Sorry arima, I'm going Bayesian

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Sommaire

Application numérique : Une alternative à l'algorithme E-M pour le mélange de gaussiennes

Échantillon $\boldsymbol{x} = (\boldsymbol{x}_1, \ldots, \boldsymbol{x}_N)$ suivant une loi $f(\boldsymbol{x}_i, \theta)$ avec des variables latentes $\boldsymbol{z} = (\boldsymbol{z}_1, \boldsymbol{z}_2, \ldots, \boldsymbol{z}_n)$

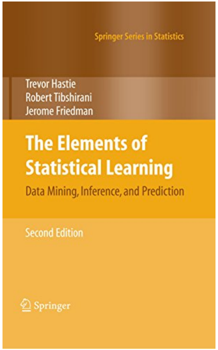
Maximisation de $L(\boldsymbol{\theta}; \mathbf{X}) = p(\mathrm{x}|\boldsymbol{\theta}) = \int p(\mathrm{x}, \mathrm{z}|\boldsymbol{\theta}) d\mathbf{Z}$

$$E\left[L(\mathrm{x};\boldsymbol{\theta})|\boldsymbol{\theta}^{(c)}\right] = E\left[L((\mathrm{x},\mathrm{z});\boldsymbol{\theta})|\boldsymbol{\theta}^{(c)}\right] - E\left[\sum_{i=1}^{n} \log f\left(z_i|\boldsymbol{x}_i, \boldsymbol{\theta}\right)|\boldsymbol{\theta}^{(c)}\right]$$

$$L(\mathrm{x};\boldsymbol{\theta}) = Q\left(\boldsymbol{\theta};\boldsymbol{\theta}^{(c)}\right) - H\left(\boldsymbol{\theta};\boldsymbol{\theta}^{(c)}\right)$$

$\boldsymbol{\theta}^{(c+1)} = \arg\max_{\boldsymbol{\theta}}\left(Q\left(\boldsymbol{\theta},\boldsymbol{\theta}^{(c)}\right)\right)$ fait tendre la suite $L\left(\mathrm{x};\boldsymbol{\theta}^{(c+1)}\right)$ vers un max local

$$p\left(\mathcal{C}_1|x\right) = \frac{p\left(\mathcal{C}_1\right) p\left(x|\mathcal{C}_1\right)}{p\left(\mathcal{C}_1\right) p\left(x|\mathcal{C}_1\right) + p\left(\mathcal{C}_2\right) p\left(x|\mathcal{C}_2\right)}$$



Application : mélange de gaussiennes

- Les données **x** viennent de gaussiennes $\{C_1, \ldots, C_M\}$
- $\boldsymbol{z}_i \in \{1, \ldots, m\}$ si un individu i vient de $C_1, \ldots, C_M$

$$\boldsymbol{x}_i | (\boldsymbol{z}_i = j) \sim \mathcal{N}\left(\mu_j, \sigma_j^2\right) \quad \mathbb{P}(\boldsymbol{z}_i = j) = \pi_j$$

- **E-M** : maximisation de

$$\ell_\theta(x) = \sum_{n=1}^{N} \log\left(\sum_{j=1}^{M} \pi_j\, \mathcal{N}_{\mu_j, \sigma_j^2}(x_n)\right)$$

$$\theta = (\pi, \mu, \sigma^2)$$

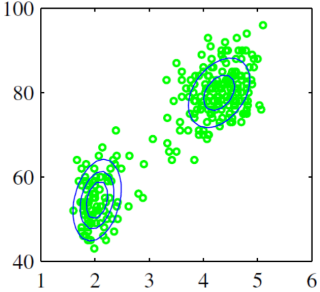


Figure – Mélange de deux gausiennes [F. Sur, Introduction à l'apprentissage automatique]

Application : Échantillonneur de Gibbs pour le mélange de 2 gaussiennes

Pour 2 gaussiennes : $\begin{cases} \boldsymbol{x}_i | (\boldsymbol{z}_i = 1) \sim \mathcal{N}\left(\mu_1, \sigma_1^2\right) & \mathbb{P}\left(\boldsymbol{z}_i = 1\right) = \pi_1 \\ \boldsymbol{x}_i | (\boldsymbol{z}_i = 2) \sim \mathcal{N}\left(\mu_2, \sigma_2^2\right) & \mathbb{P}\left(\boldsymbol{z}_i = 2\right) = \pi_2 = 1 - \pi_1 \end{cases}$

- On veut obtenir $\Theta = (\boldsymbol{z}, \mu) = (\boldsymbol{z_1}, \ldots, \boldsymbol{z_1}, \mu)$ $(\pi, \sigma$ fixés à $\hat{\pi}, \hat{\sigma})$
- $z|\mu$: utiliser $\mathbb{P}\left(\boldsymbol{z}_i = j | \mu, \boldsymbol{x}\right) = \dfrac{\hat{\pi}_j p_{\mathcal{N}}(\boldsymbol{x}_i, \mu_j, \hat{\sigma}_j)}{\hat{\pi}_1 p_{\mathcal{N}}(\boldsymbol{x}_i, \mu_1, \hat{\sigma}_1) + \hat{\pi}_2 p_{\mathcal{N}}(\boldsymbol{x}_i, \mu_2, \hat{\sigma}_2)}$ $\quad j \in \{1, 2\}$
- $\mu|z$:

(à priori)

$$\boldsymbol{x_i}|\mu, \boldsymbol{z_i} = j \sim \mathcal{N}\left(\mu_j, 1/\hat{\tau}_j\right)$$
$$\mu_j \sim \mathcal{N}\left(\mu_0, 1/\tau_0\right)$$

