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Profile control in distributed parameter systems using lexicographic optimization based MPC ☆

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Abstract

Process equipment that exhibits significant spatial variation of system properties, such as temperature or concentration in a fixed bed reactor, are typically modeled as distributed parameter systems. While some properties of the final product exiting the equipment may depend on the states concerning the endpoint, others may be a function of the history of processing within the equipment. In such instances, control of the spatial property profile may be beneficial. In this work, we explore the idea of profile control using extended MPC and outline the additional challenges that must be addressed in this context. In case that the target profile is unachievable, we present an MPC formulation that uses lexicographic optimization to prioritize the different sections of the profile. Simulation of a simple representative system namely a hypothetical plug flow reactor is used to demonstrate that the lexicographic optimization based MPC provides a systematic approach to profile control and spans between the endpoint control strategy and the whole profile control strategy. The benefits of lexicographic optimization based MPC were also demonstrated on a large-scale distributed parameter system of industrial size, namely the continuous pulp digester.

Keywords: Distributed parameter system; Extended Kalman filter; Model-based control; Continuous pulp digester; Lexicographic optimization

1. Introduction

Key equipment in large continuous plants typically represents distributed parameter systems (DPS), where the system properties exhibit a significant spatial variation. While most product quality variables are determined by the endpoint properties of the DPS, others may depend on the reaction path assumed during processing. The path dependence is also critical in situations where irreversible phenomena may occur such as catalyst poisoning or gelation. Furthermore, the endpoint itself is a manifestation of the reaction path and a particular path adopted may offer advantages over others. Thus, from an operations perspective, it may be desirable to control not only the endpoint but also the spatial property profile. Examples of DPS include fixed bed reactors, distillation towers and continuous pulp digesters. Despite these advantages of profile control, only the endpoint property is commonly controlled in large-scale distributed parameter systems. For example, control of bottom and top product compositions in a distillation column [1,2] as well as endpoint Kappa number control in a continuous pulp digester [3,4] have been reported in the literature. An obvious advantage of the endpoint property control is that we typically have an online or laboratory measurement of the property of interest at the exit and this facilitates its implementation. On the other hand, controlling the property profile requires addressing additional challenges. Firstly, the profile must be constructed from available measurements. Secondly, guaranteeing feasibility of the target profile in presence of disturbances is difficult. Thus, while a target endpoint may continue to be achievable for a particular set of disturbances, it is unlikely that the corresponding target profile

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will be achievable for the same set of disturbances. In fact the part of the target profile close to the feed is unlikely to be achieved even for very small perturbations in feed composition. In this work, we present a novel approach for control of profile in a DPS, which is particularly beneficial when the target profile is unachievable.

Early work on control of property profile in distributed parameter systems had been motivated by studies on high purity distillation columns [5–8] and fixed bed sorption [9,10]. This work acknowledged the fact that conventional linearized models of distributed parameter systems are inadequate for modeling the nonlinear dynamics particularly during non-steady state transitions. Consequently, the profile of the property of interest is modeled as a nonlinear wave, which propagates through the equipment in response to changes in the operating conditions. The wave model has the advantage of providing a low-order approximation that describes the property of interest directly and can be used in model-based control schemes [11,12]. Profile control strategies have also been used for control of packed bed reactors to ensure that the reactor hotspot is below the safety limit, which in turn avoids catalyst deactivation [13,14]. A full profile control of temperature in an FBR has been experimentally demonstrated by Yoshida and Matsumoto [15]. More recently, studies on control of distributed parameter systems such as particle size control in crystallization [16–18] and emulsion polymerization [19], and spatial control in drying [20], have been presented. In context of the Kappa profile control in a continuous pulp digester, profile reconstruction through state estimation using a multi-rate extended Kalman filter has been demonstrated by Padhiyar et al. [21]. Subsequently, they present control of the Kappa number at three different locations along the length of the digester. Doyle and Kayihan [22] show that control of Kappa number at multiple points results in a tightly constrained Kappa number profile.

System theory properties of DPS and its control have attracted a lot of attention [23]. Broadly, the DPS control approach can be classified into two types. A practical approach includes discretization of the nonlinear PDE followed by synthesis of the controller [24–26]. This approach has been criticized on the grounds that the fundamental control-theoretic properties of the original system that should depend only on the location of the actuators and sensors now depend on discretization too [27]. The other approach for control of DPS is based on maximum principle of Pontryagin [28]. Here, the controller is synthesized based on the infinite dimensional model of the distributed parameter system. Applicability of this approach is limited to small problems only because of the association of extensive analytical simplifications. In this work, we adopt the former approach for control of distributed parameter systems and employ an extended MPC [29] framework to implement profile control. When the target profile becomes unachievable, either due to disturbances or input constraints, the controller tuning plays a crucial role in determining the closed loop behaviour. For example, a higher priority may be provided to achieve the endpoint target when the whole profile cannot be achieved. In this work, the use of a lexicographic optimization [30–35] based MPC to explicitly prioritise the different parts of the profile is proposed. Here we split the profile into sections and solve the MPC control problem as a multi-tiered optimization, where the different tiers represent priorities of the different parts of the profile. Once an optimal solution is obtained for a particular tier, the next most important objective is optimised by using constraints that ensure that the objective of the preceding tier is maintained at a satisfactory level. This process is continued until the optimal value of the least important objective is achieved. A similar strategy has been implemented in the framework of linear MPC for prioritizing the various goals of the controller such as satisfaction of setpoints and feasibility of constraints [36,37]. Our work seeks to tailor the lexicographic strategy for use in profile control of distributed parameter systems. For comparison purpose, the non-lexicographic version of MPC is also considered wherein the full profile is controlled using a single weighted objective function. We will refer to this strategy as full profile control. The lexicographic optimization based MPC offers significant advantages over full profile control when the target profile becomes unachievable by explicitly prioritising the different parts of the profile in a transparent manner. We demonstrate using a simple example that the lexicographic optimization based MPC spans between endpoint property control and the full profile control strategies. The added computational expense of solving multiple optimization problems at each instant is not significant when linear MPC or extended MPC is used where the optimization problem is a quadratic program.

The paper has been organized as follows. Section 2 introduces a hypothetical plug flow reactor (PFR) example and illustrates the issues pertaining to the available degrees of freedom in the profile control problem. The three different control law formulations, namely endpoint control, profile control using a single objective function, and profile control with lexicographic method are discussed in Section 3. Simulation results for the PFR example are presented in Section 4. To verify the benefits of the lexicographic optimization based MPC for large-scale distributed parameter systems, profile control in a continuous pulp digester of industrial scale is presented as Section 5 followed by concluding remarks.

