TEORÍA DE CONTROL 1

Respuesta en el Tiempo de los Sistemas 2do Orden

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Functions of a Closed Loop System

https://www.youtube.com/watch?v=bEnRRI7XXRs&list=PLDiXbC2f4yX1MDpoxb6j26PTzmkRp5Qxi

PID Controller

https://www.youtube.com/watch?v=YLGLrEwEiTQ&list=PLDiXbC2f4yX1MDpoxb6j26PTzmkRp5Qxi&index=2

Overhead crane control with ACS880 drives

https://www.youtube.com/watch?v=3iZ4Ddh6DX4

Konecranes Smart Features - Sway Control

https://www.youtube.com/watch?v=3qdz6g1tw5A

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Sistemas de Segundo Orden

$$\frac{C(s)}{R(s)} = \frac{b_0}{s^2 + a_1 s + a_0}$$

$K = \dfrac{b_0}{a_0}$ gain

$\omega_n = \sqrt{a_0}$ natural frequency (undamped frequency)

$\zeta = \dfrac{a_1}{2\sqrt{a_0}}$ damping ratio

Re-escribiendo:

$$\frac{C(S)}{R(S)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

Sistemas de Segundo Orden: Entrada Escalón Unitario

$$\frac{C(S)}{R(S)} = \frac{wn^2}{S^2 + 2\xi wnS + wn^2}$$

$$C(S) = \frac{1}{S} + \frac{A1}{S - P1} + \frac{A2}{S - P2}$$

A1 y A2 son constantes
P1 y P2 son raíces de la Ecuación Característica: $S^2 + 2\xi wnS + wn^2 = 0$

Usando las tablas de Transformada de Laplace:

$$c(t) = 1 + A1e^{P1.t} + A2e^{P2.t}$$

Donde:

A1$= -\frac{1}{2} + \frac{\xi}{2\sqrt{(\xi^2-1)}}$ P1$= -\xi.wn + wn\sqrt{(\xi^2 - 1)}$

A2$= -\frac{1}{2} - \frac{\xi}{2\sqrt{(\xi^2-1)}}$ P2$= -\xi.wn - wn\sqrt{(\xi^2 - 1)}$

Sistemas de Segundo Orden: Entrada Escalón Unitario

$\zeta > 1$ Sistema sobre amortiguado

$\zeta = 1$ Sistema críticamente amortiguado

$\zeta < 1$ Sistema sub amortiguado

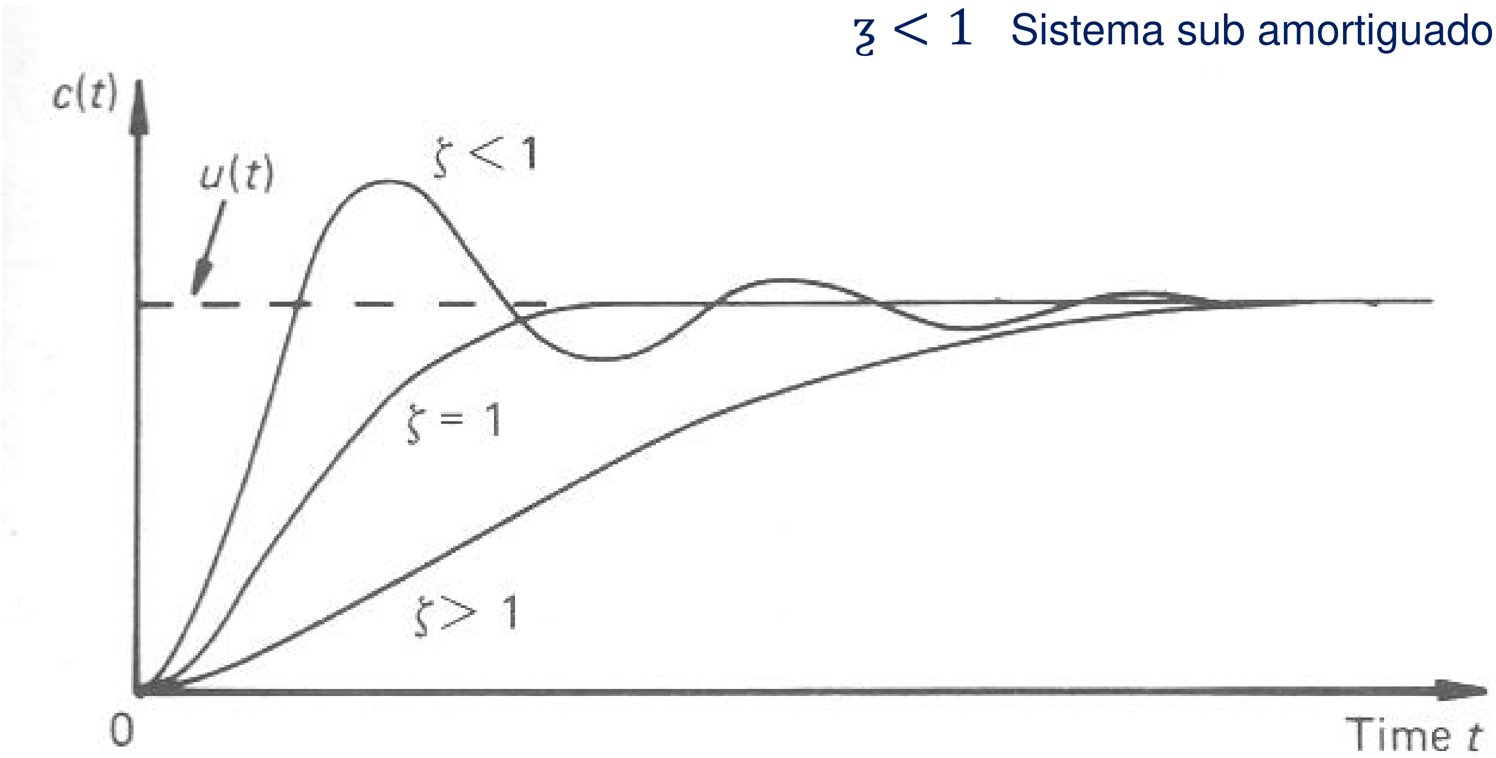


Fig. 4.5 Form of step response for second order system

Sistemas de Segundo Orden: Entrada Escalón Unitario

$\zeta > 1$ Sistema sobre amortiguado

$$c(t) = 1 + A1e^{P1.t} + A2e^{P2.t}$$

Donde:

$$A1 = -\frac{1}{2} + \frac{\zeta}{2\sqrt{(\zeta^2-1)}} \qquad P1 = -\zeta.wn + wn\sqrt{(\zeta^2-1)}$$

$$A2 = -\frac{1}{2} - \frac{\zeta}{2\sqrt{(\zeta^2-1)}} \qquad P2 = -\zeta.wn - wn\sqrt{(\zeta^2-1)}$$

