

The Effects of Sin Taxes and Advertising Restrictions in a Dynamic Equilibrium

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Abstract

We develop a dynamic equilibrium model of firm competition to study the impact of counterfactual policies, such as taxes and advertising restrictions, on pricing, advertising, consumption and welfare. We estimate the model using micro level data on the market for colas. We use consumer level exposure to television commercials to estimate the impact of advertising on product choice, model firms’ dynamic competition through their choice of advertising budgets and product prices, and exploit firms’ practice of delegating decisions over advertising slots to agencies to link the rich consumer-level advertising variation with firms’ strategic choice variables. We show that a sugar-sweetened beverage tax leads to a reduction in advertising and that the incremental effects of implementing advertising restrictions are substantially reduced with a tax in place.

JEL codes: D12, H22, I18, L13, M37

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1 Introduction

Governments often seek to reduce consumption of sin goods, such as tobacco, alcohol, and sugar-sweetened beverages, by levying taxes on them. In response firms are likely to adjust their strategic choices, including their prices and potentially their advertising expenditures. As advertising can affect product demands both contemporaneously, and into the future, the introduction of sin taxes can have dynamic effects on the market equilibrium. With many sin goods, such as tobacco and alcohol, governments have implemented restrictions to advertising in addition to taxes.¹ However, there is little work that studies the interactions of taxes and advertising restrictions, or that accounts for the possibility that firms re-optimize advertising expenditures in response. The market for sugar-sweetened beverages is one where these considerations are relevant. Taxes that aim to reduce consumption (due to its association with obesity and diet-related disease) have been implemented in many jurisdictions,² and restrictions on advertising are increasingly being introduced.³

In this paper we study the equilibrium impacts of sin taxes and advertising restrictions, accounting for firms' dynamic supply-side responses over advertising expenditures. In order to overcome the challenge of solving a dynamic game in which players have a large action space – firms' advertising strategies are potentially complicated high-dimensional objects – we exploit the organization of the advertising market to develop a tractable framework. We estimate a model of consumer demand for the cola market, which is the segment of the broader non-alcoholic drinks market that accounts for the majority of advertising. We use the demand estimates, along with a dynamic supply-side model, to simulate the impact of policy changes on firms' choices over their optimal pricing and advertising budgets.

The UK cola market has two dominant firms – Coca Cola and Pepsico – and a few lower quality, cheaper store (i.e., private label) brand alternatives. Both Coca Cola and Pepsico advertise, while store brands are not advertised. We focus on television advertising, which is the form of advertising that accounts for the highest share of spend in the cola (and broader food and drinks) market. Coca Cola and Pepsico decide their monthly advertising budgets and delegate to an advertising agency the task of choosing specific slots to maximize the exposure of consumers to their advertising. The intermediary role played by advertising agencies is an institutional feature of the UK television advertising market (see Crawford

¹For instance, see DeCicca et al. (2022).

²Widely known as soda taxes, as of April 2021 over 50 different jurisdictions had introduced taxes on soft drinks (GFRP (2021)).

³For example, in the US and UK advertising of junk foods to children is restricted. In the UK wider restrictions were going through the legislative process but were put on hold during the COVID-19 pandemic.

et al. (2017)) and is common to other advertising markets.⁴ It provides a link between the rich variation in advertising exposure of consumers and drinks firms’ strategic decision over total monthly advertising expenditure. The agencies therefore play the role of simplifying the dynamic advertising game played by the firms, reducing their action space from being highly multidimensional (entailing choices over the timing and channels of each advertising slot, given advertising prices and their expectations over viewing behavior of consumers) to a decision over total monthly expenditure, which makes solving Coca Cola and Pepsico’s intertemporal profit maximization problems feasible.

Firms’ decisions over product prices and brand advertising levels depend on how sensitive demand for their products is to these choices. We use rich longitudinal micro data on the consumption choices of UK households over the period 2010-2016 to estimate demand for cola products (as well as outside options that capture non-cola drinks consumption). We observe in our purchase data the disaggregate barcode-level products that households choose (including the transaction price), and also detailed information on household TV viewing habits. We couple this with data on the universe of UK TV adverts, which include details of when, on what channels, and during which programs specific brands were advertised. These data enable us to construct household-specific measures of exposure to brand-level advertising. To identify the impact of brand-level advertising exposure on consumer demands we exploit variation in exposure across households of the same demographic make-up and that have the same television viewing habits (across genres, times and channels). This variation arises because there are differences in brand advertising across similar shows, meaning households of the same income, composition and TV viewing preferences are exposed to different levels of brand advertising.

Our demand estimates show that there is a correlation in consumer preferences over price and advertising; on average, consumers that are particularly sensitive to a change in price also tend to be relatively sensitive to advertising changes. They also show positive spillovers in brand advertising. For instance, the own-advertising elasticity for Regular Coke is 0.12, while the cross-advertising elasticity of (demand for) Diet Coke is 0.05 and Regular Pepsi is 0.02. In other words, as well as raising demand for Regular Coke, Regular Coke advertising stimulates demand for Diet Coke and (to a lesser extent) Pepsi. These features of consumer demands play an important role in driving advertising responses to policy changes.

Our model incorporates the decisions firms take each month over their prices and brand advertising expenditures, given the shape of consumer demands and their expectations about how advertising expenditures map into exposure via the role played by advertising agencies.

⁴For instance, advertising agencies play a similar role for US television advertising (see Hristakeva and Mortimer (2023)).

Firms' decisions over prices depend on the distribution of consumers' stock of exposure to brand-level advertising, meaning that optimal prices are a function of past advertising choices. Firms' decisions over advertising budgets depend on how prices and demand for their products in the future will be affected by current advertising. Therefore, competition over advertising budgets is dynamic, and the solution concept we use to solve the game is a Markov perfect equilibrium.

We solve the model in the (observed) case where there are no taxes or advertising restrictions in place, and re-solve the model under several counterfactual policies including a prohibition of advertising for sugar-sweetened (Regular) cola products, a specific tax on sugar-sweetened products, an ad valorem tax on sugar-sweetened products, and a combination of a tax and advertising restriction. We show that in response to the introduction of either form of tax, firms lower advertising of taxed products. A key reason for this is the correlation in consumer price and advertising sensitivities, which mean that a tax leads the most advertising sensitive consumers to switch away from taxed brands, lowering the incentive firms have to invest in advertising. We find that the reduction in advertising is larger under an ad valorem tax. This is driven by the fact that the ad valorem tax reduces optimal price-cost margins (whereas, a specific tax, leads them to increase slightly), thereby reducing the profitability of the marginal consumer, which lowers firms' incentive to advertise taxed brands. Both a tax and the advertising restriction lead to reductions in advertising of diet brands. This is driven by a within-firm complementarity in advertising strategies – the returns to advertising diet products is lower the lower is advertising of taxed, sugary products, which is in part driven by our finding that brand advertising has positive spillovers to the demand of other cola brands.

Overall, our results suggest that an advertising restriction, if implemented on its own, will have a relatively small impact on total sugar consumption from drinks (2.7%). In contrast, taxes of the scale that we have seen implemented in practice around the world have a much larger impact (reducing sugar consumption by around 16.5%). If an advertising restriction is implemented on top of a tax, its impact on sugar is very small, in part because the higher prices resulting from the tax already drive away the most price and advertising sensitive consumers. While the taxes appear regressive, as the consumer surplus loss resulting from higher prices is larger for low income households, this is off-set by larger sugar reductions among low income households. Based on internalities from sugar consumption of the scale estimated by Allcott et al. (2019), the reduction in internalities achieved by the tax is broadly enough to compensate consumers (including for each income quartile) for the consumer surplus loss due to higher prices.

Our paper contributes to a literature on the ex ante evaluation of the effects of taxes on sin goods. This literature, which studies the incidence or optimal design of sin taxes, focuses on the impact that tax has on consumption through higher prices (e.g., Bonnet and Réquillart (2013), Harding and Lovenheim (2017), Griffith et al. (2019), Allcott et al. (2019), Dubois et al. (2020), O’Connell and Smith (2023)). Wang (2015) uses a demand model that incorporates dynamics through consumer stockpiling and uses it to simulate consumer responses to a tax on sugar-sweetened beverages. We contribute to this literature by studying sin taxation in an empirical equilibrium model that incorporates dynamics arising through brand-level advertising.

Our empirical approach draws on the literature on dynamic games in empirical IO, using the solution concept of Markov perfect equilibrium (Maskin and Tirole (1988)), and the solution algorithms of the general form developed in Ericson and Pakes (1995) and Pakes and McGuire (1994). We therefore relate to previous research (Dubé et al. (2005), Doraszelski and Markovich (2007)) that uses this approach to solve for a dynamic equilibrium in models in which firms choose advertising strategies. As advertising can be interpreted as a form of investment that firms make to raise future profits, our work is also related to models of dynamic investments games such as that in Ryan (2012), or the dynamic product repositioning model of Sweeting (2013). We are the first paper to use this framework to study policies aimed at reducing sin good consumption. In addition, our framework introduces a way of linking advertising exposure, which drives consumer responses, with firms’ advertising expenditure. Our approach exploits a common feature of advertising markets, so it provides a way of solving an otherwise intractable dynamic oligopoly game that will be useful in other markets and contexts.

We also relate to a literature that estimates the impact of advertising on consumer demand.⁵ Like us Dubois et al. (2018) use consumer-level exposure to advertising to estimate the impact of TV advertising on demand (in the potato chips market). However, they simulate a complete advertising ban which eliminates any dynamics in firms’ response to the policy.

A number of recent papers consider other mechanisms through which advertising can impact market equilibria. These include Murry (2017), who focuses on how advertising decisions can impact the contracting between car manufacturers and their dealers, and Gentzkow et al. (2021) and Zubanov (2021), which draw on the two-sided market theory of advertising in Rochet and Tirole (2003) to model the determination of prices in the advertising market. As we focus on policy intervention in a specific consumer goods market in which advertising

⁵This includes Erdem et al. (2008a, 2008b), Goeree (2008), Shapiro (2018) and Shapiro et al. (2021).

is dominated by manufacturers, we abstract from manufacturers-retailers vertical relations and equilibrium in the advertising market itself.

Section 2 introduces our main data sources and summarizes the key features of the cola market. Section 3 describes our dynamic equilibrium model. Sections 4 and 5 describe our empirical model, present estimates and characterize market equilibrium in the absence of tax or advertising restrictions. Section 6 presents the impact of tax and restrictions on advertising on market equilibrium.⁶

2 The market for colas

As of April 2021 sugar-sweetened beverage taxes were in place in over 50 jurisdictions (GFRP (2021)). These taxes are typically motivated as a means to tackle the negative health effects associated with consumption of these products (which may give rise to externalities if people partially ignore privately borne health costs, or externalities if some of the costs are borne by others, for instance, due to higher public health care, or health insurance premium, costs). Sugar- and artificially-sweetened beverages are highly advertised; they accounted for 4% of consumer spending on food and drinks in 2016, but 7% of television advertising. Two-thirds of advertising of, sugar and artificially-sweetened beverages for was for cola products. For this reason we focus our analysis on the cola market. We use data from the UK covering the period 2010-2016.

In the UK a sugar-sweetened beverage tax was introduced in April 2018. The structure of the tax provides firms with the opportunity to avoid it through lowering the sugar content of their products. As a result only the two main cola brands, in addition to a few niche energy drinks, pay the tax (Dickson et al. (2023)). Therefore, our focus on the UK cola market captures the majority of products subject to UK sugar-sweetened beverage taxation.

2.1 Market structure

We use micro data on the drinks purchases made by a sample of consumers living in Great Britain that is collected by the market research firm Kantar, as part of their Fast Moving Consumer Goods (FMCG) At-Home Purchase Panel. We have a sample of over 21,000 households that record all grocery purchases they make and bring into the home over 2010-

⁶A number of appendices provide additional information: A (Purchase data), B (Advertising market and data), C (Equilibrium delegation decision of advertising), D (The impact of tax on advertising in a static single-product monopoly), E (Solution to advertising agency problem), F (Additional estimation results), G (Transition function), H (Solution algorithm), I (Consumer surplus decomposition) and J (Additional counterfactual results)

2016, using a hand held scanner or mobile phone app. We observe details of all products purchased, including the transaction price, as well as demographic variables and detailed measures of household television viewing behavior. The data has a panel structure, with the average household present in the data for over 100 weeks.

The cola market is dominated by two firms, Coca Cola Enterprises, which has a market share of 60.7% and Pepsico, which has a market share of 33.4% (see Table 2.1). Each firm sells a Regular and Diet version of its cola. Coca Cola Enterprises’ market share is split approximately equally between Regular and Diet Coke (the latter comprises just under 60% of its market share), while around three-fourths of Pepsico’s market share is accounted for by Diet Pepsi. The remaining products in the market are store brand (also referred to as own brand and private label). Each brand is available in numerous different container types and sizes (for instance 4×330ml cans or 2l bottle). In total there are 42 products in the UK cola market.⁷

Table 2.1: *Firms and brands*

Firm	Brand	Expenditure share	No. of products	Average price (£ per liter)
Coca Cola Enterprises	Regular Coke	25.9%	15	0.82
	Diet Coke	34.8%	15	0.81
Pepsico	Regular Pepsi	7.6%	3	0.72
	Diet Pepsi	25.8%	5	0.73
Store brands	Regular store	2.4%	2	0.21
	Diet store	3.5%	2	0.21
All		100.0%	42	0.74

Notes: Authors’ calculations using data from Kantar FMCG At-Home Purchase Panel. Diet Coke includes Coke Zero and Diet Pepsi includes Pepsi Max.

2.2 Television advertising

We use data on television advertising of non-alcoholic beverages from the market research firm AC Nielsen for the period 2009-2016.⁸ Our data contain details on individual adverts (we observe over 1 million adverts for cola), including the brand that was advertised, when the advert was shown (date, time, channel and during/between which program(s)), and the

⁷We drop a small number of minor products. These include niche Coca Cola and Pepsi sub-brands (e.g., Diet Coke with Vitamins) that each have market shares below 0.5% and a large number of minor products that each account for fewer than 10,000 (0.67%) transactions in our data. In our product definition we aggregate together Diet Coke and Coke Zero, and Diet Pepsi and Pepsi Max. In total the 42 cola products in our analysis cover over 80% of total cola sales. See Appendix A for details of the cola products.

⁸Digital advertising is growing, however, it remains a relatively small share of total *food and drink* advertising, with it being estimated to account for only 5% of all drinks advertising spend (DCMS (2021)).

expenditure required to advertise during the slot. For 2015-2016 we have additional data on TV advertising of all food and alcohol products, and for 2015 we also observe the industry standard measures of how many people viewed each advert.

In an average month Coca Cola Enterprises spends £1.1m purchasing 9,300 slots, accounting for total advertising time of 3,515 minutes. The price of these slots varies widely depending on the expected audience number (for instance, the price of advertising on a popular channel during prime-time can be several times the price of advertising on a more niche channel). Pepsico advertises less than Coca Cola Enterprises, spending £0.2m purchasing slots in a typical month. There is no advertising for the store brand colas.

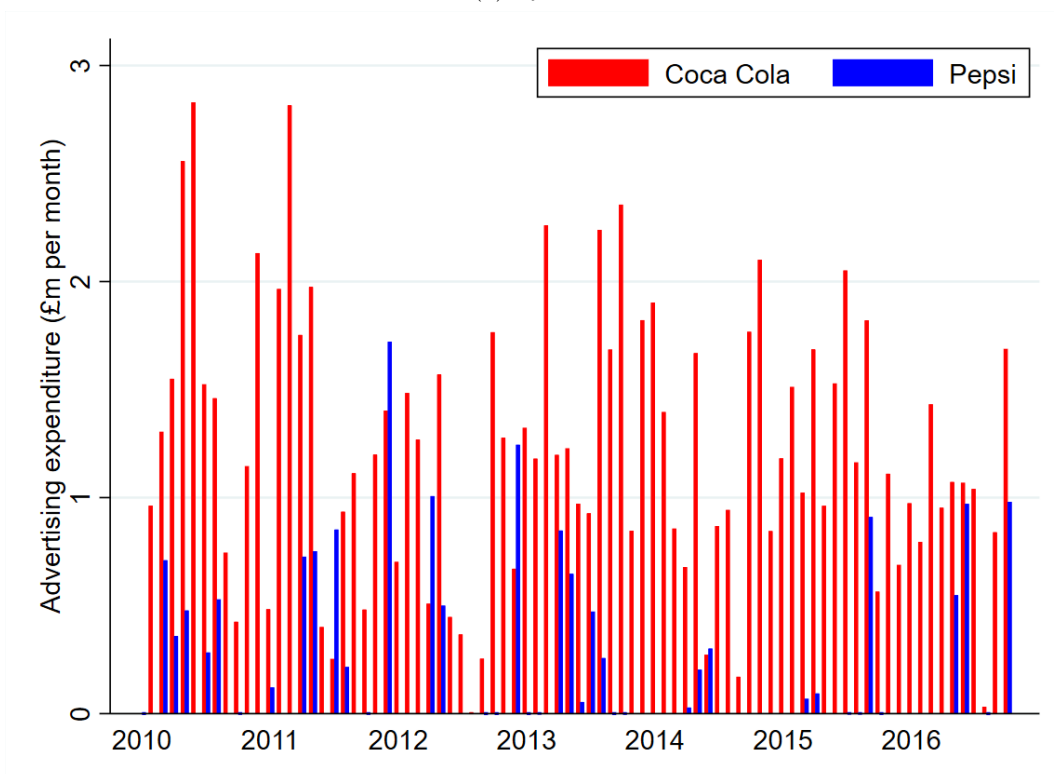
Figure 2.1 shows advertising spending over time, separately for Coca Cola Enterprises (Coca Cola) and Pepsico (Pepsi), and within firm separately by Regular and Diet brands. It illustrates that spending fluctuates over time, and that while Coca Cola Enterprises tends to invest more in advertising its Regular than Diet brand (the former accounts for 57% of their total spend), Pepsico advertises almost exclusively its Diet brand. In our analysis we focus on Coca Cola's advertising decisions over its Regular and Diet brands and Pepsi's decision over its Diet brand.

An important institutional feature of television advertising is that advertisers (i.e., Coca Cola Enterprises and Pepsico) contract with advertising agencies that purchase advertising slots from channels on their behalf. In 2016, across TV advertising of all food and drinks products, we observe 40 different agencies. We also observe that in each year Coca Cola and Pepsico each contract with only one agency, and that they use different agencies. Coca Cola Enterprises accounts for 29% of the food and drinks advertising of the agency it contracts with in 2016, and Pepsico accounts for 3% (see Appendix B.2 for further details). A second important feature of UK TV advertising is that it is primarily national in nature. For 2016, across all Coca Cola and Pepsico advertising, 73% of slots aired nationally. The remaining slots were aired on one of 11 broad regions, with the majority of regional slots running concurrently across several regions.

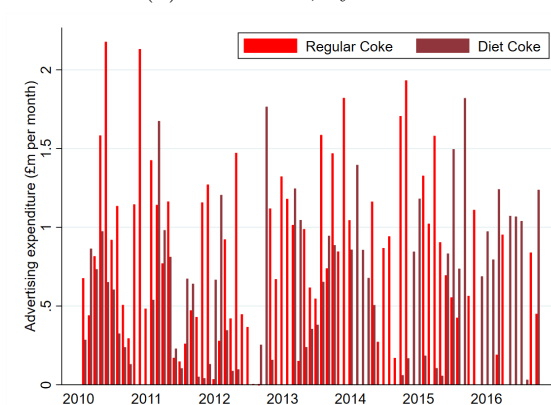
In our analysis we allow for advertising to impact consumer choice and, except when we discuss welfare effects in Sections 6.2 and 6.3, we are agnostic about whether advertising enters consumers' underlying experience utility or only decision utility. As both Coca Cola and Pepsi are universally known, and television advertising for them mainly focuses on emphasizing the pleasure associated with consuming them, we do not consider the case where advertising for Coca Cola and Pepsi is informative, either about product existence or characteristics.

Figure 2.1: *Advertising Expenditure*

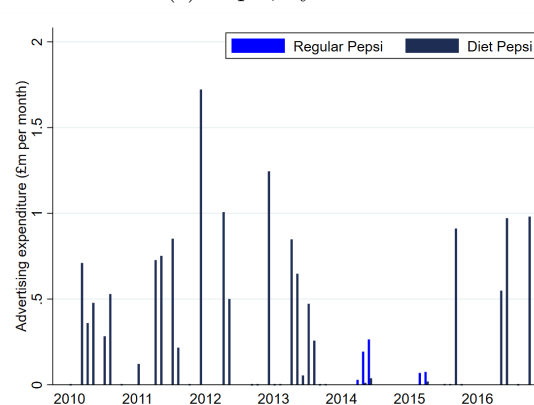
(a) by firm



(b) Coca Cola, by brand



(c) Pepsi, by brand



Notes: Authors' calculations using data from AC Nielsen Advertising data for 2010-2016.

2.3 Household exposure to TV advertising

Firms invest in advertising to influence current and future demand for their products, in order to raise their profits. The extent to which a given financial investment in advertising will influence profitability depends in part on which consumers are exposed to the advertising.

Exposure depends on when adverts are shown, and on the television viewing behavior of households.

We observe when adverts air in the advertising data. In the purchase data, we observe measures of household television viewing behavior. Specifically, each year households fill in a detailed survey stating which shows and stations they watch and during which time slots in a typical week they watch TV, and how regularly they do so. We use the combination of advertising slot information and TV viewing behavior to build a measure of a household's exposure to brand level advertising. We exploit variation in exposure across consumers to identify the impact of advertising on consumer choice (see Section 4.3 for details of our strategy for isolating exogenous variation in advertising exposure).

Let i index consumer (in our application a household), b brand (Regular Coke, Diet Pepsi, etc.) and k advertising slot. A slot refers to a specific time, date, station and broad region when an advert is shown. Within an interval of time, such as a week, the number of potential slots is very large; for instance with around 100 channels and 4 advertising breaks per hour there are over 70,000 slots each week. Let $w_{ik} \in [0, 1]$ denote the probability a household watches television during slot k , $T_{bk} \geq 0$ be the length of an advert for brand b that ran during slot k , and $\omega(\cdot)$ be some concave function that captures any diminishing returns to advertising length. The advertising exposure of consumer i during time period t (we consider a week) is given by:

$$a_{ibt} = \sum_{\{k|t(k)=t\}} w_{ik}\omega(T_{bk}), \quad (2.1)$$

where $t(k)$ is the week of slot k . We directly observe T_{bk} in the advertising data. We use the TV watching survey in the purchase data to measure w_{ik} . In order to estimate how the ordinal survey responses (households state whether they regularly/sometimes/rarely/never watch) map into probabilities we combine the information we have on total viewership of each individual advert in 2015 with the survey answers given by households in 2015 (see Appendix B.3).

3 Equilibrium model

In order to analyze the impacts of policies such as taxes and restrictions to advertising, we specify a dynamic equilibrium model. We apply this model to the market for cola, but it could be applied in other oligopoly markets in which firms compete in prices and television advertising budgets. Each period firms choose product prices and brand advertising budgets, delegating the choice of advertising slots to an advertising agency that is set the objective of

maximizing consumer exposure to brand-level advertising. Consumers choose which products to purchase based on their preferences, the prices they face, and their exposure to television advertising. If a consumer is exposed to advertising in one period, it may influence their future choices; hence firms' choice of advertising budgets affects both their current and future profits. Our equilibrium model is therefore one of dynamic competition.

