Multitask Moral Hazard, Incentive Contracts and Land Value

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Abstract

Sharecropping theory generally does not take into account land fertility. We develop a repeated Principal-Agent model under moral hazard where the Principal delegates the use and maintenance of a productive asset. In a multitask framework, we characterize the optimal spot contract focusing on the best incentives in the contract design. One of the main messages for land tenancy is that in a relationship where long term commitment between a landlord and a non monitored tenant is not possible, moral hazard on the peasant's actions leads to non efficient effort provision both on production and land quality maintenance. The land fertility maintenance tasks may mitigate or raise the optimal contract incentives according to the substitutability or complementarity of productive and investment tasks. Several important issues for development economics are discussed: discussions on technological innovation, production increase, tenancy reforms, agricultural taxation and sustainable agricultural development are revisited within this framework.

Key words : moral hazard, incentive contracts, land value, soil conservation, sharecropping.

JEL classification : D82, O13, Q15, Q24

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1. Introduction

Sharecropping has drawn many economists' interest for a long time. Perennial empirical observation of this agricultural organization brought up a lot of theoretical questions on its efficiency. Stiglitz (1974) pointed out that share contracts stand for second best optimal choice under moral hazard resulting from a trade-off between incentives to work and risk sharing with a risk averse tenant. This paper shows how second best contracts are different from renting at fixed rate when the Agent is risk neutral or from the usual second best sharecropping form when the landlord takes into account the long term effects of peasant's labor on the land. Adam Smith (1776) yet pointed out the importance of the sharecropper's short term behavior[1]:

"It could never, however, be to the interest of this last species of cultivators [the metayer] to lay out, in the further improvement of the land, any part of the little stock which they might save from their own share of the produce, because the lord, who laid out nothing, was to get one-half of whatever it produced. The tithe, which is but a tenth of the produce, is found to be a very great hindrance to improvement. A tax, therefore, which amounted to one-half must have been an effectual bar to it. It might be the interest of a metayer to make the land produce as much as could be brought out by means of the stock furnished by the proprietor; but it could never be his interest to mix any part of his own with it."

Here, careful attention is devoted to land fertility, agricultural technology and peasant's tasks multiplicity contrary to what is usually done in this literature. Share-tenancy models generally assume that the landlord's objective is to maximize his expected utility for a crop season depending on his net benefit, his labor supply and the contract shape (Stiglitz, 1974, Bardhan and Srinivasan, 1971, Eswaran and Kotwal, 1985, and surveys of Singh, 1989, or Chuma, Hayami and Otsuka, 1992). But if the landlord has a long term planning horizon, his intertemporal utility should depend on all future net benefits due to his property right. Despite the study of some long term contracts between landowners and landless peasants (see Dutta, Ray, Sengupta, 1989, where infinitely repeated relationships with threats of eviction are examined, or Bose, 1993), most models are static. Allen and Lueck (1992) study the contract choice between cash rent and crop share in a static simple model putting forward the idea that a share contract can curbs the farmer's incentive to exploit land attributes. Other models with multiple labor inputs have a quite different perspective (Bardhan and Srinivasan, 1971, Braverman and Stiglitz, 1982, 1986, Bardhan, 1984, Eswaran and Kotwal, 1985, Bose, 1993, Allen and Lueck, 1993-a&b, Roumasset and Uy, 1987, or Fafchamps,

[1]This quote of Adam Smith was already cited by Johnson (1950).

1993). In Bardhan and Srinivasan (1971) or Eswaran and Kotwal (1985), both landlord and tenant provide a labor input. In Roumasset and Uy (1987), a model with an investment task, a production one and two periods studies the reduction of agency costs by monitoring. Bardhan (1984 - chap. 7), Braverman and Stiglitz (1986) have a sharecropping model with a fertilizer input and a non observable labor effort. They determine the efficient incentives on both separable inputs by the mean of production sharing and cost sharing. Bardhan (1984 - chap. 8) shows with a two period model the trade-off between production incentives, enhanced in first period by a dismissal threat from the landlord, and land improvement incentives decreasing with a more powered contract. In Fafchamps (1993), two inputs are introduced sequentially.

None of these model turns out to consider the situation modelled here. We consider the case of full delegation where the tenant takes all decisions simultaneously. Because all actions are unobservable for the landlord cost sharing is not possible. Our multitask moral hazard problem can be compared to the model of Holmström and Milgrom (1987, 1991) applied to agriculture by Luporini and Parigi (1992). However, in Luporini and Parigi (1992), none of both inputs is like an investment decision because they consider two distinct production tasks that are a subsistence crop and a cash crop. Moreover, for Holmström and Milgrom (1987 and 1991), the Principal's behavior is static and information structure is set differently with signals for each task[2]. We model explicitly the long term objective of the landlord. This is not considered in the repeated moral hazard theory. In models of Rogerson (1985-a) and Lambert (1983), Agent's actions don't modify future production technology. Fudenberg, Holmström and Milgrom (1990) and Malcomson and Spinnewyn (1988) study the possibility in a repeated moral hazard relation to implement long term contracts, which are Pareto superior to short term agreements (Radner, 1985, Rubinstein and Yaari, 1983), by spot contracts sequences.

The present model focuses on short term contracts[3] under moral hazard. Contracts are incomplete because land fertility is assumed to be non contractible. This assumption can be based on prohibitively high contract design costs which do not allow the landlord to propose land fertility contingent contractual terms. Agricultural work complexity can also be called upon (Murrell, 1983) to justify that land use is too much complicated to be specified in a contract as well as empirical observation of actual contracts. This constraint is exogenous to the model. All production inputs, excepted land fertility, are called efforts representing labor tasks or even other agricultural non la-

[2] We use the Principal-Agent terminology for the relationship between the landlord and the tenant.

[3] However, non-overlapping finite length contracts with full commitment would not change qualitative results.

bor inputs. The output consists in the production and the final land fertility. Agricultural activity during a crop season affects future productions because land fertility is governed by an investment function and therefore depends on past inputs. As in Allen and Lueck (1992, 1993), and Allen (1985) the production function depends on land quality.

In this context we derive the notion of land value in the Principal's objective. We characterize the optimal second best contract and compare it to the first best with a given exogenous land value function. We find what conditions enable the first best to be implemented with imperfect information in the case of risk neutrality, although in general they are likely to be too restrictive. The second best linear contract is different from Stiglitz's one (Stiglitz, 1974). For example, risk neutrality being assumed, it is not a fixed rent one. High powered or low powered incentives (in the sense of Williamson, 1985) come from the properties of land improving efforts compared to marginal productivities. As in Baker (1992), the gap between the Principal's objective and the Agent's performance measure leads to efforts distortions in the second best case. We show then how the optimal sharing rule depends on land fertility, production and land investment functions (and Agent's risk aversion unless risk neutral). It provides high powered incentives when the Agent's labor supply has globally a positive effect on fertility. On the contrary, if efforts damage land quality, the contract has low powered incentives. This model is able to give a rationale to various forms of contracts observed empirically. We give some discussions about the production level and the fertility path dynamics induced by land tenancy as well as other interpretations of this Principal-Agent model.

Section 2 presents the model and its assumptions with respect to production, information, preferences. Section 3 explains the incentive problem. It gives the efficiency results and compare them to the usual share-tenancy ones. It provides some comparative statics and dynamics. The land value function is characterized. Several discussions of the model on development economics are in section 4. Section 5 concludes. In the appendix, some proofs are given and the case of risk aversion is discussed as well as the role of uncertainty.

2. The model

2.1. Production, information and preferences

We consider a linearly homogenous in land area agricultural production function (as generally admitted, see Stiglitz, 1974, and Chuma, Hayami and Otsuka, 1992) such that agricultural output of period t is $y_t = \nu_t f(x_{t-1}, e_t)$ where $e_t \in \Re^{+n}$ is a vector of peasant's work effort, x_{t-1} the land fertility at the end of period $t-1$, and ν_t a multiplicative positive random variable with mean one representing weather uncertainty. The effort can gather labor tasks or other agricultural inputs. Production increases with land fertility (x) and work effort (e), but at a decreasing rate. Formally, $f(.,.) : \Re^{+} \times \Re^{+n} \rightarrow \Re^{+}$ is twice differentiable, increasing in both arguments, concave in (x, e). An investment function controls land fertility dynamics according to $x_t = \varepsilon_t g(x_{t-1}, e_t)$ where ε_t is a positive random variable with mean one representing weather influence or other random externalities on land fertility[4]. e_t is a decision variable and x_{t-1} is a state variable. Depending on the nature of efforts, land fertility can increase or decrease in the components of e. We assume that $g(.,.) : \Re^{+} \times \Re^{+n} \rightarrow \Re^{+}$ is twice differentiable, concave in x and e and increasing in x. With this general specification x_t can be increasing or decreasing over time i.e. $x_t > x_{t-1}$ or $x_{t-1} > x_t$.

The landlord represented by a Principal rents out some land to a peasant, an Agent, under a contractual arrangement. Production is observable and verifiable. The Agent's actions are unobservable to the Principal, which generates some possible moral hazard, because monitoring tenant's tasks is prohibitively costly. Though observable, land fertility is not a contractible value because it is not verifiable. Hence, we consider incomplete contracts i.e. contracts which cannot be contingent to states land quality at the end of the production process. Murrell (1983) put forward that, given the complexity of the specification of the agricultural tasks set and the difficulty to observe and measure land quality, the contract can't be complete. Allen and Lueck (1992, 1993, 1996) simply use incomplete contracts without justification because empirically observed contracts never include fertility conditions. We could also say that the cost of completing contracts would be too high with respect to the potential inefficiency loss resulting from incompleteness.

According to the contract signed for period t, the Principal pays the Agent $t_t(y_t)$ at the end of crop season t[5]. The function $t_t(.)$ is chosen to depend only on the current production because only spot contracts are feasible. Let $U(t_t(y_t), e_t)$ be the Agent's instantaneous utility where U is increasing in its first argument and strictly decreasing in each effort component of e_t. The Principal's instantaneous utility $y_t - t_t(y_t)$ depends on his net revenue which implies risk neutrality. For many results, we will suppose that the Agent has separable utility between effort cost and

[4]We could have taken additive random variables without changing all next results but a multiplicative one seems more realistic and actually fits into the model.

[5]Generally, when $t_t(y_t) = y_t - r$ the contract is called a fixed rent agreement and r is the rent rate. When $t_t(y_t) = ay_t + b$ with $0 < a < 1$ the share rate of production and b a constant, the contract is called a sharecropping arrangement (pure sharecropping if $b = 0$). When $t_t(y_t) = b$ the contract is a fixed wage.

payment i.e. that $U(t(y), e) = U(t(y)) - C(e)$ with the cost of efforts $C(.)$ being an increasing, convex and twice differentiable function.

2.2. Land value

Taking into account land fertility dynamics for the landlord seems relevant as Johnson (1950) called to mind:

"When a man sells a bushel of wheat, he has no interest in the use to which the wheat is put and is consequently willing to sell to the highest bidder. However, when a man sells the use of land, he has a real interest in how the land will be used. Consequently, the choice of tenant is never made without considering what the impact of the tenancy will be upon the value of the asset."

We model this behavioral conjecture to derive consequences on the optimal contract type. At each crop season, the landlord can contract with a tenant for the current period but commitment is limited to one period contracts. There is competition between landless peasants and the landlord cannot commit himself to more than one period. Consequently, all contracts are signed for only one season. Though Pareto superior, a full commitment long term contract would require the tenant's payment to be contingent to current and past performances ("memory effect" for consumption smoothing shown by Rogerson, 1985-a, Chiappori, Macho, Rey, Salanié, 1994). Such a contract needs the capacity of past productions to be recorded and verifiable in the future. We consider that courts will enforce only one period contracts like in Phelan (1995) or that Agents work only for one period and the Principal changes of tenant each period. The Principal behaves to shape and propose at each crop season a contract that induces the best incentives through the Agent's incentive compatibility (IC_t) so as to maximize his welfare respecting the Agent's individual rationality (IR_t) represented here by an exogenous reservation utility $\overline{U_t}$. He cannot cultivate himself the land because he lives too far or that his marginal utility of leisure is too high.

The Principal has an infinite life time and maximizes an a time separable intertemporal utility with discount factor $\rho \in [0, 1[$[6]:

$$\begin{aligned} w_0(x_{-1}) = \quad & \underset{t_t(.)}{Max}\ E_{\nu_t, \varepsilon_t} \textstyle\sum_{t=0}^{\infty} \rho^t \left[y_t - t_t(y_t) \right] \\ & s.t.\ \forall t \geq 0, \left\{ \begin{array}{l} EU(t_t(y_t), e_t) \geq \overline{U_t} \quad (IR_t) \\ e_t \in \arg\max\limits_{e} EU(t_t(y_t), e) \quad (IC_t) \end{array} \right. \end{aligned} \tag{2.1}$$

[6]Principal's risk neutrality is not needed to derive the land value.

If random factors and reservation utility are stationary ($\forall t,\ w_t(.) \equiv w(.)$), the value function is solution of the following "Bellman equation":

$$w(x) = \underset{t(.)\ s.t.\ (IC),(IR)}{Max} \left\{ E\left[y - t(y)\right] + \rho E w(z) \right\} \tag{2.2}$$

$w(x_0)$ represents the maximal utility the Principal would benefit after the first period from his land of quality x_0. It is the utility provided by the optimal contracts path[7] $t_1^*(.)$, $t_2^*(.)$, ..

Contract theory (see Hart and Holmström, 1987) does not allow to characterize the optimal contracts in order to tell the properties of $w(.)$. As a benchmark, we define the value function with perfect information (i.e. without the incentive constraint):

Proposition 2.1. *With perfect information, the land value is an increasing and concave function of fertility x.*

The proof is omitted[8] but consists in defining an operator T generating the expected utility from the perspective of the end of period with a given value function governing the expected utility at the beginning period. The objective of the program is to maximize the expected value of net non transferred profits in terms of current monetary equivalent. The value function is the fixed point of T. It is increasing concave from the fact that T is a contraction mapping which maps the space of increasing concave functions to itself.

3. Optimal incentives

Our first purpose is to study the incentive problem faced by the landlord in the contract choice for the current season with a given land value function depending on its future fertility.

The Principal's objective is to maximize the expected sum of the current period utility and the final land value. This value function is derived from our model but it could be stated anyway, for example in the case of a competitive land market in which the price could be the inverse land demand function. Consequently, the risk neutral Principal has an intertemporal utility function $y - t(y) + v(x)$ where $v(.)$ represents the land value for the Principal depending on land fertility x after the season ($v(.)$ is supposed differentiable and strictly increasing). The Agent's utility function is $U(t(y)) - C(e)$. The increasing concave cost function is also assumed to satisfy $\overrightarrow{C_e}(0_n) = 0_n$[9].

[7] The existence of optimal contracts is not ascertained and usually difficult to prove when the set of feasible actions and of results are continuous, and when the Principal can choose the transfer function from an infinite dimension set (see Page, 1987). We will assume existence.

[8] A formal proof is given for the imperfect information cases.

[9] We note $\overrightarrow{C_e}$ the gradient of C with respect to e.

The output is (x, y) and the Agent's performance measure is y. Hence the Principal's objective is not directly linked to y because of the land value term $v(x)$ in which x is function of Agent's effort. As in Baker (1992), or Baker, Gibbons and Murphy (1994), Pareto efficiency of the optimal contract depends upon the relation between the Principal's objective and the Agent's performance measurement.