A1 y A2 reales
P1 y P2 reales negativos

Dos sistemas de primer orden:

$$\frac{C(S)}{R(S)} = \frac{K1}{1+\tau1.S} \cdot \frac{K2}{1+\tau2.S}$$

Sistemas de Segundo Orden: Entrada Escalón Unitario

$\zeta = 1$ Sistema críticamente amortiguado

$$c(t) = 1 + A1e^{P1.t} + A2e^{P2.t}$$

Donde:

$$A1 = -\frac{1}{2} + \frac{\zeta}{2\sqrt{(\zeta^2 - 1)}} \qquad P1 = -\zeta.wn + wn\sqrt{(\zeta^2 - 1)}$$

$$A2 = -\frac{1}{2} - \frac{\zeta}{2\sqrt{(\zeta^2 - 1)}} \qquad P2 = -\zeta.wn - wn\sqrt{(\zeta^2 - 1)}$$

A1 y A2 reales e iguales
P1 y P2 reales negativos e iguales

Dos sistemas de primer orden iguales:

$$\frac{C(S)}{R(S)} = \frac{K}{1+\tau.S} \cdot \frac{K}{1+\tau.S}$$

Sistemas de Segundo Orden: Entrada Escalón Unitario

$\xi < 1$ Sistema sub amortiguado

$$c(t) = 1 + A1e^{P1.t} + A2e^{P2.t}$$

Donde:

$$A1 = -\frac{1}{2} + \frac{\xi}{2\sqrt{(\xi^2-1)}} \qquad P1 = -\xi.wn + wn\sqrt{(\xi^2-1)}$$

$$A2 = -\frac{1}{2} - \frac{\xi}{2\sqrt{(\xi^2-1)}} \qquad P2 = -\xi.wn - wn\sqrt{(\xi^2-1)}$$

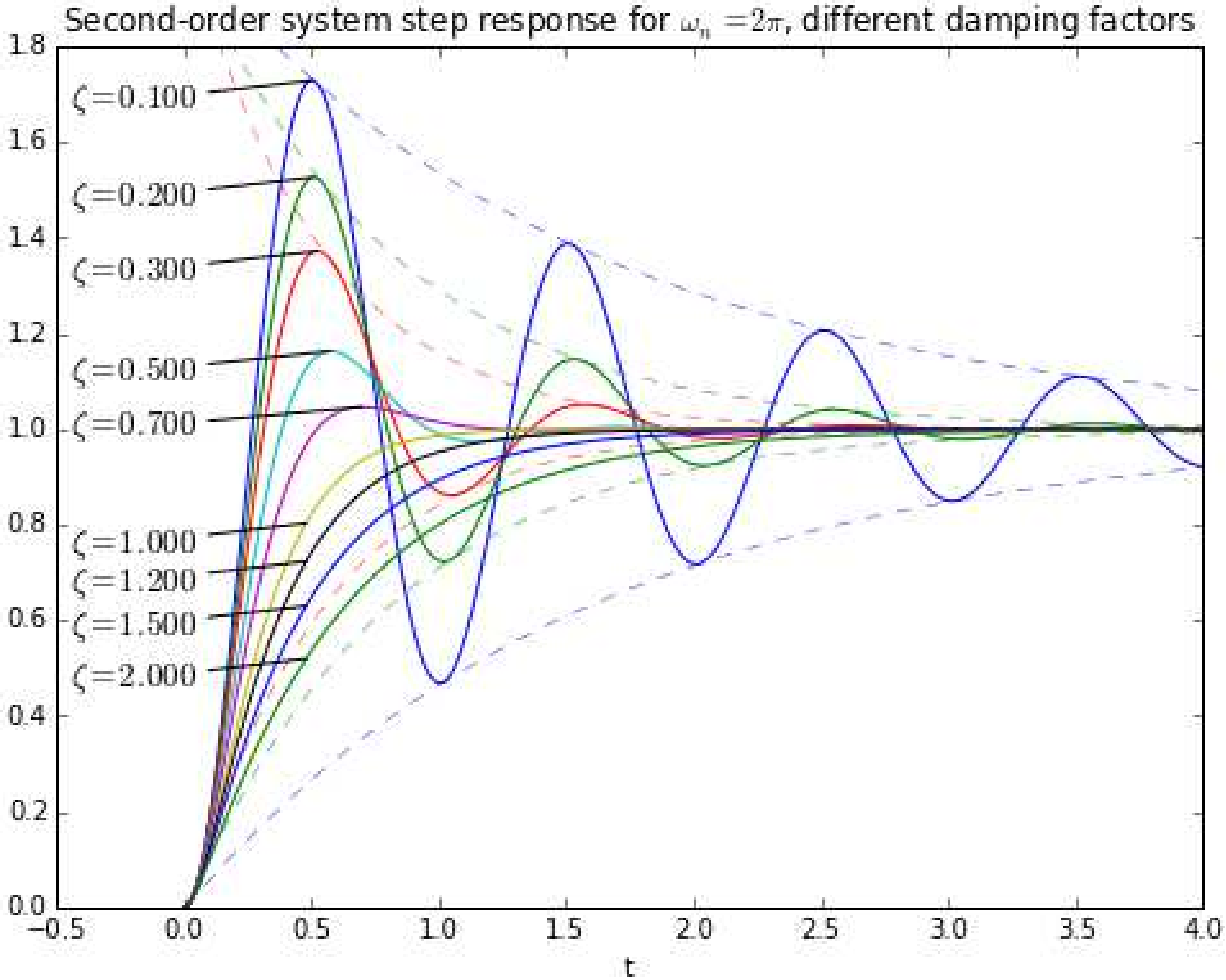
A1 y A2 imaginarios conjugados
P1 y P2 raíces imaginarios conjugados

Respuesta: $$c(t) = \left(1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_d t + \theta)\right)$$

