Multitask Moral Hazard, Incentive Contracts and Land Value

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Abstract

Sharecropping theory generally does not take into account land fertility. We develop a repeated Principal-Agent model under moral hazard where the Principal delegates the use and maintenance of a productive asset. In a multitask framework, we characterize the optimal spot contract. One of the main messages for land tenancy is that in a relationship where long term commitment between a landlord and a non monitored tenant is not possible, moral hazard on the peasant's multiple actions leads to non efficient effort provision both on production and land quality maintenance. Moreover, the land fertility maintenance tasks may not always mitigate optimal incentives but can also raise them depending on the congruence (substitutability or complementarity) of the multiple productive and investment tasks. Several important issues for development economics are discussed like the effect of technological innovation on efficiency of land tenancy.

Key words : moral hazard, incentive contracts, land value, soil conservation, sharecropping.

JEL classification : D82, O13, Q15, Q24

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1 Introduction

Sharecropping has drawn many economists' interest for a long time. Perennial empirical observation of this agricultural organization brought up a lot of theoretical questions on its efficiency. Stiglitz (1974) pointed out that share contracts stand for second best optimal choice under moral hazard resulting from a trade-off between incentives to work and risk sharing with a risk averse tenant. This paper shows what makes a difference when the landlord takes into account the long term effects of peasant's labor on the land and the multitask nature of agriculture is recognized. Adam Smith (1776) yet pointed out the importance of the sharecropper's short term behavior[1]:

"It could never, however, be to the interest of this last species of cultivators [the metayer] to lay out, in the further improvement of the land, any part of the little stock which they might save from their own share of the produce, because the lord, who laid out nothing, was to get one-half of whatever it produced. The tithe, which is but a tenth of the produce, is found to be a very great hindrance to improvement. A tax, therefore, which amounted to one-half must have been an effectual bar to it. It might be the interest of a metayer to make the land produce as much as could be brought out by means of the stock furnished by the proprietor; but it could never be his interest to mix any part of his own with it."

Share-tenancy models generally assume that the landlord's objective is to maximize his expected utility for a crop season depending on his net benefit, his labor supply and the contract shape (Stiglitz, 1974, Bardhan and Srinivasan, 1971, Eswaran and Kotwal, 1985, and surveys of Singh, 1989, or Chuma, Hayami and Otsuka, 1992). Despite the study of some long term contracts between landowners and landless peasants (see Dutta, Ray, Sengupta, 1989, where infinitely repeated relationships with threats of eviction are examined, or Bose, 1993), most models are static. Allen and Lueck (1992) study the contract choice between cash rent and crop share in a static simple model putting forward the idea that a share contract can curbs the farmer's incentive to exploit land attributes. Other models with multiple labor inputs have a quite different perspective (Bardhan and Srinivasan, 1971, Braverman and Stiglitz, 1982, 1986, Bardhan, 1984, Eswaran and Kotwal, 1985, Bose, 1993,

[1] This quote of Adam Smith was already cited by Johnson (1950).

Allen and Lueck, 1993-a&b, Roumasset and Uy, 1987, or Fafchamps, 1993). In Bardhan and Srinivasan (1971) or Eswaran and Kotwal (1985), both landlord and tenant provide some labor input. In Roumasset and Uy (1987), a model with an investment task, a production one and two periods studies the reduction of agency costs by monitoring. Bardhan (1984 - chap. 7), Braverman and Stiglitz (1986) have a sharecropping model with a fertilizer input and a non observable labor effort. They determine the efficient incentives on both separable inputs by the mean of production sharing and cost sharing. Bardhan (1984 - chap. 8) shows with a two period model the trade-off between production incentives, enhanced in first period by a dismissal threat from the landlord, and land improvement incentives decreasing with a more powered contract. In Fafchamps (1993), two inputs are introduced sequentially.
We consider the case of full delegation where the tenant takes all decisions simultaneously. Because all actions are unobservable for the landlord, cost sharing is not possible. In the multitask moral hazard problem of Holmström and Milgrom (1987, 1991), the principal's behavior is static and in the application to agriculture by Luporini and Parigi (1992), neither input is like an investment decision because they consider two distinct production tasks that are a subsistence crop and a cash crop. In the repeated moral hazard theory (Rogerson (1985-a) and Lambert (1983)), agent's actions don't modify future production technology. Fudenberg, Holmström and Milgrom (1990) and Malcomson and Spinnewyn (1988) study the possibility in a repeated moral hazard relationship to implement long term contracts, which are Pareto superior to short term agreements (Radner, 1985, Rubinstein and Yaari, 1983), by spot contracts sequences.

Therefore, nothing in the literature, turns out to consider the situation modelled here where repeated contracts under multitask moral hazard on production and investment are used. In Dubois (2002), only one task is modelled and an empirical application shows the relevance of the trade-off involving not only risk sharing and incentives to produce but also counter-incentives on land overuse due to the non-contractibility of land quality.
The present paper is meant to study the theoretical background of such facts when more general production patterns are considered, in particular because of the multitask nature of agriculture. We then derive interesting implications of such a theoretical model not underlined in the previous literature.

Contracts are short term contracts and incomplete because land fertility is assumed to be non contractible. Agricultural activity during a crop season affects future productions because land fertility is governed by an investment function and therefore depends on past inputs. We derive the notion of land value in the principal's objective. We characterize the optimal second best contract and compare it to the first best with a given exogenous land value function. We find what restrictive necessary conditions enable the first best to be implemented with imperfect information in the case of risk neutrality. The second best linear contract is different from Stiglitz's one (Stiglitz, 1974). For example, risk neutrality being assumed, it is not a fixed rent. High powered or low powered incentives (in the sense of Williamson, 1985) can be optimal depending on the congruence between the production and land investment multidimensional functions. As in Baker (1992), the gap between the principal's objective and the agent's performance measure leads to distortions of efforts in the second best case. Comparative statics and dynamics of production and land quality can be derived in this framework. Some discussions about the production level and the fertility path dynamics induced by land tenancy follow as well as other interpretations of this principal-agent model. Cycles, stagnation or development of land quality and agricultural production can be obtained and explained.

Section 2 presents the model and its assumptions with respect to production, information, preferences. Section 3 explains the incentive problem. It gives the efficiency results and compare them to the usual share-tenancy ones. Section 4 provides some comparative statics and dynamics. The land value function is characterized in Section 5. Several discussions of the model on development economics are in section 6. Section 7 concludes. In the appendix, some proofs are given and the case of risk aversion as well as the role of uncertainty are discussed.

2 The model

2.1 Production, information and preferences

We consider an agricultural production function linearly homogenous in land area (as generally admitted, see Stiglitz, 1974, and Chuma, Hayami and Otsuka, 1992) such that agricultural output of period t is $y_t = \nu_t f(x_{t-1}, e_t)$ where $e_t \in \Re^{+n}$ is a vector of peasant's work effort, x_{t-1} the land fertility at the end of period $t-1$, and ν_t a multiplicative positive random variable with mean one representing weather uncertainty. Efforts represent labor tasks or other agricultural inputs. Production increases with land fertility (x) and work effort (e), but at a decreasing rate. Formally, $f(.,.) : \Re^+ \times \Re^{+n} \rightarrow \Re^+$ is twice differentiable, increasing in both arguments, concave in (x, e).

An investment function controls land fertility dynamics according to $x_t = \varepsilon_t g(x_{t-1}, e_t)$ where ε_t is a positive random variable with mean one representing weather influence or other random externalities on land fertility[2]. The vector e_t is a decision variable and x_{t-1} is a state variable. Depending on the nature of efforts, land fertility can increase or decrease in the components of e. We assume that $g(.,.) : \Re^+ \times \Re^{+n} \rightarrow \Re^+$ is twice differentiable, concave in x and e and increasing in x. With this general specification x_t can be increasing or decreasing over time.

The landlord represented by a principal rents out some land to a peasant, an agent, under a contractual arrangement. Production is observable and verifiable. The agent's actions are unobservable to the principal because monitoring tenant's tasks is prohibitively costly. Though observable, land fertility is not a contractible value because it is not verifiable. Contracts are incomplete because they cannot be contingent to land quality at the end of the production process. Murrell (1983) put forward that, given the complexity of the specification of the set of agricultural tasks and the difficulty to observe and measure land quality, the contract can't be complete. Allen and Lueck (1992, 1993, 1996) use also incomplete contracts because contracts observed empirically never include conditions on fertility. We could also say that the cost of completing contracts would be too high with respect to the potential loss resulting of incompleteness.

We assume that only spot contracts are feasible[3]. According to the contract signed for period t, the principal pays the agent $\tau_t(y_t)$ at the end of crop season t[4]. Let $U(\tau_t(y_t), e_t)$

[2] We could have taken additive random variables without changing all next results.

[3] However, non-overlapping finite length contracts with full commitment would not change qualitative results.

[4] Generally, when $\tau(y) = y - r$ the contract is called a fixed rent agreement and r is the rent rate. When $\tau(y) = ay + b$ with $0 < a < 1$ the share rate of production and b a constant, the contract is called a sharecropping arrangement (pure sharecropping if $b = 0$). When $\tau(y) = b$ the contract is a fixed wage.

be the agent's instantaneous utility where U is increasing in its first argument and strictly decreasing in each component of effort e_t. The principal is risk neutral and his instantaneous utility is $y_t - \tau_t(y_t)$. For many results, we will suppose that the agent has separable utility between cost of effort and payment that is $U(\tau(y), e) = U(\tau(y)) - C(e)$. The cost of efforts $C(.)$ is an increasing, convex and twice differentiable function.

2.2 Land value

Taking into account land fertility dynamics for the landlord seems relevant as Johnson (1950) called to mind:

"When a man sells a bushel of wheat, he has no interest in the use to which the wheat is put and is consequently willing to sell to the highest bidder. However, when a man sells the use of land, he has a real interest in how the land will be used. Consequently, the choice of tenant is never made without considering what the impact of the tenancy will be upon the value of the asset."

We assume that the landlord contracting with a tenant cannot commit to more than one period. Though Pareto superior, a full commitment long term contract would require the tenant's payment to be contingent to current and past performances ("memory effect" for consumption smoothing shown by Rogerson, 1985-a, Chiappori, Macho, Rey, Salanié, 1994). Such a contract needs the capacity of past productions to be recorded and verifiable in the future. We consider that courts will enforce only one period contracts like in Phelan (1995) or that agents work only for one period and the principal is long life compared to the agent. The principal proposes at each season a contract that induces the best incentives through the agent's incentive compatibility constraint (IC_t) and the agent's individual rationality (IR_t) where the agent has an exogenous reservation utility $\overline{U_t}$.

The principal has an infinite life time and maximizes a time separable intertemporal utility with discount factor $\rho \in [0, 1[$:

$$\begin{aligned} w_0(x_{-1}) = \quad & \underset{\tau_t(.)}{Max} E_{\nu_t,\varepsilon_t} \sum_{t=0}^{\infty} \rho^t \left[y_t - \tau_t(y_t)\right] \\ & s.t.\ \forall t \geq 0, \begin{cases} EU(\tau_t(y_t), e_t) \geq \overline{U_t} \ \ (IR_t) \\ e_t \in \arg\max_{e} EU(\tau_t(y_t), e) \ \ (IC_t) \end{cases} \end{aligned} \tag{1}$$

If random factors and reservation utility are stationary, the value function is solution of the following "Bellman equation":

$$w(x) = \underset{\tau(.)\ s.t.\ (IC),(IR)}{Max} \{E[y - \tau(y)] + \rho E w(z)\} \tag{2}$$

$w(x)$ represents the maximal utility the principal gets from his land of quality x provided by the path of optimal contracts $\tau_1^*(.)$, $\tau_2^*(.)$, ..

As a benchmark, considering the value function with perfect information (i.e. without the incentive constraint), we can prove easily that it is an increasing and concave function of fertility x. The proof is omitted[5] but consists in defining an operator T generating the expected utility at the end of some period with a given value function at the beginning of the period. The value function is the fixed point of T. It is increasing concave from the fact that T is a contraction mapping which maps the space of increasing concave functions to itself.

3 Optimal incentives

Our first purpose is to study the incentive problem faced by the landlord with a given land value function. This value function is derived from our model but it could be stated anyway, for example in the case of a competitive land market in which the price could be equal to the inverse land demand function. Consequently, the intertemporal utility function of the principal is $y - \tau(y) + v(x)$ where $v(.)$ represents the value for the principal of the land of fertility x after the season ($v(.)$ is supposed differentiable and strictly increasing). The agent's utility function is $U(\tau(y)) - C(e)$. The increasing convex cost function is also assumed to satisfy $\overrightarrow{C_e}(0_n) = 0_n$[6].

The principal's objective is not directly linked to y because of the land value term $v(x)$ which depends on agent's effort. As in Baker (1992), or Baker, Gibbons and Murphy (1994), Pareto efficiency of the optimal contract depends upon the relation between the principal's objective and the agent's performance measurement.

[5] A formal proof is given for the imperfect information cases.

[6] We note $\overrightarrow{C_e}$ the gradient of C with respect to e.

3.1 Perfect information

When all efforts are observable and verifiable by the principal, there is no moral hazard problem. The endogenous value function is then increasing concave. The principal's maximization program is to maximize $E\left[y-\tau\left(y\right)+v\left(x\right)\right]$ by choosing $\tau\left(.\right)$ and e subject to the agent's participation constraint

$$EU\left(\tau\left(y\right)\right)-C\left(e\right)\geq\overline{U} \tag{3}$$

and we have

Proposition 1 *In the case of perfect information, the optimal contract implements* $\tau^{*}=\Phi\left(\overline{U}+C\left(e^{*}\right)\right)$ *and* e^{*} *satisfying* $\forall i$ $\frac{\partial f}{\partial e_i}+E[\varepsilon v'\left(\varepsilon g\right)]\frac{\partial g}{\partial e_i}=\Phi'\left(\overline{U}+C\left(e^{*}\right)\right)\frac{\partial C}{\partial e_i}$ *(leaving out arguments and noting* $\Phi=U^{-1}$ *the inverse utility function) .*

Proof. Appendix A.1.

