

Optimal Incentives under Moral Hazard and Heterogeneous Agents: Evidence from Production Contracts Data

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Abstract

The objective of this paper is to develop an analytical framework for the estimation of parameters of a structural model of an incentive contract under moral hazard, taking into account agents heterogeneity. We show that allowing the principal to strategically distribute the production inputs across heterogeneous agents as part of the contract design, the principal is able to change what appears to be a uniform contract into individualized contracts tailored to fit agents' preferences or characteristics. Using micro level data on swine production contract settlements, we find that contracting farmers are heterogeneous with respect to their risk aversion or the costs of effort and that this heterogeneity affects the principal's allocation of production inputs across farmers. Relying on the identifying assumption that contracts are optimal, we obtain the estimates of a lower and an upper bound of agents' reservation utilities. We show that farmers with higher risk aversion have lower outside opportunities and hence lower reservation utilities.

Keywords: Agency Contracts, Optimal Incentives, Moral Hazard, Risk Aversion, Heterogeneity.

JEL Classification: D82, L24, Q12, K32, L51.

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1 Introduction

In many business environments, including agriculture, economic agents often interact with each other repeatedly and business is conducted using a series of short-term contracts. The use of contracts to vertically coordinate the production and marketing of agricultural commodities has become common practice in many agricultural sectors including livestock, fruits and vegetables, tobacco, etc. To solve the asymmetric information problems between processors (principals) and independent farmers (agents), the majority of contracts use high powered incentives schemes to compensate farmers. Another interesting characteristic of many production contracts is that all agents contracting with the same principal are operating under formally identical contract provisions (Levy and Vukina, 2002). However, explicitly uniform contracts may not necessarily guarantee that all agents are treated equally. When the principal and agents contract repeatedly, an explicitly uniform but incomplete contract leaves a possibility for the principal to treat agents differently after learning about their types (abilities, risk aversions, costs of effort, etc.). Typically, these contracts specify a general payment formula that expresses the agent's reward as a function of his performance but in which the base payment and the incentive power of the contract depend on the provision of some inputs by the principal. Introducing the choice of these strategic variables as part of the contract design, the principal is able to change what appears to be a uniform contract into individualized contracts tailored to fit agents' preferences or characteristics.

The objective of this paper is to study this contract design problem, to present a method that would allow the identification and estimation of the structural parameters of the moral hazard model, and to test predictions aimed at assessing the empirical reliability of the model. In order to identify the heterogeneity among agents, we assume that they have different risk aversion attitudes and different costs of effort and that their characteristics are observed by the principal. In the empirical part of the paper we use a panel data containing individual

settlements of livestock production contracts. Our analysis explains an apparent anomaly frequently observed in many agricultural contracts which manifests itself in the principal's use of seemingly uniform contracts for the purposes of governing the relationships with heterogeneous agents.¹

Empirical tests of contract theory are typically performed with either cross-industry and cross-firm data or with intra-firm data. As pointed out by Chiappori and Salanié (2003), the first approach can provide more general empirical results but faces the problem of unobserved heterogeneity (for the econometrician). The second type of data will generate the results that are difficult to generalize but has the advantage of dealing with agents that operate in the same environment, which removes a lot of the potential unobserved heterogeneity. This research belongs to the second category of studies. Our data comes from payroll records of one company that contracts the production of live hogs with independent farmers.

The literature concerned with empirical testing of contract theory related to this paper follows two distinct approaches. One line of research takes contracts as given and model the behavior of the principal and the agents under the observed contractual terms without assuming optimal contract design. For example, interesting studies in labor economics of Paarsch and Shearer (2000, 2007) use the information on incentive contracts and longitudinal individual outputs in order to estimate how effort responds to incentives provided by piece rate contracts. They do not study the optimal contract design nor do they assume contract optimality to identify the model primitives. However, certain aspects of their approach is

¹A related topic more linked to the adverse selection problem in a similar environment has been studied by Leegomonchai and Vukina (2005). They test whether chicken companies allocate production inputs of varying quality by providing high ability agents with high-quality inputs or by providing low ability agents with high quality inputs. The first strategy would stimulate the career concerns type of response on the part of the growers, whereas the second strategy would generate a ratchet effect. Their results show no significant input discrimination based on grower abilities that would lead to either career concerns or ratchet effect type of dynamic incentives.

related to ours because they use an assumption about the contract design to identify the heterogeneity of agents regarding their cost of effort. Their assumption is not specifically an optimality assumption because they assume that the employer cannot discriminate between workers according to their observable cost of effort. Instead they assume that the contract is designed to satisfy at least the participation constraint of the least able worker. Other papers within the same paradigm include, for example, Abbring, Chiappori, Heckman, and Pinquet (2003); Chiappori, Durand, Geoffard (1998); Ferrall and Shearer (1999); and Shearer (2004). They take advantage of the fact that they can observe the actual contracts and eventually some changes in the contract forms, which enable them to test various implications of moral hazard.

The other line of research in empirical testing of contract theory explicitly or implicitly assumes that contracts are optimal. Then, it derives predictions about the determinants of some observed contract parameters and test those predictions empirically. This approach is often used when one does not observe all of the exact contractual terms agreed upon between a principal and an agent. A good example of this approach is the empirical work on sharecropping contracts where the goal is usually to test between the alternative theories of contract design, for example the transaction cost versus the risk sharing explanation (Allen and Lueck, 1994, Dubois, 2002, Akerberg and Botticini, 2002).

Our paper presents the combination of the above two approaches. First, we empirically check several testable implications of the incentive theory without assuming the contract optimality. The fact that we can precisely observe all relevant contract stipulations allows us to model the agent's behavior in a way that is consistent with the assumption that contracts are either optimal or suboptimal.² Second, after modeling the agent's behavior, we analyze the

²The fact that in the real economy there are many institutional constraints or bounded rationality types of behavior that may restrain the actors to use theoretically optimal contracts has been well established in the literature. For a discussion on optimal versus suboptimal contracts, see Chiappori and Salanié (2003).

principal's decisions and contract design. We confirm that contract farmers are heterogeneous with respect to their risk preferences and/or their costs of effort and that this heterogeneity affects the principal's decision of how to allocate the production inputs across farmers. Using the identifying assumption that contracts are optimal, we obtain estimates of a lower and an upper bound on agents' reservation utilities. We also show that farmers with higher risk aversion have lower outside opportunities due to lower reservation utilities.

Our paper also contributes to the literature on testing the trade-off between risk and incentives. When it comes to the determination of contract choice, the transaction cost literature (e.g., Allen and Lueck, 1992) claims the unimportance of risk. On the other hand, Pennings and Smidts (2000) found that the degree of risk aversion is important in explaining owner-managers' choice between relatively safe fixed-price contracts versus spot market transactions. Akerberg and Botticini (2002) showed that if one controls for the endogenous matching between principals and agents, the agent's risk aversion appears to significantly influence the contract choice. When it comes to testing whether risk imposes a constraint to offering incentives the evidence is also mixed, with some work finding evidence in favor of the theories, while other find little (Prendergast, 1999, 2002). In our structural model, we show that individual risk aversion or cost of effort identified with the longitudinal performance data actually affects the principal's optimal contract choice in which she must balance the incentives and the risk sharing in a moral hazard environment.

2 Industry Description and Data

Swine production in the United States is characterized by an increasing presence of vertically integrated firms (called integrators) that contract the production (grow-out) of hogs with independent farmers. The contract production is dominated by large national companies (Smithfield Foods, Tyson, etc.,) that run their businesses through smaller profit centers that

issue contracts, supply inputs and slaughter finished animals.

A production contract is an agreement between an integrator company and a farmer (grower) that binds the farmer to specific production practices. Different stages of production of animals are typically covered by different contracts and farmers generally specialize in the production of animals under one contract. The most frequently observed contracts in the swine industry are single production stage contracts such as farrowing contracts, nursery contracts and especially finishing contracts. All production contracts have two main components: one is the division of responsibility for providing inputs, and the other is the method used to determine grower compensation. Growers provide land, housing facilities, utilities (electricity and water) and labor and are also responsible for manure management and disposal of dead animals. An integrator company provides animals, feed, medications and services of field men. Companies also own and operate feed mills and processing plants and provide transportation of feed and live animals. When it comes to specifying integrator's responsibilities for providing inputs, the terms of the contract are intentionally vague. The integrator decides on the volume of production both in terms of the rotations of batches on a given farm as well as the number (density) and weight of incoming animals (feeder pigs) inside the house. A typical scheme for compensating growers in finishing contracts is based on a base plus bonus payment per pound of gain (live weight) transferred, where a bonus payment reflects some efficiency measure such as feed conversion.

The data set used in this study is an unbalanced panel from Martin (1997). It contains a sample of contract settlement data for individual growers who contracted the finishing stage of hog production with an integrator in North Carolina. The data set spans the period between December 1985 and April 1993, for a total of 802 observations. Each observation represents one contract realization, i.e., the payment received and the grower performance associated with one batch of animals delivered to the integrator's processing plant. There are 122 growers in

the data set and the number of observations per grower ranges from 2 to 37.³

The size of the grow-out operation (the number of finishing houses) varies across growers between one and five houses. All houses under contract have approximately the same maximal capacity. The median density of a house is 1,226 hogs per house and the mean density is 1,234 hogs per house. The contract coverage varies across farms and time. Sometimes one contract will cover multiple houses on a given farm, other times each house will be covered by a separate contract. In cases when multiple houses are covered by one contract, the grower payment is calculated by treating all houses as one unit. The coverage of the contract is determined by the timing of the placement and genetic composition of feeder pigs. The animals covered by the same contract have to be placed on a given farm at the same time and have to have similar genetic characteristics. The average length of the production cycle is approximately 19 weeks. Counting one additional week for the necessary cleanup gives a maximum of 2.6 batches of finished hogs per house per year. The data summary statistics are presented in Table 1.

The particular finishing contract that generated the data is fairly representative for the industry as a whole. The contract requires that growers furnish fully equipped housing facilities and that they follow the management and husbandry practices specified by the integrator. The contract guarantees the grower a minimum of 7 batches of feeder pigs and is automatically renewed unless canceled in writing. The integrator provides the grower with feeder pigs, feed, medication, veterinary services and services of the field personnel. The quality of all inputs as well as the time of placement of feeder pigs and shipment of grown animals are exclusively under control of the integrator.

The compensation to grower i for the batch of hogs under contract t , as the payment for husbandry services and the housing facilities rental, is calculated on a per pound of gain basis with bonuses earned on a per head basis. The bonus is based on the difference between

³It appears that the data sample has been extracted randomly from the population of all contracts that has been settled between this integrator and her growers during this time period.