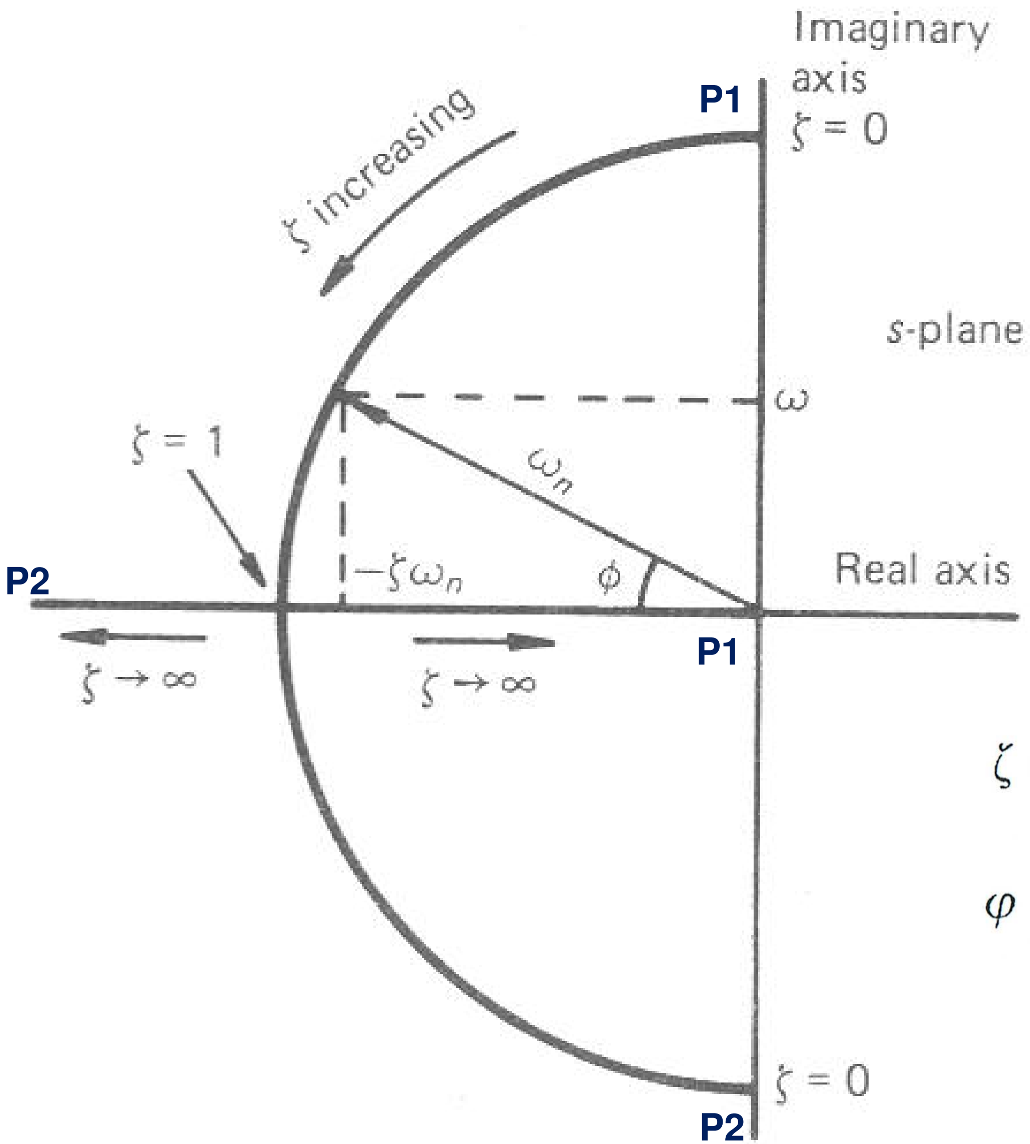
$$\theta = \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)$$

