (1+r_l)L_l(r_l) + (1+i)S - (1+r_d)D(r_d)$$

s.t.

$$\begin{aligned}
L_{sw}(r_{sw}) &= \begin{cases} > \bar{L}_{sw} & \text{if } r_{sw} < r_{out} \\ \bar{L}_{sw} & \text{if } r_{sw} = r_{out} \\ 0 & \text{if } r_{sw} > r_{out} \end{cases} \\
L_s(r_s) &= [1 - F_s(r_s - r_{out})]p_s\mathbf{L} \\
L_l(r_l) &= [1 - F_l(r_l - r_{out})](1 - p_s)\mathbf{L} \\
D(r_d) &= \left(\frac{1 + r_d}{1 + \bar{r}_d}\right)^{-\epsilon^d} \mathbf{D} \\
L_{sw} + L_s + L_l + S &= E + D && \text{(Balance sheet)} \\
\lambda(L_{sw} + L_s + L_l) &\leq N && \text{(Net worth constraint)}
\end{aligned}$$

Substituting the balance sheet constraint in the objective function, we can rewrite the maximization problem as:

$$\max_{r_{sw},r_s,r_l,r_d} \quad N = (r_{sw}-i)L_{sw}(r_{sw})+(r_s-i)L_s(r_s)+(r_l-i)L_l(r_l)+(1+i)E+(i-r_d)D(r_d)$$

s.t.

$$\begin{aligned}
L_{sw}(r_{sw}) &= \begin{cases} > \bar{L}_{sw} & \text{if } r_{sw} < r_{out} \\ \bar{L}_{sw} & \text{if } r_{sw} = r_{out} \\ 0 & \text{if } r_{sw} > r_{out} \end{cases} \\
L_s(r_s) &= [1 - F_s(r_s - r_{out})]p_s\mathbf{L} \\
L_l(r_l) &= [1 - F_l(r_l - r_{out})](1 - p_s)\mathbf{L} \\
D(r_d) &= \left(\frac{1 + r_d}{1 + \bar{r}_d}\right)^{-\epsilon^d} \mathbf{D} \\
\lambda(L_{sw} + L_s + L_l) &\leq N
\end{aligned}$$

The associated Lagrangian is:

$$\begin{aligned}
\mathcal{L} &= (r_{sw} - i)L_{sw}(r_{sw}) + (r_s - i)[1 - F_s(r_s - r_{out})]p_s\mathbf{L} + (r_l - i)[1 - F_l(r_l - r_{out})](1 - p_s)\mathbf{L} \\
&+ (1 + i)E + (i - r_d)\left(\frac{1 + r_d}{1 + \bar{r}_d}\right)^{-\epsilon^d} \mathbf{D} \\
&- \xi \Bigg((\lambda - (r_{sw} - i))L_{sw}(r_{sw}) + (\lambda - (r_s - i))[1 - F_s(r_s - r_{out})]p_s\mathbf{L} \\
&+ (\lambda - (r_l - i))[1 - F_l(r_l - r_{out})]p_l\mathbf{L} - (1 + i)E - (i - r_d)\left(\frac{1 + r_d}{1 + \bar{r}_d}\right)^{-\epsilon^d} \mathbf{D}\Bigg)
\end{aligned}$$

, where ξ is the Lagrange multiplier on the leverage constraint.
The F.O.C. with respect to r_s, $\frac{\partial \mathcal{L}}{\partial r_s} = 0$ yields:

$$\begin{aligned}
&[1 - F_s(r_s - r_{out})]\, p_s\mathbf{L} - (r_s - i)f_s(r_s - r_{out})p_s\mathbf{L} \\
-\xi(-(1 - F_s(r_s - r_{out}))p_s\mathbf{L}) &- (\lambda - (r_s - i))f_s(r_s - r_{out}))]p_s\mathbf{L}) = 0
\end{aligned}$$

Solving for $r_s - i$ yields:

$$r_s - i = \frac{1 - F_s(r_s - r_{out})}{f_s(r_s - r_{out})} + \lambda\frac{\xi}{1 + \xi}$$

Analog for the F.O.C. with respect to r_l, $\frac{\partial \mathcal{L}}{\partial r_l} = 0$.