Table 1: Descriptive Statistics

Variable	Mean	Std. Deviation
Feed conversion ratio (f_{it})	2.76	0.151
Grower's revenue in US\$ (R_{it})	18 886	10 022
Heads placed (H_{it})	2 077	1 111
Mortality rate (m_{it})	0.039	0.020
Feed used in (1000) pounds (F_{it})	1 033	553
Weight gained in (1000) pounds (q_{it})	373	195
Weight of incoming feeder pigs in pounds (κ_{0it})	44.16	5.05
Weight of outgoing finished hogs in pounds (κ_{it})	234.22	7.71
Price of feed (in US\$ per 100 pounds)	11.13	0.3
Price of hogs (in US\$ per 100 pounds)	44.85	11.1
Price of feeder pigs (in US\$ per 100 pounds)	83.51	14.1

Prices of feed, hogs and feeder pigs varied during the period sampled by the data.

the individual grower's feed conversion, expressed as pounds of feed divided by pounds of gain $\frac{F_{it}}{q_{it}}$, and a standard feed conversion ratio ϕ . If the grower's ratio is above the standard, he receives no bonus and simply earns the base piece rate α multiplied by the total pounds gained q_{it} . If the grower's ratio is below the standard ratio, the difference is multiplied by a constant β to determine the per head bonus rate. The total bonus payment is then determined by multiplying the bonus rate by the number of pigs marketed, where the marketed pigs $(1 - m_{it})H_{it}$ are those feeder pigs that survived the fattening process and m_{it} is the animal mortality rate. Algebraically, the exact formula for the total compensation is:

$$R_{it} = \alpha q_{it} + \max[0, \beta(\phi - \frac{F_{it}}{q_{it}})](1 - m_{it})H_{it}. \quad (1)$$

During the period covered by the data set some parameters of the payment mechanism (1) have changed. The base piece rate varied with the type of feeder pigs placed on a grower farm. For commingled feeder pigs $\alpha = 0.0315$, whereas for integrator's own nursery feeder pigs

$\alpha = 0.0275$.⁴ Also, as a result of technological progress in nutrition and housing design, the feed conversion standard was lowered from $\phi = 3.50$ to $\phi = 3.35$. However, after the lower feed conversion standard was introduced, the higher standard of 3.50 remained in effect for commingled pigs. Consequently, we have three different payment schemes: $(\alpha = 0.0315, \phi = 3.50)$, $(\alpha = 0.0275, \phi = 3.50)$ and $(\alpha = 0.0275, \phi = 3.35)$. All observed feed conversion ratios are below the benchmark feed conversion (ϕ), so the truncation of the bonus payment at zero will be ignored and the payment scheme simplified⁵ as

$$R_{it} = \alpha q_{it} + \beta \left(\phi - \frac{F_{it}}{q_{it}} \right) (1 - m_{it}) H_{it}. \quad (2)$$

Notice that the imposed simplification bears very little consequence on the riskiness of the payment scheme. The risk that the grower is exposed to is due to the variance of his revenue R_{it} which is the result of his performance measured by the feed conversion ratio $\frac{F_{it}}{q_{it}}$ that is affected by random shocks. Although the truncation at zero never binds, there is considerable variability in feed conversion ratios and hence payments across growers. In fact, the elimination of the truncation of the bonus payment at zero, if it were actually introduced by the integrator into the contract, might increase the variability of an individual grower payment relative to his peers, but in this particular empirical case it is inconsequential.

In addition to individual grower contract settlement data, the proposed methodology requires the integrator-level price data for the inputs and the output. However, such data is not available. Instead we use the regional market prices for feed, feeder pigs and finished hogs, also obtained from Martin (1997). The feed prices are quarterly figures for the Appalachian region, the feeder pig prices are monthly observations for North Carolina and the market prices for

⁴There are three types of feeder pigs in the data set. Commingled pigs are feeder pigs that are either bought at an auction or from an outside source. The third type are own feeder pigs which come from the breeding stock controlled by the integrator, hence are deemed to be of superior quality.

⁵In principle, the truncation at zero should modify ex-ante the behavior of the agent, but it is ignored for tractability and also because it is far from being empirically operant and thus unlikely to affect the behavior of agents.

finished hogs are monthly prices received by North Carolina farmers for barrows and gilts.⁶

3 The Model

We model the integrator-grower relationship in a principal-agent framework. The timing of the contractual game played between the principal and the agent is as follows. The principal (integrator) proposes the contract to the agents (growers) on a *take-it-or-leave-it* basis. The contract specifies the division of responsibilities for providing inputs and the payment formula. The integrator is required to provide animals (feeder pigs) and feed and the grower is required to provide housing for animals and labor (exert effort). After the grower observes the payment formula, the number and the weight of incoming feeder pigs supplied by the integrator, he accepts or rejects the contract. A grower that accepted the contract then exerts effort.

The tasks performed by the grower are not perfectly observable by the integrator, who therefore faces a moral hazard problem in the delegation of production tasks. The incentives to the grower to behave according to the principal's objective are provided through the payment scheme which always includes a particular type of bonus (premium) mechanism. In our data, the bonus depends on a perfectly observable and verifiable performance measure which is the feed conversion ratio. The agent's payment (2) can then be written as a linear function of the performance measure, i.e. the feed conversion ratio $f_{it} = \frac{F_{it}}{q_{it}}$, such that

$$R_{it} = \tilde{\alpha}_{it} - \tilde{\beta}_{it}(f_{it} - \phi) \quad (3)$$

where the fixed component ($\tilde{\alpha}_{it}$) and the slope ($\tilde{\beta}_{it}$) of this linear function depend on some parameters as

$$\tilde{\alpha}_{it} = \tilde{\alpha}_{it}(\kappa_{0it}, H_{it}) = \alpha q_{it} = \alpha[\kappa_{it}(1 - m_{it}) - \kappa_{0it}] H_{it} \quad (4)$$

$$\tilde{\beta}_{it} = \tilde{\beta}_{it}(H_{it}) = \beta(1 - m_{it})H_{it} \quad (5)$$

⁶The procedure to convert the quarterly prices into monthly figures and the exact matching of the monthly prices to contract settlement dates is explained in detail in Martin (1997).

with κ_{it} being the weight of outgoing finished hogs, κ_{0it} the weight of incoming feeder pigs, and H_{it} the number of heads of animals placed on the farm. When the principal proposes the contract to the agent, he proposes the payment scheme (3) where parameters $\tilde{\alpha}_{it}$ and $\tilde{\beta}_{it}$ are known. Thus, at the time the agent has to accept or reject the contract, the contractual payment consists of a fixed payment $\tilde{\alpha}_{it}$, and a premium part which is tied to the performance $(f_{it} - \phi)$ with the known incentive power $\tilde{\beta}_{it}$. After accepting the contract, the agent exerts effort.

We consider that the parameters of this affine function are fixed at the time the grower chooses his effort and that the only source of risk comes from the performance in terms of feed conversion. The assumption that the parameters $\tilde{\alpha}_{it}$ and $\tilde{\beta}_{it}$ depend on conditions and variables known and observed by the grower when he chooses his effort is reasonable. Actually, the grower always observes the number H_{it} and the weight κ_{0it} of feeder pigs when they arrive on the farm. The grower also knows that the pigs are grown until they reach their target weight κ_{it} . Finally, the grower can accurately judge the mortality rate m_{it} by observing the genetic make-up and the overall condition of feeder pigs delivered to the farm and the density at which they are stocked. Empirically, we see that there is actually very little variation in mortality rates given H_{it} and little variation in the weight κ_{it} of finished animals. Thus, it is true that $\tilde{\alpha}_{it}$ and $\tilde{\beta}_{it}$ are known as soon as H_{it} and κ_{0it} are known.

3.1 Theory of agent behavior

We assume that grower i 's preferences over revenue R_{it} and effort e_{it} at period t are described by the utility function $U_i(R_{it} - C_i(e_{it}))$ which is known by the principal. $C_i(\cdot)$ is a positive increasing function implying that effort is costly. We assume that growers exhibit constant absolute risk aversion such that $U_i(R_{it} - C_i(e_{it})) = \frac{-1}{\theta_i} \exp -\theta_i(R_{it} - C_i(e_{it}))$ where $\theta_i > 0$ is the absolute risk aversion parameter, and also assume that the stochastic revenue is normally distributed. Under these assumptions, grower i 's expected utility can be expressed as an

increasing concave function of the mean-variance criterion (which corresponds to the certainty equivalent value of revenue) and her maximization problem can be written equivalently as:

$$\max_{e_{it}} W_i(R_{it}, e_{it}) = ER_{it} - \frac{\theta_i}{2} Var R_{it} - C_i(e_{it}). \quad (6)$$

Notice that the curvature of the utility function is grower-specific which allows much more flexibility than when the curvature is common to all agents, i.e. when θ_i is constant across i .

First, let's specify how the observed outcome stochastically depends on the unobservable grower effort and assume that

$$f_{it}(e_{it}) - \phi = (\lambda - e_{it})u_{it} \quad (7)$$

where λ reflects growers' fixed ability parameter, e_{it} is a costly effort which improves (reduces) the feed conversion ratio, and u_{it} is an i.i.d. (across growers and periods) normal production shock with mean of 1 and σ^2 variance.⁷ This specification shows that a unit of effort is worth one unit of feed conversion ratio which gets transformed into revenue through $\tilde{\beta}_{it}$. Since the cost of effort is monetary, it must be in the same units as revenue, hence we specify

$$C(e_{it}) = \gamma_i \tilde{\beta}_{it} e_{it}$$

where $0 < \gamma_i < 1$ is the grower i 's cost of effort⁸. The cost of effort γ_i can be interpreted as a ratio expressing how much it costs in revenue to reduce the feed conversion by an amount worth one unit of money. Thus it must be below one for the agent to exert a positive value of effort. Actually the expected marginal benefit of effort is $E[-R'_{it}(e)] = \tilde{\beta}_{it}$ while the marginal cost is $\gamma_i \tilde{\beta}_{it}$, so that no effort will be provided (whatever risk aversion) if $\gamma_i \geq 1$.

⁷As seen from (7), the random term u has to be positive, which violates the normality assumption and hence the mean-variance representation of growers' preferences can only be seen as an approximation of the expected utility where the utility is exponential (CARA). This approximation is exact when u is normal even if the risk is not small. The fact that the empirical distribution of u is not too far from being normal helps us justify the mean-variance criterion approximation even if the risk is large.

⁸Notice that the apparently more general specification $f_{it} - \phi = (\lambda - \rho e_{it})u_{it}$ is not different from the chosen one because we could simply redefine effort as $\tilde{e}_{it} = \rho e_{it}$ whose cost will be $\frac{\gamma_i}{\rho} \tilde{\beta}_{it} \tilde{e}_{it}$ instead of $\gamma_i \tilde{\beta}_{it} e_{it}$.