2. An illustrative example

As a representative DPS, consider a hypothetical system of a PFR as shown in Fig. 1. The system is approximated by nine CSTRs in series. Reactant A enters the first CSTR at a rate F and concentration $C_{a,F,1}$ and flows down the column to form dimer product P through the following elementary reaction,

$$2A \xrightarrow{k} P \tag{1}$$

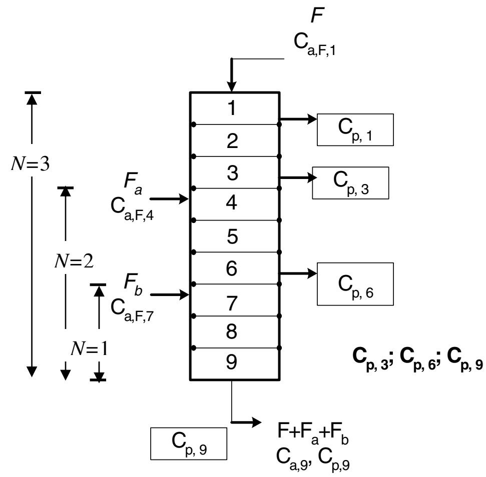

Fig. 1. Schematic diagram of a plug flow reactor. Variables in bold letters are CVs, those in italics are MVs and ones in boxes are the measurements.

Reactant A may also be introduced through trim streams located in the 4th and 7th CSTRs with rates F_a and F_b, and concentrations $C_{a,F,4}$ and $C_{a,F,7}$, respectively. We seek to control the profile of the product concentration C_p along the length of the column. The three manipulated variables (MVs), F, F_a, and F_b, can be used to control three independent points along the profile. For example, fixing one concentration from each of the sets $\{C_{p,1}, C_{p,2}, C_{p,3}\}$, $\{C_{p,4}, C_{p,5}, C_{p,6}\}$ and $\{C_{p,7}, C_{p,8}, C_{p,9}\}$ fixes the entire profile. The remaining points of the profile are determined by the interaction of the state variables and the structure of the DPS. Thus, if $C_{p,7}$ is one of the target points of the profile, then fixing $C_{p,7}$ determines $C_{p,8}$ and $C_{p,9}$ for the structure depicted in Fig. 1. Hence one cannot control all the three compositions, $C_{p,7}$–$C_{p,9}$, independently. On the other hand, had F_a, and F_b been located at 8th and 9th CSTRs, independent control of $C_{p,7}$, $C_{p,8}$ and $C_{p,9}$ would have been possible. Thus, the idea of the whole profile may be equivalently represented in terms of independent points on the profile and these points must be selected depending on the location and type of the manipulated variables. In the current work, we assume that the target profile can be converted to unique target values of $C_{p,3}$, $C_{p,6}$, and $C_{p,9}$. A representative time and spatial variation of the profile is shown in Fig. 2.

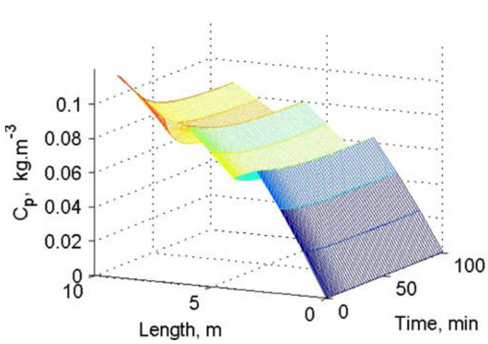

Fig. 2. Product concentration profile along the length of PFR.

3. Control methodology

The model of the distributed parameter system may be described in the state space form after lumping as follows:

$$\dot{x} = f(x, u, d) \tag{2}$$

$$y = g(x, d) \tag{3}$$

where x, u, and d are vectors of state, manipulated, and unmeasured disturbance variables, respectively. Available measurements are denoted by y. Nonlinear model predictive control of a distributed parameter system generally represents a formidable task due to requirement of integrating a set of partial differential equations during the online solution of a nonlinear program. To retain the simplicity of quadratic program, [29] suggested that the future predictions needed in MPC may be obtained by adding the nonlinear unforced response of the system with a forced response based on a linear model. Thus, the approximated model response may be written as,

$$x(t) = \int_{t_0}^{t} f(x(\tau), u(t_0), d(\tau))\mathrm{d}\tau + \int_{t_0}^{t} \left.\frac{\partial f}{\partial u}\right|_{u(t_0)} (u(\tau) - u(t_0))\mathrm{d}\tau \tag{4}$$

Use of such an approximate form of the model makes the resulting MPC formulation a quadratic program (QP). As one may not have measurements of all the state variables needed to construct the property profile, an extended Kalman filter (EKF) for state reconstruction is used (for e.g., see [21]). It was observed in the above mentioned reference that the EKF with an input disturbance model provided reasonable state estimates for a wide range of process and parametric disturbances in a continuous pulp digester. We continue using the input disturbance model, where it is assumed that the effect of the plant-model mismatch can be estimated by assuming the disturbance as a load variable. The MPC strategy calculates the control moves by an online solution of the following optimization problem,

$$\min_{\Delta U_k} \left\|(Y_{k+1/k} - R_{k+1/k})\right\|^2_{W_e} + \|\Delta U_k\|^2_{W_u} \tag{5}$$

such that

$$\Delta u_{k+q} = \cdots = \Delta u_{k+p-1} = 0$$
$$u^{\mathrm{low}}_{k+l} \leqslant u_{k+l} \leqslant u^{\mathrm{high}}_{k+l}, \quad 0 \leqslant l \leqslant q-1$$
$$\Delta u^{\mathrm{min}}_{k+l} \leqslant \Delta u_{k+l} \leqslant \Delta u^{\mathrm{max}}_{k+l}, \quad 0 \leqslant l \leqslant q-1$$
$$y^{\mathrm{low}}_{k+l/k} \leqslant y_{k+l/k} \leqslant y^{\mathrm{high}}_{k+l/k}, \quad 1 \leqslant l \leqslant p$$
$$Y_{k+1/k} = [y^{\mathrm{T}}_{k+1/k} \quad y^{\mathrm{T}}_{k+2/k} \quad \cdots \quad y^{\mathrm{T}}_{k+p/k}]^{\mathrm{T}}$$

where $Y_{k+1/k}$ is a vector of output predictions over the prediction horizon p and R_{k+1} is the corresponding target vector. If the controlled variable is chosen as only the endpoint, the above problem refers to endpoint property control. On the other hand, if $Y_{k+1/k}$ consists of prediction of the whole profile (or the points that uniquely characterize the profile) the control problem becomes one of full profile control. ΔU_k represents an appropriately defined vector of input moves over q future samples, which are optimised at every sampling instant. W_u and W_e are weighting matrices for the manipulated inputs, u_k, $u_{k+1}, \ldots, u_{k+q-1}$ and the controlled variables, y_k, y_{k+1}, $\ldots, y_{k+p}$, respectively.