$\left(\tau_j = \mathbf{1}/\sigma_j^{\mathbf{2}},\ \bar{x} = \frac{\mathbf{1}}{N} \sum_{i=\mathbf{0}}^{N} x_i\right)$

(à postériori)

$$p(\mu_j|\boldsymbol{z}_i, \mathrm{x}) \propto p(\boldsymbol{x}|\mu, \boldsymbol{z_i} = j)p(\mu_j)$$
$$\sim \mathcal{N}(\mu_0', 1/\tau_0')$$

(= maj des coefs)

$$\tau_{j_0}' = \tau_0 + N\hat{\tau}_j$$
$$\mu_{j_0}' = \frac{N\ \hat{\tau}_j\ \bar{x} + \tau_0 \mu_0}{N\hat{\tau}_j + \tau_0}$$

Échantilloneur de Gibbs pour une mixture gaussienne

1. Prendre les valeurs initiales $\Theta_0 = (\boldsymbol{z}^{(0)}, \mu_1^{(0)}, \mu_2^{(0)}),\ \mu_0, \tau_0$
2. Pour t de à ...
   - Pour i de 1 à N tirer

     $\boldsymbol{z}_i^{(t+1)} \in \{1,2\}$ avec $\mathbb{P}(\boldsymbol{z}_i^{(t+1)} = 1) = \dfrac{\hat{\pi}_1 p_{\mathcal{N}}(\boldsymbol{x}_i, \mu_1^{(t)}, \hat{\sigma}_1)}{\hat{\pi}_1 p_{\mathcal{N}}(\boldsymbol{x}_i, \mu_1^{(t)}, \hat{\sigma}_1) + \hat{\pi}_2 p_{\mathcal{N}}(\boldsymbol{x}_i, \mu_2^{(t)}, \hat{\sigma}_2)}$
   - Pour $j \in \{1,2\}$

     Estimer $\tau'_{jo} = \tau_0 + N\hat{\tau}_j \quad \mu'_{jo} = \dfrac{N\ \hat{\tau}_j\ \bar{x} + \tau_0\mu_0}{N\hat{\tau}_j + \tau_0}$

     Tirer $\mu_j^{(t+1)} \sim \mathcal{N}\left(\mu'_{jo}, 1/\tau'_{jo}\right)$

```
Z_given_mu <- function(X,Z,mu,pi_1){
    # Z variables latentes

    remove_i <- (c(1:length(X)) !=i) # enleve variable i
    estimate_sigma_1 <- sd(X[(remove_i & Z == 1)]) # classe 1
    estimate_sigma_2 <- sd(X[(remove_i & Z == 2)]) # classe 2

    for (i in 1:length(Z)){

        proba1 <- pi_1 * dnorm(X[i],mu[1],estimate_sigma_1) /
            (pi_1 * dnorm(X[i],mu[1],estimate_sigma_1) +
            (1-pi_1) * dnorm(X[i],mu[2],estimate_sigma_2))

        Z[i] = sample(1:2, size=1,prob=c(proba1, 1-proba1),
            replace=TRUE)
    }
    return(Z)
}
```

```
mu_given_Z = function(X, Z, mu_prior){
    # Z variables latentes
    # mu_prior contient parametres loi a priori de mu

    mu = rep(0,2) ; sigma = rep(0,2)

    for(j in 1:2){

        sample_j_size = sum(Z==j)
        sample_j_mean = mean(X[Z==j])
        sigma[j] = sd(X[Z==j]) ; precision_j = 1 / sigma[j]^2

        precision_post = sample_j_size * precision_j +
            mu_prior$precision

            mean_post = (sample_j_mean * sample_j_size *
            precision_j + mu_prior$mean *
            mu_prior$precision ) / precision_post

        mu[j] = rnorm(1,mean_post,sqrt(1/precision_post)) # on
               tire mu selon la loi normale a posteriori
    }
    return(list(mu = mu, sigma = sigma))
    }
```

```
echantillonneur_gibbs <- function(X,N_simu){
    # X sont les donnees
    # initalisation
    .
    .

    for (k in 1:N_simu){

        Z <- Z_given_mu(X,Z,mu,pi_1) ; param_post <- mu_given_Z(X, Z, mu_prior)
        mu = param_post$mu
    .
    .
    .
    }
    return(list(Z, mu_1, mu_2))
}
```

Listing 1 – Implémentation en R

Blocky block

Just a Block

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Blocky block

Example Block

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Blocky block

Alert Block

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How to use frame-breakings ?

In this template, and only this, I defined a "breakingframe" template frame that should not hold any useful information. The background of this frame is pinkish solid and it is not countable as a separate frame. You can use this as a transitioning page between different topics or for any funny funky stuff to release the tense of the poor audience during your presentation.

```
        \breakingframe{
            Put your contents here, such as images, text ..etc. Be as silly as
                possible .. or not!
        }
```

Look at the next slide, in code, as an example !

Motivation : Q1

1. Stone masonry walls are usually not homogeneous through the thickness
2. Leaf-separation effects on the strength capacity
3. In-plane and out-of-plane behaviours interaction
4. Internal cracking onsets and 3D crack paths (cannot be captured experimentally)

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The study main phases

How to arrange stones ?

Objective function

Packing objective

$$\text{Minimize } F(\vec{X}_i)_i = || \vec{S}_i - \vec{S}_{i-1} ||$$

$$Fitness\left(F(\vec{X}_i)\right)_i = F(\vec{X}_i)_i(1 + \xi_1 P_A)^{\xi_2}$$

- $S_i, S_{i-\mathbf{1}}$: locations of i and $i - 1$ stones
- $\xi_{\mathbf{1}}$: penalty multiplier
- $\xi_{\mathbf{2}}$: penalty exponent
- P_A : penalties summation

Merci de votre attention