Equity Structured Products

Project: Autocall Pricing with Heston

by

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Master 272 - 2021/2022
Quantitative Finance
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Introduction

As part of our studies in quantitative finance for the Master 272 program of Paris-Dauphine University, we worked on an Autocall Pricer. In business terms, it represents an "Athena", meaning that to get coupons we must autocall. This financial product is priced with Heston model in order to take into account the stochasticity of the volatility. For that, the model has been calibrated with FTSE 100 Index prices (available on https://www.eurex.com/ex-en/markets/idx/country/ftse).

Several market datas were used for this project. First, for the funding of the product (issued by Barclays), we obtained Senior CDS curve (Bloomberg) of the bank as well as SOFR Rate Curve (Bloomberg). Indeed, given that the product is issued in USD, the funding was calculated with US Rates. Then, the underlying being denominated in GBP, to discount the prices we used the United Kingdom Government Bonds - Yields Curve (available on http://www.worldgovernmentbonds.com/country/united-kingdom/). The market datas were fixed on Friday 15th of July, 2022.

For the product we priced with the following (as per the termsheet) caracteristics:

- Strike : 100%
- Maturity : 6Y
- Coupon : 9.35%
- Down-and-In barrier : 60%
- Non-Call period : 1Y
- Underlying : FTSE 100 Index
- Denomination : USD

Implementation

In order to implement the pricing, we must first introduce the Heston Model. It allows to take into account a stochastic variance (hence volatility) that has its own diffusion process and will be correlated with the spot diffusion.

$$
\begin{aligned}
\mathrm{d}S(t) &= rS(t)\mathrm{d}t + \sqrt{v(t)}S(t)\mathrm{d}W_x(t), \\
\mathrm{d}v(t) &= \kappa(\bar{v} - v(t))\mathrm{d}t + \gamma\sqrt{v(t)}\mathrm{d}W_v(t).
\end{aligned}
$$

Figure 1: Heston Model PDE

With κ being the speed of mean reversion, γ the vol of vol, responsible for the convexity of the volatility surface. $\bar{v}$ is the long term variance, and ρ is the correlation coefficient between the two processes. V_0 is the initial variance.

In order to make the calibration, we simulate spots and variances processes. The number of simulations has been set to 1000 and the time step to 1000. Unfortunately, the time consumption was quite long (several hours) so we were not able to capture the full dynamic of the market prices. However, we were able to make 10 iterations on the parameters (initially set to levels empirically observed) and we obtained the following parameters :

- κ : 3.39
- V_0 : 0.1029
- γ : 0.2896
- $\bar{v}$: 0.0766
- ρ : -0.747

Once the market was fitted (or approximately), we were able to reprice the options as following:

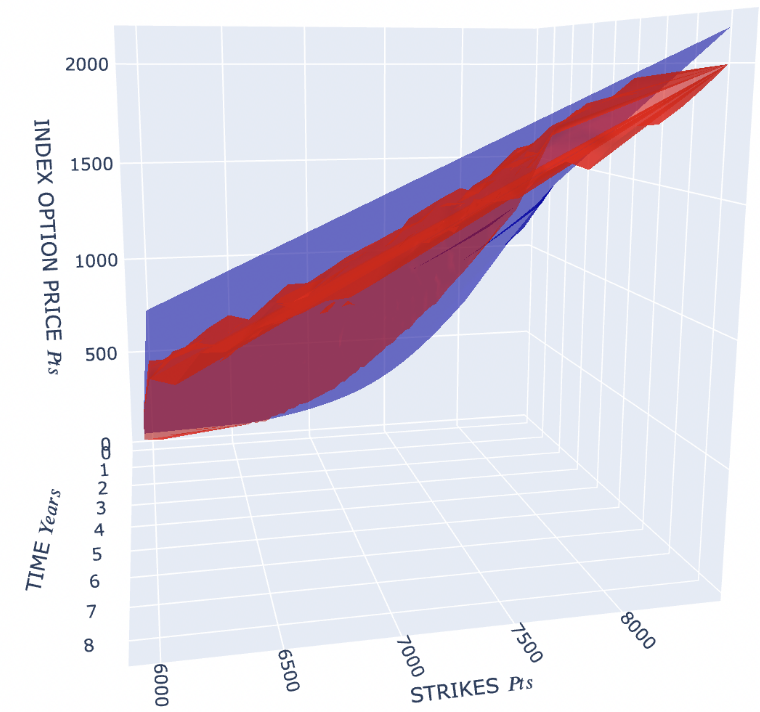

Figure 2: Market Prices vs Heston Prices

Thus, we were able to price vanilla put options. This is our best approximation tool for a Down and In Put. We then priced the DIP with the Heston process that has the calibrated parameters. We found an option value of approximately 14.8% (as a percentage of the initial spot price)