We describe the structure of the dynamic oligopoly game, the role of advertising agencies in mapping advertising budgets to slots, and hence to consumer advertising exposure, and then we outline our consumer demand model. In this section we describe the structure of the model; we provide details of the empirical specification in Section 4 and 5.

3.1 The firm's decision

We index (cola) firms by $f = 1, \dots, F$, brands by $b = 1, \dots, B$ and products by $j = 1, \dots, J$; we denote the set of products and brands owned by firm f respectively by \mathcal{J}_f and \mathcal{B}_f . Throughout we assume the sets of firms, brands and products that comprise the market are fixed. p_{jt} and c_{jt} denote the period t price and marginal cost of product j . We denote advertising expenses used to purchase television advertising slots for brand b during period t by e_{bt} . We allow for the possibility that agencies charge a markup over these expenses to cover fixed costs, and due to any market power they exercise, which we denote by $\psi_b \geq 0$, meaning a firm's total brand advertising cost is $(1 + \psi_b)e_{bt}$.

Each period firm f chooses advertising expenditures for its brands (along with prices for its products). These expenditures are used by an advertising agency to purchase advertising slots on the firm's behalf, which determines the flow of advertising exposure, a_{ibt} , of all consumers $i \in I$ for brand b in period t . We denote the period t stock of consumer brand advertising exposure by $\mathcal{A}_{ibt} = g(a_{ib0}, a_{ib1}, \dots, a_{ibt-1})$, the vector of consumer exposure stocks across brands $\mathcal{A}_{it} = (\mathcal{A}_{i1t}, \dots, \mathcal{A}_{iBt})$, and the set of exposure stocks across consumers $\mathcal{A}_t = \{\mathcal{A}_{it}\}_{i \in I}$. The market demand function for each product depends on \mathcal{A}_t , capturing the potentially persistent effects of advertising. Specifically, the share of the potential market M_t accounted for by product j , is $s_{jt}(\mathbf{p}_t, \mathcal{A}_t)$, where $\mathbf{p}_t = (p_{1t}, \dots, p_{Jt})$. Note that the dependence of demand on \mathcal{A}_t means a firm's current choice of e_{bt} will impact the advertising exposure stock in future periods (making competition between firms dynamic).

Firm f 's flow profits take the form:

$$\pi_f(\mathcal{A}_t, \mathbf{p}_t, \mathbf{e}_t) = \sum_{j \in \mathcal{J}_f} (p_{jt} - c_{jt}) s_{jt}(\mathbf{p}_t, \mathcal{A}_t) M_t - \sum_{b \in \mathcal{B}_f} (1 + \psi_b) e_{bt}. \quad (3.1)$$

The firm's problem at period $t = 0$ is to choose prices and advertising budgets to maximize the present discounted value of its stream of flow profits:

$$\max_{\{p_{jt}\}_{\forall t, j \in \mathcal{J}_f}, \{e_{bt}\}_{\forall t, b \in \mathcal{B}_f}} \sum_{t=0}^{\infty} \beta^t \pi_f(\mathcal{A}_t, \mathbf{p}_t, \mathbf{e}_t), \quad (3.2)$$

given the relationship between advertising budgets and exposure stocks, $\mathcal{A}_t(e_{t-1}, \mathcal{A}_{t-1})$. Firms simultaneously set prices to maximize profits (conditional on the distribution of advertising exposure stocks). Since prices directly impact current but not future flow profits, firm f 's first order condition for period t prices is:

$$s_{jt}(\mathbf{p}_t, \mathcal{A}_t) + \sum_{j' \in \mathcal{J}_f} (p_{j't} - c_{j't}) \frac{\partial s_{j't}(\mathbf{p}_t, \mathcal{A}_t)}{\partial p_{jt}} = 0, \quad (3.3)$$

for all $j \in \mathcal{J}_f$. Let $p_{jt}^*(\mathcal{A}_t)$ denote the optimal price given the advertising exposure stock distribution. We can re-write the flow profit, $\tilde{\pi}_f(\mathcal{A}_t, \mathbf{e}_t)$, as $\tilde{\pi}_f(\mathcal{A}_t, \mathbf{e}_t) \equiv \pi_f(\mathcal{A}_t, p_{jt}^*(\mathcal{A}_t), \mathbf{e}_t)$, and the firm's intertemporal profits as $\sum_{t=0}^{\infty} \beta^t \tilde{\pi}_f(\mathcal{A}_t, \mathbf{e}_t)$.

In solving for firms' optimal advertising strategies, we focus on Markov perfect equilibrium, where strategies are a function of payoff-relevant state variables (Maskin and Tirole (1988)). For firm f , a strategy σ_f is a mapping between state variables \mathcal{A}_t (i.e., the current advertising exposure stock distribution) and advertising expenditure for the brands it owns, $\sigma_f(\mathcal{A}_t) \equiv (\{e_{bt}\}_{b \in \mathcal{B}_f})$. Given a strategy profile of competing firms, $\sigma_{-f}(\mathcal{A}_t)$, we can write the firm's intertemporal profit maximization using a recursive formulation. Given other firms' strategies $\sigma_{-f}(\mathcal{A}_t)$, firm f solves the Bellman equation:

$$\pi_f^*(\mathcal{A}_t) = \max_{\{e_{bt}\}_{b \in \mathcal{B}_f}} \tilde{\pi}_f(\mathcal{A}_t, \mathbf{e}_t) + \beta \pi_f^*(\mathcal{A}_{t+1}). \quad (3.4)$$

A Markov perfect equilibrium is a list of strategies, σ_f^* for $f = 1, \dots, F$, such that no firm has an incentive to deviate from the action prescribed by σ_f^* in any subgame that starts at some state \mathcal{A}_t .

We solve for a Markov perfect equilibrium, restricted to pure strategies, using an approach similar to Pakes and McGuire (1994) (we describe in more details our empirical implementation in Section 4). A Markov perfect equilibrium in pure strategies of this dynamic game may not exist, and if it exists, it need not be unique.⁹ We assume that conditions for the existence of a subgame perfect Markov equilibrium of this game are satisfied, we use neces-

⁹Ericson and Pakes (1995) and Doraszelski and Satterthwaite (2003) provide conditions for existence in games with similar structures. However, the structure of our game differs meaning these conditions do not directly apply.

sary conditions to characterize an equilibrium (as suggested by Maskin and Tirole (1988)) and we check empirically for multiplicity of equilibria.

3.2 The role of advertising agencies

Firms delegate the choice over advertising slots to advertising agencies, meaning the agencies play an important role in translating the brand advertising budgets that firms choose into consumer-level advertising exposure. In exchange for a payment, the agency chooses slots to maximize consumer exposure to brand advertising subject to a budget constraint. The agencies play a role in simplifying the dynamic competition between the firms. Without them firms would be tasked with directly choosing advertising slots (rather than making decisions over total brand advertising expenditures), meaning their action space would consist of potentially many thousands of decisions over which slots to advertise in. By incorporating the intermediary role of advertising agencies in our model we capture an important feature of the advertising market, which also drastically reduces firms' action space and ensures the dynamic oligopoly game is tractable.

We take as given firms' decision to delegate slot choices to an agency. There are a couple of reasons that can rationalize this delegation choice. First, choosing advertising slots on television over thousands of possibilities may require specialized human capital in marketing and media relationships, meaning agencies may have efficiency and cost advantage compared to in-house firms. This cost advantage in itself can explain the delegation decision. Second, firms' decision to delegate slot choices to advertising agencies can arise as an equilibrium outcome, as the delegation can help soften competition in advertising. We show two examples of this in Appendix C, in a static game where there is a fixed cost of not delegating, and a dynamic (repeated) game in which delegation can arise for similar reasons as tacit collusion in prices.

3.3 The advertising agency's problem

As in Section 2.3, we use T_{bk} to denote the length of advert for brand b during slot (i.e., station-date-time) k , w_{ik} to denote the probability that consumer i watches it during slot k , and we measure the expected flow advertising exposure of consumer i for brand b in period t as in equation (2.1), $a_{ibt} = \sum_{\{k|t(k)=t\}} w_{ik}\omega(T_{bk})$, for some increasing concave function, $\omega(\cdot)$.¹⁰

Letting ρ_k denote the price of advertising during slot k ; total expenditure for buying advertising slots for brand b during period t is given by $e_{bt} = \sum_{\{k|t(k)=t\}} \rho_k T_{bk}$. Each period

¹⁰This captures the possibility of advertising effectiveness diminishing in advert length, see Dubé et al. (2005), Bagwell (2007), Gentzkow et al. (2021).

the firm that owns brand b contracts with an advertising agency to maximize a flow of advertising exposure for a budget e_{bt} . The agency chooses the set of slots, T_{bk} , to solve:

$$\begin{aligned} \max_{\{T_{bk}\}_k} \quad & \sum_i a_{ibt} \\ \text{s.t.} \quad & \sum_{\{k|t(k)=t\}} \rho_k T_{bk} \leq e_{bt}. \end{aligned} \tag{3.5}$$

The first order condition of the agency's problem implies that the ratio of total marginal impacts during two advertising slots, k and k' , is set equal to the ratio of the prices of advertising during these slots:

$$\frac{\sum_i w_{ik} \omega'(T_{bk})}{\sum_i w_{ik'} \omega'(T_{bk'})} = \frac{\rho_k}{\rho_{k'}}.$$

The optimal choice during slot k satisfies

$$T_{bk}^* = \omega'^{-1} \left(\frac{\rho_k}{\sum_i w_{ik}} \frac{1}{\lambda_{bt}^*} \right), \tag{3.6}$$

where λ_{bt}^* is the Lagrange multiplier on the constraint in the agency's problem. Concavity of $\omega(\cdot)$ means T_{bk}^* is a decreasing function of the price per viewer during slot k , $\frac{\rho_k}{\sum_i w_{ik}}$.

The optimization problem (3.5) assumes that the agencies are price-takers in the advertising slot market when purchasing slots for the cola firms. Given the small share of total advertising accounted for by cola firms,¹¹ this assumption is a natural one. Variation in advertising slots prices will be driven by the expected audience of a show or TV station (see empirical evidence in Bel and Laia Domènech (2009) and this prediction from an equilibrium model in Gentzkow et al. (2021) and Zubanov (2021)).

3.4 The consumer's problem

We model consumers as making a discrete decision over which (if any) cola product to purchase each period. At this point we take no normative stance on the relationship between advertising and consumer welfare, nor do we rule out the possibility that consumers are subject to internalities. Therefore, we refer to “decision utility” as in Bernheim (2009). We return to this point when making consumer welfare statements in Section 6.

We specify the decision utility that consumer i obtains from choosing product j in period t as:

$$U_{ijt} = V(\mathcal{A}_{it}, p_{jt}, \mathbf{x}_{jt}; \theta_i) + \epsilon_{ijt}. \tag{3.7}$$

¹¹Cola advertising accounts for 3% of total food and drink TV advertising expenditure and slots.

The decision utility for consumer i associated with product j depends on their stock of exposure to advertising for all brands, \mathcal{A}_{it} , the price of the product, p_{jt} , observable and unobservable product characteristics, \mathbf{x}_{jt} and a vector of preferences parameters, θ_i . ϵ_{ijt} is an idiosyncratic shock that we assume is distributed type I extreme value. The decision utility from choosing the non-cola outside option ($j = 0$) is $U_{i0t} = V(\theta_i) + \epsilon_{i0t}$.

The consumer level choice probability for product $j \in \{1, \dots, J\}$ is:

$$s_{ijt} = \frac{\exp(V(\mathcal{A}_{it}, p_{jt}, \mathbf{x}_{jt}; \theta_i))}{\exp(V(\theta_i)) + \sum_{j'=1}^J \exp(V(\mathcal{A}_{it}, p_{j't}, \mathbf{x}_{j't}; \theta_i))}.$$

The market share function for product $j \in \{1, \dots, J\}$ is obtained by integrating across the consumer specific preferences and the advertising exposure distribution:

$$s_{jt}(\mathbf{p}_t, \mathcal{A}_t) = \int \int \frac{\exp(V(\mathcal{A}_{it}, p_{jt}, \mathbf{x}_{jt}; \theta_i))}{\exp(V(\theta_i)) + \sum_{j'=1}^J \exp(V(\mathcal{A}_{it}, p_{j't}, \mathbf{x}_{j't}; \theta_i))} dF(\theta_i, \mathcal{A}_{it}).$$

3.5 Counterfactual policy simulations

We use our equilibrium model to simulate the introduction of two different forms of sugar-sweetened beverage tax, an advertising restriction on sugar-sweetened colas, and a combination of these policies. We consider taxes that apply to products with in excess of 5 grams of sugar per 100ml (this is similar the structure of the UK tax). Let $j \in \Omega_S$ denote the set of sugar-sweetened cola products with sugar content above this threshold and $j \in \Omega_N$ denote the set of other colas. We simulate taxes implying the following relationship between the tax-inclusive price \mathbb{p}_{jt} and the tax-exclusive price p_{jt} :

$$\mathbb{p}_{jt} = \begin{cases} p_{jt} + \text{tax}_{jt} & \forall j \in \Omega_S \\ p_{jt} & \forall j \in \Omega_N \end{cases}$$

where tax_{jt} is the tax levied on product j . We consider two common forms of tax: a specific (or volumetric) tax, $\text{tax}_{jt} = \mathbb{t}^s$, and an ad valorem tax, $\text{tax}_{jt} = \mathbb{t}^{ad} p_{jt}$.

With a tax in place the firm's flow profit function is:

$$\pi_f^{\mathbb{t}}(\mathcal{A}_t, \mathbf{p}_t, \mathbf{e}_t) = \sum_{j \in \mathcal{J}_f} (p_{jt} - c_{jt}) s_{jt}(\mathbf{p}_t, \mathcal{A}_t) M_t - \sum_{b \in \mathcal{B}_f} (1 + \psi_b) e_{bt}.$$

Solving the associated system of price first order conditions yields each product's counterfactual optimal price, conditional on the distribution of advertising exposure stocks, $p_{jt}^{\mathbb{t}}(\mathcal{A}_t)$. The associated flow profit function for each firm, $\tilde{\pi}_f^{\mathbb{t}}(\mathcal{A}_t, \mathbf{e}_t) \equiv \pi_f^{\mathbb{t}}(\mathcal{A}_t, p_{jt}^{\mathbb{t}}(\mathcal{A}_t), \mathbf{e}_t)$ can then be used to solve for the counterfactual Markov perfect equilibrium.

Both specific and ad valorem taxes are commonly used as corrective policies aimed at changing the relative prices of alcohol, cigarettes, fuels, cars, and sugar-sweetened beverages. There tends to be lower pass-through of ad valorem taxes than specific taxes, due to the fact that under an ad valorem tax (unlike a specific tax) a firm that raises its margin by implementing a marginal (tax-exclusive) price rise of dp will raise the tax-inclusive (consumer) price by $dp(1 + \mathfrak{t}) > dp$ (e.g., see Anderson et al. (2001)). The extent of pass-through will directly influence the size of consumption responses to a tax. Additionally, it will interact with firms' advertising responses. For instance, if in equilibrium the tax is under-shifted, this means the price-cost margins of taxed products are lower (than under no tax), and the profitability associated with a marginal consumer is lower, all else equal, acting to reduce the incentive a firm has to invest in advertising (see Appendix D for an illustrative example).

Under a restriction that prohibits advertising of sugary products (those in the set Ω_S), the firm's problem described in equation (3.2) becomes:

$$\max_{\{p_{jt}\}_{\forall t, j \in \mathcal{J}_f}, \{e_{bt}\}_{\forall t, b \in \mathcal{B}_f \cap \Omega_N}} \sum_{t=0}^{\infty} \beta^t \pi_f(\mathcal{A}_t, \mathbf{p}_t, \mathbf{e}_t), \quad (3.8)$$

where $\mathcal{B}_f \cap \Omega_N$ is the set of firm f 's brands that are not subject to the advertising restriction.

4 Empirical demand model

A key input to our dynamic model are product-level demand functions. We estimate these using a consumer-level discrete choice model for cola products. We define a choice occasion as a week in which a household purchases any drink product, and model the decision of which (if any) cola product the household chooses. We capture the purchase of a non-cola through two "outside options" – one that comprises non-cola drinks with sugar and one that consists of non-cola drinks that contain no sugar.¹² An important feature of our demand model is that it incorporates the impact of consumer-level advertising exposure on choice.

4.1 Advertising exposure

As discussed in Section 2.3, we measure the flow of exposure to brand advertising in week t for household i , according to $a_{ibt} = \sum_{\{k|t(k)=t\}} w_{ik} \omega(T_{bk})$, where $\omega(\cdot)$ captures diminishing returns to advert length. We assume ω is a power function, $\omega(T) = T^\gamma$, in which case the solution to the advertising agency's problem (equation (3.5)) takes a log-linear form, between the price per viewer of a slot and advert length (conditional on brand-time fixed effects).

¹²They account for 37% and 39% of choice occasions.

We use the advertising data for 2015 (where we observe the slot price, viewership and length of all food and drink TV advertising) to estimate $\hat{\gamma} = 0.64$ (the estimate p-value is smaller than 0.0001). This implies a 60 second advert is 1.56 ($= 2^{0.64}$) times as productive as a 30 second advertising in raising consumer exposure, indicating a degree of diminishing returns to advert length. See Appendix E for more details.

We model a consumer's demand for cola products as a function of their stock of exposure to brand advertising. We specify the consumer's exposure stock to brand b advertising at the beginning of week t as the discounted sum of past advertising exposure:

$$A_{ibt} = \sum_{s=0}^{t-1} \delta^{t-1-s} a_{ibs} = \delta A_{ibt-1} + a_{ibt-1}.$$

This specification implies exposure to brand advertising two weeks ago contributes δ as much to the current stock of exposure as the same amount of exposure one week ago. We set $\delta = 0.9$, which is the value estimated in Shapiro et al. (2021) at the week level for consumer good markets. We use data on advertising and household TV viewing behaviour in a pre-sample year (2009) to construct initial exposure stocks (advertising exposure older than 52 weeks has a negligible impact on stocks).

4.2 Utility specification

We specify the form of the decision utility that consumer i obtains from choosing product j in week t (i.e., the form of equation (3.7)), paying particular attention to allow heterogeneity in consumer sensitivity to price and advertising, and spillovers in the effects of advertising of one brand on demand for another.

We estimate the demand model separately for 12 demographic groups, denoted $d(i)$, based on household type (household with children, working age household with no children, pensioner household) and income quartiles (see Appendix A). This controls for demographic attributes advertisers may target.

Let $j = 1, \dots, J_1$ denote the advertised products (Coca Cola and Pepsico), $j = J_1 + 1, \dots, J$ denote the non-advertised store brands, $j = \underline{0}$ denote choosing a sugary non-cola and $j = \bar{0}$ denote choosing a non-sugary alternative to cola. Let $b(j)$ denote the brand to which product j belongs, $-b(j)$ the other brands owned by the firm that sells product j , $f(j)$ the firm that makes product j , and $-f(j)$ the rival firm. So, for instance, if j is a 2 liter bottle of Regular Coke, $b(j)$, $-b(j)$, $f(j)$ and $-f(j)$ denote, respectively, Regular Coke brand, Diet Coke brand, Coca Cola Enterprises and Pepsico.

We specify the decision utility function for product $j \in \{1, \dots, J_1\}$ as:

$$U_{ijt} = \alpha_i p_{jr(i,t)t} + \beta_i^O \sinh^{-1}(A_{ib(j)t}) + \beta_{d(i)}^W \sinh^{-1}(A_{i-b(j)t}) + \beta_{d(i)}^X \sinh^{-1}(A_{i-f(j)t}) \quad (4.1) \\ + \gamma_i \text{Sug}_{b(j)} + \phi_{d(i)} \mathbf{Z}_{if(j)} + \eta_{if(j)} + \chi_{d(i)j} + \xi_{d(i)b(j)\tau(t)} + \zeta_{d(i)b(j)r(i,t)} + \epsilon_{ijt}.$$

where $p_{jr(i,t)t}$ is the price (measured per-unit) of product j in the retailer consumer i shops with, $r(i, t)$, in week t . We allow for three distinct effects of advertising on decision utility: an own-brand advertising effect, β_i^O , a within-firm spillover effect, $\beta_{d(i)}^W$, and a cross-firm spillover effect, $\beta_{d(i)}^X$. In each case we enter the relevant advertising stock into decision utility through the inverse-hyperbolic sine function, to capture diminishing returns of advertising exposure on consumers' decision utility. Decision utility also depends on whether the brand is sugar-sweetened or not ($\text{Sug}_{b(j)}$), a vector of detailed measures of household TV viewing behavior interacted with firm, $\mathbf{Z}_{if(j)}$, consumer specific firm (i.e., Coca Cola vs. Pepsi) valuations, $\eta_{if(j)}$, and product, $\chi_{d(i)j}$, time (year-quarter) varying brand, $\xi_{d(i)b(j)\tau(t)}$ and retailer varying brand effects, $\zeta_{d(i)b(j)r(i,t)}$ (all of which are demographic group specific).

The inclusion of the three exposure stocks, $(A_{ib(j)t}, A_{i-b(j)t}, A_{i-f(j)t})$ in the decision utility function is important in enabling our model to flexibly capture the impact of advertising on consumer choice. Suppose instead we included only the own-brand effect; then, if (as is expected) the own-brand effect is positive (i.e., an increase in advertising exposure for a brand raises demand for products belonging to that brand), this specification would impose that cross-advertising effects are negative (the advertising exposure lowers demand for all other brands). By including advertising of other brands in the decision utility function, we break this restriction, allowing, for instance, that an increase in advertising for one brand raises demand for a second one. It is possible this type of spillover effect is stronger within-firm than across them, which we allow for in our specification by including separate within- and cross-firm spillover effects.

We model preferences over price, own-brand advertising, sugar and the firm effects as random coefficients. We specify that the sugar and firm coefficients $(\gamma_i, \eta_{i,b(j)})$ follow demographic-group specific independent normal distributions and that the price and own-brand coefficient distribution are such that $(\ln(-\alpha_i), \ln(\beta_i^O))$ follows a demographic-group specific joint normal distribution (with non-zero covariance).¹³ Allowing for correlation in price and advertising preferences is potentially important for modeling the impact of tax policy on advertising. A tax will raise the price consumers face for the set of taxed product.

¹³The log-normality imposes a sign restriction that means a price increase for a product cannot raise its demand, and an advertising increase cannot lower it. We have experimented with using normal distribution that do not impose the sign restriction. They result in similar price and advertising elasticities but have the undesirable property of implying some consumers have upward sloping demands.

This will act to lead the most price sensitive consumers to switch away from these products. Whether sensitivity of the post-tax marginal consumer's demand to advertising is higher or lower than the marginal consumer prior to the introduction of the tax will influence whether the firm responds to the tax by raising or lowering its advertising.