3.1. Perfect information

As a benchmark, when all efforts are observable and verifiable by the Principal, information is symmetric without moral hazard problem. We already noted that the endogenous value function is increasing concave. The Principal's maximization program is to maximize $E[y - t(y) + v(x)]$ by choosing $t(.)$ and e subject to the Agent's participation constraint

$$EU(t(y)) - C(e) \geq \overline{U} \tag{3.1}$$

Proposition 3.1. *In the case of perfect information, the optimal contract implements* $t^* = \Phi\left(\overline{U} + C(e^*)\right)$ *and* e^* *satisfying* $\forall i \; \frac{\partial f}{\partial e_i} + E\left[\varepsilon v'(\varepsilon g)\right] \frac{\partial g}{\partial e_i} = \Phi'\left(\overline{U} + C(e^*)\right) \frac{\partial C}{\partial e_i}$ *(leaving out arguments and noting* $\Phi = U^{-1}$ *the inverse utility function) .*

Proof. Appendix A.1.

The implemented first best effort satisfies $\overrightarrow{f_e} + E\left[\varepsilon v'\right] \overrightarrow{g_e} = \Phi'\left(\overline{U} + C(e^*)\right) \overrightarrow{C_e}$. The marginal disutility of Agent's effort equals the sum of expected marginal utility reached (one for a risk neutral Agent) times the marginal productivity and the marginal land value times the marginal fertility investment. A first best contract consists in enforcing the optimal effort level e^* and in paying the Agent a fixed wage leaving him with his reservation utility. This is conform to the classical result when one party is risk neutral: full insurance is provided to the risk averse Agent while the risk neutral Principal bears all the risk.

Moreover, if $U(0) \geq \overline{U}$ then the decision to leave land fallow by the landlord is included in the set of feasible contracts. We will suppose that $U(0) = \overline{U}$ i.e. that the landlord is not obliged to give a minimum wage for a peasant who would not work at all (the null effort is when none input is supplied). The contract implying a zero transfer will then be equivalent to a fallow decision of the Principal.

So as the Principal benefits from proposing a non zero contract the reservation utility must not be too high because the inequality $Ey^* + Ev(\varepsilon g(x, e^*)) - \Phi\left(\overline{U} + C^*\right) \geq Ev(\varepsilon g(x, 0_n)) + f(x, 0_n)$

i.e. $\overline{U} \leq U\left[Ey^* + Ev\left(\varepsilon g\left(x, e^*\right)\right) - Ev\left(\varepsilon g\left(x, 0_n\right)\right) - f\left(x, 0_n\right)\right] - C^*$ must be satisfied ($C^* = C\left(e^*\right)$). Else, land use has a negative effect on its value that counterbalances its net yield for the Principal. In this case, soil depletion induces the necessity to leave land fallow.

3.2. Imperfect information

With imperfect information on Agent's actions, the Principal seeks the transfer mechanism $t(.)$ maximizing his expected utility $E\left[y - t(y) + v(x)\right]$ under the Individual Rationality constraint (3.1), and the Incentive Compatibility one:

$$e \in \arg\max EU\left(t(y)\right) - C(e) \tag{3.2}$$

The optimal contract existence is not guaranteed but we will suppose it[10].

First, with a risk averse tenant, as shown for example by Stiglitz (1974), the second best cannot reach the first best because to implement the first best effort e^* the contract must both respect individual rationality constraint and give some incentives making $t(y)$ depend on y. If e^* is carried out and $Et\left(y^*\right) \leq t^*$ (otherwise the Principal's welfare is clearly less than first best one), Jensen's inequality says that $EU\left(t\left(y^*\right)\right) - C\left(e^*\right) \leq U\left(Et\left(y^*\right)\right) - C\left(e^*\right) \leq U\left(t^*\right) - C\left(e^*\right) = \overline{U}$ with equality only if the Agent is risk neutral. Thus, even if the first best effort is implementable, the Principal's welfare cannot equal first best's one if the Agent is risk averse.

But what are the conditions for a reward function to elicit the Agent's first best action e^*.

Proposition 3.2. *The first best effort e^* is implementable if and only if $\overrightarrow{f_e}$, $\overrightarrow{g_e}$ and $\overrightarrow{C_e}$ are colinear at optimum. In that case, the second best contract is Pareto efficient if and only if the Agent is risk neutral.*

Proof. The proof relies on the previous remarks and the fact that if we have an incentive differentiable transfer function $t(.)$, (3.2) is replaced by $E\nu U' t'(y) \overrightarrow{f_e}(x, e) = \overrightarrow{C_e}(e)$ with the first order differential approach[11]. Therefore, the effort implemented is such that $\overrightarrow{f_e}$ and $\overrightarrow{C_e}$ are colinear. The first best effort can be implemented if and only if $\overrightarrow{f_e}$ and $\overrightarrow{C_e}$ are colinear at e^*.

[10] Page (1987) showed that if the functional transfer space is uniformly bounded and sequentially compact for the point wise convergence topology, then an optimal contract exists. In our case, the sequential compactness is not guaranteed.

[11] The differential approach validity is difficult to prove. Rogerson (1985-b) shows it with some assumptions on the distribution function of the results according to the Agent's actions. Jewitt (1988) shows the validity by specifying the utility function inducing the Agent's objective to be concave.

Efficiency is reached if the first best optimal effort is such that all TRS[12] between efforts in production, cost and investment are equal at e^*. Of course, the case where two of the three vectors $\overrightarrow{f_e}$, $\overrightarrow{g_e}$ and $\overrightarrow{C_e}$ are never colinear brings that the second best will always be inefficient. Let's look at some sufficient conditions for the first best action to be implementable (appendix A.2).

Corollary 3.3. *With a risk neutral Agent, whenever $\overrightarrow{f_e}$ or $\overrightarrow{C_e}$ is colinear to $\overrightarrow{g_e}$, the first best is implementable with a linear contract which slope is $a^* = 1 + E\varepsilon v' \overrightarrow{g_e}/\overrightarrow{f_e}$ or $a^{*-1} = 1 - E\varepsilon v' \overrightarrow{g_e}/\overrightarrow{C_e}$.*

These conditions correspond to the *production and land fertility investment colinearity* or the *cost and land fertility investment colinearity.* The vectors $\overrightarrow{f_e}$ and $\overrightarrow{g_e}$ being colinear means that the TRS of two efforts in f and g are equal. Then, there is no distortion because efficient production incentives are also efficient land quality investment incentives. When Agent's effort is unidimensional, the first best is implementable if the Agent is risk neutral.

However, though the Principal is risk neutral and whatever be the Agent's risk aversion, the first best optimum cannot in general be implemented. In most contributions to sharecropping theory (Rao, 1971, Stiglitz, 1974, Newbery, 1977, Newbery and Stiglitz, 1979), the risk neutrality of the tenant allows to reach the first best even in imperfect information with a fixed rent contract. As we showed, it isn't always valid when the landlord accounts for his land value in a long term horizon. The Principal's objective is different because he cares about his expected future benefits from his land. His long term objective does not allow to implement the first best even with a risk neutral tenant. This result is similar to Baker (1992) where the difference between the Principal's objective and the contractible Agent's performance measure generates incentives distortions[13]. We showed how these different objectives can be generated endogenously reflecting different long term - short term interests in a particular environment. We can also make the two following interesting remarks.

First, if the land quality investment function has the structural following form: $x_t = \varepsilon\widehat{g}(x_{t-1}, Ey_t) = \varepsilon\widehat{g}(x_{t-1}, f(x_{t-1}, e_t)) = \varepsilon g(x_{t-1}, e_t)$ then $\overrightarrow{g_e} = \frac{\partial \widehat{g}}{\partial y}\overrightarrow{f_e}$. Hence $\overrightarrow{g_e}$ and $\overrightarrow{f_e}$ are always colinear and the first best is achieved. In this particular case, the expected production appears to be an exhaustive statistic of Agent efforts' effects on land fertility.

Second, if $f(x_{t-1}, e_t) = \nu\widehat{f}(x_{t-1}, g(x_{t-1}, e_t))$ then $\overrightarrow{f_e} = \nu\frac{\partial \widehat{f}}{\partial g}\overrightarrow{g_e}$, the first best is carried out and the expected fertility appears to be an exhaustive statistic of Agent efforts' effects on production.

[12] The Technical Rate of Substitution (TRS) between e_i and e_j in C is $-\frac{\partial C}{\partial e_j}/\frac{\partial C}{\partial e_i}$.

[13] Baker et al. (1994) show this inefficiency can sometimes be reduced by additional implicit contracts.

Finally, one can see easily that in the risk neutral case, as soon as there is a differentiable second best transfer function, the second best can be implemented by a linear contract.

3.3. Linear contracts

We focus our study on linear contracts, as usually done in this literature[14]. In the sequel, we consider that both parties are risk neutral.

The transfer function can be written $t(y) = ay + b$. We want to analyze the incentive constraint and its implications by the differentiable approach implying that the Incentive Constraint (IC) is:

$$a\overrightarrow{f_e}(x,e) = \overrightarrow{C_e}(e) \tag{3.3}$$

The share of production a is also called the contract slope and said to represent the incentive power of the contract. We first establish the following lemma (proof in A.3):

Lemma 3.4. *In the risk neutral case, the IC constraint defines an Agent's best answer effort supply $e(.,.)$ function of a and x, differentiable for which $\overrightarrow{e_x} = -a\overrightarrow{f_{ex}}M$ and $\overrightarrow{e_a} = -\overrightarrow{f_e}M$ with $M = [af_{ee} - C_{ee}]^{-1}$. Moreover $\left\langle \overrightarrow{f_e}, \overrightarrow{e_a} \right\rangle > 0$[15] and $\left\langle \overrightarrow{f_{ex}}, \overrightarrow{e_x} \right\rangle > 0$, i.e.$\sum_{i=1}^{n} \frac{\partial^2 f}{\partial x \partial e_i} \frac{\partial e_i}{\partial x} > 0$ and $\sum_{i=1}^{n} \frac{\partial f}{\partial e_i} \frac{\partial e_i}{\partial a} > 0$.*

According to this lemma, with only one task, effort supply is strictly increasing with the incentive power a whereas it is strictly increasing or decreasing with fertility x as fertility and effort are complementary or substitute in the production function[16]. The incentive elasticity of production is always positive in the risk neutral case.

With risk aversion, conditions on the utility function of the agent need to be introduced as shown in appendix B.1.

Proposition 3.5. *The second best optimal contract slope verifies*[17] *(proof in A.4):*

$$a^* = 1 + E\varepsilon v' \frac{\langle \overrightarrow{e_a}, \overrightarrow{g_e} \rangle}{\left\langle \overrightarrow{e_a}, \overrightarrow{f_e} \right\rangle} \tag{3.4}$$

[14] We could call upon bounded rationality to justify this behavior. But, simple incentive schemes like sharing rules are generally prevalent in economy. Hart and Holmstrom (1987) say it can be caused by prohibitive costs of writing intricate contracts but also by a need of robustness in front of other possible Agent's choices. Holmstrom and Milgrom (1987) showed they are optimal when errors are normal and the Agent has constant absolute risk aversion.

[15] We note $\langle .,. \rangle$ the canonic scalar product of $\Re^n$ i.e. for $z_1, z_2 \in \Re^n$, $\langle z_1, z_2 \rangle = \sum_{i=1}^{n} z_{1i} z_{2i}$.

[16] With n efforts completely separable in production and cost, the same result holds for each one.

[17] With an agent having utility function $U(.)$

$$a^* = 1 + E\varepsilon v' \frac{\langle \overrightarrow{e_a}, \overrightarrow{g_e} \rangle}{\left\langle \overrightarrow{e_a}, \overrightarrow{f_e} \right\rangle} - (1 - \frac{E\nu U'}{EU'}) \frac{f}{\left\langle \overrightarrow{e_a}, \overrightarrow{f_e} \right\rangle}$$

In the equation giving a^*, the first quantity is the optimal contract slope when the tenant is risk neutral and fertility is not worth for the landlord ($a^* = 1$: fixed rent). The second puts the slope to the second best trade-off between production and fertility investment incentives. The third term allows to make the optimal trade-off between production incentives and risk sharing for the sharecropper. Hence, we obtain a more complicated trade-off between production incentives, investment incentives and production risk sharing. Two kinds of inefficiency affect the optimal contract: the multidimensional incentives problem and the risk aversion one. In a static moral hazard model, the multidimensionality alone would not lead to inefficiency. Our dynamic framework introduces a distortion effect between tasks even with risk neutrality.

The optimal contract slope a^* depends at each crop season on the land fertility whatever be the Agent's risk aversion. There is a "memory effect" through the land fertility which depends on past actions. It has nothing to do with Rogerson's memory effect (1985-a) due to long term contracts provided for consumption smoothing when the Agent is risk averse. We have only spot contracts but memory in the production process. Some long term contracts would complicate the study adding the usual memory effect.

Remark: The usual share-tenancy theory can be retrieved by setting $\overrightarrow{g_e} = 0_n$ in which case a^* reflects only the optimal trade-off between incentives and risk sharing.

Remark: The sign of the scalar product $\langle \overrightarrow{e_a}, \overrightarrow{g_e} \rangle = \left\langle \overrightarrow{f_e} M, \overrightarrow{g_e} \right\rangle$ is ambiguous even if $\left\langle \overrightarrow{f_e}, \overrightarrow{g_e} \right\rangle > 0$ we could have $-\left\langle \overrightarrow{f_e} M, \overrightarrow{g_e} \right\rangle < 0$. The figure B.7 shows an intuition of what could happen. In this case, even if marginal productivity and marginal investment of efforts are positive, the distortion between incentives can lead to a low powered contract. As $-M$ is symmetric definite positive, we can write $-M = \rho' \Lambda \rho$ with ρ a rotation and Λ a diagonal matrix with positive eigenvalues. In this extreme case, the inefficiency of the multitask distortion is so bad that it is better to give low incentives.

With a risk averse landowner having instantaneous utility function V

$$a^* = 1 + \frac{E\varepsilon v'}{E\nu V'} \frac{\langle \overrightarrow{e_a}, \overrightarrow{g_e} \rangle}{\left\langle \overrightarrow{e_a}, \overrightarrow{f_e} \right\rangle} - \left(1 - \frac{E\nu U' EV'}{EU' E\nu V'}\right) \frac{f}{\left\langle \overrightarrow{e_a}, \overrightarrow{f_e} \right\rangle}$$

3.4. Comparative statics and the dynamics of incentives and land fertility

3.4.1. High powered or low powered incentives

We now give some intuitions on the second best optimal contract slope characterizing the incentives power according to the properties of cost, investment and production functions[18].

- *Production and fertility colinearity*:

  As shown in corollary 3.3, the first best is then implementable. This is always the case when Agent's effort is unidimensional and the Agent is risk neutral. If g is increasing, meaning effort has positive influence on land fertility investment, the optimal contract has high powered incentives because the transfer function slope is greater than one. If g is decreasing, meaning effort has opposite influences on land fertility investment, the optimal contract has low powered incentives because the transfer function slope is smaller than one. When the farmer's work diminishes future fertility, payment incentives make his labor supply such that marginal cost is lower than marginal production. Drawing the marginal productivities and costs (see figure B.1), shows how first best is reached in this particular case[19]. With a unique effort, though the Agent is risk neutral, the optimal contract is not a fixed rent one as usually (see Stiglitz, 1974, Newbery and Stiglitz, 1979, Chuma, Hayami and Otsuka, 1992) but a share contract with a fixed part maintaining the Agent to his reservation utility level. Therefore, the risk sharing argument is not necessary to explain sharecropping contracts because land overuse disincentive can motivate them (when $g_e < 0$, the optimal slope is smaller than one).