Frecuencia Natural Amortiguada: $\omega_d = \omega_n\sqrt{1-\zeta^2}$

Sistemas de Segundo Orden: Entrada Escalón Unitario



Sistemas de Segundo Orden: Lugar geométrico de raíces



$$S^2 + 2ʒwnS + wn^2 = 0$$

$$(S - P1)(S - P2) = 0$$

P1$= -ʒ.wn + wn\sqrt{(ʒ^2 - 1)}$

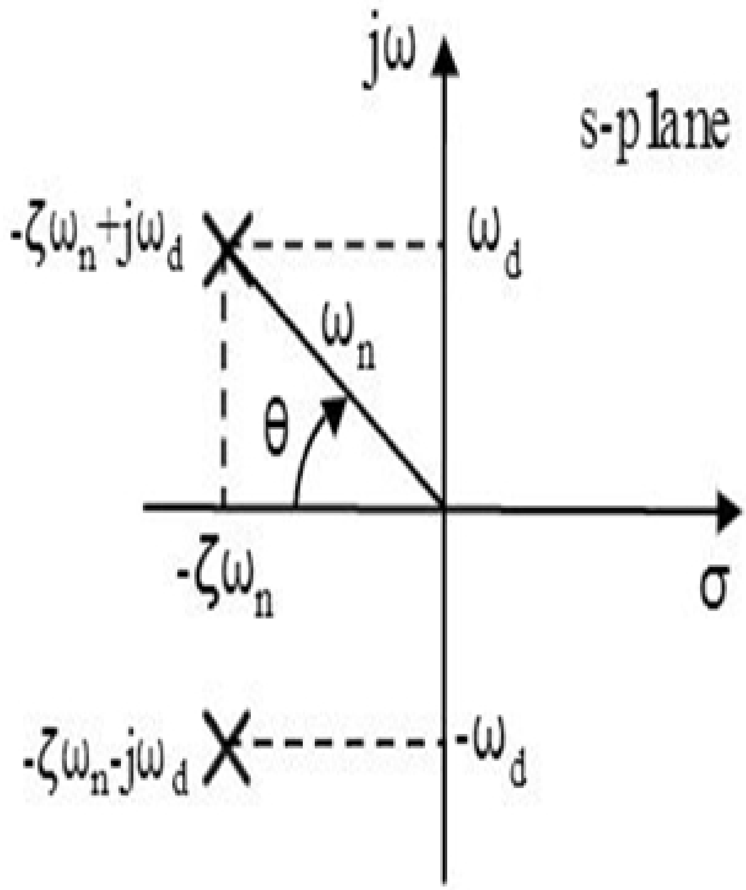
P2$= -ʒ.wn - wn\sqrt{(ʒ^2 - 1)}$

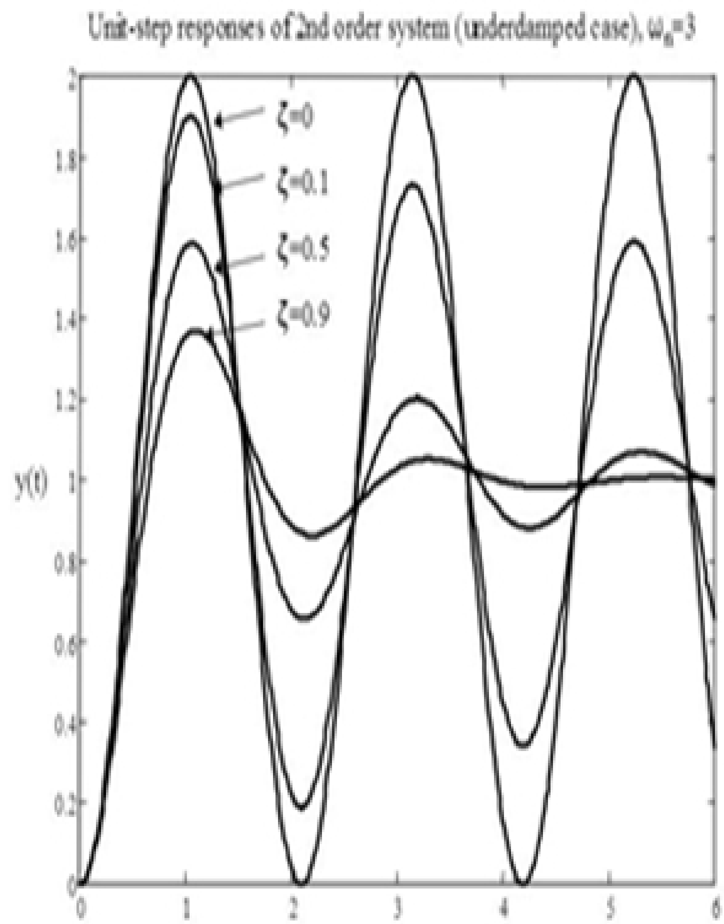
$$\zeta = \cos\varphi$$