Our rich specification for consumer preferences also helps the model capture realistic patterns of substitution across products. In addition, it allows for flexibility in the curvature of product-level market demands, which are an important determinant of tax pass-through (see Weyl and Fabinger (2013)).¹⁴

For store brands (which never advertise), $j \in \{J_1 + 1, \dots, J\}$, we specify decision utility as:

$$U_{ijt} = \alpha_i p_{jr(i,t)t} + \gamma_i \text{Sug}_{b(j)} + \chi_{d(i)j} + \xi_{d(i)b(j)\tau(t)} + \zeta_{d(i)b(j)r(i,t)} + \epsilon_{ijt}.$$

The decision utility associated with each of the two outside options is $U_{i0t} = \gamma_i + \chi_{d(i)0} + \xi_{d(i)0\tau(t)} + \epsilon_{i0t}$ and $U_{i\bar{0}t} = \epsilon_{i\bar{0}t}$.

4.3 Identification

We face two main identification challenges; pinning down the causal impact of advertising changes and price changes on product-level demands.

4.3.1 Advertising

We observe rich variation in consumer-level exposure to brand advertising in our data. Some of this variation likely reflects targeting of advertising to groups of consumers and/or time periods where demand is particularly susceptible to advertising. We include rich controls for demographics and television viewing behavior in our demand model that are designed to control for this targeting, and use the residual variation in exposure, among households belonging to the same demographic group and with similar TV viewing habits, to identify the impact of brand advertising on product-level demands.

The consumer level variation in our advertising exposure measure is driven by our consumer specific measures of TV watching behavior (the w_{ik} s in equation (2.1)) coupled with the (overwhelmingly national) brand slots chosen by advertisers (T_{bk} s in equation (2.1)). A

¹⁴A feature of logit demand models with no heterogeneity in preference parameters is that they heavily restrict demand curvature. However, the addition of preference heterogeneity breaks the link between the curvature of individual and market-level demand curves, allowing for more flexibility in the latter, as curvature now also depends on how the composition of individuals along the market demand curve changes (see, for example, Griffith et al. (2018)).

threat to identification of advertising effects from using this form of variation is that cola advertisers can target their advertising at consumers on the basis of their anticipated demand for cola products.

One possibility is that advertisers systematically target households of a particular demographic type. To control for this we estimate our demand model separately by demographic groups (based on household income and structure), thereby allowing all preference parameters to vary by demographic group. Included in these preferences are demographic specific time varying brand effects (the $\xi_{d(i)b(j)\tau(t)}$'s in equation (4.1)). These control for the possibility that placement of advertising slots is driven by time-varying (and demographic specific) shocks to brand-level demands.

A related concern is that advertisers are able to target viewers of particular TV programs. In the UK television advertising market, advertisers typically purchase exposure on the basis of achieving a certain number of impacts by demographic group within an interval of time (see Crawford et al. (2017)). Therefore, systematic variation in brand advertising across programs with similar total viewership is most likely when the composition of that viewership is correlated with demographics. We include in our demand model a detailed vector of measures of household TV watching behavior, interacted with Coca Cola and PepsiCo ($\mathbf{Z}_{if(j)}$ in equation (4.1)). This includes how regularly the household watches: (i) TV in a typical week, (ii) shows within each of six genres (e.g., sport, documentaries, entertainment), (iii) shows on different stations (the three main terrestrial channels, and the group of cable/satellite channels),¹⁵ and (iv) during different time slots (e.g., prime-time weekday, non-prime time weekend).

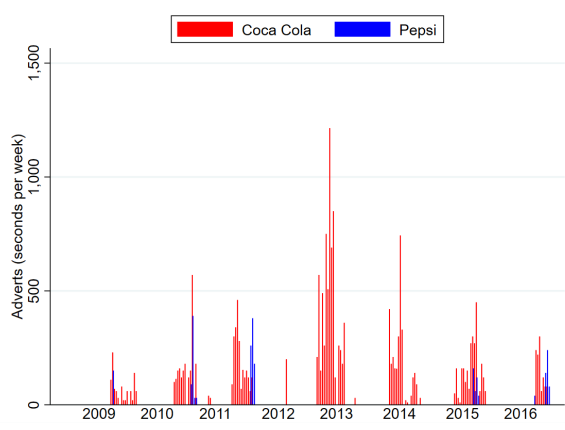
Our strategy, therefore, is to exploit variation in exposure to TV advertising across consumers within the same demographic group and with comparable average TV viewing habits. There is substantial variation in advertising exposure of this sort. For instance, a regression of individual brand exposure stocks on demographic-time-brand effects and the TV viewing behavior controls (with demographic group specific coefficients) has an R^2 of 0.54, indicating that, conditional on our controls for targeting there is substantial residual variation in advertising exposure.

¹⁵In the UK there are five terrestrial channels available to all households that pay for a TV license. Three of these – ITV, Channel 4 and Channel 5 show adverts. Other stations are available via freeview, cable and satellite. See Appendix B.1 for more details.

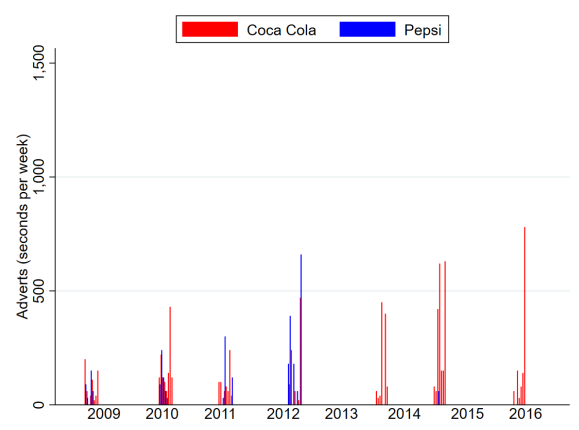
Figure 4.1: *Within genre advertising variation*

Talent contests

(a) The X Factor

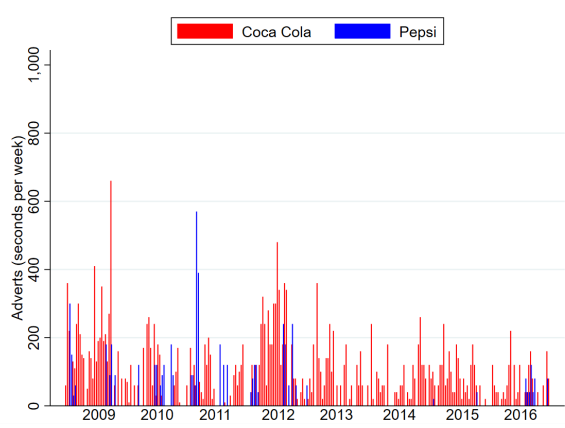


(b) Britain's Got Talent

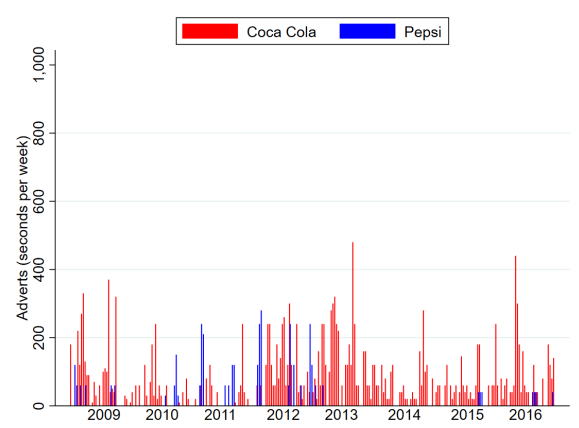


US sitcoms

(c) Frasier



(d) Everybody Loves Raymond



Notes: Authors' calculations using data from AC Nielsen Advertising data for 2010-2016. Figures show number of seconds of adverts shown during the indicated show per week.

In Figure 4.1 we illustrate graphically two examples that highlight the kind of variation that we use. In the top two panels we show variation in advertising in seconds per week separately for Coca Cola and Pepsico brands during two shows, The X Factor and Britain's Got Talent. These are popular prime-time talent contest shows, both shown on the station ITV, but at different times of the year (one in Spring, one in Autumn). According to the TV viewing data 46% of households regularly watch Britain's Got Talent (25% of which do not regularly watch The X Factor) and 39% regularly watch The X Factor (12% of which do not regularly watch Britain's Got Talent). Both Coca Cola and Pepsico adverts are aired during each show, but while Pepsico advertising makes up just 11% of the cola advertising time during The X Factor over 2009-2016, it makes up 27% of the Britain's Got Talent cola

advertising time. Households will therefore be differentially exposed to advertising by the two firms depending on whether they watch neither, one, or both shows. The bottom two panels show a similar comparison between two US sitcoms, *Frasier* and *Everybody Loves Raymond*. These shows are aired across most months of 2009-2016 with different amounts and different timing of Coca Cola and Pepsico advertising.

4.3.2 Prices

An important feature of the UK grocery market is that the main supermarkets have both national store networks and pricing policies (see UK Competition Commission (2000)). This means we do not rely on cross-sectional regional price variation, common in studies of US markets (which typically use Hausman instruments (Hausman et al. (1994))).¹⁶ Instead we exploit the fact the drinks firms (i.e., Coca Cola Enterprises and Pepsico) engage in annual negotiation with each of the main retailers to agree a recommended (national) retail price and agreements on the number, type and timings of temporary price reductions for the forthcoming year (see Competition Commission (2013)). While the recommended price for a given product tends to be similar across retailers, the timing of temporary price reductions vary. This results in shoppers facing different prices depending on when and at which retailer they shop with.

This strategy relies on the following assumptions. First, it requires that we are able to control for aggregate demand shocks that potentially are correlated with nationally set prices. To do this we include a rich set of demographic varying brand effects (including time and retailer varying effects, $\xi_{d(i)b(j)\tau(t)}$ and $\zeta_{d(i)b(j)r(i,t)}$). Second, it requires that retailer choice is exogenous from the point of view of cola choice (ruling out, for instance, a consumer visiting several retailers to find the lowest price for a particular product). We think this assumption is reasonable for two reasons. First, cola is a small share of consumer expenditure, so the gains from shopping around are small. Second, temporary price reductions in the UK market tend to be numerous, so it is likely that if a specific product is not on sale when a shopper visits a retailer, a close substitute will be (i.e., the same brand in a different size).

The third assumption underpinning our strategy is that our estimates capture intra-temporal consumer response, rather than intertemporal responses (e.g., whereby consumers stock-up in response to sales). Such intertemporal responses would likely lead us to over-estimate own price elasticities and underestimate cross price elasticities (Hendel and Nevo (2006)). We cannot rule out a priori that some consumers stockpile in responses to sales, but

¹⁶In addition, the cola market over the period of our study has a stable set of brands and products, meaning we can not use variation in the set of characteristics of all products across markets as price instruments (e.g., Berry (1994), Berry et al. (1995), Gandhi and Houde (2020)).

we can offer empirical evidence that this effect is not quantitatively important in our UK context. Using the same dataset as in this paper (i.e., the Kantar FMCG Purchase Panel) O’Connell and Smith (2023) show that when a consumer purchases a drink product on sale, they are more likely to choose a different brand, container type (i.e., can/bottle) and size relative to their previous purchase, but they do not systematically change the timing of their purchases. This is evidence that consumers respond to sales by intra-temporally substituting across products rather than stockpiling.¹⁷

4.4 Demand estimates

We estimate the demand model by simulated maximum likelihood; we present parameter estimates and product-level price elasticities in Appendix F. In Table 4.1 we report brand-level price and advertising elasticities. The price elasticities give the percent change in demand for the brand listed in the first column in response to a 1% increase in the price of all products belonging to the brand detailed in the first row. The brand own-price elasticity for Regular and Diet Coke is around -2.2 and is somewhat larger in magnitude than the own-price elasticities for Regular and Diet Pepsi. The cross-price elasticities indicate consumers are more willing to switch within Coca Cola and Pepsi brands than between them, and that they are more willing to substitute within Regular and Diet brands than between them (for instance, the cross-price elasticity of demand for Regular Pepsi, with respect to a rise in the price of Regular Coke products, is almost twice as large than for demand of Diet Pepsi).

The advertising elasticities describe the impact of a 1% increase in the stock of all consumers’ exposure to advertising of the brand in the first row on demand for the brand in the first column, and therefore should be interpreted as long-run elasticities.¹⁸ The own-brand elasticities for Regular and Diet Coke advertising are around 0.11, while the Diet Pepsi own-brand elasticity is around half this. The cross-elasticities indicate substantial within-firm advertising spillovers. For instance, a 1% increase in Regular Coke advertising raises demand for Diet Coke products by 0.05% (around half the increase in Regular Coke demand). There is also evidence for cross-firm advertising spillovers (Regular and Diet Coke advertising raising Pepsi demand and Diet Pepsi advertising raising Coke demand), however these are substantially smaller in magnitude than the within-firm spillovers.

¹⁷O’Connell and Smith (2023) also show that there is no economically meaningful change in the probability that, when buying on sale, a consumer shops at a difference retailer compared with their previous purchase, which supports our assumption of exogenous retailer choice.

¹⁸Suppose flow exposure is constant over time, so $A_{ibt} = \frac{1}{1-\delta}a_{ib}$, then a 1% increase in the stock is equivalent to a 1% permanent increase in the flow.

Table 4.1: *Brand price and advertising elasticities*

(1)	Price elasticities				Advertising elasticities		
	Coke		Pepsi		Coke		Pepsi
	Regular (2)	Diet (3)	Regular (4)	Diet (5)	Regular (6)	Diet (7)	Diet (8)
Regular Coke	-2.210 [-2.285, -2.143]	0.511 [0.492, 0.539]	0.050 [0.048, 0.054]	0.092 [0.087, 0.095]	0.115 [0.099, 0.162]	0.043 [-0.006, 0.086]	0.020 [0.008, 0.031]
Diet Coke	0.378 [0.366, 0.407]	-2.192 [-2.249, -2.126]	0.023 [0.022, 0.025]	0.142 [0.135, 0.147]	0.054 [0.009, 0.090]	0.110 [0.095, 0.147]	0.016 [0.003, 0.027]
Regular Pepsi	0.210 [0.169, 0.219]	0.128 [0.102, 0.134]	-1.842 [-1.906, -1.485]	0.552 [0.442, 0.585]	0.021 [0.002, 0.037]	0.020 [0.003, 0.035]	0.015 [-0.013, 0.039]
Diet Pepsi	0.110 [0.107, 0.117]	0.232 [0.223, 0.243]	0.157 [0.150, 0.168]	-1.679 [-1.723, -1.621]	0.015 [-0.002, 0.031]	0.011 [-0.005, 0.024]	0.057 [0.050, 0.074]
Regular Store	0.243 [0.233, 0.262]	0.155 [0.149, 0.163]	0.063 [0.060, 0.068]	0.106 [0.101, 0.111]	-0.021 [-0.030, -0.017]	-0.017 [-0.024, -0.012]	-0.011 [-0.015, -0.007]
Diet Store	0.130 [0.125, 0.140]	0.276 [0.268, 0.289]	0.031 [0.030, 0.034]	0.170 [0.165, 0.178]	-0.020 [-0.027, -0.016]	-0.021 [-0.027, -0.017]	-0.012 [-0.016, -0.009]
Regular outside	0.185 [0.180, 0.196]	0.138 [0.133, 0.144]	0.050 [0.048, 0.054]	0.095 [0.091, 0.099]	-0.020 [-0.025, -0.018]	-0.017 [-0.021, -0.015]	-0.009 [-0.012, -0.007]
Diet outside	0.104 [0.101, 0.111]	0.236 [0.228, 0.246]	0.027 [0.025, 0.029]	0.152 [0.147, 0.158]	-0.019 [-0.024, -0.015]	-0.021 [-0.025, -0.018]	-0.011 [-0.014, -0.009]

Notes: Numbers show the elasticity of demand for the brand shown in column (1) with respect to the price (columns (2)-(5)) or advertising stocks (columns (6)-(8)) of the brands shown in the first row. The price elasticities are with respect to a 1% price rise of all products comprising the brand. The advertising elasticities are with respect to a 1% rise in all consumer exposure stocks. 95% confidence bands are shown in square brackets.

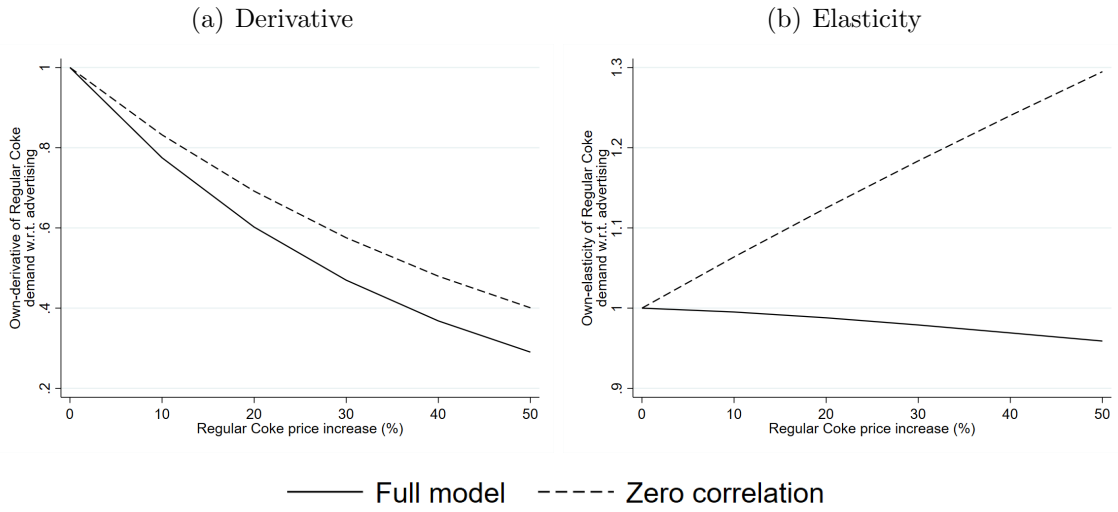
The positive cross-advertising elasticities indicate the importance of including spillover advertising effects in consumer's decision utilities. When we exclude these effects, and re-estimate the model, we find similar own advertising elasticities, but negative cross-elasticities between Coca Cola and Pepsi products (see Appendix F). Hence, with this specification we would conclude that brand advertising steals market share from all rival brands, even within firm. However, Table 4.1 shows that in fact brand advertising leads to substantial within-firm (positive) spillovers, and modest cross-firm ones.

Figure 4.2 shows how the sensitivity of brand demand to advertising varies with the brand price level. Panel (a) shows how the derivative of demand for Regular Coke with respect to Regular Coke advertising varies with a (simulated) increase in the price of all Regular Coke products. Panel (b) shows how the own-advertising elasticity for Regular Coke varies with price. In each case we plot the relationship with our full model estimates (the solid line) and when we set the within demographic group advertising and price sensitivity covariance parameters to zero (the dashed line).

The figure highlights the role the correlation parameters play in determining the shape of demands. When they are set to zero the advertising derivative declines gradually enough as price rises that the advertising elasticity rises (as the derivative falls less quickly with price than the quantity demanded of Coke Regular). However, using our estimates of the within demographic group correlation in price and advertising sensitivities, we find the ad-

vertising derivative declines sufficiently quickly as price rises, that the advertising elasticity also falls. In other words, as price rises, the consumers that substitute away from the brand are relatively advertising sensitive. This feature of demand influences how firms adjust their advertising in response to the introduction of the tax, since with a tax in place demand for the taxed products will comprise a less advertising sensitive consumer base (relative to there being no tax). Had we assumed zero correlation in price and advertising sensitivity we would have imposed that the advertising elasticity rises as we move upwards along the demand curve, whereas in fact our estimates suggest the opposite is true.

Figure 4.2: *Impact of Regular Coke price level on advertising sensitivity of demand*



Notes: Figure shows how the derivative (panel (a)) and elasticity (panel (b)) for demand for Regular Coke with respect to Regular Coke advertising varies with the price of Regular Coke products. The solid lines corresponds to our full demand model, the dashed lines correspond to when we switch off the within demographic group correlation in price and advertising preferences. In all cases we express numbers relative to 0% price increase.

5 Supply-side estimation

In the supply model we treat Coca Cola and Pepsico as the strategic players. They compete over the prices of their products and their brand advertising budgets. Store brands are not advertised, and during the time period we consider Pepsico almost never chooses to advertise Regular Pepsi; therefore we model advertising choices for Regular Coke, Diet Coke and Pepsi Diet. Prices for the store brands are much lower than for Coca Cola and Pepsico products. In policy simulations we hold fixed their prices, treating these products as if they are priced at cost.

We exploit week-to-week variation in advertising exposure in our demand model. However, firms make decisions over their advertising expenditures at lower frequency (with these

decisions generating week-to-week variation in exposure as the advertising slots arranged by agencies are aired). We assume firms make decisions over prices and advertising expenditures each month. While there is variation in prices across retailers (at a given point in time), this is primarily driven by the differential timing of temporary price reductions. Rather than complicate our framework with a formal model of vertical relations, we make the simplifying assumption that drinks firms set a single price for each product across retailers.¹⁹ To solve for the equilibrium of our model we need to specify how firms form expectations about how the distribution of consumer stocks of advertising exposure is impacted by investments in advertising expenditure. We first outline how we do this before presenting the static and dynamic equilibrium conditions in the (observed) zero-tax case.

5.1 The state transition function

Advertising agencies play the role of shrinking firms' action space to a tractable decision over product prices and brand advertising expenditures. However the state space in the firm's decision problem, outlined in Section 3.1, is still large as it consists of the joint distribution of consumer level exposure stocks for each brand (which we denote $\mathcal{A}_t = \{(A_{i1t}, \dots, A_{iBt})\}_{i \in I}$). While the behavior of advertising agencies implies that the advertising exposure distribution in the population depends on these advertising expenditures in a known way, via viewership behavior and realized television slots choices, the information burden on firms in tracking, and forming optimal expenditure strategies that depend on this entire distribution is formidable and renders the dynamic oligopoly game computationally intractable.

We therefore posit that firms track a summary statistic for the brand-specific consumer exposure distribution and present evidence that doing so results in negligible prediction error. In particular, we assume that the state space consists of the expected value of the exposure stock distribution for each brand $(\mathbb{A}_{1t}, \dots, \mathbb{A}_{Bt})$, where $\mathbb{A}_{bt} = \frac{1}{I} \sum_i \mathcal{A}_{ibt} = \delta \mathbb{A}_{bt-1} + \mathbb{a}_{bt-1}$, and where $\mathbb{a}_{bt} = \frac{1}{I} \sum_i a_{ibt}$ is the average flow exposure. By tracking the mean of the distribution firms make a prediction error in their demands equal to $s_{jt}(\mathbf{p}_t, \mathbb{A}_{1t}, \dots, \mathbb{A}_{Bt}) - E_{\mathcal{A}_t}[s_{jt}(\mathbf{p}_t, \mathcal{A}_{i1t}, \dots, \mathcal{A}_{iBt})]$. In practice this error is small, with the average absolute error (across products) being 2% of product-level demands. This is because errors are upward for consumers who are more exposed than the mean and downward for those less exposed than the mean, and thus those errors tend to compensate each other on average.

¹⁹In practice, for a given product-year a drinks firm and retailer agree on a base price \bar{p} and a sale price p_S , with the former applying ρ proportion of weeks. Instead of modelling choice over (\bar{p}, p_S, ρ) , we model choice over $p = (1 - \rho)\bar{p} + \rho p_S$. This average price exhibits little variation across retailers. Cross-retailer variation in the price of a given product at a point in time is driven by non-synchronization of sales. Hence, we specify the relationship between prices in the supply game, p_{jm} , and those faced by consumers in retailer r , week $t \in m$ as $p_{jrt} = p_{jm} + e_{jrt}$, where $\mathbb{E}[e_{jrt} | (j, m)] = 0$.