The marginal disutility of agent's effort equals the expected marginal utility (one for a risk neutral agent) times the marginal productivity plus the marginal land value times the marginal fertility investment. A first best contract consists in enforcing the optimal effort level e^{*} and in paying the agent a fixed wage leaving him with his reservation utility.
Assuming that a contract implying a zero payment ($\tau=0$) corresponds to the decision to leave land fallow (no effort is supplied), we simply need to assume that $U\left(0\right)\geq\overline{U}$ in order to have the decision to leave land fallow included in the set of feasible contracts.
The principal benefits from proposing a non zero contract if the reservation utility is not too high that is if $\overline{U}\leq U\left[Ey^{*}+Ev\left(\varepsilon g\left(x,e^{*}\right)\right)-Ev\left(\varepsilon g\left(x,0_n\right)\right)-f\left(x,0_n\right)\right]-C\left(e^{*}\right)$. Else, land use has a negative effect on its value that counterbalances its net yield for the principal. In this case, soil depletion induces the necessity to leave land fallow.

3.2 Imperfect information

With imperfect information, the principal seeks the transfer mechanism $\tau\left(.\right)$ maximizing $E\left[y-\tau\left(y\right)+v\left(x\right)\right]$ under the Individual Rationality constraint (3), and the Incentive Compatibility constraint[7]:

$$e \in \arg\max EU\left(\tau\left(y\right)\right) - C\left(e\right) \tag{4}$$

First, with a risk averse tenant, the first best cannot be reached because to implement the first best effort e^* the contract must both respect individual rationality constraint and give some incentives making $\tau\left(y\right)$ depend on y (Stiglitz, 1974). Using Jensen's inequality, it is clear that even if the first best effort is implementable, the principal's welfare cannot equal first best's one if the agent is risk averse.

However, the next proposition shows the necessary and sufficient conditions for implementing agent's first best action e^*.

Proposition 2 *The first best effort e^* is implementable if and only if $\overrightarrow{f_e}$, $\overrightarrow{g_e}$ and $\overrightarrow{C_e}$ are colinear at optimum. In that case, the second best contract is Pareto efficient if and only if the agent is risk neutral.*

Proof. The proof simply relies on the fact replacing (4) by $E[\nu U'\tau'\left(y\right)]\overrightarrow{f_e} = \overrightarrow{C_e}$ with the first order differential approach[8]. The first best effort being such that $\overrightarrow{f_e} + E[\varepsilon v'\left(\varepsilon g\right)]\overrightarrow{g_e} = \Phi'\left(\overline{U} + C\left(e^*\right)\right)\overrightarrow{C_e}$, it can be implemented if and only $\overrightarrow{f_e}$, $\overrightarrow{g_e}$ and $\overrightarrow{C_e}$ are colinear at e^*. The rest of the proposition follows from standard use of Jensen's inequality.

Efficiency can be reached only if all technical rates of substitution[9] between efforts in production, cost and investment are equal at the first best optimal effort e^*. Thus we get easily sufficient conditions for the first best to be implementable (appendix A.2).

Corollary 3 *With a risk neutral agent, whenever $\overrightarrow{f_e}$ or $\overrightarrow{C_e}$ is colinear to $\overrightarrow{g_e}$, the first best is implementable with a linear contract which slope is $a^* = 1 + E(\varepsilon v')\overrightarrow{g_e}/\overrightarrow{f_e}$ or $a^{*-1} = 1 - E(\varepsilon v')\overrightarrow{g_e}/\overrightarrow{C_e}$.*

These conditions correspond to the *production and land fertility investment colinearity or congruence* or the *cost and land fertility investment colinearity.* The vectors $\overrightarrow{f_e}$ and

[7] The optimal contract existence is not guaranteed but we will suppose it. Page (1987) showed that if the functional transfer space is uniformly bounded and sequentially compact for the point wise convergence topology, then an optimal contract exists. In our case, the sequential compactness is not guaranteed.

[8] The differential approach validity is difficult to prove. Rogerson (1985-b) shows it with some assumptions on the distribution function of the results according to the Agent's actions. Jewitt (1988) shows the validity by specifying the utility function inducing the Agent's objective to be concave.

[9] The Technical Rate of Substitution (TRS) between e_i and e_j in C is $-\frac{\partial C}{\partial e_j}/\frac{\partial C}{\partial e_i}$.

$\overrightarrow{g_e}$ being colinear means that any TRS between two efforts in f and g are equal. Then, efficient production incentives are also efficient incentives for land quality investment. This is obviously always the case when effort is unidimensional and the agent is risk neutral.

In most contributions to sharecropping theory (Rao, 1971, Stiglitz, 1974, Newbery, 1977, Newbery and Stiglitz, 1979), the risk neutrality of the tenant allows to reach the first best even in imperfect information with a fixed rent contract. As we showed, it isn't always valid when the landlord accounts for his land value in a long term horizon. Allen and Lueck (1992) already obtained this result in a transaction cost model. Here, a sort of transaction cost appears endogenously because of both the multitask technology and the long term horizon of the landlord. The principal's objective is different because he cares about his expected future benefits from his land. As in Baker (1992), the difference between the principal's objective and the contractible performance measure generates incentives distortions[10]. We showed how this can be generated endogenously. Two special cases may be noted:

First, if the expected production is an exhaustive statistic of efforts' effects on land fertility, that is if $x_t = \varepsilon g(x_{t-1}, e_t) = \varepsilon \widehat{g}(x_{t-1}, f(x_{t-1}, e_t)) = \varepsilon \widehat{g}(x_{t-1}, Ey_t)$, then $\overrightarrow{g_e}$ and $\overrightarrow{f_e}$ are always colinear and the first best is achieved.

Second, if expected fertility is an exhaustive statistic of efforts' effects on production, that is if $f(x_{t-1}, e_t) = \nu \widehat{f}(x_{t-1}, g(x_{t-1}, e_t))$ then $\overrightarrow{f_e} = \nu \frac{\partial \widehat{f}}{\partial g} \overrightarrow{g_e}$ and the first best can be obtained.

3.3 Linear contracts

We focus our study on linear contracts, as usually done in this literature[11] concentrating for simplicity on the risk neutral case.

Let the transfer function be written $\tau(y) = ay + b$. By the differentiable approach, the Incentive Constraint (IC) becomes:

$$a\overrightarrow{f_e}(x, e) = \overrightarrow{C_e}(e) \tag{5}$$

[10] Baker et al. (1994) show this inefficiency can sometimes be reduced by additional implicit contracts.

[11] We could call upon bounded rationality to justify this behavior. But, simple incentive schemes like sharing rules are generally prevalent in economy. Hart and Holmström (1987) say it can be caused by prohibitive costs of writing intricate contracts but also by a need of robustness in front of other possible Agent's choices. Holmström and Milgrom (1987) showed they are optimal when errors are normal and the Agent has constant absolute risk aversion.

The share of production a is also called the contract slope and said to represent the incentive power of the contract. We first establish that (proof in A.3):

Lemma 4 *In the risk neutral case, the IC constraint defines an agent's best answer effort supply* $e(.,.)$ *depending on* a *and* x*, differentiable, for which* $\overrightarrow{e_x} = -a\overrightarrow{f_{ex}}M$ *and* $\overrightarrow{e_a} = -\overrightarrow{f_e}M$ *with* $M = [af_{ee} - C_{ee}]^{-1}$*. Moreover* $\left\langle \overrightarrow{f_e}, \overrightarrow{e_a} \right\rangle > 0$[12] *and* $\left\langle \overrightarrow{f_{ex}}, \overrightarrow{e_x} \right\rangle > 0$*, i.e.* $\sum_{i=1}^{n} \frac{\partial^2 f}{\partial x \partial e_i} \frac{\partial e_i}{\partial x} > 0$ *and* $\sum_{i=1}^{n} \frac{\partial f}{\partial e_i} \frac{\partial e_i}{\partial a} > 0$*.*

Therefore, with only one task, the supply of effort is strictly increasing with the incentive power a whereas it is strictly increasing or decreasing with fertility x as fertility and effort are complementary or substitute in the production function[13]. With risk aversion, conditions on the utility function of the agent need to be introduced as shown in appendix B.1. Then, we can show that

Proposition 5 *The second best optimal contract slope verifies*[14] *(proof in A.4):*

$$a^* = 1 + E(\varepsilon v^{'}) \frac{\langle \overrightarrow{e_a}, \overrightarrow{g_e} \rangle}{\left\langle \overrightarrow{e_a}, \overrightarrow{f_e} \right\rangle} \tag{6}$$

Equation (6) shows how the optimal contract slope differs form the one obtained when land value is not affected by effort (in which case $a^* = 1$ implying that the contract is a fixed rent). The second term of (6) reflects the trade-off between production and fertility investment incentives. With risk aversion another term reflecting the usual trade-off between production incentives and risk sharing appears (see Appendix B.1). What is to be noted is that our dynamic framework introduces a distortion effect between tasks even with risk

[12] We note $\langle ., . \rangle$ the canonic scalar product of $\Re^n$ i.e. for $z_1, z_2 \in \Re^n$, $\langle z_1, z_2 \rangle = \sum_{i=1}^{n} z_{1i} z_{2i}$.

[13] With n efforts completely separable in production and cost, the same result holds for each one.

[14] With an agent having utility function $U(.)$

$$a^* = 1 + E\varepsilon v^{'} \frac{\langle \overrightarrow{e_a}, \overrightarrow{g_e} \rangle}{\left\langle \overrightarrow{e_a}, \overrightarrow{f_e} \right\rangle} - (1 - \frac{E\nu U^{'}}{EU^{'}}) \frac{f}{\left\langle \overrightarrow{e_a}, \overrightarrow{f_e} \right\rangle}$$

With a risk averse landowner having instantaneous utility function V

$$a^* = 1 + \frac{E\varepsilon v^{'}}{E\nu V^{'}} \frac{\langle \overrightarrow{e_a}, \overrightarrow{g_e} \rangle}{\left\langle \overrightarrow{e_a}, \overrightarrow{f_e} \right\rangle} - (1 - \frac{E\nu U^{'} EV^{'}}{EU^{'} E\nu V^{'}}) \frac{f}{\left\langle \overrightarrow{e_a}, \overrightarrow{f_e} \right\rangle}$$

neutrality.

The optimal contract slope a^* depends at each crop season on the land fertility whatever the agent's risk aversion. There is a "memory effect" through land fertility which depends on past actions. It has nothing to do with Rogerson's memory effect (1985-a) due to the desire for consumption smoothing when the agent is risk averse. We have only spot contracts but memory in the production process.

Remark: The sign of the scalar product $\langle \overrightarrow{e_a}, \overrightarrow{g_e} \rangle = \left\langle \overrightarrow{f_e} M, \overrightarrow{g_e} \right\rangle$ is ambiguous because even if $\left\langle \overrightarrow{f_e}, \overrightarrow{g_e} \right\rangle > 0$ we could have $-\left\langle \overrightarrow{f_e} M, \overrightarrow{g_e} \right\rangle < 0$. The figure 7 shows an intuition of what could happen. As $-M$ is symmetric definite positive, we can write $-M = \rho' \Lambda \rho$ with ρ a rotation and Λ a diagonal matrix with positive eigenvalues. Even if marginal productivity and marginal investment of efforts are positive, the distortion between efforts can lead to a low powered contract. In this extreme case, the inefficiency of the multitask distortion is so bad that it is better to give low incentives.

4 Comparative statics and the dynamics of incentives and land fertility

4.1 High powered or low powered incentives

Some particular cases are interesting to highlight[15].

- *Congruence of production and fertility maintenance*:

  In this case, corollary 3 shows that the first best is implementable. This is always the case when effort is unidimensional. If the effort has positive influence on land fertility, the optimal contract has high powered incentives (the slope is greater than one). If the effort has negative effect on land fertility, the optimal contract has low powered incentives (slope smaller than one). The graph 1 of the marginal productivities and costs when effort depletes future fertility shows how to reach the first best is reached

[15] Considerations about substitutability, complementarity, and land fertility issues can be found among other contributions to this literature in Binswanger and Rosenzweig (1986), Bardhan (1984, 1989), Braverman and Stiglitz (1986), Allen and Lueck (1992, 1993-a&b), Singh(1989), Chuma, Hayami and Otsuka (1992).

in this case[16]. A sharecropping contract is used because of the land overuse effect of effort.

- *Two tasks* (e_1, e_2):
  Scalar product $\left\langle \overrightarrow{f_e} M, \overrightarrow{g_e} \right\rangle$ is equal to

$$(g_1[a(f_1 f_{22} - f_2 f_{12}) - f_1 C_{22} + f_2 C_{12}] + g_2[a(f_2 f_{11} - f_1 f_{12}) - f_2 C_{11} + f_1 C_{12}])/\Delta$$

  with $\Delta = (af_{11} - C_{11})(af_{22} - C_{22}) - (af_{12} - C_{12})^2$ which is strictly positive for $a \geq 0$ because $af - C$ is strictly concave[17].

Proposition 6 *If cost and production functions verify $f_{12} \geq 0$, $C_{12} \leq 0$ then $a^* > 1$ if g is increasing in both efforts and $a^* < 1$ if g is decreasing in both efforts.*

This proposition shows that if efforts are complementary or separable in cost and production functions, the optimal contract is high powered if agent's efforts improve land fertility and low powered if they diminish land fertility.