$$\varphi = \cos^{-1}\zeta \text{ or } \varphi = \tan^{-1}\frac{\sqrt{(1-\zeta^2)}}{\zeta}$$

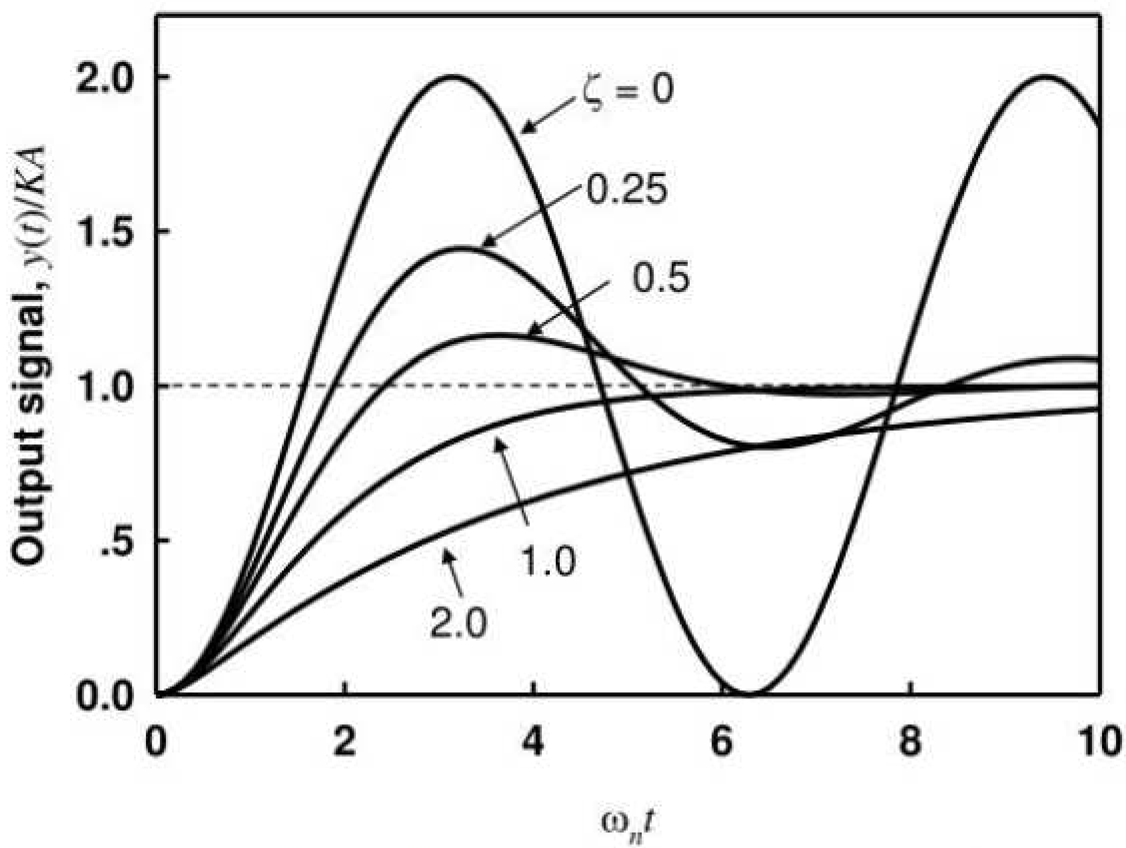
Fig. 4.6 Locus of roots of second order system with fixed ω_n as ζ varies from 0 to ∞

Sistemas de Segundo Orden





Sistemas de Segundo Orden: Mejor respuesta



Ringing frequency: $T_d = \dfrac{2\pi}{\omega_d}$

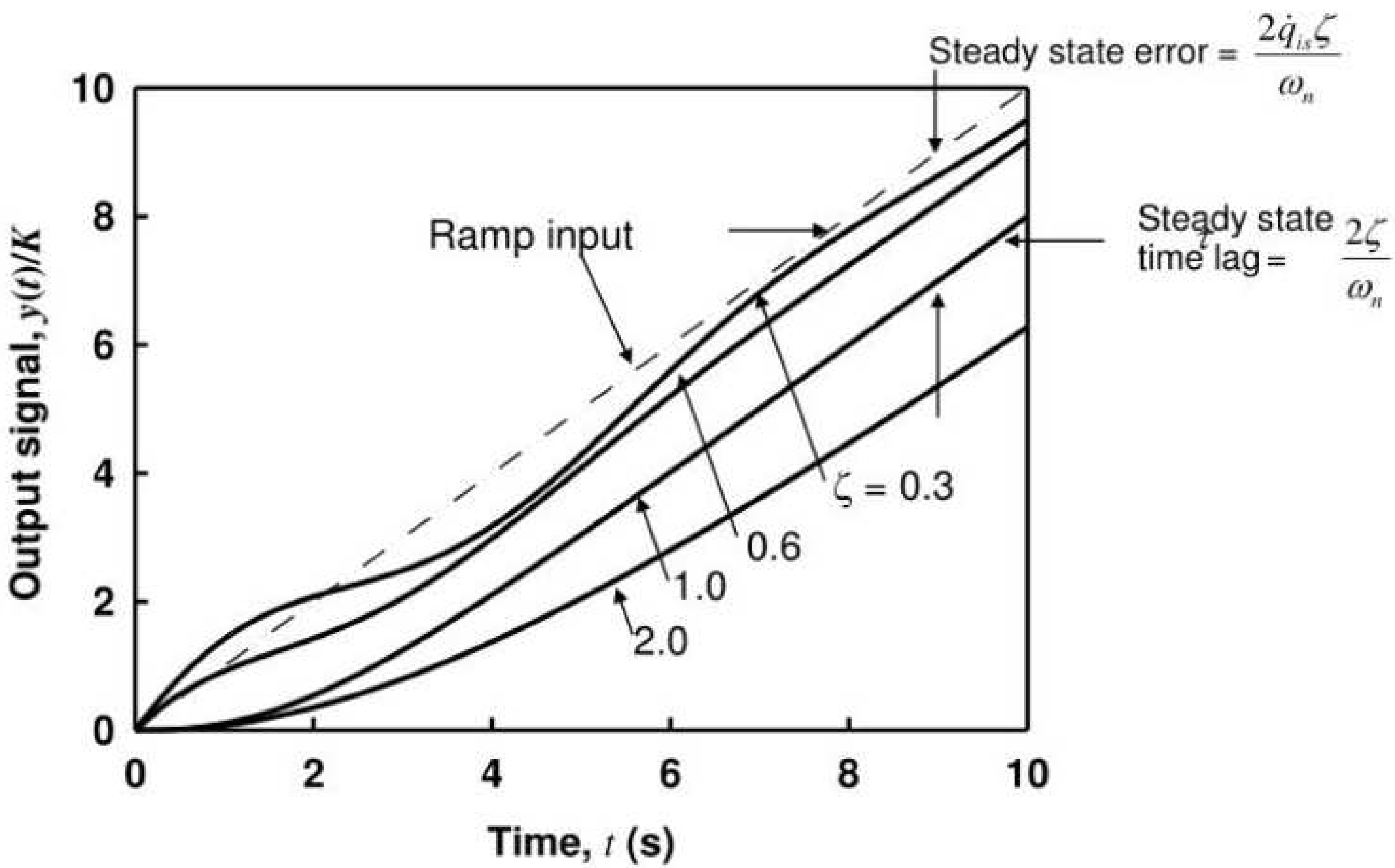
Ringing frequency: $\omega_d = \omega_n\sqrt{1-\zeta^2}$