The lexicographic method of optimization assumes that the objective function consists of trade-offs, which can be prioritised. Thus, in the full profile control problem, if the whole profile becomes unachievable, alternate solutions may exist, each of which may achieve only parts of the target profile. It may be desirable to prioritise so that certain parts of the profile are achieved at the cost of other parts. While this is heuristically attempted in conventional MPC applications by differentially weighting the multiple control objectives, a systematic procedure results from the lexicographic method of optimization. Let us assume that the profile has been split into N sections with $Y^1_{k/k}, Y^2_{k/k}, \ldots, Y^N_{k/k}$, representing the relevant profile estimates at time instant k. Note that these sections could be overlapping or non-overlapping. The control objective for each section may then be written as,

$$J^i = \sum_{j=1}^{p} W^i_e (Y^i_{k+j/k} - R^i_{k+j})^2 + \sum_{m=1}^{q} W^i_u (\Delta U_{k+m-1})^2 \tag{6}$$

where $i = 1, 2, \ldots, N$. The N sections are numbered such that Section 1 represents the most important part of the profile, Section 2 the next most important part of the profile and so on. The MPC controller based on lexicographic method solves the following problem online,

$$\begin{aligned}
&\min_{\Delta U_k, \Delta U_{k+1}, \ldots, \Delta U_{k+q-1}} && J^1 \\
&\text{s.t.} && C^1 \text{ are feasible} \\
&\min_{\Delta U_k, \Delta U_{k+1}, \ldots, \Delta U_{k+q-1}} && J^2 \\
&\text{s.t.} && C^2 \text{ are feasible} \\
& && y^1_{k+p/k} = y^{1*}_{k+p/k} \\
&\vdots && \\
&\min_{\Delta U_k, \Delta U_{k+1}, \ldots, \Delta U_{k+q-1}} && J^N \\
&\text{s.t.} && C^N \text{ are feasible} \\
& && y^1_{k+p/k} = y^{1*}_{k+p/k} \\
& && y^2_{k+p/k} = y^{2*}_{k+p/k} \\
&\vdots && \\
& && y^{N-1}_{k+p/k} = y^{N-1*}_{k+p/k}
\end{aligned} \tag{7}$$

where C^i represents the constraints relevant to the ith objective function corresponding to the ith section of the profile. Typically, C^i consist of constraints on the input and output variables. The equality constraints $y^i_{k+p/k} = y^{i*}_{k+p/k}$ used in the (i+1)st tier of the optimization problem with objective function J^{i+1} above enforce the lexicographic condition that the optimal profile at the end of the prediction horizon obtained in the ith tier is maintained. The right hand sides of these lexicographic constraints denote the optimal values obtained in the previous tier. Thus, the profile in the previous section should be at its optimal value at the end of the prediction horizon. The optimised input corresponding to the current time instant, ΔU_k, obtained at the end of the Nth optimization problem is injected into the plant. Simulations reveal that the lexicographic constraints are usually active at the optimal solution of the corresponding optimization problem. Thus, the lexicographic constraints constrain the evolution of the optimal profile at steady state in the $i-1$ section to its optimal value while solving the ith optimization problem.

To summarize, the lexicographic optimization based MPC finds an optimal solution that explicitly prioritizes the controller objectives. Since in the current work, we use an extended MPC implementation, each of the optimization problems are QPs for which efficient solvers exist. In the next section, we compare the pros and cons of the exit and full profile control strategies and demonstrate that the lexicographic method based control provides a trade-off between the two.

4. Benefits of lexicographic optimization based MPC: a case study

A simple example consisting of a PFR is presented in Section 2 to test the benefits of the lexicographic optimization based MPC. We assume the reaction in Eq. (1) as irreversible and isothermal following elementary kinetics. A mass balance over an infinitesimally small radial slice along the axial flow direction z yields the following mathematical model for the DPS,

$$\frac{\partial C_p}{\partial t} = -\frac{F(z)}{A}\frac{\partial C_p}{\partial z} + kC_a^2 \tag{8}$$

where C_a is the mass concentration of reactant A and C_p the mass concentration of product P.

As shown in Fig. 1, the DPS is stimulated assuming nine CSTRs in series. We also assume that we have access to four measurements namely product concentrations at 1st, 3rd, 6th, and 9th CSTRs. As discussed previously, one could estimate the full profile (product concentrations in the nine CSTRs) and attempt to control it. Alternatively, one could achieve the same by fixing one concentration from each of the following sets $\{C_{p,1}, C_{p,2}, C_{p,3}\}$, $\{C_{p,4}, C_{p,5}, C_{p,6}\}$ and $\{C_{p,7}, C_{p,8}, C_{p,9}\}$, since this uniquely fixes the entire profile. We have chosen $C_{p,3}$, $C_{p,6}$, and $C_{p,9}$ as the controlled variables for the two profile control strategies and $C_{p,9}$ for the exit control strategy. The three manipulated inputs include the main feed flowrate F at the top of the column and two trim flowrates F_a at the 4th CSTR, and F_b at the 7th CSTR. The three feed streams carry pure component A with the nominal feed concentrations $C_{a,F,1}$, $C_{a,F,4}$, and $C_{a,F,7}$ at 0.5 kg m^{-3}.

Table 1
Setpoints and constraints used in control of the hypothetical PFR: Case Study 1 and 2

	$C_{p,3}$ ($\text{kg m}^{-3} \times 10^2$)	$C_{p,6}$ ($\text{kg m}^{-3} \times 10^2$)	$C_{p,9}$ ($\text{kg m}^{-3} \times 10^2$)
Setpoint 1	7.00	9.17	11.73
Setpoint 2	5.68	6.98	8.77
	F ($\text{m}^3 \text{min}^{-1}$)	F_a ($\text{m}^3 \text{min}^{-1}$)	F_b ($\text{m}^3 \text{min}^{-1}$)
Min	0	0	0
Max	2	1	1

To showcase the merits of the three control approaches discussed in Section 3 both servo as well as regulatory problems are considered. The initial (Setpoint 1) and final (Setpoint 2) targets used in the servo problem are documented in Table 1. The regulatory problem results from a step disturbance in the feed concentration. In Case Study 1, the injected disturbance is small enough that the target profile (Setpoint 2) continues to be feasible. On the other hand, in Case Study 2, the target profile (Setpoint 2) cannot be achieved due to the large magnitude of the disturbance. Both of these case studies assume no mismatch between plant and controller models.