Proposition 7 *If effort 2 has no effect on fertility ($g_2 = 0$) but cost, production and investment functions verify $C_{12} \geq \frac{f_1}{f_2} C_{22}$ and $f_{12} \leq \frac{f_1}{f_2} f_{22}$ then $a^* \leq 1$ if effort 1 increases fertility ($g_1 \geq 0$) and $a^* \geq 1$ if it reduces fertility ($g_1 \leq 0$).*

This example shows that a low powered contract can happen even if efforts have a positive effect on fertility. Conditions of Proposition 7 concern the case where the marginal productivity of e_1 decreases faster in e_2 than marginal productivity of e_2 times the TRS of e_2 with e_1 and similarly for the cost function. Therefore, low incentives can sometimes be optimal by reducing distortions between efforts given by a unique performance measure such as production even if no effort depletes fertility;

4.2 Land fertility and optimal slope

- *Colinearity of production and fertility investment*: (proof in A.6)

[16] C' is the marginal cost, f' and g' are the marginal productivities. If $g' > 0$ then the optimal slope is $a_+ > 1$. If $g' < 0$ then the optimal slope is $a_- < 1$.

[17] We note $f_{12} = \frac{\partial^2 f}{\partial e_1 \partial e_2}$ and similar notations for f_{11}, f_{22}, C_{11}, C_{12}, C_{22}.

Proposition 8 *When $\overrightarrow{f_e}$ and $\overrightarrow{g_e}$ are colinear (first best is obtained):*
If $x_{t+1}(x_t)$ increases[18] *in x_t then $a^*(.)$ is decreasing at x_t (increasing) if $a^* > 1$ ($a^* < 1$).*
If $x_{t+1}(x_t)$ decreases in x_t then $a^*(.)$ is increasing at x_t (decreasing) if $a^* > 1$ ($a^* < 1$).

If the first best efforts are such that $x_{t+1}(x_t)$ increases in x_t, the optimal slope converges toward one when x increases. On the contrary, if the investment function in fertility is such that at the first best $x_{t+1}(x_t)$ decreases in x_t, then the optimal slope diverges when x grows, increasing when larger than one and decreasing towards zero when smaller.
This means that in the case where technology where efficient provision of effort leads to a growth in fertility across periods, low powered optimal contracts will need to provide less and less incentives while high powered contracts will have to give more and more incentives. In the case of "fragile lands" where even an efficient provision of efforts will lead to deplete land fertility, the contrary will happen. Actually low powered contracts will have to be more and more powerful while high powered contracts will have to be less and less powerful.

- *Separability of efforts and land fertility*: (proof in A.7)

Proposition 9 *When effort and fertility are separable ($\overrightarrow{f_{ex}} = \overrightarrow{g_{ex}} = 0_n$), if $a^* > 1$ ($a^* < 1$) then $a^*(x)$ is decreasing (increasing).*

In the case of separability between efforts and land fertility in production and investment, the larger the land fertility the closer will be the optimal contract to a fixed rent (the optimal contract form without land value). With risk aversion, instead of converging to one, the slope will converge towards the second best sharing rate generally between 0 and 1. This result means that in the case of separability, the difference in optimal incentives when taking into account future land value or not is more important for low fertility lands than for high fertility lands.

5 Endogeneity of land value

In the previous sections we considered the land value v as given exogenously. The preceding results rely only on its increasing property and sometimes on concavity. Sufficient conditions

[18] It is the case if $\overrightarrow{f_{ex}}$ and $\overrightarrow{g_e}$ are positively colinear.

ensuring that the endogenous value function will be monotone and concave in the multitask risk neutral case and the unidimensional risk averse case follow (proofs are in A.5).
With risk neutrality, the landlord's welfare remains unchanged if contracts are restricted to linear transfer functions. Noting η_a^f elasticity of production with respect to the contract incentive, η_a^g elasticity of investment with respect to the contract incentive, η_x^g the elasticity of investment with respect to fertility, η_x^f the elasticity of production with respect to fertility, we have (proof in A.5):

Proposition 10 *With risk neutrality, the land value function is increasing when $\eta_a^f \eta_x^g \geq \eta_a^g \eta_x^f$.*

- *Colinearity of production and fertility investment:* (proof in A.8).

Proposition 11 *With risk neutrality, whenever $\overrightarrow{f_e}$ and $\overrightarrow{g_e}$ are colinear or $\overrightarrow{C_e}$ and $\overrightarrow{g_e}$ are colinear, the land value function is increasing and concave.*

When all TRS between efforts in the fertility investment function and the production function (or the cost function) are equal, the land value function is increasing concave and equal to the first best one.

- *Multiplicative separability of fertility and efforts in production* (proof in A.9):

Proposition 12 *With risk neutrality, if the production function is multiplicatively separable between fertility and efforts then the value function is increasing.*

- *Additive separability of fertility and efforts in production and investment* (proof in A.9):

Proposition 13 *With risk neutrality, when effort and fertility are separable in production, the land value function is increasing. It is concave if separability also holds for investment.*

- *Two tasks*:

Proposition 14 *With two tasks and a risk neutrality, the land value is increasing if the TRS between efforts in investment and marginal productivity are both greater or smaller than that in production*[19].

[19]It happens for example when $f_{1x} \geq 0$, $f_{2x} \leq 0$, $g_1 \geq 0$, $g_2 \leq 0$ or $\frac{f_2}{f_1} \leq \frac{g_2}{g_1}$, $f_{1x} \leq 0$, $f_{2x} \geq 0$, $g_1 \geq 0$, $g_2 \geq 0$ or $\frac{f_2}{f_1} \geq \frac{f_{2x}}{f_{1x}}$, $g_1 \geq 0$, $g_2 \leq 0$.

The existence of sufficient conditions under which no *ad hoc* assumption is done for the land value and it is generated endogenously in the model is important. Actually our analysis relies on apparently reasonable properties concerning the monotonicity and concavity of the value of land according to its quality. Considering an exogenous land value function with these nice properties is sufficient for the study of contracting choices but it could be that delegating through contracts the use of land generate endogenously a value which could not satisfy the properties. In such a case, the theoretical results would be inconsistent. Therefore it is important to exhibit (at least) sufficient conditions under which the model remains internally coherent in which only technology is given exogenously and determines the endogenous contracts and resulting value of land.

6 Discussions on development and policy issues

First, farming through contractual arrangements does not imply that better lands will produce more. Actually, the production level changes with fertility according to the sign of $f_x + \left\langle \overrightarrow{e_x} + a_x \overrightarrow{e_a}, \overrightarrow{f_e} \right\rangle$ which may be positive or negative (though we assumed that $f_x \geq 0$). Actually, a land fertility increase has roughly speaking three effects on production: a direct one by the marginal productivity of x (f_x), an indirect one on farmer's effort supply which can be decomposed in a direct effect ($\overrightarrow{e_x}$) and an indirect one through the variation of incentives ($a_x \overrightarrow{e_a}$). Therefore we can expect that according to the agricultural technology employed, the production may sometimes decrease even if fertility increases. One has to be careful concerning the observed relationship between production levels and land quality because in many cases it is ambiguous. However, focusing on dynamics of fertility and then on technological innovations in this model seems rather interesting.

6.1 Fertility dynamics

One important issue in agriculture is that of fertility dynamics. Clearly, the fertility dynamics through tenancy will be different from the first best one when the conditions allowing to implement it given in proposition 2 and 3 are not satisfied.

In order to give a simple analytical example of the problem, assume the investment function takes the following form (without uncertainty):

$$x_{t+1} = (1+\delta)x_t - \widehat{g}(e_t)$$

Then $x_{t+1} - x_t \gtreqqless 0$ as $\widehat{g}(e_t(x_t)) \lesseqqgtr \delta x_t$ where $e_t(x_t) = e_t(a^*(x_t), x_t)$ is the agent's best answer to a contract slope a^*. The optimal slope determined by the landlord depends on fertility. Hence, increase of fertility over time is not compulsory. If production tasks are damaging for fertility and marginal land value is small for the landlord, he will give production incentives which are likely to decrease fertility. But, another more important feature in the multitask environment is that if incentive distortions lead the farmer not to realize the needed investment to maintain fertility then it can decrease over time. Therefore, when investment tasks are not directly linked to production, land tenancy is likely to change fertility dynamics with respect to the first best path.

The model can generate cycles for fertility, contracts, or production depending on the specification of the investment function. As contract incentives depend on fertility the dynamics of fertility will lead to some changes in the contracts signed. Figure 6 shows a possible time path for fertility.

- If x_{t+1} increases with x_t, the fertility dynamics will be monotonic (decreasing in some ranges and increasing in others). Several cases can appear. See (a), (b), (c) and (d) in figure 8.

- If x_{t+1} is decreasing in x_t at some point, non monotonic dynamics happen at some fertility levels and permanent cycles appear as soon as the curve $x_{t+1}(x_t)$ crosses the bisecting line with a decreasing slope.

As $\frac{\partial x_{t+1}}{\partial x_t} < 0 \Leftrightarrow \widehat{g}' e_x > 1+\delta$, different conditions may lead to cyclic change:

Permanent cycles will appear for all fertility levels if $\widehat{g}' e_x > 1+\delta$. Replacing e_x by its expression, the condition is $f_{ex}\widehat{g}' > (1+\delta)\left[C_{ee} - f_{ee} - v'\widehat{g}''\right]$.

If $f_{ex} \geq 0$ and $\widehat{g}' \geq 0$, it will always happen for $\delta = -1$ or for sufficiently small δ. See case (c) in figure 8.

With bounding conditions on second order derivatives of production, cost and investment functions, e_x is positive and bounded above, and as $\widehat{g}'$ is increasing, a threshold $\underline{x}(\delta)$ (increasing in δ) exists such that $\widehat{g}'e_x > 1+\delta$ for $x \geq \underline{x}(\delta)$. If $x_{t+1}(\underline{x}(\delta)) > \underline{x}(\delta)$, permanent cycles appear for some $x > \underline{x}(\delta)$. See cases (e), (f), (g) and (h) in figure 8.

When permanent cycles appear increasing and decreasing phases of fertility change alternate. As effort levels are positively correlated with fertility levels, low activity with low fertility stocks alternate with high activity and high fertility stocks.

Land tenancy can lead to various fertility dynamics rather different of that obtained if land is farmed by the owner providing efficient level of efforts.

6.2 Technological innovation

For simplicity, we consider an exogenous value function which is sufficient to study the short-run dynamics of innovation when an innovation is known to be adopted in the long term. Otherwise we would have to consider the effect of an innovation on the value function itself. The question is whether innovating or not for the current period whereas adoption is granted in the sequel (which justifies to keep the same exogenous value function for the future).

Focusing on multitask incentives allows to study innovations consisting in the introduction of a new agricultural task. We examine the introduction of two kinds of innovations improving agricultural production. First, pest control increases significantly production without any effect on fertility. Second, the use of chemical fertilizer to increase production may improve or damage future fertility[20]. Both examples fit into the framework of the model which shows that tasks' multiplicity generally prevents the first best to be reached even when the agent is risk neutral. Consider the following examples.

- *Traditional technique*: Cobb-Douglas production function $f(x, e_t) = x^n e_t^p$, linear additive cost $C(e_t) = e_t$, fertility dynamics $g(x, e_t) = (1+\delta-\mu e_t)x$, increasing concave value function $v(z) = z^r$.

With this traditional technology and risk neutrality, first best can be reached under land tenancy like in an owner-operated farm. Introducing a new task for the tenant, the first best

[20] Instead of fertilizing or pest controlling we could have considered the Green Revolution example i.e. the change of traditional seeds by High Yielding Varieties which turned out to be more sensitive to water shorts and induced other tasks in farmer's work like irrigation. The model suggests it could have made land tenancy less efficient than with past technique although welfare has been improved.

will no longer be reached because of distortions in peasant's incentives though landlord's welfare is increased (otherwise the innovation would not be adopted) unless the gradient of investment and production with respect to efforts are colinear.

- *Pest control introduction*: Let e_p be a pest control task with no effect on fertility (pesticides used for production). Controlling pest reduces the crop part usually lost in traditional technology, so production is increasing with the quantity of scattered pesticides over cultivated area. The fertility equation is unchanged from the traditional one and we consider the following specifications: $f(x, e_t, e_p) = x^n e_t^p (1 + e_p^q)$, $C(e_t, e_p) = e_t + e_p$, $g(x, e_t, e_p) = (1 + \delta - \mu e_t)x$.

First, consider the case for which fallowing allows to improve land fertility i.e. $\delta > 0$ and effort e_t depletes soil quality. Computations are done with the following parameters: $\mu = 1$, $n = 0.4$, $p = 0.3$, $q = 0.2$, $r = 0.6$, $\delta = 0.1$, $\rho = 0.9$. Though rasing effort is bad for land fertility, the pest control introduction increases the optimal contract slope. For example $a^*(1) = 0.65$ with traditional method and 0.70 with new one. For $x = 1$, the results[21] show that pest control raises incentives, production is higher but fertility is depleted compared to the use of the traditional technique. In fact, the initial technique is such that fertility improves for $x < x^* = 1.11$ and is depleted above. The new technique is such that $x_n^{*sb} = 0.21 < x^*$. Therefore, the new technique lowers the steady state level of fertility because with the pest control introduction the productivity of the first task is better, inciting the peasant to raise his effort on it. Figure 2 shows the fertility change during one period for both technologies.

As predicted, the introduction of a new task changes the fertility dynamics and introduces an inefficiency (the first best steady state is larger than the second best one: $x_n^{*fb} = 0.22$). There is more soil depletion with the new technique. Plots which quality is between the two steady states improve with the traditional technology but suffer lower fertility maintenance (because of more production incentives) with the pesticide introduction.

[21] Complete results are

$x = 1$	a^*	e_t	e_p	x_{+1}	Prod.	Welfare
Old tech.	0.65	0.09	–	1.004	0.50	1.30
New tech. (SB)	0.70	0.19	0.05	0.90	0.95	1.55
New tech. (FB)	–	0.184	0.07	0.92	0.96	1.56

.