Rise time decreases ζ with but increases ringing

Optimum settling time can be obtained from ζ ~ 0.7

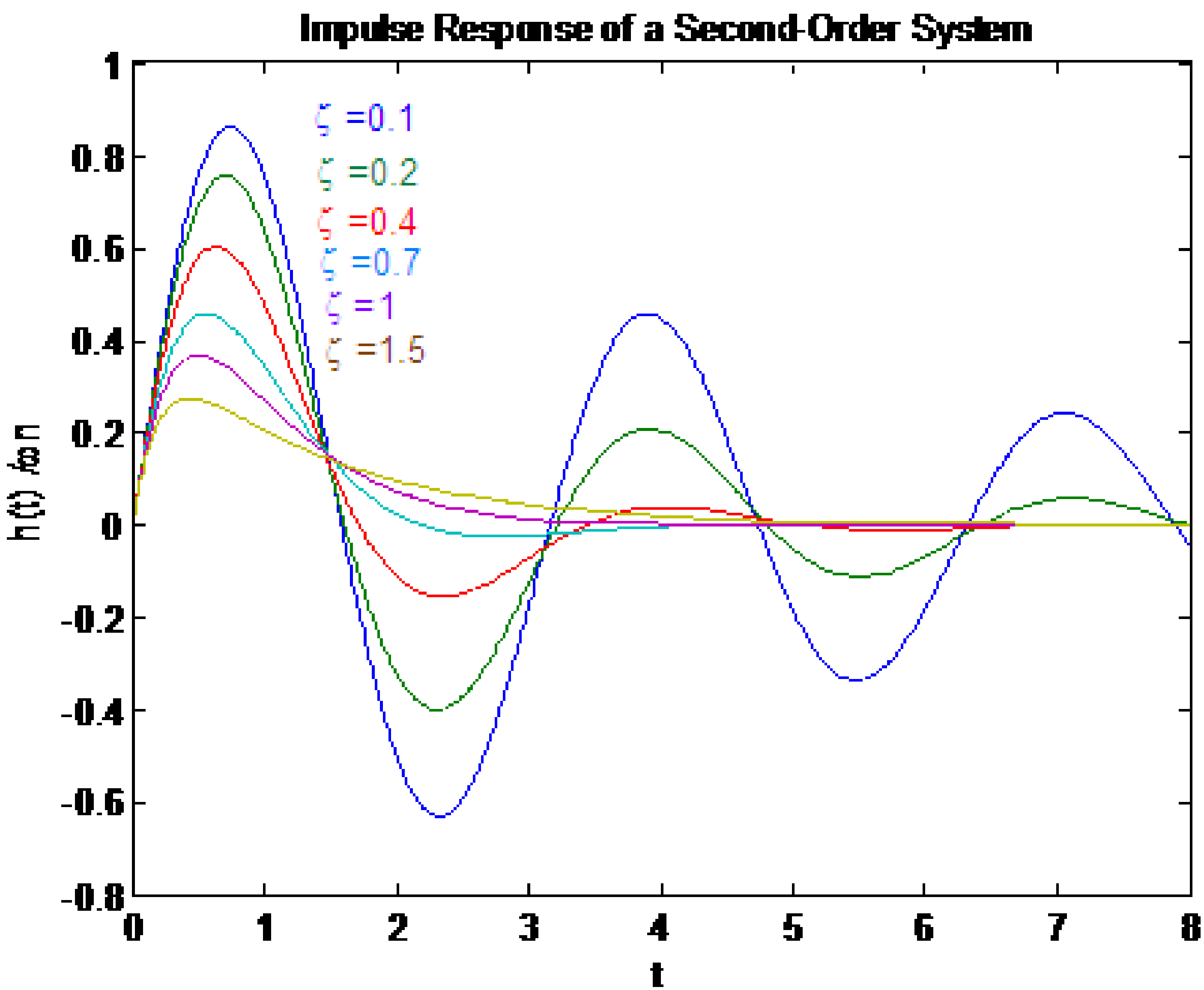
Practical systems use 0.6< ζ <0.8

Sistemas de Segundo Orden: Entrada Rampa



Typical ramp response of second-order instrument

Sistemas de Segundo Orden: Entrada Impulso



Sistemas de Segundo Orden: Sobreimpulso

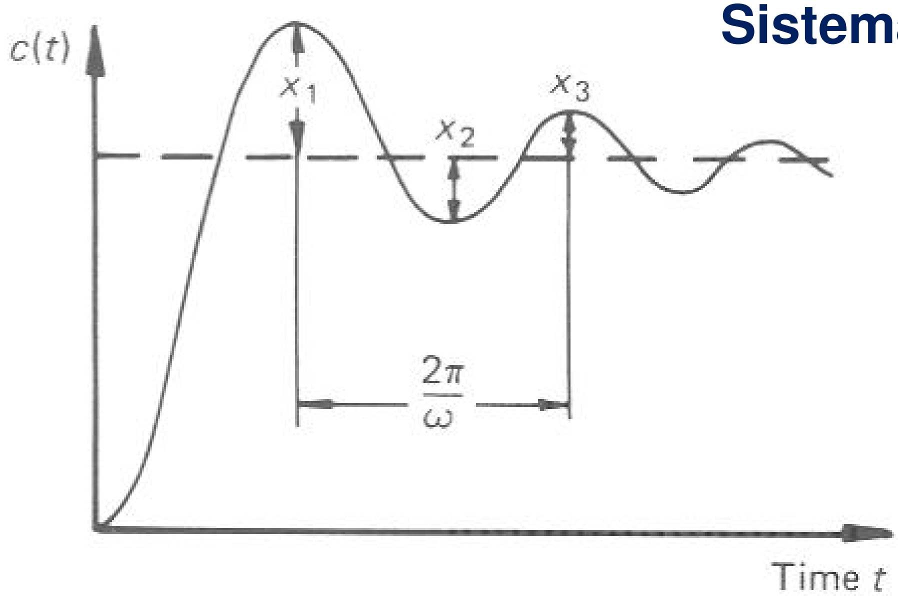


Fig. 4.10 Step response of oscillatory system

$$c(t_1) = 1 + \exp\left(-\frac{\zeta\pi}{\sqrt{(1-\zeta^2)}}\right)$$

$$c(t_2) = 1 - \exp\left(-\frac{2\zeta\pi}{\sqrt{(1-\zeta^2)}}\right)$$

$$c(t_3) = 1 + \exp\left(-\frac{3\zeta\pi}{\sqrt{(1-\zeta^2)}}\right)$$

Xi

$$c(t) = 1 - \frac{\exp(-\zeta\omega_n t)}{\sqrt{(1-\zeta^2)}} \sin\left(\omega_n\sqrt{(1-\zeta^2)}t + \tan^{-1}\frac{\sqrt{(1-\zeta^2)}}{\zeta}\right)$$

$$\therefore \quad \frac{dc(t)}{dt} = -\frac{1}{\sqrt{(1-\zeta^2)}}\{\exp(-\zeta\omega_n t)\cos[\omega_n\sqrt{(1-\zeta^2)}t + \varphi]\omega_n\sqrt{(1-\zeta^2)}$$

$$+\exp(-\zeta\omega_n t)(-\zeta\omega_n)\sin[\omega_n\sqrt{(1-\zeta^2)}t + \varphi]\}$$

$$= \frac{\exp(-\zeta\omega_n t)}{\sqrt{(1-\zeta^2)}}\{\zeta\omega_n \sin[\omega_n\sqrt{(1-\zeta^2)}t + \varphi]$$

$$-\omega_n\sqrt{(1-\zeta^2)}\cos[\omega_n\sqrt{(1-\zeta^2)}t + \varphi]\}$$

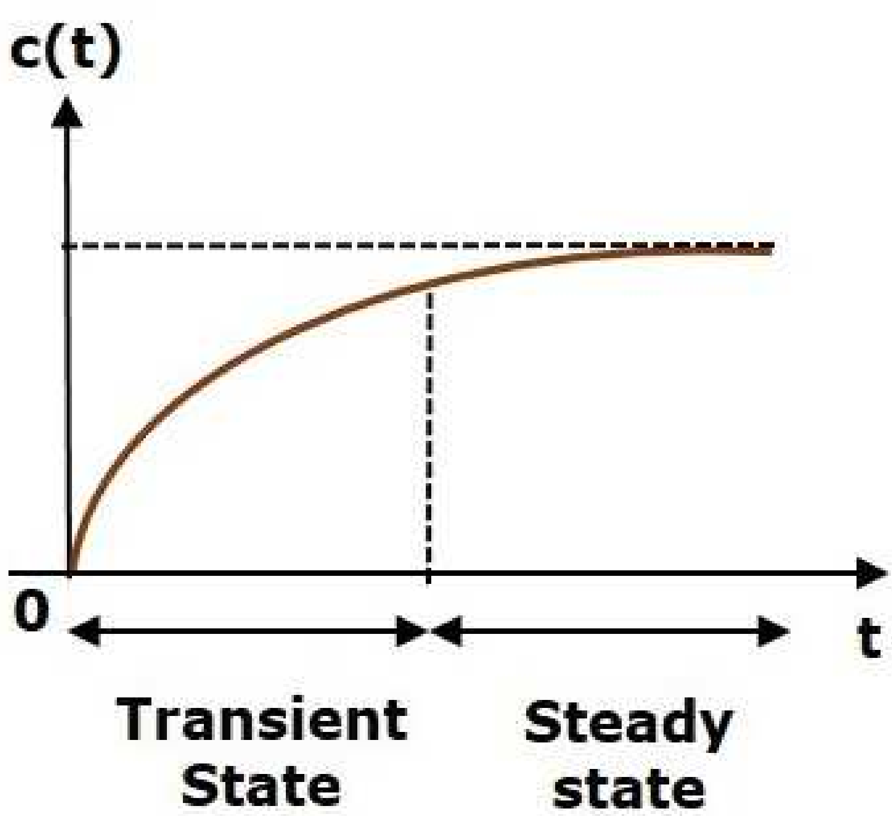
$$= 0 \quad \text{when } \tan[\omega_n\sqrt{(1-\zeta^2)}t + \varphi] = \frac{\sqrt{(1-\zeta^2)}}{\zeta}$$

But $\varphi = \tan^{-1}\dfrac{\sqrt{(1-\zeta^2)}}{\zeta}$

Hence peaks and troughs occur when $\omega_n\sqrt{(1-\zeta^2)}t = n\pi$

i.e. when $t = \dfrac{n\pi}{\omega_n\sqrt{(1-\zeta^2)}} = t_n$ where $n = 1, 2, 3 \ldots$

