Simplicially driven simple contagion

Maxime Lucas[1,2], Iacopo Iacopini[3], Thomas Robiglio[4], Alain Barrat[5] and Giovanni Petri[1,2]

[1]Mathematics and Complex Systems Research Area, ISI Foundation, Via Chisola 5, 10126 Turin, Italy
[2]CENTAI, Corso Inghilterra 3, 10138 Turin, Italy
[3]Department of Network and Data Science, Central European University, 1100 Vienna, Austria
[4]Politecnico di Torino, DISAT, 10129 Turin, Italy
[5]Aix Marseille Univ, Université de Toulon, CNRS, CPT, Marseille, 13009, France

Motivation

SPREADING PROCESSES CAN AFFECT EACH OTHER

- HIV increases susceptibility to other diseases.
- Unsafe behaviours boost pathogen spread.

HIGHER-ORDER STRUCTURES FOR SOCIAL CONTAGION

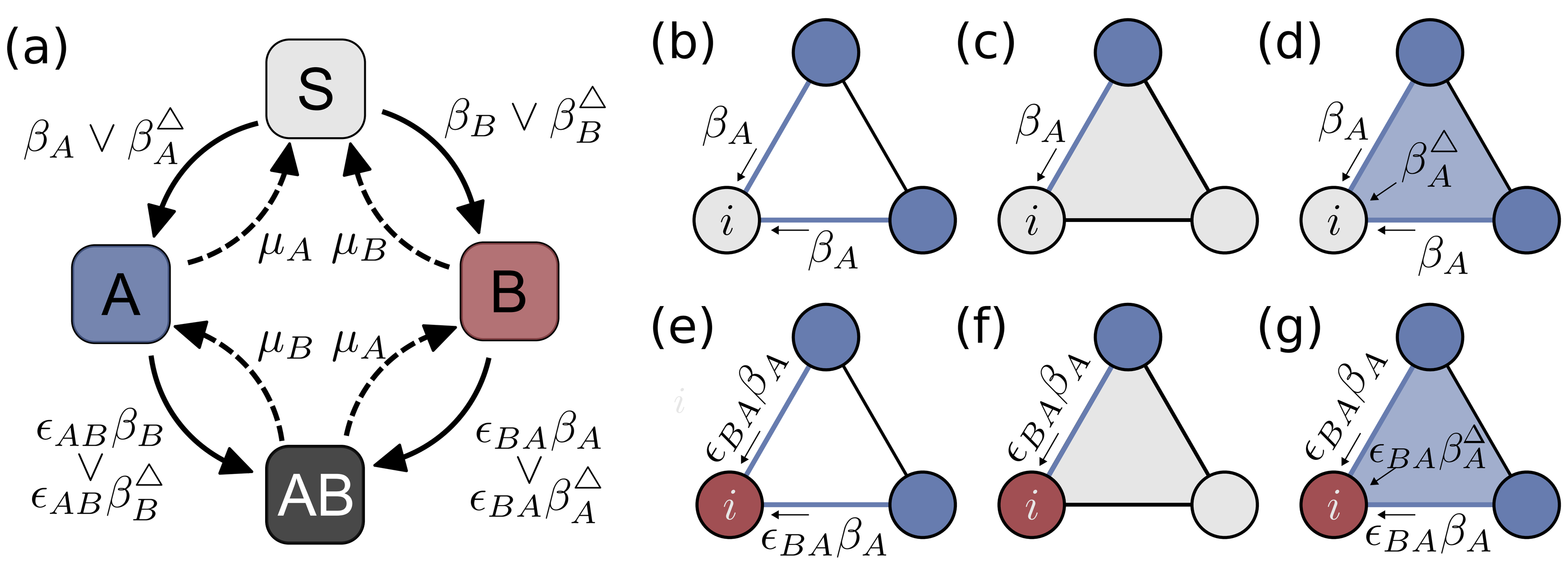
- Abrupt transition to the endemic state.
- Presence of a bi-stable region where endemic and disease free state coexist.

SO FAR:

- Modeling of processes limited to symmetric interaction and simple contagion.

Model

We consider a **model for two interacting spreading pathogens** A and B, which also include simplicial contagion [1]. Individuals can be in one of four compartments, following the standard SIS framework.



Key features of the model:

- Contagion events take place on top of a contact **structure with higher-order (non-pairwise) interactions** [2].
- For each pathogen we have **two contagion routes**: **simple** controlled by parameter $\beta_{x,1} \equiv \beta_x$ and **simplicial** controlled by parameter $\beta_{x,2} \equiv \beta_x^\triangle$. All other nodes in the simplex need to be infectious for the simplex to be considered so.
- The **interaction between the two contagion processes** is controlled via two non-negative parameters ϵ_{AB} and ϵ_{BA}. The two processes **cooperate** if $\epsilon_{xx'} > 1$ and **compete** if $\epsilon_{xx'} < 1$.

