

Environmental Regulation of Livestock Production Contracts

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Abstract

In this paper we address the problem of the optimal regulation of a vertically integrated industry where the production of an environmentally polluting output is contracted with independent agents. The provision of production inputs is divided between the principal and the agents such that the resulting production externality is the consequence of their joint actions. The stylized facts of the model are reflective of the swine and poultry industries in the U.S. The main result shows that in a three-tier hierarchy (regulator-firm-agent) involving either a single-sided or a double-sided moral hazard problem, a principle of equivalence across regulatory schemes generally obtains. The only task for the regulatory agency is to determine the optimal total fiscal revenue in each state because any sharing of the regulatory burden between the firm and the agent would result in the same optimal solution. The equivalence result is upset only when the effects of regulation on the endogenous organizational choices of the industry are explicitly taken into account.

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JEL Classification: L51, K32, D82.

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1 Introduction

During the 1980's, a substantial increase in the number of environmental clean-up cases in the U.S. has been coupled by an increase in the entry rate of small judgment proof firms into hazardous sectors (Ringleb and Wiggins, 1990). This phenomenon occurred because firms, trying to minimize the liability exposure, segregated their risky activities in small corporations. As the claimants were restricted to the assets of the small corporation typically unable to pay the associated liability damages, such segregation was found valuable. This result exposed the inefficiency of the tort liability as a primary institutional form for dealing with large-scale, long-term environmental hazards.

Given this problem, some authors have investigated the design of optimal schemes for lender's liability in the case of judgment-proof firms (e.g., Pitchford, 1995; Boyer and Laffont, 1997; and Balkenborg, 2001). However, there has been noticeably less interest in addressing these problems in a standard regulation framework. Moreover, the papers that examined the environmental regulation, similar to the above literature on vicarious liability, focused only on cases where agents alone influence the level of pollution whereas the principal has little direct means for prevention or abatement.¹

In this paper we address the problem of the optimal regulation of a vertically integrated livestock industry in which the environmentally polluting production (grow-out) of live animals is contracted with independent agents². A distinct feature of these contracts is the fact that the provision of production inputs is divided between the principal and the agents such that the resulting environmental pollution is the consequence of their joint actions. The potential impact of livestock and poultry production on environmental quality has become a major concern in areas with high density of concentrated animal feeding operations (CAFOs). It is increasingly common for environmental advocacy groups to argue that contracting is an important cause of adverse environmental quality effects in livestock production, largely because contracting increases the scale of livestock operations, simultaneously reducing op-

¹For example, Chambers and Quiggin (1996) modelled a non-point source pollution problem as a multi-task principal-agent problem where the agents are independent farmers producing corn and polluting the environment and the principal is the regulatory agency.

²A theoretical general model of regulation in this kind of three-tier relationship was developed in Bontems, Dubois and Vukina (2003).

portunities for economics of scope in livestock utilization through reduced specialization.

The production and management of animal waste generates many potential external effects that need to be considered when designing regulatory policy. The most important one is the nutrient runoff and leaching from application of manure to cropland. Accidental spills and leaks from waste storage facilities and direct ambient air pollution from feedlots and storage facilities including odors and ammonia gases are also causing serious concerns. The problems associated with the design and implementation of environmental regulation of CAFOs are different than those related to regulating traditional family farms. In the later case the standard economic prescription of taxing the externality such that the polluter pays the environmental cost of his action is not feasible due the non-point source nature of the pollution problem. On the contrary, CAFOs are more similar to point source industrial polluters, hence some of the traditional regulatory instruments may prove to be adequate. However, the fact that a significant portion of CAFOs are in fact contract operations makes the design of the regulatory policy regimes substantially more complicated because the incidence of the regulatory compliance cost is not obvious.

In the U.S., to the extent that livestock and poultry production is subject to the federal water quality regulation, it is regulated under the Clean Water Act of 1972, 33 U.S.C.A. §§ 1251-1387 (2002). The most recent changes in regulation specific to livestock and poultry farming was introduced in February 2003 in the form of National Pollutant Discharge Elimination System Permit Regulation and Effluent Limitation Guidelines and Standards for Concentrated Animal Feeding Operations (CAFOs); Final Rule (EPA, 2003). EPA's Final Rule made significant modifications to the regulation of CAFOs while maintaining the basic regulatory structure. The major changes include a) the elimination of the 25-year, 24-hour storm discharge exemption, b) requirement that chicken operations that use dry manure handling systems obtain permits, and c) subjecting wastes applied to cropland and pastures under the control of the CAFO operator to permit requirements.

EPA estimates the total monetized social cost of the final regulations at about \$335 million annually in the pre-tax 2001 dollars. These costs include compliance costs borne by CAFOs and also administrative costs to federal and State governments. EPA estimates the

total compliance costs for large CAFOs at \$283 million per year, the administrative costs to federal and State governments \$9 million per year, and the remainder are the compliance costs to medium and small CAFOs. Across all livestock sectors, an estimated 285 or 3% of existing large CAFOs may be vulnerable to facility closure as a result of complying with final regulation. These results are based on an analysis that does not consider the long-term effects on market adjustment and also available cost share assistance from federal and State governments (EPA, 2003).

The increase in cost of regulatory compliance for CAFOs as a result of the Final Rule will occur mainly through mandating the adoption of certain environmentally friendlier production practices or pollution abatement technologies and administrative costs associated with permitting. However, to keep things general and reasonably simple, in this paper we use tax as a generic regulatory instrument. We model a trilateral relationship between the Environmental Protection Agency ($\star \star \star$), the contractor (firm) and a agent (producer) with the technology characterized by a joint production of output (live animal weight) and pollution (waste). We assume that output is observable and verifiable and hence contractible whereas pollution is observable but not verifiable and hence not contractible. From a theoretical point of view, this three-tier hierarchical model can be compared to the recent modelling of supervisory problem in a hierarchy (Faure-Grimaud, Laffont and Martimort, 2000, Faure-Grimaud and Martimort, 2001) where the principal (here the $\star \star \star$) uses an intermediary agent (here the principal) to regulate a final agent (here the producer). In our regulatory model, the information structure is quite simple with either single sided moral hazard or double sided moral hazard, and the use of taxation of both contracting parties can be interpreted as a way to improve the regulation of producers only.

We found that in this three-tier hierarchy involving either a single-sided or a double-sided moral hazard problem, the principle of equivalence across regulatory schemes mostly obtains. In both situations, regardless of the tax legal incidence, for a given amount of tax revenue, the regulator can obtain the same outcome. The $\star \star \star' \star$ only task is to determine the optimal total tax revenue in each state because any sharing of the tax burden between the principal and the agent would result in the same optimal solution. In this regard our

results provide an important extension of an earlier work by Segerson and Tietenberg (1992) who studied the structure of penalties in a three-tier hierarchy under the assumption of risk neutrality for all parties and moral hazard on the agent's side and showed that the efficient outcome can be reached by imposing a penalty on either party.

However, when the effects of regulation on the endogenous organizational choices of the industry are explicitly taken into account, the equivalence principle breaks down and the design of the optimal regulatory scheme becomes more complicated. When the regulator wants to foster contracting as a dominant mode of organizing livestock production, the optimal taxation scheme prescribes the minimal and maximal shares that the agent and the principal have to pay. In a situation where the $\star\star\star$ needs to simultaneously regulate independent producers and principal-agent contract organizations without being able to discriminate, a uniquely determined optimal division of the aggregate tax burden between the principal and the agent is necessary.

The rest of the paper is organized as follows. The next section is devoted to stylized facts on contracting in animal agriculture. The basic model is developed in section 3. In section 4 we analyze the case of a single-sided moral hazard in the relationship between the processor and the agent and in section 5 the case of a double-sided moral hazard. Section 6 investigates the consequences of endogenous organizational choice for the equivalence results obtained earlier. Concluding remarks are given in section 7.

2 Contracting in Animal Agriculture: Institutions and Technology

Agricultural contracts are an integral part of the production and marketing of ever increasing number of agricultural commodities. The growth in the use of contracts as means of organizing agricultural production has been staggering. In the U.S. in 1969, only 156,400 farms, or about 6% of all farms used production and/or marketing contracts. The value of production under contract totaled \$5.4 billion or nearly 12% of the total value of commodities sold. By 1993, the percentage of farms using production and/or marketing contracts had grown to about 11%, and the total value of production under contract arrangements had increased

almost nine times to \$47 billion or 32% of the total value of commodities sold. Currently, over one-third of the contracted value is produced in conjunction with production contracts, where the contractor retains ownership of the commodity (USDA/ERS, 1999). Production contracts specify in detail the division of production inputs supplied by the two parties, the quality and quantity of a particular commodity and the type of the remuneration mechanism for the grower.

Typical examples of production contracts are livestock contracts. A production contract is an agreement between a processing firm (also known as integrator) and a farmer (grower) that binds the farmer to specific production practices. Growers provide land, production facilities, utilities (electricity and water) and labor. Housing and waste handling units have to be constructed and equipped in strict compliance with the integrator's specifications. Growers are also fully responsible for compliance with federal, state and local environmental laws regarding disposal of dead animals and manure. An integrator company provides animals to be grown to processing weight, feed, medications and services of field men who supervise the adherence to the contract stipulations and provide production and management expertise. Typically, the company also owns and operates hatcheries, feed mills and processing plants, and provides transportation of feed and live animals. The integrator also decides on the volume of production both in terms of the rotations of batches on a given farm and the density of animals inside the house.

The most notable characteristic of modern livestock production systems based on contracts has been the shift to large-scale, intensive, specialized, confined animal operations. Opponents of such productions systems cite many negative environmental impacts of increased geographic concentration of manure stocks. Among various externalities generated by the production and management of animal waste, nutrient runoff and leaching and air quality problems (ammonia emissions) are the most pervasive ones. For both of those, nutrient management plays a critical role. The nutrients of greatest concern are nitrogen and phosphorus.

The amount of nutrients from animal waste that ends up deposited in the environment is directly related to the type of animals raised, the composition of animal feed, and the waste

management technology that farmers use. Once feed composition and the waste handling and storing technology is fixed, the amount of pollution (nutrient content in manure) generated by a particular type of animal (e.g., a sow, a feeder pig, or a finished hog) is more or less deterministic.

