Equity Structured Products

Project: Autocall Pricing with Heston

by

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Introduction

As part of our studies in quantitative finance for the Master 272 program of Paris-Dauphine University, we worked on an Autocall Pricer. In business terms, it represents an "Athena", meaning that to get coupons we must Autocall. This financial product is priced with Heston model in order to take account the stochasticity of the volatility. For that, the model has been calibrated with FTSE 100 Index prices (available on https://www.eurex.com/ex-en/markets/idx/country/ftse).

Several market datas were used for this project. First, for the funding of the product (issued by Barclays), we did get Senior CDS curve of the bank as well as SOFR Rate Curve. Indeed, given that the product is issued in USD, the funding was calculated with US Rate. Then, the underlying being denominated in GBP, to discount the prices we used the United Kingdom Government Bonds - Yields Curve (available on http://www.worldgovernmentbonds.com/country/united-kingdom/). The market datas were fixed on Friday 15th of July, 2022.

The product we priced has the following caracteristics:

- Strike : 100%
- Maturity : 6Y
- Coupon : 9.35%
- Down-and-In barrier : 60%
- Non-Call period : 1Y
- Underlying : FTSE 100 Index
- Denomination : USD

Implementation

In order to implement the pricing, we must first introduce the Heston Model. It allows to take into account a stochastic variance (hence volatility) that has its own diffusion process and will be correlated with the spot diffusion.

$$\begin{aligned}
\mathrm{d}S(t) &= rS(t)\mathrm{d}t + \sqrt{v(t)}S(t)\mathrm{d}W_x(t), \\
\mathrm{d}v(t) &= \kappa(\bar{v} - v(t))\mathrm{d}t + \gamma\sqrt{v(t)}\mathrm{d}W_v(t).
\end{aligned}$$

Figure 1: Heston Model PDE

With κ being the speed of mean reversion, γ the vol of vol, responsible for the convexity of the volatility surface. $\bar{v}$ is the long term variance, and ρ is the correlation coefficient between the two processes. V_0 is the initial variance.

In order to make the calibration, we simulate spots and variances processes. The number of simulations has been set to 1000 and the time step to 1000. Unfortunately, the time consumption was quite long (several hours) so we were not able to capture the full dynamic of the market prices. However, we were able to make 10 iterations on the parameters (initially set to levels empirically observed) and we obtained the following parameters :

- κ : 3.39
- V_0 : 0.1029
- γ : 0.2896
- $\bar{v}$: 0.0766
- ρ : -0.747

Once the market was fitted (or approximately), we were able to reprice the options as following:

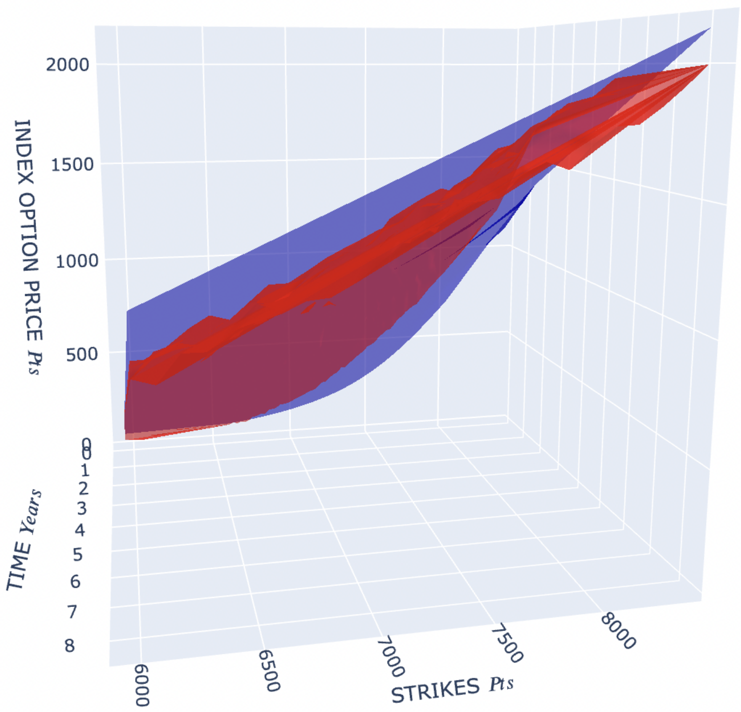

Figure 2: Market Prices vs Heston Prices

Thus, we were able to price vanilla put options. This is our best approximation for a Down and In Put. We then prices the DIP with the Heston process that has the calibrated parameters. We found an option value of approximately 14.8% (as a percentage of the initial spot price)

The client will bring his notional to the bank, meaning that the later will be able to invest it as a rate composed of (SOFR + Credit spread). Given the uncertainty of the maturity of the product (autocallability), we weighted by the Autocallable probabilities bullet funding as following :

$$Funding = 2*P(AC_{2Y})*(SOFR_{2Y}+CDS_{2Y})+3*P(AC_{3Y})*(SOFR_{3Y}+CDS_{3Y})+...+6*P(AC_{6Y})*(SO$$

With $P(AC_2)$ for example being the probability of Autocall at the second year (the first observation as we have a non call period of 1Y)
As anticipated, given that the Autocallable probabilities are conditional on no Autocall previously, they are decreasing. Except for the last one as if we don't Autocall before maturity, the product will terminate anyway at year 6.

