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# HUMAN SELF-ROTATION BY MEANS OF LIMB MOVEMENTS*

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**Abstract** – A 'weightless' astronaut can alter the orientation of his body in space by moving his limbs in an appropriate manner. The feasibility of employing two particular limb maneuvers, one producing pitch motion, and the other yaw, is established analytically, and quantitative results are presented.

## INTRODUCTION

To PERFORM certain tasks that arise in connection with manned space flight, an astronaut must be able to change the orientation of his body while in a state of 'weightlessness'. This is a relatively easy matter in situations that permit the astronaut to maintain direct contact with some part of a space vehicle, for he can then exploit forces that come into play as a result of such contacts. When forces of interaction between the astronaut and a space vehicle cannot be employed, two alternatives present themselves: The astronaut can either carry a device capable of producing forces that change his orientation, such as a gas gun, or he can use relative motions of parts of his body, as do trampolinists, trapeze performers, etc. The latter option is attractive because it eliminates both the danger of mechanical failure and the necessity to carry a supply of fuel. These considerations motivated the study described in the present paper.

It is not difficult to establish qualitative relationships between certain limb movements and associated rotations of the torso. For example, it is apparent that symmetrical, rotational motions of the arms of a man whose torso is initially without rotational motion will lead to a pitch rotation of the torso. What is more difficult, is to arrive at a quantitative description of such a maneuver. This, however, is precisely what is needed in order to assess any proposed method of reorientation from a practical point of view. It was decided, therefore, to undertake analytical studies of specific maneuvers. Now, it is a fact that any change whatsoever in the orientation of a rigid body can be produced by three successive rotations about two body-fixed axes, the first and the third rotation being performed about the same axis. Thus, if a man can perform both a pitch rotation and a yaw rotation, he can acquire any desired orientation. Accordingly, maneuvers leading to such rotations were analyzed. (Further maneuvers have been discussed in primarily qualitative terms by Kulwicki *et al.*(1962) and by Stepantsov (1966).) The results of the analyses justify considerable optimism regarding the use of limb movements as a self-rotation technique.

Before turning to the consideration of particular maneuvers, it may be helpful to elaborate on the meaning of the terms 'pitch' and 'yaw' and to define certain other terms used in the sequel.

Let three intersecting, mutually perpendicular lines be fixed relative to the torso, the first of these, called the yaw axis, having the same general orientation as the spine, the second, called the pitch axis, passing from right to left, and the third being called the roll axis. The phrases 'pitch motion', 'yaw motion', and 'roll motion' then refer to rotations of the torso about the pitch, yaw, and roll axes, respectively. The location of the

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point of intersection of the axes is selected in such a way as to facilitate the particular analysis under consideration; and the plane determined by any pair of axes is designated by the name of the axis normal to it; e.g. the pitch plane is normal to the pitch axis.

## PITCH

The pitch maneuver under consideration was suggested by Kulwicki (1962) and was analyzed by McCrank (1964) who, however, failed to investigate its effectiveness and postulated an arm motion which cannot be performed for physiological reasons. During this maneuver, the arms are held straight at the elbows and perform a rotary motion with respect to the torso, remaining symmetrically disposed with respect to the pitch plane at all times. The longitudinal axis of each arm travels on the surface of an imaginary, torso-fixed cone whose vertex is at the shoulder. The arm motion can be described in terms of the cone semi-vertex angle and the orientation of the cone axis relative to the torso, any physically attainable orientation being permissible. The legs must be kept fixed relative to the torso and symmetrically located with respect to the pitch plane. The sense of pitching of the torso is opposite to that in which the arms travel on the surfaces of the cones.

To obtain an analytical description of this motion, the human is modelled as a system $S$ of three rigid bodies. One of these, designated $A$, represents the torso, head, and legs. The remaining two, $B$ and $B'$, each represent an arm and are connected to $A$ at points $O$ and $O'$, respectively, which represent shoulder joints (see Fig. 1).

Body $A$ is assumed to be symmetric with respect to the pitch plane, so that its mass center $A^*$ lies thereon. Points $O$ and $O'$ are symmetrically located at a distance $a_2$ to either side of the pitch plane, and the line $P$ passing through them is the pitch axis. The yaw axis $Y$ intersects $P$ at point $C$; and the position of $A^*$ relative to $C$ is specified by a distance $a_3$, measured along $Y$, and a distance $a_1$ measured from the roll plane. The mass of $A$ is $m_A$, and the moment of inertia of $A$ about a line through $A^*$ parallel to $P$ is $I^A$. Unit vectors $\mathbf{a}_1$, $\mathbf{a}_2$, $\mathbf{a}_3$ are fixed in $A$ parallel to the roll, pitch, and yaw axes, respectively. Since only pitch motions of $A$ are to be considered, the orientation of $A$ in an inertial reference frame $F$ can be described by means of a single angle, such as the angle $\xi$ between $Y$ and a line fixed in $F$ and normal to $P$.

Body $B$ possesses an axis of symmetry designated $L$. The mass center $B^*$ of $B$ is located on $L$ at a distance $b$ from point $O$. The mass of $B$ is $m_B$, the moment of inertia of

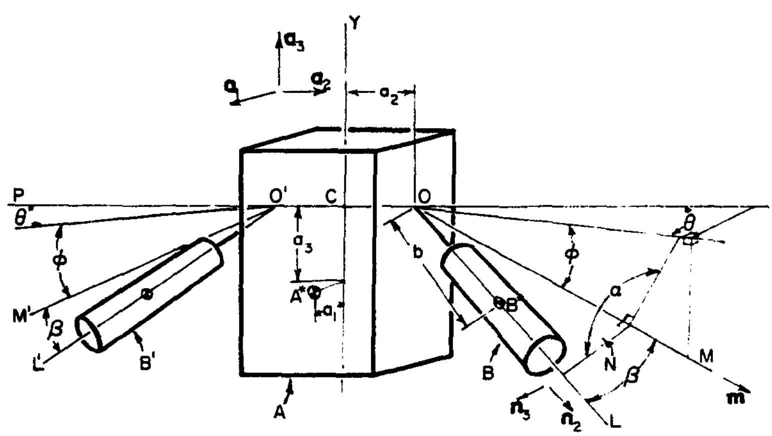


Fig. 1. Pitch motion model.