The amount of nitrogen in manure can be reduced by substituting synthetic amino-acids for crude proteins (corn, soybeans) in animal feed. When deciding on the feed rations, the integrator's objective is to minimize costs per pound of feed subject to a series of nutritional constraints. Consequently, the cost-efficiency of feed formulations is highly dependable on relative prices of competing inputs. When the prices of corn and soybeans are high, it may be actually profitable to replace crude proteins with synthetic ones. When the prices of feed grains are low, absent any regulatory or other incentives, responding only to price signals, the feed rations will be based on crude proteins with high nitrogen in manure.

Phytic acid is the predominant form of phosphorus in corn and soybeans. However, phytic acid phosphorus cannot be digested by the monogastric animals (pigs, poultry) and is excreted in manure. Therefore, producers add inorganic phosphorus to diets to meet nutritional requirements of these animals. One way to increase the availability of phytic acid phosphorus is to add phytase to the ration with an objective to convert phytic acid into inorganic phosphorus. When phytase is added to the diets, inorganic phosphorus supplementation can be reduced and still maintain optimal performance. The rations based on phytase are more expensive than the regular inorganic phosphorus diets. While the use of phytase in animal rations benefits the society by reducing phosphorus pollution, the cost is exclusively born by the integrators.

Different types of waste management practices that growers use can produce dramatically different outcomes. Nitrogen losses like volatilization, denitrification and nitrate leaching that occur between excretion and crop uptake can vary from as little as 30% in liquid pit systems where manure is injected into the soil to greater than 90% in multistage lagoons where manure is irrigated.³ On the other hand, phosphorus is not subject to losses between excretion and land application, so all phosphorus excreted by animals remains in the manure.

³For details see Vukina, 2003.

Excessive phosphorus levels are known to degrade surface water quality by causing algae blooms and accelerating the eutrophication process.

An obvious solution to manure nutrient management problem is the source reduction. Conceptually, the pollution can be reduced by restricting the output (for example via a tax on output), or by reducing the amount of unusable nutrients in feed by somehow regulating the nutrient content in feed. The former regulatory scheme is easily implementable because the output is readily observable by all interested parties. The later scheme is considerably more complicated because the precise feed composition is known only to the integrator and could be discovered by the growers and the regulator only after bearing the costs of laboratory analysis. The regulatory objective can be however achieved by providing the integrator with the incentives to use environmentally friendly feed instead of the traditional environmentally unfriendly mix, even when this type of feed is less productive (more costly) in terms of feed efficiency.

3 The Basic Model

3.1 Assumptions and notation

We consider an institutional structure with three players: the regulator ($\star \star \star$), the principal (\star) and the agent (\star). This corresponds to the case where an integrator firm contracts the production of live animals with independent producers (growers). The production of output generates a negative externality that is subject to $\star \star \star$'s regulation.

The production process can be described as follows. An agent exerts effort $\star \in \{\underline{\star}, \bar{\star}\}$ that is unobservable to the principal. Also the principal supplies a production input $\star \in \{\underline{\star}, \bar{\star}\}$. In the case of livestock production this input is animal feed. The principal can choose either good feed $\underline{\star}$ which is less efficient in the production of output (live weight) but environmentally friendlier or bad feed $\bar{\star}$ which is highly productive but more polluting. Effort \star and input \star together generate output $\star \in \{\underline{\star}, \bar{\star}\}$ with $\underline{\star} \leq \bar{\star}$ and pollution $\star = \star(\star) \in \{\underline{\star}, \bar{\star}\}$ with $\underline{\star} \leq \bar{\star}$.

Hence, the production technology depends on input \star provided by \star and effort \star provided

by \star and is described by the following stochastic process

$$\star(\bar{\star} | \star\star\star) = 1 - \star(\underline{\star} | \star\star\star) = \star(\star)\star(\star)$$

with the following normalization $\star(\bar{\star}) = 1$, $\star(\underline{\star}) = \star - 1$, and notation $\star(\underline{\star}) = \underline{\star}$ and $\star(\bar{\star}) = \bar{\star} - \underline{\star}$. Pollution is a production externality that is jointly determined with the production state of nature. We call the high state of nature the case where production and pollution are high and the low state of nature the case where they are low.

The cost of effort for the agent is normalized to \star , the level of effort. The input cost for the principal is $\star(\star)$ with the normalization $\star(\bar{\star}) = 0$ and $\star(\underline{\star}) = \star - 0$.

We assume that production is observable and verifiable which implies that it is contractible. In addition we assume that pollution is also observable but not verifiable implying that neither the $\star\star\star$ nor the principal can write contracts contingent on pollution damages. Note however that the optimal regulatory scheme would be exactly the same if pollution were verifiable because it is simply a joint outcome of the production process.

The principal and the agent contract the production of output \star . Because of the moral hazard problem, the wage \star received by the agent needs to be contingent on production. The contract is then simply $\{\star(\underline{\star}) = \underline{\star}\star(\bar{\star}) = \bar{\star}\}$. Before contracting between \star and \star occurs, the $\star\star\star$ commits to some regulatory scheme to control pollution. Given the pollution level, \star and \star are required to pay a fee to the $\star\star\star$ in the amount of $\star(\star) \in \{\star(\underline{\star}) = \underline{\star}\star(\bar{\star}) = \bar{\star}\}$ and $\star(\star) \in \{\star(\underline{\star}) = \underline{\star}\star(\bar{\star}) = \bar{\star}\}$. Total tax revenue is then $\star(\star) = \star(\star) + \star(\star) \in \{\star(\underline{\star}) = \underline{\star}\star(\bar{\star}) = \bar{\star}\}$. The objective of the $\star\star\star$ is to maximize the expected difference between the tax revenue and the environmental damage $\mathbb{E}(\star - \star)$.⁴

The agent's utility function is separable in consumption and effort and equal to $\star(\star - \star) - \star$ where \star is increasing concave and \star is the cost of effort. The principal's utility function is $\star(\star - \star - \star - \star(\star))$ where \star is also increasing concave. Both \star and \star are risk averse. The exogenous reservation utilities of the principal and the agent are respectively \star_0 and \star_0 . Throughout the paper, we will sometimes consider a special case where \star and \star have constant absolute risk aversion utility functions of the form $\star(\star) = -\frac{1}{\star_\star} \exp(-\star_\star \star)$

⁴This objective is quite general and can take into account any net social surplus obtained in each state of nature simply by redefining the value of \star .

and $\star(\star) = -\frac{1}{\star\star} \exp(-\star\star\star)$.

3.2 First-Best

The first-best outcome obtains in case where there is no asymmetry of information among the players, that is, both agent's effort and principal's input are observable by everybody. Assuming that it is socially optimal to always produce with low effort $\underline{\star}$ and environmentally friendly feed $\underline{\star}$, the regulatory agency can simply mandate the use of these inputs. In a standard procedure, the problem can be solved in two steps. First, solve the principal-agent optimal contracting problem given some regulatory scheme $\{\underline{\star}\star\star\star\star\star\}$ and then solve the $\star\star\star$'s problem taking into account the endogenous optimal contract from the first step. It is easy to see that perfect risk sharing between the principal and the agent is then achieved meaning that the payment to the agent $\bar{\star}_0(\bar{\star}\star\star)$ and $\underline{\star}_0(\underline{\star}\star\star)$ must solve the following equation

$$\frac{\star'(\bar{\star} - \star - \bar{\star}_0(\bar{\star}\star\star) - \star 1_{\star=\underline{\star}})}{\star'(\underline{\star} - \underline{\star} - \underline{\star}_0(\underline{\star}\star\star) - \star 1_{\star=\underline{\star}})} = \frac{\star'(\bar{\star}_0(\bar{\star}\star\star) - \bar{\star})}{\star'(\underline{\star}_0(\underline{\star}\star\star) - \underline{\star})} \star$$

Next, we study two cases, one in which input \star is observable by everybody and the other where \star is private information of the principal and hence unobservable by the $\star\star\star$ and the agent. In the first case there is a single moral hazard problem associated with the agent's effort, whereas in the second case there is a double moral hazard problem associated with the agent's effort and the principal's input.

4 Optimal Regulation under Single Moral Hazard

Absent any regulation, the principal would always selects the more polluting input $\bar{\star}$ that results in the environmental damage $\bar{\star}$. The regulatory intervention can be justified on the ground that some optimal scheme can implement the less polluting input $\underline{\star}$ resulting in $\underline{\star}$ at a cost lower than the expected benefit from damage reduction.⁵ We start by assuming that \star is observable by the $\star\star\star$ and therefore the principal is required to supply $\underline{\star}$. The other case will be studied in section 5.

⁵The remaining case where using the bad input $\bar{\star}$ is better for society is less interesting and will be ignored, although it can be solved in the same fashion as other problems in this paper.

With respect to effort \star , there are two possible cases. In the first case, the regulator wants to implement low effort $\underline{\star}$ which is in conflict with the principal's preference under no regulation scenario. In the second case, the $\star \star \star$ wants to implement $\bar{\star}$, which is the same effort level that \star would implement absent any regulation, hence there is no conflict of interest. The latter is of course the case if the damage is the same in each state and, by a continuity argument, when the gap between damages $\bar{\star}$ and $\underline{\star}$ is sufficiently low. Then, the $\star \star \star$'s problem is only to extract the principal's rent knowing his reservation utility \star_0 . The case of no conflict of interest is less interesting and will be ignored in the following.

We solve the problem in two steps. First, we solve the optimal contracting problem between \star and \star given some regulatory scheme $\{\underline{\star} \star \star \bar{\star} \star \star \bar{\star}\}$. Then, we solve the $\star \star \star$'s problem taking into account the endogenous optimal contract.