Now, consider that fertility decreases if land is left fallow (i.e. it needs maintenance) and that the first task improves soil quality. Computations are now done with the following parameters: $\mu = -1$, $n = 0.4$, $p = 0.3$, $q = 0.2$, $r = 0.6$, $\delta = -0.1$, $\rho = 0.9$. Then[22], though incentives are good for land fertility, the pest control introduction lowers the optimal contract slope, but making first task more productive the production however increases. The initial technique is such that we have fertility improvement for $x > x^* = 0.22$ and depletion below. The new technique is such that $x_n^* = 0.12 < x^*$. There is less soil depletion with the new technique. Plots which quality is between x_n^* and x^* are depleted with the traditional technology but improve after the pesticide introduction.
We now consider another kind of innovation.

- *Fertilizer introduction*: Let e_f be the fertilizing task (fertilizer input scattered over cultivated area), the fertility equation is now different from the traditional one. Consider the following specifications: $f(x, e_t, e_f) = x^n e_t^p (1 + e_f^q)$, $C(e_t, e_f) = e_t + e_f$, $g(x, e_t, e_f) = (1 + \delta - \mu e_t + \eta e_f)x$. The optimal contract slope depends on values of p and q and different patterns can be found. Here, we focus only on fertility dynamics according to signs of μ and η.

First, we consider the case for which fallowing improves fertility, the first task decreases fertility while the second increases it. Computations are done with the following parameters: $\mu = 1$, $\eta = 1$, $n = 0.4$, $p = 0.3$, $q = 0.2$, $r = 0.6$, $\delta = 0.1$, $\rho = 0.9$. Then, fertility dynamics are shown by figure 4.

We see that the fertility level converges towards x_1 with the traditional technique which is greater than x_2 with the new technique ($x_1 = 1.15$ and $x_2 = 0.29$). Surprisingly, the introduction of a fertility improving task decreases the steady state soil quality and soils endowed with $x \in]x_2, x_1[$ improve with first technology whereas they are damaged under the new one. In fact, under the new technique, incentives (a^*) are larger because of the

[22] Some results:

x	Tech.	a^*	e_t	e_p	x_{+1}	Prod.	Welfare
0.1	Old	1.16	0.06	–	0.096	0.17	0.33
0.1	New	1.12	0.10	0.02	0.099	0.29	0.40
0.2	Old	1.26	0.10	–	0.19	0.26	0.51
0.2	New	1.19	0.17	0.04	0.21	0.46	0.62

introduction of a fertilizing task. Raising first task productivity, its level is higher[23]. Though the second best welfare is increased with the innovation, the steady state fertility level is lowered.

Secondly, we consider the case for which the first task improves land (fallowing being bad for soil) while the second damages it. Computations are done with the following parameters: $\mu = -1$, $\eta = -1$, $n = 0.4$, $p = 0.3$, $q = 0.2$, $r = 0.6$, $\delta = -0.1$, $\rho = 0.9$. Then, fertility dynamics are shown by figure 3.

Fertility improves above a threshold x_1 with the traditional technique whereas with the new technique this threshold is lower $x_2 < x_1$[24]. However, for large fertility levels (above $\underline{x}$), the adoption of innovation can slow down fertility improvements. This pattern is similar to the introduction of a fertility neutral pesticide excepted that for high soil quality (i.e. $x > \underline{x}$) the direct effect of depletion of fertility overcomes the reduction of e_f through lower incentives.

We have seen that technological innovations can have different opposite effects on efficiency, production and fertility dynamics particularly because the introduction of multiple tasks may introduce inefficiencies in land tenancy compared to owner operated farms. Short run and long run effects can be very different in terms of production growth and sustainable growth.

6.3 A digression about taxation policies in agriculture

The principal-agent model we developed can give rise to various interpretations. I can be done in terms of taxation and pricing policies giving a particular attention to the fertility dynamics induced by these policies

As done by Hoff (1993) and Braverman and Stiglitz (1989), assume the agent is the repre-

[23] We have:

$x = 1$	a^*	e_t	e_f	x_{+1}	Prod.	Welfare
Old tech.	0.65	0.09	–	1.003	0.49	1.30
New tech.	0.76	0.23	0.05	0.93	1.0	1.58

[24] Some results:

x	Tech.	a^*	e_t	e_p	x_{+1}	Prod.	Welfare
0.1	Old	1.16	0.06	–	0.096	0.17	0.33
0.1	New	1.09	0.09	0.02	0.097	0.28	0.39
0.2	Old	1.26	0.10	–	0.19	0.26	0.51
0.2	New	1.14	0.15	0.035	0.204	0.45	0.61
1	Old	1.91	0.45	–	1.35	0.79	1.41
1	New	1.34	0.57	0.16	1.32	1.43	1.76

sentative farmer of an agricultural economy while the principal is the state planner designing agricultural taxation. In a less developed country where the regulator's objective is indeed to maximize expected sum of agricultural returns, the taxation policy may be important with respect to incentives to produce and incentives to invest. The intertemporal social welfare is the expected sum of raised taxes in agricultural sector plus the farmer's reservation utility which is constant. The regulator has the choice between taxing production at rate $1-a$ and taxing cultivated land areas at rate $-b$ (it is a subvention if $b>0$). Of course, the risk neutral regulator has an infinite planning horizon while the representative farmer has a short term life. Though endowed with the property right, the agent is more myopic than the regulator.

Not caring about investment and future productivity (myopic planner), the optimal tax in a risk neutral agricultural economy is a land property tax or land owning subsidy (if negative). The agricultural taxation literature (Hoff, 1993) argues that a production tax can be better than a land tax when institutions for spreading and pooling risk are imperfect. This result applies whether land is tilled by owners, wage-earners or sharecroppers. Hoff shows that a mix of land and production taxes are Pareto superior. Here, even with perfect risk pooling, the tax system is not neutral when the fertility dynamics are important.

Even if the farmer is risk neutral, because of distorted incentives to invest, the optimal taxation is not only a fixed price per hectare (subvention if positive or tax if negative, which depends on the value of reservation utility) but also involves a production tax. Hence taxation is important for investment incentives of farmers.

If we assume that the labor intensive method used by farmers depletes soil fertility, the optimal taxation will influence agricultural intensification. Assume that fertility dynamics is given by equation: $x_{t+1}-x_t=(\delta-e)\,x_t$ (following agricultural models of optimal control of erosion, McConnell, 1983, Barrett, 1991). We can consider that x is the soil depth which has a positive effect on output[25]. Then the dynamics of soil depth depend on e whereas δ is the natural regeneration rate. Agricultural output can be increased by clearing, weeding by hand or scattering herbicides, but the gains are then short lived because the soil will be quickly

[25] Because in deeper soils there is more room for plant roots to take hold and more nutrients available for plant growth.

eroded away unless other tasks such that building terraces on hillsides are undertaken.

What the model shows is also that optimal taxation depends crucially on land quality which may change across periods. Moreover, the multiplicity of tasks involved by technological constraints may affect the efficiency of taxation.

7 Conclusion

Finally, this principal-agent model of moral hazard an endogenous dynamics of some production factor shows the importance of land fertility in the determination of the optimal contract between a landlord and a tenant. The opposition of long term and short term objectives of contracting parties leads to a second best contract which is a sharecropping agreement even with risk neutrality. The model is able to explain many kinds of tenancy contracts or workers' payment schemes: sharecropping, all mixed contracts with a fixed part and a sharing rule. The optimal contract depends on production and land fertility investment technologies (in addition to the usual risk sharing motives when agents are risk averse). Non intuitive optimal contracts may appear in the multitask case. Even with risk neutrality and a technology in which efforts are productively worth and land improving, the second best contract can have low powered incentives. In general, task multiplicity generates inefficiencies in contracting (unless marginal investment and productivities are perfectly colinear) which may affect the fertility dynamics compared to the one which would be observed for an efficient cultivation. In particular, technological innovations may change the fertility time path which can deviate importantly form the first best one when innovations introduce new task that create some non congruence between productive and investment efforts. Finally, development policy designed to cope with problems of technical efficiency and income insurance in risky rural regions generally proposes simply to improve access to formal insurance and credit markets for peasants favouring fixed rental contracts instead of sharecropping contracts. However, we showed that sharecropping may be used also to reduce distortions between efforts in a multitask activity. Forbidding share contracts could increase production in the short run but become very bad in the long run, leading to diminish land quality. In fragile land regions, policy design should take care of long term environmental effects of restricting the feasible set of contracts.

A Proofs

A.1 Proof of proposition 1

The first order conditions of the Lagrangian are (λ being the Lagrange multiplier associated to (3)): $EU(\tau) = \overline{U} + C(e)$, $\lambda EU'(\tau) = 1$, $\overrightarrow{f_e} - \lambda \overrightarrow{C_e}(e^*) + E\varepsilon v' \overrightarrow{g_e} = 0_n$. Hence the first best (e^*, τ^*) is characterized by $\overrightarrow{f_e}(x, e^*) - (EU'(\tau^*))^{-1} \overrightarrow{C_e}(e^*) + E\varepsilon v'(\varepsilon g(x, e^*)) \overrightarrow{g_e}(x, e^*) = 0_n$ and $EU(\tau^*) = \overline{U} + C(e^*)$. With $\tau^* = \Phi\left(\overline{U} + C(e^*)\right)$ we have a first best solution. To respect the second order condition the following symmetric matrix: $f_{ee}(x, e^*) - \Phi'\left(\overline{U} + C(e^*)\right) C_{ee}(e^*) - \Phi'\left(\overline{U} + C(e^*)\right) \overrightarrow{C_e}(e^*)' \overrightarrow{C_e}(e^*)$
$+E\left[\varepsilon v'(\varepsilon g(x, e^*)) g_{ee}(x, e^*) + v''(\varepsilon g(x, e^*)) \varepsilon^2 \overrightarrow{g_e}(x, e^*)' \overrightarrow{g_e}(x, e^*)\right]$[26] must be semi-definite negative. f, g and U being concave, C and Φ convex, $\overrightarrow{C_e}(e^*)' \overrightarrow{C_e}(e^*)$ and $\overrightarrow{g_e}(x, e^*)' \overrightarrow{g_e}(x, e^*)$ symmetric semi-definite positive, the second order condition is satisfied if $E\varepsilon v'(\varepsilon g(x, e^*)) \geq 0$ and $E\varepsilon^2 v''(\varepsilon g(x, e^*)) \leq 0$ i.e. v increasing and concave. So, if v is assumed concave there is always a unique solution to this problem.

A.2 Proof of corollary 3

Let $\lambda \in \Re$ such that $\overrightarrow{g_e} = \lambda \overrightarrow{f_e}$ (the proof with $\overrightarrow{C_e}$ and $\overrightarrow{g_e}$ colinear is similar). Consider the linear contract of slope a, then the incentive constraint gives $a\overrightarrow{f_e}(x, e) = \overrightarrow{C_e}(e)$ which defines a differentiable function $e(x, a)$ (implicit functions theorem) verifying $\forall a$, $a\overrightarrow{f_e}(x, e(x, a)) = \overrightarrow{C_e}(e(x, a))$. Then the solution of $\lambda E\varepsilon v'(\varepsilon g(x, e(x, a))) + 1 = a$ allows to implement the first best. Actually, call a^* its solution, then e is solution of $a^* \overrightarrow{f_e}(x, e) = \overrightarrow{C_e}(e)$ hence $e(x, a^*)$ verifies
$(1 + \lambda E\varepsilon v'(\varepsilon g(x, e(x, a^*)))) \overrightarrow{f_e}(x, e(x, a^*)) = \overrightarrow{C_e}(e(x, a^*))$ i.e.
$\overrightarrow{f_e}(x, e(x, a^*)) + E\varepsilon v'(\varepsilon g(x, e(x, a^*))) \overrightarrow{g_e}(x, e(x, a^*)) = \overrightarrow{C_e}(e(x, a^*))$. Consequently $e(x, a^*)$ is solution of the first order condition to the first best problem. We have $a^* = 1 + E\varepsilon v' \overrightarrow{g_e} / \overrightarrow{f_e}$. Note $S(a) = 1 - a + \lambda E\varepsilon v'(\varepsilon g(x, e(x, a)))$. S is continuous and $S(0) = 1 + \lambda E\varepsilon v'(\varepsilon g(x, e(x, 0))) = 1 + \lambda E\varepsilon v'(\varepsilon g(x, 0_n))$ and $\underset{a \to +\infty}{Lim} S(a) = -\infty$ by supposing that

[26] We note $f_{ee} = \begin{bmatrix} \frac{\partial^2 f}{\partial e_1 \partial e_1} & \cdots & \frac{\partial^2 f}{\partial e_1 \partial e_n} \\ . & & . \\ \frac{\partial^2 f}{\partial e_n \partial e_1} & \cdots & \frac{\partial^2 f}{\partial e_n \partial e_n} \end{bmatrix}$ the Hessian matrix of f with respect to e, and similarly for g_{ee} and C_{ee}.

v' is bounded. If $1+\lambda E\varepsilon v'(\varepsilon g(x,0_n))<0$ then $a^*<0$ and $e=0_n$ so no effort happens. If $1+\lambda E\varepsilon v'(\varepsilon g(x,0_n))\geq 0$, $S(a)=0$ has always a solution. If $\lambda>0$ i.e. $\overrightarrow{f_e}$ and $\overrightarrow{g_e}$ have the same direction then $a^*>1$. If $\lambda<0$ i.e. if $\overrightarrow{f_e}$ and $\overrightarrow{g_e}$ have opposite directions then $a^*<1$.