$B$ about $L$ is $I_2^B$, and the moment of inertia of $B$ about any line passing through $B^*$ and normal to $L$ is $I_1^B$. The geometrical and inertia properties of $B'$ are identical to those of $B$.

The kinematical analysis in the sequel involves the cone axes, $M$ and $M'$, which are fixed with respect to $A$ and are located symmetrically with respect to the pitch plane. Line $M$ passes through $O$ and its orientation is determined by angles $\theta$ and $\varphi$, as shown in Fig. 1. (The lines forming $\theta$ lie in the yaw plane, and those forming $\varphi$ lie in a plane normal to the yaw plane.) Lines $L$ and $M$ intersect, thereby forming the fixed semi-vertex angle $\beta$, and also determining a plane $N$ which rotates once about $M$ per cycle of the maneuver. Thus, the angle $\alpha$ between $N$ and a plane which is normal to the yaw plane and contains $M$ increases from 0 to $2\pi$ during one cycle. (Symmetry considerations permit one to locate $L'$ and $M'$.) A unit vector $\mathbf{m}$ remains parallel to $M$, and mutually perpendicular unit vectors $\mathbf{n}_1$, $\mathbf{n}_2$, $\mathbf{n}_3$ are fixed in $N$, with $\mathbf{n}_2$ parallel to $L$ and $\mathbf{n}_1$ normal to $N$.

The Lagrange equation governing the motion of $S$ when $S$ is in free fall and has no initial rotational motion is

$$\frac{\partial K}{\partial \dot{\xi}} = 0 \tag{1}$$

where $\dot{\xi}$ denotes the time derivative of $\xi$ and $K$ is the kinetic energy associated with motion of $S$ relative to $S^*$, the mass center of $S$. $K$ can be expressed as

$$K = K_\omega{}^A + K_\omega{}^B + K_\omega{}^{B'} + K_v{}^A + K_v{}^B + K_v{}^{B'} \tag{2}$$

where the first three terms on the right-hand-side represent the rotational kinetic energies of $A$, $B$ and $B'$; e.g. as only pitch motions are considered, so that the angular velocity of $A$ in $F$ is given by

$${}^F\boldsymbol{\omega}^A = \dot{\xi}\mathbf{a}_2 \tag{3}$$

one can express $K_\omega{}^A$ as

$$K_\omega{}^A = \tfrac{1}{2} I^A \dot{\xi}^2 \tag{4}$$

and, if the angular velocity of $B$ in $F$ is expressed as

$$F\boldsymbol{\omega}^B = \omega_1{}^B\mathbf{n}_1 + \omega_2{}^B\mathbf{n}_2 + \omega_3{}^B\mathbf{n}_3 \tag{5}$$

then

$$K_\omega{}^B = \tfrac{1}{2}[I_1{}^B(\omega_1{}^B)^2 + I_2{}^B(\omega_2{}^B)^2 + I_1{}^B(\omega_3{}^B)^2]. \tag{6}$$

The last three terms in (2) reflect the motions of the mass centers of $A$, $B$ and $B'$; e.g. if $\mathbf{v}_A$ and $\mathbf{v}_B$ are the velocities of $A^*$ and $B^*$, respectively, relative to $S^*$, then

$$K_v{}^A = \tfrac{1}{2} m_A \mathbf{v}_A{}^2 \tag{7}$$

and

$$K_v{}^B = \tfrac{1}{2} m_B \mathbf{v}_B{}^2. \tag{8}$$

To obtain expressions for $\omega_1{}^B$, $\omega_2{}^B$, $\omega_3{}^B$, $\mathbf{v}_A$ and $\mathbf{v}_B$, suitable for insertion into (6), (7) and (8), the angular velocity of $B$ in $F$ is expressed as

$${}^F\boldsymbol{\omega}^B = {}^F\boldsymbol{\omega}^A + {}^A\boldsymbol{\omega}^N + {}^N\boldsymbol{\omega}^B \tag{9}$$

where ${}^A\boldsymbol{\omega}^N$, the angular velocity of $N$ in $A$, is

$${}^A\boldsymbol{\omega}^N = \dot{\alpha}\mathbf{m} \tag{10}$$

and ${}^N\boldsymbol{\omega}^B$, the angular velocity of $B$ in $N$, must be given by

$${}^N\boldsymbol{\omega}^B = -\dot{\alpha}\mathbf{n}_2 \tag{11}$$

if the arm, while moving about $M$, rotates in $N$ in such a way as to avoid twisting at the shoulder in a physically impossible manner. As

$$\mathbf{m} = \cos\beta\,\mathbf{n}_2 - \sin\beta\,\mathbf{n}_3 \tag{12}$$

and

$$\mathbf{n}_i = \sum_{j=1}^{3} c_{ij}\mathbf{a}_j \quad i = 1,2,3 \tag{13}$$

where the $c_{ij}$ of interest are

$$\left.\begin{aligned}
c_{12} &= -\sin\theta\cos\alpha - \cos\theta\sin\varphi\sin\alpha \\
c_{21} &= +\cos\theta\sin\beta\sin\alpha \\
&\quad +\sin\theta\cos\varphi\cos\beta \\
&\quad +\sin\theta\sin\varphi\sin\beta\cos\alpha \\
c_{22} &= -\sin\theta\sin\beta\sin\alpha \\
&\quad +\cos\theta\cos\varphi\cos\beta \\
&\quad +\cos\theta\sin\varphi\sin\beta\cos\alpha \\
c_{23} &= -\sin\varphi\cos\beta + \cos\varphi\sin\beta\cos\alpha \\
c_{32} &= -\sin\theta\cos\beta\sin\alpha \\
&\quad -\cos\theta\cos\varphi\sin\beta \\
&\quad +\cos\theta\sin\varphi\cos\beta\cos\alpha
\end{aligned}\right\} \quad (14)$$