The client will bring his notional to the bank, meaning that the later will be able to invest it at a rate composed of (SOFR + Credit spread). Given the uncertainty of the maturity of the product (autocallability), we weighted by the autocallable probabilities the bullet funding as following :

$$
\begin{aligned}
Funding = 2 * P(AC_{2Y}) * (SOFR_{2Y} + CDS_{2Y}) + \\
3 * P(AC_{3Y}) * (SOFR_{3Y} + CDS_{3Y}) + ... + \\
6 * P(AC_{6Y}) * (SOFR_{6Y} + CDS_{6Y})
\end{aligned}
$$

With $P(AC_2)$ for example being the probability of Autocall at the second year (the first observation as we have a non call period of 1Y)

As anticipated, given that the Autocallable probabilities are conditional on no Autocall previously, they are decreasing. Except for the last one as if we don't Autocall before maturity, the product will terminate anyway at year 6.

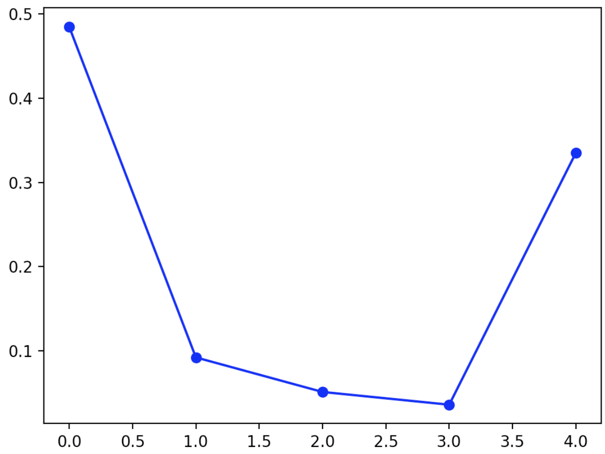

Figure 3: Autocall probabilities

As a result we found 14.78% for the funding.

Finally, once, the funding and DIP were priced, we had to price the Coupon part of the product. For that, we used the simulated paths (with calibrated parameters). For each path, we calculate the coupon (or none) that is paid to the client in %. For example, if the spot is above 100% of its initial level at year 2, the client will receive 2 * the annual coupon, meaning 18.70%. One additional thing to take into account is the discount factor. So we had to discount with the appropriate rate (UK interest rate yield curve) and appropriate period. In the above example we would have used the 2Y rate with a period of 2. Additionally, if the spot is not above, the product remains alive and the coupon (with unique periodicity for each path) can be paid/not later.