4.1. Case Study 1: achievable target profile

Here the system is switched from Setpoint 1 to the Setpoint 2 at 120 min. A +10% step disturbance in the concentration of the feed stream at top of the column, $C_{a,F,1}$, is injected at 520 min. The closed loop response for such a servo and a regulatory problem using the end point and full profile strategy is shown in Fig. 3. The error penalty matrix for the endpoint property control strategy is $W_e = 2 \times 10^5$ and for the full profile control strategy $W_e = \text{diag}(5 \times 10^5\ 3 \times 10^5\ 2 \times 10^5)$ corresponding to $C_{p,3}$, $C_{p,6}$, and $C_{p,9}$, respectively. These values of the weight matrix were selected to normalize the deviations from the setpoints for each of the three controlled variables thus ensuring equal contribution in the error penalty term of the objective function. The weighting matrix for inputs is $W_u = \text{diag}(2.5 \times 10^3\ 10^4\ 10^4)$. The corresponding manipulated input trajectories are shown in Fig. 4.

It is noted that the controlled variables using endpoint control (dashed line) and profile control (solid line) strategies successfully switched from their old targets to new targets even in presence of the disturbance. Indeed, the endpoint control strategy does not attempt to control $C_{p,3}$ and $C_{p,6}$ resulting in the apparent offset. This clearly demonstrates that the full profile control strategy may successfully control the reaction path and thus offer advantages for overall property control. Fig. 3d shows the steady state profile of the product composition along the length of the column at $t = 1000$ min. Simulation times for exit control and profile control are 0.02 s, and 0.021 s, respectively. These values correspond to a P-IV®; 1 Gb RAM and 1.8 GHz processor.

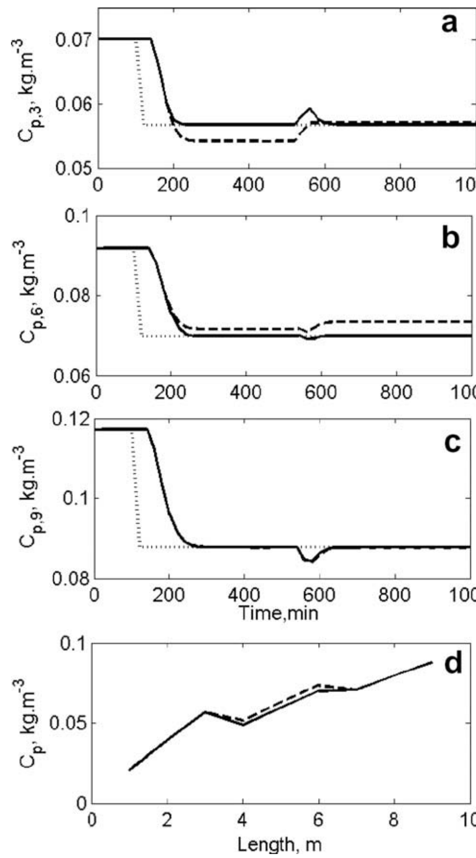

Fig. 3. Case Study 1: Closed loop response for endpoint control (dashed) and profile control (solid): reference value (dotted) for the hypothetical PFR.

Thus, if the targets are feasible, both profile and endpoint control strategies are able to achieve their objectives satisfactorily. As available degrees of freedom are efficiently utilized in profile control, it may be preferable over endpoint property controller. Next we present a case where a severe disturbance results in infeasibility of Setpoint 2.

4.2. Case Study 2: unachievable target profile

As discussed previously, the target profile may become unachievable in presence of large disturbances or under limited scope of manipulation. To simulate such a situation, we increase $C_{a,F,1}$ from 0.5 Kg m^{-3} to 1 Kg m^{-3} with constraints in manipulated inputs as shown in Table 1. Using identical controller tuning as in Case Study 1, the simulation results for controlled and manipulated variables are depicted in Figs. 5 and 6, respectively. The full profile control strategy (solid line) attempts to reject the disturbance by reducing the residence time of the reactant and hence increasing the three manipulated variables as seen from Fig. 6. However, the MVs saturate and an offset is observed in all three sections of the PFR. More importantly, the endpoint target properties are also not achieved. On the other hand, since the endpoint controller attempts

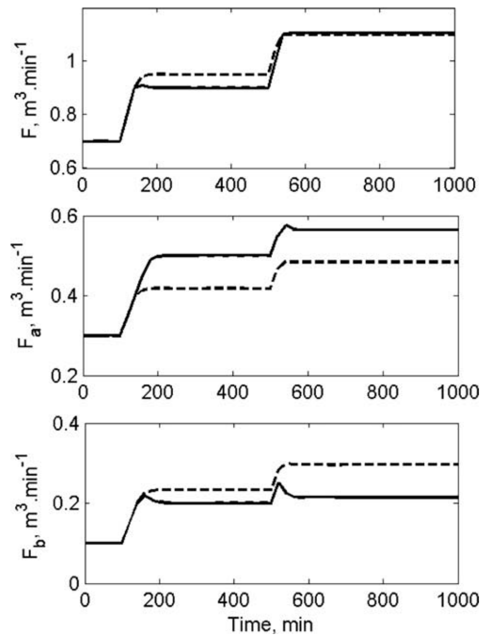

Fig. 4. Case Study 1: Manipulated variable profile for endpoint control (dashed) and profile control (solid) strategies for the hypothetical PFR.

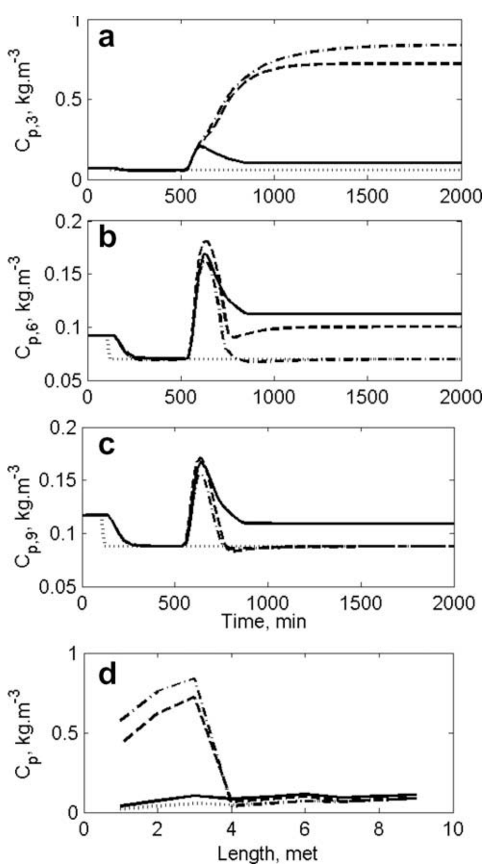

Fig. 5. Case Study 2: Closed loop response for endpoint control (dashed), profile control with a whole profile control (solid), and profile control with lexicographic optimization (dashed dotted) strategies; reference value (dotted) for the hypothetical PFR.

