

Systemic risk and contagion effects in Australian financial institutions and sectors

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1. Literature review

The starting point for the proposed research is the recent literature and the CIFR targeted areas and APRA aims and functions. This recent literature includes the following Adrian and Brunnermeier (2011), Acharya et al. (2012), Acharya et al. (2012) and Brownlees and Engle (2010). The proposed research aims to extend and apply these techniques particularly in relation to the entities regulated by APRA. Thus our broad aim is to develop, implement and bring to bear recent developments in stress testing on the aims of APRA and the CIFR targeted research areas detailed above.

2. Improved measures of contagion and systematic risk

CoVaR_q as proposed in Adrian and Brunnermeier (2011) is a basis for proposed measures contagion, exposure and systemic risk. It suffers from a number of drawbacks:

- Couched in terms of VaR_q containing the scale of the original measurements. It is worthwhile to have measures and techniques robust to scale.
- Conditioning on VaR_{0.5} is undesirable and relatively intractable. In our proposal we reference stress with respect to the unconditional VaR_q. This permits a more transparent analysis and estimation.
- Our proposed approach separates out scale effects and interdependence effects and aims to relates these separately to external variables including shocks and drivers of systemic risk. Thus VaR_q movements due to scale are disentangled from movements due to codependence with separate driver responses.

3. Significance of the project and policy implications

Understanding the impact of external shocks and their propagation through the financial system is vital for managing and remediating systemic risk. Effective regulation is dependent upon the development of a robust and reliable set of appropriate risk measures. We propose new measures of systemic risk that relate marginal and joint distributions separately to external drivers. This allows for more cogent and coherent stress testing as it includes the estimation of contagion effects, exposure effects and systemic risk across related entities and different financial sectors. Improved stress testing, estimation of risk effects and transmission of shocks through the financial system will make for more cogent prudential policy, prudential margin setting and better identify sources of risk to the financial system.

4. Percentile sensitivity and contagion

4.1. Theorem

Suppose x is a random vector with marginal distributions¹

$$F(x) \equiv \{F(x_1), \dots, F(x_p)\} \equiv (u_1, \dots, u_p) \equiv u .$$

Further suppose $0 \leq q \leq 1$ is given and $Q(x)$ is the vector of q -quantiles

$$F\{Q(x)\} = q1 = Q(u) ,$$

where 1 is a vector of p ones. Define the stress vector with respect to x_j as²

$$\frac{dQ(x)}{dx_j} \equiv Q(x|u_j > q) - Q(x) , \quad (1)$$

where $Q(x|u_j > q)$ is the vector of q -quantiles of x given $u_j > q$. Then if $Q(x)$ is linear in q then

$$\frac{dQ(x_i)}{dQ(x_j)} \equiv \frac{dQ(x_i)/dx_j}{dQ(x_j)/dx_j} = \frac{f_j}{f_i} \frac{q_{ij}}{q(1-q)} , \quad q_{ij} = C_{ij}(q + q_{ij}, q) - q^2 , \quad (2)$$

where f_i and f_j are the densities of x_i and x_j evaluated at their q -quantiles and C_{ij} is the copula of (u_i, u_j) . Further if

$$s_{ij} \equiv \frac{q_{ij}}{q(1-q)} ,$$

then $-1 \leq s_{ij} \leq 1$ with $s_{ij} = \pm 1$ if x_i and x_j are comonotonic and counter monotonic, respectively. If x_i and x_j are independent then $s_{ij} = 0$. If u_i and u_j are exchangeable then $s_{ji} = s_{ij}$.

¹To economise on notation, write $F_j(x) \equiv F_j(x_j) \equiv F(x_j)$ and

²In Adrian and Brunnermeier (2011) $\Delta CoVar_q \equiv q_{y|x=q_x} - q_y$. Variable y is generally the “financial system” and hence considered is the change in the VaR_q of the system when institution x stressed, with stress interpreted as $x = q_x$. On page 10 of their paper they incorrectly state “... *CoVaR* conditions on the event that [an] institution is at its *VaR* level, which occurs with probability q .”