4.1 The optimal contract between \star and \star

With incentives to implement low effort $\underline{\star}$ and the mandate from the $\star \star \star$ to use the good input $\underline{\star}$, the principal's program is

$$\begin{aligned} \max_{\underline{\star} \star} \quad & \star \star \star (\bar{\star} - \star - \bar{\star} - \star) + (1 - \star \star) \star (\underline{\star} - \underline{\star} - \underline{\star} - \star) \\ \star \star \star \star \star \star (\bar{\star} - \bar{\star}) + (1 - \star \star) \star (\underline{\star} - \underline{\star}) - \underline{\star} & \geq \star_0 \quad (\star) \\ \star \star \star (\bar{\star} - \star) + (1 - \star \star) \star (\underline{\star} - \underline{\star}) - \underline{\star} & \geq \star \star \star (\bar{\star} - \bar{\star}) + (1 - \star \star) \star (\underline{\star} - \underline{\star}) - \bar{\star} \quad (\star) \end{aligned} \quad (1)$$

where \star and \star are Lagrange multipliers associated with the participation and incentive constraints. To avoid notational clutter, let $\star (\bar{\star} - \star - \bar{\star} - \star) = \bar{\star}$, $\star (\underline{\star} - \underline{\star} - \underline{\star} - \star) = \underline{\star}$, $\star (\bar{\star} - \bar{\star}) = \bar{\star}$, $\star (\underline{\star} - \underline{\star}) = \underline{\star}$, and $\star' (\bar{\star} - \star - \bar{\star} - \star) = \bar{\star}'$, $\star' (\underline{\star} - \underline{\star} - \underline{\star} - \star) = \underline{\star}'$, $\star' (\bar{\star} - \bar{\star}) = \bar{\star}'$, $\star' (\underline{\star} - \underline{\star}) = \underline{\star}'$. The program is concave, so that first order conditions are sufficient and give

$$\star = \frac{(1 - \star \star) \underline{\star} \bar{\star}' \underline{\star}' - \underline{\star} \bar{\star}'}{\underline{\star} - \bar{\star}} \quad (2)$$

$$\star = \frac{\star \star \star \underline{\star}' + (1 - \star \star) \underline{\star} \bar{\star}'}{\underline{\star} \bar{\star}'} \star_0 \quad (3)$$

because \star and \star are increasing.

The participation constraint is thus binding ($\star \star 0$), that is $\star \star (\bar{\star} - \underline{\star}) + \underline{\star} = \star_0 + \underline{\star}$. If, in addition, the incentive constraint is also binding, i.e. if $\star \star 0$ (which is the case provided

that $\bar{\star}$ is not too large),⁶ we obtain the following results

$$\begin{aligned}\underline{\star} &= \star_0 + \frac{\bar{\star}\underline{\star} - \underline{\star}\bar{\star}}{\bar{\star} - \underline{\star}} \\ \bar{\star} &= \underline{\star} + \frac{\bar{\star} - \underline{\star}}{\star(\bar{\star} - \underline{\star})} \star \underline{\star}\end{aligned}$$

since $\underline{\star} \star \bar{\star}$ and $\bar{\star} \star \underline{\star}$. Because the incentive constraint is binding, the net utility of the agent in the low state is lower than that in the high state (in the opposite case, $\bar{\star} \star \underline{\star}$, the incentive constraint would be trivially strictly satisfied). Notice that $\underline{\star}$ does not depend on \star whereas $\bar{\star}$ is decreasing in \star indicating that the more the good input reduces the expected production, the less powerful will be the incentives given to the agent. Moreover, looking at the way the contract depends on the taxes proposed by the $\star \star \star$, we get the following proposition:

Proposition 1 *The optimal wages $\bar{\star}^*$ and $\underline{\star}^*$ are such that $\bar{\star}^*$ depends only on $(\bar{\star} \star \star)$, $\underline{\star}^*$ depends only on $(\underline{\star} \star \star)$ and*

$$\frac{\star \bar{\star}^*}{\star \star} = 1 + \frac{\star \bar{\star}^*}{\star \star} = \frac{\bar{\star}_\star}{\bar{\star}_\star + \bar{\star}_\star} \in (0 \star 1) \quad (4)$$

$$\frac{\star \underline{\star}^*}{\star \underline{\star}} = 1 + \frac{\star \underline{\star}^*}{\star \underline{\star}} = \frac{\underline{\star}_\star}{\underline{\star}_\star + \underline{\star}_\star} \in (0 \star 1) \quad (5)$$

where $\bar{\star}_\star = -\frac{\bar{\star}''}{\bar{\star}'}$, $\bar{\star}_\star = -\frac{\bar{\star}''}{\bar{\star}'}$, $\underline{\star}_\star = -\frac{\underline{\star}''}{\underline{\star}'}$, $\underline{\star}_\star = -\frac{\underline{\star}''}{\underline{\star}'}$ are the rates of absolute risk aversion of \star and \star at consumption levels in good or bad states.

Agent's net wages $(\bar{\star}^* - \bar{\star}$ and $\underline{\star}^* - \underline{\star})$ depend only on the total tax revenue \star , that is

$$\begin{aligned}\frac{\star(\bar{\star}^* - \bar{\star})}{\star \star} &= \frac{\star(\bar{\star}^* - \bar{\star})}{\star \star} = \frac{\star(\bar{\star}^* - \bar{\star})}{\star \star} = \frac{-\bar{\star}_\star}{\bar{\star}_\star + \bar{\star}_\star} \in (-1 \star 0) \\ \frac{\star(\underline{\star}^* - \underline{\star})}{\star \underline{\star}} &= \frac{\star(\underline{\star}^* - \underline{\star})}{\star \underline{\star}} = \frac{\star(\underline{\star}^* - \underline{\star})}{\star \underline{\star}} = \frac{-\underline{\star}_\star}{\underline{\star}_\star + \underline{\star}_\star} \in (-1 \star 0)\end{aligned}$$

Similarly, the principal's profits $(\bar{\star} - \bar{\star} - \bar{\star}^* - \star$ and $\underline{\star} - \underline{\star} - \underline{\star}^* - \star)$ depend only on the total tax revenue \star

$$\begin{aligned}\frac{\star(\bar{\star}^* + \bar{\star})}{\star \star} &= \frac{\star(\bar{\star}^* + \bar{\star})}{\star \star} = \frac{\star(\bar{\star}^* + \bar{\star})}{\star \star} = \frac{\bar{\star}_\star}{\bar{\star}_\star + \bar{\star}_\star} \in (0 \star 1) \\ \frac{\star(\underline{\star}^* + \underline{\star})}{\star \underline{\star}} &= \frac{\star(\underline{\star}^* + \underline{\star})}{\star \underline{\star}} = \frac{\star(\underline{\star}^* + \underline{\star})}{\star \underline{\star}} = \frac{\underline{\star}_\star}{\underline{\star}_\star + \underline{\star}_\star} \in (0 \star 1)\end{aligned}$$

⁶Alternatively, the incentive constraint could be non-binding (for example, when $\bar{\star}$ tends to $+\infty$), in which case the incentive problem would disappear and the first-best would be achieved.

Proof. See Appendix A. ■

Proposition 1 provides a set of important results. First, wages in each state depend only on taxes corresponding to the same state. Second, an increase in taxes \star on \star increases the wage but the agent is not fully compensated. Therefore, when taxes increase, net wages decrease. Similarly, a reduction in taxes decreases wages but the wage reduction is less than the tax reduction. Third, the changes in wages with respect to taxes \star depend on the ratio of the absolute risk aversions of the agent and the principal. The more risk averse \star relative to \star , the less net wages respond to taxes on \star , that is the more insurance will the principal provide to agent's net wage against changing taxes. In other words, the principal absorbs the larger part of the net wage change coming from the changes in taxes when the agent is more risk averse. Fourth, wages also depend on taxes \star paid by the principal. Finally, the agent's net wage $\star - \star$ changes exactly in the same way with respect to taxes \star or \star . Actually, wages respond to the change in taxes in such a way that an increase in taxes $\bar{\star}$ can be compensated exactly by a decrease in $\bar{\star}$ to leave net wages $\bar{\star}^*(\bar{\star} \bar{\star}) - \bar{\star}$ unchanged. The same is true for the other state of nature. This result follows from $\frac{\bar{\star}^*}{\star\star} - \frac{\star\star^*}{\star\star} = \frac{\star\star^*}{\star\star} - \frac{\star\star^*}{\star\star} = 1$.

With the incentive constraint binding, i.e. $\star \star 0$, one can show the following inequality⁷

$$\frac{\bar{\star}'}{\star} \star \frac{\star'}{\star} \quad (6)$$

Setting $\bar{\star}_0(\bar{\star} \bar{\star})$ and $\star_0(\star \star)$ equal the wages that would accomplish the perfect risk sharing between \star and \star for a given regulatory scheme, the inequality (6) shows that wages $\bar{\star}^*(\bar{\star} \bar{\star})$ and $\star^*(\star \star)$ are such that $\bar{\star}^*(\bar{\star} \bar{\star}) \star \bar{\star}_0(\bar{\star} \bar{\star})$ or $\star^*(\star \star) \star \star_0(\star \star)$.

With CARA utility functions for \star and \star , (6) implies that $\bar{\star}^*(\bar{\star} \bar{\star}) - \star^*(\star \star) \star \bar{\star}_0(\bar{\star} \bar{\star}) - \star_0(\star \star)$. Therefore, the gap between wages in good and bad states is reduced compared to the case with the perfect risk sharing. Given the fact that the risk aversion coefficients are constant, we obtain

$$\frac{\bar{\star}^*}{\star\star} = \frac{\star\star^*}{\star\star} = 1 + \frac{\bar{\star}^*}{\star\star} = 1 + \frac{\star\star^*}{\star\star} = \frac{\star\star}{\star\star + \star\star} \in (0, 1)$$

and the optimal wage contracts are linear with respect to taxes, that is $\bar{\star}^*(\bar{\star} \bar{\star}) = \frac{\star\star}{\star\star + \star\star} \bar{\star} - \frac{\star\star}{\star\star + \star\star} \bar{\star} + \bar{\star}^*(0 \star 0)$ and $\star^*(\star \star) = \frac{\star\star}{\star\star + \star\star} \star - \frac{\star\star}{\star\star + \star\star} \star + \star^*(0 \star 0)$. In this case, $\bar{\star}^*(0 \star 0)$ and

⁷Notice that the reverse inequality in (6) is obtained when the principal wants to implement $\bar{\star}$. For example, this will happen if there is no regulation.