Comparación sistemas de primer y segundo Orden



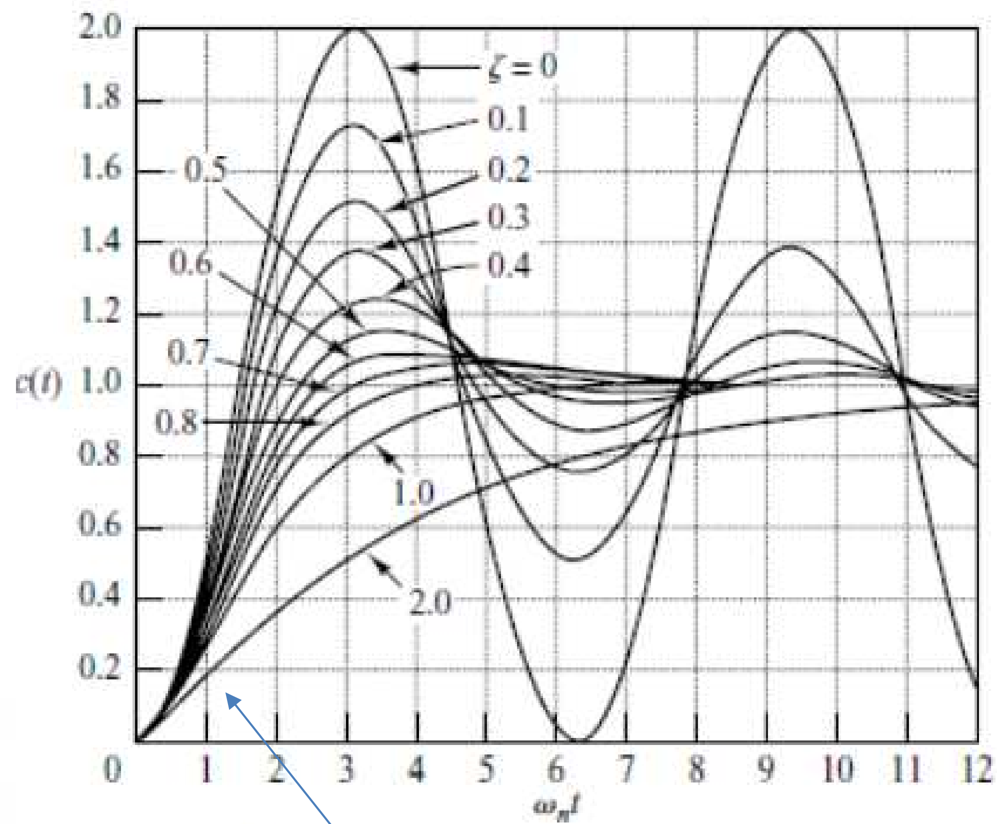
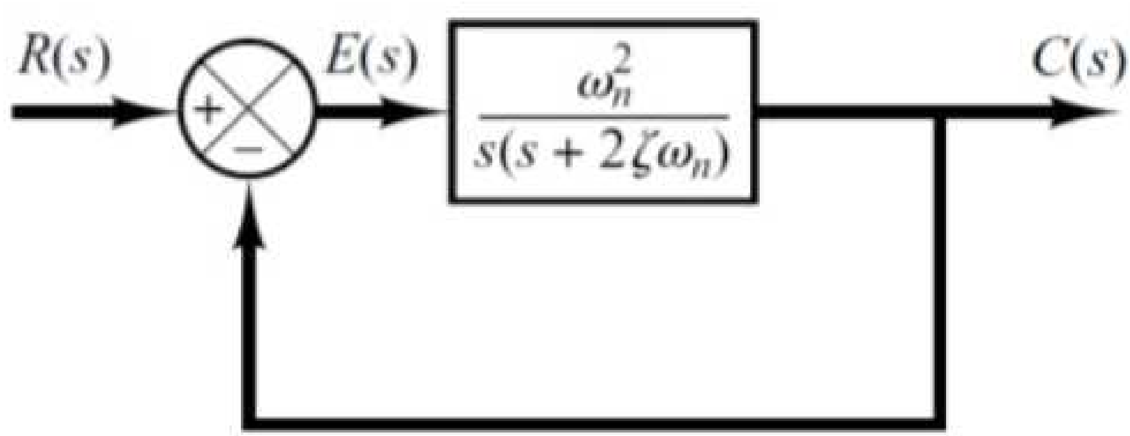


Figure 1: Unit step response curves of the system

Punto de inflexión

Sistemas de Segundo Orden: Diagrama de bloques



$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Sistemas de Segundo Orden: Especificación de respuesta

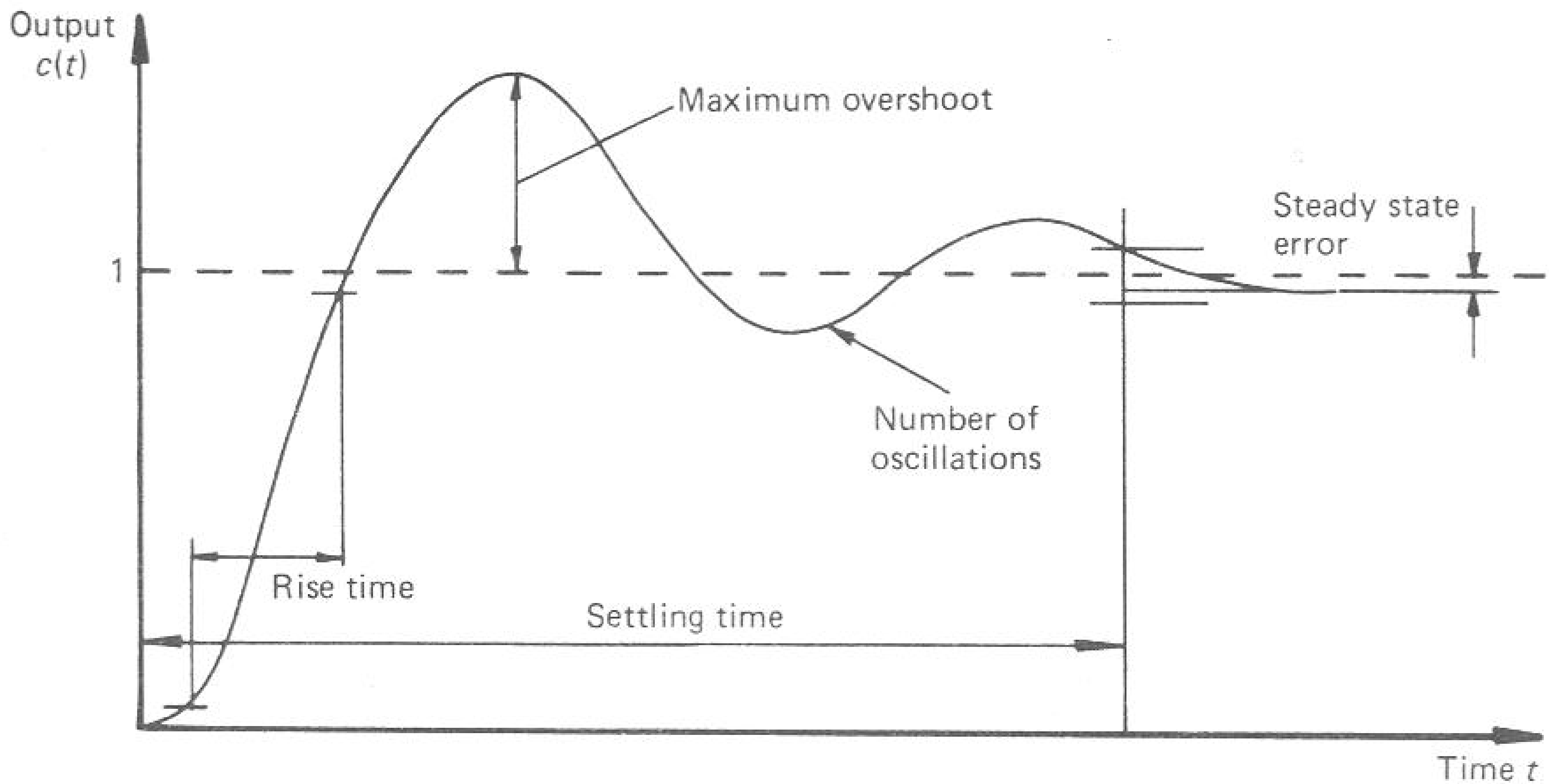
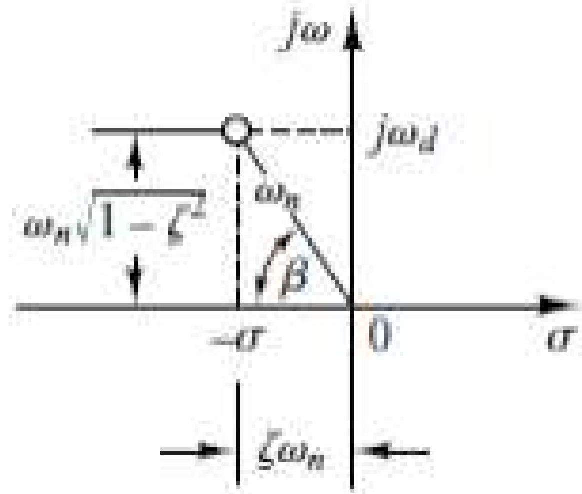