A.3 Proof of lemma 4

According to (5), $\forall a\geq 0, a\overrightarrow{f_e}(x,e)=\overrightarrow{C_e}(e)$. The implicit functions theorem tells that $\exists e(.,.):\Re^{+2}\rightarrow\Re^{+n}$ such that $\forall a\geq 0, e(x,a)$ is solution of (5). $e(.,.)$ is continuously differentiable. Differentiating IC with respect to a and x: $\overrightarrow{e_a}(x,a)=-\overrightarrow{f_e}(x,e(x,a))M$, $\overrightarrow{e_x}(x,a)=-a\overrightarrow{f_{ex}}(x,e(x,a))M$ with $M=[af_{ee}(x,e(x,a))-C_{ee}(e(x,a))]^{-1}$. M is regular since it is symmetric definite negative because f is strictly concave, C is strictly convex and $a\geq 0$. We have $\forall\overrightarrow{u}\in\Re^n, \langle\overrightarrow{u}M,\overrightarrow{u}\rangle<0$ so $\left\langle\overrightarrow{e_a},\overrightarrow{f_e}\right\rangle>0$ and $\left\langle\overrightarrow{e_x},\overrightarrow{f_{ex}}\right\rangle>0$.

A.4 Proof of proposition 5

The principal's maximization problem can be written:

$\underset{(a,b)\in\Re^+\times\Re}{Max}(1-a)E\nu f(x,e(x,a))-b+Ev(\varepsilon g(x,e(x,a)))$ subject to

$\underset{e}{Max}EU[a\nu f(x,e)+b]-C(e)\geq\overline{U}$ and $E[U'a\nu]\overrightarrow{f_e}=\overrightarrow{C_e}$ with the first order approach. We will suppose that the fixed payment b is adjusted so that the individual rationality constraint binds. Actually, for example with one task, the Lagrange multiplier associated to the participation constraint which equals

$\lambda=\frac{1}{EU'}f-(\frac{E\nu U'+aE\nu^2U''f_e}{aE\nu U''})/(\frac{E\nu U'}{EU'}f-\frac{E\nu U'+aE\nu^2U''f_e}{aE\nu U''})$ is likely to be non zero (and equals one in the risk neutral case). Therefore we have $\forall a, b(x,a)$ such that $\underset{e}{\max}(EU[a\nu f(x,e)+b]-C(e))=\overline{U}$. Also the incentive constraint defines an agent's best answer to contract $(a,b(a))$: $e(x,a)$ is the implicit function defined by $E[U'a\nu]\overrightarrow{f_e}=\overrightarrow{C_e}$. It gives the program

$\underset{a\in\Re^+}{Max}H(a)=(1-a)E\nu f(x,e(x,a))-b(x,a)+Ev(\varepsilon g(x,e(x,a)))$

So, $\frac{\partial H}{\partial a}(a^*)=0$ leads to $-f+(1-a^*)\left\langle\overrightarrow{e_a},\overrightarrow{f_e}\right\rangle-(\frac{\partial b}{\partial a})_{|\overline{U}}+E[\varepsilon v']\langle\overrightarrow{e_a},\overrightarrow{g_e}\rangle=0$. The definition of b implies that $(\frac{\partial b}{\partial a})_{|\overline{U}}=-fE\nu U'/EU'$ ($=-f$ with risk neutrality), and with lemma 4, $\overrightarrow{e_a}=-\overrightarrow{f_e}M$. Differentiating the implicit equation defining $e(.,.)$ we obtain for the risk averse case more complicated formulae:

$\overrightarrow{e_a}=[E\nu U'+af(E\nu^2U''-E\nu U''\frac{E\nu U'}{EU'})]\overrightarrow{f_e}\widetilde{M}$ and

$\overrightarrow{e_x} = a[E\nu U'\overrightarrow{f_{ex}} + af_x(E\nu^2U'' - E\nu U''\frac{E\nu U'}{EU'})\overrightarrow{f_e}]\widetilde{M}$, with the matrix $\widetilde{M}^{-1} = [C_{ee} - aE\nu U' f_{ee} - a^2E\nu^2U''\overrightarrow{f_e}'\overrightarrow{f_e}]$ being symmetric positive definite because $U'' \leq 0$.

Remark: The second order condition is difficult to check in the risk averse case, with risk neutrality, it is: $\left\langle \overrightarrow{e_a}, \overrightarrow{e_a}[f_{ee} - C_{ee} + E\varepsilon v' g_{ee}]\right\rangle + E\varepsilon^2 v'' \left\langle \overrightarrow{e_a}, \overrightarrow{g_e}\right\rangle^2 \leq 0$. As $[f_{ee} - C_{ee} + E\varepsilon v' g_{ee}]$ is a definite negative matrix, $\left\langle \overrightarrow{e_a}, \overrightarrow{e_a}[f_{ee} - C_{ee} + E\varepsilon v' g_{ee}]\right\rangle < 0$ and either v concave or $E\varepsilon^2 v'' \left\langle \overrightarrow{e_a}, \overrightarrow{g_e}\right\rangle^2$ not large in magnitude if positive.

A.5 Monotonicity of the land value function

Formally[27]: $w(x) \geq \underset{\tau(.)\in\widehat{\Theta}\ s.t.\ (IC),(IR)}{Max} E[y - \tau(y)] + \rho Ew(z)$ since $\widehat{\Theta} \subset \Theta$. With help of proposition **??**, we define $\theta_x : \Theta_x \to \widehat{\Theta}_x$ which associates the linear second best transfer $d \in \widehat{\Theta}_x$ to the second best differentiable one $\tau \in \Theta_x$. And then, $w(x) = E[y - \tau(y)] + \rho Ew(z) = E[y - d(y)] + \rho Ew(z) \leq \underset{\tau(.)\in\widehat{\Theta}\ s.t.\ (IC),(IR)}{Max} E[y - \tau(y)] + \rho Ew(z)$ where $\tau \in \Theta_x$, $d = \theta(\tau) \in \widehat{\Theta}_x \subset \widehat{\Theta}$.

In some special cases we prove that the land value is indeed monotone increasing i.e. that a better plot brings a higher welfare to its owner. We must show that the solution to the functional equation is increasing i.e. defining the operator T by: $w \to Tw$ such that $Tw(x) = \underset{\tau(.)\ s.t.\ (IC),(IR)}{Max} E[y - \tau(y)] + \rho Ew$ that it is a contraction mapping which fixed point is increasing. We must show that T maps increasing functions set into itself.

The land value function is solution of the functional equation:

$$w(x) = \underset{\tau(.)\in\widehat{\Theta}\ s.t.\ (IC),(IR)}{Max} E[y - \tau(y)] + \rho Ew(z)$$

where $\widehat{\Theta}$ is the functional space of choice i.e. the linear functions from $\Re^+$ to $\Re^+$. It is equivalent to: $w(x) = \underset{\tau(.)\in\widehat{\Theta}}{Max}\{F(x, \tau(.)) + \rho Ew(G(x, \tau(.), \varepsilon))\}$ because e depends on x and $\tau(.)$ through (IC) and (IR), with $G(x, \tau(.), \varepsilon) = g(\varepsilon(x, e(x, \tau(.))))$ and $F(x, \tau(.)) = E[\nu f(x, e(x, \tau(.))) - \tau(\nu f(x, e(x, \tau(.))))]$ where $e(x, \tau(.))$ is the implemented agent's effort vector by contract $\tau(.)$.

We note $C(X, \Re)$ the X to $\Re$ continuous functions set and $\widetilde{C}(X, \Re)$ its increasing functions subset. Let $\|.\|_\infty$ be the superior norm[28] of $C(X, \Re)$ which gives to $C(X, \Re)$ a structure

[27] Θ is the differentiable functions space and $\widehat{\Theta}$ is the linear functions subset of Θ.

[28] For $f(.) \in C(X, \Re)$, we have $\|f\|_\infty = \underset{x\in X}{Sup}|f(x)|$.

of complete metric space. We note d the corresponding distance[29]. Let the operator T: $C(X,\Re) \to C(X,\Re)$ such that:

$\forall w \in C(X,\Re)\,,\, Tw : x \in X \to Tw(x) = \underset{\tau(.)\in\widehat{\Theta}}{Max}\left[F(x,\tau(.)) + \rho Ew(G(x,\tau(.),\varepsilon))\right]$

It is obvious that $Tw \in C(X,\Re)$ because Tw is continuous by continuous functions composition (supposing that the maximum is reached and continuous).

i) Show that T is a contraction of modulus ρ:

Let $w, \widetilde{w} \in C(X,\Re)$: $\forall x \in X$, we have:

$Tw(x) = \underset{\tau(.)\in\widehat{\Theta}}{Max}\left[F(x,\tau(.)) + \rho Ew(G(x,\tau(.),\varepsilon))\right] = F(x,\tau) + \rho Ew(z)$

where $\tau \in \underset{\tau(.)\in\widehat{\Theta}}{\arg\max}\left[F(x,\tau(.)) + \rho Ew(G(x,\tau(.),\varepsilon))\right]$ and $z = G(x,\tau,\varepsilon)$.

And $T\widetilde{w}(x) = \underset{\tau(.)\in\widehat{\Theta}}{Max}\left[F(x,\tau(.)) + \rho E\widetilde{w}(G(x,\tau(.),\varepsilon))\right] = F(x,\widetilde{\tau}) + \rho E\widetilde{w}(\widetilde{z})$

where $\widetilde{\tau} \in \underset{\tau(.)\in\widehat{\Theta}}{\arg\max}\left[F(x,\tau(.)) + \rho E\widetilde{w}(G(x,\tau(.),\varepsilon))\right]$ and $\widetilde{z} = G(x,\widetilde{\tau},\varepsilon)$.

Hence according to the definition of Tw and $T\widetilde{w}$:

$T\widetilde{w}(x) \geq F(x,\tau) + \rho E\widetilde{w}(z)$ and $Tw(x) \geq F(x,\widetilde{\tau}) + \rho Ew(\widetilde{z})$

So $\rho E\left[\widetilde{w}(z) - w(z)\right] \leq T\widetilde{w}(x) - Tw(x) \leq \rho E\left[\widetilde{w}(\widetilde{z}) - w(\widetilde{z})\right]$

Hence: $\forall x \in X,\ |T\widetilde{w}(x) - Tw(x)| \leq \rho \underset{y\in X}{Sup}|\widetilde{w}(y) - w(y)| = \rho\|\widetilde{w} - w\|_\infty$

and $\|T\widetilde{w} - Tw\|_\infty \leq \rho\|\widetilde{w} - w\|_\infty$

T is indeed a contraction with modulus ρ.

ii) Show that the fixed point of T is increasing under some conditions:

Let $w \in C(X,\Re)$: $\forall x \in X$, (simplifying notations obviously, using previous results):

$$\begin{aligned} Tw(x) &= \underset{\tau(.)\in\widehat{\Theta}}{Max} E\left[\nu f(x,e(x,\tau(.))) - \tau(\nu f(x,e(x,\tau(.))))\right] + \rho Ew(\varepsilon g(x,e(x,\tau(.)))) \\ &= \underset{a\geq 0}{Max} f(x,e(x,a)) - C(e(x,a)) - \overline{U} + \rho Ew(\varepsilon g(x,e(x,a))) \\ &= f(x,e(x,a^*(x))) - C(e(x,a^*(x))) - \overline{U} + \rho Ew(\varepsilon g(x,e(x,a^*(x)))) \end{aligned}$$

where $a^*(x) = \underset{a}{\arg\max} f(x,e(x,a)) - C(e(x,a)) - \overline{U} + \rho Ew(\varepsilon g(x,e(x,a)))$ and $e(x,a) = \underset{e}{\arg\max} aE[\nu f(x,e)] - C(e)$. With the envelope theorem $\frac{\partial}{\partial x}Tw(x) = \frac{\partial f}{\partial x} + \rho E\varepsilon w'\frac{\partial g}{\partial x} + \left\langle \overrightarrow{e_x}, \overrightarrow{f_e} - \overrightarrow{C_e} + \rho E\varepsilon w'\overrightarrow{g_e}\right\rangle = \frac{\partial f}{\partial x} + \rho E\varepsilon w'\frac{\partial g}{\partial x} + \left\langle \overrightarrow{e_x}, (1-a)\overrightarrow{f_e} + \rho E\varepsilon w'\overrightarrow{g_e}\right\rangle$ because $a\overrightarrow{f_e} = \overrightarrow{C_e}$. As $\frac{\partial f}{\partial x} + \rho E\varepsilon w'\frac{\partial g}{\partial x} > 0$, $\left\langle \overrightarrow{e_x}, (1-a)\overrightarrow{f_e} + \rho E\varepsilon w'\overrightarrow{g_e}\right\rangle \geq 0$ will suffice for $\frac{\partial}{\partial x}Tw(x) > 0$. With proposition 5 we have $\left\langle \overrightarrow{e_x}, (1-a)\overrightarrow{f_e} + \rho E\varepsilon w'\overrightarrow{g_e}\right\rangle = \rho E\varepsilon w'\left\langle \overrightarrow{e_x}, \left\langle \overrightarrow{e_a}, \overrightarrow{f_e}\right\rangle\overrightarrow{g_e} - \left\langle \overrightarrow{e_a}, \overrightarrow{g_e}\right\rangle\overrightarrow{f_e}\right\rangle$. But $\rho E\varepsilon w' / \left\langle \overrightarrow{e_a}, \overrightarrow{f_e}\right\rangle > 0$, so

[29] The distance d is defined by : $\forall f,g \in C(X,\Re)\ d(f,g) = \|f-g\|_\infty = \underset{x\in X}{Sup}|f(x) - g(x)|$.

$\left\langle \overrightarrow{e_x}, \left\langle \overrightarrow{e_a}, \overrightarrow{f_e} \right\rangle \overrightarrow{g_e} - \langle \overrightarrow{e_a}, \overrightarrow{g_e} \rangle \overrightarrow{f_e} \right\rangle \geq 0$ is sufficient. When this condition is checked, Tw is strictly increasing i.e. $Tw \in \widetilde{C}(X, \Re)$.