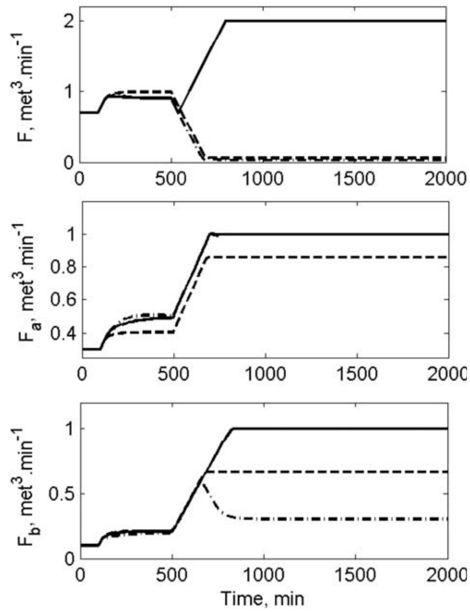

Fig. 6. Case 2: Manipulated variable profiles for endpoint control (dashed), for profile control with a single objective function (solid), and for profile control with lexicographic optimization (dashed dotted) strategies for the hypothetical PFR.

control of $C_{p,9}$ alone, it successfully uses the degrees of freedom to meet its objective (dashed lines in Figs. 5 and 6). Although an apparent offset in $C_{p,3}$ and $C_{p,6}$ is observed, the controller at least provides the endpoint targets. Since $C_{p,9}$ uniquely characterizes the profile in CSTRs 7–9, it implies that at least in this section, the target has been met. Fig. 5a and b shows that the target profiles are not met in CSTRs 1–3 and 4–6, respectively. However, from an operations perspective, the exit control strategy may be superior to the full profile strategy. The lexicographic method, on the other hand, offers a systematic method to assign the degrees of freedom to the conflicting control objectives. Here, the PFR is first divided into multiple sections. We choose three overlapping sections with: Section 1 ($N = 1$) consists of CSTRs 7–9, Section 2 ($N = 2$) with CSTRs 4–9 and Section 3 ($N = 3$) with CSTRs 1–9 (see Fig. 1). The profile in Section 1 has the highest priority and thus the corresponding control problem with objective function J^1 (see Eq. (7)) is solved first. This is followed by solution corresponding to Sections 2 and 3 along with the appropriate lexicographic constraint. The performance of the lexicographic method is shown as the dashed dotted line in Figs. 5 and 6. It is observed from Fig. 5c that the lexicographic optimization based controller achieves its target in Section 1 even in presence of the disturbance, while maintaining the target profile in the Section 2 (see Fig. 5b). Finally, it attempts to obtain the target profile in the top part of the PFR. However, this is unachievable as observed from Fig. 5a. While the infeasibility of the profile is due to fundamental process limitations, the lexicographic optimization based MPC systematically achieved the control objectives by satisfying Sections 1 and 2 while failing to meet the target in Section 3.

Although the lexicographic method solved three QPs, the integration of the PDEs necessary to provide future predictions in Eq. (4) need to be performed only once. The average simulation times for solving a single iteration in the full profile control and lexicographic optimization based profile control are 0.5 s and 0.54 s, respectively. Thus, in our problem the lexicographic method was only 8% more computationally expensive than the full profile control problem.

5. Profile control in a pulp digester

The continuous pulp digester is a long tubular reactor, which represents a large distributed parameter system. Usually pulping in the digester is carried out by treating the wood chips with an aqueous solution of sodium hydroxide and sodium sulfide called white liquor. This process is called Kraft process. While there are multiple variations of continuous pulp digesters, we have used a dual vessel pulp digester as shown in Fig 7. Pre-steamed wood chips and white liquor are introduced in the impregnation vessel. Here white liquor diffuses into the pores of wood chips. The chip–liquor mixture flows into the digester vessel wherein the majority of the delignification occurs due to higher temperature. The digester vessel is divided into three functional zones namely cook, modified continuous cook (mcc), and extended modified continuous cook (emcc). The wood chips and white liquor flow concurrently within the cook zone, at the end of which the spent liquor is extracted and sent for recovery. The chips then encounter a countercurrent flow of dilute liquor.

5.1. Mathematical model for the continuous pulp digester

A mechanistic model of the continuous pulp digester [38] has been used in our study. The mathematical model represents the DPS and is approximated with CSTRs in series by the early lumping approach. We approximate

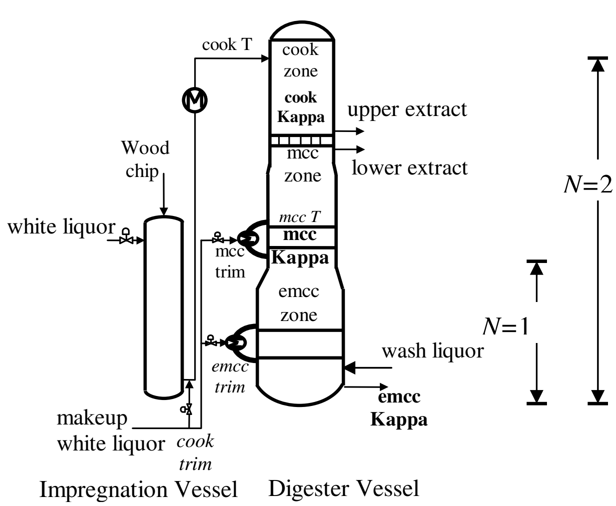

Fig. 7. Schematic of dual vessel pulp digester. Variables in italics represent manipulated variables, in bold font represent controlled outputs.

the digester by 45 axial CSTRs in series of unequal volumes to represent the mass and energy conservation laws. As in the Purdue model, each CSTR is assumed to consist of three phases namely solid phase, entrapped liquor and free liquor phase. The entrapped liquor phase resides in the pores of the chips, while free liquor surround the chip phase. The solid phase consists of five components: (1) high reactive lignin, (2) low reactive lignin, (3) cellulose, (4) araboxylan, and (5) galactoglucomman. Entrapped and free liquor are assumed to consist of six components each: (1) active effective alkali (2) passive effective alkali, (3) active hydrosulfide (HS), (4) passive HS, (5) dissolved solids, and (6) dissolved lignin. Apart from the five solid states, six entrapped liquor and six free liquor states, there are two more states, namely free liquor temperature and chip temperature in each of the 45 CSTRs. Transient behaviour of all the 19 states in each CSTR is represented by mass and energy balances. The resulting 855 states model is integrated in MATLAB by the *ode45* solver. The Kappa number measures the extent of delignification and represents one of the key property variables that need to be controlled. For the detailed process model, the reader should refer to Padhiyar et al. [21] and Wisnewski et al. [38].

