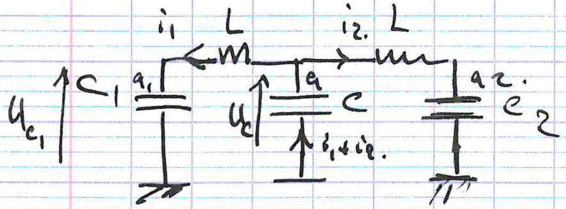

$q_1 = C_1 u_{c_1}$ $\quad q_2 = C_2 u_{c_2}$

$i_1 = \dfrac{dq_1}{dt}$ $\quad i_2 = \dfrac{dq_2}{dt}$

$q = C u_c \quad \dfrac{dq}{dt} = -(i_1 + i_2)$

$u_{c_1} + u_{L_1} = u_c$

$$\frac{q_1}{C_1} + L\frac{di_1}{dt} = u_c = \frac{q}{C} \qquad \text{avec } -\frac{dq}{dt} = i_1 + di_2$$

$$L\frac{d^2q_1}{dt^2} + \frac{q_1}{C_1} = -\frac{1}{C}(q_1 + q_2)$$

$$\Rightarrow L\frac{d^2q_1}{dt} + \left(\frac{1}{C_1} + \frac{1}{C}\right)q_1 + \frac{1}{C}q_2 = 0.$$

de m̂ $\quad L\dfrac{d^2q_2}{dt} + \left(\dfrac{1}{C_2} + \dfrac{1}{C}\right)q_2 + \dfrac{1}{C}q_1 = 0.$

$$\Rightarrow \begin{pmatrix} \dfrac{d^2q_1}{dt^2} \\ \dfrac{d^2q_2}{dt} \end{pmatrix} + \begin{pmatrix} \omega_1^2 + \omega_c^2 & \omega_c^2 \\ \omega_c^2 & \omega_2^2 + \omega_c^2 \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \end{pmatrix} = 0.$$

avec $\quad \omega_1 = \dfrac{1}{\sqrt{LC_1}} \quad \omega_2 = \dfrac{1}{\sqrt{LC_2}} \quad \omega_c = \dfrac{1}{\sqrt{LC}}$

On suppose $\omega_1 > \omega_2$.

valeur propre. $\quad (\omega_1^2 + \omega_c^2 - \lambda)(\omega_2^2 + \omega_c^2 - \lambda) - \omega_c^4 = 0.$

$$\omega_1^2\,\omega_2^2 + \omega_1^2\,\omega_c^2 - \lambda\,\omega_1^2$$
$$\omega_c^2\,\omega_2^2 + \cancel{\omega_c^2\,\omega_c^2} - \lambda\,\omega_c^2$$
$$-\lambda\,\omega_2^2 - \lambda\,\omega_c^2 + \lambda^2 \quad \cancel{-\,\omega_c^4} = 0$$

$$\lambda^2 - \lambda\left[\omega_1^2 + \omega_2^2 + 2\omega_c^2\right] + \omega_1^2\omega_2^2 + \omega_c^2\left(\omega_1^2 + \omega_2^2\right) = 0$$

$$\Delta = \left(\omega_1^2 + \omega_2^2 + 2\omega_c^2\right)^2 - 4\,\omega_1^2\omega_2^2 - 4\,\omega_c^2\left(\omega_1^2 + \omega_2^2\right) =$$
$$= \left(\omega_1^2 + \omega_2^2\right)^2 + \cancel{4\,\omega_c^2\left(\omega_1^2 + \omega_2^2\right)} + 4\,\omega_c^4 - 4\,\omega_1^2\omega_2^2$$
$$\cancel{-4\,\omega_c^2\left(\omega_1^2 + \omega_2^2\right)}.$$
$$= \left(\omega_1^2 - \omega_2^2\right)^2 + 4\,\omega_c^4$$

Soit $$\lambda_1 = \frac{1}{2}\left[\omega_1^2 + \omega_2^2 + 2\omega_c^2 + \sqrt{\left(\omega_1^2 - \omega_2^2\right)^2 + 4\omega_c^4}\right]$$

$$\lambda_2 = \frac{1}{2}\left[\omega_1^2 + \omega_2^2 + 2\omega_c^2 - \sqrt{\left(\omega_1^2 - \omega_2^2\right)^2 + 4\,\omega_c^4}\right]$$

+ Si $\omega_c = 0$ $\left|\begin{array}{l}\lambda_1 = \omega_1^2 \\ \lambda_2 = \omega_2^2\end{array}\right.$ ok.

+ si $4\omega_c^2 \ll \left(\omega_1^2 - \omega_2^2\right)^2$

$$\lambda_1 = \frac{1}{2}\left[\omega_1^2 + \omega_2^2 + 2\omega_c^2 + \left(\omega_1^2 - \omega_2^2\right)\left[1 + \frac{2\,\omega_c^4}{\left(\omega_1^2 - \omega_2^2\right)^2}\right]\right]$$
$$= \omega_1^2 + \omega_c^2 + \frac{\omega_c^4}{\left(\omega_1^2 - \omega_2^2\right)^2}.$$

$$\lambda_2 = \omega_2^2 + \omega_c^2 - \frac{\omega_c^4}{\left(\omega_1^2 - \omega_2^2\right)^2}.$$