substitution of (3), (13), (10), (12) and (11) into (9) leads to

$$\left.\begin{aligned}
\omega_1{}^B &= \dot{\xi} c_{12} \\
\omega_2{}^B &= \dot{\xi} c_{22} + \dot{\alpha}(\cos\beta - 1) \\
\omega_3{}^B &= \dot{\xi} c_{32} - \dot{\alpha}\sin\beta.
\end{aligned}\right\} \quad (15)$$

Next, $\mathbf{v}_A$ is formed as the time derivative in $F$ of $\mathbf{r}^{A^*/S^*}$, the position vector of $A^*$ relative to $S^*$, which, due to symmetry, may be expressed as

$$\mathbf{r}^{A^*/S^*} = -\frac{2m_B}{m_A + 2m_B}[(\mathbf{r}^{B^*/A^*}\cdot\mathbf{a}_1)\mathbf{a}_1 + (\mathbf{r}^{B^*/A^*}\cdot\mathbf{a}_3)\mathbf{a}_3] \quad (16)$$

where $\mathbf{r}^{B^*/A^*}$, the position of $B^*$ relative to $A^*$, is (see Fig. 1) given by

$$\mathbf{r}^{B^*/A^*} = -a_1\mathbf{a}_1 + a_2\mathbf{a}_2 + a_3\mathbf{a}_3 + b\mathbf{n}_2. \quad (17)$$

Consequently,

$$\mathbf{v}_A = \frac{2m_B}{m_A + 2m_B}\left\{-\left[b\frac{dc_{21}}{d\alpha}\dot{\alpha} + \dot{\xi}(bc_{23} + a_3)\right]\mathbf{a}_1 + \left[\dot{\xi}(bc_{21} - a_1) - b\frac{dc_{23}}{d\alpha}\dot{\alpha}\right]\mathbf{a}_3\right\}. \quad (18)$$

Finally, $\mathbf{v}_B$ is given by

$$\mathbf{v}_B = \mathbf{v}_A + {}^F\boldsymbol{\omega}^B \times b\mathbf{n}_2 + {}^F\boldsymbol{\omega}^A \times (-a_1\mathbf{a}_1 + a_2\mathbf{a}_2 + a_3\mathbf{a}_3) \quad (19)$$

or, after substitution from (18), (5), (15), (13) and (3), by

$$\begin{aligned}
\mathbf{v}_B = &\left\{\frac{m_A}{m_A + 2m_B}\left[\dot{\xi}(bc_{23} + a_3) + \dot{\alpha} b\frac{dc_{21}}{d\alpha}\right]\right\}\mathbf{a}_1 \\
&+ \{\dot{\alpha} b \sin\beta\, c_{12}\}\mathbf{a}_2 \\
&+ \left\{\frac{m_A}{m_A + 2m_B}\left[\dot{\xi}(a_1 - bc_{21}) + \dot{\alpha} b\frac{dc_{23}}{d\alpha}\right]\right\}\mathbf{a}_3.
\end{aligned} \quad (20)$$

Substitution from (4), (6), (15), (7), (18), (8) and (20) into (2) now leads to

$$\begin{aligned}
K = &\dot{\xi}^2\left\{\frac{I^A}{2} + I_1{}^B(c_{12}{}^2 + c_{32}{}^2) + I_2{}^B c_{22}{}^2 \right.\\
&+ \frac{m_A m_B}{m_A + 2m_B}[b^2(c_{21}{}^2 + c_{23}{}^2) \\
&\left. + 2b(a_3 c_{23} - a_1 c_{21}) + (a_1{}^2 + a_3{}^2)]\right\} \\
&+ 2\dot{\xi}\dot{\alpha}\left\{I_2{}^B c_{22}(\cos\beta - 1) - I_1{}^B c_{32}\sin\beta \right.\\
&+ \frac{m_A m_B}{m_A + 2m_B}\left[b^2\left(c_{23}\frac{dc_{21}}{d\alpha} - c_{21}\frac{dc_{23}}{d\alpha}\right)\right. \\
&\left.\left. + b\left(a_3\frac{dc_{21}}{d\alpha} + a_1\frac{dc_{23}}{d\alpha}\right)\right]\right\} \\
&+ \dot{\alpha}^2\left\{2I_2{}^B(1 - \cos\beta) + (I_1{}^B - I_2{}^B \right.\\
&+ m_B b^2)\sin^2\beta + \frac{m_A m_B}{m_A + 2m_B} b^2\left[\left(\frac{dc_{21}}{d\alpha}\right)^2 \right.\\
&\left.\left. + \left(\frac{dc_{23}}{d\alpha}\right)^2\right]\right\}
\end{aligned} \quad (21)$$

