Estimating systemic risk measures

1. SRISK

SRISK is a Systemic Risk measure based on the Marginal Expected Shortfall [MES] due to Acharya et al. (2012). SRISK_{jt} is the expected capital shortfall of financial institution j conditional on a systemic event

$$\text{SRISK}_{jt} = \max(0, [kL_{jt} - (1-k)\text{LRMES}_{jt} - 1]W_{jt})$$

where k is a prudential capital requirement; W is market value of equity; D is book value of debt; $L = (D+W)/W$ is financial leverage; LRMES is long run MES.

$$\text{LRMES}_{jt} = E_t(R_{jt+1:t+h} | R_{mt+1:t+h} < C).$$

where R_{jt} is equity return on financial institution j; R_{mt} is return on the financial system, C is crisis threshold return.

Aggregate systemic risk of financial system is

$$\sum_{j=1}^{N} \text{SRISK}_{jt}.$$

1.1. Brownlees and Engle (2015)

- Data: 95 large US financial firms; daily equity returns and market capitalization from CRSP; quarterly book values for equity and debt from Compustat
- Period: 2000-2012
- Econometric methods:
  - estimate Dynamic Conditional Correlation (DCC) model for firm and market with threshold GARCH volatilities
  $$\begin{bmatrix} R_{jt} \\ R_{mt} \end{bmatrix} \sim \mathcal{D}\left(0, \begin{bmatrix} \sigma_{jt}^2 & \rho\sigma_{jt}\sigma_{mt} \\ \rho\sigma_{jt}\sigma_{mt} & \sigma_{mt}^2 \end{bmatrix}\right)$$
  - obtain LRMES using Monte Carlo simulation of returns based on model estimates. Use S repetitions and compute
  $$\text{LRMES}_{jt} = \frac{\sum_{s=1}^{S} R_{jt+1:t+h}^{s} \times I(R_{mt+1:t+h}^{s} < C)}{\sum_{s=1}^{S} I(R_{mt+1:t+h}^{s} < C)}$$
  where $I(x) = 1$ if true and 0 otherwise.
- Results:
  - SRISK successfully identifies systemically risky firms during the GFC
  - SRISK has predictive value: predicts capital injections by Fed Reserve
  - aggregate SRISK provides early warning of declines in industrial production and higher unemployment

1.2. Engle et al. (2014)

- Data: 196 large financial institutions in Europe: banks, insurance companies, financial services firms and real estate firms; daily equity returns and market capitalization from Datastream; quarterly book values for equity and debt from Compustat; world and Europe equity indexes from MSCI
- Period: 1990-2012
- Econometric methods:
  - estimate multi-factor, time varying model of returns using Dynamical Conditional Betas, Engle (2014)
  - model volatility of errors using univariate asymmetric GARCH model
  - use skewed t distribution for marginals of the innovations and a t copula for the dependence structure between the innovations
  - estimation proceeds recursively from an international model [World and European indexes]; then country models [add in respective country index and use the parameters from the international model] then firm model [include firm returns and use parameters from country model]
  - directly estimate LRMES
    * simulate forward 125 days returns from model estimates
    * use $S = 50,000$ draws to compute

$$\text{LRMES}_{jt} = \frac{\sum_{s=1}^{S} R^s_{jt+1:t+h} \times I(R^s_{mt+1:t+h} < -40\%)}{\sum_{s=1}^{S} I(R^s_{mt+1:t+h} < -40\%)}$$

where $I(x) = 1$ if true and 0 otherwise. where $I(x) = 1$ if true and 0 otherwise.

- Results:
  - rank firms, firm types and countries by SRISK: banks contribute 83% insurance companies 15% of systemic risk in Europe; highest countries - France and UK contribute 52%
  - aggregate SRISK Granger-causes industrial production and business confidence index
  - SRISK is positively related to changes in 3-month interbank rate but not significantly related to stock market return and volatility.

2. CoVaR

Adrian and Brunnermeier (2011) propose CoVaR which is the VaR of the financial system m when financial institution j is in distress which is operationally defined as its VaR(q) level.

$$\Pr(R_t^m \leq \mathrm{CoVaR}_{qt}^{m|j} | R_t^j = \mathrm{VaR}_{qt}^j) = q.$$

Increased risk to system m when financial institution j is in distress

$$\Delta\mathrm{CoVaR}_{qt} = \mathrm{CoVaR}_{qt}^{m|j} - \mathrm{CoVaR}_{qt}^{m|b^j}$$

where b^j denotes the benchmark state for j and equals its median return.