The above specification of the stochastic production technology with the feed conversion effort being the only source of moral hazard can be justified on several grounds. First, the reason for not modeling the weight of the finished animals as the moral-hazard component of the production is because the final weight of market hogs varies very little, not only within farmers that grow hogs for the same integrator, but also across the entire industry. There are two reasons for this: consumers' preferences for a particular type of pork and the automatization of the slaughterhouse processing lines. Given consumers' preferences for various pork items, there has to be one ideal weight of finished hogs that gives the highest yield and optimal mix of primary and secondary products. Because of the fact that modern slaughterhouses are highly automatized, the variability of sizes of animals that need to be processed would require frequent calibration of the equipment and would considerably slow down the speed of the processing line.

Second, the animal grow-out technology is described by animals eating *ad libitum* or "at will". This means that the feed is always there for them to eat. Given the fact that the targeted weight is given (and the weight of incoming feeder pigs vary), the growers' objective is to produce the targeted weight with as little feed as possible. The aggregate consumption of feed depends on many factors, some of which growers cannot control (animals genetics and the quality of feed rations), and others which they can (optimal in-house environment, feed management, spoilage, pilfering, etc.). Most importantly, the total feed utilization depends on the number of days that animals spent on the farm, so the trick is to fatten them to the target weight as quickly as possible. It is very hard to precisely say what exactly farmers do to improve feed conversion, but the fact of the matter is that some farmers are frequently better than others. The substantial variation in feed conversion among farms convincingly testifies to that effect.

Finally, the mortality variations across growers are small and mainly explained by the

number of feeder pigs placed in growers' houses as well as their genetic make-up, both variables being under control of the integrator and not growers.

Then, using (3) and (7) we can write the agent's certainty equivalent of net revenue as

$$W_i(R_{it}, e_{it}) = \tilde{\alpha}_{it} - \tilde{\beta}_{it} [Ef_{it} - \phi] - \frac{\theta_i}{2} \tilde{\beta}_{it}^2 Var[f_{it}] - \gamma_i \tilde{\beta}_{it} e_{it} \quad (8)$$

and the first order condition for the maximization problem in (6) becomes

$$-\frac{\partial}{\partial e_{it}} Ef_{it} - \frac{\theta_i}{2} \tilde{\beta}_{it} \frac{\partial}{\partial e_{it}} Var[f_{it}] = \gamma_i.$$

Given (7), it is clear that $Ef_{it} - \phi = \lambda - e_{it}$ and $Var[f_{it}] = (e_{it} - \lambda)^2 \sigma^2$ which gives the following expression for the optimal effort level:

$$e_{it}^* = \frac{1 - \gamma_i}{\sigma^2 \theta_i \tilde{\beta}_{it}} + \lambda. \quad (9)$$

A careful inspection of equation (9) reveals three important characteristics of this production technology and the contract payment mechanism. First, equilibrium effort decreases feed conversion ratio (recall that a good feed conversion is a low one) but it also increases its variance. This is the result of the multiplicative error structure in the production function (7). The justification for this specification comes from the necessity to model a technology where exerting low/zero effort would result in the feed conversion ratio attributable only to the default ability parameter corrected for the shock. Notice that the risk associated with this strategy is also low. The feed conversion ratio will be predictably bad, but it cannot be worse than the pigs' metabolism dictates. Choosing very high effort, on the other hand, improves expected feed conversion significantly, but it is at the same time risky as it generates high variability in performance.⁹

⁹Imagine an anxious grower inspecting his pigs every hour just to make sure that they are doing well, whereas the more normal procedure would be to come only twice a day. By doing this he can possibly introduce some infectious disease into the housing environment that otherwise with less frequent visits would have not happened. So, without the disease, the feed conversion is phenomenal because everything was functioning meticulously all the time. With the disease, however, the feed conversion went sky-high because pigs got diarrhea.

Second, the optimal effort level depends monotonically on the heterogeneity parameter $\tilde{\theta}_i = \frac{\theta_i}{1-\gamma_i}$, which, for ease of exposition, we will sometimes simply refer to as the risk aversion, despite the fact that it actually represents the risk aversion adjusted for the cost of effort. Risk aversion and cost of effort (higher $\tilde{\theta}_i$) both reduce the optimal effort whereas stronger incentives ($-\tilde{\beta}_{it}$) increase effort. Lower $\tilde{\theta}_i$ indicates more efficient growers because of either lower risk aversion or lower cost of effort, so we will refer to this parameter as the growers' efficiency parameter.

Thirdly, equation (9) also implies that optimal effort is only affected by the incentives power of the contract ($-\tilde{\beta}_{it}$) and not by the constant part of the payment $\tilde{\alpha}_{it}$. This result has a simple consequence for the equilibrium strategy that the integrator (principal) would pursue when it comes to deciding how many feeder pigs to allocate to each grower (agent) according to his efficiency parameter.

3.2 Theory of principal behavior - Model 1

Now, we model the principal's behavior taking into account the agent's optimal response. We assume that the principal is risk neutral and maximizes the expected profit per grower. We start with Model 1 where we assume that the principal has to choose within the class of payment functions (2) that are empirically observed, but not necessarily optimal. In other words, here we are not taking into account the participation constraint of the growers that already signed the contract. The approach that relies on the observed contracts being optimal will be exploited in section (3.3).

The integrator's profit function is given by:

$$\pi_{it} = pQ_{it} - w_F F_{it} - R_{it}(H_{it}, \kappa_{0it}) - w_H(\kappa_{0it}) H_{it} \quad (10)$$

where p is the market price of hogs, $Q_{it} = \kappa_{it}(1 - m_{it})H_{it}$ is the total live weight removed from the grower's farm, $R_{it}(H_{it}, \kappa_{0it})$ is the grower payment, w_F is the market price of feed

and $w_H(\kappa_{0it})$ is the market price of feeder pigs of weight κ_{0it} .

By deciding how many feeder pigs (H_{it}) of weight κ_{0it} to place on a grower's farm, the principal can vary the contract parameters $\tilde{\alpha}_{it}$ and $\tilde{\beta}_{it}$. As mentioned before, the contracts between the integrator and all agents have the same structure (summarized by the payment scheme (2)), but the allocation of integrator-supplied inputs among growers of different characteristics is not stipulated in the general contract and the integrator can choose them unilaterally in his dealings with each individual grower. Within the class of contractual payments that are observed in the data, varying the quantity and quality of production inputs across growers allows the integrator to use his bargaining power in designing individual incentive contracts for each grower. This approach will generate some predictions about the principal's "optimal choices".

As required by the theory, optimal contracts should depend on agent's preferences and her outside opportunities. In particular, the incentive power of the contract in a moral hazard environment should depend on the particular trade-off between risk sharing and incentives that depends on the agent's preferences, whereas the fixed component of the contractual payment should depend on the agent's reservation utility. As agent's preferences (risk aversion) and reservation utilities (depending on outside options and preferences) are likely to be heterogeneous, we expect that the principal will tailor particular incentive contracts according to the agent's types. The specification used enables the partitioning of the effects of risk aversion and reservation utilities into the constant and variable parts of the payment.

The problem faced by the integrator is to choose the contract parameters in order to maximize his profit under the incentive compatibility and individual rationality constraints of the agent. Assuming that reservation utilities are not time-varying, this can be formally described as follows:

$$\max_{H_{it}, \kappa_{0it}} E\pi_{it} = E[pQ_{it} - w_F F_{it} - R_{it}(H_{it}, \kappa_{0it}) - w_H(\kappa_{0it}) H_{it}] \quad (11)$$

subject to

$$EU_i(R_{it} - C(e_{it}^*)|\kappa_{0it}, H_{it}) \geq \bar{U}_i$$

and

$$e_{it}^* = \arg \max_{e_{it}} EU_i(R_{it}(H_{it}, \kappa_{0it}) - C(e_{it})|\kappa_{0it}, H_{it})$$

where $R_{it}(H_{it}, \kappa_{0it}) = \tilde{\alpha}_{it}(\kappa_{0it}, H_{it}) - \tilde{\beta}_{it}(H_{it})(f_{it}(e_{it}^*) - \phi)$ and \bar{U}_i is the reservation utility of agent i .

Using the certainty equivalent of the agent's utility like in Section 3.1, and incorporating the incentive constraint in the profit function of the principal by replacing the effort level by its optimal value, the principal's maximization program is thus equivalent to:

$$\max_{H_{it}, \kappa_{0it}} E\pi_{it}^*(H_{it}, \kappa_{0it}) = E[pQ_{it} - w_F F_{it} - R_{it}(H_{it}, \kappa_{0it}) - w_H(\kappa_{0it}) H_{it}] \quad (12)$$

subject to

$$W_i(R_{it}(H_{it}, \kappa_{0it})) \geq \bar{W}_i$$

where $\bar{W}_i = U_i^{-1}(\bar{U}_i)$, the function $W_i(\cdot)$ is defined as in (8), and π_{it}^* denotes the profit function that incorporates the optimal effort (given by the incentive constraint).

In order to characterize the principal's maximization program $\max_{H_{it}, \kappa_{0it}} E\pi_{it}^*(H_{it}, \kappa_{0it})$ we need to examine the functional forms of the cost function for feeder pigs $w_H(\kappa_0)$ and the mortality function $m_{it}(H)$. Towards this objective, we introduce two assumptions:

- Assumption 1: $w_H(\cdot)$ is increasing convex.

Assumption 1 is likely to be satisfied if $w_H(\kappa_0)$ reflects the cost of raising live animals to weight κ_0 because feed conversion rapidly worsens (increases) with heavier animals and therefore the feeding costs progressively increase as animals grow larger. Price data on different weights of feeder pigs show that this assumption is generally satisfied.

- Assumption 2: $m_{it}(H_{it})$ is increasing concave with $m''(1-m) + 2m'^2 \geq 0$ and $2m' + m''H > 0$.

In Assumption 2 we assume that the mortality rate function $m_{it}(H_{it})$ is such that the profit function has a unique maximum $(H_{it}^*(\tilde{\theta}_i), \kappa_{0it}^*(\tilde{\theta}_i))$. It is obvious that the number of animals placed on a grower's farm cannot be infinite given that the housing facilities are of finite size. The mortality rate will be increasing and necessarily approaching 100% when H approaches infinity. This implies that profits will obtain at a maximum for $H < \infty$.