$\underline{x}^*(0,0)$ are the optimal wages with both taxes set to zero obtained as

$$\begin{aligned}\underline{x}^*(0,0) &= x^{-1}(x_0 + \frac{\bar{x}\underline{x} - \underline{x}\bar{x}}{\bar{x} - \underline{x}}) \\ \bar{x}^*(0,0) &= x^{-1}(\underline{x} + \frac{\bar{x} - \underline{x}}{x(\bar{x} - \underline{x})})x\end{aligned}$$

4.2 The Regulator's problem

Given the optimal contract between the principal and the agent, the x, x^* 's program is to choose taxes x and x^* to maximize the expected tax revenue net of environmental damage $\mathbb{E}(x - x^*)$, with $x = x + x^*$, subject to the participation constraint of the principal:

$$\begin{aligned}\max_{x, x^*} & \bar{x}(\bar{x} + x - x^*) + (1 - x)(\underline{x} + x - x^*) \\ & x x^* \bar{x}^* + (1 - x)x^* \geq x_0\end{aligned}\quad (7)$$

where $\bar{x}^* = x(\bar{x} - x - \bar{x}^*(\bar{x} - x) - x^*)$, $\underline{x}^* = x(\underline{x} - x - \underline{x}^*(\underline{x} - x) - x^*)$ and $\bar{x}^*(\bar{x} - x)$ and $\underline{x}^*(\underline{x} - x)$ are the solutions to (1).

The program being concave, the first order conditions are sufficient

$$x = x\{x(1 + \frac{\bar{x}x^*}{x x^*})\bar{x}^* + (1 - x)\frac{x x^*}{x x^*}\underline{x}^*\} \quad (8)$$

$$1 - x = x\{x\frac{\bar{x}x^*}{x x^*}\bar{x}^* + (1 - x)(1 + \frac{x x^*}{x x^*})\underline{x}^*\} \quad (9)$$

$$x = x\{x\frac{\bar{x}x^*}{x x^*}\bar{x}^* + (1 - x)\frac{x x^*}{x x^*}\underline{x}^*\} \quad (10)$$

$$1 - x = x\{x\frac{\bar{x}x^*}{x x^*}\bar{x}^* + (1 - x)\frac{x x^*}{x x^*}\underline{x}^*\} \quad (11)$$

Substituting conditions (4) and (5) from Proposition 1 into (8)-(11) we get

$$x = \frac{1}{(1 + \frac{\bar{x}x^*}{x x^*})\bar{x}^*} = \frac{\bar{x} + x}{\bar{x}} \frac{1}{\bar{x}^*} x_0 \quad (12)$$

$$\frac{\bar{x}^*}{\underline{x}^*} = \frac{\frac{x x^*}{x x^*}}{\frac{x x^*}{x x^*}} = \frac{x}{\bar{x}} \frac{\bar{x} + x}{\underline{x} + x} \quad (13)$$

indicating that the principal's participation constraint (7) is binding because $x > 0$.

Condition (13) enables us to derive several important results. First, in equilibrium the principal's ex post utility levels are insensitive to the choice of regulatory instruments selected by the x, x^* that is

$$\frac{\bar{x}^*}{x x^*} = \frac{x x^*}{x x^*} = \frac{\bar{x}^*}{x x^*} = \frac{x x^*}{x x^*} = \frac{\bar{x}^*}{x x^*} = \frac{x x^*}{x x^*}$$

Second, (13) also implies that

$$\frac{\frac{\star \mathbb{E} \star^*}{\star \star}}{\star \star} = \frac{\frac{\star \mathbb{E} \star^*}{\star \star}}{\star \star} = \frac{\frac{\star \mathbb{E} \star^*}{\star \star}}{\star \star}$$

$$\frac{\frac{\star \mathbb{E} \star^*}{\star \star}}{\star \star} = \frac{\frac{\star \mathbb{E} \star^*}{\star \star}}{\star \star} = \frac{\frac{\star \mathbb{E} \star^*}{\star \star}}{\star \star}$$

meaning that as far as the principal's expected utility is concerned, the choice of tax instruments in both states of nature is irrelevant. From the perspective of the principal, taxing either \star or \star contingently on the state of nature is equivalent. Finally, the ratio of marginal utilities of the principal in high and low states depends on the ratio of absolute risk aversions of the agent and the principal in high and low states.

If the principal's utility function \star exhibits increasing absolute risk aversion which includes in particular the CARA (Constant Absolute Risk Aversion) case, regardless of utility function \star , we have $\frac{\bar{\star}^*}{\underline{\star}^*} \geq 1$ ($= 1$ if CARA). Actually, if $\frac{\bar{\star}^*}{\underline{\star}^*} > 1$, then with increasing absolute risk aversion $\bar{\star}^* \geq \underline{\star}^*$ implying that $\frac{\bar{\star}^* + \bar{\star}^*}{\underline{\star}^* + \underline{\star}^*} \geq \frac{\bar{\star}^*}{\underline{\star}^*} > \frac{\bar{\star}^*}{\underline{\star}^*}$, because $\frac{\bar{\star}^*}{\underline{\star}^*} > \frac{\bar{\star}^*}{\underline{\star}^*}$ $\forall \star, \star, \star > 0$ which would imply $\frac{\bar{\star}^*}{\underline{\star}^*} > 1$. Moreover, because of the conflict of interest between $\star \star \star$ and \star about effort, we have $\frac{\bar{\star}^*}{\underline{\star}^*} > \frac{\bar{\star}^*}{\underline{\star}^*}$. Thus $1 > \frac{\bar{\star}^*}{\underline{\star}^*} > \frac{\bar{\star}^*}{\underline{\star}^*}$ implies that $\bar{\star} - \bar{\star}^* - \bar{\star}^*(\bar{\star}^* \bar{\star}^*) - \star \star \underline{\star} - \underline{\star}^* - \underline{\star}^*(\underline{\star}^* \underline{\star}^*) - \star$ and $\bar{\star}^*(\bar{\star}^* \bar{\star}^*) - \bar{\star}^* > \underline{\star}^*(\underline{\star}^* \underline{\star}^*) - \underline{\star}^*$ which implies $\bar{\star}^* + \bar{\star}^*(\bar{\star}^* \bar{\star}^*) - \underline{\star}^* - \underline{\star}^*(\underline{\star}^* \underline{\star}^*) > \bar{\star} - \underline{\star} > 0$ and therefore $\bar{\star}^* = \bar{\star}^* + \bar{\star}^* > \underline{\star}^* = \underline{\star}^* + \underline{\star}^*$. This implies that total taxes in the high state (higher production and higher pollution) are greater than that in the low state.

Using Proposition 1, we can show the following

Proposition 2 (Equivalence Principle - I) *Given some total tax revenue $(\bar{\star}^* \underline{\star}^*)$ that the $\star \star \star$ wants to raise, taxing \star or \star or both is equivalent. Any taxation scheme satisfying $\bar{\star} + \bar{\star} = \bar{\star}^*$ and $\underline{\star} + \underline{\star} = \underline{\star}^*$ results in the same outcome and generates the same utility levels to all parties and hence the same welfare level. $\star \star \star$'s regulation only determines the optimal total tax revenue in each state and any sharing of total optimal taxes between \star and \star results in the same optimal solution.*

Proof. See Appendix B. ■

This equivalence principle implies, for instance, that the optimal taxation scheme $(\bar{\star}^* \underline{\star}^*)$ can be implemented by taxing only \star ($\bar{\star}^* = \bar{\star}^*$, $\underline{\star}^* = \underline{\star}^*$), or \star ($\bar{\star}^* = \bar{\star}^*$, $\underline{\star}^* = \underline{\star}^*$). Also,

the $\star\star\star$ could subsidize \star and tax \star (for example, $\overline{\star}^* = -\star$, $\underline{\star}^* = -\star$, $\overline{\star}^* = \overline{\star} + \star$, $\underline{\star}^* = \underline{\star} + \star$). Consequently, only total tax revenue matters. When the total tax burden increases, it is shared between \star and \star according to their relative risk aversion (see Proposition 1). It is to be noted that this equivalence principle is quite strong and robust. In particular, it can be straightforwardly extended to a version of the model with many or continuous states of nature and more or continuous levels of effort.

Of course, if $\star\star\star$ values the tax revenue collected from the principal and the agent differently (for example because of different administrative costs) then the equivalence principle would disappear and designing the optimal regulatory scheme would require placing the full tax burden on the party for which tax collection is the least costly. The equivalence principle is also derived under the assumption that the principal and the agent agreed upon an optimal contract after the regulatory scheme was announced by the $\star\star\star$. Any rigidity or impediment in the implementation of this optimal contract would break the equivalence principle.

In the CARA case, (13) implies that $\star^{*'}(\overline{\star} - \overline{\star} - \overline{\star}^*(\overline{\star}\overline{\star}) - \star) = \star^{*'}(\underline{\star} - \underline{\star} - \underline{\star}^*(\underline{\star}\underline{\star}) - \star)$ that is

$$\begin{aligned}\overline{\star} - \underline{\star} &= (\overline{\star} - \underline{\star}) + (\overline{\star}^*(\overline{\star}\overline{\star}) - \underline{\star}^*(\underline{\star}\underline{\star})) \\ \overline{\star} - \underline{\star} &= \frac{\star\star}{\star\star + \star\star} [(\overline{\star} + \overline{\star}) - (\underline{\star} + \underline{\star})] + \overline{\star}^*(0\star 0) - \underline{\star}^*(0\star 0)\end{aligned}$$

Moreover, (6) implies $\frac{\star'}{\underline{\star}} \star 1$, that is, $\exp -\star\star [\overline{\star}^*(\overline{\star}\overline{\star}) - \underline{\star}^*(\underline{\star}\underline{\star}) - (\overline{\star} - \underline{\star})] \star 1$ implying $\overline{\star}^*(\overline{\star}\overline{\star}) - \overline{\star} \star \underline{\star}^*(\underline{\star}\underline{\star}) - \underline{\star}$. Therefore, net wage is lower in the high state case than in the low state case.