The F.O.C. with respect to r_d, $\frac{\partial \mathcal{L}}{\partial r_d} = 0$ yields:

$$(1+\xi)\left(-\left(\frac{1+r_d}{1+\bar{r}_d}\right)^{-\epsilon^d}\mathbf{D} - (i-r_d)\epsilon^d\left(\frac{1+r_d}{1+\bar{r}_d}\right)^{-\epsilon^d-1}\mathbf{D}\frac{1}{1+\bar{r}_d}\right) = 0$$

Solving for r_d yields $1+r_d^* = \frac{\epsilon^d}{\epsilon^d-1}(1+i)$.

Proof of Proposition 1

Apply the implicit function theorem on FOCs (24) and (25) to get how the optimal rates r_s^* and r_l^* react to a change in the policy rate i. We show this explicitly for r_s.

Define the function $G(r_s, i)$ from FOC (24) and consider the case where the leverage constraint does not bind s.t. $\xi = 0$:

$$G(r_s,i) = r_s - i - \frac{1-F(r_s - r_{out}(i))}{f(r_s - r_{out}(i))} = 0$$

We have:

$$\begin{aligned}
\frac{\partial G}{\partial r_s} &= 1 - \left(\frac{-f(r_s - r_{out}(i))^2 - (1-F(r_s - r_{out}(i)))f'(r_s - r_{out}(i))}{f(r_s - r_{out}(i))^2}\right) \\
&= 2 + \frac{(1-F(r_s - r_{out}(i)))f'(r_s - r_{out}(i))}{f(r_s - r_{out}(i))^2}
\end{aligned}$$

and:

$$\begin{aligned}
\frac{\partial G}{\partial i} &= -1 - \left(\frac{f(r_s - r_{out})^2 r'_{out}(i) + (1-F(r_s - r_{out}(i)))f'(r_s - r_{out}(i))r'_{out}(i)}{f(r_s - r_{out}(i))^2}\right) \\
&= -1 - r'_{out}(i) - \frac{(1-F(r_s - r_{out}(i)))f'(r_s - r_{out}(i))r'_{out}(i)}{f(r_s - r_{out}(i))^2}
\end{aligned}$$

By the implicit function theorem, it therefore follows:

$$\frac{dr_s^*}{di} = -\frac{\partial G}{\partial i}\Big/\frac{\partial G}{\partial r_s} = \frac{1 + r'_{out}(i)\left(1 + \frac{(1-F(r_s - r_{out}(i)))f'(r_s - r_{out}(i))}{f(r_s - r_{out}(i))^2}\right)}{1 + \left(1 + \frac{(1-F(r_s - r_{out}(i)))f'(r_s - r_{out}(i))}{f(r_s - r_{out}(i))^2}\right)}$$

Using the FOC (24), this can be rewritten:

$$\frac{dr_s^*}{di} = \frac{1 + r'_{out}(i)\left(1 + (r_s^* - i)\frac{f'(r_s - r_{out}(i))}{f(r_s - r_{out}(i))}\right)}{1 + \left(1 + (r_s^* - i)\frac{f'(r_s - r_{out}(i))}{f(r_s - r_{out}(i))}\right)}$$

Analog derivation for $\frac{dr_l^*}{di}$.

Proof of Proposition 2

Let $i < \bar{i}$ so that $\frac{\partial r_{out}}{\partial i} = g'(i) < 1$. Rewrite $\frac{dr_s^*}{di}$:

$$\frac{dr_s^*}{di} = \frac{1 + g'(i)x_s}{1 + x_s},$$

where $x_s := 1 + (r_s^* - i)\frac{f'(r_s - r_{out}(i))}{f(r_s - r_{out}(i))}$.