and, by substituting from (21) into (1), eliminating time, and defining

$$\left.\begin{aligned}
M &= \frac{m_A m_B}{m_A + 2m_B} \\
J &= I_1{}^B - I_2{}^B + Mb^2 \\
E_1 &= \cos\theta\cos\varphi\,[J\sin^2\beta - I_2{}^B(\cos\beta - 1)] \\
E_2 &= \sin\beta\,[\sin\theta\,(J\cos\beta \\
&\qquad + I_2{}^B - Mba_3\sin\varphi) - \cos\varphi\, Mba_1] \\
E_3 &= \cos\theta\sin\beta\,[Mba_3 - \sin\varphi\,(J\cos\beta \\
&\qquad + I_2{}^B)] \\
F_1 &= \frac{I^A}{2} + M(a_1{}^2 + a_3{}^2) + I_2{}^B \\
&\quad + J[\cos^2\beta(\sin^2\theta - \cos^2\varphi\cos^2\theta) \\
&\quad + \cos^2\theta] - 2Mb\cos\beta\,(\sin\varphi a_3 \\
&\quad + \cos\varphi\sin\theta a_1) \\
F_2 &= J\sin^2\beta\,(\sin^2\theta - \sin^2\varphi\cos^2\theta) \\
F_3 &= 2J\sin\theta\cos\theta\sin\varphi\sin^2\beta \\
F_4 &= 2\sin\beta\cos\theta \\
&\quad \times (J\cos\beta\sin\theta\cos\varphi - Mba_1) \\
F_5 &= 2\sin\beta\,[Mb(a_3\cos\varphi - a_1\sin\theta\sin\varphi) \\
&\quad - J\cos^2\theta\sin\varphi\cos\varphi\cos\beta]
\end{aligned}\right\} \quad (22)$$

one can express the equation of motion as

$$\frac{d\xi}{d\alpha} = -\frac{E_1 + E_2\sin\alpha + E_3\cos\alpha}{F_1 + F_2\cos^2\alpha + F_3\sin\alpha\cos\alpha + F_4\sin\alpha + F_5\cos\alpha}. \quad (23)$$

Integration of (23) in the interval $0 \leqslant \alpha \leqslant 2\pi$ yields $\Delta\xi$, the pitch reorientation per cycle of the maneuver. An analytical solution is not readily available except in the special case of cone axes parallel to the pitch axis. In this case, $\theta = \varphi = 0$, and (23) yields $\Delta\xi$ (in rad.) as

$$\Delta\xi = \pi\left[1 + \frac{2E - F}{\sqrt{F^2 - 4G^2}}\right] \quad (24)$$

where

$$\left.\begin{aligned}
E &= J\sin^2\beta + I_2{}^B(1 - \cos\beta) \\
F &= \frac{I^A}{2} + I_2{}^B + J\sin^2\beta + M(a_1{}^2 + a_3{}^2) \\
G &= Mb\sin\beta\sqrt{a_1{}^2 + a_3{}^2}.
\end{aligned}\right\} \quad (25)$$

Neither (23) nor (24) can be used until suitable values have been selected for the inertia properties. Moreover, $I^A$, $a_1$ and $a_3$ depend on the position in which the legs are held, as well as on the inertia properties of the limbs. Table 1 contains values based on a model of an average man as proposed by Hanavan (1964).

It is of interest to observe how the pitch obtained per cycle, $\Delta\xi$, varies as a function of the cone semi-vertex angle, $\beta$. In Fig. 2, $\Delta\xi$ is plotted as a function of $\beta$ for three cases: (1) the maneuver performed with the legs straight; (2) the maneuver performed with the legs tucked close to the body; and (3) the maneuver performed with the legs tucked and a 5 lb weight held in each hand. In all three cases, the cone axes are parallel to the pitch axis, equation (24) is used, and the requisite inertia properties are taken from Table 1.

It can be seen that pitch increases monotonically with $\beta$. Since the construction of the shoulder joint places an upper limit on $\beta$ once

Table 1

| Symbol | Arm | Arm with 5 lb weight in hand | Units |
|---|---|---|---|
| $m_B$ | 0·289 | 0·445 | slugs |
| $b$ | 0·899 | 1·292 | ft |
| $I_1{}^B$ | 0·1331 | 0·261 | slug-ft² |
| $I_2{}^B$ | 0·0030 | 0·0030 | slug-ft² |
| **Symbol** | **Legs straight** | **Legs tucked*** | **Units** |
| $m_A$ | 4·45 | 4·45 | slugs |
| $a_1$ | 0 | 0·309 | ft |
| $a_3$ | 1·483 | 1·028 | ft |
| $I^A$ | 8·13 | 3·87 | slug-ft² |

*In the 'legs tucked' position, the longitudinal axes of the thighs are normal to the roll plane and the knees are bent through 150°.

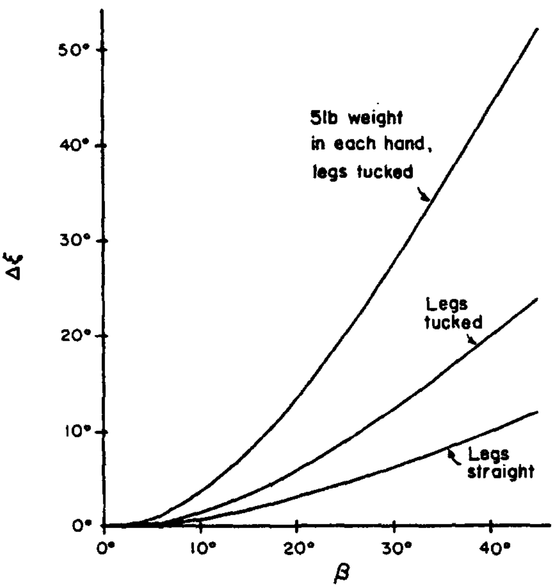


Fig. 2. Pitch reorientation as a function of $\beta$ ($\theta = \varphi = 0$).

a particular cone axis has been chosen, the limit being about 45° when $\theta = \varphi = 0$, a man with his legs straight can expect only 12° of pitch when performing a cycle of the maneuver in this fashion. Tucking the legs markedly improves the effectiveness of the maneuver, as does holding 5 lb weights in the hands; e.g. when $\beta$ is 45° and the legs are tucked, 24° of pitch can be obtained per cycle and, if the weights are added as well, the amount of pitch per cycle becomes 52°.

