Sorry arima, I'm going Bayesian

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Sommaire

Approche bayésinne pour la modélisation des séries temporelles

- Les modèle espace-états

		bruit blanc gaussien
equation d'observation	$y_t = Z_t^T \alpha_t + \epsilon_t$	$\epsilon_t \sim N(0, H_t)$
equation de transition	$\alpha_{t+1} = T_t \alpha_t + R_t \eta_t$	$\eta_t \sim N(0, Q_t)$

- y_t obervations
- α_t variables d'états / latentes / cachées
- Z_t matrice de mesure
- T_t matrice de transitions

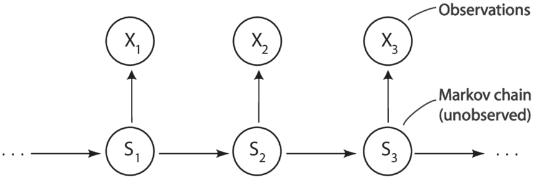

Figure – [researchgate.net]

Approche bayésinne pour la modélisation des séries temporelles

→ Bayesian structural time series (BSTS)

		bruit blanc gaussien
observation	$y_t = \mu_t + \beta^T x_t + \tau_t + \varepsilon_t$	$\varepsilon_t \sim N\left(0, \sigma_\varepsilon^2\right)$
regression	$\beta^T x_t$	
tendance + marche aléatoire	$\mu_t = \mu_{t-1} + \delta_{t-1} + u_t$	$u_t \sim N\left(0, \sigma_u^2\right)$
marche aléatoire	$\delta_t = \delta_{t-1} + v_t$	$v_t \sim N\left(0, \sigma_v^2\right)$
sainsonnalité	$\tau_t = -\sum_{s=1}^{s-1} \tau_{t-s} + w_t$	$w_t \sim N\left(0, \sigma_w^2\right)$

→ Bayesian structural time series (BSTS)

observation	$y_t = Z_t^T \alpha_t + \epsilon_t$	$\epsilon_t \sim N(0, H_t)$
	Z_t^T $\begin{pmatrix} 1 & 0 & \beta^T \mathbf{x}_t \end{pmatrix}$	α_t^T $\begin{pmatrix} \mu_t & \delta_t & 1 \end{pmatrix}^T$
equation de transition	$\alpha_{t+1} = T_t \alpha_t + R_t \eta_t$	$\eta_t \sim N(0, Q_t)$
α_t $\begin{pmatrix} \mu_t \\ \delta_t \\ 1 \end{pmatrix}$	T_t $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$N_t \eta_t$ $\begin{pmatrix} u_t \\ v_t \\ w_t \end{pmatrix}$

→ estimation des paramètres

Loi à postériori états cachés α_t : Le filtre de Kalman

Itérations sur l'estimation $p(\alpha_t | y_{1:t}) \sim \mathcal{N}(\hat{\alpha}_t, P_t)$

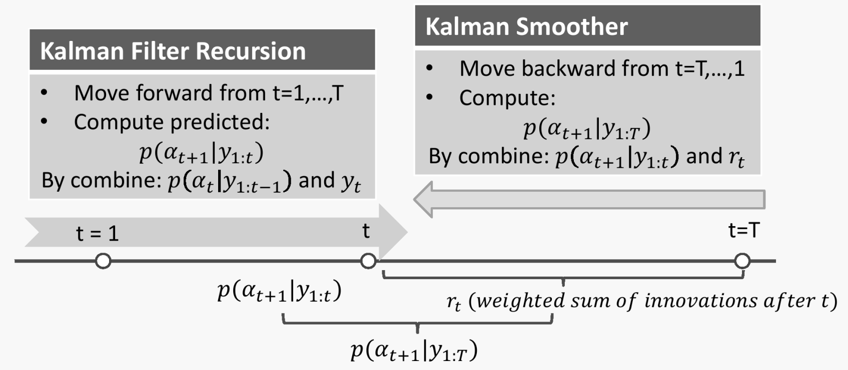

Figure – [github : anhdanggit/nowcasting-google-queries/]

Loi à postériori de β : *spike-and-slab prior*

- On prend la partie regression $y_t^* = y_t - \mu_t$
- On utilise pour β une distribution à priori *spike-and-slab* :
  - $p(\gamma) = \prod_{k=1}^{N} \pi^{\gamma_k}(1-\pi)^{1-\gamma_k}$, $\gamma_k \in \{0,1\}$ $N = Card(\mathbf{x})$
  - $p\left(\beta, \gamma, \sigma_\varepsilon^2\right) = p\left(\beta_\gamma | \gamma, \sigma_\varepsilon^2\right) p\left(\sigma_\varepsilon^2 | \gamma\right) p(\gamma)$
  - $\beta_\gamma \middle| \sigma_\epsilon^2, \gamma \sim \mathcal{N}\left(b_\gamma, \sigma_\epsilon^2 \left(\Omega_\gamma^{-1}\right)^{-1}\right) \quad \sigma_\epsilon^2 \middle| \gamma \sim IG\left(\frac{\nu}{2}, \frac{ss}{2}\right)$

On utilise les propriété des lois conjugé pour obtenir les loi à postériori $\beta_\gamma | \sigma_\epsilon, \gamma, \mathbf{y}^*$, $\gamma_\epsilon^2 | \gamma, \mathbf{y}^*$,$\gamma | \mathbf{y}^*$

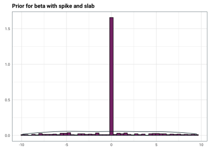

Figure – [batisengul.co.uk]

Conclusion

- Auteur prèfère mettre incertitude dans la prior que sur l'estimation des coéfficientss

Blocky block

Just a Block

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Blocky block

Example Block

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Blocky block

Alert Block

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How to use frame-breakings ?

In this template, and only this, I defined a "breakingframe" template frame that should not hold any useful information. The background of this frame is pinkish solid and it is not countable as a separate frame. You can use this as a transitioning page between different topics or for any funny funky stuff to release the tense of the poor audience during your presentation.

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\breakingframe{
    Put your contents here, such as images, text ..etc. Be as silly as
        possible .. or not!
}
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Look at the next slide, in code, as an example !

Motivation : Q1

1. Stone masonry walls are usually not homogeneous through the thickness
2. Leaf-separation effects on the strength capacity
3. In-plane and out-of-plane behaviours interaction
4. Internal cracking onsets and 3D crack paths (cannot be captured experimentally)

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The study main phases

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How to arrange stones ?

Objective function

Packing objective

$$\text{Minimize } F(\vec{X_i})_i \ = || \ \vec{S_i} - \vec{S}_{i-1} \ ||$$

$$Fitness\Big(F(\vec{X_i})\Big)_i = F(\vec{X_i})_i(1 + \xi_1 P_A)^{\xi_2}$$

- $S_i, S_{i-\mathbf{1}}$: locations of i and $i-1$ stones
- $\xi_{\mathbf{1}}$: penalty multiplier
- $\xi_{\mathbf{2}}$: penalty exponent
- P_A : penalties summation

Merci de votre attention