Similarly, rewrite $\frac{dr_l^*}{di}$:

$$\frac{dr_l^*}{di} = \frac{1 + g'(i)x_l}{1 + x_l},$$

where $x_l := 1 + (r_l^* - i)\frac{f'(r_l - r_{out}(i))}{f(r_l - r_{out}(i))}$.

The function $f(z) = \frac{1+g'(i)z}{1+z}$ is decreasing in z. It follows that $\frac{dr_s^*}{di} > \frac{dr_l^*}{di}$ if $x_s < x_l$:

$$1 + (r_s^* - i)\frac{f'(r_s - r_{out}(i))}{f(r_s - r_{out}(i))} < 1 + (r_l^* - i)\frac{f'(r_l - r_{out}(i))}{f(r_l - r_{out}(i))} \tag{44}$$

using the definition of the elasticity of the density function $f(x)$ at x^*:

$$\epsilon_f|_{x^*} = \frac{f'(x^*)}{f(x^*)}x^*$$

, we can rewrite condition (44) as:

$$\frac{r_s^* - i}{r_s^* - g(i)}\epsilon_{f_s|_{r_s^* - g(i)}} < \frac{r_l^* - i}{r_l^* - g(i)}\epsilon_{f_l|_{r_l^* - g(i)}},$$

Proof of Lemma 2

Let $f_k \sim GPD(\mu_k, \sigma_k, \xi_k)$, where $k \in \{s, l\}$. Plugging in the associated probability density functions and cumulative density functions in the first order conditions derived in Lemma 1 yields:

$$r_k - i = \frac{\left(1 + \xi_k \frac{(r_k - r_{out} - \mu_k)}{\sigma_k}\right)^{-\frac{1}{\xi_k}}}{\frac{1}{\sigma_k}\left(1 + \xi_k \frac{(r_k - r_{out} - \mu_k)}{\sigma_k}\right)^{-\frac{1}{\xi_k}-1}}$$

Solving for r_k yields the result.

Proof of Proposition 3

Let $f_s \sim GPD(\mu_s, \sigma_s, \xi_s)$ and $f_l \sim GPD(\mu_l, \sigma_l, \xi_l)$. The threshold $\bar{i}$ is implicitly defined as the highest policy rate i at which the bank's net worth constraint exactly binds (i.e. with the multiplier equal to zero). The binding net worth constraint is given by:

$$\lambda L = L_{sw}(r_{sw} - i) + L_s(r_s - i) + L_l(r_l - i) + E(1 + i) + D(i - r_d)$$

The threshold $\bar{i}$ is the highest policy rate i such that this equality holds. We use the facts that $r_{sw} = r_{out}$ in equilibrium and $r_k - i = \sigma_k + \xi_k(r_k - r_{out} - \mu_k)$ for $k \in \{s, l\}$ from Lemma 2. Furthermore, since we are looking at the highest rate i such that the constraint exactly binds, it holds $r_{out} = i$ and again from Lemma 2: $r_k - i = \frac{\sigma_k - \xi_k \mu_k}{1 - \sigma_k}$. Plugging this in the binding constraint yields:

$$\lambda L = L_s\left(\frac{\sigma_s - \xi_s \mu_s}{1 - \xi_s}\right) + L_l\left(\frac{\sigma_l - \xi_l \mu_l}{1 - \xi_l}\right) + E(1 + i) + D * \max(i - \frac{1 + i\epsilon}{\epsilon - 1}, i)$$

We then use the expressions for loan demands with $r_{out} = i$:

$$L_k = [1 - F_k(r_k - i)]p_k$$

where $k \in \{s, l\}$ Replacing F_k with the GPD cumulative distribution function and using $r_k - i = \frac{\sigma_k - \xi_k \mu_k}{1 - \sigma_k}$, loan demands simplify to:

$$L_k = \left(1 + \frac{\xi_k}{1 - \xi_k}\frac{\sigma_k - \mu_k}{\sigma_k}\right)^{-\frac{1}{\xi_k}} p_k$$