The location of the cone axes relative to the torso has a significant effect on the amount of pitch obtained per cycle. In Fig. 3, $\Delta\xi$ is plotted as a function of $\theta$, with $\varphi = 0$ (see Fig. 1 for $\theta$ and $\varphi$), for two values of $\beta$, i.e. 20° and 45°, when the legs are tucked. (A digital computer was used to integrate equation (23).) $\Delta\xi$ is seen to decrease with increasing $\theta$. This suggests that is is advantageous to maintain the cone axes in or near the roll plane.

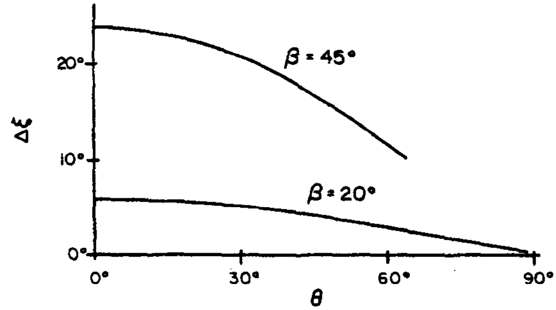


Fig. 3. Pitch reorientation as a function of $\theta$ ($\varphi = 0$, legs tucked).

When the cone axes are lowered in the roll plane, $\Delta\xi$ may increase or decrease. This can be seen in Fig. 4, which shows the pitch per cycle as a function of $\varphi$, with $\theta = 0$, the legs

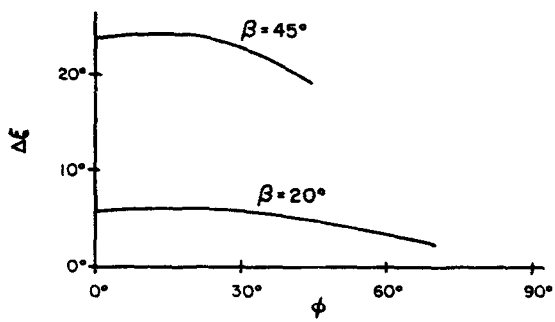


Fig. 4. Pitch reorientation as a function of $\varphi$ ($\theta = 0$, legs tucked).

tucked, and $\beta$ again equal to 20° and 45°. Both curves possess a maximum when $\varphi$ is about 15°. The relative flatness of the curves between 0° and 30° is important since physiological constraints at the shoulder joint are such that the semi-vertex angles $\beta$ that can be used become larger as $\varphi$ is increased; e.g. while the upper bound on $\beta$ is about 45° with $\varphi = 0$, it is closer to 60° when $\varphi = 30°$. Consequently, the greatest amount of pitch per cycle obtained without weights in the tucked position is about 33° and this is achieved when $\beta = 60°, \varphi = 30°$ and $\theta = 0$.

## YAW

The yaw maneuver discussed in the sequal was suggested to the authors by James L. Jones of the NASA Ames Research Center. This maneuver can be performed either with the arms or the legs. In both cases, the limbs remain straight at the knees and elbows, and the pair of limbs that is not used must remain fixed relative to the torso in such a way that the yaw axis is a principal axis of inertia.

The maneuver is performed in two phases. For definiteness, suppose that the legs are used and that rotation of the torso to the left is desired. Then phase 1 begins with the right leg extended forward from the torso, and the left leg extended rearward, through equal angles $\beta_0$. The right leg is swept to the right and then to the rear (relative to the torso) while the left leg is swept leftward and forward; that is, the longitudinal axis of each leg moves on the surface of an imaginary, torso-fixed cone whose vertex is at the hip and whose axis is parallel to the yaw axis. In the course of the 'coning' motion, no twisting of the leg occurs. Thus, the toes always point nearly forward. At the conclusion of phase 1, the right leg is extended rearward and the left leg forward relative to the torso.

In phase 2, the legs travel simultaneously in planes parallel to the pitch plane until each leg has returned to the position it occupied (with respect to the torso) at the beginning of phase 1. During both phases, the two legs move at the same rate so that their longitudinal axes maintain digonal symmetry about the yaw axis.

Consider the behavior of the torso during this maneuver. As phase 1 progresses, the torso rotates in an inertial reference frame to its left about the yaw axis as desired, while the orientation of the yaw axis remains fixed. During phase 2, the torso turns back to the right, but this regression is not sufficient to nullify the rotation obtained in phase 1. A net rotation to the left thus results from the performance of one complete cycle of the maneuver. The direction of rotation can be reversed by starting phase 1 with the left foot forward, rather than the right one.

For the purposes of analysis, the human is again modelled as a system $S$ of three rigid bodies. One of these, designated $A$, represents the torso, head, and arms. The remaining two, $B$ and $B'$, each represent a leg, and these are connected to $A$ at points $O$ and $O'$, respectively, which represent the hips (see Fig. 5).

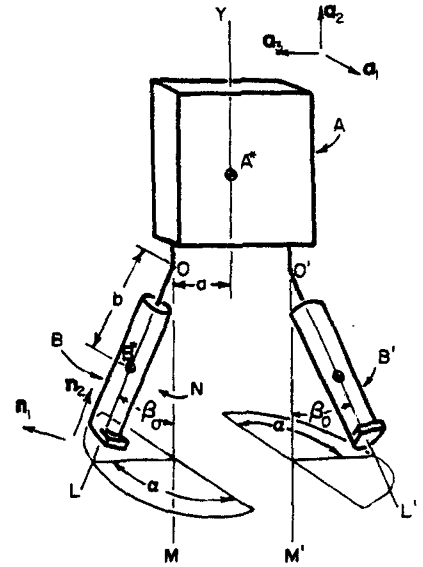


Fig. 5. Yaw motion model, phase 1.