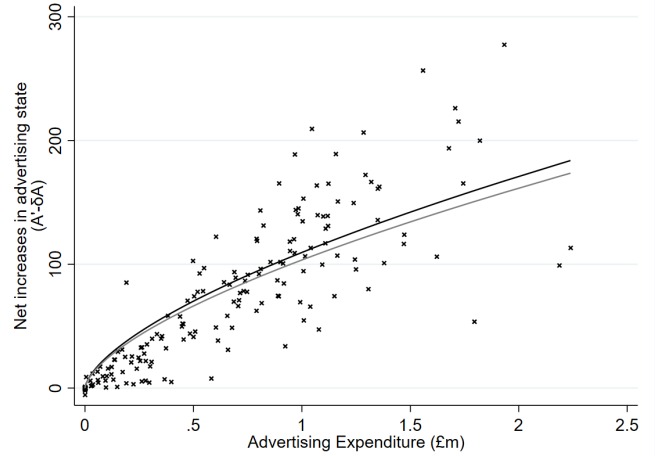
Combining the consumer-level advertising exposure (equation (2.1)) with our estimate of the optimal condition for the choice of advertising slots (captured by our estimate of the curvature parameter for $\omega(\cdot)$ in equation (3.6), γ), the evolution of the brand b state variable can be re-written $\mathbb{A}_{bt} = \delta \mathbb{A}_{bt-1} + \lambda_{t-1} e_{bt-1}^\gamma$, where λ_{t-1} is a period specific rate of transformation of advertising expenses into additional brand level advertising exposure, and depends on advertising slot prices (see Appendix G). Firms do not observe the realization of λ_{t-1} when making decisions over their advertising budgets e_{bt-1} (as slot advertising prices are not yet known), and therefore at this point in time λ_{t-1} is a random variable. We assume that firms form expectations of the changes in the advertising state conditional on expenditure, which implies the stock satisfies:

$$\mathbb{A}_{bt} - \delta \mathbb{A}_{bt-1} = \lambda e_{bt-1}^\gamma + v_{bt-1}, \quad (5.1)$$

where $v_{bt-1} = (\lambda_{t-1} - \lambda) e_{bt-1}^\gamma$. We estimate this equation with linear methods (as γ is already known).

Table 5.1: *Advertising state law of motion*

	$\mathbb{A}_{bt} - \delta \mathbb{A}_{bt-1}$ (1)	$\mathbb{A}_{bt} - \delta \mathbb{A}_{bt-1}$ (2)
$e_{bt-1} \quad (\hat{\lambda})$	0.0153 (0.0004)	0.0145 (0.0006)
$\text{var}(v_{bt-1})$	776	800
r^2	0.8751	0.8728
N	249	246
Instrument	No	Yes



Notes: Table shows estimates of equation (5.1). Column (1) are OLS estimates, column (2) are IV estimates instrumenting e_{bt-1}^γ with \mathbb{A}_{bt-2} . The figure shows a scatter plot of monthly advertising expenditure, e_{bt-1} , and net changes in the advertising state, $\mathbb{A}_{bt} - \delta \mathbb{A}_{bt-1}$ (across brands and year-months). The black line is based on the OLS estimate and the grey line on the IV estimate (in both cases with $\gamma = 0.64$).

Column (1) in Table 5.1 shows estimates of λ and the variance of the error term under the assumption that $\mathbb{E}[v_{bt-1}|e_{bt-1}] = 0$ (which would be the case if $\mathbb{E}[\lambda_{t-1}|e_{bt-1}] = \lambda$). In column (2) we allow for this possibility that $\mathbb{E}[v_{bt-1}|e_{bt-1}] \neq 0$ by instrumenting e_{bt-1}^γ with the two period lagged mean advertising stock \mathbb{A}_{bt-2} . This is observed and therefore in firms' information sets when they choose advertising expenditure e_{bt-1} , and as there is likely to be diminishing returns to investment in a brand's advertising stock, it is likely to influence the

firm's chosen level of flow investment. We find that instrumenting leads to a modest decline in $\hat{\lambda}$ relative to column (1). We also include a scatter plot of the underlying data and plot the relationship implied between the change in net stock and advertising investment, which makes clear the implied relationship is very similar across both sets of estimates.

To solve for a Markov perfect equilibrium we discretize the state space. Specifically, for a set of evenly spaced discrete values $\{A_1, \dots, A_K\}$, where $A_1 = 0$, we use the state transition function:

$$P(A_{bt} = A_{k'} | A_{bt-1} = A_k, e_{bt-1}) = \int_{A_{k'-1}}^{A_{k'}} f_v(A_{bt} - \delta A_k - \lambda e_{bt-1}^\gamma) \frac{A_{bt} - A_{k-1}}{A_{k'} - A_{k'-1}} dA_{bt} \quad (5.2) \\ + \int_{A_{k'}}^{A_{k'+1}} f_v(A_{bt} - \delta A_k - \lambda e_{bt-1}^\gamma) \frac{A_{k'+1} - A_{bt}}{A_{k'+1} - A_{k'}} dA_{bt}.$$

As there are three advertising states, one for Regular Coke, Diet Coke and Diet Pepsi, the state grid $\{A_1, \dots, A_K\}^3$ is of dimension K^3 . We set a value for A_K above the 99th percentile of observed mean stocks in the data and check ex post that the maximum state has zero probability mass in the equilibrium ergodic distribution. We use an evenly spaced grid and set $K = 21$, meaning there are 9,261 points in the discretized state space.

5.2 State-specific optimal prices

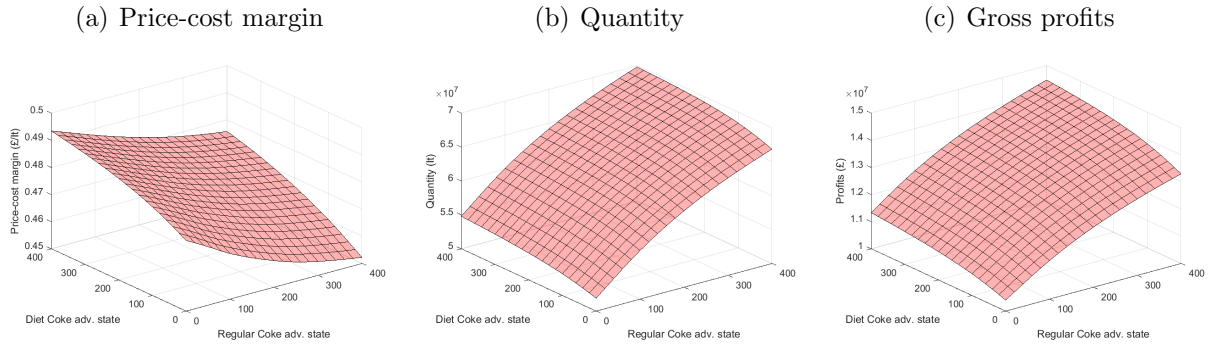
We use the advertising state-specific optimal pricing conditions of equation (3.3), evaluated at the observed prices and advertising state variables, to infer product-level marginal costs. The average (quantity-weighted) marginal cost and price-cost margin per liter among Coca Cola products are 0.45 and 0.38, and the average (expenditure-weighted) Lerner index is 0.46. For Pepsico products the average cost, margin and Lerner index are 0.25, 0.41 and 0.62. Hence, on average Pepsico products have lower costs and similar price-cost margins (meaning higher Lerner indexes) than Coca Cola products.²⁰

We use estimates of product-level demands and marginal costs, along with the price first order conditions (equation (3.3)) to solve for the vector of optimal prices at each point of the advertising state space. Figure 5.1(a) shows how the average price-cost margins of Regular Coke products varies across the advertising state space. The state space is three dimensional; the figure holds fixed the Diet Pepsi state and shows how the average margins of Regular Coke products vary with the Diet Coke and Regular Coke advertising states. It shows that, conditional on the Pepsi and Diet Coke states, the average margin of Regular Coke products is decreasing in the Regular Coke advertising state. The mechanism underlying this

²⁰We report product-level costs, margins and Lerner indexes in Appendix F.

is the negative correlation in consumer's price and advertising sensitivities (reflected in the covariance parameters in the demand model random coefficient distribution); as the Regular Coke advertising state increases, the composition of demand for Regular Coke is increasingly made up of more price sensitive consumers, which lowers the (conditional on state) optimal Regular Coke prices. In contrast there is a (weaker) positive relationship between the Diet Coke advertising state and Regular Coke margins. This reflects the fact that, as Diet Coke is advertised more, relatively advertising sensitive consumers shift from Regular Coke toward Diet Coke, which lowers the advertising and price sensitivity of the Regular Coke consumer base.

Figure 5.1: *Variation in Regular Coke Nash equilibrium with Coca Cola advertising states*



Notes: Panel (a) shows variation in the average price-cost margin for Regular Coke products. Panels (b) and (c) show variation in total quantity and gross profits for Regular Coke. In each panel we hold fixed the Diet Pepsi advertising state at the highest probability state in the (dynamic) equilibrium distribution.

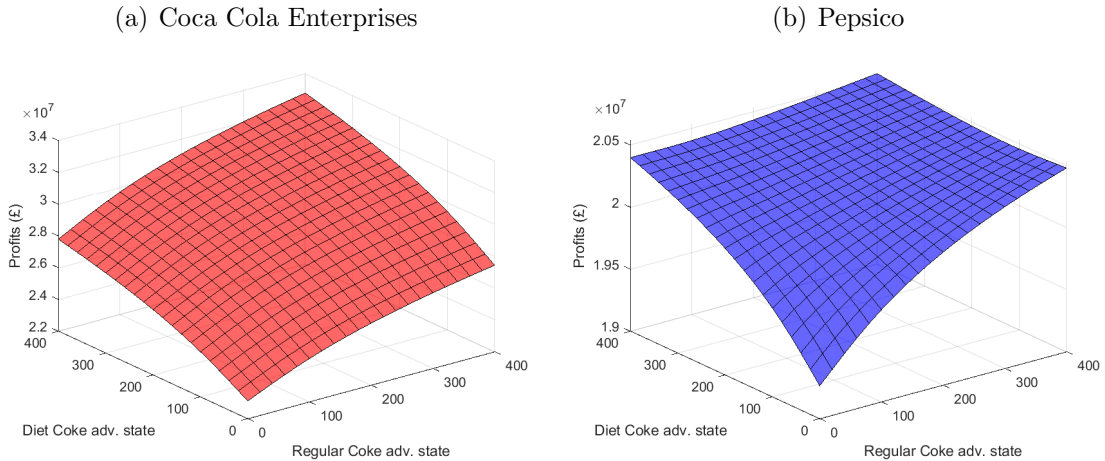
Figure 5.1(b) shows how demand for Regular Coke products varies across the Coca Cola advertising states. Variation in the demand function across the state space reflects both the direct effect of different advertising levels on demand and the indirect effect of the impact of different advertising states on demand via the optimal prices. Demand for Regular Coke products increases in the Regular Coke advertising state, both due to the direct channel and the indirect channel (Regular Coke prices are lower at higher states). Regular Coke demand is also increasing in the Diet Coke advertising state, though less strongly. This reflects a demand spillover (Diet Coke advertising stimulates Regular Coke demand in addition to Diet Coke demand – an effect that comes through the within firm advertising spillover effects in our decision utility specification), which is strong enough to overcome an offsetting indirect effects (Regular Coke prices are rising in Diet Coke advertising).

Figure 5.1(c) shows how gross profits (i.e., excluding advertising expenses) for Regular Coke products vary with the two Coca Cola advertising states. As the Regular Coke advertising state rises there are two off-setting forces, demand rises but margins fall – the former dominates and hence profits rise. Regular Coke profits are also increasing in Diet Coke ad-

vertising (due to the within firm demand spillover), but comparatively less strongly with the Regular Coke state.

In Figure 5.2 we plot how the Coca Cola and Pepsico gross profit functions (which sum across all products they own) vary with the two Coca Cola advertising states, holding the Pepsico state fixed. Coca Cola gross profits are increasing in both Coca Cola advertising states. Pepsico profits are increasing in each dimension of Coca Cola advertising (though much less strongly than Coca Cola profit). This largely reflects a cross-firm spillover effect of advertising in demand – Coca Cola advertising raises decision utility from Pepsico products which act to raise demand for them. At higher levels of Coca Cola advertising Pepsico profits are less sensitive to further increases in Coca Cola advertising. These firm-level profit functions, which incorporate strategic pricing competition, serve as an input into the dynamic advertising game.

Figure 5.2: *Variation in firm-level gross profits with Coca Cola advertising states*



Notes: Panel (a) shows variation in total Coca Cola Enterprises gross profits and panel (b) shows variation in Pepsico gross profits. In each panel we hold fixed the Diet Pepsi advertising state at the highest probability state in the (dynamic) equilibrium distribution.

5.3 Markov perfect equilibrium

We use the Bellman equations for Coca Cola and Pepsico (equation (3.4)) to solve for the Markov perfect equilibrium (see Appendix H for details of the solution algorithm). We fix the brand level agency mark-up over expenses so that our model's equilibrium predictions about average advertising expenditures matches their levels in the data. This implies Pepsico, who advertise less, pay a mark-up that is 1.5 times higher than the average paid by Coca Cola, which is consistent with the mark-up partly being driven by fixed cost recovery for the advertising agency. We set firms' monthly discount factor to $\beta = 0.992$.

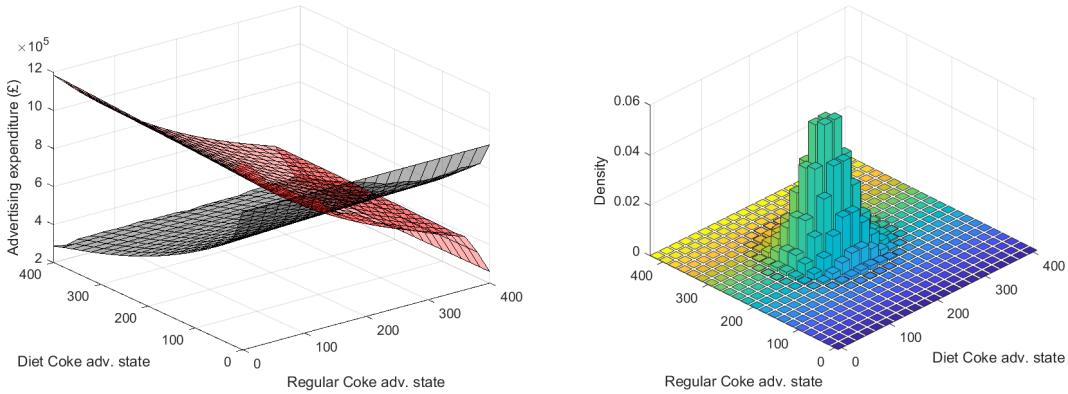
We obtain Markov perfect equilibrium strategies (policy functions) for each advertised brand, which prescribes the optimal choice of advertising expenditure at each point in the advertising state space. In Figure 5.3(a) we plot how the policy functions for Regular Coke (red) and Diet Coke (grey) vary across the Coca Cola advertising states. As in the previous figures, we hold the Diet Pepsi advertising state fixed. The policy functions show that for both Regular and Diet Coke, when the average of consumers' stock of advertising exposure is depleted, the returns from investing in more advertising are relatively high and therefore optimal expenditures are higher, whereas as stocks become large the returns decline so optimal expenditure is lower. The cross-brand relationship between states and optimal expenditures is much weaker, with optimal advertising expenditure for Regular Coke being relatively insensitive to the Diet Coke state (and the converse).

Firms' optimal policy functions, coupled with the state-to-state transition function (equation (5.2)) generate a Markov perfect equilibrium (ergodic) distribution over the state space. In Figure 5.3(b) we plot the ergodic distribution of the equilibrium over the Coca Cola advertising states (integrating across the Pepsi state).

Figure 5.3: *Optimal policy function for Coca Cola Enterprises*

(a) Advertising expenditure

(b) Equilibrium distribution



Notes: In panel (a), the red surface shows Regular Coke advertising expenditure and the grey surface shows Diet Coke expenditure, where we hold fixed the Diet Pepsi advertising state at the highest probability state in the (dynamic) equilibrium distribution. In the panel (b) we integrate over the Diet Pepsi advertising state space.

6 Counterfactual policy analysis

We use our model to simulate a series of counterfactual policies. We characterize their impact on equilibrium prices, advertising expenditure and quantities, and on aggregate profits and consumer surplus to show their distributional consequences. We consider a regulation that prohibits advertising of sugar-sweetened cola, and a specific and ad valorem sugar-sweetened

beverage tax. We assume the tax is levied on Regular Coke and Pepsi, with a rate for the specific tax of £0.22 per liter and a rate for the ad valorem tax calibrated to achieve the same reduction in equilibrium quantity as the specific tax, holding advertising fixed. We also consider the combination of advertising restriction and tax.²¹

Our model generates a set of functions that describe how static objects (e.g., state-specific optimal prices, quantities, profits, consumer surplus) vary across the advertising state space ($\mathbb{A} = \{\mathbb{A}_b\}$), which we denote by $y_\chi(\mathbb{A})$ for $\chi \in \{0, \mathfrak{s}, \mathfrak{a}\}$, and an equilibrium (ergodic) distribution over the state space, which we denote by $g_\chi(\mathbb{A})$ for $\chi \in \{0, \mathfrak{r}, \mathfrak{s}, \mathfrak{s}\mathfrak{r}, \mathfrak{a}, \mathfrak{a}\mathfrak{r}\}$. 0 denotes no policy in place and \mathfrak{s} and \mathfrak{a} denote the counterfactual imposition of a specific and ad valorem tax respectively. \mathfrak{r} denotes the counterfactual imposition of an advertising restriction (which we consider both in the absence of tax and in combination with each type of tax). The (average) equilibrium outcome is given by $\bar{Y}_\chi = \int_{\mathbb{A}} y_\chi(\mathbb{A}) g_\chi(\mathbb{A})$.

6.1 Impact on market equilibrium

In Table 6.1 we summarize the impact of each counterfactual policy on equilibrium (tax-inclusive) prices, price-cost margins, advertising expenditures, quantities and sugar consumption. The numbers are percent changes relative to the no policy (observed) equilibrium. Column (1) reports the impact of an advertising restriction that prohibits advertising of sugar-sweetened cola. Column (2) reports the impact of the introduction of a specific tax, holding fixed the equilibrium distribution across advertising states (and hence holding fixed firms' advertising expenditures). Columns (3) and (4) show the *incremental* impact of accounting for equilibrium advertising responses and adding to the tax the advertising restriction, respectively. Columns (5)-(7) repeat columns (2)-(4) for an ad valorem tax. We discuss each policy in turn.

²¹We consider a tax levied on the cola advertisers, since one of our aims is to characterize advertising responses to tax policy. The specific tax that we simulate is similar in spirit to the one introduced in the UK on April 2018, which entailed a rate of £0.24 pence per liter for products with in excess of 8g of sugar per 100ml and £0.18 pence per liter for products with 5-8g of sugar per 100ml. In that instance store brand colas and most non-colas avoided the tax by reformulating their products to just below the 5g sugar threshold. In our counterfactual analysis we therefore assume store brand cola and the sugar outside option have 5g of sugar per 100ml and are untaxed. Coca Cola and Pepsi, which had approximately 10.5g of sugar when the tax was introduced, did not reformulate and were therefore subject to the higher tax rate.

Table 6.1: *Aggregate impact of counterfactual policies*

	No tax	Specific tax			Ad valorem tax		
	Adv. restrict. (1)	Fixed adv. (2)	+ Eq. adv. response (3)	+ Adv. restrict. (4)	Fixed adv. (5)	+ Eq. adv. response (6)	+ Adv. restrict. (7)
Δ price							
Regular Coke/Pepsi	0.7%	28.8%	0.1%	0.5%	37.0%	0.1%	0.4%
Diet Coke/Pepsi	-1.0%	-1.4%	-0.1%	-0.7%	-1.4%	-0.2%	-0.6%
Δ margin							
Regular Coke/Pepsi	1.6%	5.1%	0.2%	1.1%	-34.8%	0.2%	0.5%
Diet Coke/Pepsi	-2.1%	-2.8%	-0.2%	-1.4%	-2.8%	-0.4%	-1.2%
Δ advertising exp.							
Regular Coke/Pepsi	-100.0%	-	-33.1%	-100.0%	-	-47.3%	-100.0%
Diet Coke/Pepsi	-7.7%	-	-3.3%	-10.8%	-	-8.5%	-15.1%
Δ quantity							
Regular Coke/Pepsi	-13.0%	-55.1%	-1.0%	-4.5%	-55.2%	-1.5%	-3.9%
Diet Coke/Pepsi	-3.8%	11.2%	-0.9%	-4.7%	10.8%	-1.7%	-4.2%
Δ sugar							
All drinks	-2.7%	-16.2%	-0.1%	-0.4%	-16.5%	-0.1%	-0.3%

Notes: Numbers are expressed as a percentage of the pre-policy (i.e., pre tax and advertising restriction) level. Columns (1), (2) and (5) show changes relative to the pre-policy level. Column (3) (column (6)) shows the incremental change relative to column (2) (column (5)) and column (4) (column (7)) shows the incremental change relative to column (3) (column (6)).

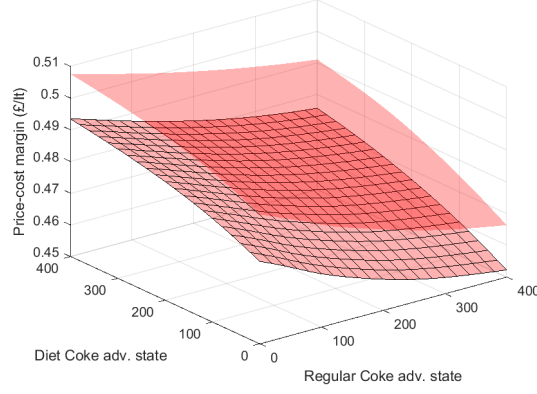
Advertising restriction. Column (1) shows that a ban on advertising sugar-sweetened cola (which directly impacts Regular Coke advertising) leads to a reduction in consumption of Regular Coke and Pepsi products of 13.0% and a fall in sugar consumption (from all drinks) of 2.7%.²² It also leads to a reduction in consumption of Diet Coke and Pepsi of 3.8%. While price and margins change relatively little, the advertising restriction leads to 7.7% reduction in Diet advertising (almost entirely driven by a reduction in Diet Coke advertising). The ban on Regular advertising and the decline in investment in Diet Coke advertising leads to a change in the equilibrium distribution (see panels (b) and (c) of Figure 6.1 where we plot the pre and post ban equilibrium distributions).

The decline in the equilibrium quantity of Diet Coke and Pepsi products reflects two channels. First, as advertising on Regular products has positive spillovers to demand for Diet products, banning it, all else equal, acts to reduce demand for Diet Coke and Pepsi. Second, the equilibrium response of Coca Cola to the policy is to reduce advertising of Diet Coke which directly acts to lower Diet Coke demand.

²²This accounts for changes in sugar from Regular Coke and Pepsi – which each have 106g of sugar per liter, and regular store brands and the sugary outside drink – which have 50g of sugar per liter. We assume the size (in liters) of the sugary outside option is equal to the mean size of the inside (cola) products.