Fig. 4.9 Parameters describing unit step response

Sistemas de Segundo Orden: Especificación de respuesta

Tiempo de subida: $$t_r = \frac{\pi - \beta}{\omega_d}$$



Máximo sobreimpulso: $$\exp\left(-\frac{\zeta\pi}{\sqrt{(1-\zeta^2)}}\right) \qquad t_n = \frac{n\pi}{\omega_n\sqrt{(1-\zeta^2)}} \qquad n = 1.$$

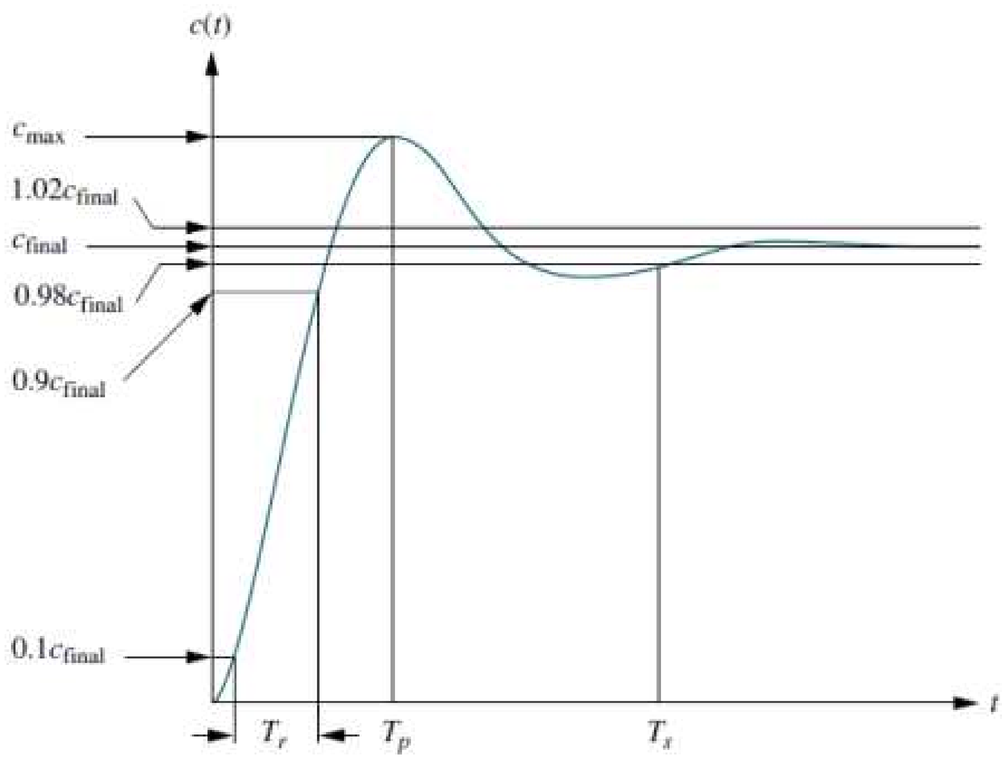
Tiempo de establecimiento:

$t_s = \frac{4}{\zeta\omega_n}$	(2% criterion)
$t_s = \frac{3}{\zeta\omega_n}$	(5% criterion)

Sistemas de Segundo Orden: Especificación de respuesta

Número de oscilaciones n:

Hasta sobreimpulso n, tal que c(tn) < 2%



Second-order underdamped response specifications

Especificación de respuesta: Sistemas de Orden n

Error en estado estable E(s) = U(s) –C(s):

$$\frac{C(s)}{U(s)} = G(s)$$

$$\lim_{t \to \infty} c(t) = \lim_{s \to 0} sC(s) \quad \text{by the final value theorem}$$

$$= \lim_{s \to 0} G(s) \quad \text{for a steady unit input, } U(s) = \frac{1}{s}$$

$$\text{e.g.} \quad \text{if } \frac{C(s)}{U(s)} = \frac{50(1 + 5s)}{(s^2 + 3s + 16)(1 + s)}$$

$$[c(t)]_{t \to \infty} = \lim_{s \to 0} \frac{50(1 + 5s)}{(s^2 + 3s + 16)(1 + s)} = \frac{50}{16}$$

Error en estado estable E(s) = U(s) –C(s) = 1 – 50/16 = - 34/16

Sistemas de orden superior

Dada la función transferencia

$$\frac{C(s)}{U(s)} = \frac{P(s)}{Q(s)}$$

Donde P(s) y Q(s) son polinomios
Q(s)=0 es la ecuación característica

$$\frac{C(s)}{U(s)} = \frac{P(s)}{(s-p_1)(s-p_2)(s-p_3)\ldots(s-p_N)}$$

$$c(t) = 1 + A_1 e^{p_1 t} + A_2 e^{p_2 t} + A_3 e^{p_3 t} + \ldots A_N e^{p_N t}$$

$$c(t) = 1 + \sum_{n=1}^{N} A_n e^{p_n t}$$

Gracias

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