- *Two tasks* (e_1, e_2):

  Let's examine some comparative statics about the sign of the scalar product $\left\langle \overrightarrow{f_e} M, \overrightarrow{g_e} \right\rangle$ equal to

  $$(g_1[a(f_1 f_{22} - f_2 f_{12}) - f_1 C_{22} + f_2 C_{12}] + g_2[a(f_2 f_{11} - f_1 f_{12}) - f_2 C_{11} + f_1 C_{12}])/\Delta$$

  with $\Delta = (af_{11} - C_{11})(af_{22} - C_{22}) - (af_{12} - C_{12})^2$ which is strictly positive for $a \geq 0$ because $af - C$ is strictly concave[20].

[18] Considerations about substitutability, complementarity, and land fertility issues can be found among other contributions to this literature in Binswanger and Rosenzweig (1986), Bardhan (1984, 1989), Braverman and Stiglitz (1986), Allen and Lueck (1992, 1993-a&b), Singh(1989), Chuma, Hayami and Otsuka (1992).

[19] C' is the marginal cost, f' and g' are the marginal productivities. If $g' > 0$ then the optimal slope is $a_+ > 1$. If $g' < 0$ then the optimal slope is $a_- < 1$.

[20] We note $f_{12} = \frac{\partial^2 f}{\partial e_1 \partial e_2}$ and similar notations for f_{11}, f_{22}, C_{11}, C_{12}, C_{22}.

Proposition 3.6. *If cost and production functions verify $f_{12} \geq 0$, $C_{12} \leq 0$ then $a^* > 1$ if g is increasing in both efforts and $a^* < 1$ if g is decreasing in both efforts.*

This proposition shows clearly that if efforts are completely separable in cost and production functions ($f_{12} = C_{12} = 0$) or if efforts are complementary in production and cost, then the optimal second best contract has a high powered slope if Agent's efforts are land fertility improving. At the contrary, if they diminish land fertility then the second best contract is low powered i.e. its slope belongs to $[0, 1]$.

Proposition 3.7. *If cost, production and land fertility investment functions verify $g_2 = 0$, $C_{12} \geq \frac{f_1}{f_2} C_{22}$, $f_{12} \leq \frac{f_1}{f_2} f_{22}$ then $a^* \leq 1$ if g is increasing ($g_1 \geq 0$) and $a^* \geq 1$ if g is decreasing ($g_1 \leq 0$).*

This example shows that the scalar product $\left\langle \overrightarrow{f_e} M, \overrightarrow{g_e} \right\rangle$ can be positive and induce a low powered second best contract whereas g remains increasing. These conditions say that when f_1 (marginal productivity of e_1) decreases in e_2 faster than f_2 times the TRS of e_2 by e_1 and when C_1 (marginal cost of e_1) increases in e_2 faster than C_2 times the TRS of e_2 by e_1 in the production function, then the second best slope is smaller than one when $g_1 > 0$ (e_2 being land fertility neutral). This example allows to understand that low incentives can sometimes reduce incentive distortions between efforts given by a unique performance measure such as production.

3.4.2. Land fertility and optimal slope

- *Production and fertility investment colinearity*: (proof in A.6)

Proposition 3.8. *When $\overrightarrow{f_e}$ and $\overrightarrow{g_e}$ are colinear then:*
If the implemented first best[21] *is such that $x_{t+1}(x_t)$ increases in x_t then $a^*(.)$ is decreasing at x_t (increasing) if $a^* > 1$ ($a^* < 1$).*
If the implemented first best is such that $x_{t+1}(x_t)$ decreases in x_t then $a^*(.)$ is increasing at x_t (decreasing) if $a^* > 1$ ($a^* < 1$).

Assuming the first best is such that $x_{t+1}(x_t)$ increases in x_t i.e. it raises the next period fertility level for higher initial fertility, we obtain that the optimal slope converges toward one when x increases. This is certainly the more realistic assumption, but on the contrary, if the fertility investment function is such that at the first best $x_{t+1}(x_t)$ decreases in x_t, then the optimal slope

[21] It is the case if $\overrightarrow{f_{ex}}$ and $\overrightarrow{g_e}$ are positively colinear.

diverges when x grows i.e. it increases if it is larger than one and decreases towards zero if it is smaller.

- *Separability of land fertility and efforts*: (proof in A.7)

Proposition 3.9. *When effort and fertility are separable ($\overrightarrow{f_{ex}} = \overrightarrow{g_{ex}} = 0_n$), if $a^* > 1$ ($a^* < 1$) then $a^*(x)$ is decreasing (increasing).*

With risk aversion, instead of converging to one, the slope will converge towards the second best sharing rate generally between 0 and 1.

3.5. The land value function

In the previous paragraphs we considered v as given exogenously. The preceding results rely only on its increasing property (sometimes on concavity). What are the conditions ensuring that the endogenous value function will be monotone and eventually concave. We give some results in the multitask risk neutral case and the unidimensional risk averse case (proofs are in A.5).
With risk neutrality, the landlord's welfare remains unchanged if he is restricted to linear transfer functions. According to appendix A.5, the land value function is increasing if $f_x + \rho w' g_x + \left\langle \overrightarrow{e_x}, \overrightarrow{f_e} - \overrightarrow{C_e} + \rho w' \overrightarrow{g_e} \right\rangle \geq 0$[22]. Noting η_a^f elasticity of production with respect to the contract incentive , η_a^g elasticity of investment with respect to the contract incentive, η_x^g the elasticity of investment with respect to fertility, η_x^f the elasticity of production with respect to fertility, we have (proof in A.5):

Proposition 3.10. *With risk neutrality, the land value function is increasing when $\eta_a^f \eta_x^g \geq \eta_a^g \eta_x^f$.*

- *Production and fertility investment colinearity:* (proof in A.8).

Proposition 3.11. *With risk neutrality, whenever $\overrightarrow{f_e}$ and $\overrightarrow{g_e}$ are colinear or $\overrightarrow{C_e}$ and $\overrightarrow{g_e}$ are colinear, the land value function is increasing and concave.*

When all TRS between efforts in the fertility investment function and the production function (or the cost function) are equal, the land value function is increasing concave and equal to the first best one.

[22] These three quantities are respectively: the marginal productivity of land fertility x, the marginal value of x and the marginal production profit obtained by effort variation $\overrightarrow{e_x}$ plus the marginal value of land fertility investment provided by $\overrightarrow{e_x}$.

- *Multiplicative separability of fertility and efforts in production* (proof in A.9):

Proposition 3.12. *With risk neutrality, if the production function is multiplicatively separable between fertility and efforts then the value function si increasing.*

- *Additive separability of fertility and efforts in production and investment* (proof in A.9):

Proposition 3.13. *With risk neutrality, when marginal effort productivity is constant in x i.e. effort and fertility are separable in production, then the land value function is increasing and it is concave if separability is true in investment.*

These two cases where the land value function is formally proved to be increasing and concave are quite simple but they give examples where our assumption of concavity of v in the previous sections is valid..

- *Two tasks*:

Proposition 3.14. *With two tasks and a risk neutral Agent, the land value is increasing if the TRS between efforts in investment and marginal productivity are both greater or smaller than that in production*[23].

The case with risk aversion is more complex but we can comment some interesting results when there is one task (proof in A.10).

Proposition 3.15. *The land value function is increasing when f_e/f is increasing in x (for example a Cobb-Douglas function) or when $f_{ex} \geq 0$ and $g_e \leq 0$ or when the Agent is not too risk averse and $f_{ex} \leq 0$, $g_e \geq 0$*[24].

This case with risk aversion shows that if the effort decreases the fertility (for example if the productive effort leads to land overuse) but increases its marginal productivity then the value function is increasing. Furthermore, if the Agent is not too risk averse and his efforts increase fertility but reduce its marginal productivity, then the value function is increasing.

[23] It happens for example when $f_{1x} \geq 0$, $f_{2x} \leq 0$, $g_1 \geq 0$, $g_2 \leq 0$ or $\frac{f_2}{f_1} \leq \frac{g_2}{g_1}$, $f_{1x} \leq 0$, $f_{2x} \geq 0$, $g_1 \geq 0$, $g_2 \geq 0$ or $\frac{f_2}{f_1} \geq \frac{f_{2x}}{f_{1x}}$, $g_1 \geq 0$, $g_2 \leq 0$.

[24] We hint that in some extreme situations where the Agent is very risk averse, it can happen with $f_{ex} \geq 0$ and $g_e \geq 0$ that the land value decreases with fertility. In a very risky environment and with a very risk averse farmer, better land may sometimes not make its owner better. If the landowner must design a very incentive contract because Agent's effort increase fertility (with the assumption of complementarity of fertility and effort supplied) then Agent's risk aversion can make him reduce his effort and hence production and fertility. In this case, land tenancy is not a good agricultural organization because farmers are too risk averse and it is not possible to incite them for fertility investment and production.

4. Discussions on development and policy issues

In this section, we develop discussions around the model implications in terms of production levels, dynamics of fertility, technological innovations. Finally, we also give non exhaustive other interpretations of this Principal-Agent model.

4.1. Production level

Farming through contractual arrangements does not imply better lands to produce more. Actually, the production evolves with fertility according to the sign of $f_x + \left\langle \overrightarrow{e_x} + a_x \overrightarrow{e_a}, \overrightarrow{f_e} \right\rangle$ which may be positive or negative (though we assumed that $f_x \geq 0$). A land fertility improvement has roughly speaking three effects on production: a direct one by the marginal productivity of x, an indirect one on farmer's effort supply which can be decomposed in a direct effect on effort behavior $\overrightarrow{e_x}$ and an indirect other through the contract incentives variation $a_x \overrightarrow{e_a}$. Comparative statics are quite complicated and ambiguous, unless we have only one task:

Proposition 4.1. *With risk neutrality and one task, if effort damages fertility and is complementary with it in production and investment functions ($f_{ex} \geq 0$, $g_{ex} \geq 0$, $g_e \leq 0$) then better tenanted lands produce more. For opposite conditions ($f_{ex} < 0$, $g_{ex} < 0$, $g_e > 0$) and a sufficiently small marginal productivity of fertility, the reverse is true.*

Proof. With a risk neutral Agent and only one task, the first best is implementable and the derivative of production with respect to fertility x is $f_x + f_e e_x$ where $f_x \geq 0$, $f_e \geq 0$, $-e_x = (f_{ex} + E[\varepsilon v^{'}]g_{ex} + E[\varepsilon^2 v^{''}]g_e g_x)/(f_{ee} + E[\varepsilon v^{'}]g_{ee} + E[\varepsilon^2 v^{''}]g_e^2 - C_{ee})$ and v is increasing concave.

We note that the implemented effort increases or decreases with fertility as effort and fertility are complementary or substitute. In the multitask risk averse framework, the production variation with fertility depends on effort reaction and on the contractual incentive change with fertility. Therefore we can expect that according to the agricultural technology employed, the production may sometimes decrease even if fertility increases.

4.2. Fertility dynamics

One important issue in agriculture is that of fertility dynamics. Clearly, the fertility dynamics through tenancy will be different from the first best one when the conditions allowing to implement it given in proposition 3.2 and 3.3 are not satisfied. In the following, we analyze some particular case to show how fertility dynamics will be modified because of the conflicting interests between the landlord and the tenant.

In order to give a simple analytical example of the problem, we assume the investment function takes the following form (without uncertainty): $x_{t+1} = (1+\delta)x_t - \widehat{g}(e_t)$ i.e. it is separable between labor and fertility. Then $x_{t+1} - x_t \gtreqqless 0$ as $\widehat{g}(e_t(x_t)) \lesseqqgtr \delta x_t$ where $e_t(x_t) = e_t(a^*(x_t), x_t)$ is the Agent's best answer to an a^* contract slope. The optimal slope determined by the landlord depends on fertility. Hence, fertility improvement over time is not compulsory. If production tasks are damaging for fertility and marginal land value is small for the landlord, he will give production incentives which are likely to decrease fertility. But, another more important feature in the multitask environment is that if incentive distortions lead the farmer not to realize the needed investment to maintain fertility then it can decrease over time. Therefore, when investment tasks are not directly linked to production, land tenancy is likely to change fertility dynamics with respect to the first best path.

The model can generate fertility, contract, or production cycles according to the fertility investment equation. We already noted the possibility of a convergence toward some steady state(s) (without uncertainty on x). The convergence can be monotonic or with oscillations around the limit. Therefore, as the contract shape depends on fertility, we can obtain converging cycles to stable steady state till an uncertainty or technological innovation shock removes the steady state. The figure B.6 shows a possible fertility time path.

- If x_{t+1} increases with x_t then for each fertility level, the fertility dynamics will be monotonic (decreasing in some ranges and increasing in others). Several cases can appear. See (a), (b), (c) and (d) in figure B.8.

- If x_{t+1} is decreasing in x_t at some point, then non monotonic dynamics arise for some fertility levels and permanent cycles appear in the fertility dynamics if the curve $x_{t+1}(x_t)$ crosses the bisecting line with a decreasing slope.

As $\frac{\partial x_{t+1}}{\partial x_t} < 0 \Leftrightarrow \widehat{g}' e_x > 1+\delta$, different conditions may lead to cyclical evolution :
Permanent cycles will appear for all fertility levels if $\widehat{g}' e_x > 1+\delta$. As $e_x = \frac{-f_{ex} + v'' \widehat{g}'(1+\delta)}{f_{ee} + v' \widehat{g}'' + v'' \widehat{g}'^2 - C_{ee}}$, the condition is $f_{ex}\widehat{g}' > (1+\delta)\left[C_{ee} - f_{ee} - v' \widehat{g}''\right]$. If $f_{ex} \geq 0$ and $\widehat{g}' \geq 0$, it will always happen for $\delta = -1$ or for sufficiently small δ. See case (c) in figure B.8.

With bounding conditions on second order derivatives of production, cost and investment functions,

e_x is positive and bounded above, and as $\widehat{g}$ is assumed convex $\widehat{g}'$ is increasing: so $\exists \underline{x}(\delta)$ ($\underline{x}(\delta)$ is increasing with δ) such that $\forall x \geq \underline{x}(\delta)$, $\widehat{g}' e_x > 1 + \delta$ i.e. $\frac{\partial x_{t+1}}{\partial x_t} < 0$. So if $x_{t+1}(\underline{x}(\delta)) > \underline{x}(\delta)$, then permanent cycles appear for some $x > \underline{x}(\delta)$. See cases (e), (f), (g) and (h) in figure B.8.
Of course $\frac{\partial x_{t+1}}{\partial x_t}$ can be negative or positive for smaller fertility levels but with bounding conditions, as $e_x \geq 0$, we know that $\underline{x}(\delta)$ exists.

When permanent cycles appear, we see that increasing and decreasing phases of fertility evolution alternate. Moreover, effort levels are positively correlated with fertility levels. In this case, low activity with low fertility stocks alternate with high activity and high fertility stocks.
The delegation of management achieved here by land tenancy can lead to various fertility dynamics and rather different of that in a non delegated activity where non productive investment tasks would be implemented by the owner-operator.