If we label the number of animals shipped (i.e., the number of animals that survived the fattening process) as $H_{it}^s = (1 - m_{it}(H_{it}))H_{it}$, then the condition $2m' + m''H > 0$ is simply equivalent to assuming that the number of animals survived $H_{it}^s(H_{it})$ is a concave function of the number of animals placed H_{it} ($H_{it}^{s''}(H_{it}) < 0$). For example, this assumption is satisfied on $[0, 2\eta]$ with the mortality rate function¹⁰

$$m_{it}(H_{it}) = 1 - \exp\left(-\frac{H_{it}}{\eta}\right); \text{ with } \eta > 0. \quad (13)$$

Now we are in the position to state the following two results:

Proposition 1: The optimal decisions $(H_{it}^*(\tilde{\theta}_i), \kappa_{0it}^*(\tilde{\theta}_i))$ made by the integrator are such that $\frac{\partial \kappa_{0it}^*}{\partial \tilde{\theta}_i}$ is positive if and only if the elasticity of survived animals with respect to the efficiency parameter $\tilde{\theta}_i$ is larger than -1 , that is:

$$\frac{\partial \kappa_{0it}^*}{\partial \tilde{\theta}_i} > 0 \Leftrightarrow \frac{\partial \ln H_{it}^{s*}}{\partial \ln \tilde{\theta}_i} > -1$$

where $\frac{\partial \ln H_{it}^{s*}}{\partial \ln \tilde{\theta}_i} = \frac{\partial \ln[(1 - m_{it}(H_{it}^*))H_{it}^*]}{\partial \ln \tilde{\theta}_i}$.

Proof: See Appendix 6.1. \square

¹⁰In fact, the condition $2m' + m''H > 0$ is satisfied in this case if $H_{it} < 2\eta$. We will check empirically that η is sufficiently large compared to the range of values of H .

Proposition 2: If the following conditions are satisfied:

$$\begin{aligned}
p - \phi w_F + \alpha &> 0 \\
\phi w_F - \alpha - w'_H(\kappa_{0it}) &< 0 \\
\frac{\partial \ln \kappa_{0it}^*}{\partial \ln \tilde{\theta}_i} + \frac{\partial \ln}{\partial \ln \tilde{\theta}_i} \left(\frac{\partial}{\partial H_{it}^*} \left(\frac{1}{1 - m_{it}(H_{it}^*)} \right) \right) &< 1
\end{aligned}$$

then, the optimal decisions $(H_{it}^*(\tilde{\theta}_i), \kappa_{0it}^*(\tilde{\theta}_i))$ of the principal are such that

$$\frac{\partial H_{it}^*}{\partial \tilde{\theta}_i} < 0 \Rightarrow \frac{\partial \kappa_{0it}^*}{\partial \tilde{\theta}_i} > 0.$$

Proof: See Appendix 6.2. \square

Propositions 1 and 2 show that the optimal decisions of the integrator in terms of the number and weight of animals (feeder pigs) placed with contract growers must satisfy some equilibrium conditions such that, under some mild conditions that will be empirically verified, less efficient growers (either more risk averse or growers with a higher cost of effort) should either receive both fewer and heavier animals or more and lighter animals. This means that if less efficient growers are charged with less demanding task of tending for fewer animals, then they will also receive a "head start" in terms of their feeder pigs being heavier such that they will take less time to grow to market weight. This optimal strategy results from the principal's objective to minimize the total cost of moral hazard due to the delegation of tasks to heterogenous agents with their effort being unobservable.

In order to test these propositions, $\tilde{\theta}_i$ needs to be identified at least up to a scale. The results provide a test of the model since the structure can be rejected if, for example, $\frac{\partial H_{it}^*}{\partial \tilde{\theta}_i} < 0$ and $\frac{\partial \kappa_{0it}^*}{\partial \tilde{\theta}_i} < 0$, or if $\frac{\partial \kappa_{0it}^*}{\partial \tilde{\theta}_i}$ and $(1 + \frac{\partial \ln H_{it}^{s*}}{\partial \ln \tilde{\theta}_i})$ have opposite signs.

3.3 Theory principal behavior - Model 2

Recall that in Model 1 of the previous section, we ignored the workings of the growers' participation constraint. This could have been justified by either implicitly assuming that the

participation constraint is automatically satisfied for all growers who signed such contracts, or alternatively, by arguing that principals are not legally constrained to use any particular remuneration scheme for agents and therefore those payment schedules that we observe are in fact optimal.

When explicitly assuming that contracts are optimal, one has to determine whether the participation constraint is binding or not. If the principal can only choose κ_{0it} and H_{it} to maximize profit and if he has to use the payment formula in (2), then there is no reason for the participation constraint to be binding. Actually, one can see that the choice of κ_{0it} and H_{it} moves the parameters of the linear payment $\tilde{\alpha}_{it}$ and $\tilde{\beta}_{it}$ the same way as the principal could do by choosing $\tilde{\alpha}_{it}$ and $\tilde{\beta}_{it}$ directly, but unlike in the standard principal-agent models, κ_{0it} and H_{it} also change some other component of the principal's profit function. However, if manipulating the choice variables makes the participation constraint not binding, the principal can easily make it binding by adding or subtracting a fixed transfer to the agent's revenue R_{it} . Adding such a constant does not change the incentive constraint (as shown by the expression for the optimal effort (9)), thus the principal can perform the maximization program by incorporating only the incentive constraint and then ask for a fixed transfer from the agent in case the participation constraint is not binding.

Therefore, since we exactly observe the contract agreed between the principal and the agent, assuming that the observed contract is optimal, one can deduce that the optimal solution of the program (11) is such that $(H_{it}^*, \kappa_{0it}^*) = \arg \max_{H_{it}, \kappa_{0it}} E\pi_{it}^*(H_{it}, \kappa_{0it})$ and that the participation constraint is binding

$$W_{it}(R_{it}(H_{it}^*, \kappa_{0it}^*), e_{it}^*) = \overline{W}_i. \quad (14)$$

Replacing e_{it}^* by its analytical expression from (9) we obtain the expression for the certainty

equivalent measure of an agent's utility

$$\begin{aligned}
W_{it}(R_{it}(H_{it}, \kappa_{0it}), e_{it}^*) &= \tilde{\alpha}_{it} - \tilde{\beta}_{it}[Ef_{it} - \phi] - \frac{\theta_i}{2}\tilde{\beta}_{it}^2 Var[f_{it}] - \gamma_i \tilde{\beta}_{it} e_{it}^* \\
&= \tilde{\alpha}_{it} + \frac{(1 - \gamma_i)^2}{2\sigma^2\theta_i} - \gamma_i \lambda \tilde{\beta}_{it},
\end{aligned} \tag{15}$$

that will be subsequently used to identify the heterogeneity in growers' reservation utilities.¹¹

4 Identification and Estimation Results

Using the panel data described before, we can now estimate the structural model we developed so far. Remark that the identification that follows is obtained because of the variation of contracts across periods for a given grower in addition to the variation across growers. Actually, the number and weight of animals provided to growers at each period is varying for a given grower and thus make the payment scheme vary through variations of $\tilde{\alpha}_{it}$ and $\tilde{\beta}_{it}$. We assume that this variation is exogenous (shocks u_{it} in production function are uncorrelated with $\tilde{\alpha}_{it}$ and $\tilde{\beta}_{it}$) and comes from the upstream variations in the total demand for the Principal.

Then, substituting (9) in (7) yields the formula for the difference between the benchmark feed conversion ϕ and the equilibrium feed conversion f_{it}^* as

$$\phi - f_{it}^* = \frac{1 - \gamma_i}{\tilde{\beta}_{it}\sigma^2\theta_i} u_{it} \tag{16}$$

which by taking logs gives the following equation

$$\ln((\phi - f_{it}^*) \tilde{\beta}_{it}) = -\ln(\sigma^2\tilde{\theta}_i) + \ln(u_{it}). \tag{17}$$

As $(\phi - f_{it}^*) \tilde{\beta}_{it}$ is observed, the individual level parameters $\ln(\sigma^2\tilde{\theta}_i)$ in (17) can be estimated with a linear regression including growers fixed effects. Moreover, as $\sigma^2 = var(u_{it})$, we can

¹¹Of course, the identification of Model 2 critically depends on our assertion that in the empirically observed contracts the transfer payments are exactly zero. Based on our substantial practical knowledge of this industry, we are reasonably confident that this claim is correct. However, the existence of some side payments that are not reported on contract settlement sheets would render our identification of reservation utilities invalid.

identify σ^2 as the variance of the exponential of the error terms, which leads to the identification of the efficiency parameter $\tilde{\theta}_i = \frac{\theta_i}{1-\gamma_i}$. As $0 < \gamma_i < 1$, implies that $\theta_i < \tilde{\theta}_i$, $\tilde{\theta}_i$ allows the identification of an upper bound of the risk aversion parameter θ_i . Once the estimates of $\tilde{\theta}_i$ are known, one can test for the heterogeneity of grower types (their risk aversions normalized by the cost of effort). Keep in mind that the identification of $\tilde{\theta}_i$ comes from the assumption that shocks u_{it} are uncorrelated with the heterogeneity parameter and that the growers' efficiency parameters $\tilde{\theta}_i$ are constant across time and contracts.

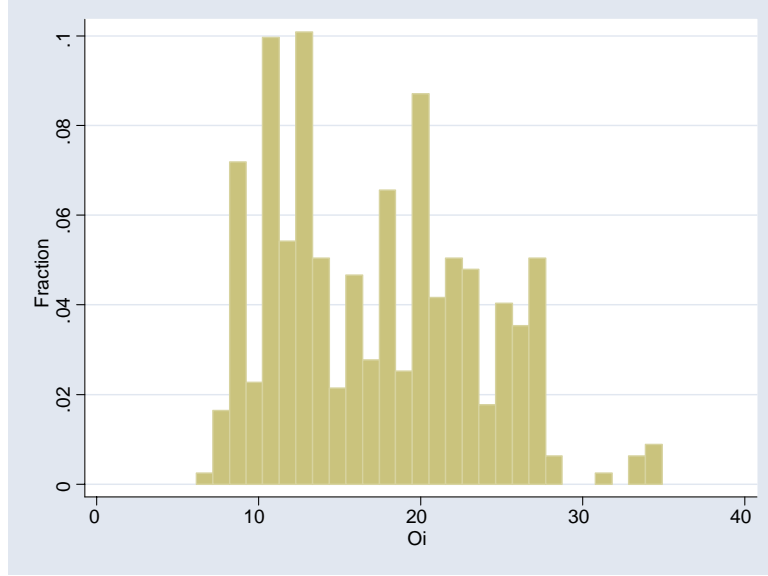
The estimation of (17) shows that the unexplained variance accounts for around 50% of the total variance. We do not show the full set of parameter estimates but testing that all $\ln(\tilde{\theta}_i)$ are equal strongly rejects the homogeneity of growers with respect to their risk aversions or the costs of effort ($F(121, 680) = 5.34$). Remark that even when we remove the set of growers that have the fewer number of contracts in the data, our main results stay the same. The distribution of efficiency parameters $\tilde{\theta}_i$ displayed in Figure 1 is characterized by the fact that the median efficiency parameter equals 17.32 while the 25th percentile value of the distribution equals 12.0 and the 75th percentile equals 21.84. In addition to showing substantial heterogeneity of grower types, the results also indicate that the estimated heterogeneity parameters are negatively correlated with the size of the growers' production facilities. The average of $\tilde{\theta}_i$ for growers that operates only one housing unit is 21.4, whereas for others (who have mostly 2 or 3 houses), it is 12.9.¹²

4.1 Performance

Our next objective is to test whether the theoretical implications of the model are consistent with the data. We first check whether the sufficient conditions on the mortality function

¹²As suggested by one referee, it would be interesting to see how the estimated risk aversion parameters are correlated with other growers' characteristics. However, in our data set, growers appear only with their ID numbers, so, other than the size of their production facilities (number of houses), we don't have their socio-economic characteristics.