With risk neutrality and one task, the first order condition on the agent's effort implies that $\frac{\partial}{\partial x} Tw(x) = \frac{\partial f}{\partial x} + \rho E \varepsilon w' \frac{\partial g}{\partial x} > 0$.

iii) As T is a contraction, it has a unique fixed point w satisfying $w = Tw$. Since $\forall v_0 \in C(X, \Re)$, $\forall k = 0, 1, 2, ..., d(T^k v_0, w) \leq \rho^k d(v_0, w)$, with $v_0 \in \widetilde{C}(X, \Re)$ the sequence $T^k v_0$ converges towards w. As $\widetilde{C}(X, \Re)$ is a closed subset of $C(X, \Re)$, $w \in \widetilde{C}(X, \Re)$. Consequently, the solution of the functional equation is an increasing function[30].Proof of proposition 10: obvious with $\frac{\partial Tw(x)}{\partial x} = (\eta_a^f \eta_x^g - \eta_a^g \eta_x^f) f g w' \left\langle \overrightarrow{e_a}, \overrightarrow{f_e} \right\rangle / ax + a f_x$.As $f_x \geq 0$ and $\rho w' g_x \geq 0$, a useful sufficient condition for $\frac{\partial Tw(x)}{\partial x} \geq 0$ appears: $\Gamma = \left\langle \overrightarrow{e_a}, \overrightarrow{f_e} \right\rangle \langle \overrightarrow{e_x}, \overrightarrow{g_e} \rangle - \langle \overrightarrow{e_a}, \overrightarrow{g_e} \rangle \left\langle \overrightarrow{e_x}, \overrightarrow{f_e} \right\rangle \geq 0$.

For the formal concavity study of the land value function, we must prove that the solution to the functional equation is concave. In some cases, it is quite easily proved. A first in mind condition is the one which allows to implement the first best.

Two tasks: We find several properties allowing to infer the land value function is increasing. When we restrict to two efforts, as $(\overrightarrow{f_e}, \overrightarrow{g_e})$ makes up a base of $\Re^2$ (otherwise we are in the colinear case), we can write $\overrightarrow{f_{ex}} = \lambda \overrightarrow{f_e} + \mu \overrightarrow{g_e}$. With lemma 4, we obtain $\Gamma = a\mu(\left\langle \overrightarrow{f_e} M, \overrightarrow{f_e} \right\rangle \langle \overrightarrow{g_e} M, \overrightarrow{g_e} \rangle - \left\langle \overrightarrow{f_e} M, \overrightarrow{g_e} \right\rangle^2)$. By Schwartz's inequality, $\left\langle \overrightarrow{f_e} M, \overrightarrow{f_e} \right\rangle \langle \overrightarrow{g_e} M, \overrightarrow{g_e} \rangle \geq \left\langle \overrightarrow{f_e} M, \overrightarrow{g_e} \right\rangle^2$. Therefore $\Gamma \geq 0$ if and only if $\mu \geq 0$. Since $\mu = (f_1 g_2 - f_2 g_1)/(f_1 f_{2x} - f_2 f_{1x})$ it needs that numerator and denominator be of the same sign i.e. $f_1/f_2 \geq g_1/g_2$ and $f_1/f_2 \geq f_{1x}/f_{2x}$ or $f_1/f_2 \leq g_1/g_2$ and $f_1/f_2 \leq f_{1x}/f_{2x}$.

A.6 Proof of proposition 8

Like in A.7, $\frac{\partial a^*(x)}{\partial x} = -\frac{\partial^2 H}{\partial a \partial x} / \frac{\partial^2 H}{\partial a^2}$ with $\frac{\partial^2 H}{\partial a^2} < 0$ and $\frac{\partial^2 H}{\partial a \partial x} = \left\langle \overrightarrow{e_{ax}}, \overrightarrow{f_e} - \overrightarrow{C_e} + E\varepsilon v' \overrightarrow{g_e} \right\rangle + E\varepsilon^2 v'' \langle \overrightarrow{e_a}, \overrightarrow{g_e} \rangle [g_x + \langle \overrightarrow{e_x}, \overrightarrow{g_e} \rangle] + \left\langle \overrightarrow{e_a}, \overrightarrow{f_{ex}} + E\varepsilon v' \overrightarrow{g_{ex}} + \overrightarrow{e_x}[f_{ee} - C_{ee} + E\varepsilon v' g_{ee}] \right\rangle$.

According to the assumption, we write $\overrightarrow{g_e} = \lambda \overrightarrow{f_e}$. With proposition 3, lemma 4, and the incentive constraint: $a^* - 1 = \lambda E \varepsilon v'$, $\overrightarrow{e_x} = -a^* \overrightarrow{f_{ex}} M$, $a^* \overrightarrow{f_e} = \overrightarrow{C_e}$. So $\overrightarrow{f_e} - \overrightarrow{C_e} + E\varepsilon v' \overrightarrow{g_e} = 0_n$, the matrix $[f_{ee} - C_{ee} + E\varepsilon v' g_{ee}]$ is equal to $M^{-1} = a^* f_{ee} - C_{ee}$, $\overrightarrow{e_x}[f_{ee} - C_{ee} + E\varepsilon v' g_{ee}] = -a^* \overrightarrow{f_{ex}}$, $\overrightarrow{f_{ex}} + E\varepsilon v' \overrightarrow{g_{ex}} + \overrightarrow{e_x}[f_{ee} - C_{ee} + E\varepsilon v' g_{ee}] = 0_n$. Then $\frac{\partial^2 H}{\partial a \partial x} = E\varepsilon^2 v'' \langle \overrightarrow{e_a}, \overrightarrow{g_e} \rangle [g_x + \langle \overrightarrow{e_x}, \overrightarrow{g_e} \rangle]$. As

[30] See Stokey, Lucas, Prescott (1989) for further methodological details.

the first best can be implemented, we have $e(x, a^*(x)) = e^*(x)$ ($e^*(x)$ being the effort supply defined by the first best program). Therefore, $g_x + \langle \overrightarrow{e_x}, \overrightarrow{g_e} \rangle \geq 0$ means that at first best, the fertility level reached at the end of the season increases with the initial fertility. So, we have the result. In the particular case where $\overrightarrow{f_{ex}} = \mu \overrightarrow{g_e}$, with $\mu \geq 0$, $\frac{\partial^2 H}{\partial a \partial x} = E\varepsilon^2 v'' \langle \overrightarrow{e_a}, \overrightarrow{g_e} \rangle [g_x - a^* \mu \langle \overrightarrow{g_e} M, \overrightarrow{g_e} \rangle]$ has obviously same sign than $-\langle \overrightarrow{e_a}, \overrightarrow{g_e} \rangle$ because M is definite negative.

A.7 Proof of proposition 9

Writing $H(a, x) = f(x, e(x, a)) - C(e(x, a)) - \overline{U} + Ev(\varepsilon g(x, e(x, a)))$, $a^*(x) = \arg\max_a H(a, x)$. According to the envelope theorem $\forall x$, $\frac{\partial H}{\partial a}(a^*(x), x) = 0$ implying that $\frac{\partial a^*(x)}{\partial x} = -\frac{\partial^2 H}{\partial a \partial x} / \frac{\partial^2 H}{\partial a^2}$. But, $\frac{\partial^2 H}{\partial a^2} = E\varepsilon^2 v'' \langle \overrightarrow{e_a}, \overrightarrow{g_e} \rangle^2 + \langle \overrightarrow{e_a}, \overrightarrow{e_a}[f_{ee} - C_{ee} + E\varepsilon v' g_{ee}] \rangle + \left\langle \overrightarrow{e_{aa}}, \overrightarrow{f_e} - \overrightarrow{C_e} + E\varepsilon v' \overrightarrow{g_e} \right\rangle$ and $\frac{\partial^2 H}{\partial a \partial x} = \left\langle \overrightarrow{e_a}, \overrightarrow{f_{ex}} + E\varepsilon v' \overrightarrow{g_{ex}} + \overrightarrow{e_x}[f_{ee} - C_{ee} + E\varepsilon v' g_{ee}] \right\rangle + \left\langle \overrightarrow{e_{ax}}, \overrightarrow{f_e} - \overrightarrow{C_e} + E\varepsilon v' \overrightarrow{g_e} \right\rangle + E\varepsilon^2 v'' \langle \overrightarrow{e_a}, \overrightarrow{g_e} \rangle [g_x + \langle \overrightarrow{e_x}, \overrightarrow{g_e} \rangle]$

According to the second order condition of the principal's maximization: $\frac{\partial^2 H}{\partial a^2} \leq 0$ which is assumed satisfied (to be proved we only need that $\left\langle \overrightarrow{e_{ax}}, \overrightarrow{f_e} - \overrightarrow{C_e} + E\varepsilon v' \overrightarrow{g_e} \right\rangle$ be negative or small in magnitude if positive because $\langle \overrightarrow{e_a}, \overrightarrow{e_a}[f_{ee} - C_{ee} + E\varepsilon v' g_{ee}] \rangle + E\varepsilon^2 v'' \langle \overrightarrow{e_a}, \overrightarrow{g_e} \rangle^2 \leq 0$ since v is increasing concave, f, g, concave, C convex, $[f_{ee} - C_{ee} + E\varepsilon v' g_{ee}]$ definite negative). As $\overrightarrow{e_x} = -a \overrightarrow{f_{ex}} M$, when $\overrightarrow{f_{ex}} = 0_n$ then $\overrightarrow{e_x} = 0_n$ and $\overrightarrow{e_{ax}} = 0_n$. So $\frac{\partial^2 H}{\partial a \partial x} = E\varepsilon^2 v'' \langle \overrightarrow{e_a}, \overrightarrow{g_e} \rangle g_x + E\varepsilon v' \langle \overrightarrow{e_a}, \overrightarrow{g_{ex}} \rangle$. Moreover if $\overrightarrow{g_{ex}} = 0_n$: $\frac{\partial^2 H}{\partial a \partial x} = E\varepsilon^2 v'' \langle \overrightarrow{e_a}, \overrightarrow{g_e} \rangle g_x$. g being increasing in x, v increasing and concave: $a^{*'}(x)$ has same sign than $-\langle \overrightarrow{e_a}, \overrightarrow{g_e} \rangle$. So, since $a^* \gtreqless 1$ whether $\langle \overrightarrow{e_a}, \overrightarrow{g_e} \rangle \gtreqless 0$, we have the desired result. We note that in the separable case : $\frac{\partial a^*(x)}{\partial x} = -E\varepsilon^2 v'' \langle \overrightarrow{e_a}, \overrightarrow{g_e} \rangle g_x /$
$(E\varepsilon^2 v'' \langle \overrightarrow{e_a}, \overrightarrow{g_e} \rangle^2 + \langle \overrightarrow{e_a}, \overrightarrow{e_a}[f_{ee} - C_{ee} + E\varepsilon v' g_{ee}] \rangle + \left\langle \overrightarrow{e_{aa}}, \overrightarrow{f_e} - \overrightarrow{C_e} + E\varepsilon v' \overrightarrow{g_e} \right\rangle)$.

A.8 Proof of proposition 11

At the implemented efforts, the incentive compatibility condition says that $a \overrightarrow{f_e} = \overrightarrow{C_e}$, so with the assumption, we can write: $\overrightarrow{f_e} = \mu \overrightarrow{g_e}$ with $\mu \in \Re$. Thus: $\left\langle \overrightarrow{e_a}, \overrightarrow{f_e} \right\rangle \overrightarrow{g_e} - \langle \overrightarrow{e_a}, \overrightarrow{g_e} \rangle \overrightarrow{f_e} = \langle \overrightarrow{e_a}, \mu \overrightarrow{g_e} \rangle \overrightarrow{g_e} - \langle \overrightarrow{e_a}, \overrightarrow{g_e} \rangle \mu \overrightarrow{g_e} = 0_n$, so $\Gamma = 0$ which gives the monotonicity. For concavity, using notations of A.5, we remark that $Tw(x) = \underset{a \geq 0}{Max} f(x, e(x, a)) - C(e(x, a)) - \overline{U} + \rho Ew(\varepsilon g(x, e(x, a))) = \underset{e}{Max} f(x, e) - C(e) - \overline{U} + \rho Ew(\varepsilon g(x, e))$. Noting $K(x, e) = f(x, e) -$

$C(e) - \overline{U} + \rho E w(\varepsilon g(x,e))$ and $e(x) = \arg\max_e K(x,e)$, we know that $K(x,e)$ is concave in (x,e) because f, g are concave in (x,e) and C is convex in e. We have $\forall x, K_e(x, e(x)) = 0$ so $\forall x,\ e_x(x) = -K_{ex}(x,e(x))/K_{ee}(x,e(x))$. In addition, $Tw(x) = K(x,e(x))$, so $\frac{\partial^2}{\partial x^2}Tw(x) = (K_{xx}K_{ee} - K_{ex}^2)/K_{ee} < 0$ because $K_{ee} < 0$ and $K_{xx}K_{ee} - K_{ex}^2 > 0$ (concavity of K in (x,e)). Then, Tw is of course concave in x.

A.9 Proof of proposition 13

In this case, with results of lemma 4: $\overrightarrow{e_x} = -a\overrightarrow{f_{ex}}[af_{ee} - C_{ee}]^{-1} = -a\overrightarrow{f_{ex}}M$. If we suppose $\overrightarrow{f_{ex}} = 0_n$: $\overrightarrow{e_x} = 0_n$ and hence $\Gamma = 0$ which is sufficient for monotonicity. In order to prove that the solution of the functional equation is concave, using appendix A.5, we must prove that if w is concave then Tw is also.