4.3. Technological innovation

For simplicity, assume an exogenous value function, because the computation of innovation effects on land value would require simulations. Taking v exogenous is however sufficient to study the short-run dynamics when an innovation is known to be adopted in the long term. Here, we compare the situation of whether innovating or not for the current period whereas there is no alternative in the sequel which justifies to keep the same exogenous value function for the future.
The model focuses on the multitask incentives problems and therefore allows to study innovations consisting in the introduction of a new agricultural task. We examine the introduction of two kinds of production improving labor intensive innovations in agriculture. First, the pest control which increases significantly production without any effect on fertility. Secondly, the fertilizer use improving or damaging fertility according to its nature[25]. These two examples fit into the larger framework of the model which shows that the tasks multiplicity generally prevents the first best to be reached even when the Agent is risk neutral. For example, defining an initial traditional agricultural technique with:

- *Traditional technique*: It has a Cobb-Douglas production function, linear additive cost, exogenous fixed increasing concave value function: $f(x, e_t) = x^n e_t^p$, $C(e_t) = e_t$, $g(x, e_t) = (1 + \delta - \mu e_t)x$ and $v(z) = z^r$ is fixed.

[25] Instead of fertilizing or pest controlling we could have considered the Green Revolution example i.e. the change of traditional seeds by High Yielding Varieties which turned out to be more sensitive to water shorts and induced other tasks in farmer's work like irrigation. The model suggests it could have made land tenancy less efficient than with past technique although welfare has been improved.

With the traditional technology, land tenancy affords to achieve the first best and Pareto efficiency like in an owner-operated farm. If a technological innovation happens and introduces a new task for the tenant, the first best will probably no longer be reached because of distortions in peasant's incentives though landlord's welfare is increased (otherwise the innovation would not be adopted). Two examples illustrate this idea: pest control and fertilizer use.

- *Pest control introduction*: Let e_p be the pest control task with no effect on fertility (pesticides used for production). Controlling pest reduces the crop part usually lost in traditional technology, so production is increasing with the quantity of scattered pesticides over cultivated area. The fertility equation is unchanged from the traditional one. We have: $f(x, e_t, e_p) = x^n e_t^p (1 + e_p^q)$, $C(e_t, e_p) = e_t + e_p$, $g(x, e_t, e_p) = (1 + \delta - \mu e_t)x$.

First, consider the case for which fallowing allows to improve land fertility i.e. $\delta > 0$ **and effort depletes soil quality**. The computing is done with the following parameters: $\mu = 1$, $n = 0.4$, $p = 0.3$, $q = 0.2$, $r = 0.6$, $\delta = 0.1$, $\rho = 0.9$. Though effort incentives are bad for land fertility, the pest control introduction allows to raise the optimal contract slope whatever be x. For example $a^*(1) = 0.65$ with traditional method and 0.70 with new one. For $x = 1$, the results[26] show the pest control raises incentives and production and therefore depletes fertility. In fact, the initial technique is such that we have fertility improvement for $x < x^* = 1.11$ and depletion above. The new technique is such that $x_n^{*sb} = 0.21 < x^*$. Therefore, the new technique lowers the steady state level of fertility because with the pest control introduction the productivity of the first task is better, inciting the peasant to raise his effort on it. The figure B.2 shows the fertility change during one period for both technologies.

As predicted, the introduction of a new task changes the fertility dynamics and introduces an inefficiency (the first best steady state is larger than the second best one: $x_n^{*fb} = 0.22$). The convergence point (or steady state) is lower. There is more soil depletion with the new technique. Plots which quality is between the two steady states improve with the traditional technology but suffer lower fertility maintenance (because of more production incentives) with the pesticide introduction.

Now, if we consider that fertility decreases if land is left fallow (i.e. it needs maintenance) and that the first task improves soil quality. The computing is now done with

[26] Complete results are

$x = 1$	a^*	e_t	e_p	x_{+1}	Prod.	Welfare
Old tech.	0.65	0.09	–	1.004	0.50	1.30
New tech. (SB)	0.70	0.19	0.05	0.90	0.95	1.55
New tech. (FB)	–	0.184	0.07	0.92	0.96	1.56

.

the following parameters: $\mu = -1$, $n = 0.4$, $p = 0.3$, $q = 0.2$, $r = 0.6$, $\delta = -0.1$, $\rho = 0.9$. Then[27], though effort incentives are good for land fertility, the pest control introduction allows to lower the optimal contract slope whatever be x but making first task more productive the production however increases. The initial technique is such that we have fertility improvement for $x > x^* = 0.22$ and depletion below. The new technique is such that $x_n^* = 0.12 < x^*$. Therefore, the new technique widens the range of increasing fertility levels. By controlling pests, production increase as well as first task productivity implying less incentives needed to enhance fertility maintenance by first task. There is less soil depletion with the new technique. Plots which quality is between x_n^* and x^* depletes with the traditional technology but improves after the pesticide introduction.

With the introduction of a production specific task substitute with the first one, welfare is improved but fertility can be either damaged even with over-incentives on the traditional fertility damaging task or improved even with disincentives on the fertility maintaining task. We now consider another kind of innovation introducing a new task which maintains or damages soil resources. The fertilizing task introduces inefficiencies which can modify fertility path.

- *Fertilizer introduction*: Let e_f be the fertilizing task (fertilizer input scattered over cultivated area), the fertility equation is now different from the traditional one. We have: $f(x, e_t, e_f) = x^n e_t^p (1 + e_f^q)$, $C(e_t, e_f) = e_t + e_f$, $g(x, e_t, e_f) = (1 + \delta - \mu e_t + \eta e_f)x$. The optimal incentive slope depends on values of p and q and different patterns can be found. Here, we focus only on fertility dynamics according to signs of μ and η.

First, we consider the case for which fallowing improves fertility, the first task decreases x while the second increases it. The computing is done with the following parameters: $\mu = 1$, $\eta = 1$, $n = 0.4$, $p = 0.3$, $q = 0.2$, $r = 0.6$, $\delta = 0.1$, $\rho = 0.9$. Then, fertility dynamics are shown by figure B.4.

We see that the fertility level converges towards a steady state x_1 with the traditional technique which is greater than with the new technique $x_2 < x_1$ ($x_1 = 1.15$ and $x_2 = 0.29$). Hence, we surprisingly see that the introduction of a fertility improving task decreases the steady state soil resource and soil endowed with $x \in]x_2, x_1[$ improves with first technology whereas they are damaged

[27]Some results:

x	Tech.	a^*	e_t	e_p	x_{+1}	Prod.	Welfare
0.1	Old	1.16	0.06	–	0.096	0.17	0.33
0.1	New	1.12	0.10	0.02	0.099	0.29	0.40
0.2	Old	1.26	0.10	–	0.19	0.26	0.51
0.2	New	1.19	0.17	0.04	0.21	0.46	0.62

under the new one with fertilizer. In fact, under the new technique, incentives (i.e. a^*) are larger because of the introduction of a fertilizing task. Raising first task productivity, it is more engaged[28]: there are distortions in Agent's effort. Though the second best welfare is increased with the innovation, the steady state fertility level is lowered.

Secondly, we consider the case for which the first task improves land (fallowing being bad for soil) while the second damages it. The computing is done with the following parameters: $\mu = -1$, $\eta = -1$, $n = 0.4$, $p = 0.3$, $q = 0.2$, $r = 0.6$, $\delta = -0.1$, $\rho = 0.9$. Then, fertility dynamics are shown by figure B.3.

We see that fertility improves above a threshold. It is x_1 with the traditional technique whereas with the new technique this threshold is lower $x_2 < x_1$[29]. However, for large fertility levels (above $\underline{x}$), the adoption of innovation can slow down fertility improvements. This pattern is similar to the introduction of a fertility neutral pesticide use excepted that for large soil resources (i.e. $x > \underline{x}$) the direct fertility damaging effect wins against the reducing of e_f through lower incentives.

We have seen that technological innovations can have different opposite effects on efficiency, production and fertility dynamics. Agricultural policies should also take care of that when introducing a new technology. Short run and long run effects can be very different in terms of production growth and sustainable growth.

4.4. Policy implications on land tenancy

Development policies are often designed to improve agricultural efficiency and production levels by regulating agricultural markets and rural organization. Sharecropping has long been thought as inefficient and many land policies forbid its practice. The imperfection of insurance markets in developing countries advocated for the sharecropping institution because it is considered as improving social welfare by spreading and pooling income risk of peasants. However, a possible development policy designed to cope with these problems of technical efficiency and income insurance in risky rural regions has been to improve access to formal insurance and credit markets for peasants while

[28]We have:

$x = 1$	a^*	e_t	e_f	x_{+1}	Prod.	Welfare
Old tech.	0.65	0.09	–	1.003	0.49	1.30
New tech.	0.76	0.23	0.05	0.93	1.0	1.58

[29]Some results:

x	Tech.	a^*	e_t	e_p	x_{+1}	Prod.	Welfare
0.1	Old	1.16	0.06	–	0.096	0.17	0.33
0.1	New	1.09	0.09	0.02	0.097	0.28	0.39
0.2	Old	1.26	0.10	–	0.19	0.26	0.51
0.2	New	1.14	0.15	0.035	0.204	0.45	0.61
1	Old	1.91	0.45	–	1.35	0.79	1.41
1	New	1.34	0.57	0.16	1.32	1.43	1.76

forbidding share tenancy contracts. With the new point of view of this paper, the policy implications are not so clear. Actually, sharecropping may be used also as a mean to reduce land overuse of the tenant and so as preserving soil quality. Even without the need for risk sharing, sharecropping contracts may be optimal. Forbidding share contracts could increase production in the short run but become very bad in the long run, leading to diminish sensitively the land quality. The dynamic framework of this model, where moral hazard on the unobserved actions of the agent applies not only on production but also on capital maintenance, shows that land tenancy policies should take into account the environmental and long term effects of resource allocation. In fragile land regions, policy designs should take care of long term environmental effects of restricting the feasible set of contracts. Generally, one thinks that fixed rent contracts allow to achieve efficiency though they make the farmer bear all the production risk. Here, the transfer of land property rights to the tiller appears to be the unique solution to implement an efficient allocation of resources.

4.5. A digression about taxation and pricing policies in agriculture

The Principal-Agent model we developed can give rise to various interpretations. We can interpret the results in terms of taxation and pricing policies giving a particular attention to the fertility dynamics induced by these policies

As made in Hoff (1993) and Braverman and Stiglitz (1989), we assume that the Agent is the representative farmer of an agricultural economy while the Principal is the state planner designing agricultural taxation. In a less developed country where the regulator's objective is indeed to maximize expected sum of agricultural returns, the taxation policy may be important with respect to incentives to produce and incentives to invest. As we consider the representative farmer is always held at his reservation utility, the intertemporal social welfare is the expected sum of raised taxes in agricultural sector plus the farmer's reservation utility which is constant. The regulator has the choice between taxing production at rate $(1-a)$ and taxing cultivated land areas at rate $-b$ (it is a subvention if $b>0$). Of course, the regulator is considered as risk neutral and with an infinite planning horizon while the representative farmer has a short term view and behaves so as to maximize his expected net profit utility. Though endowed with the property right, the Agent is myopic unless we consider that a period is entire time life and that we look at long term evolution of agriculture.

Without caring about investment and future productivity or with a myopic planner, the optimal tax in a risk neutral agricultural economy is a land property tax or land owning subsidy. The agricultural taxation literature (Hoff, 1993) argue that a production tax can be better than a land tax when institutions for spreading and pooling risk are imperfect. This result applies whether land is tilled by owners, wage-earners or sharecroppers. Hoff shows that a mix of land and production taxes are Pareto superior. Here, even with perfect risk pooling, the tax system is not neutral when the fertility dynamics are important.

According to the results of the model, even if the farmer is risk neutral, because of distorted incentives to invest, the optimal taxation is not only a fixed price per hectare (subvention or tax if positive or negative according to the reservation utility) but also involves a production tax. We hence show the importance of taxation on investment incentives for farmers.

If we assume that the agricultural effort represents a labor intensive method used by farmers that depletes soil fertility, the optimal taxation will influence agricultural intensification. If the fertility dynamics is given by the equation: $x_{t+1} - x_t = (\delta - e)\, x_t$, following agricultural models of optimal control of erosion (McConnell, 1983, Barrett, 1991), we can consider that x is the soil depth which has a positive effect on output[30], the agricultural output can be increased by clearing, weeding by hand or scattering herbicides, but the gains are then short lived because the soil will be quickly eroded away unless other tasks such that building terraces on hillsides are undertaken. Then the dynamics of soil depth depends on e which induces a soil loss because of cultivation whereas δ is the natural regeneration rate.

Another interpretation of the model in the risk neutral environment is that of a state planned agriculture through a fixed price guarantee policy. Actually, assume that a is the guaranteed price contractually between government and farmers whereas b is a property tax or subsidy according to its sign which depends on the reservation utility level. The social welfare is indeed $f - C$. The planner's objective is therefore to maximize the expected intertemporal welfare by setting agricultural guaranteed price and subsidy rate to farmers who don't care about fertility (for example in a urbanizing environment) or because of myopic behavior in a poor environment.

5. Conclusion

Finally, this Principal-Agent model of moral hazard with a stock variable in the production function shows the importance of land fertility in the optimal contract determination. Because of limited

[30]Because in deeper soils there is more room for plant roots to take hold and more nutrients available for plant growth.

commitment, the opposition of long term and short term objectives of contracting parties leads to a second best contract which is a sharecropping agreement even with risk neutrality. This model is able to explain many kinds of tenancy contracts or workers' payment schemes: sharecropping, all mixed contracts with a fixed part and a sharing rule, and even very high powered incentives interpreted as fixed price contracts with premia or penalties to over-production. The optimal contract depends on production and land fertility investment technologies as well as risk aversion, for the purpose of risk sharing. Coming from efforts distortions, non intuitive optimal contracts appear in the multitask case. Even with risk neutrality and a technology in which efforts are productively worth and land improving, the second best contract can be a share one (i.e. with low powered incentives compared to a fixed rent contract). Moreover, according to production and investment technologies, better lands do not always produce more because it can be optimal for the landlord not to incite production when it does not improve fertility. The fertility dynamics can be sensitively modified through land tenancy from the self cultivation or first best time path. At last, numerous discussions show some implications of the model. We saw that technological innovation may change the fertility time path. We gave examples to prove that the model can generate endogenous agricultural cycles. Finally, land fertility seems to be an important economic factor in the design of tenancy agreements and should be taken into account for policy design with respect to land conservation and production levels.

A. Proofs

A.1. Proof of proposition 3.1

The first order conditions of the Lagrangian are (λ being the Lagrange multiplier associated to (3.1)): $EU(t) = \overline{U} + C(e)$, $\lambda EU'(t) = 1$, $\overrightarrow{f_e} - \lambda \overrightarrow{C_e}(e^*) + E\varepsilon v' \overrightarrow{g_e} = 0_n$. Hence the first best (e^*, t^*) is characterized by $\overrightarrow{f_e}(x, e^*) - (EU'(t^*))^{-1}\overrightarrow{C_e}(e^*) + E\varepsilon v'(\varepsilon g(x, e^*))\overrightarrow{g_e}(x, e^*) = 0_n$ and $EU(t^*) = \overline{U} + C(e^*)$. With $t^* = \Phi\left(\overline{U} + C(e^*)\right)$ we have a first best solution. To respect the second order condition the following symmetric matrix: $f_{ee}(x, e^*) - \Phi'\left(\overline{U} + C(e^*)\right)C_{ee}(e^*) - \Phi'\left(\overline{U} + C(e^*)\right)\overrightarrow{C_e}(e^*)'\overrightarrow{C_e}(e^*)$
$+E\left[\varepsilon v'(\varepsilon g(x, e^*))g_{ee}(x, e^*) + v''(\varepsilon g(x, e^*))\varepsilon^2\overrightarrow{g_e}(x, e^*)'\overrightarrow{g_e}(x, e^*)\right]$[31] must be semi-definite negative. f, g and U being concave, C and Φ convex, $\overrightarrow{C_e}(e^*)'\overrightarrow{C_e}(e^*)$ and $\overrightarrow{g_e}(x, e^*)'\overrightarrow{g_e}(x, e^*)$ symmetric semi-definite positive, the second order condition is satisfied if $E\varepsilon v'(\varepsilon g(x, e^*)) \geq 0$ and $E\varepsilon^2 v''(\varepsilon g(x, e^*)) \leq 0$ i.e. v increasing and concave. So, if v is assumed concave there is always a unique solution to this problem.