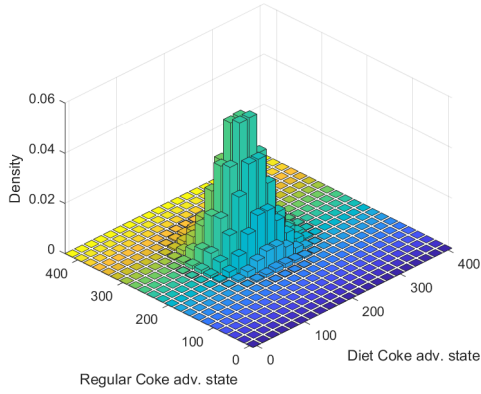
Figure 6.1: *Impact of specific tax and advertising restriction
on state-specific optimal margins*

(a) Average Regular Coke margins

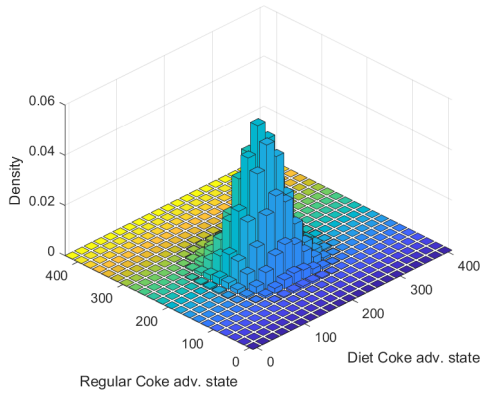


on equilibrium distribution

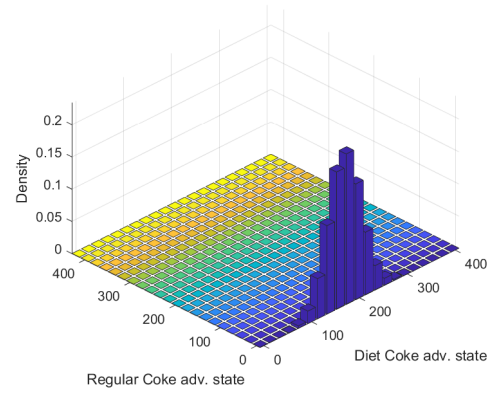
(b) Pre-policy



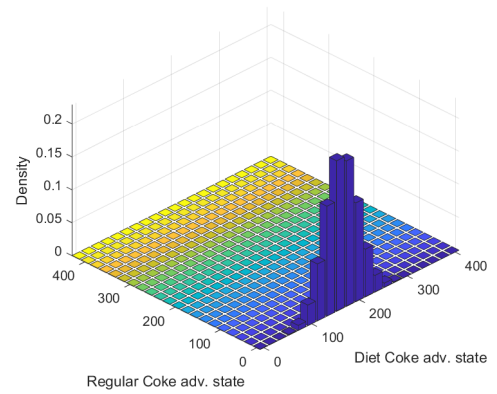
(d) Tax



(c) Advertising restriction



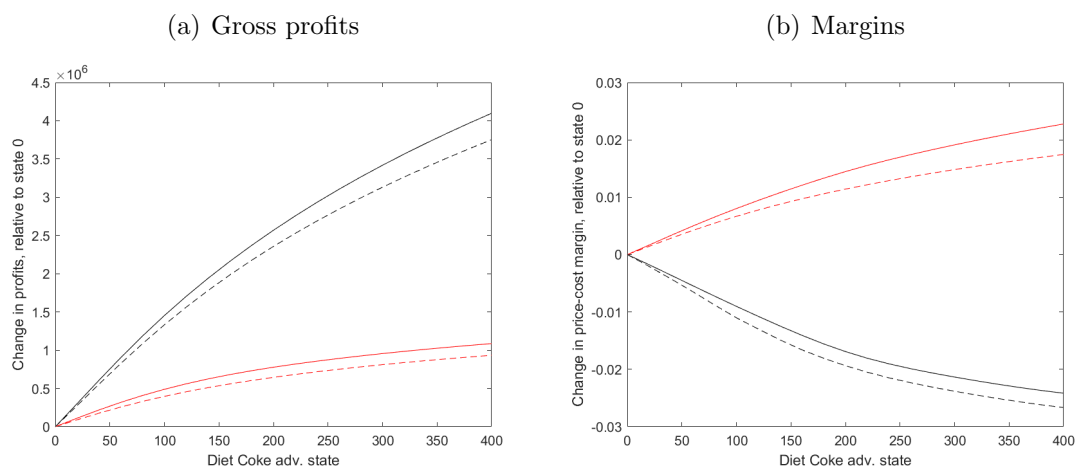
(e) Tax and advertising restriction



Notes: Panel (a) shows variation in the average price-cost margin for Regular Coke products. The hatched surface is pre-policy (and repeats Figure 5.1(a)) and the smooth surface corresponds to when a specific tax is in place. In each case we hold fixed the Diet Pepsi advertising state at the highest probability state in the pre-policy equilibrium distribution. Panels (b)-(e) show the ergodic distribution, integrating over the Diet Pepsi advertising state space. Panel (b) repeats Figure 5.3(b). In Appendix J we show the equivalent figure for the ad valorem tax.

In Figure 6.2 we illustrate why, in equilibrium, Coca Cola lowers advertising of its Diet brand. Panel (a) shows how equilibrium gross profits for Regular (red lines) and Diet (grey lines) Coke vary with the Diet Coke advertising state. We show this relationship holding the Regular Coke advertising state at its modal “no policy” equilibrium value (solid lines) and at 0 (dashed lines), corresponding to the advertising restriction. The graph shows that after the ban is in place the returns to advertising Diet Coke, both in terms of Regular and Diet Coke profits are lower. This is what leads Coca Cola to lower its equilibrium expenditure on Diet advertising. Panel (b) shows the main reason why the restriction leads to a fall in the returns to Diet advertising. In particular, it shows how the average price-cost margin for Regular and Diet Coke products change with the Diet Coke advertising state. Moving to higher Diet Coke advertising states results in the equilibrium margin for Diet Coke falling and for Regular Coke rising (reflecting a sorting of the most advertising, and due to correlation in preferences, the most price sensitive consumers towards Diet Coke). However, when the restriction is in place, the extent to which higher Diet Coke advertising lowers the average equilibrium margins for Diet Coke products rises and the extent to which it raises margins for Regular Coke products falls. This happens because, when there is zero Regular Coke advertising, raising Diet Coke advertising attracts particularly advertising (and hence price) sensitive consumers who (had Regular Coke advertising been positive) may have remained Regular Coke consumers.

Figure 6.2: *Return to Diet Coke advertising*



Notes: Figure shows how the equilibrium profits (panel (a)) and average price-cost margin (panel (b)) of Regular Coke (red lines) and Diet Coke (grey lines) vary with the Diet Coke advertising state. The dashed line holds the Regular Coke advertising state fixed at the highest probability state in the pre-policy intervention equilibrium distribution. The dashed lines hold fixed the Regular Coke advertising state at 0. In all cases the Pepsi Diet advertising state is held fixed at the highest probability state in the pre-policy intervention equilibrium distribution.

Specific tax. Column (2) of Table 6.1 shows the impact of a £0.22 per liter specific tax on Regular Coke and Pepsi, holding firms’ advertising policy functions (and hence the equilibrium distribution over states) at its pre-tax level. The tax results in a 28.8% rise in the average price of Regular Coke and Pepsi (i.e., the taxed products’) prices. This reflects both the mechanical impact of the tax on prices and firms’ equilibrium margin adjustment; on average the pass-through of the tax is around 110%, which corresponds to an increase in equilibrium price-cost margins for taxed products of 5% (see panel (a) of Figure 6.1 where we show how the average Regular Coke price-cost margins vary across the advertising state space with no tax in place (hatched surface) and the tax in place (smooth surface)). The corresponding change in Regular Coke and Pepsi equilibrium quantity is 55.1%, with overall sugar intake from drinks falling by 16.2%.²³

Column (3) shows the incremental impact of accounting for Coca Cola and Pepsi’s change in optimal advertising resulting from the introduction of the specific tax (by re-solving for the Markov perfect equilibrium). The tax results in a 33.1% reduction in spending on Regular Coke advertising. A key mechanism driving this effect is the correlation in consumers’ price and advertising sensitivities; the tax induces a large increase in Regular products’ prices, which drives away price and advertising sensitive consumers and lowers the returns to further advertising. The tax also results in a modest reduction in advertising of Diet products (panels (b) and (d) of Figure 6.1 show the implication for the equilibrium distribution over states). This lower level of advertising expenditure induces a further modest reduction in demand for Regular products of around 1%.

Column (4) shows the impact of coupling the specific tax with the advertising restriction that prohibits advertising of Regular brands. With no tax in place the advertising restriction lowers Regular Coke and Pepsi consumption by 13% and total sugar intake by 2.7%. With a tax in place, the effect of the restriction is attenuated; it leads to a reduction in Regular Coke and Pepsi consumption of 4.5% and a small fall of 0.4% in total sugar intake.

Ad valorem tax. We calibrate the ad valorem tax such that it results in approximately the same reduction in equilibrium quantity for Regular Coke and Pepsi as the specific tax, holding fixed advertising strategies. Hence, by construction, in column (5), we see the same reduction in Regular Coke and Pepsi quantity of 55.2% as in column (2). The tax rate

²³Seiler et al. (2021) study the introduction of a beverage tax (levied on both sugar and artificially sweetened drinks) in Philadelphia, a setting where a natural control group (nearby counties) exists. They find the tax raised average prices by 34%, led to 46% reduction in consumption of taxed products, and a 22% fall once cross-border shopping is accounted for. The tax we consider has a narrower base and results in larger quantity fall for taxed goods.

required to achieve this reduction is 64%. Average pass-through of the tax is around 55%, which is reflected in the 34.8% fall in equilibrium price-cost margins of the taxed products.

Column (6) shows the incremental impact of accounting for firms' advertising responses to the tax. Equilibrium advertising expenditure on Regular products falls by 47.3%, which is significantly larger than the 33.1% fall under the specific tax. This larger advertising response is linked to the under-shifting of the tax. An ad valorem (unlike a specific) tax puts a multiplicative wedge between the tax-inclusive consumer price and the tax-exclusive firm price; to increase the latter by 1% requires a 1.64% increase in the former. This puts downwards pressure on prices, inducing firms to lower their margins. Lower margins, in turn, reduce the profitability of attracting additional consumers, which acts to lower the return on advertising. As advertising of Diet products has a positive spillover to demand for Regular products, this same mechanism lowers (though to a lesser extent) the desirability of advertising Diet products – hence the ad valorem tax also results in a sizeable fall in Diet advertising. As a consequence of these larger advertising responses (relative to under a specific tax), the impact on equilibrium quantities is larger. Similarly, to the specific tax, the incremental impact of adding the advertising restriction on top of the ad valorem tax is smaller than the advertising restrictions' impact in the absence of tax.

6.2 Impact on economic surplus

In Table 6.2 we summarize the impact of each policy on economic surplus. We express numbers as percent changes relative to total consumer spending (or equivalently, firm revenue) in the no policy (observed) equilibrium. We report tax revenue, the change in Coca Cola and Pepsico profits and consumer surplus, and the sum of three, which we refer to as gross surplus. For consumer surplus we report two numbers – the static and total (i.e., static plus dynamic) effect. The static effect reflects the change in optimal prices, conditional on advertising state, and the total effect reflects both this and the change in the equilibrium distribution over states due to firms reoptimizing their advertising expenditures (see Appendix I for details). As the main channel through which policy impacts prices is through the state-specific optimal prices,²⁴ this provides an approximate decomposition of consumer surplus changes into price and advertising effects. A policymaker that wishes to discount the apparent impact of reduced advertising on utility that is based on revealed preferences (as advertising may not in fact enter the consumer's underlying experience utility function), is best using the “Static effect” numbers. The primary motivation behind policies that aim

²⁴For instance, the specific tax results in a 28.9% increase in the average price of Regular Coke and Pepsi product. 28.8% is down the state-specific price equilibrium and 0.1% due to the change in equilibrium distribution over states – see columns (2) and (3) of Table 6.1

to reduce sugar-sweetened beverage consumption is to lower the social costs of sugar consumption (which may arise through an externality due to higher health care costs, or people imposing internalities on themselves by under weighting private costs arising from future health problems). The reduction in gross surplus (which we report both based on the total and static consumer surplus numbers) must be weighed against the reduction in social costs achieved by the policies.

The advertising restriction leads to a reduction in firm profits of 2.1%. Its impact on consumer and gross surplus depends on whether or not advertising is viewed as directly contributing to consumer welfare. In the case that it is, consumer surplus fall by 4.5% and gross surplus by 6.5%. On the other hand, stripping out any consumer surplus changes resulting from the change in the distribution over advertising state leads to a fall in gross surplus of 2.1%. The advertising restriction results in a reduction in sugar from drinks of 2.7%. Both the specific and ad valorem taxes result in larger reductions in profits (of 5.6% and 9.0% respectively) and consumer surplus (which falls by around 6.5% in each case from the static pricing effect alone). These larger losses are partially offset by the fact that the taxes raise revenue, and that they achieve much larger reductions in sugar from drinks (of around 16.5%). The addition of the advertising restriction on top of either tax leads to only a small additional fall in sugar (though, under the view that advertising does not directly contribute to consumer welfare, the additional fall in gross surplus is also small).

Table 6.2: *Aggregate surplus impact of counterfactual policies*

	No tax	Specific tax		Ad valorem tax	
	Adv. restrict. (1)		Adv. restrict. (3)		Adv. restrict. (5)
	(2)	(4)			
Tax revenue	-	4.3%	3.8%	7.1%	6.4%
Δ profits	-2.1%	-5.6%	-7.0%	-9.0%	-10.1%
Consumer surplus					
Static effect	0.0%	-6.5%	-6.2%	-6.5%	-6.2%
Total effect	-4.5%	-7.2%	-10.3%	-7.7%	-10.6%
Gross surplus					
Static effect	-2.1%	-7.9%	-9.4%	-8.5%	-9.9%
Total effect	-6.5%	-8.6%	-13.6%	-9.7%	-14.2%
Δ sugar	-2.7%	-16.3%	-16.7%	-16.6%	-16.8%

Notes: Numbers (with the exception of the final row) are expressed as a percentage of pre-policy total consumer expenditure and show changes relative to the pre-policy level. We report consumer surplus changes that result from a “static effect”, which strips out advertising responses, and a “total effect” which does not. We also report gross surplus (the sum of tax revenue, profits changes and consumer surplus changes) under these two versions of consumer surplus. The final row shows the percent change in sugar from all drinks relative to pre-policy, repeating information in Table 6.1.

The main lessons from Table 6.2 are that the specific and ad valorem taxes do a similar job at reducing sugar consumption. The ad valorem tax results in a somewhat larger reduction

in gross surplus than the specific tax, however it also results in higher tax revenue (7.1% vs. 4.3%), which comes at the expense of larger reductions in firms profits as it acts to lower firms' market power. The advertising restriction (alone) results in a much more modest fall in sugar than either of the taxes. However, as long as advertising does not directly contribute to consumer welfare, the gross surplus loss from the restriction is relatively small. The case for adding an advertising restriction on top of a tax is relatively weak as it results in only a small additional reduction in sugar.

6.3 Distributional impact

The aggregate consumer surplus numbers in Table 6.2 mask heterogeneity across households. In Table 6.3 we show how each policy changes the sugar consumption and consumer surplus in each household income quartile. The numbers reflect the heterogeneity we incorporate in our demand model, by allowing all preferences parameters to vary by household income quartiles (interacted with household type). In this table we focus on the static consumer surplus effect (stripping out the effects of advertising).²⁵

A distributional analysis of the impact of advertising restrictions and taxes for sin goods, will be affected by any internalities savings the policies generate, and how these savings vary across the income distribution. To illustrate the potential importance of this channel, in Table 6.2, we also report changes in consumer surplus net of externality savings. We base our measure of internalities on the estimates in Allcott et al. (2019). They find that the externality per fl oz of sugar-sweetened beverage consumption ranges, linearly, from 1.10 cents for the lowest income group to 0.83 cents for the highest income groups. This translates to £0.0029, £0.0027, £0.0025 and £0.0022 per gram of sugar for our income quartiles 1 to 4.²⁶

Under all policies, the reduction in consumer surplus (both as a fraction of total spending, and in monetary terms) is largest for households that belong to the bottom income quartile. However, under both the specific and ad valorem taxes sugar reductions are also largest for this group. Given this, and the fact that their externality per sugar gram is higher, the taxes (whether or not they are coupled with advertising restrictions) are no longer regressive once these externality savings are accounted for.

²⁵We reproduce the table based on the total effect in Appendix J.

²⁶A fluid ounce equals 0.03l. Regular Coke and Pepsi have around 100g of sugar per 1l, so 1.10 cents per fl oz, at a 1.25 £-\$ exchange rate, corresponds to 0.29 pence per gram of sugar.

Table 6.3: *Distributional impact of counterfactual policies*

Income quartile	No tax	Specific tax		Ad valorem tax	
	Adv. restrict. (1)	(2)	Adv. restrict. (3)	(4)	Adv. restrict. (5)
Change in sugar					
Bottom	-2.88%	-17.64%	-18.12%	-17.88%	-18.25%
2nd	-2.78%	-17.07%	-17.45%	-17.23%	-17.45%
3rd	-2.32%	-17.29%	-17.63%	-17.70%	-17.96%
Top	-2.83%	-12.22%	-12.73%	-12.56%	-12.83%
Change in consumer surplus					
Bottom	0.00%	-8.13%	-7.69%	-8.07%	-7.68%
2nd	0.00%	-6.52%	-6.19%	-6.47%	-6.18%
3rd	0.00%	-7.14%	-6.87%	-7.23%	-6.99%
Top	0.00%	-4.00%	-3.74%	-4.10%	-3.86%
Change in consumer surplus net of internalities					
Bottom	1.22%	-0.68%	-0.03%	-0.52%	0.03%
2nd	1.01%	-0.36%	0.11%	-0.25%	0.12%
3rd	0.71%	-1.87%	-1.50%	-1.84%	-1.52%
Top	0.69%	-1.02%	-0.64%	-1.04%	-0.73%

Notes: Change in sugar is expressed as a percent of the income quartile specific pre-policy total drink sugar consumption. Change in consumer surplus (including net of internalities) is expressed as a percent of income quartile specific pre-policy total expenditure. The consumer surplus measure strips out advertising responses.

7 Conclusion

In this paper we develop a model of firm competition in advertising and prices, which we use to quantify the impact of sin taxes and advertising restrictions, accounting for the dynamic equilibrium response of firms' advertising strategies. We incorporate the role of advertising agencies in our model, which provides a link between the rich consumer level variation in advertising exposure and the strategic advertising expenditures which enter firms action space. We apply the model to the cola segment of the UK non-alcoholic drinks market (which is the segment of the market in which most advertising expenditures are made). We exploit variation in advertising exposure across households of the same demographic makeup and TV viewing behavior to estimate the impact of advertising on demand and solve for the Markov perfect equilibrium of the dynamic advertising game played by firms. We use our model to simulate the introduction of different forms of sin tax and a restriction on advertising.

We show that in response to the introduction of a specific or an ad valorem tax, firms lower advertising of taxed products. An important driver of this result is our finding that consumers who are price sensitive also tend to be more advertising sensitive, meaning a tax induces the most advertising sensitive consumers to switch away from taxed brands, lowering

the incentive to advertise. The reduction in advertising is larger under an ad valorem tax, as, unlike a specific tax, it leads to lower price-cost margins reducing the profitability of the marginal consumer, which lowers the incentive to advertise. We also show that both taxes and a restriction that prohibits advertising of brands that contain sugar acts to lower advertising of diet brands. This is driven by a within-firm complementarity in advertising strategies – the returns to advertising diet products is lower the lower is advertising of taxed, sugary products, which is in part driven by our finding that brand advertising has positive spillovers to the demand of other cola brands. Overall, we show that the specific and ad valorem taxes we consider lead to similar reduction in sugar and gross surplus, though the ad valorem tax raises more revenue and reduces firm profits by more, and, once externalities are accounted for the taxes are not regressive. An advertising restriction leads to a smaller reduction in sugar, and its incremental effectiveness is reduced if a tax is already in place.

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APPENDIX: FOR ONLINE PUBLICATION

The Effects of Sin Taxes and Advertising Restrictions in a Dynamic Equilibrium

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A Purchase data

In Table A.1 we report the set of cola products over which we model demand and supply. A product is defined as a firm-brand-pack combination. For each product we report its share of total cola expenditure and its average price per liter. We model consumer demand over this set of products and two outside options that are other (non-cola) drinks (either with or without sugar).

In Table A.2 we report 12 demographic groups over which we allow all consumer preference parameters to vary. These are based on the interaction of household type and income. The household types are: whether the household is working age with no child present, a pensioner households with no child present, or a household with a child present. We define working age household as one with at least one member aged 18-65 and a household with a child as one with any member aged 18 or less. We also group households based on what quartile of the equivalized income distribution they belong to. We define equivalized income as household income divided by the OECD equivalence scale. The table reports the number of households and transactions (cola and outside option purchases) for each household type.

Table A.1: *Firms and brands*

Firm	Brand	Pack	Expenditure share	Average price (£ per liter)
Coca Cola Enterprises	Regular Coke	Bottle(s): 1.25l: Single	0.6%	0.83
		Bottle(s): 1.5l: Single	0.3%	0.72
		Bottle(s): 1.75l: Single	0.5%	0.83
		Bottle(s): 1.75l: Multiple	2.7%	0.63
		Cans: 10x330ml: Single	0.9%	0.99
		Cans: 12x330ml: Single	2.5%	0.96
		Cans: 15x330ml: Single	0.6%	0.88
		Cans: 24x330ml: Single	2.1%	0.84
		Bottle(s): 2l: Single	0.9%	0.83
		Bottle(s): 2l: Multiple	4.7%	0.61
		Cans: 30x330ml: Single	1.1%	0.76
		Bottle(s): 3l: Single	1.0%	0.61
		Bottle(s): 4x1.5l: Single	0.4%	0.65
		Cans: 6x330ml: Single	1.4%	1.10
		Cans: 8x330ml: Single	6.1%	0.99
	Diet Coke	Bottle(s): 1.25l: Single	0.5%	0.84
		Bottle(s): 1.5l: Single	0.3%	0.73
		Bottle(s): 1.75l: Single	0.4%	0.85
		Bottle(s): 1.75l: Multiple	3.1%	0.62
		Cans: 10x330ml: Single	1.5%	1.02
		Cans: 12x330ml: Single	4.6%	0.97
		Cans: 15x330ml: Single	1.0%	0.88
		Cans: 24x330ml: Single	2.8%	0.83
		Bottle(s): 2l: Single	0.9%	0.80
		Bottle(s): 2l: Multiple	5.4%	0.62
		Cans: 30x330ml: Single	1.3%	0.76
		Bottle(s): 3l: Single	0.6%	0.61
		Bottle(s): 4x1.5l: Single	0.4%	0.65
		Cans: 6x330ml: Single	1.8%	1.00
		Cans: 8x330ml: Single	10.3%	0.99
Pepsico	Regular Pepsi	Bottle(s): 2l: Single	5.1%	0.52
		Cans: 6x330ml: Single	0.4%	0.82
		Cans: 8x330ml: Single	2.1%	0.82
	Diet Pepsi	Bottle(s): 1.5l: Single	0.2%	0.63
		Cans: 12x330ml: Single	0.6%	0.82
		Bottle(s): 2l: Single	15.0%	0.52
		Cans: 6x330ml: Single	0.9%	0.84
		Cans: 8x330ml: Single	9.2%	0.83
Store brands	Regular store	Bottle(s): 2l: Single	2.1%	0.18
		Bottle(s): 4x2l: Single	0.2%	0.24
	Diet store	Bottle(s): 2l: Single	3.0%	0.19
		Bottle(s): 4x2l: Single	0.5%	0.24
All			100.0%	0.74

Notes: Authors' calculations using data from Kantar FMCG At-Home Purchase Panel. Diet Coke includes Coke Zero and Diet Pepsi includes Pepsi Max.