5.2. Profile control in the pulp digester

Several obstacles hamper the efficient control of a pulp digester. Large transport delay, multi-rate measurements, complex nonlinear behaviour, and the stochastic variability in the biological feedstock are a few of them. Kappa number at the exit of the digester has been studied by various researchers [39,3]. As discussed previously, control of Kappa profile using MPC [21,22] suggests that controlling the profile instead of the exit Kappa number can provide a tighter control of the pulp quality. Moreover, material properties of the pulp such as fiber length depend on the history of processing in the digester. Certain safety related issues such as plugging of the digester vessel are also related to the reaction path [40], thereby motivating the need for profile control. Though both the profile control related studies [21,22] show a successful control of Kappa profile, they implicitly assume that the full Kappa profile is achievable. While in normal conditions, Kappa profile remains achievable, in presence of plant-model mismatch, unmeasured disturbances, and input limitations, the target Kappa profile may become unachievable. In this section, the lexicographic optimization based MPC is used to control the Kappa profile in the digester. Through two case studies, it is shown that the lexicographic MPC solution is identical to the full profile solution when the target profile is achievable (Case Study 3). However if the target profile becomes unachievable, the lexicographic optimization based MPC enforces priorities to achieve the target exit Kappa number (Case Study 4). As before, the Kappa number profile is represented by Kappa number at the end of cook zone, mcc zone, and emcc zone in the digester. We have used the endpoint Kappa number along with various components of liquor at lower and upper extract as measurements available every 10 min. The list of these measurements is summarized in Table 2. Kappa number at cook and mcc zones are inferentially estimated using EKF with an input disturbance model as discussed in [21]. Cook trim, mcc trim, and emcc temperature are used as manipulated inputs for the 3×3 control problem. Mismatches in heat transfer coefficient and heat of reaction are introduced to simulate plant-model mismatch. The heat transfer coefficient of the plant-model is increased from 744.3 to 748.2 kJ min^{-1} K^{-1} m^{-3} and the heat of reaction was reduced from −639.1 to −636.5 kJ kg^{-1}. Error penalty and move suppression weighting matrices W_e and W_u, are provided in Table 3. White noise with a standard deviation of 0.3% of the nominal measurement value was injected in all measurements. As discussed in [21], with the 17 measurements used, the system comprising of 855 states is not observable. To implement lexicographic optimization based MPC, the pulp digester is divided into two overlapping sections. The first section ($N = 1$, see Fig. 7) consists of the emcc zone whereas the second section ($N = 2$) consists of the entire digester vessel. Section 1 is accorded the highest priority.

Table 2
List of measurements used in the control of the digester

Measurement
Kappa number at the end of emcc zone (1)
DS of free liquor at upper and lower extract (2)
DL of free liquor at upper and lower extract (2)
Chip temp at the end of IZ, cook, mcc, emcc zone (4)
Free liquor temp at the end of IZ, cook, mcc, emcc zone (4)
EA concentration at upper and lower extract (2)
HS concentration at upper and lower extract (2)

Table 3
Constraints and controller tuning parameters used for the continuous pulp digester control: Case Study 3 and 4

	Cook Kappa	Mcc Kappa	Exit Kappa
Setpoints			
Setpoint 1	93.5	50.5	23.8
Setpoint 2	98.7	65.4	28.5
Error and move suppression weights			
W_e	22.9	78.3	351.3
W_u	0.0625	625	1907
Output constraints			
Max	*103.5*	*60.5*	*30.9*
Min	*83.5*	*40.5*	*16.8*
	Emcc T (K)	Cook trim (m^3 min^{-1})	Mcc trim (m^3 min^{-1})
Input constraints			
Min	400	0	0
Case Study 3:			
Max	450	1	1
Case Study 4:			
Max	440	1	1

5.3. Case Study 3: achievable profile for the continuous pulp digester

Here, the digester is at a given target Kappa profile when a setpoint change is introduced in the three Kappa numbers at cook, mcc and emcc at 6.5 h. The old and new setpoints are documented in Table 3 along with the input and output constraints. In this case study, the new setpoints are achievable. Fig. 8 shows the closed loop response of all the three control strategies, namely endpoint control (dashed line), full profile control (solid) and lexicographic optimization based profile control (dashed dotted). The corresponding manipulated variables are shown in Fig. 9. As can be seen from Fig. 8, both the full profile control as well as lexicographic optimization based control strategies result in nearly identical controlled variable trajectories. On the other hand, while endpoint control provides a superior endpoint Kappa number control, the remainder of the Kappa profile is not at the desired profile. Also, the solution of the endpoint control could not converge to a steady controller output (see Fig. 9). The manipulated inputs and the inferential controlled variables, namely cook and mcc Kappa numbers do not settle down at a steady value even after 33.5 h of operation.

While the full profile is achievable by both the profile control strategies, input limitations have a definite impact on the feasibility of the profile. This has been demonstrated in the next case study.

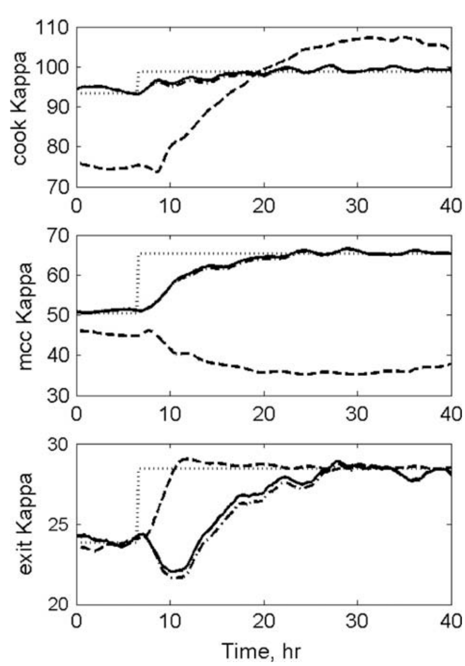

Fig. 8. Case Study 3: Closed loop response for full profile control (solid), lexicographic optimization based profile control (dashed dotted) and endpoint control strategies for the continuous pulp digester. The setpoints are shown as dotted lines.