A.2. Proof of corollary 3.3

Let $\lambda \in \Re$ such that $\overrightarrow{g_e} = \lambda \overrightarrow{f_e}$ (the proof with $\overrightarrow{C_e}$ and $\overrightarrow{g_e}$ colinear is similar). Consider the linear contract of slope a, then the incentive constraint gives $a\overrightarrow{f_e}(x, e) = \overrightarrow{C_e}(e)$ which defines a differentiable function $e(x, a)$ (implicit functions theorem) verifying $\forall a$, $a\overrightarrow{f_e}(x, e(x, a)) = \overrightarrow{C_e}(e(x, a))$. Then the solution of $\lambda E\varepsilon v'(\varepsilon g(x, e(x, a))) + 1 = a$ allows to implement the first best. Actually, call a^* its solution, then e is solution of $a^*\overrightarrow{f_e}(x, e) = \overrightarrow{C_e}(e)$ hence $e(x, a^*)$ verifies
$(1 + \lambda E\varepsilon v'(\varepsilon g(x, e(x, a^*))))\overrightarrow{f_e}(x, e(x, a^*)) = \overrightarrow{C_e}(e(x, a^*))$ i.e.
$\overrightarrow{f_e}(x, e(x, a^*)) + E\varepsilon v'(\varepsilon g(x, e(x, a^*)))\overrightarrow{g_e}(x, e(x, a^*)) = \overrightarrow{C_e}(e(x, a^*))$. Consequently $e(x, a^*)$ is solution of the first order condition to the first best problem. We have $a^* = 1 + E\varepsilon v'\overrightarrow{g_e}/\overrightarrow{f_e}$. Note $S(a) = 1 - a + \lambda E\varepsilon v'(\varepsilon g(x, e(x, a)))$. S is continuous and $S(0) = 1 + \lambda E\varepsilon v'(\varepsilon g(x, e(x, 0))) = 1 + \lambda E\varepsilon v'(\varepsilon g(x, 0_n))$ and $\underset{a \to +\infty}{Lim} S(a) = -\infty$ by supposing that v' is bounded. If $1 + \lambda E\varepsilon v'(\varepsilon g(x, 0_n)) < 0$ then $a^* < 0$ and $e = 0_n$ so no effort happens. If $1 + \lambda E\varepsilon v'(\varepsilon g(x, 0_n)) \geq 0$, $S(a) = 0$ has always a solution. If $\lambda > 0$ i.e. $\overrightarrow{f_e}$ and $\overrightarrow{g_e}$ have the same direction then $a^* > 1$. If $\lambda < 0$ i.e. if $\overrightarrow{f_e}$ and $\overrightarrow{g_e}$ have opposite directions then $a^* < 1$.

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[31] We note $f_{ee} = \begin{bmatrix} \frac{\partial^2 f}{\partial e_1 \partial e_1} & \cdots & \frac{\partial^2 f}{\partial e_1 \partial e_n} \\ \cdot & & \cdot \\ \frac{\partial^2 f}{\partial e_n \partial e_1} & \cdots & \frac{\partial^2 f}{\partial e_n \partial e_n} \end{bmatrix}$ the Hessian matrix of f with respect to e, and similarly for g_{ee} and C_{ee}.

A.3. Proof of lemma 3.4

According to (3.3), $\forall a \geq 0, a\overrightarrow{f_e}(x,e) = \overrightarrow{C_e}(e)$. The implicit functions theorem tells that $\exists e(.,.) : \Re^{+2} \rightarrow \Re^{+n}$ such that $\forall a \geq 0, e(x,a)$ is solution of (3.3). $e(.,.)$ is continuously differentiable. Differentiating IC with respect to a and x: $\overrightarrow{e_a}(x,a) = -\overrightarrow{f_e}(x,e(x,a))\,M$, $\overrightarrow{e_x}(x,a) = -a\overrightarrow{f_{ex}}(x,e(x,a))\,M$ with $M = [af_{ee}(x,e(x,a)) - C_{ee}(e(x,a))]^{-1}$. M is regular since it is symmetric definite negative because f is strictly concave, C is strictly convex and $a \geq 0$. We have $\forall \overrightarrow{u} \in \Re^n, \langle \overrightarrow{u}M, \overrightarrow{u}\rangle < 0$ so $\left\langle \overrightarrow{e_a}, \overrightarrow{f_e} \right\rangle > 0$ and $\left\langle \overrightarrow{e_x}, \overrightarrow{f_{ex}} \right\rangle > 0$.

A.4. Proof of proposition 3.5

The Principal's maximization problem can be written:

$\underset{(a,b)\in\Re^+\times\Re}{Max}\ (1-a)\,E\nu f(x,e(x,a)) - b + Ev(\varepsilon g(x,e(x,a)))$ subject to

$\underset{e}{Max}\ EU[a\nu f(x,e)+b] - C(e) \geq \overline{U}$ and $E[U'a\nu]\overrightarrow{f_e} = \overrightarrow{C_e}$ with the first order approach. We will suppose that the fixed payment b is adjusted so that the individual rationality constraint binds. Actually, for example with one task, the Lagrange multiplier associated to the participation constraint which equals

$\lambda = \frac{1}{EU'}f - (\frac{E\nu U' + aE\nu^2 U'' f_e}{aE\nu U''})/(\frac{E\nu U'}{EU'}f - \frac{E\nu U' + aE\nu^2U''f_e}{aE\nu U''})$ is likely to be non zero (and equals one in the risk neutral case). Therefore we have $\forall a$, $b(x,a)$ such that $\max\limits_{e}\,(EU[a\nu f(x,e)+b] - C(e)) = \overline{U}$. Also the incentive constraint defines an Agent's best answer to contract $(a, b(a))$: $e(x,a)$ is the implicit function defined by $E[U'a\nu]\overrightarrow{f_e} = \overrightarrow{C_e}$. It gives the program

$\underset{a\in\Re^+}{Max}\ H(a) = (1-a)\,E\nu f(x,e(x,a)) - b(x,a) + Ev(\varepsilon g(x,e(x,a)))$

So, $\frac{\partial H}{\partial a}(a^*) = 0$ leads to $-f + (1-a^*)\left\langle \overrightarrow{e_a}, \overrightarrow{f_e} \right\rangle - (\frac{\partial b}{\partial a})_{|\overline{U}} + E[\varepsilon v']\,\langle \overrightarrow{e_a}, \overrightarrow{g_e}\rangle = 0$. The definition of b implies that $(\frac{\partial b}{\partial a})_{|\overline{U}} = -fE\nu U'/EU'$ $(= -f$ with risk neutrality), and with lemma 3.4, $\overrightarrow{e_a} = -\overrightarrow{f_e}M$. Differentiating the implicit equation defining $e(.,.)$ we obtain for the risk averse case more complicated formulae:

$\overrightarrow{e_a} = [E\nu U' + af(E\nu^2U'' - E\nu U''\frac{E\nu U'}{EU'})]\overrightarrow{f_e}\widetilde{M}$ and

$\overrightarrow{e_x} = a[E\nu U'\overrightarrow{f_{ex}} + af_x(E\nu^2U'' - E\nu U''\frac{E\nu U'}{EU'})\overrightarrow{f_e}]\widetilde{M}$, with the matrix

$\widetilde{M}^{-1} = [C_{ee} - aE\nu U' f_{ee} - a^2E\nu^2U''\overrightarrow{f_e}'\overrightarrow{f_e}]$ being symmetric positive definite because $U'' \leq 0$.

Remark: The second order condition is difficult to check in the risk averse case, with risk neutrality, it is: $\left\langle \overrightarrow{e_a}, \overrightarrow{e_a}[f_{ee} - C_{ee} + E\varepsilon v' g_{ee}] \right\rangle + E\varepsilon^2 v''\,\langle \overrightarrow{e_a}, \overrightarrow{g_e}\rangle^2 \leq 0$. As $[f_{ee} - C_{ee} + E\varepsilon v' g_{ee}]$ is a definite negative matrix, $\left\langle \overrightarrow{e_a}, \overrightarrow{e_a}[f_{ee} - C_{ee} + E\varepsilon v' g_{ee}] \right\rangle < 0$ and either v concave or $E\varepsilon^2 v''\,\langle \overrightarrow{e_a}, \overrightarrow{g_e}\rangle^2$ not large in magnitude if positive.

A.5. Monotonicity of the land value function

Formally[32]: $w(x) \geq \underset{t(.)\in\widehat{\Theta}\ s.t.\ (IC),(IR)}{Max} E[y - t(y)] + \rho E w(z)$ since $\widehat{\Theta} \subset \Theta$. With help of proposition ??, we define $\theta_x : \Theta_x \to \widehat{\Theta}_x$ which associates the linear second best transfer $d \in \widehat{\Theta}_x$ to the second best differentiable one $t \in \Theta_x$. And then, $w(x) = E[y - t(y)] + \rho E w(z) = E[y - d(y)] + \rho E w(z) \leq \underset{t(.)\in\widehat{\Theta}\ s.t.\ (IC),(IR)}{Max} E[y - t(y)] + \rho E w(z)$ where $t \in \Theta_x$, $d = \theta(t) \in \widehat{\Theta}_x \subset \widehat{\Theta}$.

In some special cases we prove that the land value is indeed monotone increasing i.e. that a better plot brings a higher welfare to its owner. We must show that the solution to the functional equation is increasing i.e. defining the operator T by: $w \to Tw$ such that $Tw(x) = \underset{t(.)\ s.t.\ (IC),(IR)}{Max} E[y - t(y)] + \rho E w$ that it is a contraction mapping which fixed point is increasing. We must show that T maps increasing functions set into itself.

The land value function is solution of the functional equation:

$w(x) = \underset{t(.)\in\widehat{\Theta}\ s.t.\ (IC),(IR)}{Max} E[y - t(y)] + \rho E w(z)$

where $\widehat{\Theta}$ is the functional space of choice i.e. the linear functions from $\Re^{+}$ to $\Re^{+}$. It is equivalent to:

$w(x) = \underset{t(.)\in\widehat{\Theta}}{Max}\{F(x, t(.)) + \rho E w(G(x, t(.), \varepsilon))\}$ because e depends on x and $t(.)$ through (IC) and (IR), with $G(x, t(.), \varepsilon) = g(\varepsilon(x, e(x, t(.))))$ and $F(x, t(.)) = E[\nu f(x, e(x, t(.))) - t(\nu f(x, e(x, t(.))))]$ where $e(x, t(.))$ is the implemented Agent's effort vector by contract $t(.)$.

We note $C(X, \Re)$ the X to $\Re$ continuous functions set and $\widetilde{C}(X, \Re)$ its increasing functions subset. Let $\|.\|_\infty$ be the superior norm[33] of $C(X, \Re)$ which gives to $C(X, \Re)$ a structure of complete metric space. We note d the corresponding distance[34]. Let the operator T: $C(X, \Re) \to C(X, \Re)$ such that:

$\forall w \in C(X, \Re),\ Tw : x \in X \to Tw(x) = \underset{t(.)\in\widehat{\Theta}}{Max}[F(x, t(.)) + \rho E w(G(x, t(.), \varepsilon))]$

It is obvious that $Tw \in C(X, \Re)$ because Tw is continuous by continuous functions composition (supposing that the maximum is reached and continuous).

i) Show that T is a contraction of modulus ρ:

Let $w, \widetilde{w} \in C(X, \Re)$: $\forall x \in X$, we have:

$Tw(x) = \underset{t(.)\in\widehat{\Theta}}{Max}[F(x, t(.)) + \rho E w(G(x, t(.), \varepsilon))] = F(x, t) + \rho E w(z)$

where $t \in \underset{t(.)\in\widehat{\Theta}}{\arg\max}[F(x, t(.)) + \rho E w(G(x, t(.), \varepsilon))]$ and $z = G(x, t, \varepsilon)$.

[32] Θ is the differentiable functions space and $\widehat{\Theta}$ is the linear functions subset of Θ.

[33] For $f(.) \in C(X, \Re)$, we have $\|f\|_\infty = \underset{x\in X}{Sup}|f(x)|$.

[34] The distance d is defined by : $\forall f, g \in C(X, \Re)\ d(f, g) = \|f - g\|_\infty = \underset{x\in X}{Sup}|f(x) - g(x)|$.

And $Tw̃ (x) = \underset{t(.)\in\widehat{\Theta}}{Max} \left[F(x,t(.)) + \rho E\widetilde{w}(G(x,t(.),\varepsilon))\right] = F\left(x,\widetilde{t}\right) + \rho E\widetilde{w}(\widetilde{z})$

where $\widetilde{t} \in \underset{t(.)\in\widehat{\Theta}}{\arg\max} \left[F(x,t(.)) + \rho E\widetilde{w}(G(x,t(.),\varepsilon))\right]$ and $\widetilde{z} = G\left(x,\widetilde{t},\varepsilon\right)$.