Figure 1: Distribution of Estimated $\tilde{\theta}_i$



$m_{it}(H_{it})$ that we introduced in Assumption 2 are satisfied. The data does not allow us to estimate function $m(\cdot)$ and its first and second derivatives non-parametrically because the sample size is not large enough for such a demanding estimation but one can use the parametric form (13) for mortality from which it follows that

$$H_{it} = -\eta \ln(1 - m_{it})$$

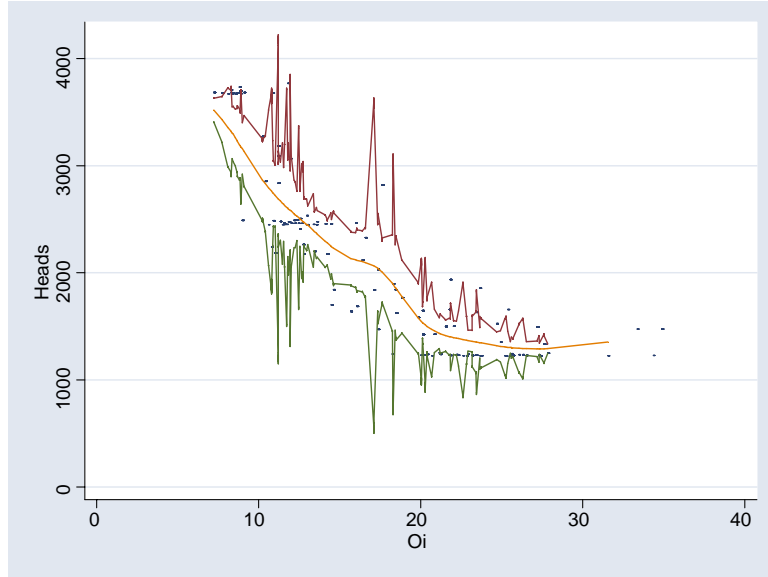
and then estimate the parameter η by least squares.¹³ The results show that $\hat{\eta} = 26,300$ (with the standard error of 445) and the functional fit is quite good with $R^2 = 79\%$. When estimating η 's that vary across feeder pigs type, the R^2 goes up to 85% while the estimates of η are 26,000 (s.e. 638); 27,300 (s.e. 724); and 15,100 (s.e. 708) for the three different types of animals. Notice that for the mortality function in (13), the assumption that led to our Propositions, i.e., $2m' + m''H > 0$ is satisfied if $H < 2\eta$. Since the observed values of H_{it} are between 1,100 and 1,500 per house, this condition is easily satisfied. Controlling for the

¹³Here we assumed that the mortality rate is exogenous to the number of animals placed, i.e., no other unobserved factors jointly affect the number of feeder pigs placed and their subsequent mortality.

density of animals in the housing facilities, the prediction of the mortality rate is even better showing that mortality and H are almost deterministically related.

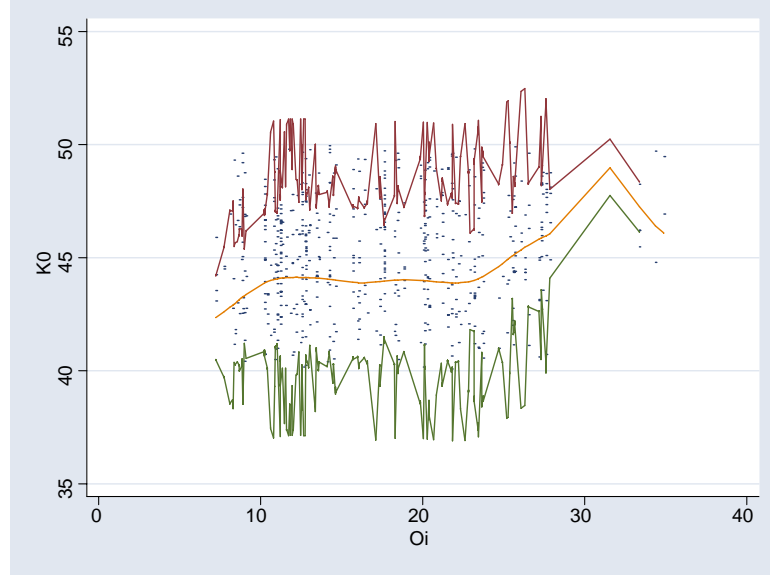
Next, using the structural estimates of adjusted risk aversion parameters $\tilde{\theta}_i$, we want to test the main propositions of the paper. We want to test whether the integrator supplies more feeder pigs to less risk averse growers by looking at the relationship between H_{it} and $\tilde{\theta}_i$. First, non-parametric tests of independence between H_{it} and $\tilde{\theta}_i$, or the average over contracts of H_{it} for grower i and $\tilde{\theta}_i$ show that independence is strongly rejected. The Spearman rank correlation coefficient is negative and strongly significant. Next, a non-parametric estimate of $E(H_{it} | \tilde{\theta}_i)$ obtained by using a standard kernel regression method (shown in Figure 2) clearly indicates that $E(H_{it} | \tilde{\theta}_i)$ is a strictly decreasing function of $\tilde{\theta}_i$, and so does a linear regression model (whose results are not reported here).

Figure 2: Non parametric estimate of $E(H_{it} | \tilde{\theta}_i)$ and confidence interval



Next, the elasticity of the number of animals placement with respect to adjusted risk aversion is uniquely identified. A non parametric estimation of $E(\ln H_{it} | \ln \tilde{\theta}_i)$ shows that

Figure 3: Non parametric estimate of $E(\kappa_0 | \tilde{\theta}_i)$ and confidence interval



we cannot reject that this function is linear (see Appendix 6.4) and the linear regression gives the estimate $\frac{\partial E(\ln H_{it} | \ln \tilde{\theta}_{ii})}{\partial \ln \tilde{\theta}_i} = -0.84$ with a robust standard error of 0.02. This result shows that a 10% increase in absolute risk aversion results in a 8.4% decrease in the number of animals that the integrator would place on the grower's farm. Based on Proposition 2, this result suggests that the weight of feeder pigs should increase with growers' risk aversion. The result is confirmed by looking at the elasticity of survived animals with respect to risk aversion, i.e., $\frac{\partial E(\ln H_{it}^s | \ln \tilde{\theta}_i)}{\partial \ln \tilde{\theta}_i} = -0.85(0.02) > -1$, which based on Proposition 1, says that the weight of the incoming feeder pigs (κ_0) that the integrator places on a grower's farm would increase with risk aversion if and only if the elasticity of survived animals with respect to $\tilde{\theta}_i$ is greater than -1 . A non-parametric estimate of the weight of incoming feeder pigs conditional on the risk aversion parameter shown in Figure 3 seems to indicate that $E(\kappa_{0it} | \tilde{\theta}_i)$ is an increasing function of $\tilde{\theta}_i$. In fact, a parametric estimate of the elasticity provide quite precise results since $\widehat{\frac{\partial \ln \kappa_{0it}}{\partial \ln \tilde{\theta}_i}} = 0.04(0.01)$.

4.2 Cost of moral hazard

The welfare cost of moral hazard emanates from the observability problem and the fact that contract growers are risk averse and face uncertain income streams. The volatility of income constitutes a direct real cost to growers and can be thought of as the cost of moral hazard in the sense that without moral hazard, integrators could pay growers constant wages to compensate them for their effort in case effort were observable and verifiable. However, obtaining the exact welfare estimates of the cost of moral hazard is impossible because the marginal cost of effort (γ_i) and the absolute risk aversion coefficient (θ_i) are not identified. Nevertheless, it is interesting to look at the relationship between the mean and the variance of growers' revenues and their adjusted risk aversion parameters $\tilde{\theta}_i$. First, 60% of the variance of total payments to growers, R_{it} , is explained by the *between-growers variance*. Second, a linear regression shows a significant negative relationship between the *within-grower variance* (estimated for each grower along the time dimension of the panel data) and adjusted risk aversion $\tilde{\theta}_i$. Also, the mean payment is significantly decreasing with adjusted risk aversion. The grower level variability of income is such that the average standard deviation is \$3,960 with a median of \$2,856. The above results point out that the cost of moral hazard to growers is likely to be substantial.

Moreover, it is important to note that the costs of asymmetric information arise not only from the fact that part of the performance risk (in terms of feed conversion) has to be borne by growers (because they have to be given the correct incentives), but also from the fact that the integrator allocates different number of animals to different growers according to their types. We anticipate that more risk averse growers (or growers with higher cost of effort) would have lower revenues because, *ceteris paribus*, they perform worse in terms of the feed conversion ratio (which reduces their bonus payment), but also because they receive fewer animals compared to the less risk averse growers.

Notice however that the relationship between grower risk aversion and his expected revenue is theoretically ambiguous. Looking at the equilibrium effort equation (9), it follows that the optimal effort decreases with higher risk aversion but also with $\tilde{\beta}$ and hence H_{it} . Therefore, since more risk averse growers receive fewer animals (H_{it}), the overall comparative statics effect of risk aversion on the unconditional optimal effort and hence on the expected revenue is undetermined.

The empirical results show that the revenues of more risk-averse growers are less volatile but, also, on average lower. Table 2 shows the average of the means and standard deviations of each grower's revenue R_{it} for different percentiles of the distribution of $\tilde{\theta}_i$. Except for the 50-60 percentiles of the distribution, the relationship shows a negative link between the mean and the variance of grower revenue and adjusted risk aversion. This empirical result shows that the net effect of adjusted risk aversion on revenue is negative. This net effect is a combination of the indirect effect of $\tilde{\theta}_i$ on the equilibrium values of H and κ_0 via the fixed component and the incentive power of the payment, and the direct effect of $\tilde{\theta}_i$ on performance through effort provision.