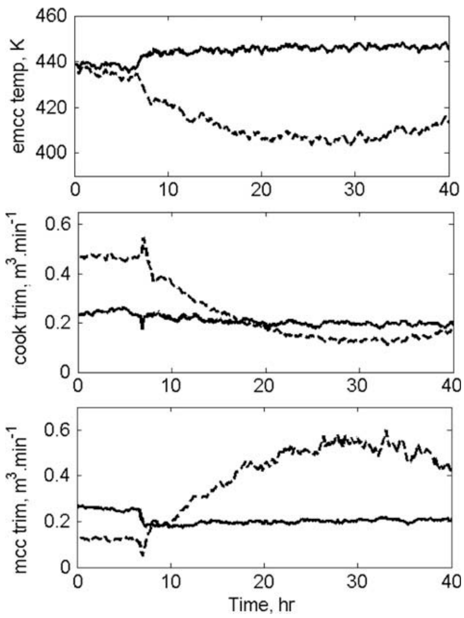

Fig. 9. Case Study 3: Manipulated inputs profiles for full profile control (solid), lexicographic optimization based profile control (dashed dotted) strategies and endpoint control strategies (dashed) for the continuous pulp digester.

5.4. Case Study 4: unachievable profile in the continuous pulp digester

To make the new target profile unachievable, the upper limit for emcc temperature is reduced from 450 K to 440 K while maintaining all the remaining specifications as in Case Study 3. The closed loop response of all the three control strategies is shown in Fig. 10 and corresponding manipulated inputs in Fig. 11. As can be seen from Fig. 10, the full profile control fails to achieve any of the three setpoints. On the other hand, lexicographic optimization based MPC using the same tuning parameters that were used in full profile control could bring the exit Kappa number to its set point, while selectively transferring the offset to profiles in the cook and mcc sections. Although the manipulated inputs for the two strategies appear identical in Fig. 11, the lexicographic optimization based MPC strategy uses slightly larger amounts of cook and mcc trim flowrates. One should note that the endpoint control strategy resulted in a superior control of exit Kappa number, although there is a large deviation in cook and mcc Kappa numbers from their respective setpoints. Further they have not settled down even at the end of 40 h.

In this example, the incremental computational expense in using the lexicographic optimization based control is due to the solution of an additional QP (two sections). Typically, the most expensive step in using extended MPC for a large-scale distributed parameter system lies in integrating the nonlinear ODEs (see Eq. (4)). However, it is noted that while solution of additional QPs is required in the lexicographic approach, the integration is performed only once. Hence, the increase in the computational burden in the lexicographic approach is minimal. The average simulation times for computation of the control moves in the full profile control and lexicographic optimization based profile control are 63 s and 63.5 s, respectively or approximately 11% of the sampling interval (10 min). In contrast, the average time needed to solve each QP in either of the two control strategies was only 0.5 s.

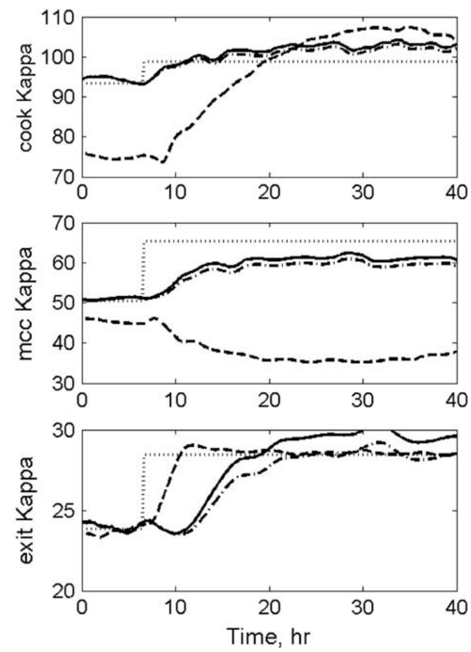

Fig. 10. Case Study 4: Closed loop response for profile control with full profile control (solid), lexicographic optimization based profile control (dashed dotted) and endpoint control strategies (dashed) for the continuous pulp digester. The setpoints are shown as dotted lines.

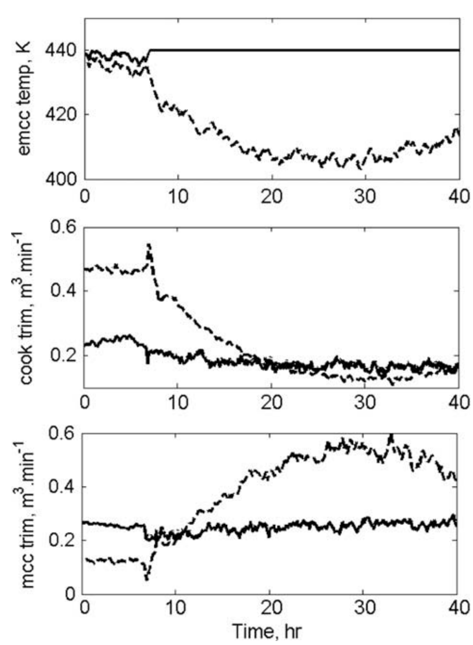

Fig. 11. Case Study 4: Manipulated inputs profiles for full profile control (solid), and lexicographic optimization based profile control (dashed dotted) and endpoint control strategies (dashed) strategies for the continuous pulp digester.

6. Conclusions

In this work, we demonstrated a novel formulation of MPC using lexicographic optimization. This formulation has an advantage of explicitly prioritising the conflicting control objectives, which is of particular concern when the targets become infeasible. We demonstrated the benefits for profile control in a DPS using a simple PFR example and a large-scale system namely, the continuous pulp digester. We also showed that the added computation burden is insignificant when using extended MPC.