Mean-field description

We **assume identical recovery rates** $\mu_A = \mu_B = \mu$ and **introduce the rescaled infectivity parameters** $\lambda_x = \beta_x \langle k \rangle / \mu$ and $\lambda_x^\triangle = \beta_x^\triangle \langle k_\triangle \rangle / \mu$. The general mean-field equations describing the evolution of the densities are:

$$\dot\rho_A = -\rho_A + \lambda_A \rho_S(\rho_A + \rho_{AB}) + \lambda_A^\triangle \rho_S(\rho_A + \rho_{AB})^2 + \rho_{AB} - \epsilon_{AB}\lambda_B\rho_A(\rho_B + \rho_{AB}) - \epsilon_{AB}\lambda_B^\triangle \rho_A(\rho_B + \rho_{AB})^2 \tag{1a}$$

$$\dot\rho_B = -\rho_B + \lambda_B \rho_S(\rho_B + \rho_{AB}) + \lambda_B^\triangle \rho_S(\rho_B + \rho_{AB})^2 + \rho_{AB} - \epsilon_{BA}\lambda_A\rho_B(\rho_A + \rho_{AB}) - \epsilon_{BA}\lambda_A^\triangle \rho_B(\rho_A + \rho_{AB})^2 \tag{1b}$$

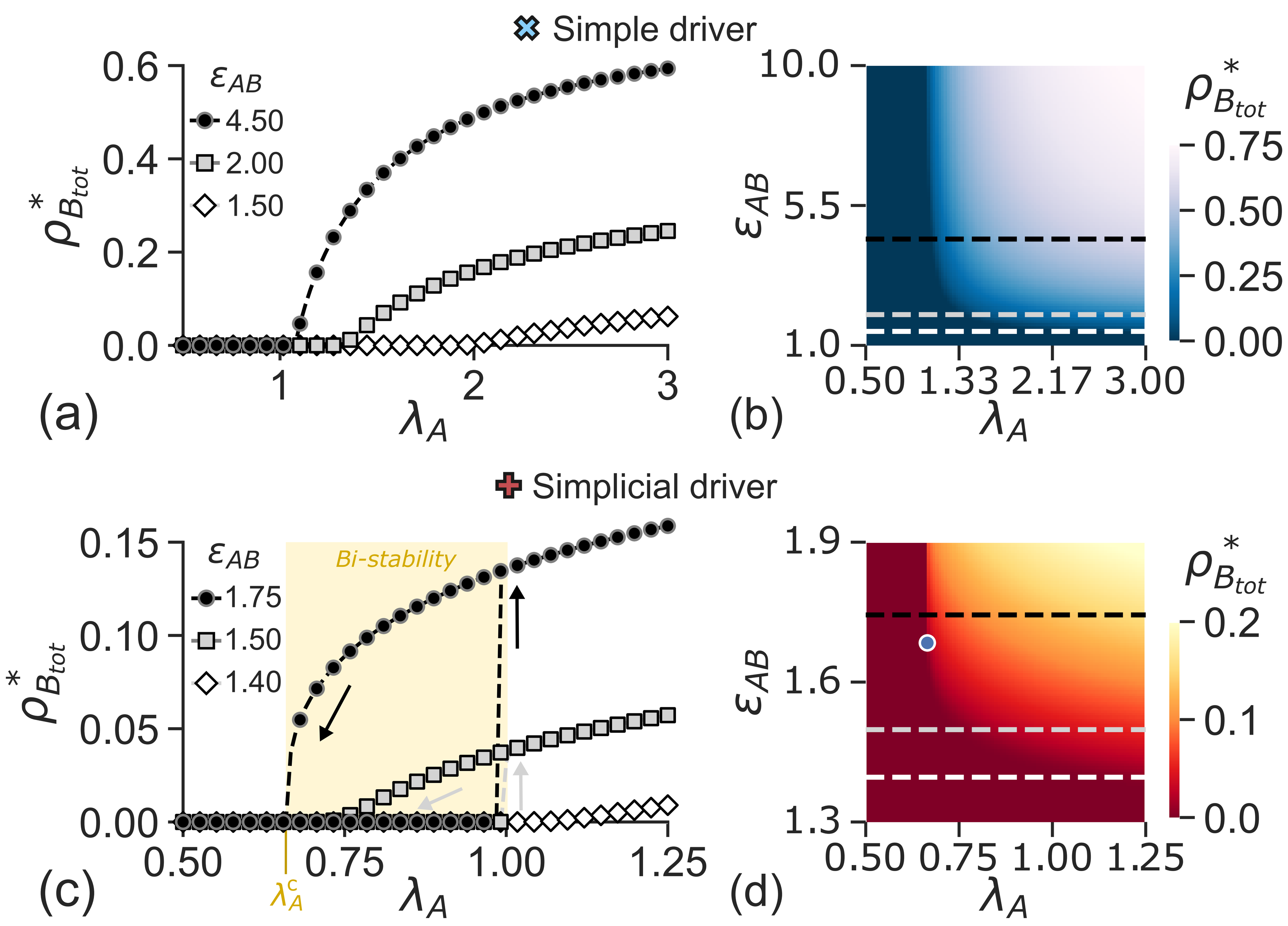
$$\dot\rho_{AB} = -2\rho_{AB} + \epsilon_{AB}\lambda_B\rho_A(\rho_B + \rho_{AB}) + \epsilon_{AB}\lambda_B^\triangle \rho_A(\rho_B + \rho_{AB})^2 + \epsilon_{BA}\lambda_A\rho_B(\rho_A + \rho_{AB})\epsilon_{BA}\lambda_A^\triangle \rho_B(\rho_A + \rho_{AB})^2 \tag{1c}$$