Girardi and Ergün (2013) suggest to condition on when financial institution j is at its VaR level at best:

$$\Pr(R_t^m \leq \mathrm{CoVaR}_{qt}^{m|j} | R_t^j \leq \mathrm{VaR}_{qt}^j) = q.$$

They define the benchmark state b^j as a one standard deviation around the mean

$$\mu_t^j - \sigma_t^j \leq R_t^j \leq \mu_t^j + \sigma_t^j.$$

and measure the percentage systemic risk contribution of j by

$$\Delta\mathrm{CoVaR}_{q,t}^{m|j} = 100 \times (\mathrm{CoVaR}_{qt}^{m|j} - \mathrm{CoVaR}_{qt}^{m|b^j})/\mathrm{CoVaR}_{qt}^{m|b^j}.$$

Girardi and Ergün (2013) argue their measure improves on Adrian and Brunnermeier (2011) as it

- is more general and relevant as it takes into account severity of tail losses
- facilitates back-testing of CoVaR using standard back-tests for VaR.

Mainik and Schaanning (2012) also show the Girardi and Ergün (2013) measure is better

- it is consistent with respect to the dependence parameter unlike the Adrian and Brunnermeier (2011) measure which is not, e.g.in a bivariate Gaussian model, the Adrian and Brunnermeier (2011) measure is decreasing in the correlation!
- consistency property also extends to Conditional Expected Shortfall [CoES].

2.1. Adrian and Brunnermeier (2011)

- Data: weekly equity returns of 357 US bank holding companies from CRSP; quarterly balance sheet data from Compustat; state variables: VIX, liquidity spread; change in T-bill rate; change in slope of yield curve; change in credit spread; controls - market equity return; return on real estate sector.

- Period: 1986-2010
- Econometric methods:
  - use quantile regressions of asset returns to estimate $CoVaR$
  - add lagged state variables for conditional results
  - construct out-of-sample forward ΔCoVaR by projecting ΔCoVaR on lagged size, leverage, maturity mismatch and industry dummies
- Results:
  - VaR and ΔCoVaR are weakly related in the cross section but strongly related over time
  - ΔCoVaR larger for firms with higher leverage, more maturity mismatch and larger size
  - strong negative correlation between contemporaneous ΔCoVaR and forward ΔCoVaR

2.2. Girardi and Ergün (2013)

- Data: estimate $CoVaR$ and conduct back-tests using daily equity returns of 74 large US financial firms
- Period: 2000-20087
- Econometric methods:
  - bivariate GARCH-DCC models assuming (i) bivariate Gaussian (ii) bivariate skewed-t distributions
  - 3-step procedure: 1. estimate individual VaR using univariate GARCH; 2. estimate bivariate density for returns using bivariate GARCH; 3. numerically solve CoVaRs

$$\int_{-\infty}^{\mathrm{CoVaR}_{qt}^{m|j}} \int_{-\infty}^{\mathrm{VaR}_{qt}^{j}} pdf_t(x,y)dydx = q^2$$

$$\int_{-\infty}^{\mathrm{CoVaR}_{qt}^{m|b^j}} \int_{\mu_t^j-\sigma_t^j}^{\mu_t^j+\sigma_t^j} pdf_t(x,y)dydx = p_t^j q.$$

- Results:
  - backtests favour skewed-t distribution
  - rank of contribution to systemic risk: depositary institutions [largest]; broker-dealers; insurance companies; non-depositary institutions [smallest]
  - VaR and ΔCoVaR are weakly related cross sectionally and over time
  - all industry groups showed substantial increase in pre crisis ΔCoVaR
  - ΔCoVaR is positively related to size and equity beta, and to leverage during down markets.

3. CATFIN

Allen et al. (2012) create a macroindex of systemic risk designated CATFIN which is the arithmetic average of three VaR measures using the Generalized Pareto distribution [GPD], skewed generalized error distribution [SGED] and a nonparametric approach being the relevant quantile of the empirical distribution.

$$\mathrm{VaR}_{\mathrm{GPD}} = \mu + \left(\frac{\sigma}{\xi}\right)\left[\left(\frac{\alpha N}{n}\right)^{-\xi} - 1\right]$$

where μ, σ, ξ are location, scale and shape parameters. α is loss probability level, n is number of extremes and N is total data points.

Obtain $\mathrm{VaR}_{\mathrm{SGED}}$ by solving numerically

$$\int_{-\infty}^{\mathrm{VaR}_{\mathrm{SGED}(\alpha)}} f_{\mu,\sigma,\kappa,\lambda}(z)dz = \alpha$$

where f is SGED probability density function; κ controls height and tails of the density and λ is skewness.

Allen et al. (2012) also create analogous measures for expected shortfall but report that the predictability results are similar.

3.1. Allen et al. (2012)

- Data: monthly returns and market capitalization for US financial companies; Chicago Fed National Activity Index [CFNAI] and alternative macro-economic performance indicators; collect similar stock price data for EU and Asia and GDP growth rates
- Period: 1973-2009
- Econometric methods:
  - use maximum likelihood to estimate GPD and SGED parameters from monthly returns
  - estimate predictive auto-regressions of CFNAI using CATFIN as a predictor plus other control variables
- Results:
  - CATFIN is negatively related to the future CFNAI for 1- to 6-month ahead forecast horizons
  - this predictability comes from banks
  - results are robust to alternative measures of economic performance
  - regional CATFIN also has predictive power for GDP growth in EU and Asia

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