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References

[1] S. Gruner, K.-D. Mohl, A. Kienle, E.D. Gilles, G. Fernholz, M. Friedrich, Nonlinear control of a reactive distillation column, Control Eng. Pract. 11 (2003) 915–925.
[2] A.K. Jana, A.N. Samanta, S. Ganguly, Observer-based control algorithms for a distillation column, Chem. Eng. Sci. 61 (2006) 4071–4085.
[3] P.A. Wisnewski, F.J. Doyle, Model-based predictive control studies for a continuous pulp digester, IEEE Trans. Contr. Syst. 9 (2001) 435–444.
[4] P.A. Wisnewski, F.J.I. Doyle, Control structure selection and model predictive control of the Weyerhaeuser digester problem, J. Process Contr. 8 (1998) 487–495.
[5] W.L. Luyben, Profile position control of distillation columns with sharp temperature profiles, AIChE J. 18 (1972) 238–240.
[6] W. Marquardt, Concentration profile estimation and control in binary distillation, in: T.J. McAvoy, Y. Arkun, E. Zafiriou (Eds.), Proceedings of the IFAC Workshop on Model Based Process Control, Atlanta, June 13–14, 1988, pp. 137–144.
[7] W. Marquardt, Nonlinear model reduction for binary distillation, in: Proceedings of the IFAC Symposium, McGreavy, Bournemouth, UK, December 8–10, 1986, pp. 123–128.
[8] Y.-L. Hwang, Nonlinear wave theory for dynamics of binary distillation columns, AIChE J. 37 (1993) 705–723.
[9] E.D. Gilles, U. Epple, Non-linear wave propagation and model reduction of the fixed-bed reactor, in: K.H. Ebert, P. Deuflhard, W. Jager (Eds.), Proceedings of International Workshop on Modelling of Chemical Reaction Systems, Heidelberg, September 1–5, 1980, 1981, pp. 312–336.
[10] Y.-L. Hwang, F.G. Helfferich, Dynamics of continuous countercurrent mass transfer processes: II. Single component systems with nonlinear equilibria, Chem. Eng. Sci. 43 (1988) 1099–1114.
[11] F.J. Doyle, H.M. Budman, M. Morari, Linearizing controller design for a packed-bed reactor using a low-order wave propagation model, Ind. Eng. Chem. Res. 35 (1996) 3567–3580.
[12] M. Han, S. Park, Control of high-purity distillation column using a nonlinear wave theory, AIChE J. 39 (1993) 787–796.
[13] D.J. Kozub, J.F. MacGregor, J.D. Wright, Application of LQ and IMC controllers to a packed-bed reactor, AIChE J. 33 (1987) 1496–1506.
[14] L.C. Windes, W.H. Ray, A control scheme for packed bed reactors having a changing catalyst activity profile. I. On-line parameter estimation and feedback control, J. Process Contr. 2 (1992) 23–42.
[15] M. Yoshida, S. Matsumoto, Control of axial temperature distribution in a packed-bed reactor, J. Chem. Eng. Jpn. 31 (1998) 381–390.
[16] P.D. Christofides, N.H. El-Frarra, M. Li, P. Mhaskar, Model-based control of particulate processes, Chem. Eng. Sci. (2007), doi:10.1016/j.ces.2007.07.017.
[17] R.D. Braatz, S. Hasebe, Particle size and shape control in crystallization processes, in: J.B. Rawlings, et al. (Eds.), AIChE Symposium Series: Proceedings of the 6th International Conference on Chemical Process Control, 2002, pp. 307–327.
[18] J.B. Rawlings, S.M. Miller, W.R. Witkowski, Model identification and control of solution crystallization process—a review, Ind. Eng. Chem. Res. 32 (1993) 1275–1296.
[19] C.D. Immanuel, F.J. Doyle, Hierarchical multiobjective strategy for particle-size distribution control, AIChE J. 49 (2003) 2383–2399.
[20] P. Dufour, Y. Toure, D. Blanc, P. Laurent, On nonlinear distributed parameter model predictive control strategy: on-line calculation time reduction and application to an experimental drying process, Comp. Chem. Eng. 27 (2003) 1533–1542.
[21] N. Padhiyar, A. Gupta, A. Gautam, S. Bhartiya, F.J. Doyle, S. Dash, S. Gaikwad, Nonlinear inferential multi-rate control of Kappa number at multiple locations in a continuous pulp digester, J. Process Contr. 16 (2006) 1037–1053.
[22] F.J. Doyle, F. Kayihan, Reaction profile control of the continuous pulp digester, Chem. Eng. Sci. 54 (1999) 2679–2688.
[23] R. Teman, Infinite Dimensional Dynamical Systems in Mechanics and Physics, Springer-Verlag, New York, 1988.
[24] M.J. Balas, Finite-dimensional control of distributed parameter systems by Galerkin approximation of infinite dimensional controllers, J. Math. Anal. Appl. 114 (1986) 17–36.
[25] R.F. Curtain, Finite-dimensional compensator design for parabolic distributed systems with point sensors and boundary input, IEEE Trans. Automat. Contr. 27 (1982) 98–104.
[26] D.H. Gay, W.H. Ray, Identification and control of distributed parameter systems by means of the singular value decomposition, Chem. Eng. Sci. 50 (1995) 1519–1539.
[27] W.H. Ray, Advanced Process Control, McGraw-Hill, New York, 1981.
[28] X. Li, J. Yong, Optimal control for a class of distributed parameter systems, in: Proceedings of the IEEE Conference on Decision and Control, 5–7 December 1990.
[29] C.E. Garcia, Quadratic/dynamic matrix control of nonlinear processes – an application to a batch reaction process, Annu. Meet. Am. Inst. Chem. Eng. (1984).
[30] J.P. Ignizio, Goal Programming and Extensions, Lexington, Massachusetts, 1976.
[31] J.P. Ignizio, Linear Programming in Single and Multiple-Objective Systems, Prentice-Hall, Englewood Cliffs, New Jersey, 1982.
[32] Y. Ijiri, Managerial Goals and Accounting for Control, Rand McNally, Chicago, 1965.
[33] V. Jääskelainen, Linear Programming and Budgeting, Petrocelli/Charter, New York, 1976.
[34] H.D. Sherali, Equivalent weights for lexicographic multi-objective programs: characterizations and computations, Eur. J. Oper. Res. 11 (1982) 367–379.
[35] H.D. Sherali, A.L. Soyster, Preemptive and nonpreemptive multiobjective programming: relationships and counterexamples, J. Optim. Theory Appl. 39 (1983) 173–186.
[36] T.A. Meadowcroft, G. Stephanopoulos, C. Brosilow, The modular multivariable controller: I. Steady-state properties, AIChE J. 38 (1992) 1254–1278.
[37] J. Vada, O. Slupphaug, T.A. Johansen, B.A. Foss, Linear MPC with optimal prioritized infeasibility handling: application, computational issues and stability, Automatica 37 (2001) 1835–1843.
[38] P.A. Wisnewski, F.J.I. Doyle, F. Kayihan, Fundamental continuous-pulp-digester model for simulation and control, AIChE J. 43 (1997) 3175–3192.
[39] R. Amirthalingam, J.H. Lee, Subspace identification based inferential control applied to a continuous pulp digester, J. Process Contr. 9 (1999) 397–406.
[40] S. Bhartiya, F.J. Doyle, Mathematical model predictions of a plugging phenomenon in an industrial single vessel pulp digester, Ind. Eng. Chem. Res. 43 (2004) 5225–5232.