Hence according to the definition of Tw and $T\widetilde{w}$:

$T\widetilde{w}(x) \geq F(x,t) + \rho E\widetilde{w}(z)$ and $Tw(x) \geq F\left(x,\widetilde{t}\right) + \rho Ew(\widetilde{z})$

So $\rho E\left[\widetilde{w}(z) - w(z)\right] \leq T\widetilde{w}(x) - Tw(x) \leq \rho E\left[\widetilde{w}(\widetilde{z}) - w(\widetilde{z})\right]$

Hence: $\forall x \in X$, $\left|T\widetilde{w}(x) - Tw(x)\right| \leq \rho \underset{y\in X}{Sup} \left|\widetilde{w}(y) - w(y)\right| = \rho \left\|\widetilde{w} - w\right\|_\infty$

and $\left\|T\widetilde{w} - Tw\right\|_\infty \leq \rho \left\|\widetilde{w} - w\right\|_\infty$

T is indeed a contraction with modulus ρ.

ii) Show that the fixed point of T is increasing under some conditions:

Let $w \in C(X,\Re)$: $\forall x \in X$, (simplifying notations obviously, using previous results):

$$\begin{aligned} Tw(x) &= \underset{t(.)\in\widehat{\Theta}}{Max}\, E\left[\nu f(x,e(x,t(.))) - t(\nu f(x,e(x,t(.))))\right] + \rho Ew(\varepsilon g(x,e(x,t(.)))) \\ &= \underset{a\geq 0}{Max}\, f(x,e(x,a)) - C(e(x,a)) - \overline{U} + \rho Ew(\varepsilon g(x,e(x,a))) \\ &= f(x,e(x,a^*(x))) - C(e(x,a^*(x))) - \overline{U} + \rho Ew(\varepsilon g(x,e(x,a^*(x)))) \end{aligned}$$

where $a^*(x) = \underset{a}{\arg\max}\, f(x,e(x,a)) - C(e(x,a)) - \overline{U} + \rho Ew(\varepsilon g(x,e(x,a)))$ and $e(x,a) = \underset{e}{\arg\max}\, aE\left[\nu f(x,e)\right] - C(e)$. With the envelope theorem $\frac{\partial}{\partial x}Tw(x) = \frac{\partial f}{\partial x} + \rho E\varepsilon w' \frac{\partial g}{\partial x} + \left\langle \overrightarrow{e_x}, \overrightarrow{f_e} - \overrightarrow{C_e} + \rho E\varepsilon w' \overrightarrow{g_e} \right\rangle = \frac{\partial f}{\partial x} + \rho E\varepsilon w' \frac{\partial g}{\partial x} + \left\langle \overrightarrow{e_x}, (1-a)\overrightarrow{f_e} + \rho E\varepsilon w' \overrightarrow{g_e} \right\rangle$ because $a\overrightarrow{f_e} = \overrightarrow{C_e}$. As $\frac{\partial f}{\partial x} + \rho E\varepsilon w' \frac{\partial g}{\partial x} > 0$, $\left\langle \overrightarrow{e_x}, (1-a)\overrightarrow{f_e} + \rho E\varepsilon w' \overrightarrow{g_e} \right\rangle \geq 0$ will suffice for $\frac{\partial}{\partial x}Tw(x) > 0$. With proposition 3.5 we have $\left\langle \overrightarrow{e_x}, (1-a)\overrightarrow{f_e} + \rho E\varepsilon w' \overrightarrow{g_e} \right\rangle = \rho E\varepsilon w' \left\langle \overrightarrow{e_x}, \left\langle \overrightarrow{e_a}, \overrightarrow{f_e} \right\rangle \overrightarrow{g_e} - \left\langle \overrightarrow{e_a}, \overrightarrow{g_e} \right\rangle \overrightarrow{f_e} \right\rangle / \left\langle \overrightarrow{e_a}, \overrightarrow{f_e} \right\rangle$. But $\rho E\varepsilon w' / \left\langle \overrightarrow{e_a}, \overrightarrow{f_e} \right\rangle > 0$, so

$\left\langle \overrightarrow{e_x}, \left\langle \overrightarrow{e_a}, \overrightarrow{f_e} \right\rangle \overrightarrow{g_e} - \left\langle \overrightarrow{e_a}, \overrightarrow{g_e} \right\rangle \overrightarrow{f_e} \right\rangle \geq 0$ is sufficient. When this condition is checked, Tw is strictly increasing i.e. $Tw \in \widetilde{C}(X,\Re)$.

With risk neutrality and one task, the first order condition on the agent's effort implies that $\frac{\partial}{\partial x}Tw(x) = \frac{\partial f}{\partial x} + \rho E\varepsilon w' \frac{\partial g}{\partial x} > 0$.

iii) As T is a contraction, it has a unique fixed point w satisfying $w = Tw$. Since $\forall v_0 \in C(X,\Re)$, $\forall k = 0,1,2,...$, $d(T^k v_0, w) \leq \rho^k d(v_0,w)$, with $v_0 \in \widetilde{C}(X,\Re)$ the sequence $T^k v_0$ converges towards w. As $\widetilde{C}(X,\Re)$ is a closed subset of $C(X,\Re)$, $w \in \widetilde{C}(X,\Re)$. Consequently, the solution of the functional equation is an increasing function[35].Proof of proposition 3.10: obvious with $\frac{\partial Tw(x)}{\partial x} = (\eta_a^f \eta_x^g - \eta_a^g \eta_x^f) fgw' \left\langle \overrightarrow{e_a}, \overrightarrow{f_e} \right\rangle / ax + af_x$.As $f_x \geq 0$ and $\rho w' g_x \geq 0$, a useful sufficient condition for $\frac{\partial Tw(x)}{\partial x} \geq 0$ appears: $\Gamma = \left\langle \overrightarrow{e_a}, \overrightarrow{f_e} \right\rangle \left\langle \overrightarrow{e_x}, \overrightarrow{g_e} \right\rangle - \left\langle \overrightarrow{e_a}, \overrightarrow{g_e} \right\rangle \left\langle \overrightarrow{e_x}, \overrightarrow{f_e} \right\rangle \geq 0$. For the formal concavity study of the land value function, we must prove that the solution to the functional equation is concave. In

[35]See Stokey, Lucas, Prescott (1989) for further methodological details.

some cases, it is quite easily proved. A first in mind condition is the one which allows to implement the first best.

Two tasks: We find several properties allowing to infer the land value function is increasing. When we restrict to two efforts, as $(\overrightarrow{f_e}, \overrightarrow{g_e})$ makes up a base of $\Re^2$ (otherwise we are in the colinear case), we can write $\overrightarrow{f_{ex}} = \lambda \overrightarrow{f_e} + \mu \overrightarrow{g_e}$. With lemma 3.4, we obtain $\Gamma = a\mu(\left\langle \overrightarrow{f_e} M, \overrightarrow{f_e} \right\rangle \langle \overrightarrow{g_e} M, \overrightarrow{g_e} \rangle - \left\langle \overrightarrow{f_e} M, \overrightarrow{g_e} \right\rangle^2)$. By Schwartz's inequality, $\left\langle \overrightarrow{f_e} M, \overrightarrow{f_e} \right\rangle \langle \overrightarrow{g_e} M, \overrightarrow{g_e} \rangle \geq \left\langle \overrightarrow{f_e} M, \overrightarrow{g_e} \right\rangle^2$. Therefore $\Gamma \geq 0$ if and only if $\mu \geq 0$. Since $\mu = (f_1 g_2 - f_2 g_1)/(f_1 f_{2x} - f_2 f_{1x})$ it needs that numerator and denominator be of the same sign i.e. $f_1/f_2 \geq g_1/g_2$ and $f_1/f_2 \geq f_{1x}/f_{2x}$ or $f_1/f_2 \leq g_1/g_2$ and $f_1/f_2 \leq f_{1x}/f_{2x}$.

A.6. Proof of proposition 3.8

Like in A.7, $\frac{\partial a^*(x)}{\partial x} = -\frac{\partial^2 H}{\partial a \partial x} / \frac{\partial^2 H}{\partial a^2}$ with $\frac{\partial^2 H}{\partial a^2} < 0$ and $\frac{\partial^2 H}{\partial a \partial x} = \left\langle \overrightarrow{e_{ax}}, \overrightarrow{f_e} - \overrightarrow{C_e} + E\varepsilon v' \overrightarrow{g_e} \right\rangle + E\varepsilon^2 v'' \langle \overrightarrow{e_a}, \overrightarrow{g_e} \rangle [g_x + \langle \overrightarrow{e_x}, \overrightarrow{g_e} \rangle] + \left\langle \overrightarrow{e_a}, \overrightarrow{f_{ex}} + E\varepsilon v' \overrightarrow{g_{ex}} + \overrightarrow{e_x}[f_{ee} - C_{ee} + E\varepsilon v' g_{ee}] \right\rangle$.

According to the assumption, we write $\overrightarrow{g_e} = \lambda \overrightarrow{f_e}$. With proposition 3.3, lemma 3.4, and the incentive constraint: $a^* - 1 = \lambda E\varepsilon v'$, $\overrightarrow{e_x} = -a^* \overrightarrow{f_{ex}} M$, $a^* \overrightarrow{f_e} = \overrightarrow{C_e}$. So $\overrightarrow{f_e} - \overrightarrow{C_e} + E\varepsilon v' \overrightarrow{g_e} = 0_n$, the matrix $[f_{ee} - C_{ee} + E\varepsilon v' g_{ee}]$ is equal to $M^{-1} = a^* f_{ee} - C_{ee}$, $\overrightarrow{e_x}[f_{ee} - C_{ee} + E\varepsilon v' g_{ee}] = -a^* \overrightarrow{f_{ex}}$, $\overrightarrow{f_{ex}} + E\varepsilon v' \overrightarrow{g_{ex}} + \overrightarrow{e_x}[f_{ee} - C_{ee} + E\varepsilon v' g_{ee}] = 0_n$. Then $\frac{\partial^2 H}{\partial a \partial x} = E\varepsilon^2 v'' \langle \overrightarrow{e_a}, \overrightarrow{g_e} \rangle [g_x + \langle \overrightarrow{e_x}, \overrightarrow{g_e} \rangle]$. As the first best can be implemented, we have $e(x, a^*(x)) = e^*(x)$ ($e^*(x)$ being the effort supply defined by the first best program). Therefore, $g_x + \langle \overrightarrow{e_x}, \overrightarrow{g_e} \rangle \geq 0$ means that at first best, the fertility level reached at the end of the season increases with the initial fertility. So, we have the result. In the particular case where $\overrightarrow{f_{ex}} = \mu \overrightarrow{g_e}$, with $\mu \geq 0$, $\frac{\partial^2 H}{\partial a \partial x} = E\varepsilon^2 v'' \langle \overrightarrow{e_a}, \overrightarrow{g_e} \rangle [g_x - a^* \mu \langle \overrightarrow{g_e} M, \overrightarrow{g_e} \rangle]$ has obviously same sign than $-\langle \overrightarrow{e_a}, \overrightarrow{g_e} \rangle$ because M is definite negative.

A.7. Proof of proposition 3.9

Writing $H(a, x) = f(x, e(x, a)) - C(e(x, a)) - \overline{U} + Ev(\varepsilon g(x, e(x, a)))$, $a^*(x) = \arg\max_a H(a, x)$. According to the envelope theorem $\forall x$, $\frac{\partial H}{\partial a}(a^*(x), x) = 0$ implying that $\frac{\partial a^*(x)}{\partial x} = -\frac{\partial^2 H}{\partial a \partial x} / \frac{\partial^2 H}{\partial a^2}$. But, $\frac{\partial^2 H}{\partial a^2} = E\varepsilon^2 v'' \langle \overrightarrow{e_a}, \overrightarrow{g_e} \rangle^2 + \left\langle \overrightarrow{e_a}, \overrightarrow{e_a}[f_{ee} - C_{ee} + E\varepsilon v' g_{ee}] \right\rangle + \left\langle \overrightarrow{e_{aa}}, \overrightarrow{f_e} - \overrightarrow{C_e} + E\varepsilon v' \overrightarrow{g_e} \right\rangle$ and $\frac{\partial^2 H}{\partial a \partial x} = \left\langle \overrightarrow{e_a}, \overrightarrow{f_{ex}} + E\varepsilon v' \overrightarrow{g_{ex}} + \overrightarrow{e_x}[f_{ee} - C_{ee} + E\varepsilon v' g_{ee}] \right\rangle + \left\langle \overrightarrow{e_{ax}}, \overrightarrow{f_e} - \overrightarrow{C_e} + E\varepsilon v' \overrightarrow{g_e} \right\rangle + E\varepsilon^2 v'' \langle \overrightarrow{e_a}, \overrightarrow{g_e} \rangle [g_x + \langle \overrightarrow{e_x}, \overrightarrow{g_e} \rangle]$

According to the second order condition of the Principal's maximization: $\frac{\partial^2 H}{\partial a^2} \leq 0$ which is assumed satisfied (to be proved we only need that $\left\langle \overrightarrow{e_{ax}}, \overrightarrow{f_e} - \overrightarrow{C_e} + E\varepsilon v' \overrightarrow{g_e} \right\rangle$ be negative or small in magnitude if positive because $\left\langle \overrightarrow{e_a}, \overrightarrow{e_a}[f_{ee} - C_{ee} + E\varepsilon v' g_{ee}] \right\rangle + E\varepsilon^2 v'' \langle \overrightarrow{e_a}, \overrightarrow{g_e} \rangle^2 \leq 0$ since v is increasing concave, f, g, concave, C convex, $[f_{ee} - C_{ee} + E\varepsilon v' g_{ee}]$ definite negative). As $\overrightarrow{e_x} = -a \overrightarrow{f_{ex}} M$, when $\overrightarrow{f_{ex}} = 0_n$ then $\overrightarrow{e_x} = 0_n$ and $\overrightarrow{e_{ax}} = 0_n$. So $\frac{\partial^2 H}{\partial a \partial x} = E\varepsilon^2 v'' \langle \overrightarrow{e_a}, \overrightarrow{g_e} \rangle g_x + E\varepsilon v' \langle \overrightarrow{e_a}, \overrightarrow{g_{ex}} \rangle$. Moreover if

$\overrightarrow{g_{ex}} = 0_n$: $\frac{\partial^2 H}{\partial a \partial x} = E\varepsilon^2 v'' \langle \overrightarrow{e_a}, \overrightarrow{g_e} \rangle g_x$. g being increasing in x, v increasing and concave: $a^{*'}(x)$ has same sign than $-\langle \overrightarrow{e_a}, \overrightarrow{g_e} \rangle$. So, since $a^* \gtreqless 1$ whether $\langle \overrightarrow{e_a}, \overrightarrow{g_e} \rangle \gtreqless 0$, we have the desired result. We note that in the separable case : $\frac{\partial a^*(x)}{\partial x} = -E\varepsilon^2 v'' \langle \overrightarrow{e_a}, \overrightarrow{g_e} \rangle g_x /$
$(E\varepsilon^2 v'' \langle \overrightarrow{e_a}, \overrightarrow{g_e} \rangle^2 + \left\langle \overrightarrow{e_a}, \overrightarrow{e_a}[f_{ee} - C_{ee} + E\varepsilon v' g_{ee}] \right\rangle + \left\langle \overrightarrow{e_{aa}}, \overrightarrow{f_e} - \overrightarrow{C_e} + E\varepsilon v' \overrightarrow{g_e} \right\rangle)$.

A.8. Proof of proposition 3.11

At the implemented efforts, the incentive compatibility condition says that $a\overrightarrow{f_e} = \overrightarrow{C_e}$, so with the assumption, we can write: $\overrightarrow{f_e} = \mu \overrightarrow{g_e}$ with $\mu \in \Re$. Thus: $\left\langle \overrightarrow{e_a}, \overrightarrow{f_e} \right\rangle \overrightarrow{g_e} - \langle \overrightarrow{e_a}, \overrightarrow{g_e} \rangle \overrightarrow{f_e} = \langle \overrightarrow{e_a}, \mu \overrightarrow{g_e} \rangle \overrightarrow{g_e} - \langle \overrightarrow{e_a}, \overrightarrow{g_e} \rangle \mu \overrightarrow{g_e} = 0_n$, so $\Gamma = 0$ which gives the monotonicity. For concavity, using notations of A.5, we remark that $Tw(x) = \underset{a \geq 0}{Max}\ f(x, e(x,a)) - C(e(x,a)) - \overline{U} + \rho E w(\varepsilon g(x, e(x,a))) = \underset{e}{Max}\ f(x,e) - C(e) - \overline{U} + \rho E w(\varepsilon g(x,e))$. Noting $K(x,e) = f(x,e) - C(e) - \overline{U} + \rho E w(\varepsilon g(x,e))$ and $e(x) = \arg\max_e K(x,e)$, we know that $K(x,e)$ is concave in (x,e) because f, g are concave in (x,e) and C is convex in e. We have $\forall x, K_e(x, e(x)) = 0$ so $\forall x,\ e_x(x) = -K_{ex}(x, e(x)) / K_{ee}(x, e(x))$. In addition, $Tw(x) = K(x, e(x))$, so $\frac{\partial^2}{\partial x^2} Tw(x) = \left(K_{xx}K_{ee} - K_{ex}^2\right) / K_{ee} < 0$ because $K_{ee} < 0$ and $K_{xx}K_{ee} - K_{ex}^2 > 0$ (concavity of K in (x,e)). Then, Tw is of course concave in x.