Results

PATHOGEN A → UNSAFE BEHAVIOUR
PATHOGEN B → DISEASE

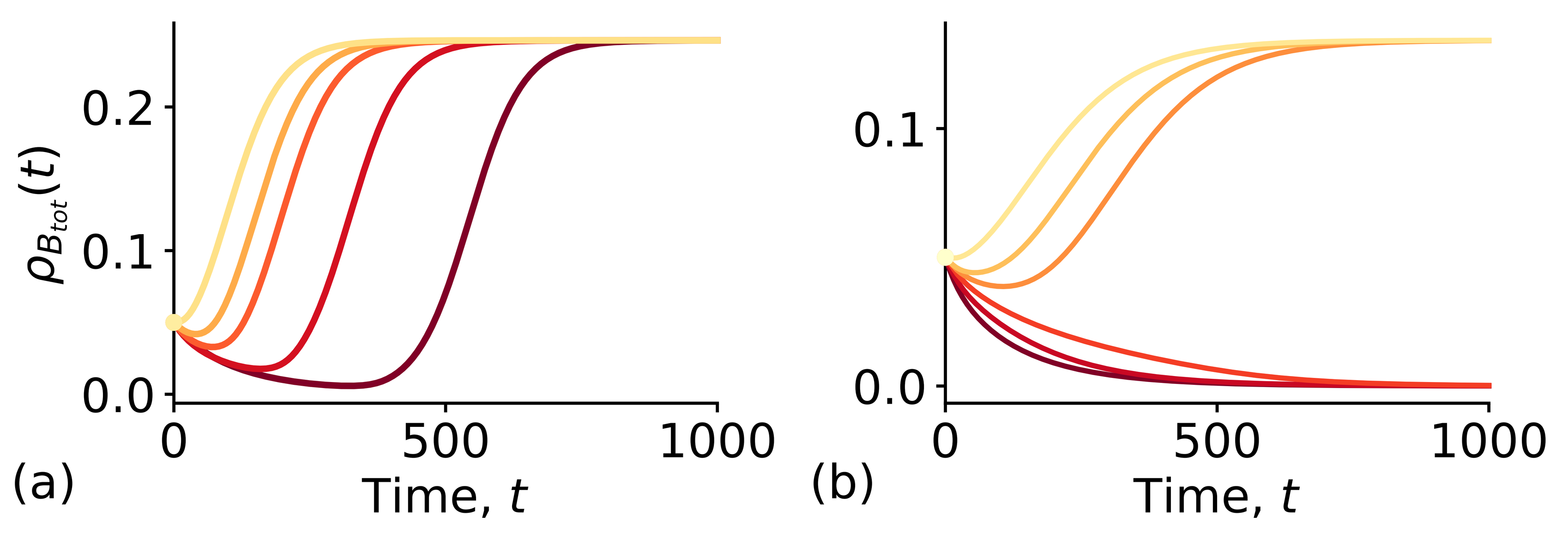
Pathogen A drives pathogen B ($\epsilon_{AB} > 1$)
Pathogen B does not affect pathogen A ($\epsilon_{BA} = 1$)

Abrupt phase transition induced by a simplicial driver



A simplicial driver with $\lambda_A^\triangle = 2.5$ can induce a discontinuous transition in pathogen B, contrary to a simple driver with $\lambda_A^\triangle = 0$. The transition of the simple contagion B becomes discontinuous above a critical value of cooperation ϵ_{AB}^c.

Temporal properties



The temporal properties of the driven proces depends on the initial conditions of the driver. Observing only the driven pathogen B we see a bistability in B by changing the initial conditions of the simplicial driver A.

References

[1] I. Iacopini *et al.*, Simplicial models of social contagion, *Nat. Comm.*

[2] F. Battiston *et al.*, Networks beyond pairwise interactions: Structure and dynamics, *Phys. Rep.*

Spring College on the Physics of Complex Systems, 20 February - 17 March 2023, Trieste, Italy