Multiplicative separability in production: $f(e,x) = k(e)h(x)$

$\overrightarrow{f_{ex}} = \overrightarrow{k_e}h' = \frac{h'}{h}\overrightarrow{f_e}$, so $\overrightarrow{e_x} = -a\overrightarrow{f_{ex}}M = -a\frac{h'}{h}\overrightarrow{f_e}M = a\frac{h'}{h}\overrightarrow{e_a}$ and then $\frac{\partial}{\partial x}Tw(x) = \frac{\partial f}{\partial x} + \rho E\varepsilon w'\frac{\partial g}{\partial x} + a\frac{h'}{h}\left\langle \overrightarrow{e_a}, \overrightarrow{f_e} - \overrightarrow{C_e} + \rho E\varepsilon w'\overrightarrow{g_e}\right\rangle = \frac{\partial f}{\partial x} + \rho E\varepsilon w'\frac{\partial g}{\partial x}$ because $\left\langle \overrightarrow{e_a}, \overrightarrow{f_e} - \overrightarrow{C_e} + \rho E\varepsilon w'\overrightarrow{g_e}\right\rangle = 0$.

Additive separability in production and investment:

With $\overrightarrow{e_x} = 0_n$ $\frac{\partial}{\partial x}Tw(x) = \frac{\partial f}{\partial x} + \rho E\varepsilon w'\frac{\partial g}{\partial x}$ and with $\overrightarrow{g_{ex}} = 0_n$ $\frac{\partial^2}{\partial x^2}Tw(x) = \frac{\partial^2 f}{\partial x^2} + \rho E\varepsilon w'\frac{\partial^2 g}{\partial x^2} + \rho\frac{\partial g}{\partial x}E\varepsilon^2 w''[\frac{\partial g}{\partial x} + \frac{\partial a}{\partial x}\langle\overrightarrow{e_a}, \overrightarrow{g_e}\rangle]$. But with proposition 9 proved in A.7, we have $\frac{\partial g}{\partial x} + \frac{\partial a}{\partial x}\langle\overrightarrow{e_a}, \overrightarrow{g_e}\rangle = g_x(1 - \frac{1}{1+F}) \geq 0$ because

$F = [\langle\overrightarrow{e_a}, \overrightarrow{e_a}[f_{ee} - C_{ee} + E\varepsilon v' g_{ee}]\rangle + \left\langle\overrightarrow{e_{aa}}, \overrightarrow{f_e} - \overrightarrow{C_e} + E\varepsilon v'\overrightarrow{g_e}\right\rangle]/E\varepsilon^2 w''\langle\overrightarrow{e_a}, \overrightarrow{g_e}\rangle^2 \geq 0$ (appendix A.7). Since we know that f, g, are concave in x and that w is increasing and concave, so $\frac{\partial^2}{\partial x^2}Tw(x) \leq 0$.

B Extensions

B.1 The case of a risk averse agent

When the agent is risk averse, the incentive constraint is $aE\nu U'\overrightarrow{f_e}(x,e) = \overrightarrow{C_e}(e)$ and different propositions appear.

With a risk averse agent the optimal sharing rule derived in proposition 5 becomes

$$a^* = 1 + E\varepsilon v' \frac{\langle \overrightarrow{e_a}, \overrightarrow{g_e} \rangle}{\left\langle \overrightarrow{e_a}, \overrightarrow{f_e} \right\rangle} - (1 - \frac{E\nu U'}{EU'}) \frac{f}{\left\langle \overrightarrow{e_a}, \overrightarrow{f_e} \right\rangle} \tag{7}$$

If the landowner is also risk averse, with V its instantaneous utility function we have

$$a^* = 1 + \frac{E\varepsilon v'}{E\nu V'} \frac{\langle \overrightarrow{e_a}, \overrightarrow{g_e} \rangle}{\left\langle \overrightarrow{e_a}, \overrightarrow{f_e} \right\rangle} - (1 - \frac{E\nu U' EV'}{EU' E\nu V'}) \frac{f}{\left\langle \overrightarrow{e_a}, \overrightarrow{f_e} \right\rangle} \tag{8}$$

The effect of an increase of incentives is different in the case of a risk averse agent because of income effects.

Proposition 15 *The production increases with the slope contract a whenever* $af\sigma_A \leq \widetilde{E}\nu / \widetilde{V}\nu$ *or equivalently* $\widetilde{E}(a\nu f) \geq 2\widetilde{\pi}$ *where* $\sigma_A = \sigma_A(a\nu f + b)$ *is the absolute risk aversion coefficient of the agent and* $\widetilde{E}\nu / \widetilde{V}\nu$ *is the mean variance ratio in a* ν *equivalent probability measure,* $\widetilde{\pi}$ *is the Arrow's risk premium at agent's income. It means that if risk aversion is small enough, increasing the production share of the agent will make his effort supply rise production.*

Proof. We see that $\left\langle \overrightarrow{e_a}, \overrightarrow{f_e} \right\rangle > 0$ if and only if $\Lambda \geq 0$ where $\Lambda = E[\nu U' + af(\nu^2 U'' - \nu U'' E\nu U' / EU')]$. From the Radon-Nikodym's theorem, we can write $E(a\nu f)^2 U'' = EU'' \widetilde{E}(a\nu f)^2$, $Ea\nu f U'' = EU'' \widetilde{E} a\nu f$, $Ea\nu f U' = EU' \widetilde{E} a\nu f$ where $\widetilde{E}$ refers to a ν equivalent probability measure. Then, $\Lambda = E[(\widetilde{E}\nu / af\widetilde{V}\nu - \sigma_A) a^2 f^2 \widetilde{V}\nu U']$ with $\widetilde{V}\nu = \widetilde{E}\nu^2 - (\widetilde{E}\nu)^2$ and $\sigma_A(a\nu f + b) = -U''(a\nu f + b) / U'(a\nu f + b)$. Also $\Lambda \geq 0$ as $\widetilde{E}(a\nu f) \geq 2\widetilde{\pi}$ where $\widetilde{\pi} = \frac{1}{2}\widetilde{V}(a\nu f)\sigma_A$ is the risk premium at the agent's income. However, we can also say that:

Lemma 16 *If the agent has a relative risk aversion less than one and if* $U''' \leq 0$ *then* $\left\langle \overrightarrow{e_a}, \overrightarrow{f_e} \right\rangle > 0$ *i.e.* $\sum_{i=1}^{n} \frac{\partial f}{\partial e_i} \frac{\partial e_i}{\partial a} > 0$. *With one task, it means that the higher incentives power the greater the agent's effort supply*[31].

Proof. We need $\Lambda \geq 0$. Noting $X = a\nu f$, $\Lambda = EXU'(X + b) + EX^2 U''(X + b) - EXU''(X + b) \frac{EXU'(X+b)}{EU'(X+b)}$. Since U is increasing concave, the unique negative term is $EU''(X + b) X^2$

[31] The condition $U''' \leq 0$ implies increasing absolute and relative risk aversion which is not very intuitive but we cannot derive general formula in the other cases. It seems important to remark that in the standard sharecropping literature, this problem is generally avoided either by neglecting second order effects or by setting b to zero, or by doing a mistake as in Singh (1989) where he deduces falsely that $U''' \leq 0$ implies decreasing absolute risk aversion.

Defining $\gamma(X) = -XU''(X)/U'(X)$ the relative risk aversion, we have: If $b \geq 0$, $\gamma \leq 1$ is sufficient because $E[XU' + X^2U''] = E[XU'(1 - \gamma(X+b) - bU''/U')] \geq 0$. If $b \leq 0$ and $U''' \leq 0$ then $EX[U'(X+b) + XU''(X+b)] \geq E[XU'(X)(1-\gamma(X))] \geq 0$ if $\gamma \leq 1$.[32]

Risk averse agent:

Noting that $1 - E\nu U'/EU' > 0$ with the utility concavity, the risk sharing term tends to diminish a^*, as intuition predicts, to alleviate the tenant from risk bearing. We know that $\left\langle \overrightarrow{e_a}, \overrightarrow{f_e} \right\rangle > 0$ in the risk neutral case.

Risk averse principal:

When, the principal (instantaneous utility function V) is risk averse and the agent is not, we have $a^* = 1 + \frac{E\varepsilon v'}{E\nu V'} \frac{\left\langle \overrightarrow{e_a}, \overrightarrow{g_e} \right\rangle}{\left\langle \overrightarrow{e_a}, \overrightarrow{f_e} \right\rangle}$. Hence, whatever be the principal's risk aversion, $a^* \gtreqless 1$ depends on the sign of $\langle \overrightarrow{e_a}, \overrightarrow{g_e} \rangle$ as when he is risk neutral. The trade-off between production and fertility uncertainty for the landlord appears and is multiplicative with the effort distortion ratio.

B.2 The role of uncertainty

Mean and variance of principal's income:

Noting the random variable $v(\varepsilon g) = \widetilde{\omega}$, the mean and variance of landlord's welfare are:

$$\begin{aligned}
\overline{Y_l} &= (1-a^*) f - b^* + E\widetilde{\omega} = f - \Phi\left(\overline{U} + C\right) + E\widetilde{\omega} \\
\sigma^2_{Y_l} &= (1-a^*)^2 f^2 \sigma^2_\nu + \sigma^2_{\widetilde{\omega}} + 2(1-a^*) f \sigma_{\nu\widetilde{\omega}}
\end{aligned}$$

because $E\nu = 1$. We see that the variance of landlord's welfare depends on variance and covariance of production and fertility uncertainties. Assuming that $a^* \leq 1$ because incentives to production would be bad for future fertility. If these random shocks are climatic shocks, we expect that production and fertility shocks owed to weather will be positively correlated meaning that landlord's income variance will be increased with the covariance of the shocks. On the contrary, if the ν is the production price uncertainty and the fertility shock is climatic, then the covariance is expected to be negative because a negative agricultural shock will be followed by a positive market price shock. Then the landlord's income variance will decrease

[32] We remark that when $b = 0$, non increasing relative risk aversion is sufficient because defining $\pi = EXU'(X)/EU'(X)$, we have $EX^2U'' - EXU''EXU'/EU' = E[U'(X)(X-\pi)(\gamma(\pi) - \gamma(X))] \geq 0$ with γ non increasing and because $E[U'(X)(X-\pi)] = 0$.

with the covariance of the shocks.

With risk neutrality, we have $E\nu U' = EU'$, and $a^* = 1 + E\varepsilon v' \frac{\langle \overrightarrow{e_a}, \overrightarrow{g_e} \rangle}{\langle \overrightarrow{e_a}, \overrightarrow{f_e} \rangle} = 1 + E\varepsilon v' \frac{\langle \overrightarrow{f_e} M, \overrightarrow{g_e} \rangle}{\langle \overrightarrow{f_e} M, \overrightarrow{f_e} \rangle}$.
The sign of $\left\langle \overrightarrow{f_e} M, \overrightarrow{g_e} \right\rangle$ is needed to know if a^* is greater, equal or smaller than one. We can meet various cases studied in the next section.

Fertility uncertainty:

The random factor on land fertility x has an influence on the optimal contract slope. The production uncertainty is taken into account through the risk sharing expression $(1 - E\nu U'/EU')f/\left\langle \overrightarrow{e_a}, \overrightarrow{f_e} \right\rangle$. Even with risk neutrality of principal and agent, the land fertility uncertainty is important because though the landlord is risk neutral on production, he is not on fertility since the value function is not linear. If the value function of future fertility for the landlord exhibits a relative prudence[33] less than one (which is the case if $v''' \leq 0$ i.e. v'' is decreasing) then by Jensen's inequality, $E\varepsilon v'(\varepsilon g) \leq v'(g)$ i.e. that with fertility uncertainty, the distortion ratio is smaller than with deterministic fertility. On the contrary if the value function exhibits a relative prudence higher than one, by Jensen's inequality, the distortion ratio is greater than with deterministic fertility.

Linearizing v' in the neighborhood of $g(x,e)$ states $E\varepsilon v' \simeq v' + Var(\varepsilon) v''$ because $E\varepsilon = 1$. Therefore, when v is increasing concave, a^* increases as $Var(\varepsilon)$ increases if $\langle \overrightarrow{e_a}, \overrightarrow{g_e} \rangle < 0$, and a^* decreases as $Var(\varepsilon)$ increases if $\langle \overrightarrow{e_a}, \overrightarrow{g_e} \rangle > 0$.

In the risk neutral case: when $a^* < 1$, a^* increases as $Var(\varepsilon)$ increases, when $a^* > 1$, a^* decreases as $Var(\varepsilon)$ increases. Uncertainty on land fertility tends to bring the optimal contract towards the usual second best one (a fixed rent contract without risk aversion). The intuition is that when land fertility improvements are too hazardous, they become worthless and so the incentive trade-off tends to favor production.

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[33] The relative prudence v is defined by $\frac{-xv'''(x)}{v''(x)}$ as in the case of a utility function.