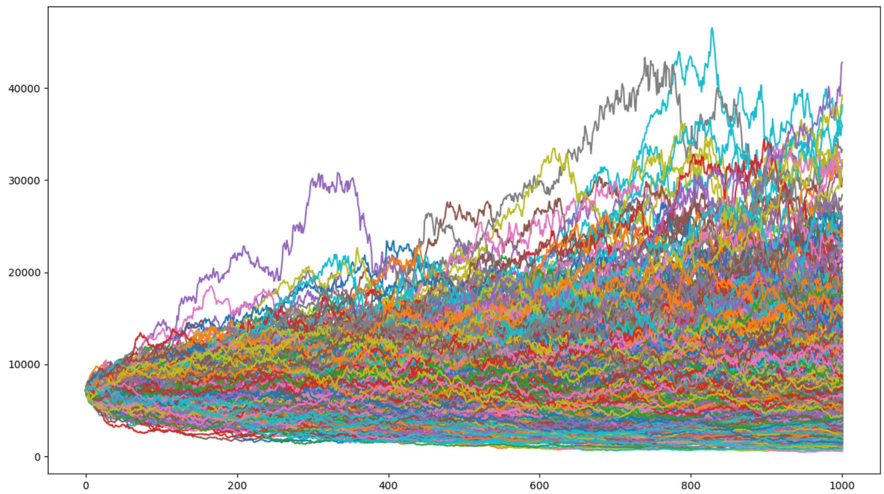

Figure 4: Calibrated Spots

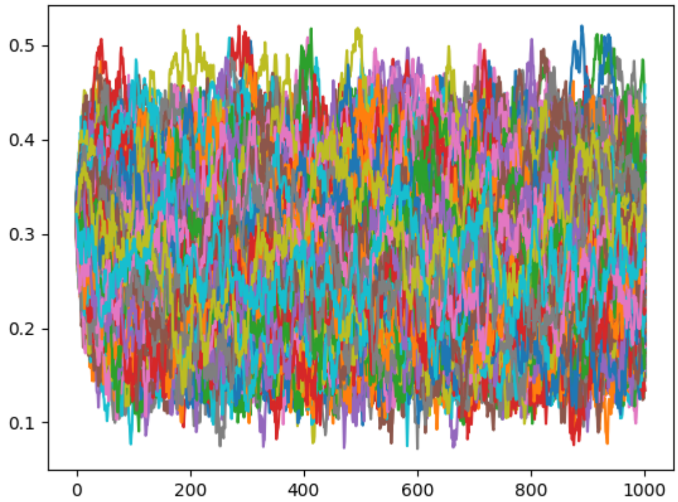

Figure 5: Calibrated variances

Finally, the Note price can be decomposed as following:

$$Price = CouponPrice - Funding - DownandInPut$$

As a result, given that the Note is sold at 100% (or less for commercial fees) to the client, the price will be the difference between 100% and the previous price

$$Price = 100\% - (14.86\% - 14.78\% - 14.83\%)$$

$$Price = 85.24\%$$

Code explanation

In order to implement in Python this pricing, several Classes have been created.

First, the main Autocall class that instances the product specificities such as DIP barrier, strike, coupon, etc. It calculates the autocall probabilities that are used for the funding and computes the Monte-Carlo price of the coupon leg.

It also uses the inherited class Numerics and Heston. The Numerics class is first used to get all the market datas for the Funding leg or the coupon price or the DIP. CDS spreads, SOFR rates and FTSE 100 Index option prices are loaded to make all the calculations. It also allows to make the calibration with the objectve function that can be minimized.

It is to be noted that **the user can either load specific parameters for the Heston processed with calibrate = False either make the calibration without specificying the parameters with calibrate = True**. Finally, this class allows to make a plot of the two price surfaces (market vs Heston).

Then the Heston class. It is the one responsible for the diffusion of the equity spot price. Two correlated processes : the spot and the variance. The CIR, a mean-reverting process, is used to modeling the variance.

Regarding the performance of the code. If for example we compute the price of the structure from the termsheet with 1000 number of steps and 10000 simulations, the final price of the structure is found in 2.5 seconds, which is quite fast. If however we want to find the calibrated parameters, several hours are required depending on the stopping criterion.

Overall, the performance is quite good even if the calibration takes a lot of time.

Risk analysis and hedge

This complex product goes with lots of risks for a trader to hold it in its books. The bank will be the seller of the product, so their are several sensitivities for the trader given this position. Usually, traders accumulates large amount of this type of structures, so potentially, this reduces the cross effect if the trader has different positions (seller or buyer) for those products.

First, it is to be noted that given that the trader is short of the product, he is long of the down-and-in put. He will then be short of the forward, so short spot, short interest rates but long repo and dividends. With the Covid pandemic and the decrease/cancellation of the dividends, the banks holding structured products were hit by this. This product has also discontinuities in its payoff. Hence, in order to hedge the product, the trader will have to replicate the DIP as an example. For example, to replicate a long DIP barrier 60% strike 100%, the trader can buy Put-Spread 59-60 and buy another Put 70. Doing so, the trader will be more conservative meaning that he will buy a product that has a smaller value compared to its "true" value, as per in the termsheet. Given the discontinuities, the will also be a risk of skew given that the trader will be long skew. He will also be long volatility (on the put and on the digital coupons).

In this product, given that their is an uncertain anticipated redemption, the bank does not now when/if it will give the notional back to the client. Hence, the funding period is uncertain which why we weighted bullet funding with autocall probabilities. So their will also be a risk of Equity/Interest rate correlation. A positive correlation will increase the hedge of the trader. If the spot price increases, the interest rate and the chances to autocall at the next period increase. The trader will buy more of bonds with short maturity and sell bonds with longer maturity. Given that the longer ones have a higher duration and that their value will decreases, the trader loses money.

To hedge this interest rate risk, the trader can use interest swap weighted on autocallable probabilities. To hedge the volatility, the trader can sell volatility/variance swap in order to benefit from a decrease in volatility only (without carry and gamma effect).

The hedges of the trader will be monitored continuously, the delta hedged each day (the trader will first buy underlyings and then rebalance).

Conclusion

This product does not reflect the market conditions at the time of the issuance (several years ago). The idea was to reprice in current market conditions. Fortunately, the option prices were free to download which was in our advantage. However, our Heston prices are quite accurate but only for short term and ITM. Given that the DIP has a barrier at 60%, the price might not reflect the real price of the option leg. However, the coupon and fundings legs reflect better the current market reality.

This project was quite demanding as the pricing methods of structured products and especially autocallable structures are not well documented the, and the technicity of the pricers in banks is much more elaborated.