The yaw axis $Y$ is presumed to coincide with a principal axis of $A$ for its mass center $A^*$. The pitch axis is chosen so that points $O$ and $O'$ lie thereon, each at a distance $a$ from $Y$. The analysis in the sequel involves two additional lines fixed in $A$, namely $M$ and $M'$, which are parallel to $Y$ and pass through $O$ and $O'$, respectively. Body $A$ has a mass $m_A$ and a moment of inertia $I^A$ about $Y$. Mutually perpendicular unit vectors $\mathbf{a}_1$, $\mathbf{a}_2$, $\mathbf{a}_3$ are fixed in $A$ parallel to the roll, yaw, and pitch axes, respectively. Since only yaw motions of $A$ are to be considered, the attitude of $A$ in an inertial reference frame $F$ can be described by a single angle $\xi$ between the pitch axis and a line fixed in $F$ and normal to $Y$.

It is assumed that bodies $B$ and $B'$ each possess an axis of symmetry. The axis of minimum moment of inertia of $B$ for its mass center $B^*$ is designated $L$, and the associated principal moment of inertia has the value $I_2^B$. The moment of inertia of $B$ about any line perpendicular to $L$ and passing through $B^*$ is $I_1^B$, and the mass of $B$ is $m_B$. Line $L$ passes through $O$, and the distance from $B^*$ to $O$ is $b$. The inertia properties of leg $B'$ are identical with those of $B$.

As different variables are used for the mathematical description of the two phases of the maneuver, two analyses are required. However, equations (1) through (8) and the remarks that were made in connection with these equations are applicable to both analyses, provided that the following points are kept in mind: Only yaw motions of the torso are being considered so that $\xi$ now describes the yaw attitude; $\mathbf{a}_2$ is parallel to the yaw axis; and $\mathbf{n}_1$, $\mathbf{n}_2$, $\mathbf{n}_3$ may represent any set of three, mutually perpendicular unit vectors parallel to the centroidal principal axes of $B$. A specific choice of $\mathbf{n}_1$, $\mathbf{n}_2$, $\mathbf{n}_3$ will be made for each of the analyses in the sequel.

In phase 1, lines $M$ and $M'$ serve as axes of cones on whose surfaces $L$ and $L'$ move while the semi-vertex angle $\beta_0$ of each cone remains constant. $L$ and $M$ determine a plane $N$ and the angle between $N$ and the pitch plane is designated $\alpha$. As the motion progresses, $N$ rotates through 180° about $M$, and $\alpha$ increases from 0 to $\pi$. We specify that mutually perpendicular unit vectors $\mathbf{n}_1$, $\mathbf{n}_2$, $\mathbf{n}_3$ be fixed in $N$ with $\mathbf{n}_2$ parallel to $L$ and $\mathbf{n}_3$ normal to $N$. Consequently, when the angular velocity of $B$ in $F$ is again expressed as in (9), the angular velocity of $N$ in $A$ is now given by

$$^{A}\boldsymbol{\omega}^{N} = -\dot{\alpha}\mathbf{a}_2 \tag{26}$$

and, to avoid twisting at the hip in a physically impossible manner, leg $B$ must rotate in $N$ with an angular velocity given by

$$^{N}\boldsymbol{\omega}^{B} = \dot{\alpha}\mathbf{n}_2. \tag{27}$$

Now, $^{F}\boldsymbol{\omega}^{A}$ and $^{A}\boldsymbol{\omega}^{N}$ can be resolved into components parallel to $\mathbf{n}_1$, $\mathbf{n}_2$, $\mathbf{n}_3$ with the aid of the relationships in Table 2,

Table 2

| | $\mathbf{a}_1$ | $\mathbf{a}_2$ | $\mathbf{a}_3$ |
|---|---|---|---|
| $\mathbf{n}_1$ | $\cos\beta_0\cos\alpha$ | $\sin\beta_0$ | $\cos\beta_0\sin\alpha$ |
| $\mathbf{n}_2$ | $-\sin\beta_0\cos\alpha$ | $\cos\beta_0$ | $-\sin\beta_0\sin\alpha$ |
| $\mathbf{n}_3$ | $-\sin\alpha$ | 0 | $\cos\alpha$ |

and substitution of (3), (26) and (27) into (9) then leads to

$$\left.\begin{aligned} \omega_1{}^B &= (\dot{\xi}-\dot{\alpha})\sin\beta_0 \\ \omega_2{}^B &= \dot{\alpha}+(\dot{\xi}-\dot{\alpha})\cos\beta_0 \\ \omega_3{}^B &= 0. \end{aligned}\right\} \tag{28}$$

Next, since $S^*$, the mass center of $S$, remains fixed in $A$ during phase 1, $\mathbf{v}_A$, the velocity of $A^*$ relative to $S^*$, and, consequently, $K_v{}^A$ are equal to zero. Finally, $\mathbf{v}_B$, the velocity of $B^*$ relative to $S^*$, can be expressed as

$$\mathbf{v}_B = \mathbf{v}^{B*/O} + \mathbf{v}^{O/S*} \tag{29}$$

where

$$\mathbf{v}^{B*/O} = {}^{F}\boldsymbol{\omega}^{B}\times(-b\mathbf{n}_2) \tag{30}$$

and, since $S^*$ is fixed in $A$ and lies on $Y$,

$$\mathbf{v}^{O/S*} = {}^{F}\boldsymbol{\omega}^{A}\times(a\mathbf{a}_3). \tag{31}$$