A.9. Proof of proposition 3.13

In this case, with results of lemma 3.4: $\overrightarrow{e_x} = -a\overrightarrow{f_{ex}}[af_{ee} - C_{ee}]^{-1} = -a\overrightarrow{f_{ex}}M$. If we suppose $\overrightarrow{f_{ex}} = 0_n$: $\overrightarrow{e_x} = 0_n$ and hence $\Gamma = 0$ which is sufficient for monotonicity. In order to prove that the solution of the functional equation is concave, using appendix A.5, we must prove that if w is concave then Tw is also.

Multiplicative separability in production: $f(e,x) = k(e)h(x)$
$\overrightarrow{f_{ex}} = \overrightarrow{k_e}h' = \frac{h'}{h}\overrightarrow{f_e}$, so $\overrightarrow{e_x} = -a\overrightarrow{f_{ex}}M = -a\frac{h'}{h}\overrightarrow{f_e}M = a\frac{h'}{h}\overrightarrow{e_a}$ and then $\frac{\partial}{\partial x}Tw(x) = \frac{\partial f}{\partial x} + \rho E \varepsilon w' \frac{\partial g}{\partial x} + a\frac{h'}{h}\left\langle \overrightarrow{e_a}, \overrightarrow{f_e} - \overrightarrow{C_e} + \rho E\varepsilon w' \overrightarrow{g_e} \right\rangle = \frac{\partial f}{\partial x} + \rho E \varepsilon w' \frac{\partial g}{\partial x}$ because $\left\langle \overrightarrow{e_a}, \overrightarrow{f_e} - \overrightarrow{C_e} + \rho E\varepsilon w' \overrightarrow{g_e} \right\rangle = 0$.

Additive separability in production and investment:

With $\overrightarrow{e_x} = 0_n$ $\frac{\partial}{\partial x}Tw(x) = \frac{\partial f}{\partial x} + \rho E\varepsilon w' \frac{\partial g}{\partial x}$ and with $\overrightarrow{g_{ex}} = 0_n$ $\frac{\partial^2}{\partial x^2}Tw(x) = \frac{\partial^2 f}{\partial x^2} + \rho E\varepsilon w' \frac{\partial^2 g}{\partial x^2} + \rho \frac{\partial g}{\partial x} E\varepsilon^2 w'' [\frac{\partial g}{\partial x} + \frac{\partial a}{\partial x}\langle \overrightarrow{e_a}, \overrightarrow{g_e} \rangle]$. But with proposition 3.9 proved in A.7, we have $\frac{\partial g}{\partial x} + \frac{\partial a}{\partial x}\langle \overrightarrow{e_a}, \overrightarrow{g_e} \rangle = g_x(1 - \frac{1}{1+F}) \geq 0$ because
$F = [\left\langle \overrightarrow{e_a}, \overrightarrow{e_a}[f_{ee} - C_{ee} + E\varepsilon v' g_{ee}] \right\rangle + \left\langle \overrightarrow{e_{aa}}, \overrightarrow{f_e} - \overrightarrow{C_e} + E\varepsilon v' \overrightarrow{g_e} \right\rangle] / E\varepsilon^2 w'' \langle \overrightarrow{e_a}, \overrightarrow{g_e} \rangle^2 \geq 0$ (appendix A.7). Since we know that f, g, are concave in x and that w is increasing and concave, so $\frac{\partial^2}{\partial x^2}Tw(x) \leq 0$.

A.10. Proof of proposition 3.15

According to appendix A.5, we need to prove that if w is increasing then Tw defined by $Tw(x) = \max_a (1-a) E[\nu f(x, e(x,a))] - b(x,a) + \rho E w(\varepsilon g(x, e(x,a)))$ is also increasing. With the envelope theorem, $\frac{\partial}{\partial x} Tw(x) = (1-a)[f_x + e_x f_e] - (\frac{\partial b}{\partial x})_{|\overline{U}} + \rho E \varepsilon w'(g_x + g_e e_x)$. Differentiating equation defining $b(x,a)$ we have $(\frac{\partial b}{\partial x})_{|\overline{U}} = -a f_x E\nu U'/EU'$. So $\frac{\partial}{\partial x} Tw(x) = f_x(1 - a(1 - E\nu U'/EU')) + (1-a) e_x f_e + \rho E \varepsilon w' g_x$ with $(1-a) f_e = (1 - E\nu U'/EU')\frac{f}{e_a} - E\varepsilon w' g_e$ by proposition 3.5. Moreover, we have $e_a = \frac{E\nu U' + a(E\nu^2 U'' - E\nu U'' E\nu U'/EU')f}{C_{ee} - aE\nu U' f_{ee} - a^2 E\nu^2 U'' f_e^2} f_e$, $e_x = a\frac{E\nu U' f_{ex} + a(E\nu^2 U'' - E\nu U'' E\nu U'/EU') f_e f_x}{C_{ee} - aE\nu U' f_{ee} - a^2 E\nu^2 U'' f_e^2}$ and $C_{ee} - aE\nu U' f_{ee} - a^2 E\nu^2 U'' f_e^2 > 0$. With the risk aversion, we know that $0 < E\nu U'/EU' < 1$. Hence supposing $E\nu^2 U'' - E\nu U'' E\nu U'/EU'$ is positive or small in magnitude if negative, we have $e_a > 0$ and e_x of the sign of f_{ex}. If $f_{ex} > 0$ and $g_e < 0$ then $a < 1, e_x > 0$ so $\frac{\partial}{\partial x} Tw(x) \geq 0$. If $f_{ex} < 0$, $g_e > 0$ and if the Agent is not too risk averse so that $1 - E\nu U'/EU'$ is small and $a(1 - E\nu U'/EU') < 1$, then $a > 1$, $e_x < 0$ and finally $\frac{\partial}{\partial x} Tw(x) \geq 0$. Also we have, $\frac{\partial}{\partial x} Tw(x) = f_x + \frac{(1 - E\nu U'/EU') a E\nu U' [f_{ex} f - f_e f_x]}{E\nu U' + a(E\nu^2 U'' - E\nu U'' E\nu U'/EU')f} + \rho E\varepsilon w' g_x > 0$ if $f_{ex} f - f_e f_x \geq 0$ and $e_a > 0$. In other cases, the land value function is likely to be increasing because $f_x > 0$ but the sign of $\frac{\partial}{\partial x} Tw(x)$ is ambiguous.

B. Extensions

B.1. The case of a risk averse agent

When the agent is risk averse, the incentive constraint is $aE\nu U' \overrightarrow{f_e}(x,e) = \overrightarrow{C_e}(e)$ and different propositions appear.

With a risk averse agent the optimal sharing rule derived in proposition 3.5 becomes

$$a^* = 1 + E\varepsilon v' \frac{\langle \overrightarrow{e_a}, \overrightarrow{g_e} \rangle}{\left\langle \overrightarrow{e_a}, \overrightarrow{f_e} \right\rangle} - (1 - \frac{E\nu U'}{EU'}) \frac{f}{\left\langle \overrightarrow{e_a}, \overrightarrow{f_e} \right\rangle} \tag{B.1}$$

If the landowner is also risk averse, with V its instantaneous utility function we have

$$a^* = 1 + \frac{E\varepsilon v'}{E\nu V'} \frac{\langle \overrightarrow{e_a}, \overrightarrow{g_e} \rangle}{\left\langle \overrightarrow{e_a}, \overrightarrow{f_e} \right\rangle} - (1 - \frac{E\nu U' EV'}{EU' E\nu V'}) \frac{f}{\left\langle \overrightarrow{e_a}, \overrightarrow{f_e} \right\rangle} \tag{B.2}$$

The effect of an increase of incentives is different in the case of a risk averse agent because of income effects.

Proposition B.1. *The production increases with the slope contract a whenever $af\sigma_A \leq \widetilde{E}\nu/\widetilde{V}\nu$ or equivalently $\widetilde{E}(a\nu f) \geq 2\widetilde{\pi}$ where $\sigma_A = \sigma_A(a\nu f + b)$ is the absolute risk aversion coefficient of* *the Agent and $\widetilde{E}\nu/\widetilde{V}\nu$ is the mean variance ratio in a ν equivalent probability measure, $\widetilde{\pi}$ is the Arrow's risk premium at Agent's income. It means that if risk aversion is small enough, increasing the production share of the agent will make his effort supply rise production.*

Proof. We see that $\left\langle \overrightarrow{e_a}, \overrightarrow{f_e} \right\rangle > 0$ if and only if $\Lambda \geq 0$ where $\Lambda = E[\nu U' + af(\nu^2 U'' - \nu U'' E\nu U'/EU')]$. From the Radon-Nikodym's theorem, we can write $E\,(a\nu f)^2\,U'' = EU''\widetilde{E}\,(a\nu f)^2$, $Ea\nu f U'' = EU''\widetilde{E}a\nu f$, $Ea\nu f U' = EU'\widetilde{E}a\nu f$ where $\widetilde{E}$ refers to a ν equivalent probability measure. Then, $\Lambda = E[(\widetilde{E}\nu/af\widetilde{V}\nu - \sigma_A)a^2 f^2 \widetilde{V}\nu U']$ with $\widetilde{V}\nu = \widetilde{E}\nu^2 - (\widetilde{E}\nu)^2$ and $\sigma_A\,(a\nu f + b) = -U''\,(a\nu f + b)\,/U'\,(a\nu f + b)$. Also $\Lambda \geq 0$ as $\widetilde{E}\,(a\nu f) \geq 2\widetilde{\pi}$ where $\widetilde{\pi} = \frac{1}{2}\widetilde{V}\,(a\nu f)\,\sigma_A$ is the risk premium at the agent's income. However, we can also say that:

Lemma B.2. *If the Agent has a relative risk aversion less than one and if $U''' \leq 0$ then $\left\langle \overrightarrow{e_a}, \overrightarrow{f_e} \right\rangle > 0$ i.e. $\sum_{i=1}^{n} \frac{\partial f}{\partial e_i}\frac{\partial e_i}{\partial a} > 0$. With one task, it means that the higher incentives power the greater the Agent's effort supply*[36].

Proof. We need $\Lambda \geq 0$. Noting $X = a\nu f$, $\Lambda = EXU'\,(X+b) + EX^2 U''\,(X+b) - EXU''\,(X+b)\,\frac{EXU'(X+b)}{EU'(X+b)}$ Since U is increasing concave, the unique negative term is $EU''\,(X+b)\,X^2$. Defining $\gamma\,(X) = -XU''\,(X)\,/U'\,(X)$ the relative risk aversion, we have: If $b \geq 0$, $\gamma \leq 1$ is sufficient because $E[XU' + X^2 U''] = E[XU'(1 - \gamma\,(X+b) - bU''/U')] \geq 0$. If $b \leq 0$ and $U''' \leq 0$ then $EX[U'\,(X+b) + XU''\,(X+b)] \geq E[XU'\,(X)\,(1-\gamma\,(X))] \geq 0$ if $\gamma \leq 1$.[37]

Risk averse Agent:

Noting that $1 - E\nu U'/EU' > 0$ with the utility concavity, the risk sharing term tends to diminish a^*, as intuition predicts, to alleviate the tenant from risk bearing. We know that $\left\langle \overrightarrow{e_a}, \overrightarrow{f_e} \right\rangle > 0$ in the risk neutral case.

Risk averse Principal:

When, the Principal (instantaneous utility function V) is risk averse and the Agent is not, we have $a^* = 1 + \frac{E\varepsilon v'}{E\nu V'}\frac{\langle \overrightarrow{e_a}, \overrightarrow{g_e} \rangle}{\left\langle \overrightarrow{e_a}, \overrightarrow{f_e} \right\rangle}$. Hence, whatever be the Principal's risk aversion, $a^* \gtreqless 1$ depends on the sign of $\langle \overrightarrow{e_a}, \overrightarrow{g_e} \rangle$ as when he is risk neutral. The trade-off between production and fertility uncertainty for the landlord appears and is multiplicative with the effort distortion ratio.

[36]The condition $U''' \leq 0$ implies increasing absolute and relative risk aversion which is not very intuitive but we cannot derive general formula in the other cases. It seems important to remark that in the standard sharecropping literature, this problem is generally avoided either by neglecting second order effects or by setting b to zero, or by doing a mistake as in Singh (1989) where he deduces falsely that $U''' \leq 0$ implies decreasing absolute risk aversion.

[37]We remark that when $b = 0$, non increasing relative risk aversion is sufficient because defining $\pi = EXU'\,(X)\,/EU'\,(X)$, we have $EX^2 U'' - EXU'' EXU'/EU' = E[U'\,(X)\,(X-\pi)(\gamma\,(\pi) - \gamma\,(X))] \geq 0$ with γ non increasing and because $E[U'\,(X)\,(X-\pi)] = 0$.

B.2. The role of uncertainty

Mean and variance of Principal's income:

Noting the random variable $v(\varepsilon g) = \widetilde{\omega}$, the mean and variance of landlord's welfare are:

$$\begin{aligned}\overline{Y_l} &= (1-a^*) f - b^* + E\widetilde{\omega} = f - \Phi\left(\overline{U}+C\right) + E\widetilde{\omega} \\ \sigma^2_{Y_l} &= (1-a^*)^2 f^2 \sigma^2_\nu + \sigma^2_{\widetilde{\omega}} + 2(1-a^*) f \sigma_{\nu\widetilde{\omega}}\end{aligned}$$

because $E\nu = 1$. We see that the variance of landlord's welfare depends on variance and covariance of production and fertility uncertainties. Assuming that $a^* \leq 1$ because incentives to production would be bad for future fertility. If these random shocks are climatic shocks, we expect that production and fertility shocks owed to weather will be positively correlated meaning that landlord's income variance will be increased with the covariance of the shocks. On the contrary, if the ν is the production price uncertainty and the fertility shock is climatic, then the covariance is expected to be negative because a negative agricultural shock will be followed by a positive market price shock. Then the landlord's income variance will decrease with the covariance of the shocks.
With risk neutrality, we have $E\nu U' = EU'$, and $a^* = 1 + E\varepsilon v' \frac{\langle \overrightarrow{e_a}, \overrightarrow{g_e} \rangle}{\left\langle \overrightarrow{e_a}, \overrightarrow{f_e} \right\rangle} = 1 + E\varepsilon v' \frac{\left\langle \overrightarrow{f_e} M, \overrightarrow{g_e} \right\rangle}{\left\langle \overrightarrow{f_e} M, \overrightarrow{f_e} \right\rangle}$. The sign of $\left\langle \overrightarrow{f_e} M, \overrightarrow{g_e} \right\rangle$ is needed to know if a^* is greater, equal or smaller than one. We can meet various cases studied in the next section.

Fertility uncertainty:

The random factor on land fertility x has an influence on the optimal contract slope. The production uncertainty is taken into account through the risk sharing expression $(1 - E\nu U'/EU')f / \left\langle \overrightarrow{e_a}, \overrightarrow{f_e} \right\rangle$.
Even with risk neutrality of Principal and Agent, the land fertility uncertainty is important because though the landlord is risk neutral on production, he is not on fertility since the value function is not linear. If the value function of future fertility for the landlord exhibits a relative prudence[38] less than one (which is the case if $v''' \leq 0$ i.e. v'' is decreasing) then by Jensen's inequality, $E\varepsilon v'(\varepsilon g) \leq v'(g)$ i.e. that with fertility uncertainty, the distortion ratio is smaller than with deterministic fertility. On the contrary if the value function exhibits a relative prudence higher than one, by Jensen's inequality, the distortion ratio is greater than with deterministic fertility.
Linearizing v' in the neighborhood of $g(x,e)$ states $E\varepsilon v' \simeq v' + Var(\varepsilon) v''$ because $E\varepsilon = 1$. Therefore, when v is increasing concave, a^* increases as $Var(\varepsilon)$ increases if $\langle \overrightarrow{e_a}, \overrightarrow{g_e} \rangle < 0$, and a^* decreases as $Var(\varepsilon)$ increases if $\langle \overrightarrow{e_a}, \overrightarrow{g_e} \rangle > 0$.