Table A.2: *Households' demographic groups*

		Number of:	
		households	transactions
Working age	Bottom income quartile	1660	184536
	2nd income quartile	1718	192576
	3rd income quartile	1398	163288
	Top income quartile	2550	257582
Pensioner	Bottom income quartile	1455	177450
	2nd income quartile	1154	134867
	3rd income quartile	568	71455
	Top income quartile	411	46172
Household with children	Bottom income quartile	3015	385244
	2nd income quartile	3447	448110
	3rd income quartile	1950	242701
	Top income quartile	2384	281669

Notes: Numbers are for our analysis sample from the Kantar FMCG At-Home Purchase Panel.

B Advertising market and data

B.1 The UK TV market

The UK TV market is heavily regulated. Four large public service broadcasters – the BBC, ITV1, Channel 4 (C4) and Channel 5 (C5) – face restrictions on how much they advertise. The BBC is funded by an annual license fee and is not allowed to show any adverts. ITV1, C4 and C5 can show adverts and do not receive license fee income, but face some restrictions regarding programming, including the total amount of adverts shown. These public broadcasters have relatively large audience shares – BBC1 has a viewing share of around 20%, ITV around 16%, BBC2 and C4 around 7% and C5 around 5%. These channels compete for consumers by offering programs designed for broad audience appeal (see Crawford et al. (2017) for a detailed discussion of the UK television advertising market).

There are also a large number of commercial channels that do not face any specific restrictions to their programming.²⁷ Access to these additional channels is through TV subscriptions. Households can view TV in four ways: free to air, freeview, satellite or cable. All households with a TV have to pay the license fee that funds the BBC. Free to air does not require any additional payment, but gives access to only the public service broadcasters. Freeview requires purchasing a box (or freeview-ready TV) to decode the digital signal, but does not require any additional payment, and gives access to a small number of additional channels. Satellite and cable require subscriptions and provide access to a broader range of

²⁷There are also additional BBC channels (e.g., BBC3, BBC4, BBC News, BBC Parliament), which have low viewing figures and are legally prohibited from advertising.

mainly commercial channels. Any household subscribing to satellite or cable will have access to all of the free to air and freeview channels.

B.2 Advertising agencies

Table B.1: *Advertising agencies in 2016*

	Total agency advertising spend (£m) on		
	All food & drink	Coca Cola	Pepsi
Omd	94.75	-	2.52
Zenith	77.35	-	-
Carat	57.04	-	-
Mediacom	37.93	10.87	-
Um	27.49	-	-
Blue 449	24.68	-	-
Mec	20.42	-	-
Mindshare Media Uk Ltd	16.80	-	-
Rocket	15.86	-	-
Initiative Media London	8.79	-	-
Arena Media	7.59	-	-
M/six	7.51	-	-
Phd	5.65	-	-
Maxus	4.13	-	-
The7stars	4.07	-	-
Starcom	3.85	-	-
Mnc	3.69	-	-
Spirit Media Scotland Ltd	1.17	-	-
Spark Foundry	0.92	-	-
Goodstuff Communications	0.77	-	-
Direct (In House) Advertising	0.64	-	-
Specialist Works Ltd	0.62	-	-
Ams Media Group Ltd	0.43	-	-
The Lane Agency	0.36	-	-
Nick Stewart Media Consultancy	0.22	-	-
Overseas Agency - Ireland	0.21	-	-
Bray Leino	0.19	-	-
Anderson Spratt Group	0.14	-	-
Not Allocated	0.11	-	-
We Are Boutique	0.10	0.01	-
Republic Of Media	0.09	-	-
Genesis Advertising Ltd	0.05	-	-
Rla Group	0.02	-	-
Morvah	0.02	-	-
John Ayling & Associates Ltd	0.01	-	-
Juice Media Uk Ltd	0.01	-	-
Hello Starling	0.01	-	-
Di5 Ltd	0.01	-	-
Walker Communications	0.01	-	-
Tcs Media Ltd	0.00	-	-

B.3 Estimating advertising impact probability

For one year, 2015, we have data on advertising impacts, the industry standard measure of viewership. This is collected by the Broadcasters Audience Research Board (BARB).²⁸

²⁸BARB collects these data as follows: a sample of households are given a remote control with a button on it for each member of the household (and a button to register the presence of guests); each individual

Impacts are measured based on Ratecard Weighted TVR (where TVR stands for TV ratings, and is sometimes alternatively known as Gross Rating Points (GRPs)). TVRs are numbers of impacts divided by the total target audience. Ratecard weighted TVRs is the metric used by broadcasters to sell advertising slots. They apply weights to the unweighted impacts to account for differences in cost by slot length contained within that minute. Ordinarily, 1 impact refers to 1 viewer watching one 30-second advertising slot, but as a pair of 15-second slots may be of higher value to an advertiser than a single 30-second slot, unweighted impacts would be insufficient to accurately account for the value of an advert. Ratecard weighted impacts account for these differences and allow comparisons to be made in terms of advertising revenue – e.g., one slot generating 50 ratecard weighted impacts can be said to generate half as much revenue as another slot generating 100 ratecard weighted impacts.

In Table B.2 we provide some descriptive statistics on the match between our purchase data (where we have information on what shows, stations, and time slots households typically watch TV during) and our advertising data. We undertake this match for all years on our data, but in 2015 we additionally observe impacts – therefore we focus on the 2015 match in Table B.2. It shows that for all Coca Cola and Pepsico adverts in 2015, there are 35,481 adverts, that we are able to match on show. This means that we observe whether households watch the show during which the advert aired. As households in the purchase data are asked about the most popular set of shows, there are some shows in the advertising data that we are unable to match at the show level. In this case we match on station and slot (we match 77,083 adverts on this basis). There are some minor stations that households in the purchase data are not asked their viewing behavior for. In this case we can only match on the basis of slot – but as Table B.2 these adverts account for a small fraction of spending and have very low measured impacts.

Table B.2: *Match in 2015 between Kantar media data and AC Nielsen advert data*

Matched on	Total agency advertising spend (£m) on		
	No. adverts	Mean impacts (TVR)	Total expenditure (£m)
Show	35481	0.0534	7.58
Station & Time slot	77083	0.0170	8.10
Time slot only	62270	0.0007	0.83

Consumer advertising exposure (equation (2.1)) depends on measures of whether a household seen an advert during slot k , w_{ik} . Our measure of w_{ik} is based on the answers to the

must press their button each time they enter or leave the room while the television is on. Each household's TV is fitted with a meter, which records 15 seconds of audio from the TV advert and matches this to a reference library. (See <https://www.barb.co.uk/about-us/how-we-do-what-we-do/>)

show, station and time slot viewing questions that households in our purchase data provide. Households' answers to these questions are qualitative and range from "never" watch, "hardly ever", "sometimes", and "regularly". We use the fact that in 2015 we also observe impacts to estimate the viewing probabilities that correspond to these qualitative answers. Denote by $q = \{1, 2, 3\}$ the three alternative answers {"hardly ever", "sometimes", and "regularly"} and let v_{ik} denote the household i 's answer regarding slot k and w_q denote the probability corresponding to answer q .

We estimate w_q by constrained nonlinear least squares applied to:

$$TVR_k = \sum_q w_q \left(\frac{1}{N} \sum_i 1_{\{v_{ik}=q\}} \right) + e_k$$

subject to

$$0 \leq w_1 \leq w_2 \leq w_3 \leq 1$$

We estimate this separately for slots that are matched on the basis of show, and slots matched on the basis of station and time slot. Table B.3 presents the estimates.

Table B.3: *Estimates of w*

	<i>TVR</i>	
	show	station slot
w_1	0.0352 (0.0223)	0.0274 (0.0040)
w_2	0.0352 (0.0223)	0.0274 (0.0040)
w_3	0.4975 (0.1153)	0.4454 (0.0159)
N	88	1208

C Equilibrium delegation decision of advertising

To simplify notation and without loss of generality, we assume each firm sells a single product. The problem facing a firm that chooses advertising slots, without delegating choices to advertising agencies, and price is:

$$\max_{\{p_{jt}\}_{\forall t}, \{T_{jkt}\}_{\forall k,t}} \sum_{t=0}^{\infty} \beta^t \pi_{jt}(p_{1t}, \dots, p_{Jt}, (T_{11\tau}, \dots, T_{JK\tau})_{\tau \leq t}) \quad (C.1)$$

Note, this depends on other firms' decisions. We seek a Markov perfect equilibrium.

If the firm delegates advertising decisions to an advertising agency, its problem is:

$$\max_{\{p_{jt}, e_{jt}\}_{\forall t}} \sum_{t=0}^{\infty} \beta^t \pi_{jt}(p_{1t}, \dots, p_{Jt}, (T_{11t}^*(e_{1t}), \dots, T_{JKt}^*(e_{jt}))_{\tau \leq t}), \quad (\text{C.2})$$

where $T_{jk}^*(e_{jt})$ represents the optimal choice of an advertising agency given the objective to maximize impacts and the budget e_{jt} .

A firm can choose either to set prices and advertising to maximize its discounted sum of profit or choose to delegate advertising choices to an agency who maximizes impacts subject to a budget. We consider first a static game and then its dynamic extension.

C.1 Static endogenous choice of delegation of advertising

Price and advertising competition without delegation Denote the profit of firm j whose product is sold at price p_j and advertised for time T_{jk} on slot k as:

$$\pi_j(p_j, T_j, p_{-j}, T_{-j}) = (p_j - c_j)q_j(p_j, T_j, p_{-j}, T_{-j}) - \sum_k \rho_k T_{jk}$$

where T_j is the vector of $(T_{jk})_{k=1, \dots, K}$ and ρ_k is the price of adverts on channel k (k denotes channels and time slots but here we use the term channel for simplicity).

Denoting with $*$ the Nash equilibrium when firms don't delegate advertising, a Nash equilibrium $(p_j^*, T_j^*, p_{-j}^*, T_{-j}^*)$ will be solution of:

$$\max_{p_j, T_j} \pi_j(p_j, T_j, p_{-j}^*, T_{-j}^*) \equiv \pi_j^*$$

and symmetrically for firm $-j$.

Price and advertising competition with advertising delegation When the firm delegates to an advertising agency, providing an impact function $\omega(T_{j1}, \dots, T_{jK})$ to maximize (independent of prices and of the competing firm's choices), the firm's problem consists in choosing prices and an advertising budget to solve:

$$\begin{aligned} \max_{p_j, e_j} \pi_j(p_j, \tilde{T}_j(e_j), p_{-j}^{**}, \tilde{T}_{-j}(e_{-j}^{**})) &\equiv \pi_j^{**} \\ \text{where } \tilde{T}_j(e_j) &= \arg \max \omega(T_{j1}, \dots, T_{jK}) \\ \text{s.t. } \sum_k \rho_k T_{jk} &\leq e_j \end{aligned}$$

given the optimal choices of competing firms p_{-j}^{**} and e_{-j}^{**} . The Nash equilibrium $(p_j^{**}, T_j^{**}, p_{-j}^{**}, T_{-j}^{**})$ are solutions of the above problem with $T_j^{**} \equiv \tilde{T}_j(e_{-j}^{**})$.

Note that depending on the own and cross demand effects of advertising, it can be that

$$\pi_j^* \leq \pi_j^{**} \text{ or that } \pi_j^* \geq \pi_j^{**}$$

Choice of delegation of advertising Now suppose each firm can choose whether or not to delegate its advertising decisions. Assume that each firm has an additional fixed cost κ_j to solve the price and advertising game in house, which it does not face when choosing only prices and advertising budgets, while delegating to an advertising agency the slot choices maximizing impact.²⁹

If κ_j are both zero, there is only one equilibrium which is not to delegate the advertising decisions to an agency because it is always a best response to choose both price and advertising to maximize profit, given the competitors' choices. Note that this is the case even if $\pi_j^{**} \geq \pi_j^*$ because if the firm can choose to delegate or not, the equilibrium decision will be not to delegate but compete more fiercely on both prices and advertising. The reason is that if the competing firm delegates to an advertising agency, the best response should be not to delegate as the firm can then do better by not delegating. Thus in this simultaneous game, all firms will not delegate to an advertising agency.

However, when $\kappa_j > 0$, both firms choosing to delegate to an agency³⁰ can be a Nash equilibrium and firms can get higher profits by delegating. The reason is that the shape of demand can be such that delegation lowers competition in advertising, which otherwise can be strong and harmful in a business stealing market environment.

To see this in more details, denote:

- $p_j^*(p_{-j}, T_{-j})$ and $T_j^*(p_{-j}, T_{-j})$ the price and advertising best responses of j to the competing price and competing advertising if not delegating to an agency.
- $p_j^{**}(p_{-j}, T_{-j})$ and $T_j^{**}(p_{-j}, T_{-j})$ the price and advertising best responses of j through delegating to an agency, in which case $T_j^{**}(p_{-j}, T_{-j}) \equiv \tilde{T}_j(e_j^{**}(p_{-j}, T_{-j}))$ and $e_j^{**}(p_{-j}, T_{-j})$ is part of the best response of firm j to firm $-j$ as follows: $\max_{p_j, e_j} (p_j - c_j) q_j(p_j, \tilde{T}_j(e_j), p_{-j}, T_{-j}) - \sum_k \rho_k \tilde{T}_{jk}(e_j)$

²⁹We do not explicitly add here the cost that the firm may also have to incur to solve the advertising agency problem directly (choosing slots to maximize impacts) that may be part of the reason why advertising agencies charge a markup. κ_j is the extra cost from not delegating which may arise if advertising agencies have efficiency gains in solving advertising choices, specialized marketing human capital and/or knowledge of television advertising markets.

³⁰Only one firm delegating can also be an equilibrium, but we do not investigate this particular case.

We then denote $\pi_j^*(p_{-j}, T_{-j})$ the profit of firm j in case of best response to (p_{-j}, T_{-j}) without delegating and $\pi_j^{**}(p_{-j}, T_{-j})$ in case of delegation, that is:

$$\pi_j^*(p_{-j}, T_{-j}) \equiv (p_j^*(p_{-j}, T_{-j}) - c_j)q_j(p_j^*(p_{-j}, T_{-j}), T_j^*(p_{-j}, T_{-j})), p_{-j}, T_{-j}) - \sum_k \rho_k T_{jk}^*(p_{-j}, T_{-j})$$

and

$$\pi_j^{**}(p_{-j}, T_{-j}) \equiv (p_j^{**}(p_{-j}, T_{-j}) - c_j)q_j(p_j^{**}(p_{-j}, T_{-j}), T_j^{**}(p_{-j}, T_{-j})), p_{-j}, T_{-j}) - \sum_k \rho_k T_{jk}^{**}(p_{-j}, T_{-j})$$

By construction $\pi_j^{**}(p_{-j}, T_{-j}) \leq \pi_j^*(p_{-j}, T_{-j})$ for any vector (p_{-j}, T_{-j}) , thus delegating to an agency cannot be a Nash equilibrium of this static game if $\kappa_j = 0$, but can be if κ_j and κ_{-j} satisfy:

$$\pi_j^{**}(p_{-j}^{**}, T_{-j}^{**}) \geq \pi_j^*(p_{-j}^{**}, T_{-j}^{**}) - \kappa_j \quad \text{and} \quad \pi_{-j}^{**}(p_j^{**}, T_j^{**}) \geq \pi_{-j}^*(p_j^{**}, T_j^{**}) - \kappa_{-j}$$

delegating can also be an equilibrium if

$$\pi_j^*(p_{-j}^*, T_{-j}^*) - \kappa_j \geq \pi_j^{**}(p_{-j}^*, T_{-j}^*) \quad \text{and} \quad \pi_{-j}^*(p_j^*, T_j^*) - \kappa_{-j} \geq \pi_{-j}^{**}(p_j^*, T_j^*)$$

Hence firms can choose endogenously to delegate to an advertising agency and obtain higher profits than without delegation as soon as there are some fixed cost attached to solving the full intractable model of competition in prices and advertising slots. Without fixed cost, it cannot be an equilibrium of the static game, though as we show this is not the case in a dynamic game.

C.2 Endogenous choice of delegation of advertising in the repeated game

For simplicity, we consider the case where advertising has no dynamic effect on demand (because consumers are memoryless).

Consider the repeated game in which firms seek to maximize their intertemporal sum of profits with discount factor $\beta \in (0, 1)$. In this game delegating to an agency can be a subgame perfect Nash equilibrium even if $\kappa_j = \kappa_{-j} = 0$ provided firms are patient enough (β large enough). Indeed, the standard trigger strategy, which entails delegate to an advertising agency as long as the competitor delegates and deviate to the no delegation forever as soon as the competing firm does not delegate, can support tacitly the delegation equilibrium. For this, we need β large enough such that (assuming for simplicity that everything is stationary

so that all demand and profit function are time independent)

$$\frac{1}{1-\beta} \underbrace{\pi_j^{**}(p_{-j}^{**}, T_{-j}^{**})}_{\text{Profit of } j \text{ with delegation given}(p_{-j}^{**}, T_{-j}^{**})} \geq \underbrace{\pi_j^*(p_{-j}^{**}, T_{-j}^{**})}_{\text{Profit of } j \text{ without delegation given}(p_{-j}^{**}, T_{-j}^{**})} + \frac{\beta}{1-\beta} \underbrace{\pi_j^*(p_{-j}^*, T_{-j}^*)}_{\text{profit of } j \text{ under no delegation equilibrium}}$$

and symmetrically for firm $-j$:

$$\frac{1}{1-\beta} \pi_{-j}^{**}(p_j^{**}, T_j^{**}) \geq \pi_{-j}^*(p_j^{**}, T_j^{**}) + \frac{\beta}{1-\beta} \pi_{-j}^*(p_j^*, T_j^*)$$

We know that it must be that $\pi_j^*(p_{-j}^{**}, T_{-j}^{**}) \geq \pi_j^{**}(p_{-j}^{**}, T_{-j}^{**})$ but as $\frac{1}{1-\beta} > 1$ and $\frac{\beta}{1-\beta} < \frac{1}{1-\beta}$ the inequality above will be satisfied whenever

$$\beta \geq \frac{\pi_j^*(p_{-j}^{**}, T_{-j}^{**}) - \pi_j^{**}(p_{-j}^{**}, T_{-j}^{**})}{\pi_j^*(p_{-j}^{**}, T_{-j}^{**}) - \pi_j^*(p_{-j}^*, T_{-j}^*)}$$

which is always true if $\pi_j^*(p_{-j}^{**}, T_{-j}^{**}) - \pi_j^*(p_{-j}^*, T_{-j}^*) < 0$, but could be impossible if

$$\pi_j^*(p_{-j}^{**}, T_{-j}^{**}) - \pi_j^{**}(p_{-j}^{**}, T_{-j}^{**}) \geq \pi_j^*(p_{-j}^{**}, T_{-j}^{**}) - \pi_j^*(p_{-j}^*, T_{-j}^*)$$

that is $\pi_j^{**}(p_{-j}^{**}, T_{-j}^{**}) \leq \pi_j^*(p_{-j}^*, T_{-j}^*)$ meaning that the delegation can be an equilibrium of the dynamic game only if the per period profit under joint delegation of all manufacturers are larger than the per period profit under joint no delegation. If that is the case, then there exists a discount factor $\beta^* < 1$ above which the delegation is a subgame perfect Nash equilibrium of the dynamic game.

In conclusion, this simple model shows that the observed delegation of advertising to advertising agency can be rationalized as an equilibrium strategy and can be a more profitable equilibrium than the no delegation strategy.

D The impact of tax on advertising in a static single-product monopoly

In the case of a static single-product monopolist, we illustrate how tax policy impacts the profit-maximizing advertising choice. This serves to highlight two important mechanisms that determine the incentives a firm faces to alter advertising in response to the introduction (or change in the level of) a tax, which in turn can impact of equilibrium outcomes including consumption.

The static single-product monopolist chooses its price, p , and its level of advertising, A , to maximize its profits. It faces the demand function $Q(p, A)$ (where $Q_p < 0$ and $Q_A > 0$), a constant marginal cost of production, c , a specific tax, τ , and a constant marginal cost of advertising, k . The monopolist's problem is therefore to choose: $(p^*, A^*) = \arg \max_{p, A} (p - c - \tau)Q(p, A) - kA$.³¹ Denote optimal output by $Q^* \equiv Q(p^*, A^*)$, the optimal price-cost margin by $\mu^* \equiv p^* - \tau - c$ and pass-through of a marginal tax increase (holding advertising fixed) on the tax exclusive price ($p^* - \tau$), relative to the tax inclusive price, by $\rho^* \equiv \left(\frac{dp^*}{d\tau}\big|_{A^*} - 1\right) / \frac{dp}{d\tau}\big|_{A^*}$. Note $\rho^* > / < 0$ if a marginal tax rise is over/under-shifted to prices – i.e., if the monopolist increases/decreases its margin in response (holding advertising fixed). The impact of a marginal increase in the tax rate on optimal advertising depends on the following condition:³²

$$\text{sign} \left\{ \frac{dA^*}{d\tau} \right\} = \text{sign} \{ \mu^* Q_{Ap}^* + \rho^* Q_A^* \}.$$

To interpret this condition, first assume the monopolist sets an exogenous fixed margin (meaning $\frac{dp^*}{d\tau} = 1$ and $\rho^* = 0$). In this case whether the tax raises advertising depends on the cross derivative of demand, Q_{Ap}^* . A tax rise increases the (tax-inclusive) price, meaning the firm is forced to produce further up its demand curve. If, at this new higher point of the demand curve, consumers are more/less responsive to advertising then the firm is incentivized to raise/lower its level of advertising. When the firm can adjust its margin (meaning price is also a choice variable), there is a second force at play. If the firm responds to the tax by raising its margin (so $\rho^* > 0$) this will increase the profitability of the marginal consumer and, all else equal, incentivize the firm to raise advertising (with the converse being the case if $\rho^* < 0$). Hence, in the monopoly case, how the composition of demand responsiveness to advertising varies along the demand curve, and whether, in equilibrium, taxes are under- or over-shifted (which depends, inter alia, on the structure of the tax and the curvature of demand) will determine advertising responses to taxes. In addition, the fact that the monopolist can vary advertising, leads to a feedback effect on price-setting, and therefore will have direct and indirect effects on the impact of tax on equilibrium consumption.³³

³¹We assume that the profit function is concave in (p, A) .

³²The condition stated in terms of demand primitives is: $\text{sign} \left\{ \frac{dA^*}{d\tau} \right\} = \text{sign} \left\{ -\frac{Q_A^*}{Q_p^*} Q_{Ap}^* + \left(-1 + \frac{Q_{pp}^*}{(Q_p^*)^2}\right) Q_A^* \right\}$.