At this point, it is worth studying the case where the principal is risk neutral. It is frequently (but not always) the case that the principal offering production contracts to independent farmers is a large publicly traded company. Public companies are known to spread risk among their shareholders, so the assumption of risk neutrality is plausible in many situations. To carry out the analysis for the risk neutral case, we can simply set

$\star' \equiv 1$ (and $\star'' \equiv 0$) which implies that

$$\begin{aligned} \frac{\overline{\star\star}^*}{\star\star} &= \frac{\star\star^*}{\star\star} = 0 \\ \frac{\overline{\star\star}^*}{\star\star} &= \frac{\star\star^*}{\star\star} = 1\star \end{aligned}$$

These results show that the wages do not depend on taxes \star that the principal pays but rather only on taxes that the agent pays \star . Agent's net wages are constant with respect to all taxes. In contrast to the risk averse principal case where total taxes affect net wages, the $\star\star\star$'s taxation policy has no bearing on \star 's behavior and on his wage. The principal insures agent's revenue from any tax change although \star 's payment varies across states of nature.

5 Optimal Regulation under Double Sided Moral Hazard

So far in the paper, we have assumed that \star was observable by all parties and it was easy for the $\star\star\star$ to mandate the use of good input. However, if \star is unobservable, mandating the use of environmentally friendly input $\underline{\star}$ is not possible, the principal will always choose bad input $\overline{\star}$. If the regulatory agency wants to stimulate the use of good input it has to design a scheme such that the principal will voluntarily choose the good input.⁸

As before, we are going to analyze only the case with conflict of interest on effort. When there is a conflict of interest between \star and $\star\star\star$ on both effort \star and input \star , the $\star\star\star$ chooses taxes to induce the principal to implement $\underline{\star}$ and $\underline{\star}$. The principal's program ends up being the same as in (1) where \star was observable. Therefore, Proposition 1 still holds. However with unobservable \star , the $\star\star\star$'s problem is augmented by an additional incentive constraint that guarantees the correct response of the principal regarding the utilization of good input

$$\begin{aligned} \max_{\star\star\star} & \overline{\star\star}(\overline{\star} + \star - \star) + (1 - \star\star)(\underline{\star} + \underline{\star} - \underline{\star}) \\ & \star\star\star\star\star + (1 - \star\star)\star\star \geq \star_0 \quad (\star) \\ & \star\star\star\star\star + (1 - \star\star)\star\star \geq \star\star + (1 - \star)\star \quad (\star) \end{aligned} \tag{14}$$

⁸Of course, it will not always make sense for the $\star\star\star$ to give incentives to the principal to choose the good input because this can be too costly compared to the environmental benefits. Here we assume that it is always valuable to the $\star\star\star$ to elicit the use of environmentally friendly input.

For notational convenience, we use \star^{**} to indicate solutions when \star is not observable whereas we keep \star^* to label solutions of Section 4 where \star is observable. Following this convention, $\overline{\star}^{**} = \star (\overline{\star} - \overline{\star} - \overline{\star}^{**} - \star)$, $\underline{\star}^{**} = \star (\underline{\star} - \underline{\star} - \underline{\star}^{**} - \star)$, where $\overline{\star}^{**}$ and $\underline{\star}^{**}$ are the solutions to (1) and $\overline{\star}^* = \star (\overline{\star} - \overline{\star} - \overline{\star}^*)$, $\underline{\star}^* = \star (\underline{\star} - \underline{\star} - \underline{\star}^*)$ and wages $\overline{\star}^*$ and $\underline{\star}^*$ are the solutions to the principal's problem when he is not required to use the good input. These are the same wage functions as in Proposition 1 with $\star = 0$ and $\star = 1$.

Using Proposition 1, we can extend the equivalence principle to the unobservable input case:

Corollary 3 (*Equivalence Principle - II*) *Given some total tax revenue $\overline{\star}^{**}$, $\underline{\star}^{**}$ that the $\star\star\star$ wants to raise, taxing \star or \star or both is equivalent, that is, any taxation scheme satisfying $\overline{\star} + \star = \overline{\star}^{**}$ and $\underline{\star} + \star = \underline{\star}^{**}$ results in the same outcome and generates the same utility levels to all parties. The $\star\star\star$'s regulation only determines the optimal total tax revenue in each state and any sharing of total optimal taxes between \star and \star results in the same optimal solution.*

Proof. See Appendix C. ■

However, in spite of the validity of the equivalence principle in the unobservable input case, the optimal regulation will change. Using the first order conditions of the problem stated above and the results of Proposition 1, we state the following proposition:

Proposition 4 *The ratio of marginal effects of taxes on the principal's utility in high and low states satisfies*

$$\frac{\frac{\overline{\star\star}^{**}}{\star\star}}{\frac{\underline{\star\star}^{**}}{\star\star}} = \frac{\overline{\star\star}^{**}}{\underline{\star\star}^{**}} = \left(\frac{\star(1 - \underline{\star})}{1 - \star\underline{\star}} + \frac{\star(1 - \star)}{(1 - \star\underline{\star})(\star + \star\star \frac{\overline{\star\star}^{**}}{\star\star})} \right)^{-1} \quad (15)$$

$$1 \leq \frac{\overline{\star\star}^{**}}{\underline{\star\star}^{**}} = \frac{\frac{\overline{\star\star}^{**}}{\star\star} \star}{\frac{\underline{\star\star}^{**}}{\star\star} \underline{\star}} \star \frac{1 - \star\underline{\star}}{\star(1 - \underline{\star})}$$

and the ratio of marginal utilities satisfies

$$\frac{\frac{\underline{\star}^{**}}{\star} \frac{\overline{\star}^{**}}{\star} + \frac{\overline{\star}^{**}}{\star}}{\frac{\underline{\star}^{**}}{\star} \frac{\underline{\star}^{**}}{\star} + \frac{\underline{\star}^{**}}{\star}} \leq \frac{\overline{\star}^{**}}{\underline{\star}^{**}} \star \frac{1 - \star\underline{\star}}{\star(1 - \underline{\star})} \frac{\underline{\star}^{**}}{\star} \frac{\overline{\star}^{**}}{\star} + \frac{\overline{\star}^{**}}{\star}$$

Proof. See Appendix D. ■

Proposition 4 indicates that the ratio of marginal effects of taxes on \star 's utility in high and low states is bounded below by 1 and bounded above by $\frac{1-\star\star}{\star(1-\star)} = \frac{\star(\bar{\star}(\underline{\star}\star))}{\star(\underline{\star}(\bar{\star}\star))} \star 1$. Compared to the case where \star was observable, the additional incentive constraint introduces a distortion in the optimal taxation. Note that if $\star = 0$ (incentive constraint not binding) or $\star = 1$ (no difference between inputs $\bar{\star}$ and $\underline{\star}$ in the production process) in (15), then we obtain the same ratio as in the observable input case

$$\frac{\bar{\star}^{\star\star}}{\star^{\star\star}} = \frac{\frac{\star\star}{\star\star}}{\frac{\star\star}{\star\star}}$$

and the distortion disappears.

Notice also that the result about the inequality of total tax revenue in the two states of nature $(\bar{\star}^{\star} \star \underline{\star}^{\star})$ with non-decreasing absolute risk aversion of the principal and conflict of interest on effort still holds because $\frac{\bar{\star}^{\star\star}}{\star^{\star\star}} \frac{\bar{\star}^{\star\star} + \bar{\star}^{\star\star}}{\underline{\star}^{\star\star} + \underline{\star}^{\star\star}} \star \frac{\bar{\star}^{\star\star}}{\underline{\star}^{\star\star}}$ and $\frac{\bar{\star}^{\star\star}}{\underline{\star}^{\star\star}} \star \frac{\bar{\star}^{\star\star}}{\underline{\star}^{\star\star}}$. In the CARA case, equation (15) implies $\frac{\bar{\star}^{\star\star}}{\underline{\star}^{\star\star}} = \exp^{-\star\star} (\bar{\star} - \underline{\star} - (\bar{\star}^{\star\star} - \underline{\star}^{\star\star}) - (\bar{\star}^{\star\star} - \underline{\star}^{\star\star})) \star 1 = \frac{\bar{\star}^{\star\star}}{\underline{\star}^{\star\star}} = \exp^{-\star\star} (\bar{\star} - \underline{\star} - (\bar{\star}^{\star\star} - \underline{\star}^{\star\star}) - (\bar{\star}^{\star\star} - \underline{\star}^{\star\star})) \star$ that is, $\bar{\star} - (\bar{\star}^{\star\star} + \bar{\star}^{\star\star}) - (\underline{\star} - (\underline{\star}^{\star\star} + \underline{\star}^{\star\star})) \star \bar{\star} - (\bar{\star}^{\star\star} + \bar{\star}^{\star\star}) - (\underline{\star} - (\underline{\star}^{\star\star} + \underline{\star}^{\star\star})) = 0$ indicating that the utility of \star in high state is lower than in bad state $(\bar{\star}^{\star\star} \star \underline{\star}^{\star\star})$ whereas they are equal when \star is observable $(\bar{\star}^{\star} = \underline{\star}^{\star})$. The difference in net utility of \star between low and high state is increased in absolute value compared to the case where \star is observable. It means that in case where \star is not observable by the $\star\star\star$, the incentives to induce the implementation of the low effort are more powerful.

If the principal is risk neutral, then $\star = 1$ and $\star = 0$, meaning that the principal's participation constraint is binding but not the incentive constraint. This is to say that in the equilibrium the risk neutral principal strictly prefers to use the good input rather than the bad input. With risk neutrality of the principal the distortion introduced by the fact that the regulator cannot observe \star disappears.

6 Endogenous Contractual Organization and regulation

It is usually reasonable to assume that the regulator is the leader of the game in the sense of first proposing a regulatory scheme to which the principal and the agent optimally respond

by agreeing on a contract. In the analysis above, we have implicitly assumed that \star and \star would always sign a contract to jointly produce the output regardless of the regulation that the $\star\star\star$ imposed provided they get at least their exogenous reservation utility. The $\star\star\star$ took this optimal response into account but could not ex-post adjust the regulatory scheme it had committed to implement. As the contract signed between \star and \star is endogenous, the equivalence principle turns out to be a robust property of the optimal taxation scheme.