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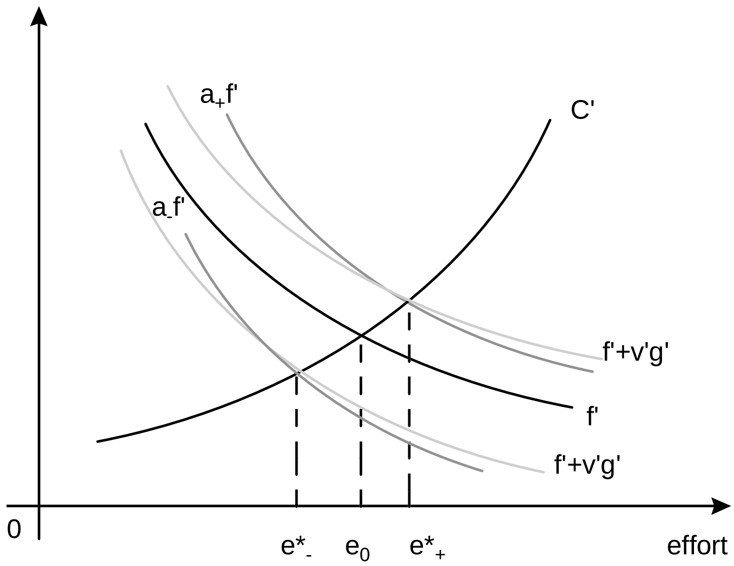

Figure 1: Optimal Contract Determination

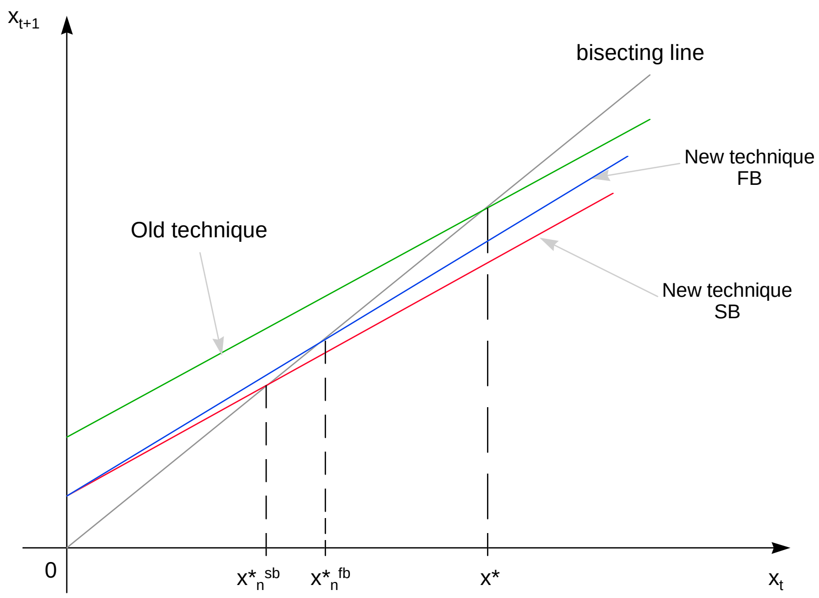

Figure 2: Innovation and fertility change

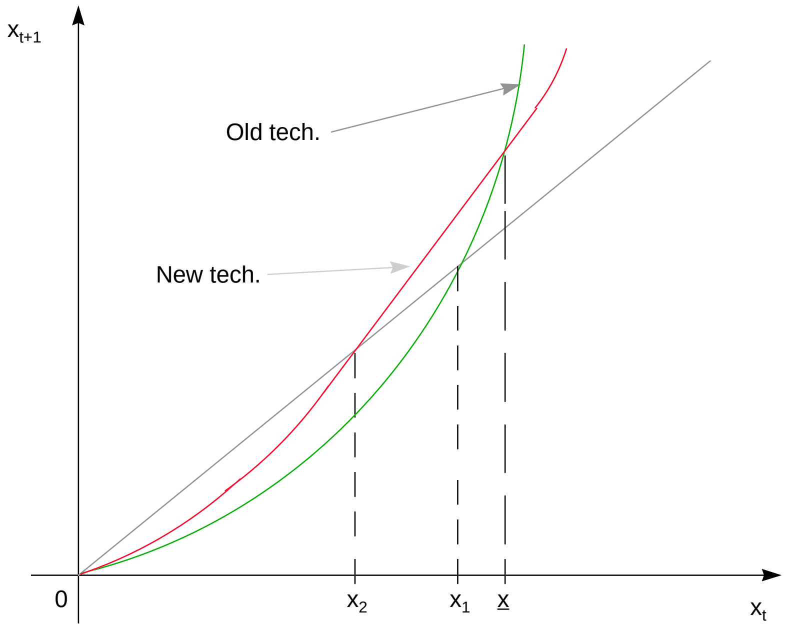

Figure 3: Innovation and fertility

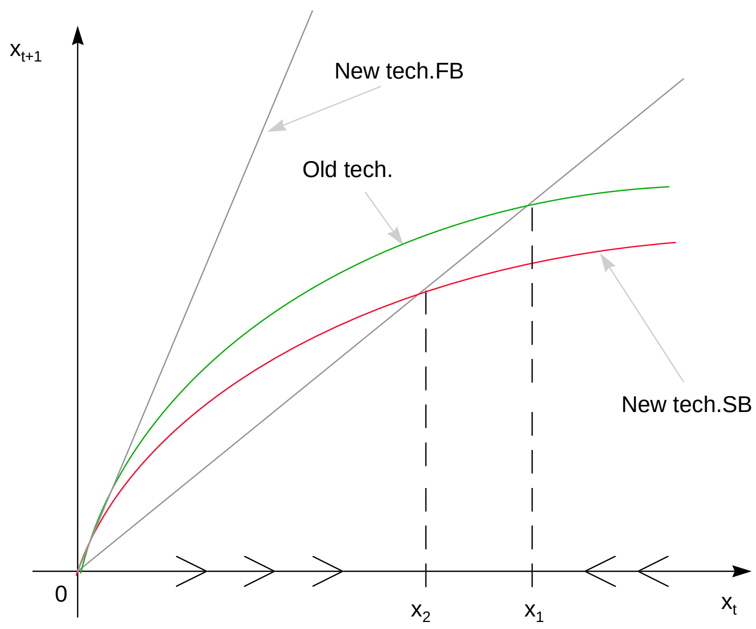

Figure 4: Innovation and fertility

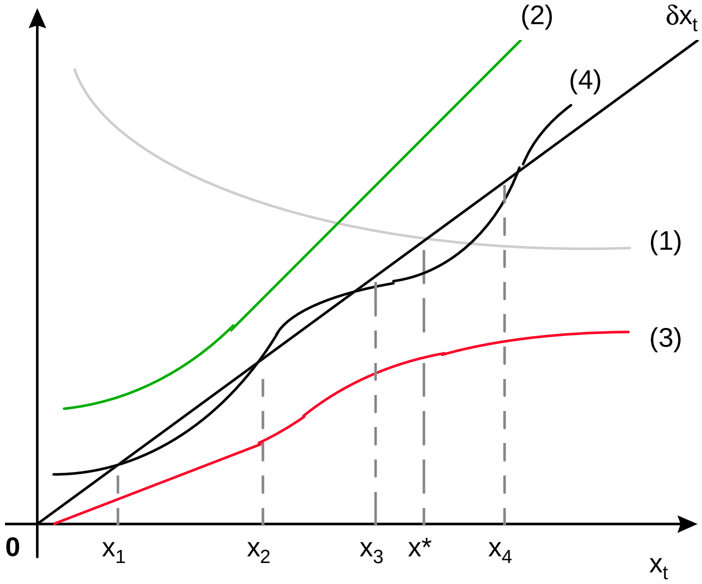

Figure 5: Fertililty dynamics examples

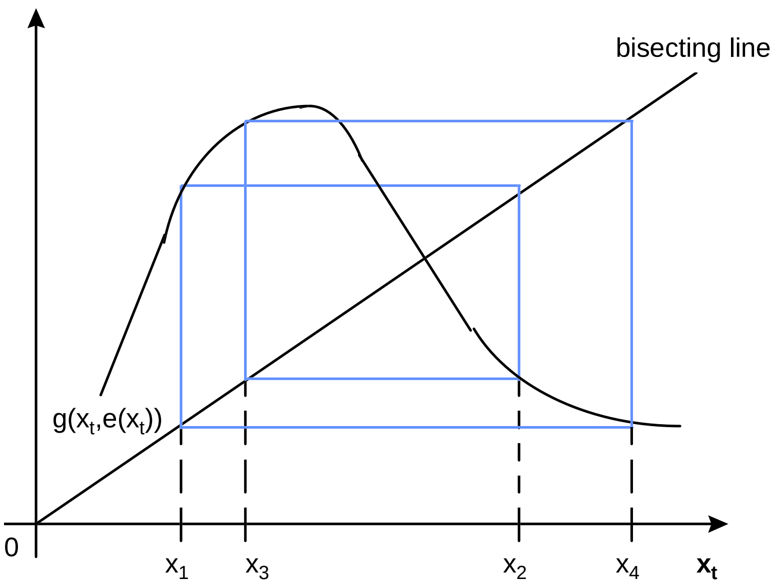

Figure 6: Fertility Path

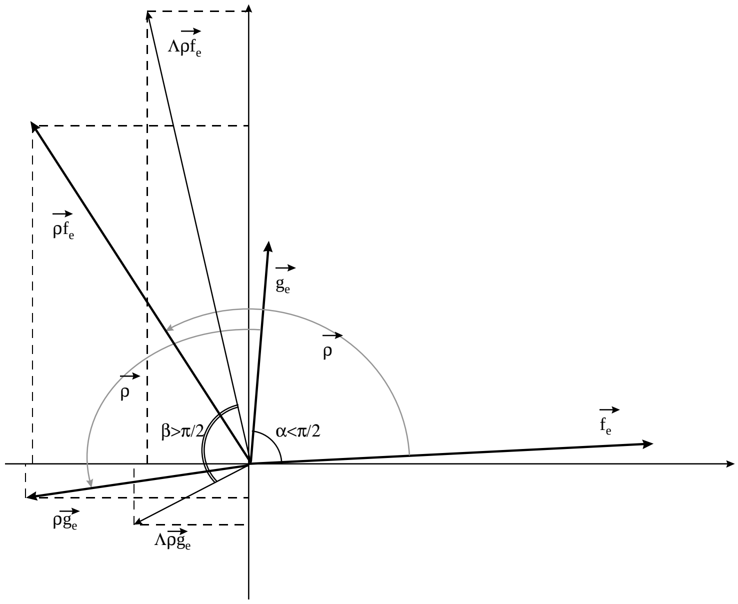

Figure 7: An intuition about a non intuitive case

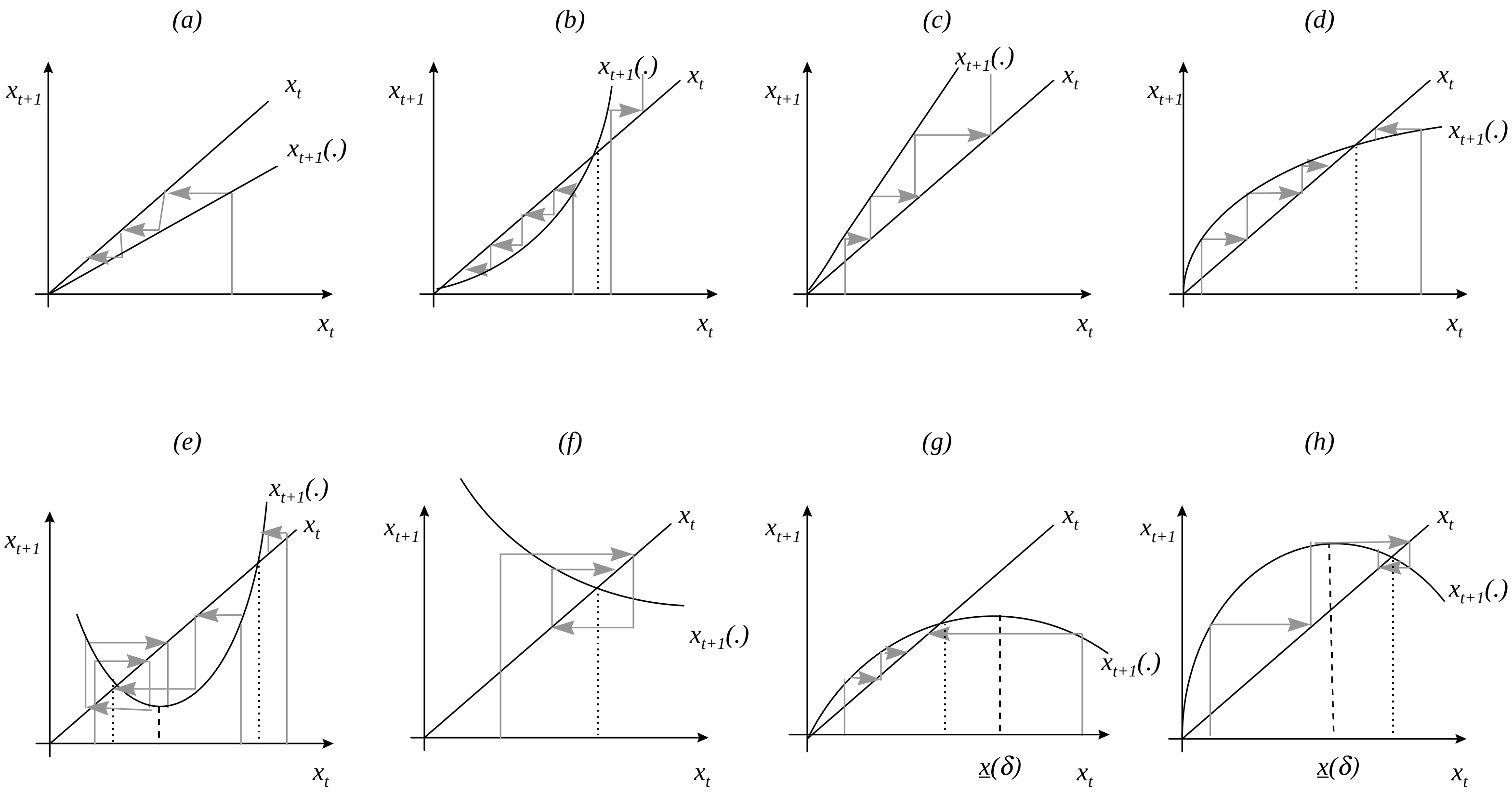

Figure 8: Fertility Dynamics : $x_{t+1} = (1+\delta)\, x_t - \widehat{g}\,(e_t)$