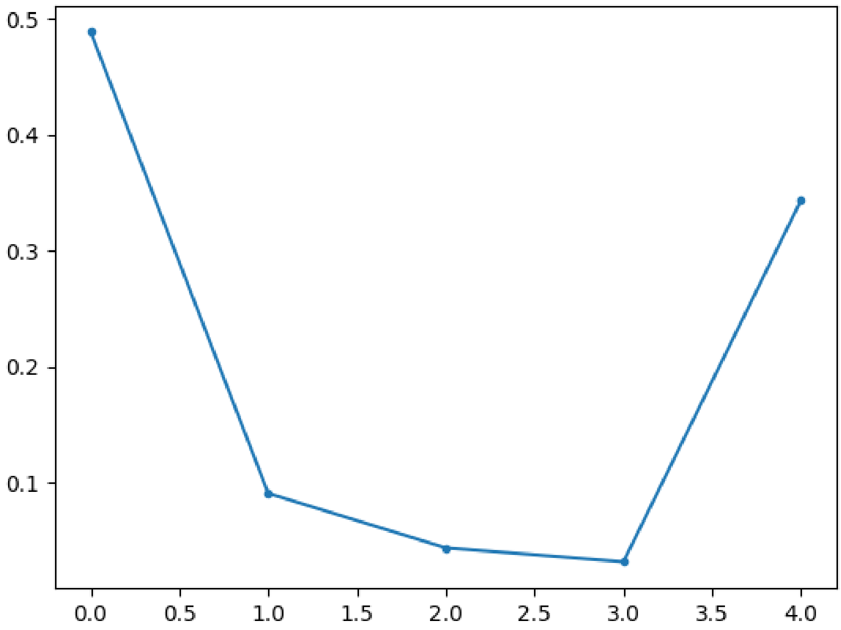

Figure 3: Autocallable probabilities

As a result we found 14.78% for the funding.

Finally, once, the funding and DIP were priced, we had to price the Coupon part of the product. For that, we used the simulated paths (with calibrated parameters). For each path, we calculate the coupon (or none) that is paid to the client in %. For example, if the spot is above 100% of its initial level at year 2, the client will receive 2 * the annual coupon, meaning 18.70%. One additional thing to take into account is the discount factor. So we had to discount with the appropriate rate (from the UK interest rate yield curve) and appropriate period. In the above example we would have used the 2Y rate with a period of 2. Additionally, if the spot is not above, the product remains alive and the coupon (with unique periodicity for each path) can be paid/not later.

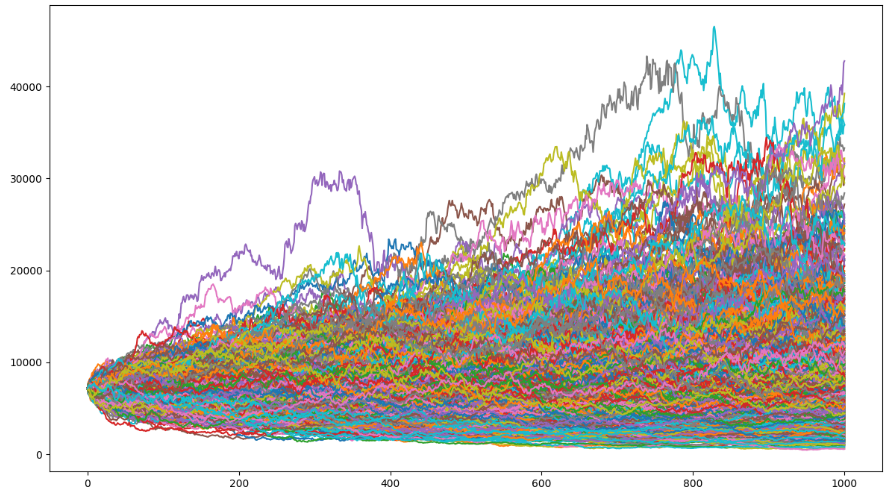

Figure 4: Calibrated Spots

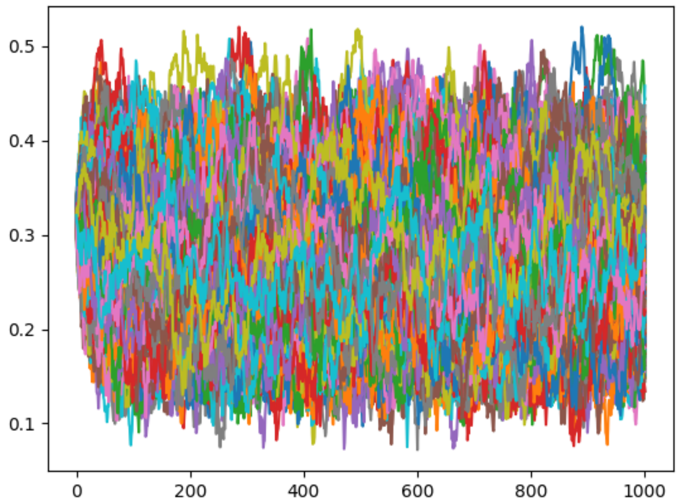

Figure 5: Calibrated variances

Finally, the Note price can be decomposed as following:

$$Price = CouponPrice - Funding - DownandInPut$$

As a result, given that the Note is sold at 100% to the client, the price will be the difference between 100% and the previous price

$$Price = 100\% - (14.86\% - 14.78\% - 14.83\%)$$

$$Price = 85.24\%$$

Conclusion

This product does not reflects the markets conditions at the time of the issuance (several years ago). This idea was to reprice in current market conditions. Fortunately, the option prices were free to download which was in our advantage. However, our Heston prices are quite accurate but only for short term and ITM. Given that the DIP has a barrier at 60%, the price might not reflect the real price of the option leg. However, the coupon and fundings legs reflect better the current market reality.

This project was quite demanding as the pricing methods of structured are not well documented the technicity of the pricers in banks is much more elaborated.