However, we have neglected the possibility that after observing the regulation, the parties to the contract may decide to go their separate ways instead of contracting and prefer to produce by themselves. When the regulatory agency is able to discriminate contract producers from independent producers, the optimal regulatory scheme would tax the parties contingently on whether they contract or independently produce. In this case, obviously the equivalence principle still holds. Nevertheless, when the contract producers cannot be distinguished from the independents (or if the output produced under contract cannot be disentangled from the output produced outside the contract), or if the law does not allow tax discrimination between contract and independent producers, then the regulator has to take into account that agents, after observing the regulatory scheme, may prefer to exit the contract with the principal and continue producing independently.

6.1 A regulation that induces contract participation

We consider first the interesting situation where the regulator may prefer contracts over independent production in the targeted industry. For example, it is conceivable that due to economies of scale in feed mixing, the marginal cost of supplying environmentally friendly feed for the integrator may be smaller than for the small independent producer. In this case the $\star\star\star$ would design a regulatory scheme which is incentive compatible with the endogenous choice to contract given that any agent has always the opportunity to produce independently and pay only taxes \star . Hence, the reservation utility of the contracting agent is endogenous as it depends on taxes \star .

Let us define a “contracting compatible” regulatory scheme as follows: When facing the regulation, agents should always prefer to produce under a contract with an integrator

rather than independently. If the agent produces independently his expected utility $\star_\star(\underline{\star}\overline{\star})$ is equal to

$$\star_\star(\underline{\star}\overline{\star}) = \max_{\star \in \{\underline{\star}\overline{\star}\} \star \in \{\underline{\star}\overline{\star}\}} \left\{ \star(\star) \star(\star) \star (\overline{\star} - \star - \star \mathbf{1}_{\star=\underline{\star}}) + (1 - \star(\star) \star(\star)) \star (\underline{\star} - \underline{\star} - \star \mathbf{1}_{\star=\underline{\star}}) - \star \right\}$$

which is clearly decreasing in $\underline{\star}$, $\overline{\star}$. The agent's reservation utility in the optimal wage contract between \star and \star writes now $\star_0(\underline{\star}\overline{\star}) = \max(\star_0 \star_\star(\underline{\star}\overline{\star}))$ but this does not change the properties of the optimal contract as described in Proposition 1. According to the equivalence principle, the optimal regulation under exogenous reservation utility is always implementable whatever the taxes $\underline{\star}\overline{\star}$ because only total taxes matter and increasing taxes on the agent can be compensated by the reduced taxes on the principal. Since $\star_\star(\underline{\star}\overline{\star})$ is decreasing in $\underline{\star}$ and $\overline{\star}$, it is always possible to choose taxes $(\underline{\star}\overline{\star})$ such that the agent's endogenous contract participation is trivially satisfied ($\star_0 \geq \star_\star(\underline{\star}\overline{\star})$). Then, one simply needs to choose taxes $(\underline{\star}\overline{\star})$ such that the sum of taxes in each state is equal to the optimal taxes required by optimal regulation.

Proposition 5 *In the optimal “contracting compatible” regulatory scheme, the agent has to pay a minimum tax. Formally, given the optimal total tax revenues $(\overline{\star}^* \star_\star^*)$, there exist two levels of taxation for the agent $\underline{\star}_{\min}^*$, $\overline{\star}_{\min}^*$ such that any taxation scheme $(\underline{\star}\overline{\star} \star_\star \star_\star)$ satisfying $\underline{\star} \geq \underline{\star}_{\min}^*$, $\overline{\star} \geq \overline{\star}_{\min}^*$ and $\overline{\star} = \overline{\star}^* - \underline{\star}$ and $\underline{\star} = \underline{\star}^* - \underline{\star}$ is optimal.*

This result indicates that here all shares of the total taxation scheme $(\overline{\star}^* \star_\star^*)$ between the principal and the agent are no longer optimal. Instead, the optimal scheme is described by a minimal share that the agent has to pay and consequently a maximal share that the principal has to pay.

6.2 Nondiscrimination between contract and independent producers

Let us consider now the situation where the $\star \star \star$ has to simultaneously regulate independent producers and principal-agent contracts without being able to discriminate. Consider first the regulation of independent producers only. Given the optimal contract between the principal and the agent, the $\star \star \star$'s problem is now to maximize the expected tax revenue

net of environmental damage under the incentive and participation constraints of the agent:

$$\max_{\underline{\star}, \underline{\star}} (\bar{\star} - \bar{\star}) + (1 - \star \underline{\star})(\underline{\star} - \underline{\star})$$

$$\begin{aligned} \star \underline{\star} (\bar{\star} - \bar{\star} - \star) + (1 - \star \underline{\star}) \star (\underline{\star} - \underline{\star} - \star) - \underline{\star} &\geq \star \bar{\star} (\bar{\star} - \bar{\star} - \star) + (1 - \star \bar{\star}) \star (\underline{\star} - \underline{\star} - \star) - \bar{\star} \\ \star \underline{\star} (\bar{\star} - \bar{\star} - \star) + (1 - \star \underline{\star}) \star (\underline{\star} - \underline{\star} - \star) - \underline{\star} &\geq \star 0 \end{aligned}$$

with \star observable. The incentive and participation constraints are binding and therefore the optimal taxes $\bar{\star}^*$, $\underline{\star}^*$ are uniquely determined as:

$$\begin{aligned} \underline{\star}^* &= \underline{\star}^* (\bar{\star} \star \underline{\star} \star \star \star \star \star \star 0 \star \star \star \underline{\star}) = \underline{\star} - \star - \star^{-1} \left(\star 0 + \underline{\star} + \frac{\underline{\star}(\bar{\star} - \underline{\star})}{\bar{\star} - \underline{\star}} \right) \\ \bar{\star}^* &= \bar{\star}^* (\bar{\star} \star \star \star \star \star \star \star \star 0 \star \star \star \underline{\star} \star \star) = \bar{\star} - \star - \star^{-1} \left(\star 0 + \underline{\star} + \frac{\underline{\star}(\bar{\star} - \underline{\star})}{\bar{\star} - \underline{\star}} + \frac{\bar{\star} - \underline{\star}}{\star(\bar{\star} - \underline{\star})} \right) \end{aligned}$$

The equivalence principle implies that optimal regulation can now be implemented in the absence of discrimination but in a unique way as follows:

Proposition 6 *The optimal regulation is uniquely determined such that taxes imposed on contracting agents are also the optimal taxes to be imposed on independent producers: $\underline{\star}^*$, $\bar{\star}^*$. The optimal tax imposed on the principal is the difference between the optimal total tax revenue in each state and the optimal tax imposed on the agents, that is $\bar{\star} = \bar{\star}^* - \bar{\star}^*$ and $\underline{\star} = \underline{\star}^* - \underline{\star}^*$.*

Like in the previous case, all shares of the total taxation scheme between the principal and the agent are no longer optimal, causing the equivalence principle to break down. Instead, there exists a unique optimal share of the aggregate tax burden ($\bar{\star}^* \star \underline{\star}^*$) between the principal and the contract producer. Finally, notice that here we have implicitly assumed that the optimal regulation scheme should preserve the industry structure intact because the regulator is not able to discriminate among producers. Consequently the taxes imposed by the $\star \star \star$ are such that producers will get the same expected utility regardless of whether they are contract operators or independent producers, so there is no incentive for any of them to switch to a different mode of organization.

7 Conclusion

In this paper we have studied the optimal regulation of a vertically integrated polluting industry characterized by private production contracts between firms and independent agents (producers). These contractual arrangements are typical in animal agriculture, notably in poultry and swine industries. The main result shows that in a three-tier hierarchy (regulator-firm-agent) involving either a single-sided or a double-sided moral hazard problem, a principle of equivalence across regulatory schemes generally obtains.

The analysis was carried out in two steps, first by looking at the situation where there is only a moral hazard problem on the agent's side and second where there is also a moral hazard problem on the firm's side. In both situations, regardless of the tax legal incidence, for a given amount of tax revenue, the regulator can obtain the same provision of inputs and effort. Once the $\star \star \star$ commits to a regulatory scheme, the private production contract between the firm (principal) and the producer (agent) is such that the ex-post utility levels of both parties do not depend on the particular structure of the taxation scheme. Hence, taxing only the principal or only the agent generates the same outcome from all parties' viewpoint. The only task of the $\star \star \star$ is to determine the optimal total tax revenue in each state because any sharing of the tax burden between the principal and the agent would result in the same optimal solution. The way the optimal wage changes with respect to taxes is intimately related to the relative risk aversion degree between the principal and the agent.

In the CARA case, we derive some comparative statics results. We show that the optimal taxation provides full insurance to the principal because he gets the same utility levels in the high state and in the low state of nature when his provision of input is observable. However, when the regulator can not observe the principal's provision of input, the full insurance situation is not attainable. The principal receives the higher utility level in the low state due to incentives required to induce him to use the good input and therefore increase the probability of obtaining the low pollution. This result is specific to the CARA case, but is nevertheless interesting. It shows that the cost of moral hazard with respect to effort is fully loaded on the agent when the principal's provision of input is observable, whereas the cost of moral hazard is shared (i.e., the principal is not fully insured) when the case of double-sided

moral hazard is considered.

It is also important to realize that neither the double-sided moral hazard nor the risk aversion impede the equivalence principle of regulatory schemes. Thus, our result can be seen as an extension of earlier results by Segerson and Tietenberg (1992) where they study the structure of penalties in a three-tier hierarchy under the assumption of risk neutrality for all parties and a moral hazard problem only on the agent's side. Their main result is that the efficient outcome can be reached by imposing a penalty on either party, which corresponds exactly to our equivalence result. Of course, the equivalence result relies heavily on the assumption that the contract between the firm and the agent can be optimally revised after any changes in the tax structure has been introduced.

The policy implications of the equivalence principle are important. It means that the EPA can implement the optimal regulation in different ways. Indeed, the optimal regulation is attainable with subsidies for one party and taxes for the other. What really matters is the total tax revenue and not the particular levels of taxes or subsidies levied on each party. However, the optimal total tax revenue that must be imposed on the contractual organization depends itself on the preferences of both parties, on their reservation utilities, and the parameters of the cost and production functions.