4.3 Heterogeneity in reservation utilities

Referring back to expressions for contract parameters (4) and (5), the measurement error in the weight of animals at the end of the production period implies that $\tilde{\alpha}_{it}$ is observed with an error but not $\tilde{\beta}_{it}$. Let's assume that κ_{it} is thus measured with an i.i.d. error ε_{it} that is supposed to be uncorrelated with κ_{0it} and H_{it} . The observed weight of finished animals is therefore $\widetilde{\kappa}_{it} = \kappa_{it} + \varepsilon_{it}$ and then the observed fixed component of the payment is $\tilde{\alpha}_{it}^* = \tilde{\alpha}_{it} + \varsigma_{it}$ where $\varsigma_{it} = \alpha(1 - m_{it})H_{it}\varepsilon_{it}$ because

$$\tilde{\alpha}_{it}^* = \tilde{\alpha}_{it} + \varsigma_{it} = \alpha[\widetilde{\kappa}_{it}(1 - m_{it}) - \kappa_{0it}]H_{it} = \alpha[\kappa_{it}(1 - m_{it}) - \kappa_{0it}]H_{it} + \alpha(1 - m_{it})H_{it}\varepsilon_{it}.$$

Table 2: Risk Aversion and Revenue

Distribution of $\tilde{\theta}_i$ percentiles (interval)	Mean R_{it} (in US\$)	Average Per Grower Standard Deviation of R_{it}
0-10% (6.1-11.2)	32 709	6 491
10-20% (11.2-12.2)	25 087	5 914
20-30% (11.2-12.5)	23 623	3 969
30-40% (12.5-14.2)	21 227	3 195
40-50% (14.2-17.3)	17 947	2 197
50-60% (17.3-19.8)	18 408	5 971
60-70% (19.8-21.1)	12 906	2 570
70-80% (21.1-23.1)	12 651	3 164
80-90% (23.1-26.2)	11 466	1 999
90-100% (26.2-52.9)	10 995	1 949

It follows that (15) becomes

$$W_{it}(R_{it}(H_{it}^*, \kappa_{0it}^*), e_{it}^*) = \tilde{\alpha}_{it}^* + \frac{(1 - \gamma_i)^2}{2\sigma^2\theta_i} - \gamma_i\lambda\tilde{\beta}_{it}^* - \varsigma_{it}.$$

Taking into account the fact that the participation constraint (14) is binding, we obtain that

$$\tilde{\alpha}_{it}^* = \Omega_i + \gamma_i\lambda\tilde{\beta}_{it}^* + \varsigma_{it} \quad (18)$$

with $E(\varsigma_{it}|\kappa_{0it}^*, H_{it}^*, \Omega_i) = \alpha(1 - m_{it})H_{it}E(\varepsilon_{it}|\kappa_{0it}^*, H_{it}^*, \Omega_i) = 0$ and $\Omega_i = \overline{W}_i - \frac{(1-\gamma_i)^2}{2\sigma^2\theta_i}$.

Remark that this expression comes from the certainty equivalent utility of the agent (15) and the assumption of binding participation constraint (14). It allows to identify the reservation utility of the agent thanks to variations in the contractual terms $(\tilde{\alpha}_{it}, \tilde{\beta}_{it})$ across contracts for a given grower (in addition to the variation across growers). As already stated, we assume that the variation in contracts for a given grower is exogenous and comes from varying total demand at the upstream level for the principal, implying that the Principal varies the total number of animals raised per period.

Then, with data on performance f_{it} , $\tilde{\alpha}_{it}^*$, κ_{0it} , H_{it}^* , and $\tilde{\beta}_{it}^*$, equation (18) allows, in principle, the identification of all coefficients $\gamma_i \lambda$ if one has an infinite number of contracts per grower. However, the number of contracts per grower will be small relative to the number of growers and the estimates of $\gamma_i \lambda$ based on short time series of contracts per grower will be very imprecise. Thus we prefer to introduce an additional assumption that all growers have the same cost of effort, namely that $\forall i : \gamma_i = \gamma$, then the parameter $\gamma \lambda$ is identified with (18) and we can test the implications of the following proposition (remark that now $\tilde{\theta}_i = \frac{\theta_i}{1-\gamma}$ and thus $\tilde{\theta}_i$ and θ_i can be confounded up to a scale factor):

Proposition 3:

- The agent's reservation utility is a weighted sum (with unknown weights $\gamma \in (0, 1)$) of Ω_i identified from (18) and $\Psi_i = \frac{1}{2\sigma^2\theta_i}$ identified from (17) using performance data:

$$\overline{W}_i = \Omega_i + (1 - \gamma) \Psi_i.$$

- A lower bound \overline{W}_{\inf}^i and upper bound \overline{W}_{\sup}^i on the reservation utility of agent i , \overline{W}_i , are identified because

$$\Omega_i = \overline{W}_{\inf}^i \leq \overline{W}_i \leq \overline{W}_{\sup}^i = \Omega_i + \Psi_i. \quad (19)$$

- If $\Omega_i(\tilde{\theta}_i)$ is decreasing in $\tilde{\theta}_i$, then one can reject that $\overline{W}_i(\tilde{\theta}_i)$ is increasing in $\tilde{\theta}_i$ (even weakly).

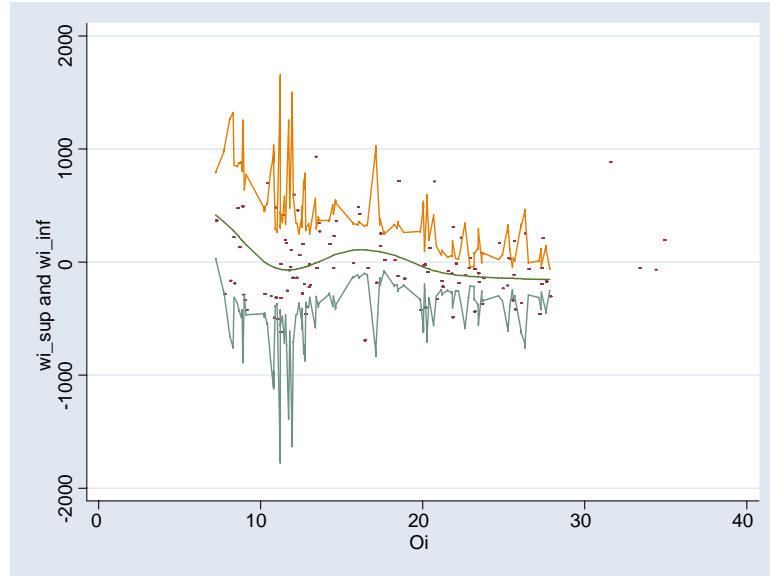
Proof: See Appendix 6.3. \square

Proposition 3 shows that invoking the assumption that contracts are indeed optimal allows the identification of a lower and an upper bound on the agents' reservation utility. This enables testing the model restriction $\gamma \lambda > 0$, and exploring the correlation between Ω_i and $\tilde{\theta}_i$, as well as the relationship between \overline{W}_{\inf}^i , \overline{W}_{\sup}^i and $\tilde{\theta}_i$.

To address the issue of growers' reservation utilities we estimate equation (18) with ob-

servations on $\tilde{\alpha}_{it}^*$ and $\tilde{\beta}_{it}^*$.¹⁴ Using generalized least squares, we obtain consistent estimates of $\{\Omega_i\}_{i=1,\dots,I}$ and $\gamma\lambda$. The estimate of $\gamma\lambda$ shows a significant and positive value ($\widehat{\gamma\lambda} = 0.80$ (0.006)), indirectly confirming the validity of the model. Recall that both the cost of effort and the ability parameters need to be positive, so their estimated positive product does not reject the model. An F test that all Ω_i are equal strongly rejects the null hypothesis, with $F(121, 679)=16.11$ and $p\text{-value}=0.000$. Also, remembering that the parameter γ could be grower specific, we estimated equation (18) using grower specific coefficients for $\tilde{\beta}_{it}^*$. Unfortunately, due to insufficient number of observations all these coefficients are imprecisely estimated. A test of homogeneity across individuals is not rejected but not very convincingly given the test's lack of power.

Figure 4: Nonparametric estimate of $E(\overline{W}_{\text{inf}}^i | \theta_i)$ and $E(\overline{W}_{\text{sup}}^i | \theta_i)$ with confidence intervals



¹⁴ Assuming that the reservation utility of a given farmer is constant over time, whereas the offered contract terms $\tilde{\alpha}_{it}^*$ and $\tilde{\beta}_{it}^*$ vary for a given farmer across time, are crucial aspects of our identification strategy. Such within-farmer variance in contract terms is coming from shocks to the principal's costs as prices of both feeder pigs and feed grains vary over time.

With the obtained estimates, we look at the relationship between Ω_i and θ_i . A linear regression shows that they are strongly negatively correlated; the same goes for the relationship between \overline{W}_{\inf}^i or \overline{W}_{\sup}^i and θ_i . A non parametric estimate of the relationship between Ω_i and θ_i shows that it is clearly decreasing. Since Ω_i consists of two components, the reservation utility \overline{W}_i and $-(1 - \gamma)\Psi_i$ which is increasing in θ_i , it follows that the reservation utility \overline{W}_i has to be decreasing in θ_i . This result implies that agents with higher risk aversion have lower reservation utilities because of lower outside opportunities. Figure 4 shows a non parametric estimate of the upper and lower bound estimates of the reservation utility¹⁵.

Finally, notice that if we considered the fact that agents could take into account the riskiness of the final weight of animals κ_{it} , then we should have modified the agent's revenue certainty equivalent by adding the mean-variance value of this additional risk denoted as η_{it} . Assuming that this random shock is of mean zero and constant variance across agents, the agent's certainly equivalent revenue (15) would become

$$W_{it}(R_{it}(H_{it}^*, \kappa_{0it}^*), c_{it}^*) = \tilde{\alpha}_{it}^* + \frac{(1 - \gamma)^2}{2\sigma^2\theta_i} - \gamma\lambda\tilde{\beta}_{it}^* - \theta_i var[\eta_{it}]$$

One can show easily that in this new model, Ω_i would become $\Omega_i = \overline{W}_i - \frac{(1-\gamma)^2}{2\sigma^2\theta_i} + \theta_i var[\eta_{it}]$. Although, this approach would weaken the possibility to identify the agents' reservation utilities (because the absolute value of θ_i is not identified), the additional term $\theta_i var[\eta_{it}]$ being increasing in θ_i , would reinforce the fact that \overline{W}_i has to be decreasing in θ_i when Ω_i decreases in θ_i , which has been empirically confirmed. Thus, this additional complexity would confirm the negative relationship between risk aversion θ_i and reservation utility Ω_i .

¹⁵The reason why it seems that only one curve appears on the graph is that, given the scale, these two curves are extremely close. This is due to the fact that the term Ψ_i which is the difference between $\Omega_i = \overline{W}_{\inf}^i$ and \overline{W}_{\sup}^i is very small compared to Ω_i .