Substituting this expression back into the binding net worth constraint yields:

$$\begin{aligned}
\lambda L \quad = \quad & \left(1 + \frac{\xi_s}{1 - \xi_s}\frac{\sigma_s - \mu_s}{\sigma_s}\right)^{-\frac{1}{\xi_s}} p_s \left(\frac{\sigma_s - \xi_s \mu_s}{1 - \xi_s}\right) \\
+ \quad & \left(1 + \frac{\xi_l}{1 - \xi_l}\frac{\sigma_l - \mu_l}{\sigma_l}\right)^{-\frac{1}{\xi_l}} p_l \left(\frac{\sigma_l - \xi_l \mu_l}{1 - \xi_l}\right) \\
+ \quad & E(1 + i) + D * \max(i - \frac{1 + i\epsilon}{\epsilon - 1}, i)
\end{aligned}$$

Defining:

$$\begin{aligned}\tau_s &= \left(1+\frac{\xi_s}{1-\xi_s}\frac{\sigma_s-\mu_s}{\sigma_s}\right)^{-\frac{1}{\xi_s}} p_s\left(\frac{\sigma_s-\xi_s\mu_s}{1-\xi_s}\right)\\ \tau_l &= \left(1+\frac{\xi_l}{1-\xi_l}\frac{\sigma_l-\mu_l}{\sigma_l}\right)^{-\frac{1}{\xi_l}} p_l\left(\frac{\sigma_l-\xi_l\mu_l}{1-\xi_l}\right)\end{aligned}$$

and assuming the zero lower bound constraint on the deposit rate binds, one can rewrite the binding net worth constraint as:

$$\lambda L = \tau_s + \tau_l + E(1+i) + Di$$

Finally, using the balance sheet identity $D + E = L + S$ and solving for i yields $\bar{i}$:

$$\bar{i} = \frac{\lambda L - \tau_s - \tau_l - E}{1+S}$$

Proof of Proposition 4

Let $f_s \sim GPD(\mu_s, \sigma_s, \xi_s)$ and $f_l \sim GPD(\mu_l, \sigma_l, \xi_l)$. We are seeking the monetary policy pass-through to the outside rate $r_{out} = g(i)$ when $i < \bar{i}$.

In the region $i < \bar{i}$, the outside rate is implicitly defined by the binding bank's net worth constraint:

$$\lambda L = N$$

The first step of the proof consists of rewriting the binding constraint as a function of i and $g(i)$ to apply the implicit function theorem. That is, our goal is to rewrite the constraint under the form:

$$N - \lambda L = h(i, g(i)) = 0$$

We have:

$$h(i, g(i)) = L_{sw}(g(i) - i) + L_s(r_s - i) + L_l(r_l - i) + D(i - r_d) + E(1+i) - \lambda L$$

We use the FOC from Lemma 2, which establishes $r_k - i = \frac{\sigma_k - \xi_k(g(i)+\mu_k - i)}{1-\xi_k}$ for $k \in \{s, l\}$ and rewrite:

$$\begin{aligned}h(i, g(i)) &= L_{sw}(g(i) - i) + L_s\left(\frac{\sigma_s - \xi_s(g(i)+\mu_s - i)}{1-\xi_s}\right) + L_l\left(\frac{\sigma_l - \xi_l(g(i)+\mu_l - i)}{1-\xi_l}\right)\\ &+ D * \min(\frac{1+i}{1-\epsilon}, i) + E(1+i) - \lambda L\end{aligned}$$

We then rewrite the optimal quantities L_s and L_l in terms of i and $g(i)$.

$$\begin{aligned}
L_s &= [1 - F_s(r_s - g(i))]p_s L \\
&= \left(1 + \xi_s \frac{r_s - g(i) - \mu_s}{\sigma_s}\right)^{-\frac{1}{\xi_s}} p_s L
\end{aligned}$$

and again using the FOC from Lemma 2 for $r_s - g(i)$:

$$L_s = \left(1 + \frac{\xi_s}{\sigma_s} \frac{(\sigma_s + i - g(i) - \mu_s)}{1 - \xi_s}\right)^{-\frac{1}{\xi_s}} p_s L$$