Referring to Table 2, one thus finds upon substitution of (30) and (31) into (29) that

$$\begin{aligned} \mathbf{v}_B = {} & [\dot{\xi}a+(\dot{\xi}-\dot{\alpha})b\sin\beta_0\sin\alpha]\mathbf{a}_1 \\ & -[(\dot{\xi}-\dot{\alpha})b\sin\beta_0\cos\alpha]\mathbf{a}_3. \end{aligned} \tag{32}$$

The kinetic energy during phase 1, found in part by substitution of (4), (6), (28), (8) and (32) into (2), is given by

$$\begin{aligned} K = {} & \dot{\xi}^2\Big[\frac{I^A}{2}+I_2{}^B+m_Ba^2+(I_1{}^B-I_2{}^B+m_Bb^2) \\ & \times\sin^2\beta_0+2m_Bab\sin\beta_0\sin\alpha\Big] \\ & -2\dot{\xi}\dot{\alpha}[I_2{}^B(1-\cos\beta_0)+(I_1{}^B-I_2{}^B \\ & +m_Bb^2)\sin^2\beta_0+m_Bab\sin\beta_0\sin\alpha] \\ & +\dot{\alpha}^2[2I_2{}^B(1-\cos\beta_0)+(I_1{}^B-I_2{}^B \\ & +m_Bb^2)\sin^2\beta_0]. \end{aligned} \tag{33}$$

Insertion of (33) into (1), elimination of time from the resulting equation of motion, and integration from 0 to $\pi$ leads to the following expression for $\Delta\xi_1$, the change in yaw during phase 1:

$$\Delta\xi_1 = \frac{\pi}{2} + \frac{2p_2 - p_1}{\sqrt{p_1^2 - 1}}\left[\frac{\pi}{2} - \tan^{-1}\left(\frac{1}{\sqrt{p_1^2-1}}\right)\right] \quad (34)$$

where

$$\left.\begin{aligned} p_1 &= \frac{\left(\frac{I^A}{2} + I_2^B + m_B a^2\right) + (I_1^B - I_2^B + m_B b^2)\sin^2\beta_0}{2m_B ab\sin\beta_0} \\ p_2 &= \frac{I_2^B(1-\cos\beta_0) + (I_1^B - I_2^B + m_B b^2)\sin^2\beta_0}{2m_B ab\sin\beta_0}\end{aligned}\right\} . \quad (35)$$

In phase 2, the legs move parallel to the pitch plane, and it is convenient to use as a variable the angle $\beta$ between $L$ and $M$ (or $L'$ and $M'$) as shown in Fig. 6. This angle ranges from $-\beta_0$ to $\beta_0$. Furthermore, it is now helpful to define $\mathbf{n}_1$, $\mathbf{n}_2$, $\mathbf{n}_3$ as mutually perpendicular unit vectors fixed in $B$ with $\mathbf{n}_2$ again parallel to $L$ and with $\mathbf{n}_1$ now parallel to the pitch plane.

${}^F\boldsymbol{\omega}^A$ and $K_\omega{}^A$ are again furnished by (3) and (4). Now, however, the angular velocity of $B$ in $F$ may be expressed as

$$ {}^F\boldsymbol{\omega}^B = {}^F\boldsymbol{\omega}^A + {}^A\boldsymbol{\omega}^B \quad (36)$$

where ${}^A\boldsymbol{\omega}^B$, the angular velocity of $B$ in $A$, is given by

$$ {}^A\boldsymbol{\omega}^B = \dot{\beta}\mathbf{a}_3. \quad (37)$$

Since the relationships between $\mathbf{a}_1$, $\mathbf{a}_2$, $\mathbf{a}_3$ and $\mathbf{n}_1$, $\mathbf{n}_2$, $\mathbf{n}_3$ are now

$$\left.\begin{aligned} \mathbf{a}_1 &= \cos\beta\mathbf{n}_1 - \sin\beta\mathbf{n}_2 \\ \mathbf{a}_2 &= \sin\beta\mathbf{n}_1 + \cos\beta\mathbf{n}_2 \\ \mathbf{a}_3 &= \mathbf{n}_3 \end{aligned}\right\} \quad (38)$$

${}^F\boldsymbol{\omega}^B$ can be expressed in the form of (5) with

$$\left.\begin{aligned} \omega_1{}^B &= \dot{\xi}\sin\beta \\ \omega_2{}^B &= \dot{\xi}\cos\beta \\ \omega_3{}^B &= \dot{\beta}. \end{aligned}\right\} \quad (39)$$

Next, $\mathbf{v}^A$ and, consequently, $K_v{}^A$ do not vanish. However $K_v{}^A$ is independent of $\dot{\xi}$ and, therefore, does not contribute to the equation of motion. Finally, $\mathbf{v}_B$ is again given by (29), and $\mathbf{v}^{B^*/O}$ by (30), but (31) must be replaced

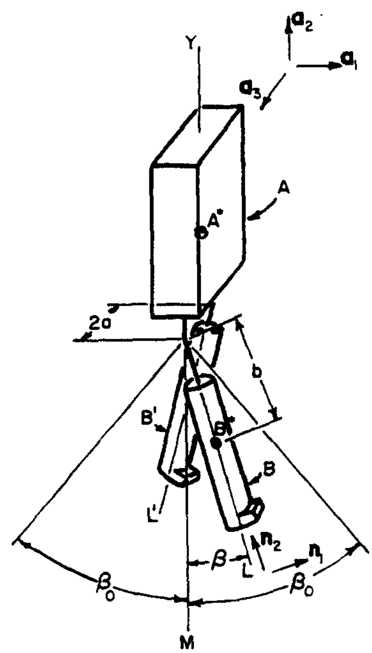


Fig. 6. Yaw motion model, phase 2.