5 Conclusion

In this paper we studied the question of optimal contracting under moral hazard when agents are heterogenous. In this case, heterogeneity calls for individually designed contracts, which stands in sharp contrast to what have been frequently observed in the real world. The examples of principals using seemingly uniform contracts when dealing with heterogenous agents are found in many agricultural sectors, particularly in livestock production contracts for broilers, turkeys, and hogs. Two main elements of all agricultural production contracts are the payment mechanism and the division of responsibilities for providing inputs. The payment mechanism consists almost always of a variable piece rate with bonuses for the efficient use of the principal-supplied inputs and is always the same for all agents. However, contracts never specify the quantity and quality of the integrator-supplied inputs to each grower. We show that the observed contracts are only nominally uniform, and that the principals are using their discretion when it comes to matching inputs with agents of different characteristics (risk aversion and/or cost of effort). Using this variation in contract variables, the principal in fact manages to design the individualized contracts that are tailored to fit the individual growers' preferences or characteristics.

The paper has two conceptually distinct parts. In the first part (Model 1) we develop an analytical framework for the econometric estimation of the heterogeneity parameter (risk aversion corrected by the cost of effort) of the contracting producers and carry out its empirical estimation using the individual growers performance data from the swine industry contracts. We found that contract farmers are heterogenous with respect to their risk aversions (and/or cost of effort) and that this heterogeneity affects the principal's allocation of production inputs across farmers. This results fits well with some of the earlier findings from the channel contract behavior literature. For example, the difference in farmers' risk attitudes has been discovered by Pennings and Wansink (2004) who found wide variation in risk attitudes among Dutch

hog producers: 39% were risk averse, 4% risk neutral, and 57% were risk seeking. The main characteristic of this part of the paper is that it takes the observed contract as given and model the behavior of the agents under the observed contractual terms without using any optimality argument about the contract design.

The obtained results are then used to look at the cost of moral hazard associated with this heterogeneity of grower types. We show that the costs of asymmetric information arise not only from the fact that part of the performance risk has to be borne by growers (because they have to be given the correct incentives to perform), but also from the fact that the integrator allocates different number of animals to different growers according to their efficiency. More risk averse growers (or those with higher cost of effort) will have lower expected revenues because on average they perform worse, but also because they receive fewer animals compared to the less risk averse growers (or those with lower cost of effort). These results were confirmed in a variety of different empirical tests. They provide evidence about the risk sharing - incentives trade-off underlying contractual relationships under moral hazard and uncertainty.

In this context an interesting question would be whether the principal could diversify risk for growers with payments indexed to some pooled performance, which would seem to be cheaper than choosing the right number and size of hogs for each farm.¹⁶ In fact, tournament-type payments are frequently used in production contracts where the common shock is important. For example, the contracts for production of broiler chickens are almost exclusively settled via cardinal tournaments. In such a scheme the payment to the contract grower consists of a base payment per pound of live weight plus the bonus payment that is determined by comparing an individual grower's performance (say feed conversion) to the group average performance. The average performance is determined by all growers whose chickens were harvested within the same week. Since the main reason for the use of tournaments is the elimination of the

¹⁶This interesting question was raised by one of the referees.

common production shock, it is important that the groups are formed such that all growers in the same group are exposed to the same common shock (temperature and humidity are very important determinants of the chicken metabolism). Having said this, it is obvious why the tournament-type settlements are never used in hog production contracts. First, the cycle is too long (19-20 weeks) and the scale of operation tends to be large, so the principal contracts with fewer growers which makes the assembly of groups that would be exposed to the same common shock difficult. Second, the available empirical evidence shows that common shock dominates the idiosyncratic shock in the production of chickens (see Levy and Vukina, 2004), however some preliminary empirical evidence points to the conjecture that the result is opposite in the production of hogs.

In the second part of the paper (Model 2) we use both the assumption of contract optimality and the fact that the contract payments are accurately observed in the data. Using the contract optimality assumption as an identifying restriction, we were able to obtain estimates of the bounds on agents' reservation utilities (although point estimates are not obtained because the cost parameter remains unidentified). We show that farmers with higher risk aversion have lower outside opportunities and hence lower reservation utilities. The obtained results fit well with some of the earlier literature on unemployment. An inverse relationship between the degree of risk aversion and the level of reservation wages has been backed up by experimental evidence on individual job search behavior (Cox and Oaxaca, 1992), and more recently, with empirical data (Pannenberg, 2007).

Finally, some interesting research directions can be outlined. Given access to adequate data, adding the problem of adverse selection to the existing problem of moral hazard would present an interesting extension. Although the assumption that the principal can perfectly observe agents' types in this industry seems realistic, given the repetitive nature of contracting between the principal and the same group of agents, the question of endogenizing the distribution of

agent types willing to contract with the principal would be interesting. This would amount to allowing agents to choose between different types of contracts. For example, keeping the research focus on contracting in agriculture, it would be interesting to look into choices that farmers make when deciding to specialize in the contract production of various types of animals or crops. One example could be signing a contract for the production of hatching eggs or the production of broiler chickens in cases where both contracts are offered by the same integrator in the same area. Another example may be in the swine sector where the choices can be made among signing a contract for the production of finished hogs, versus signing a farrow-to-finish, or a wean-to-finish contract. Besides the methodological difficulties, the main problem with this type of research is to find data on multiple contracts settlements from the same geographical area. With appropriate data one could fully analyze the initial matching between agents characteristics and the types of activity, making the distribution of agents's preferences and reservation utilities endogenous. This new step in the empirical research on contract theory will help understand the full industry structure of vertical contracts in many areas of agriculture and beyond.

References

Abbring, J.H., P.A. Chiappori and J. Pinquet (2003). "Moral hazard and dynamic insurance data," *Journal of the European Economic Association* 1(4), 767-820.

Abbring, J.H., P.A. Chiappori, J.J. Heckman and J. Pinquet (2003). "Adverse selection and moral hazard in insurance: Can dynamic data help to distinguish?" *Journal of the European Economic Association, Papers and Proceedings*, 1(2-3), 512-521.

Akerberg, D.A. and M. Botticini (2002). "Endogenous Matching and the Empirical Determinants of Contract Form," *Journal of Political Economy* 110 (3), 564-591.

Allen, D.W. and D. Lueck (1992). "Contract Choice in Modern Agriculture: Cash Rent versus Cropshare," *Journal of Law and Economics* 35 (October), 397-426.

Chiappori, P.A., F. Durand and P.Y. Geoffard (1998). "Moral hazard and the demand for physician services: First lessons from a French natural experiment - Lessons from the Rand Health Insurance Experiment," *European Economic Review* 42 (3), 499-513.

Chiappori, P.A. and B. Salanié (2003). "Testing Contract Theory: A Survey of Some Recent Work", in M. Dewatripont, L. Hansen, and P. Turnovsky, eds. *Advances in Economics and Econometrics - Theory and Applications*, 8th World Congress, Econometric Society Monographs. Cambridge University Press, Cambridge, pp. 115-149.

Cox, J.C. and R.L. Oaxaca (1992). "Direct Tests of the Reservation Wage Property," *Economic Journal* 102, 1423-1432.

Dubois, P. (2002). "Moral Hazard, Land Fertility and Sharecropping in a Rural Area of the Philippines," *Journal of Development Economics* 68 (1), 35-64.

Dubois, P. and T. Vukina (2004). "Grower Risk Aversion and the Cost of Moral Hazard in Livestock Production Contracts," *American Journal of Agricultural Economics* 86 (August), 835-841.

Ferrall C. and B. Shearer (1999) "Incentives and Transactions Costs within the Firm:

Estimating an Agency Model using Payroll Records," *Review of Economic Studies*, 66, 309-338.

Leegomomchai, P. and T. Vukina (2005). "Dynamic Incentives and Agent Discrimination in Broiler Production Tournaments," *Journal of Economics and Management Strategy* 14 (4), 849-877.

Levy, A. and T. Vukina (2002). "Optimal Linear Contracts with Heterogeneous Agents," *European Review of Agricultural Economics* 29, 205-217.

Levy A. and T. Vukina (2004). "League Composition Effect in Tournaments with Heterogeneous Players: An Empirical Analysis of Broiler Contracts." *Journal of Labor Economics* 22, 353-377

Martin, L. L. (1997). "Production Contracts, Risk Shifting, and Relative Performance Payments in the Pork Industry," *Journal of Agricultural and Applied Economics*, 29 (December), 267-278.

Pannenberg, M. (2007). "Risk Aversion and Reservation Wages," *The Institute for the Study of Labor (IZA)*, Discussion paper No. 2806, Bonn, Germany.

Paarsch, H. J. and B. Shearer (2000). "Piece Rate, Fixed Wages, and Incentive Effects: Statistical Evidence from Payroll Records," *International Economic Review* 41 (1), 59-92.

Paarsch, H. J. and B. Shearer (2007). "Do women react differently to incentives? Evidence from experimental data and payroll records." *European Economic Review* 51 (7): 1682-1707.

Pennings and Smidts (2000). "Assessing the Construct Validity of Risk Attitude." *Management Science* 46, 1337-1348.

Pennings and Wansink (2004). "Channel Contract Behavior: The Role of Risk Attitudes, Risk perceptions, and Channel Members' Market Structures." *Journal of Business*, 77, 697-723.

Prendergast, C. (1999). "The provision of incentives in firms," *Journal of Economic*

Literature, 37, 7-63.

Prendergast, C. (2002). "The tenuous trade-off between risk and incentives," *Journal of Political Economy* 110 (5), 1071-1102.

Shearer, B. (2004). "Piece Rates, Fixed Wages and Incentives: Evidence from a Payroll Experiment," *Review of Economic Studies*, 71 (2), 513-534.