By symmetry, the same holds for L_l. Using $L_{sw} = L - L_s - L_l$, one can rewrite:

$$\begin{aligned}
h(i, g(i)) &= L(g(i) - i) \\
&+ \left(1 + \frac{\xi_s}{\sigma_s} \frac{(\sigma_s + i - g(i) - \mu_s)}{1 - \xi_s}\right)^{-\frac{1}{\xi_s}} p_s L \left(\frac{\sigma_s - \xi_s(g(i) + \mu_s - i)}{1 - \xi_s} - (g(i) - i)\right) \\
&+ \left(1 + \frac{\xi_l}{\sigma_l} \frac{(\sigma_l + i - g(i) - \mu_l)}{1 - \xi_l}\right)^{-\frac{1}{\xi_l}} p_l L \left(\frac{\sigma_l - \xi_l(g(i) + \mu_l - i)}{1 - \xi_l} - (g(i) - i)\right) \\
&+ D * \min(\frac{1+i}{1-\epsilon}, i) + E(1+i) - \lambda L
\end{aligned}$$

Simplifying further yields:

$$\begin{aligned}
h(i, g(i)) &= L(g(i) - i) \\
&+ \left(1 + \frac{\xi_s}{\sigma_s} \frac{(\sigma_s + i - g(i) - \mu_s)}{1 - \xi_s}\right)^{-\frac{1}{\xi_s}} p_s L \left(\frac{\sigma_s + i - g(i) - \xi_s \mu_s}{1 - \xi_s}\right) \\
&+ \left(1 + \frac{\xi_l}{\sigma_l} \frac{(\sigma_l + i - g(i) - \mu_l)}{1 - \xi_l}\right)^{-\frac{1}{\xi_l}} p_l L \left(\frac{\sigma_l + i - g(i) - \xi_l \mu_l}{1 - \xi_l}\right) \\
&+ D * \min(\frac{1+i}{1-\epsilon}, i) + E(1+i) - \lambda L \\
&= 0
\end{aligned}$$

We can now get the partial derivatives $\frac{\partial h(i,g(i))}{\partial g(i)}$ and $\frac{\partial h(i,g(i))}{\partial i}$ and apply the implicit function theorem.

$$\begin{aligned}
\frac{\partial h(i, g(i))}{\partial g(i)} &= L(1 + p_s \left(1 + \frac{\xi_s}{\sigma_s} \frac{(\sigma_s + i - g(i) - \mu_s)}{1 - \xi_s}\right)^{-\frac{1}{\xi_s} - 1} \frac{i - g(i)}{\sigma_s(1 - \xi_s)} \\
&+ p_l \left(1 + \frac{\xi_l}{\sigma_l} \frac{(\sigma_l + i - g(i) - \mu_l)}{1 - \xi_l}\right)^{-\frac{1}{\xi_l} - 1} \frac{i - g(i)}{\sigma_l(1 - \xi_l)})
\end{aligned}$$

and

$$\begin{aligned}\frac{\partial h(i,g(i))}{\partial i} &= -L(1+p_s\left(1+\frac{\xi_s}{\sigma_s}\frac{(\sigma_s+i-g(i)-\mu_s)}{1-\xi_s}\right)^{-\frac{1}{\xi_s}-1}\frac{i-g(i)}{\sigma_s(1-\xi_s)}\\ &+ p_l\left(1+\frac{\xi_l}{\sigma_l}\frac{(\sigma_l+i-g(i)-\mu_l)}{1-\xi_l}\right)^{-\frac{1}{\xi_l}-1}\frac{i-g(i)}{\sigma_l(1-\xi_l)})+D+E\end{aligned}$$