³³In particular, tax pass-through depends on advertising adjustment, with $\frac{d(p-\tau)^*}{d\tau} > 0$ if and only if $\left(-1 + \frac{Q_{pp}^*}{(Q_p^*)^2}\right) > \frac{1}{(-Q_p)(-Q_A)} \left(-Q_{Ap}^2 - Q_{Ap} Q_{Ap} - Q_A Q_{Ap}\right)$. In contrast, with fixed advertising, $\frac{d(p-\tau)^*}{d\tau} > 0$ if and only if $\left(-1 + \frac{Q_{pp}^*}{(Q_p^*)^2}\right) > 0$.

In reality in most markets firms sell multiple products, tax liability varies across products, firms engage in competition, and advertising has persistent impacts on consumer choice meaning that competition is dynamic in nature. Our model captures these additional determinants of advertising choice, as well as the two forces highlighted in this simple example.

E Solution to advertising agency problem

The optimal advertising length during slot k satisfies equation (3.6), which we repeat here

$$T_{bk}^* = \omega'^{-1} \left(\frac{\rho_k}{\sum_i w_{ik}} \frac{1}{\lambda_{bt}^*} \right).$$

We specify that ω is a power function, $\omega(T) = T^\gamma$, hence $(\omega')^{-1}(x) = (\frac{x}{\gamma})^{\frac{1}{\gamma-1}}$, and therefore:

$$T_{bk}^* = \left(\frac{1}{\gamma} \frac{\rho_k}{\sum_i w_{ik}} \frac{1}{\lambda_{bt}^*} \right)^{\frac{1}{\gamma-1}}.$$

Note total brand advertising expenditure is

$$e_{bt} = \sum_{\{k|t(k)=t\}} \rho_k T_{bk}^* = \sum_{\{k|t(k)=t\}} \rho_k \left(\frac{\rho_k}{\gamma \sum_i w_{ik}} \right)^{\frac{1}{\gamma-1}} \left(\frac{1}{\lambda_{bt}^*} \right)^{\frac{1}{\gamma-1}}$$

Hence, combining the last two equations, we obtain:

$$T_{bk}^* = \left(\frac{\rho_k}{\sum_i w_{ik}} \right)^{\frac{1}{\gamma-1}} \left(\sum_{\{k|t(k)=t\}} \rho_k \left(\frac{\rho_k}{\sum_i w_{ik}} \right)^{\frac{1}{\gamma-1}} \right)^{-1} e_{bt} \quad (\text{E.1})$$

Allowing for a multiplicative error in the measurement of ρ_k , this implies

$$\begin{aligned} \ln \left(\frac{\rho_k}{\sum_i w_{ik}} \right) &= \tau_{t(k)} - (1 - \gamma) \log(T_{bk}^*/e_{bt(k)}) + \omega_k \\ &= \tau_{t(kb)} - (1 - \gamma) \log(T_{bk}^*) + \omega_k \end{aligned} \quad (\text{E.2})$$

where $\tau_{t(kb)}$ is a slot-brand fixed effect.

We estimate equation (E.2) using 2015 television advertising data for all food and drink brands. We aggregate the data slightly to the level of brand-station-week-slot type, where slot type is the interaction of weekday/Saturday/Sunday with 1am-6am/6am-9.30am/9.30am-12pm/12pm-2pm/2pm-4pm/4pm-6pm/6pm-10pm/10pm-10.30pm/10.30pm-1.00am. We mea-

sure price per views, $\frac{\rho_k}{\sum_i w_{ik}}$, as advertising spend for brand-station-week-slot type divided by rate card weighted television rating among adult viewers. We measure advertising length, T_{bk}^* , as advertising duration in seconds. We report estimates in Table E.1. These correspond to the $\hat{\gamma} = 0.64$ (with p-value is smaller than 0.0001) reported in the paper.

Table E.1: *Estimation of γ*

	$\ln \left(\frac{\rho_k}{\sum_i w_{ik}} \right)$
$-(1 - \gamma)$	-0.358
	0.001
Constant	10.268
	0.005
Brand-week fixed effects	Yes
R-Square	0.08
N	2,503,591

F Additional estimation results

Table F.1: *Coefficient estimates*

Inc. qrt	No kids				Pensioner			
	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4
Price	0.173 (0.040)	0.174 (0.033)	0.050 (0.034)	-0.087 (0.037)	0.017 (0.039)	0.086 (0.040)	-0.130 (0.054)	0.012 (0.058)
Adv	-1.074 (0.147)	-1.591 (0.215)	-2.217 (0.279)	-1.415 (0.191)	-1.637 (0.304)	-1.215 (0.200)	-0.981 (0.188)	-0.981 (0.272)
Price (σ^2)	0.180 (0.017)	0.129 (0.012)	0.164 (0.016)	0.151 (0.016)	0.147 (0.014)	0.172 (0.019)	0.340 (0.045)	0.198 (0.029)
Adv (σ^2)	0.475 (0.088)	0.597 (0.104)	1.766 (0.281)	0.642 (0.151)	0.559 (0.186)	0.517 (0.137)	0.426 (0.091)	0.383 (0.185)
Price-Adv (COV)	0.283 (0.031)	0.276 (0.027)	0.463 (0.040)	0.311 (0.041)	0.079 (0.021)	0.293 (0.044)	0.348 (0.049)	0.207 (0.057)
Coke (σ^2)	2.390 (0.192)	2.062 (0.148)	1.921 (0.139)	2.385 (0.171)	2.640 (0.215)	1.563 (0.134)	2.354 (0.209)	1.834 (0.221)
Pepsi (σ^2)	3.834 (0.240)	3.943 (0.260)	3.556 (0.248)	5.882 (0.358)	5.451 (0.385)	3.831 (0.302)	4.448 (0.359)	2.941 (0.390)
Sugary (σ^2)	1.731 (0.088)	2.029 (0.099)	1.898 (0.098)	2.702 (0.130)	2.150 (0.104)	2.079 (0.105)	2.358 (0.153)	2.254 (0.161)
Adv within firm	0.126 (0.062)	0.076 (0.057)	0.142 (0.053)	0.066 (0.056)	0.234 (0.065)	0.299 (0.065)	0.118 (0.081)	0.364 (0.097)
Adv across firm	0.190 (0.061)	-0.028 (0.060)	0.096 (0.057)	0.107 (0.062)	0.440 (0.070)	0.303 (0.071)	0.093 (0.089)	-0.292 (0.108)
Entertainment \times Coke	1.156 (0.454)	-0.858 (0.440)	0.234 (0.353)	-1.477 (0.500)	0.393 (0.515)	1.418 (0.544)	-0.997 (0.564)	1.765 (0.720)
Shows \times Coke	-0.101 (0.335)	-0.130 (0.299)	-0.505 (0.225)	0.023 (0.271)	0.479 (0.297)	-1.428 (0.371)	1.306 (0.354)	0.680 (0.570)
Factual \times Coke	0.797 (0.314)	0.699 (0.289)	-0.498 (0.279)	0.705 (0.297)	0.114 (0.271)	-0.106 (0.320)	-0.298 (0.451)	-0.484 (0.492)
Drama \times Coke	-1.260 (0.361)	-0.031 (0.315)	0.326 (0.374)	-0.936 (0.323)	-0.272 (0.324)	-0.088 (0.308)	1.318 (0.378)	-1.430 (0.504)
Reality \times Coke	-1.157 (0.434)	1.698 (0.456)	0.810 (0.437)	-0.862 (0.461)	0.533 (0.536)	-1.309 (0.604)	1.034 (0.716)	2.575 (0.946)
Sports \times Coke	1.057 (0.175)	0.602 (0.186)	-0.031 (0.169)	-0.197 (0.167)	-1.221 (0.182)	-0.273 (0.159)	-0.513 (0.193)	0.025 (0.270)
Entertainment \times Pepsi	-0.909 (0.463)	0.380 (0.517)	0.056 (0.447)	0.558 (0.521)	-2.768 (0.624)	1.830 (0.585)	-2.161 (0.731)	-2.044 (0.924)
Shows \times Pepsi	0.865 (0.297)	-0.880 (0.362)	-1.200 (0.420)	-1.648 (0.394)	-0.199 (0.399)	-2.538 (0.403)	0.806 (0.445)	3.575 (0.448)
Factual \times Pepsi	-1.052 (0.340)	-1.120 (0.347)	1.006 (0.405)	1.785 (0.514)	0.679 (0.442)	0.612 (0.397)	-0.597 (0.501)	-2.840 (0.703)
Drama \times Pepsi	-0.498 (0.387)	0.791 (0.369)	-0.057 (0.476)	0.642 (0.476)	-0.365 (0.368)	-0.293 (0.365)	1.336 (0.489)	2.083 (0.604)
Reality \times Pepsi	1.210 (0.450)	3.152 (0.662)	2.082 (0.727)	0.588 (0.602)	1.341 (0.604)	3.091 (0.590)	2.704 (0.787)	0.546 (1.267)
Sports \times Pepsi	0.628 (0.177)	0.728 (0.217)	-0.042 (0.235)	-0.226 (0.197)	-1.301 (0.226)	0.754 (0.204)	0.356 (0.253)	-0.262 (0.326)
ITV \times Coke	0.480 (0.169)	-0.237 (0.118)	0.126 (0.097)	0.188 (0.114)	-0.180 (0.110)	0.216 (0.128)	-0.376 (0.129)	-0.600 (0.183)
C4 \times Coke	-0.105 (0.123)	0.007 (0.126)	0.192 (0.102)	-0.222 (0.105)	-0.388 (0.124)	-0.428 (0.109)	0.015 (0.178)	-0.515 (0.196)
C5 \times Coke	-0.166 (0.123)	-0.635 (0.130)	-0.219 (0.110)	-0.191 (0.108)	-0.239 (0.120)	-0.024 (0.106)	-0.239 (0.160)	0.132 (0.180)
Cable \times Coke	0.984 (0.138)	0.380 (0.119)	0.331 (0.112)	0.633 (0.111)	-0.141 (0.121)	0.273 (0.116)	0.202 (0.130)	-0.082 (0.181)
ITV \times Pepsi	-0.257 (0.153)	-0.681 (0.141)	-0.335 (0.118)	0.327 (0.176)	0.097 (0.143)	-0.087 (0.161)	-0.200 (0.201)	-0.262 (0.266)
C4 \times Pepsi	0.035 (0.118)	0.020 (0.138)	0.233 (0.134)	0.516 (0.152)	-0.348 (0.143)	-0.571 (0.154)	0.144 (0.227)	0.441 (0.327)
C5 \times Pepsi	0.089 (0.124)	0.243 (0.132)	-0.312 (0.202)	-0.926 (0.169)	0.044 (0.138)	0.120 (0.148)	-1.001 (0.186)	-0.031 (0.314)
Cable \times Pepsi	-0.102 (0.134)	0.157 (0.133)	0.097 (0.144)	1.079 (0.151)	0.806 (0.144)	0.073 (0.158)	-0.097 (0.149)	0.694 (0.220)
Wkend-prime \times Coke	0.289 (0.222)	-0.152 (0.170)	-0.054 (0.140)	-0.369 (0.168)	-0.781 (0.229)	-1.306 (0.238)	0.818 (0.311)	-0.244 (0.307)
Wkend-non prime \times Coke	-0.337 (0.168)	-0.394 (0.127)	-0.513 (0.113)	0.505 (0.134)	-0.155 (0.170)	0.777 (0.162)	0.490 (0.211)	-0.298 (0.252)
Wkday-prime \times Coke	-0.368 (0.277)	0.380 (0.203)	0.403 (0.183)	-0.169 (0.168)	0.140 (0.281)	0.326 (0.300)	0.007 (0.267)	-0.479 (0.313)
Wkday-non prime \times Coke	-0.500 (0.168)	0.145 (0.144)	0.278 (0.105)	-0.106 (0.117)	-0.066 (0.181)	-0.390 (0.187)	0.379 (0.194)	-0.198 (0.183)
Wkend-prime \times Pepsi	-0.092 (0.206)	-0.496 (0.209)	-0.173 (0.216)	-0.607 (0.207)	0.290 (0.357)	-0.239 (0.293)	0.595 (0.352)	0.604 (0.504)
Wkend-non prime \times Pepsi	0.065 (0.162)	0.383 (0.175)	0.533 (0.152)	-0.226 (0.187)	-0.372 (0.241)	0.821 (0.219)	-0.569 (0.220)	0.544 (0.284)
Wkday-prime \times Pepsi	0.517 (0.220)	0.570 (0.281)	-0.208 (0.231)	-1.041 (0.281)	1.133 (0.422)	0.511 (0.383)	0.428 (0.341)	-0.548 (0.406)
Wkday-non prime \times Pepsi	0.233 (0.150)	0.062 (0.161)	-0.236 (0.152)	-0.183 (0.155)	-0.844 (0.241)	-0.360 (0.215)	0.295 (0.211)	-0.031 (0.277)
Viewing hours \times Coke	-0.125 (0.087)	0.007 (0.077)	-0.060 (0.072)	-0.043 (0.063)	-0.389 (0.087)	-0.048 (0.087)	-0.105 (0.112)	0.072 (0.079)
Viewing hours \times Pepsi	-0.262 (0.064)	-0.188 (0.075)	-0.141 (0.074)	0.238 (0.103)	-0.600 (0.107)	-0.170 (0.117)	-0.219 (0.148)	-0.039 (0.158)

Table F.2: *Coefficient estimates cont.*

Inc. qrt	Family			
	Q1	Q2	Q3	Q4
Price	0.154 (0.031)	0.149 (0.032)	0.092 (0.033)	-0.036 (0.033)
Adv	-2.754 (0.652)	-1.658 (0.232)	-2.210 (0.332)	-1.372 (0.166)
Price (σ^2)	0.145 (0.012)	0.118 (0.011)	0.159 (0.014)	0.118 (0.013)
Adv (σ^2)	0.777 (0.424)	0.659 (0.194)	0.889 (0.257)	0.451 (0.082)
Price-Adv (COV)	-0.015 (0.013)	0.229 (0.040)	0.339 (0.053)	0.230 (0.027)
Coke (σ^2)	2.448 (0.172)	2.401 (0.174)	2.059 (0.156)	1.983 (0.136)
Pepsi (σ^2)	3.169 (0.229)	3.999 (0.251)	4.178 (0.338)	3.677 (0.238)
Sugary (σ^2)	1.773 (0.088)	1.904 (0.096)	1.909 (0.096)	1.720 (0.088)
Adv within firm	0.063 (0.053)	0.065 (0.055)	0.046 (0.054)	0.123 (0.054)
Adv across firm	0.134 (0.057)	0.034 (0.058)	0.080 (0.057)	-0.124 (0.058)
Entertainment \times Coke	-0.283 (0.331)	0.325 (0.375)	-1.250 (0.392)	-0.065 (0.402)
Shows \times Coke	0.346 (0.259)	-0.789 (0.295)	0.825 (0.248)	-0.050 (0.250)
Factual \times Coke	0.391 (0.279)	0.297 (0.261)	-0.422 (0.256)	-0.842 (0.252)
Drama \times Coke	-1.472 (0.389)	0.862 (0.349)	-0.222 (0.422)	0.330 (0.444)
Reality \times Coke	1.619 (0.357)	-0.915 (0.367)	1.702 (0.452)	1.238 (0.441)
Sports \times Coke	-0.610 (0.154)	0.016 (0.177)	-0.819 (0.210)	0.434 (0.153)
Entertainment \times Pepsi	0.598 (0.372)	0.219 (0.489)	-0.825 (0.403)	0.230 (0.500)
Shows \times Pepsi	0.402 (0.254)	0.518 (0.353)	0.338 (0.303)	-1.426 (0.309)
Factual \times Pepsi	-0.759 (0.308)	-1.878 (0.309)	0.383 (0.311)	0.998 (0.390)
Drama \times Pepsi	-1.698 (0.370)	0.193 (0.486)	-0.452 (0.401)	0.691 (0.852)
Reality \times Pepsi	3.237 (0.414)	-0.486 (0.418)	-0.024 (0.669)	1.898 (0.528)
Sports \times Pepsi	-0.086 (0.196)	0.017 (0.210)	-0.173 (0.212)	0.152 (0.192)
ITV \times Coke	0.109 (0.113)	0.083 (0.112)	-0.105 (0.161)	-0.308 (0.107)
C4 \times Coke	-0.493 (0.119)	0.452 (0.108)	0.001 (0.119)	-0.559 (0.105)
C5 \times Coke	-0.358 (0.113)	-0.390 (0.108)	-0.090 (0.125)	-0.273 (0.146)
Cable \times Coke	0.188 (0.117)	0.134 (0.129)	0.339 (0.146)	-0.051 (0.102)
ITV \times Pepsi	0.103 (0.123)	0.002 (0.131)	-0.766 (0.167)	0.400 (0.140)
C4 \times Pepsi	-0.635 (0.144)	0.472 (0.127)	0.393 (0.119)	-1.129 (0.134)
C5 \times Pepsi	-0.160 (0.137)	0.223 (0.122)	0.427 (0.153)	0.135 (0.145)
Cable \times Pepsi	0.174 (0.131)	0.616 (0.125)	-0.031 (0.141)	0.568 (0.150)
Wkend-prime \times Coke	-0.167 (0.157)	0.234 (0.163)	-0.518 (0.198)	-0.038 (0.141)
Wkend-non prime \times Coke	0.069 (0.122)	-0.115 (0.128)	0.477 (0.146)	-0.023 (0.123)
Wkday-prime \times Coke	0.293 (0.171)	-0.073 (0.213)	0.327 (0.193)	0.082 (0.149)
Wkday-non prime \times Coke	-0.241 (0.113)	-0.059 (0.113)	0.190 (0.130)	0.402 (0.104)
Wkend-prime \times Pepsi	0.338 (0.183)	-0.182 (0.218)	0.608 (0.236)	-0.515 (0.184)
Wkend-non prime \times Pepsi	-0.280 (0.128)	0.216 (0.135)	-0.221 (0.216)	-0.076 (0.188)
Wkday-prime \times Pepsi	0.352 (0.192)	0.543 (0.226)	-0.080 (0.203)	0.478 (0.203)
Wkday-non prime \times Pepsi	0.213 (0.122)	-0.400 (0.130)	0.852 (0.190)	0.069 (0.170)
Viewing hours \times Coke	0.014 (0.087)	0.118 (0.087)	-0.103 (0.079)	0.059 (0.056)
Viewing hours \times Pepsi	-0.074 (0.104)	0.158 (0.078)	0.001 (0.074)	-0.031 (0.080)

Table F.3: *Price-level price elasticities*

	Reg Coke		Diet Coke		Reg Pepsi		Diet Pepsi	
	2l	10×330ml	2l	10×330ml	2l	8×330ml	2l	10×330ml
Regular Coke: 1.5l	0.047	0.041	0.024	0.034	0.037	0.012	0.062	0.024
Regular Coke: 2l	-1.915	0.044	0.024	0.040	0.039	0.013	0.061	0.024
Regular Coke: 10×330ml	0.023	-3.829	0.013	0.044	0.035	0.014	0.058	0.033
Regular Coke: 24×330ml	0.012	0.051	0.006	0.044	0.029	0.015	0.046	0.037
Diet Coke: 1.5l	0.024	0.021	0.049	0.059	0.018	0.006	0.099	0.038
Diet Coke: 2l	0.023	0.024	-1.793	0.069	0.020	0.006	0.097	0.038
Diet Coke: 10×330ml	0.012	0.026	0.021	-3.844	0.016	0.007	0.085	0.051
Diet Coke: 24×330ml	0.007	0.026	0.011	0.078	0.014	0.007	0.072	0.056
Reg Pepsi: 2l	0.008	0.013	0.004	0.011	-2.019	0.091	0.361	0.156
Regular Pepsi: 8×330ml	0.007	0.015	0.004	0.012	0.242	-2.890	0.332	0.171
Diet Pepsi: 1.5l	0.005	0.006	0.008	0.014	0.117	0.037	0.565	0.214
Diet Pepsi: 2l	0.005	0.007	0.008	0.019	0.119	0.041	-1.951	0.240
Diet Pepsi: 8×330ml	0.004	0.008	0.006	0.022	0.101	0.042	0.473	-3.302
Regular store: 2l	0.011	0.015	0.006	0.012	0.047	0.016	0.073	0.030
Diet store: 2l	0.006	0.008	0.011	0.022	0.024	0.008	0.116	0.048
Regular outside	0.011	0.012	0.007	0.011	0.039	0.012	0.068	0.026
Diet outside	0.007	0.007	0.012	0.019	0.021	0.007	0.108	0.040

Table F.4: *Brand price and advertising elasticities, with no advertising spillovers*

(1)	Price elasticities				Advertising elasticities		
	Coke		Pepsi		Coke		Pepsi
	Regular (2)	Diet (3)	Regular (4)	Diet (5)	Regular (6)	Diet (7)	Diet (8)
Regular Coke	-2.210	0.511	0.050	0.092	0.115	0.043	0.020
Diet Coke	0.378	-2.192	0.023	0.142	0.054	0.110	0.016
Regular Pepsi	0.210	0.128	-1.842	0.552	0.021	0.020	0.015
Diet Pepsi	0.110	0.232	0.157	-1.679	0.015	0.011	0.057
Regular Store	0.243	0.155	0.063	0.106	-0.021	-0.017	-0.011
Diet Store	0.130	0.276	0.031	0.170	-0.020	-0.021	-0.012
Regular outside	0.185	0.138	0.050	0.095	-0.020	-0.017	-0.009
Diet outside	0.104	0.236	0.027	0.152	-0.019	-0.021	-0.011

Notes: Numbers repeat those in Table 4.1, but based on demand estimates with no spillover effects (i.e., where we re-estimate the model constraining $\beta_d^W = \beta_d^X = 0$ for all d .)