USDA (2004). "National Direct Feeder Pig Report", USDA-Iowa Department of Agricultural Market News, Des Moines Iowa, various issues. http://www.ams.usda.gov/mnreports/NW_LS255.txt

6 Appendix

6.1 Proof of Proposition 1

Using (3) and the optimal grower effort (9), removing the argument of m_{it} for notational convenience, the integrator's expected profit becomes

$$\begin{aligned} E\pi_{it} &= p\kappa_{it}(1 - m_{it})H_{it} + w_F \left(\frac{1}{\tilde{\beta}_{it}\sigma^2\tilde{\theta}_i} - \phi + \frac{\alpha}{w_F} \right) [\kappa_{it}(1 - m_{it}) - \kappa_{0it}]H_{it} - w_H(\kappa_{0it})H_{it} \\ &= \left[p - w_F \left(\phi - \frac{\alpha}{w_F} \right) \right] \kappa_{it}(1 - m_{it})H_{it} - w_F \frac{\kappa_{0it}}{\beta(1 - m_{it})\sigma^2\tilde{\theta}_i} \\ &\quad + \left[w_F \left(\phi - \frac{\alpha}{w_F} \right) \kappa_{0it} - w_H(\kappa_{0it}) \right] H_{it} + w_F \frac{\kappa_{it}}{\beta\sigma^2\tilde{\theta}_i}. \end{aligned}$$

The first order condition for the integrator's expected profit maximization with respect to H_{it} and κ_{0it} are

$$\begin{aligned} \frac{\partial E\pi_{it}}{\partial H_{it}} &= 0 = \left[p - w_F \left(\phi - \frac{\alpha}{w_F} \right) \right] \kappa_{it} \left(\frac{\partial}{\partial H_{it}} [(1 - m_{it})H_{it}] \right) \\ &\quad - w_F \frac{\kappa_{0it}}{\beta\sigma^2\tilde{\theta}_i} \frac{m'_{it}}{(1 - m_{it})^2} + \left[w_F \left(\phi - \frac{\alpha}{w_F} \right) \kappa_{0it} - w_H(\kappa_{0it}) \right] \\ \frac{\partial E\pi_{it}}{\partial \kappa_{0it}} &= 0 = w_F \left[\left(\phi - \frac{\alpha}{w_F} \right) H_{it} - \frac{1}{\beta(1 - m_{it})\sigma^2\tilde{\theta}_i} \right] - w'_H(\kappa_{0it})H_{it} \end{aligned}$$

Taking derivative of the condition $\frac{\partial E\pi_{it}}{\partial \kappa_{0it}} = 0$ with respect to $\tilde{\theta}_i$ gives

$$\begin{aligned} \frac{\partial \kappa_{0it}}{\partial \tilde{\theta}_i} &= - \frac{w_F}{\beta\sigma^2 w''_H(\kappa_{0it})} \frac{\partial}{\partial \tilde{\theta}_i} \left[\frac{1}{\tilde{\theta}_i(1 - m_{it})H_{it}} \right] \\ &= \frac{w_F}{\beta\sigma^2 w''_H(\kappa_{0it}) (\tilde{\theta}_i(1 - m_{it})H_{it})^2} \left[(1 - m_{it})H_{it} + \tilde{\theta}_i \frac{\partial}{\partial H_{it}} [(1 - m_{it})H_{it}] \frac{\partial H_{it}}{\partial \tilde{\theta}_i} \right] \end{aligned}$$

Since $w''_H(\kappa_{0it}) > 0$, $\frac{\partial \kappa_{0it}}{\partial \tilde{\theta}_i}$ has the sign of

$$\begin{aligned} &(1 - m_{it})H_{it} + \tilde{\theta}_i \frac{\partial}{\partial H_{it}} [(1 - m_{it})H_{it}] \frac{\partial H_{it}}{\partial \tilde{\theta}_i} \\ &= (1 - m_{it})H_{it} \left[1 + \frac{\tilde{\theta}_i}{(1 - m_{it})H_{it}} \frac{\partial}{\partial H_{it}} [(1 - m_{it})H_{it}] \frac{\partial H_{it}}{\partial \tilde{\theta}_i} \right] \\ &= (1 - m_{it})H_{it} \left[1 + \frac{\partial \ln[(1 - m_{it})H_{it}]}{\partial \ln \tilde{\theta}_i} \right] \end{aligned}$$

Therefore $\frac{\partial \kappa_{0it}}{\partial \tilde{\theta}_i} > 0$ if and only if $\frac{\partial \ln[(1 - m_{it})H_{it}]}{\partial \ln \tilde{\theta}_i} > -1$. \square

6.2 Proof of Proposition 2

Taking the derivative of the first order condition $\frac{\partial E\pi_{it}}{\partial H_{it}} = 0$ with respect to $\tilde{\theta}_i$, we have

$$\begin{aligned} 0 = & [p - \phi w_F + \alpha] \kappa_{it} \frac{\partial^2}{\partial H_{it}^2} [(1 - m_{it}) H_{it}] \frac{\partial H_{it}}{\partial \tilde{\theta}_i} \\ & - \frac{w_F}{\beta \sigma^2} \frac{\partial}{\partial \tilde{\theta}_i} \left[\frac{\kappa_{0it}}{\tilde{\theta}_i} \frac{m'_{it}}{(1 - m_{it})^2} \right] + [\phi w_F - \alpha - w'_H(\kappa_{0it})] \frac{\partial \kappa_{0it}}{\partial \tilde{\theta}_i} \end{aligned} \quad (20)$$

Given Assumption 2, we know that the number of survived animals is a concave function of the number of placed animals, $\frac{\partial^2}{\partial H_{it}^2} [(1 - m_{it}) H_{it}] < 0$. Also, prices and parameters are such that $[p - \phi w_F + \alpha] > 0$ (confirmed by the data) and $[\phi w_F - \alpha - w'_H(\kappa_{0it})] < 0$, due to the properties of the cost function for feeder pigs. Notice that in order for the second condition to hold, it is sufficient that the marginal cost of producing feeder pigs be at least as large as the feeding cost (i.e. the target feed conversion ratio (ϕ) times the price of feed (w_F)). This assumption cannot be checked within the existing data set because, as explained before, the price data has been constructed from secondary sources and the feeder pigs prices are market averages across all weights that were transacted in that time period. However, a casual inspection of the feeder pig prices for various weight categories published by USDA (2004) confirms the assumption that $w'_H(\kappa)$ is large enough to offset the feed cost ϕw_F observed in our data. Having said this, we see that

$$\begin{aligned} \frac{\partial}{\partial \tilde{\theta}_i} \left[\frac{\kappa_{0it}}{\tilde{\theta}_i} \frac{m'_{it}}{(1 - m_{it})^2} \right] &= -\frac{\kappa_{0it}}{\tilde{\theta}_i^2} \frac{m'_{it}}{(1 - m_{it})^2} + \frac{1}{\tilde{\theta}_i} \frac{m'_{it}}{(1 - m_{it})^2} \frac{\partial \kappa_{0it}}{\partial \tilde{\theta}_i} + \frac{\kappa_{0it}}{\tilde{\theta}_i} \frac{\partial}{\partial H_{it}} \left[\frac{m'_{it}}{(1 - m_{it})^2} \right] \frac{\partial H_{it}}{\partial \tilde{\theta}_i} \\ &= \frac{\kappa_{0it}}{\tilde{\theta}_i^2} \frac{m'_{it}}{(1 - m_{it})^2} \left[-1 + \frac{\tilde{\theta}_i}{\kappa_{0it}} \frac{\partial \kappa_{0it}}{\partial \tilde{\theta}_i} + \tilde{\theta}_i \frac{(1 - m_{it})^2}{m'_{it}} \frac{\partial}{\partial H_{it}} \left[\frac{m'_{it}}{(1 - m_{it})^2} \right] \frac{\partial H_{it}}{\partial \tilde{\theta}_i} \right] \\ &= \frac{\kappa_{0it}}{\tilde{\theta}_i^2} \frac{m'_{it}}{(1 - m_{it})^2} \left[\frac{\partial \ln \kappa_{0it}}{\partial \ln \tilde{\theta}_i} + \frac{\partial \ln \frac{m'_{it}}{(1 - m_{it})^2}}{\partial \ln \tilde{\theta}_i} - 1 \right] \\ &< 0 \text{ if } \frac{\partial \ln \kappa_{0it}}{\partial \ln \tilde{\theta}_i} + \frac{\partial \ln \frac{m'_{it}}{(1 - m_{it})^2}}{\partial \ln \tilde{\theta}_i} < 1 \end{aligned}$$

Thus, $\frac{\partial H_{it}}{\partial \tilde{\theta}_i} < 0$ implies $\frac{\partial \kappa_{0it}}{\partial \tilde{\theta}_i} > 0$ as soon as $\frac{\partial \ln \kappa_{0it}}{\partial \ln \tilde{\theta}_i} + \frac{\partial \ln \left[\frac{m'_{it}}{(1 - m_{it})^2} \right]}{\partial \ln \tilde{\theta}_i} < 1$ (because also $\frac{w_F}{\beta \sigma^2} > 0$).

Empirically, the mortality function is such that $\frac{\partial \ln}{\partial \ln \tilde{\theta}_i} \left[\frac{m'_{it}}{(1-m_{it})^2} \right] = \frac{\partial \ln}{\partial \ln \tilde{\theta}_i} \left(\frac{\partial}{\partial H_{it}} \left(\frac{1}{1-m_{it}} \right) \right)$ is almost zero and the elasticity of κ_{0it} with respect to $\tilde{\theta}_i$ is $\widehat{\frac{\partial \ln \kappa_{0it}}{\partial \ln \tilde{\theta}_i}} = 0.04$ (0.01). The estimated mortality function is such that $\frac{m'_{it}}{(1-m_{it})^2}$ is very small. If the mortality function is such that we can cancel this term because $\frac{m'_{it}}{(1-m_{it})^2} \simeq 0$ then equation (20) implies that $\frac{\partial \kappa_{0it}}{\partial \theta_i}$ and $\frac{\partial H_{it}}{\partial \theta_i}$ will be of opposite signs. \square

6.3 Proof of Proposition 3

By definition of Ω_i

$$\overline{W}_i = \Omega_i + \frac{(1-\gamma_i)^2}{2\sigma^2\theta_i} = \Omega_i + (1-\gamma_i) \frac{1}{2\sigma^2 \frac{\theta_i}{1-\gamma_i}} = \Omega_i + (1-\gamma_i) \Psi_i$$

where $\Psi_i = \frac{1-\gamma_i}{2\sigma^2\theta_i} = \frac{1}{2\sigma^2\tilde{\theta}_i}$.

Then, as $0 < \gamma_i < 1$, we obtain the inequality (19). As Ω_i is identified from (18) and $\frac{1-\gamma_i}{\sigma^2\theta_i}$ is identified by (17), which also identifies $\Psi_i = \frac{1-\gamma_i}{2\sigma^2\theta_i}$, the bounds on inequality (19) are identified.

When $\gamma_i = \gamma$ for all i , $\overline{W}_i(\tilde{\theta}_i) = \Omega_i(\tilde{\theta}_i) + (1-\gamma) \Psi_i(\tilde{\theta}_i)$ where $0 < \gamma < 1$ and $\Psi_i(\tilde{\theta}_i)$ is a decreasing function of $\tilde{\theta}_i = \frac{\theta_i}{1-\gamma_i}$. Thus, one can reject that $\overline{W}_i(\theta_i)$ is increasing in θ_i (even weakly) if $\Omega_i(\theta_i)$ is non increasing in θ_i . \square

6.4 Additional results

Figure 5: Nonparametric estimate of $E\left(\ln H_{it} \mid \ln \tilde{\theta}_i\right)$