[38] The relative prudence v is defined by $\frac{-xv'''(x)}{v''(x)}$ as in the case of a utility function.

In the risk neutral case: when $a^* < 1$, a^* increases as $Var(\varepsilon)$ increases, when $a^* > 1$, a^* decreases as $Var(\varepsilon)$ increases. Uncertainty on land fertility tends to bring the optimal contract towards the usual second best one (a fixed rent contract without risk aversion). The intuition is that when land fertility improvements are too hazardous, they become worthless and so the incentive trade-off inclines to favor production.

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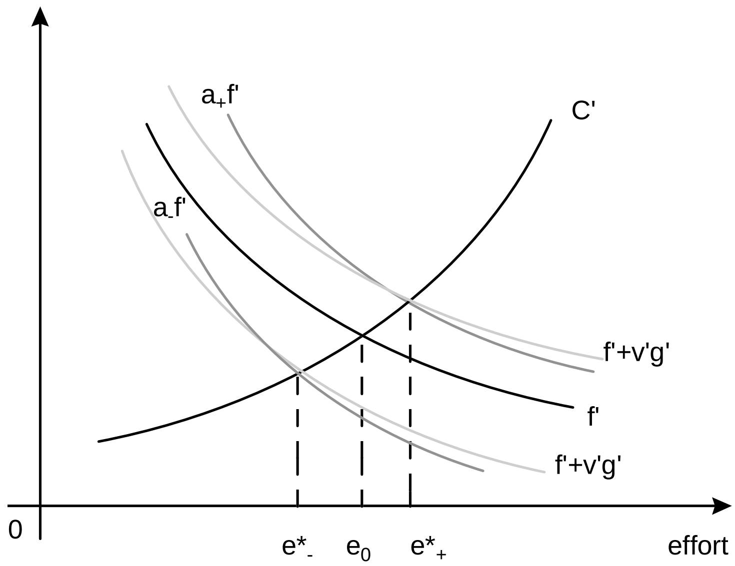

Figure B.1: Optimal Contract Determination

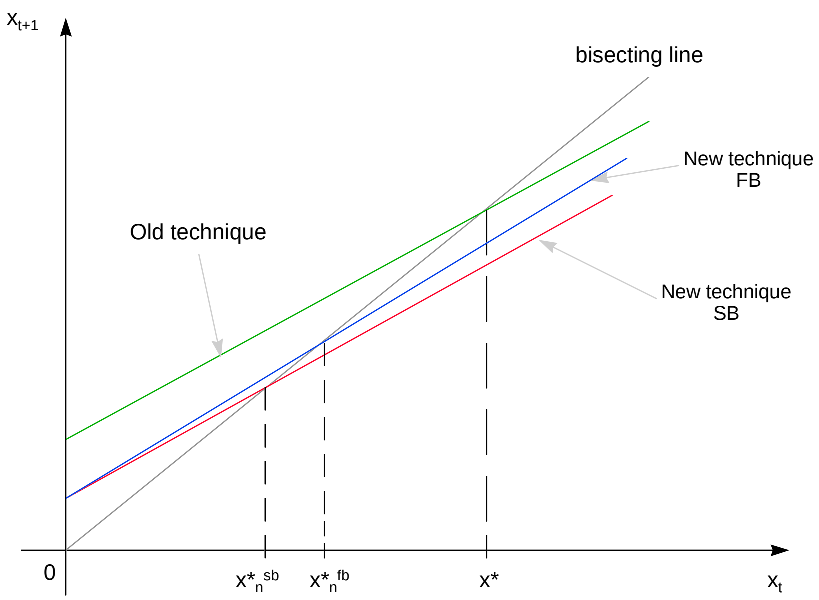

Figure B.2: Innovation and fertility change

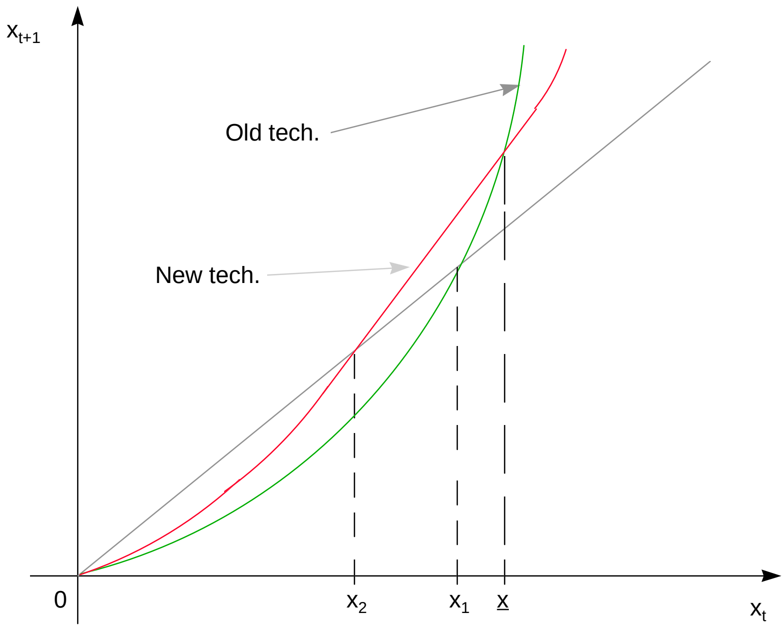

Figure B.3: Innovation and fertility

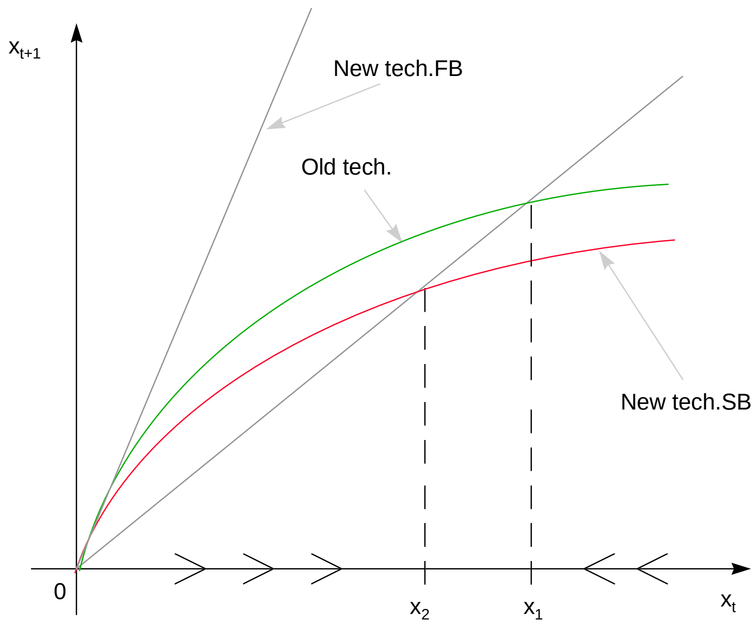

Figure B.4: Innovation and fertility

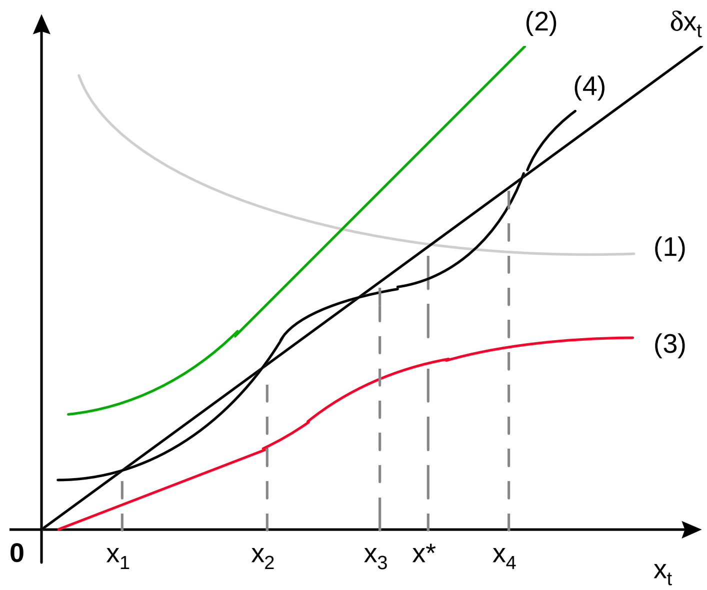

Figure B.5: Fertililty dynamics examples

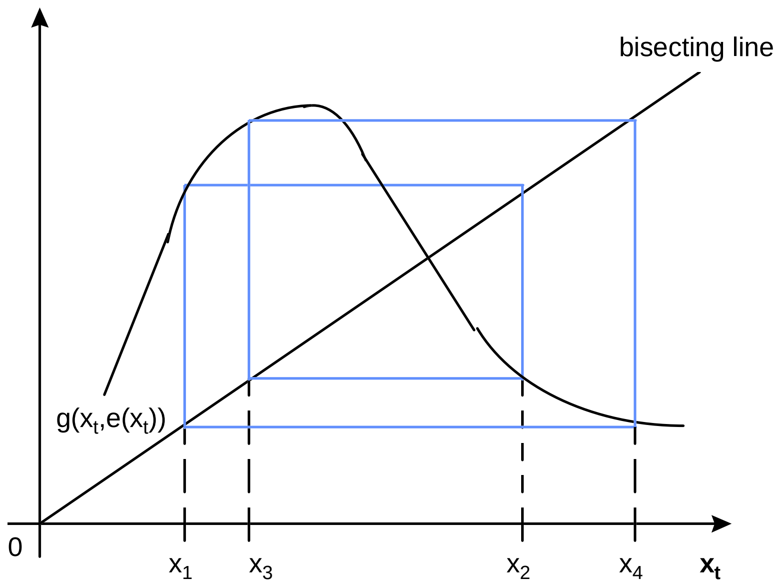

Figure B.6: Fertility Path

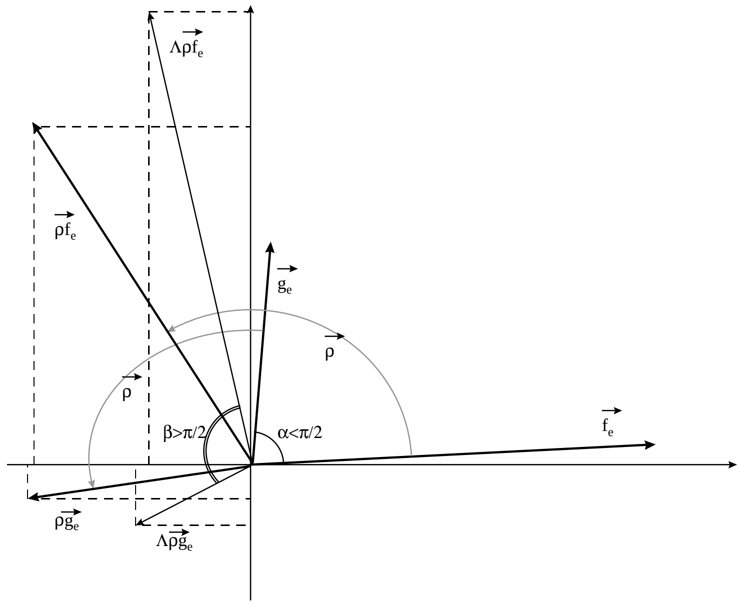

Figure B.7: An intuition about a non intuitive case

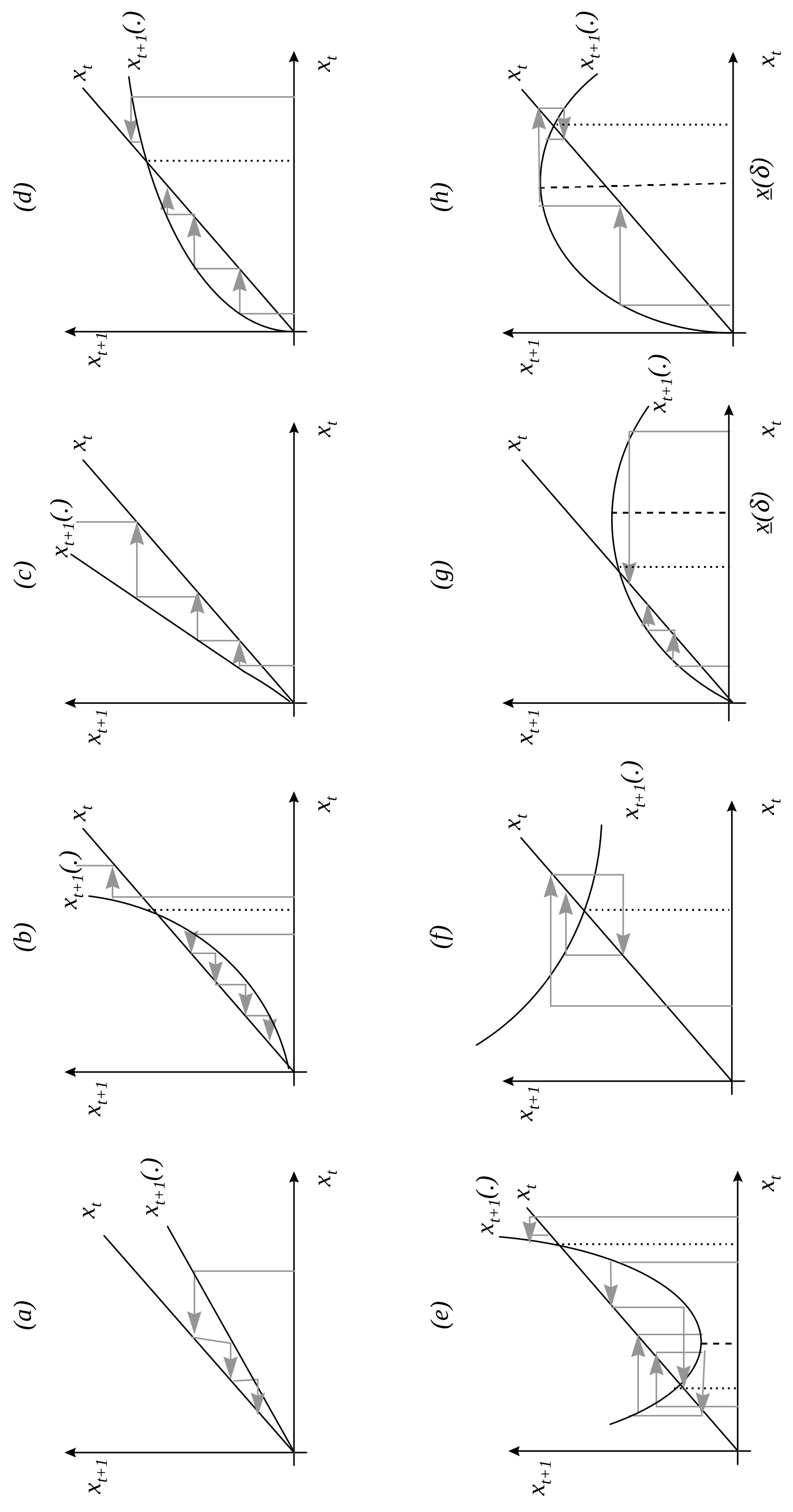

Figure B.8: Fertility Dynamics : $x_{t+1} = (1+\delta)\, x_t - \widehat{g}\,(e_t)$