Table F.5: *Product level markups*

Firm	Brand	Pack	Marginal cost (£/l)	Price-cost margin (£/l)	Lerner index
Coca Cola Enterprises	Regular Coke	Bottle(s): 1.25l: Single	0.07	0.77	0.92
		Bottle(s): 1.5l: Single	0.21	0.71	0.77
		Bottle(s): 1.75l: Single	0.12	0.78	0.87
		Bottle(s): 1.75l: Multiple	0.33	0.41	0.56
		Cans: 10x330ml: Single	0.60	0.42	0.41
		Cans: 12x330ml: Single	0.57	0.38	0.40
		Cans: 15x330ml: Single	0.58	0.39	0.40
		Cans: 24x330ml: Single	0.58	0.24	0.29
		Bottle(s): 2l: Single	0.17	0.70	0.80
		Bottle(s): 2l: Multiple	0.30	0.34	0.53
		Cans: 30x330ml: Single	0.56	0.24	0.30
		Bottle(s): 3l: Single	0.29	0.30	0.50
		Bottle(s): 4x1.5l: Single	0.41	0.31	0.43
		Cans: 6x330ml: Single	0.73	0.64	0.47
		Cans: 8x330ml: Single	0.57	0.42	0.42
	Diet Coke	Bottle(s): 1.25l: Single	0.03	0.82	0.96
		Bottle(s): 1.5l: Single	0.10	0.70	0.88
		Bottle(s): 1.75l: Single	0.09	0.79	0.90
		Bottle(s): 1.75l: Multiple	0.31	0.41	0.56
		Cans: 10x330ml: Single	0.59	0.42	0.42
		Cans: 12x330ml: Single	0.56	0.37	0.40
		Cans: 15x330ml: Single	0.50	0.39	0.44
		Cans: 24x330ml: Single	0.58	0.25	0.30
		Bottle(s): 2l: Single	0.03	0.67	0.96
		Bottle(s): 2l: Multiple	0.26	0.33	0.56
		Cans: 30x330ml: Single	0.56	0.24	0.30
		Bottle(s): 3l: Single	0.30	0.28	0.48
		Bottle(s): 4x1.5l: Single	0.44	0.32	0.42
		Cans: 6x330ml: Single	0.69	0.55	0.44
		Cans: 8x330ml: Single	0.58	0.41	0.42
Pepsico	Regular Pepsi	Bottle(s): 2l: Single	0.14	0.38	0.74
		Cans: 6x330ml: Single	0.27	0.59	0.68
		Cans: 8x330ml: Single	0.36	0.47	0.56
	Diet Pepsi	Bottle(s): 1.5l: Single	-0.03	0.66	1.04
		Cans: 12x330ml: Single	0.49	0.48	0.49
		Bottle(s): 2l: Single	0.16	0.37	0.70
		Cans: 6x330ml: Single	0.28	0.59	0.68
		Cans: 8x330ml: Single	0.44	0.41	0.48

G Transition function

The mean exposure flow for brand b advertising is

$$\mathfrak{a}_{bt} = \frac{1}{I} \sum_i \sum_{\{k|t(k)=t\}} w_{ik} \omega(T_{bk}^*),$$

and the mean exposure stock is

$$\mathbb{A}_{bt} = \sum_{s=0}^{t-1} \delta^{t-1-s} \mathfrak{a}_{bs} = \delta \mathbb{A}_{bt-1} + \mathfrak{a}_{bt-1}.$$

Given our power function specification for $\omega(\cdot)$, $\omega(T_{bk}^*) = T_{bk}^{*\gamma}$, and the optimality condition for T_{bk}^* (equation (E.1)), this implies that

$$\begin{aligned} \mathbb{A}_{bt} - \delta \mathbb{A}_{bt-1} &= \frac{1}{I} \sum_i \sum_{\{k|t(k)=t-1\}} w_{ik} T_{bk}^{*\gamma} \\ &= \frac{1}{I} \sum_i \sum_{\{k|t(k)=t-1\}} w_{ik} \left(\left(\frac{\rho_k}{\sum_i w_{ik}} \right)^{\frac{1}{\gamma-1}} \left(\sum_{\{k|t(k)=t\}} \rho_k \left(\frac{\rho_k}{\sum_i w_{ik}} \right)^{\frac{1}{\gamma-1}} \right)^{-1} \right)^{\gamma} e_{bt-1}^{\gamma} \\ &\equiv \lambda_{t-1} e_{bt-1}^{\gamma} \end{aligned}$$

Defining λ as $\mathbb{E}[\mathbb{A}_{bt} - \delta \mathbb{A}_{t-1}] = \lambda e_{bt-1}^{\gamma}$, we get

$$\mathbb{A}_{bt} - \delta \mathbb{A}_{bt-1} = \lambda e_{bt-1}^{\gamma} + \nu_{bt-1}$$

with $\nu_{bt-1} = (\lambda_{t-1} - \lambda) e_{bt-1}^{\gamma}$.

H Solution algorithm

Our solution algorithm is similar in spirit to that of Pakes and McGuire (1994).

State space discretization. The state space consists of the expected value of the exposure stock for each of brand, $(\mathbb{A}_{1t}, \dots, \mathbb{A}_{Bt})$ (see Section 5.1). In our application $B = 3$ (corresponding to Regular Coke (*RC*), Diet Coke (*DC*) and Diet Pepsi (*DP*)). For each b we discretize the state spaced into $K = 21$ evenly spaced values, A_1, \dots, A_K . We set a value for A_K above the 99th percentile of observed mean stocks in the data and check ex post that the maximum state has zero probability mass in the equilibrium ergodic distribution. The state space is of dimension $21^3 = 9,261$. Denote by $a_{\mathbf{k}}$ a single point in the state space grid (this corresponds to discrete advertising levels for each brand $((A_{RC,k}, A_{DC,k'}, A_{DP,k''})$ where $k, k', k'' \in \{1, \dots, 21\}$).

Profit function. In our application there are two firms, $f = \{C, P\}$, which correspond to Coca Cola Enterprises and Pepsico. Denote the state-specific gross profit function (i.e., prior to deducting any advertising expenditure) of firm f by $\pi_f(a_{\mathbf{k}})$. Note, $\pi_f(a_{\mathbf{k}})$ is evaluated at the state specific equilibrium price vector $\mathbf{p}(a_{\mathbf{k}})$. We compute $\pi_f(a_{\mathbf{k}})$ for $f \in \{C, P\}$ in each of the 9,261 states. This entails, at each point in the state space grid, solving the price vector that satisfies the set of first order conditions (equation (3.3)). In matrix notation,

these conditions are:

$$\mathbf{p}(a_k) = \mathbf{c} - \left[\mathbf{\Gamma} \circ \left(\frac{\partial \mathbf{q}(a_k, \mathbf{p}(a_k))}{\partial \mathbf{p}} \right) \right]^{-1} \mathbf{q}(a_k, \mathbf{p}(a_k))$$

where $\mathbf{\Gamma}$ is the product ownership matrix. Re-write this as $\mathbf{p}_k = f_k(\mathbf{p}_k)$. We start with an initial guess of \mathbf{p}_k^r , compute $\mathbf{p}_k^{r+1} = f_k(\mathbf{p}_k^r)$ and continue updating until $\|\mathbf{p}_k^{r+1} - \mathbf{p}_k^r\| = \max |\mathbf{p}_k^{r+1} - \mathbf{p}_k^r| < 10^{-4}$. Once we have obtained state-specific equilibrium prices we also compute the state-specific equilibrium quantity vector, $\mathbf{q}(a_k)$, and consumer surplus, $\text{CS}(a_k)$.

Our counterfactual simulations entail the imposition of a specific and (separately) an ad valorem tax. In order to implement these counterfactuals we must repeat the computation of the state-specific profit functions with each tax in place.

Bellman equations. Let $a = (a_{RC}, a_{DC}, a_{DP})$ denote the current levels of the Regular Coke, Diet Coke and Diet Pepsi advertising states. The two firms value functions are joint solutions of:

$$V_C(a, e_{RC}, e_{DC}) = \pi_C(a) + \max_{e_{RC}, e_{DC} \in \mathbb{R}^+} \left\{ -(\psi_{RC}e_{RC} + \psi_{DC}e_{DC}) + \beta \sum_{a'_{RC}, a'_{DC}} \right. \quad (\text{H.1})$$

$$\left. \bar{V}_C(a'_{RC}, a'_{DC}, e_{RC}, e_{DC}) p(a'_{RC}|a_{RC}, e_{RC}) p(a'_{DC}|a_{DC}, e_{DC}) \right\}$$

$$V_P(a, e_{DP}) = \pi_P(a) + \max_{e_{DP} \in \mathbb{R}^+} \left\{ -\psi_{DP}e_{DP} + \beta \sum_{a'_{DP}} \bar{V}_P(a'_{DP}, e_{DP}) p(a'_{DP}|a_{DP}, e_{DP}) \right\}, \quad (\text{H.2})$$

where

$$\bar{V}_C(a'_{RC}, a'_{DC}, e_{RC}, e_{DC}) = \sum_{a'_{DP}} V_C(a', e_{RC}, e_{DC}) p(a'_{DP}|a_{DP}, e_{DP})$$

$$\bar{V}_P(a'_{DP}, e_{DP}) = \sum_{a'_{RC}, a'_{DC}} V_P(a', e_{RC}, e_{DC}) p(a'_{RC}|a_{RC}, e_{RC}) p(a'_{DC}|a_{DC}, e_{DC}),$$

and the transition function, $p(a'_b|a_b, e_b)$, is given by equation (5.2).

Solving for the MPE. The solution algorithm is as follows:

1. Start with an initial guess of optimal advertising expenditures and value functions in each advertising state. When solving for the no tax equilibrium we use as starting

values, for all \mathbb{k} :

$$e_{RC}^l(a_{\mathbb{k}}) = e_{DC}^l(a_{\mathbb{k}}) = 0.3e^6, \quad e_{DP}^l(a_{\mathbb{k}}) = 0.2e^6 \quad V_C^l(a_{\mathbb{k}}) = \frac{\pi_C}{1-\beta} \quad V_P^l(a_{\mathbb{k}}) = \frac{\pi_P}{1-\beta}$$

When solving for the specific or ad valorem tax equilibrium we use the optimal values from the no tax equilibrium as starting values.

2. For each point in the state space, \mathbb{k} , use equations (H.1) and (H.2), evaluated at the initial guess of $(V_C^l(a_{\mathbb{k}}), V_R^l(a_{\mathbb{k}}), e_{CR}^l(a_{\mathbb{k}}), e_{CD}^l(a_{\mathbb{k}}), e_{PD}^l(a_{\mathbb{k}}))$ to solve for the optimal advertising expenditures $\tilde{e}_{CR}^{l+1}(a_{\mathbb{k}}), \tilde{e}_{CD}^{l+1}(a_{\mathbb{k}}), \tilde{e}_{PD}^{l+1}(a_{\mathbb{k}})$.
3. Use as the iteration $l+1$ advertising expenditures $e_b^{l+1}(a_{\mathbb{k}}) = (1-\lambda)e_b^l(a_{\mathbb{k}}) + \lambda\tilde{e}_b^{l+1}(a_{\mathbb{k}})$ with dampening parameter $\lambda = 0.5$.
4. Use these expenditures to evaluate the right hand side equations (H.1) and (H.2) and thereby update the value functions $(V_C^{l+1}(a_{\mathbb{k}}), V_P^{l+1}(a_{\mathbb{k}}))$.
5. Repeat steps 2-4 until the stopping criteria, for $f = \{C, P\}$:

$$\left\| \frac{V_f^{l+1} - V_f^l}{1 + |V_f^l|} \right\| = \max_{\mathbb{k}} \left| \frac{V_f^{l+1} - V_f^l}{1 + |V_f^l|} \right| < 10^{-6}$$

is satisfied.

I Consumer surplus decomposition

Denote the advertising state-specific consumer surplus under regime $\chi \in \{0, \mathfrak{s}, \mathfrak{a}\}$ (corresponding to no-tax, specific tax and ad valorem tax), by $\text{cs}_{\chi}(\mathbb{A}, \mathbf{p}_{\chi}(\mathbb{A}))$, where $\mathbb{A} = \{\mathbb{A}\}_b$ denotes the value of the brand advertising state and $\mathbf{p}_{\chi}(\mathbb{A})$ the optimal price vector. Denote the equilibrium distribution over states in regime $\chi \in \{0, \mathfrak{r}, \mathfrak{s}, \mathfrak{s}\mathfrak{r}, \mathfrak{a}, \mathfrak{a}\mathfrak{r}\}$ (where \mathfrak{r} corresponds to advertising restriction) by $g_{\chi}(\mathbb{A})$. Consider the change in equilibrium consumer surplus that results from the introduction of a specific tax (relative to when no tax is in place, and where advertising is unrestricted). This is given by:

$$\Delta \text{CS}_s = \int_{\mathbb{A}} \text{cs}_s(\mathbb{A}, \mathbf{p}_s(\mathbb{A})) g_s(\mathbb{A}) - \int_{\mathbb{A}} \text{cs}_0(\mathbb{A}, \mathbf{p}_0(\mathbb{A})) g_0(\mathbb{A}).$$

We decompose this into a static component, which reflects the change in the state-specific consumer surplus function, and a dynamic component, which reflects the change in the

equilibrium distribution over states. In particular:

$$\Delta \text{CS}_s = \underbrace{\int_{\mathbb{A}} \left(\frac{1}{2} g_0(\mathbb{A}) + \frac{1}{2} g_s(\mathbb{A}) \right) \left(\text{cs}_s(\mathbb{A}, \mathbf{p}_s(\mathbb{A})) - \text{cs}_0(\mathbb{A}, \mathbf{p}_0(\mathbb{A})) \right)}_{\text{static effect}} + \underbrace{\int_{\mathbb{A}} \left(\frac{1}{2} \text{cs}_s(\mathbb{A}, \mathbf{p}_s(\mathbb{A})) + \frac{1}{2} \text{cs}_0(\mathbb{A}, \mathbf{p}_0(\mathbb{A})) \right) \left(g_s(\mathbb{A}) - g_0(\mathbb{A}) \right)}_{\text{dynamic effect}}.$$

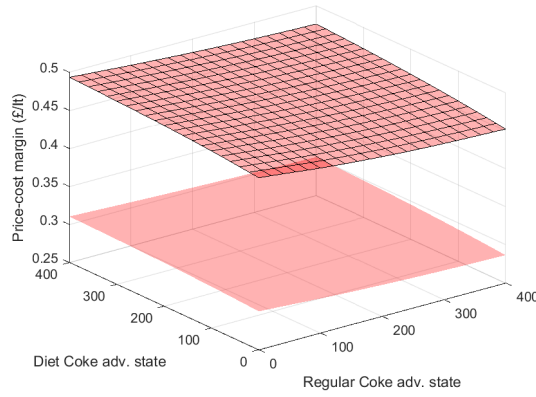
We decompose the consumer surplus effects of the other policy interventions analogously. Notice that the advertising restriction only impacts the equilibrium distribution, so the impact of an advertising restriction (in the absence of any tax) engenders zero static effect.

J Additional counterfactual results

Figure J.1: *Impact of ad valorem tax and advertising restriction*

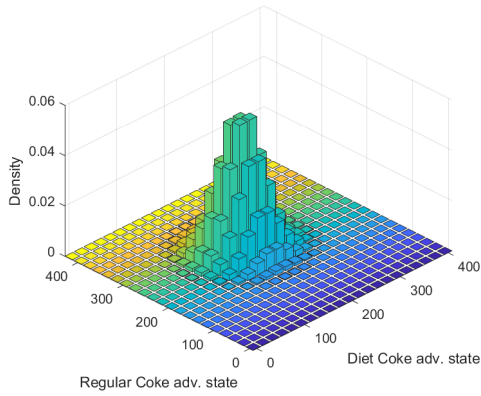
On static-specific optimal margins

(a) Average Regular Coke margins

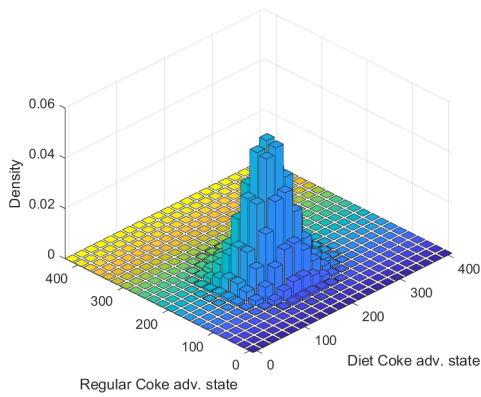


On equilibrium distribution

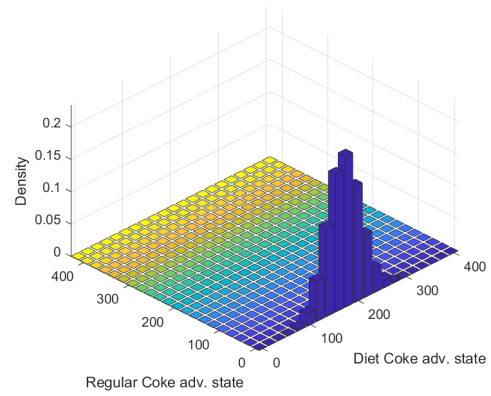
(b) Pre-policy



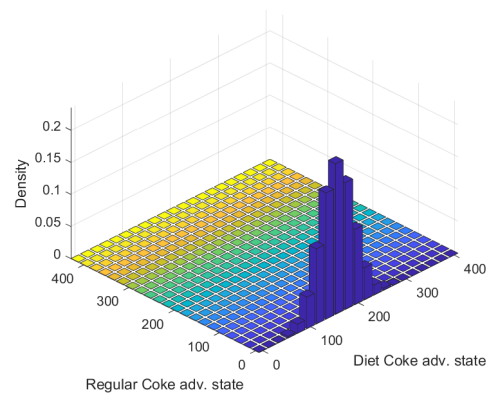
(d) Tax



(c) Advertising restriction



(e) Tax and advertising restriction



Notes: Panel (a) shows variation in the average price-cost margin for Regular Coke products. The hatched surface is pre-policy (and repeats Figure 5.1(a)) and the smooth surface corresponds to when an ad valorem tax is in place. In each case we hold fixed the Diet Pepsi advertising state at the highest probability state in the pre-policy equilibrium distribution. Panels (b)-(e) show the ergodic distribution, integrating over the Diet Pepsi advertising state space. Panel (b) repeats Figure 5.3(b).

Table J.1: *Aggregate impact of counterfactual policies, by brand*

	No tax	Specific tax			Ad valorem tax		
	Adv. restrict. (1)	Fixed adv. (2)	+ Eq. adv. response (3)	+ Adv. restrict. (4)	Fixed adv. (5)	+ Eq. adv. response (6)	+ Adv. restrict. (7)
Δ price							
Reg Coke	0.9%	28.2%	0.1%	0.6%	38.4%	0.1%	0.5%
Diet Coke	-1.3%	-1.6%	-0.1%	-0.8%	-1.6%	-0.2%	-0.7%
Reg Pepsi	-0.1%	34.2%	-0.0%	-0.1%	25.6%	-0.1%	-0.1%
Diet Pepsi	-0.0%	-0.6%	-0.0%	-0.0%	-0.2%	-0.0%	-0.0%
Reg Store	-	-	-	-	-	-	-
Diet Store	-	-	-	-	-	-	-
Reg Outside	-	-	-	-	-	-	-
Diet Outside	-	-	-	-	-	-	-
Δ margin							
Reg Coke	1.9%	5.0%	0.3%	1.3%	-34.6%	0.2%	0.7%
Diet Coke	-2.8%	-3.4%	-0.3%	-1.8%	-3.6%	-0.5%	-1.6%
Reg Pepsi	-0.1%	5.7%	-0.0%	-0.2%	-35.9%	-0.1%	-0.1%
Diet Pepsi	-0.0%	-0.9%	-0.0%	-0.0%	-0.3%	-0.1%	-0.0%
Reg Store	-	-	-	-	-	-	-
Diet Store	-	-	-	-	-	-	-
Reg Outside	-	-	-	-	-	-	-
Diet Outside	-	-	-	-	-	-	-
Δ advertising exp.							
Reg Coke	-100.0%	-	-33.1%	-100.0%	-	-47.3%	-100.0%
Diet Coke	-12.0%	-	-6.4%	-17.5%	-	-13.7%	-23.5%
Reg Pepsi	-	-	-	-	-	-	-
Diet Pepsi	0.1%	-	2.3%	1.6%	-	1.0%	0.3%
Reg Store	-	-	-	-	-	-	-
Diet Store	-	-	-	-	-	-	-
Reg Outside	-	-	-	-	-	-	-
Diet Outside	-	-	-	-	-	-	-
Δ quantity							
Reg Coke	-16.4%	-55.6%	-1.2%	-5.6%	-62.0%	-1.9%	-4.7%
Diet Coke	-6.0%	14.2%	-1.6%	-7.3%	15.5%	-2.9%	-6.7%
Reg Pepsi	-1.8%	-53.6%	-0.2%	-0.9%	-33.0%	-0.5%	-1.2%
Diet Pepsi	-1.6%	8.0%	-0.2%	-1.9%	5.7%	-0.5%	-1.7%
Reg Store	3.2%	7.9%	0.4%	2.0%	7.6%	0.7%	1.9%
Diet Store	2.8%	3.5%	0.4%	2.1%	3.4%	0.8%	1.9%
Reg Outside	3.1%	5.8%	0.4%	1.9%	5.4%	0.7%	1.7%
Diet Outside	2.6%	2.7%	0.4%	1.9%	2.5%	0.7%	1.7%

Notes: Numbers are expressed as a percentage of the pre-policy (i.e., pre tax and advertising restriction) level. Columns (1), (2) and (5) show changes relative to the pre-policy level. Column (3) (column (6)) shows the incremental change relative to column (2) (column (5)) and column (4) (column (7)) shows the incremental change relative to column (3) (column (6)).

Table J.2: *Aggregate impact of counterfactual policies, by brand*

	No tax	Specific tax			Ad valorem tax		
	Adv. restrict. (1)	Fixed adv. (2)	+ Eq. adv. response (3)	+ Adv. restrict. (4)	Fixed adv. (5)	+ Eq. adv. response (6)	+ Adv. restrict. (7)
Δ profits							
Reg Coke	-2.2%	-23.1%	1.1%	0.6%	-33.9%	1.9%	1.3%
Diet Coke	-3.4%	4.7%	-0.6%	-3.7%	5.1%	-1.0%	-3.3%
Reg Pepsi	-1.3%	-33.7%	-0.2%	-0.7%	-39.2%	-0.2%	-0.6%
Diet Pepsi	-1.0%	4.2%	-0.3%	-1.1%	3.3%	-0.4%	-1.0%
Reg Store	-	-	-	-	-	-	-
Diet Store	-	-	-	-	-	-	-
Reg Outside	-	-	-	-	-	-	-
Diet Outside	-	-	-	-	-	-	-

Notes: Numbers for price, margins, advertising expenditure and quantities are expressed as a percentage of the pre-policy (i.e., pre tax and advertising restriction) level; numbers for profits are expressed as a percentage of pre-policy total consumer expenditure. Columns (1), (2) and (5) show changes relative to the pre-policy level. Column (3) (column (6)) shows the incremental change relative to column (2) (column (5)) and column (4) (column (7)) shows the incremental change.

Table J.3: *Distributional impact of counterfactual policies (under “Total effect”’ consumer surplus)*

Income quartile	No tax	Specific tax		Ad valorem tax	
	Adv. restrict. (1)	(2)	Adv. restrict. (3)	(4)	Adv. restrict. (5)
Change in sugar					
Bottom	-2.88%	-17.64%	-18.12%	-17.88%	-18.25%
2nd	-2.78%	-17.07%	-17.45%	-17.23%	-17.45%
3rd	-2.32%	-17.29%	-17.63%	-17.70%	-17.96%
Top	-2.83%	-12.22%	-12.73%	-12.56%	-12.83%
Change in consumer surplus					
Bottom	-6.20%	-9.11%	-13.50%	-9.78%	-13.72%
2nd	-3.87%	-7.13%	-9.73%	-7.52%	-9.85%
3rd	-4.10%	-7.81%	-10.73%	-8.38%	-11.03%
Top	-3.60%	-4.60%	-7.11%	-5.15%	-7.33%
Change in consumer surplus net of internalities					
Bottom	-4.98%	-1.66%	-5.84%	-2.22%	-6.01%
2nd	-2.86%	-0.96%	-3.43%	-1.29%	-3.55%
3rd	-3.40%	-2.54%	-5.36%	-2.99%	-5.56%
Top	-2.91%	-1.63%	-4.00%	-2.08%	-4.20%

Notes: Change in sugar is expressed as a percent of the income quartile specific pre-policy total drink sugar consumption. Change in consumer surplus (including net of internalities) is expressed as a percent of income quartile specific pre-policy total expenditure. The consumer surplus measure includes both the static impact of policy on the state-specific optimal prices and the impact of the changes in the equilibrium distribution over advertising state due to changes in optimal advertising expenditure.