How can these results be interpreted in light of the new CAFO regulation? First notice that the new CAFO regulation did not fundamentally change the responsibilities of contracting parties for the provision of production inputs. Contract growers still have full responsibility for compliance with federal, state and local environmental laws regarding disposal of dead animals and animal waste. Consequently, the legal incidence of the increased costs of environmental compliance with the new CAFO rules falls entirely on contract growers. However, the economic incidence of this regulatory burden will be almost certainly different. How much different is impossible to say without analyzing individual contracts that govern particular relationships between growers and integrators. However, one can claim with certainty that, for a given total compliance cost increase, the welfare consequences for the integrator, contract growers, and the society (from the perspective of achieved environmental quality improvements) will be the same had the legal incidence of the compliance cost fallen

entirely on the integrator.

Finally, there are instances where the equivalence result breaks down and the design of the optimal regulatory scheme becomes substantially more subtle. One of such examples is the case where the effects of regulation on the endogenous organizational choices of the industry are explicitly taken into account. In case where the regulator may be interested in preserving contracts as a dominant mode of organizing livestock production, the taxation scheme needs to be modified such that it becomes incentive compatible with the agent's endogenous choice to contract in the presence of the alternative opportunity to produce independently. Contrary to the equivalence result obtained previously where all shares of the total taxation scheme between the principal and the agent were optimal, in this case the optimal scheme is described by the minimal and maximal shares that the agent and the principal have to pay respectively.

What are the situations where the equivalence principle of regulatory schemes would fail in the context of new CAFO regulation? Our results show that this will always happen in cases where there are some rigidities in the implementation of the optimal integrator-grower contracts, such as the limited liability or the bankruptcy constraint of the grower.

It is important to mention that prior to the passage of the Final Rule most people in the industry and in environmental circles anticipated that some form of shared responsibility for the removal and disposal of manure between the integrators and the growers will be implemented. To a big dismay of environmental groups, this did not happen. This issue of co-permitting may be important keeping in mind that most of the contract growers are in fact judgment proof firms. Facing increasingly stringent environmental regulation, growers are exposed to substantial risks of large penalties for environmentally hazardous disposal practices and especially catastrophic waste spills. Because growers generally have limited assets, the likelihood of bankruptcy is much larger for them than for the integrators who are large, sometimes publicly owned, companies. The potential insolvency can cause a reduction in care levels under strict liability because the contract operators would care only about the costs that they might actually have to pay. Also, wealthier growers may take greater care than poorer ones because they have more to lose and are less likely to escape paying damages

through bankruptcy.

Ignoring externalities associated with animal waste, the observed contracts should be efficient in the sense of maximizing joint integrator plus grower surplus. Also, absent any rigidities in contracting, the legal incidence of regulation (i.e. whose name is on the permit) should be irrelevant because any form of new regulation will be endogenized via a new (redefined) optimal contract. However, with the simultaneous presence of environmental externalities and grower's bankruptcy constraint, the legal incidence of regulation is no longer irrelevant but rather matters for efficiency. For the internalization of animal waste externalities where contract operators are judgment proof entities, co-permitting may in fact be required.

8 References

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A Proof of Proposition 1

To get the partial derivatives of $\bar{\star}^*$ and $\underline{\star}^*$ with respect to $\underline{\star}$, $\bar{\star}$, $\underline{\star}$, $\bar{\star}$, we differentiate the first order conditions with respect to $\underline{\star}$, $\bar{\star}$, $\underline{\star}$, $\bar{\star}$ and use them to replace \star and \star .

$$\begin{aligned}
\frac{\bar{\star}\bar{\star}^*}{\bar{\star}\bar{\star}} &= \frac{-\underline{\star}\bar{\star}''}{\underline{\star}\bar{\star}'' + [\star(\underline{\star} - \bar{\star}) + \star\underline{\star}]\bar{\star}''} = -\frac{\bar{\star}''\bar{\star}'}{-\bar{\star}''\bar{\star}' - \bar{\star}''\bar{\star}'} \\
\frac{\bar{\star}\bar{\star}^*}{\bar{\star}\bar{\star}} &= \frac{[\star(\underline{\star} - \bar{\star}) + \star\underline{\star}]\bar{\star}''}{\underline{\star}\bar{\star}'' + [\star(\underline{\star} - \bar{\star}) + \star\underline{\star}]\bar{\star}''} = \frac{-\bar{\star}''\bar{\star}'}{-\bar{\star}''\bar{\star}' - \bar{\star}''\bar{\star}'} \\
\frac{\bar{\star}\bar{\star}^*}{\bar{\star}\bar{\star}} &= \frac{\bar{\star}\bar{\star}^*}{\bar{\star}\bar{\star}} = 0 \\
\frac{\bar{\star}\bar{\star}^*}{\bar{\star}\bar{\star}} &= \frac{-(1 - \star\underline{\star})\bar{\star}''}{[\star\bar{\star}(\bar{\star} - \underline{\star}) + \star(1 - \star\underline{\star})]\bar{\star}'' + (1 - \star\underline{\star})\bar{\star}''} = -\frac{-\bar{\star}''\bar{\star}'}{-\bar{\star}''\bar{\star}' - \bar{\star}''\bar{\star}'} \\
\frac{\bar{\star}\bar{\star}^*}{\bar{\star}\bar{\star}} &= \frac{[\star\bar{\star}(\bar{\star} - \underline{\star}) + \star(1 - \star\underline{\star})]\bar{\star}''}{(1 - \star\underline{\star})\bar{\star}'' + [\star\bar{\star}(\bar{\star} - \underline{\star}) + \star(1 - \star\underline{\star})]\bar{\star}''} = \frac{-\bar{\star}''\bar{\star}'}{-\bar{\star}''\bar{\star}' - \bar{\star}''\bar{\star}'} \\
\frac{\bar{\star}\bar{\star}^*}{\bar{\star}\bar{\star}} &= \frac{\bar{\star}\bar{\star}^*}{\bar{\star}\bar{\star}} = 0
\end{aligned}$$

B Proof of Proposition 2

Given a taxation scheme $\bar{\star}$, $\bar{\star}$, $\underline{\star}$, $\underline{\star}$, denote $\bar{\star}^*$, $\underline{\star}^*$ solution of (1) and assume that the $\star\star\star$ can choose $\bar{\star}'$, $\bar{\star}'$, $\underline{\star}'$, $\underline{\star}'$ such that $\bar{\star}' + \bar{\star}' = \bar{\star} + \bar{\star}$ and $\underline{\star}' + \underline{\star}' = \underline{\star} + \underline{\star}$. Then, the principal can set wages $\bar{\star}'^*$, $\underline{\star}'^*$ such that $\bar{\star}'^* - \bar{\star}' = \bar{\star}^* - \bar{\star}$ and $\bar{\star}' + \bar{\star}'^* = \bar{\star} + \bar{\star}^*$, which is possible because $\bar{\star}' + \bar{\star}' = \bar{\star} + \bar{\star}$. Then, it is clear from (1) that the participation and incentive constraints are unchanged (because ex-post utility levels are unchanged) and the principal's objective is the same. Therefore, $(\bar{\star}'^*\bar{\star}'^*\bar{\star}'^*\bar{\star}'^*\bar{\star}'^*\bar{\star}'^*)$ implements the same outcome as $(\bar{\star}^*\bar{\star}^*\bar{\star}^*\bar{\star}^*\bar{\star}^*\bar{\star}^*)$. Actually, taxes \star and \star are perfect substitutes in the $\star\star\star$'s objective (only total taxes matter), wages \star and taxes \star are also perfect substitutes in the principal's objective, and optimal wages chosen by \star are such that net wages $(\star - \star)$ of the agent are constant (do not depend on taxes \star and \star). Therefore, the same outcome can be implemented with $\bar{\star} = \underline{\star} = 0$ or $\underline{\star} = \bar{\star} = 0$ (in fact, there is an infinity of solutions in $(\underline{\star}\bar{\star}\bar{\star}\bar{\star})$ including the cases where $\underline{\star} = \bar{\star} = 0$ or $\underline{\star} = \bar{\star} = 0$). Therefore, $\star\star\star$'s regulation problem can be written as

$$\begin{aligned}
&\max_{\bar{\star}\bar{\star}} \bar{\star}(\bar{\star} - \bar{\star}) + (1 - \star\underline{\star})(\underline{\star} - \underline{\star}) \\
&\star\star\star \bar{\star}\bar{\star}^* + (1 - \star\underline{\star})\bar{\star}^* \geq \star_0 \quad (\star)
\end{aligned}$$

where $\bar{\star}^* = \star (\bar{\star} - \bar{\star}^*(\bar{\star}) - \star)$, $\underline{\star}^* = \star (\underline{\star} - \underline{\star}^*(\underline{\star}) - \star)$, $\bar{\star}^*(\bar{\star}) = \bar{\star}^*(\bar{\star} \star \bar{\star}) + \bar{\star}$, and $\underline{\star}^*(\underline{\star}) = \underline{\star} + \underline{\star}^*(\underline{\star} \star \underline{\star})$ are total charges, $\bar{\star}^*(\bar{\star} \star \bar{\star})$ and $\underline{\star}^*(\underline{\star} \star \underline{\star})$ are solutions to (1).