with a relationship that reflects the motion of $S^*$ in $A$ and, thus

$$\mathbf{v}_B = (\dot{\xi}a + \dot{\beta}b\cos\beta)\mathbf{a}_1 + \left(\frac{m_A}{m_A + 2m_B}b\dot{\beta}\sin\beta\right) \times \mathbf{a}_2 - (\dot{\xi}b\sin\beta)\mathbf{a}_3. \quad (40)$$

The kinetic energy during phase 2 is obtained, in part, by substitution into (2) from (4), (6), (39), (8) and (40):

$$K=\dot{\xi}^2\left[\frac{I^A}{2}+m_Ba^2+I_2{}^B+(I_1{}^B-I_2{}^B+m_Bb^2)\times\sin^2\beta\right]+2\dot{\xi}\dot{\beta}\,[m_Bab\cos\beta]+\dot{\beta}^2\left[I_1{}^B+m_Bb^2\cos^2\beta+\left(\frac{m_Am_B}{m_A+2m_B}\right)b^2\times\sin^2\beta\right]. \tag{41}$$

It was mentioned earlier that the maneuver could be performed either with the arms or with the legs. Table 3 presents appropriate values of the inertia properties of an average man for the two cases.

The yaw obtained per cycle, $\Delta\xi$, varies as a function of the semi-angle of leg spread, $\beta_0$. In Fig. 7, $\Delta\xi$ is plotted as a function of $\beta_0$ for three cases: (1) the maneuver performed with the legs while the arms are held at the sides; (2) the maneuver performed with the arms while the legs are parallel to the yaw axis; and (3) the maneuver performed with the arms when a 5 lb weight is held in each hand.

Table 3

| Symbol | Leg maneuver (arms at sides) | Arm maneuver (legs parallel to yaw axis) | Units |
|---|---|---|---|
| $I^A$ | 0·519 | 0·413 | slug-ft² |
| $a$ | 0·253 | 0·664 | ft |
| $m_B$ | 0·836 | 0·289 | slugs |
| $b$ | 1.352 | 0.899 | ft |
| $I_1{}^B$ | 0·565 | 0.1331 | slug-ft² |
| $I_2{}^B$ | 0·0243 | 0·0030 | slug-ft² |

As before, substitution of (41) into (1), elimination of time, and, now, integration from $-\beta_0$ to $\beta_0$ leads to an expression for $\Delta\xi_2$, the change in yaw during phase 2:

$$\Delta\xi_2=\frac{-2q_1}{\sqrt{q_2}}\tan^{-1}\left(\frac{\sin\beta_0}{\sqrt{q_2}}\right) \tag{42}$$

where

$$\left.\begin{aligned} q_1&=\frac{m_Bab}{I_1{}^B-I_2{}^B+m_Bb^2}\\ q_2&=\frac{\frac{I^A}{2}+I_2{}^B+m_Ba^2}{I_1{}^B-I_2{}^B+m_Bb^2}.\end{aligned}\right\} \tag{43}$$

The total yaw rotation per cycle of the maneuver, $\Delta\xi$, is the sum of $\Delta\xi_1$ and $\Delta\xi_2$. With $p_1$, $p_2$, $q_1$, $q_2$ as defined in equations (35) and (43), the yaw per cycle (in rad.) is thus given by

$$\Delta\xi=\frac{\pi}{2}+\frac{2p_2-p_1}{\sqrt{p_1{}^2-1}}\left[\frac{\pi}{2}-\tan^{-1}\left(\frac{1}{\sqrt{p_1{}^2-1}}\right)\right]-\frac{2q_1}{\sqrt{q_2}}\tan^{-1}\left(\frac{\sin\beta_0}{\sqrt{q_2}}\right). \tag{44}$$

It can be seen that yaw increases monotoni-

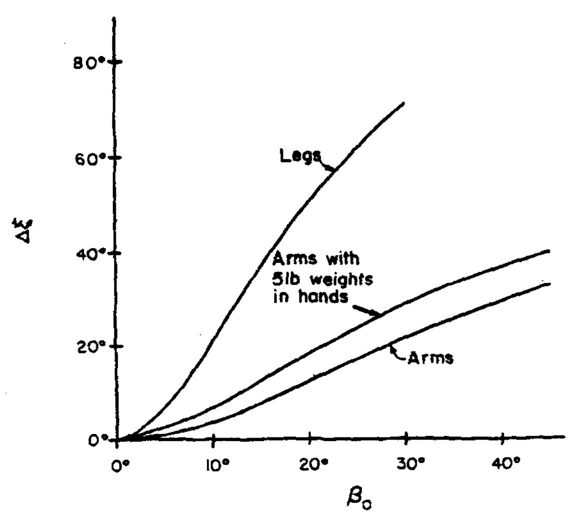


Fig. 7. Yaw reorientation as a function of $\beta_0$.

cally with $\beta_0$. Of course, the construction of the human hip and shoulder joints places an upper bound on $\beta_0$. For the legs, a maximum of 30° is reasonable, whereas for the arms $\beta_0$ may be as large as 45°. Consequently, realistic values for the maximum yaw obtained per cycle are 71° with the legs and 33° with the arms. Holding a 5 lb weight in each hand improves the performance, but not spectacularly, a reasonable upper limit for the yaw per cycle obtainable with 5 lb weights being 40°.

## CONCLUSION

Numerical results obtained from the foregoing analyses show that significant pitch and yaw rotations can be obtained by means of limb movements. By using his arms, the average man can produce about 30° of either pitch or yaw per cycle of such a maneuver. The legs can be used effectively in yaw, producing reorientations of about 70° per cycle; and hand-held weights may, but do not always, significantly enhance the effectiveness of a maneuver.

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