Finally, since $\frac{dg(i)}{di}=-\frac{\partial h/\partial i}{\partial h/\partial g(i)}$:

$$\begin{aligned}\frac{dg(i)}{di} &= -\frac{-L\left(1+p_s\kappa_s^{-\frac{1}{\xi_s}-1}\frac{i-g(i)}{\sigma_s(1-\xi_s)}+p_l\kappa_l^{-\frac{1}{\xi_l}-1}\frac{i-g(i)}{\sigma_l(1-\xi_l)}\right)+D+E}{L\left(1+p_s\kappa_s^{-\frac{1}{\xi_s}-1}\frac{i-g(i)}{\sigma_s(1-\xi_s)}+p_l\kappa_l^{-\frac{1}{\xi_l}-1}\frac{i-g(i)}{\sigma_l(1-\xi_l)}\right)}\\ &= 1-\frac{D+E}{L\left(1+p_s\kappa_s^{-\frac{1}{\xi_s}-1}\frac{i-g(i)}{\sigma_s(1-\xi_s)}+p_l\kappa_l^{-\frac{1}{\xi_l}-1}\frac{i-g(i)}{\sigma_l(1-\xi_l)}\right)}\\ &= 1-\frac{L+S}{L\left(1-(g(i)-i)\left[\frac{p_s}{\sigma_s(1-\xi_s)}\kappa_s^{-\frac{1}{\xi_s}-1}+\frac{p_l}{\sigma_l(1-\xi_l)}\kappa_l^{-\frac{1}{\xi_l}-1}\right]\right)}\end{aligned}$$

where we used the balance sheet identity: $D+E=L+S$ and

$$\begin{aligned}\kappa_s &= \left(1+\frac{\xi_s}{\sigma_s}\frac{(\sigma_s+i-g(i)-\mu_s)}{1-\xi_s}\right)\\ \kappa_l &= \left(1+\frac{\xi_l}{\sigma_l}\frac{(\sigma_l+i-g(i)-\mu_l)}{1-\xi_l}\right).\end{aligned}$$

Proof of Proposition 5

Let $f_s \sim GPD(\mu_s,\sigma_s,\xi_s)$ and $f_l \sim GPD(\mu_l,\sigma_l,\xi_l)$. Using the general expression for pass-through derived in Proposition 1, the GPD probability density function and its derivative

$$\begin{aligned}f(x) &= \frac{1}{\sigma}\left(1+\xi\left(\frac{x-\mu}{\sigma}\right)\right)^{-(\frac{1}{\xi}+1)}\\ f'(x) &= -\frac{\xi}{\sigma^2}\left(\frac{1}{\xi}+1\right)\left(1+\xi\frac{x-\mu}{\sigma}\right)^{-\frac{1}{\xi}-2}\end{aligned}$$

yields:

$$\frac{dr_k}{di}=\frac{1+g'(i)\left(1-(r_k-i)\frac{1+\xi_k}{\sigma_k+\xi_k(r_k-g(i)-\mu_k)}\right)}{1+\left(1-(r_k-i)\frac{1+\xi_k}{\sigma_k+\xi_k(r_k-g(i)-\mu_k)}\right)}$$

for $k\in\{s,l\}$.

Further substituting $r_k - i$ with $\sigma_k + \xi_k(r_k - g(i) - \mu_k)$ from the FOC derived in Lemma 2 yields the result

$$\frac{dr_k}{di} = \frac{1 - g'(i)\xi_k}{1 - \xi_k}$$

for $k \in \{s, l\}$.

Proof of Proposition 6

Let $f_s \sim GPD(\mu_s, \sigma_s, \xi_s)$ and $f_l \sim GPD(\mu_l, \sigma_l, \xi_l)$. Let $i < \bar{i}$ so that $\frac{\partial r_{out}}{\partial i} = g'(i) < 1$. Using the pass-through result from Proposition 5, it is clear that $\frac{dr_l}{di} < \frac{dr_s}{di}$ if and only if $\xi_s > \xi_l$:

$$\frac{1 - g'(i)\xi_l}{1 - \xi_l} < \frac{1 - g'(i)\xi_s}{1 - \xi_s} \Longleftrightarrow \xi_s > \xi_l$$