C Proof of Corollary 3

The proof is similar as in section B except that now the $\star \star \star$'s program has an additional incentive constraint. However, it is clear, with the same arguments on the optimal wage contract between \star and \star which keeps the same properties, that given a taxation scheme $\bar{\star}, \bar{\star}, \underline{\star}, \underline{\star}$, the $\star \star \star$ can choose $\bar{\star}', \bar{\star}', \underline{\star}', \underline{\star}'$ such that $\bar{\star}' + \bar{\star}' = \bar{\star} + \bar{\star}$ and $\underline{\star}' + \underline{\star}' = \underline{\star} + \underline{\star}$ without changing any of the net utility levels of \star and \star . The participation and incentive constraints of \star are unchanged (because ex-post utility levels are unchanged) and the principal's objective is the same. Moreover, $\bar{\star}^*, \underline{\star}^*, \bar{\star}^{**}, \underline{\star}^{**}$ are unchanged with $(\bar{\star} \star \bar{\star} \star \underline{\star} \star \underline{\star})$ or $(\bar{\star}' \star \bar{\star}' \star \underline{\star}' \star \underline{\star}')$. This implies also that the Lagrange multipliers \star and \star will not change and $\star \star \star$'s regulation problem can be written as function of total taxes only

$$\begin{aligned} \max_{\star \star} & \quad \bar{\star}(\bar{\star} - \bar{\star}) + (1 - \star \underline{\star})(\underline{\star} - \underline{\star}) \\ \star \star \star \star \star \star & \quad \bar{\star}^{**} + (1 - \star \underline{\star})\underline{\star}^{**} \geq \star_0 \quad (\star) \\ \star \star \star & \quad \bar{\star}^{**} + (1 - \star \underline{\star})\underline{\star}^{**} \geq \underline{\star} \star + (1 - \underline{\star})\underline{\star}^* \quad (\star) \end{aligned}$$

D Proof of Proposition 4

The program being concave, first order conditions are sufficient

$$\star \underline{\star} = (\star + \star) \left\{ \star \underline{\star} \left(1 + \frac{\star \bar{\star}^{**}}{\star \star} \right) \bar{\star}^{**'} + (1 - \star \underline{\star}) \frac{\star \underline{\star}^{**}}{\star \star} \underline{\star}^{**'} \right\} \quad (16)$$

$$\begin{aligned} & - \star \left\{ \underline{\star} \left(1 + \frac{\star \bar{\star}^*}{\star \star} \right) \bar{\star}^{*'} + (1 - \underline{\star}) \frac{\star \underline{\star}^*}{\star \star} \underline{\star}^{*'} \right\} \\ 1 - \star \underline{\star} & = (\star + \star) \left\{ \star \underline{\star} \frac{\star \bar{\star}^{**}}{\star \underline{\star}} \bar{\star}^{**'} + (1 - \star \underline{\star}) \left(1 + \frac{\star \underline{\star}^{**}}{\star \underline{\star}} \right) \underline{\star}^{**'} \right\} \quad (17) \\ & - \star \left\{ \underline{\star} \frac{\star \bar{\star}^*}{\star \underline{\star}} \bar{\star}^{*'} + (1 - \underline{\star}) \left(1 + \frac{\star \underline{\star}^*}{\star \underline{\star}} \right) \underline{\star}^{*'} \right\} \end{aligned}$$

$$\begin{aligned} \star \underline{\star} & = (\star + \star) \left\{ \star \underline{\star} \frac{\star \bar{\star}^{**}}{\star \star} \bar{\star}^{**'} + (1 - \star \underline{\star}) \frac{\star \underline{\star}^{**}}{\star \star} \underline{\star}^{**'} \right\} \quad (18) \\ & - \star \left\{ \underline{\star} \frac{\star \bar{\star}^*}{\star \star} \bar{\star}^{*'} + (1 - \underline{\star}) \frac{\star \underline{\star}^*}{\star \star} \underline{\star}^{*'} \right\} \end{aligned}$$

$$\begin{aligned} 1 - \star \underline{\star} & = (\star + \star) \left\{ \star \underline{\star} \frac{\star \bar{\star}^{**}}{\star \underline{\star}} \bar{\star}^{**'} + (1 - \star \underline{\star}) \frac{\star \underline{\star}^{**}}{\star \underline{\star}} \underline{\star}^{**'} \right\} \quad (19) \\ & - \star \left\{ \underline{\star} \frac{\star \bar{\star}^*}{\star \underline{\star}} \bar{\star}^{*'} + (1 - \underline{\star}) \frac{\star \underline{\star}^*}{\star \underline{\star}} \underline{\star}^{*'} \right\} \end{aligned}$$

From proposition 1 we know that $\frac{\overline{\star\star}^*}{\star\star} = \frac{\overline{\star\star}^*}{\star\star} = \frac{\star\star^*}{\star\star} = \frac{\star\star^*}{\star\star} = 0$, $\frac{\overline{\star\star}^{**}}{\star\star} = \frac{\star\star^{**}}{\star\star} = \frac{\star\star^{**}}{\star\star} = \frac{\star\star^{**}}{\star\star} = 0$, $\frac{\star\star^{**}}{\star\star} - \frac{\star\star^{**}}{\star\star} = \frac{\star\star^{**}}{\star\star} - \frac{\star\star^{**}}{\star\star} = -1$ and $\frac{\star\star^*}{\star\star} - \frac{\star\star^*}{\star\star} = \frac{\star\star^*}{\star\star} - \frac{\star\star^*}{\star\star} = -1$ therefore constraints (16) and (18) are the same and (17) and (19) are the same. Simplifying, we get

$$\begin{aligned}\star &= (\star + \star) \star \frac{\star\star^{**} \overline{\star\star}^{**}}{\star\star} - \star \frac{\star\star^{**} \overline{\star\star}^{**}}{\star\star} \\ 1 - \star\star &= (\star + \star)(1 - \star\star) \frac{\star\star^{**}}{\star\star} \star^{**'} - \star(1 - \star\star) \frac{\star\star^{**}}{\star\star} \star^{**'}\end{aligned}$$

Using also $\frac{\star\star^{**} \overline{\star\star}^{**}}{\star\star} \star^{**'} = \frac{\star\star^{**}}{\star\star} \star^{**'}$, we get

$$\begin{aligned}\star + \star &= \frac{1 - \star}{(1 - \star\star) \frac{\star\star^{**}}{\star\star} \star^{**'} - \star(1 - \star\star) \frac{\star\star^{**} \overline{\star\star}^{**}}{\star\star} \star^{**'}} \\ \star &= \frac{\frac{\star\star^{**}}{\star\star} \star^{**'} - \frac{\star\star^{**} \overline{\star\star}^{**}}{\star\star} \star^{**'}}{\star(1 - \star\star) \frac{\star\star^{**} \overline{\star\star}^{**}}{\star\star} \star^{**'} - (1 - \star\star) \frac{\star\star^{**}}{\star\star} \star^{**'}} \frac{\star(1 - \star\star)}{\frac{\star\star^{**} \overline{\star\star}^{**}}{\star\star} \star^{**'}}\end{aligned}$$

That is

$$\star = \frac{\star(1 - \star\star) \left(\frac{\star\star^{**}}{\star\star} \star^{**'} - \frac{\star\star^{**} \overline{\star\star}^{**}}{\star\star} \star^{**'} \right) + (1 - \star) \frac{\star\star^{**} \overline{\star\star}^{**}}{\star\star} \star^{**'}}{\left[(1 - \star\star) \frac{\star\star^{**}}{\star\star} \star^{**'} - \star(1 - \star\star) \frac{\star\star^{**} \overline{\star\star}^{**}}{\star\star} \star^{**'} \right] \frac{\star\star^{**} \overline{\star\star}^{**}}{\star\star} \star^{**'}}$$

As $\star(1 - \star\star) \star (1 - \star\star)$, if $\frac{\star\star^{**}}{\star\star} \star^{**'} \star \frac{\star\star^{**} \overline{\star\star}^{**}}{\star\star} \star^{**'}$ then $(1 - \star\star) \frac{\star\star^{**} \overline{\star\star}^{**}}{\star\star} \star^{**'} \star (1 - \star\star) \frac{\star\star^{**}}{\star\star} \star^{**'}$ implying $\star \star = 0$ which is not possible. Therefore $\frac{\star\star^{**}}{\star\star} \star^{**'} \star \frac{\star\star^{**} \overline{\star\star}^{**}}{\star\star} \star^{**'}$. In addition, it must be that $(1 - \star\star) \frac{\star\star^{**} \overline{\star\star}^{**}}{\star\star} \star^{**'} \star (1 - \star\star) \frac{\star\star^{**}}{\star\star} \star^{**'}$ to get $\star \geq 0$.

We have $\star = (\star + \star) \star \frac{\star\star^{**} \overline{\star\star}^{**}}{\star\star} \star^{**'} - \star \frac{\star\star^{**} \overline{\star\star}^{**}}{\star\star} \star^{**'}$ and $\frac{\star\star^{**} \overline{\star\star}^{**}}{\star\star} \star^{**'} = \frac{\star(1 - \star\star)}{1 - \star\star} - \frac{\star(\star - 1)}{(1 - \star\star)(\star + \star\star) \frac{\star\star^{**}}{\star\star}}$ then

$$\frac{\star\star^{**} \overline{\star\star}^{**}}{\star\star} \star^{**'} = \frac{\star\star^{**}}{\star\star} \left(\frac{\star(1 - \star\star)}{(1 - \star\star)} + \frac{\star(1 - \star)}{(1 - \star\star)(\star + \star\star) \frac{\star\star^{**}}{\star\star}} \right)^{-1} \geq \frac{\star\star^{**}}{\star\star} = \frac{\star\star^{**} \overline{\star\star}^{**} + \overline{\star\star}^{**}}{\star\star \star\star^{**} + \star\star^{**}}$$

In the CARA case,

$$\begin{aligned}\star + \star &= \frac{1 - \star}{(1 - \star\star) \frac{\star\star^{**}}{\star\star} \star^{**'} - \star(1 - \star\star) \frac{\star\star^{**} \overline{\star\star}^{**}}{\star\star} \star^{**'}} \\ \star &= \frac{\frac{\star\star^{**}}{\star\star} \star^{**'} - \frac{\star\star^{**} \overline{\star\star}^{**}}{\star\star} \star^{**'}}{\star(1 - \star\star) \frac{\star\star^{**} \overline{\star\star}^{**}}{\star\star} \star^{**'} - (1 - \star\star) \frac{\star\star^{**}}{\star\star} \star^{**'}} \frac{\star(1 - \star\star)}{\frac{\star\star^{**} \overline{\star\star}^{**}}{\star\star} \star^{**'}}\end{aligned}$$

That is

$$\star = \frac{\star(1 - \star\star) \left(\frac{\star\star^{**}}{\star\star} \star^{**'} - \frac{\star\star^{**} \overline{\star\star}^{**}}{\star\star} \star^{**'} \right) + (1 - \star) \frac{\star\star^{**} \overline{\star\star}^{**}}{\star\star} \star^{**'}}{\left[(1 - \star\star) \frac{\star\star^{**}}{\star\star} \star^{**'} - \star(1 - \star\star) \frac{\star\star^{**} \overline{\star\star}^{**}}{\star\star} \star^{**'} \right] \frac{\star\star^{**} \overline{\star\star}^{**}}{\star\star} \star^{**'}}$$