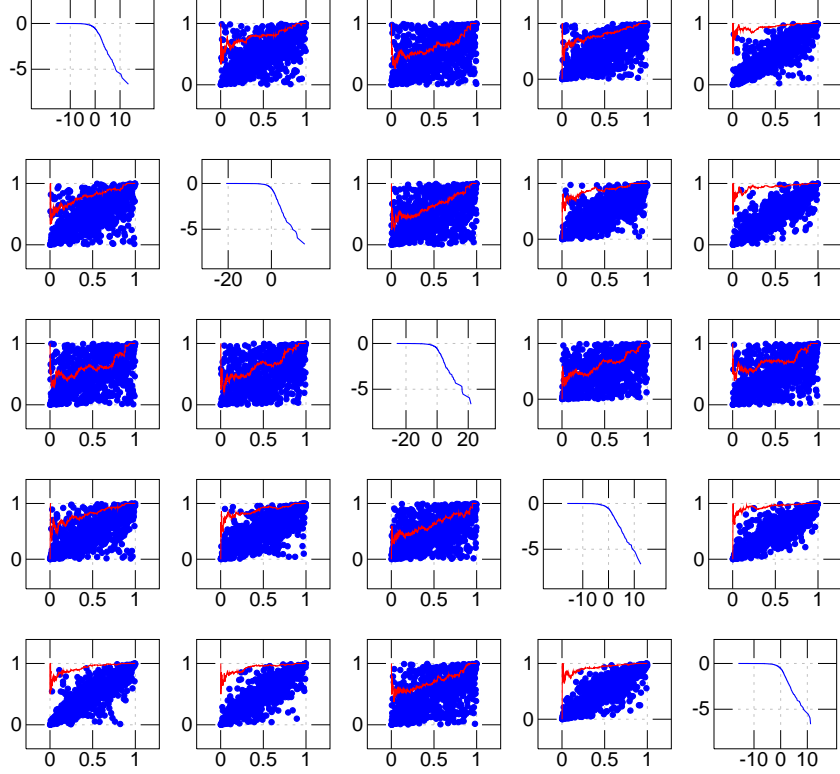


Figure 1: Pairwise copulas of bank stocks cba, anz, mcq, wbc, and the overall bank index. Red lines plot sensitivity s_{ij} as a function of q .

4.2. Implementation

Figure 1 displays the empirical copulas calculated from four weekly closing stock prices labelled anz, cba, mcq and wbc for $n = 761$ weeks from 2000 April 12 through to 2014 October 29. The empirical copulas are calculated by converting each observation to an empirical percentile and plotting the same against each of the other series percentiles.

To estimate q_{ij} at a particular q , the second equation in (2) is iterated³ starting from $q_{ij} = 0$ where copulas are estimated as

$$\hat{C}_{ij}(u_i, u_j) \equiv \hat{\mathbb{E}}\{(p_{ik} \leq u_i)(p_{jk} \leq u_j)\}.$$

Here $\hat{\mathbb{E}}$ computes the empirical mean over the cases $k = 1, \dots, n$ and p_{ik} and p_{jk} are the empirical percentiles of case k of x_i and x_j , respectively.

³The second equation in (2) is a contraction mapping and hence has a fixed point.

4.3. Discussion

The critical aspect of the result is that sensitivities factor into contributions from the ratios of the marginal densities and quantities calculated from the pairwise copulas. The via the implicit equation for q_{ij} in (2), solved using a root finding algorithm. The quantities q_{ij} are implicitly defined from the pairwise copulas. In summary

$$S \equiv \frac{dQ(x)}{dQ'(x)} ,$$

where $'$ denotes transposition. The matrix S is called the sensitivity matrix and displays the sensitivity of the q -quantile of each component of x to stress in the same or other components. Stress in x_j implies that the density of x_j changes from $f(x_j)$ to $f(x_j)/(1 - q)$ where $x_j > Q(x_j)$, called q -stressing x_j .

The ratio of the densities can be written as the ratio of hazards:

$$\frac{f_j}{f_i} = \frac{\mu_j}{\mu_i} ,$$

since both are evaluated at their respective q -quantiles. This result is useful since the hazard may be better behaved, or easier to model, than the density.

Note that the new density evaluated at $Q(x_j)$ is the hazard at $Q(x_j)$:

$$\mu_j \equiv \frac{f_j}{1 - q}$$

Hence

$$\frac{dQ(x_i)/\mu_i}{dQ(x_j)/\mu_j} = \frac{q_{ij}}{q(1 - q)}$$

4.4. Proof

By definition

$$\frac{dQ(u_i)}{du_j} \equiv Q(u_i|u_j > q) - q \equiv q_{ij}$$

and q_{ij} is such that

$$q = \frac{P(u_i \leq q + q_{ij}, u_j > q)}{1 - q} = \frac{q + q_{ij} - C_{ij}(q + q_{ij}, q)}{1 - q} , \quad (3)$$

Rearranging yields the second equation in (2). Now

$$\frac{dQ(x_i)}{dx_j} \equiv F_i^-(q + q_{ij}) - Q(x_i) \approx (F_i^-)'(q)q_{ij} = \frac{q_{ij}}{f_i} , \quad (4)$$

where $'$ denotes differentiation. The approximation follows from a first order Taylor expansion and is exact if the quantile is linear in q . Similarly

$$\frac{dQ(x_j)}{dx_j} \equiv F_j^-\{q + q(1 - q)\} - Q(x_j) \approx \frac{q(1 - q)}{f_j} . \quad (5)$$

Dividing (4) by (5) yields the first equation in (2) and completes the proof.

4.5. Further remarks

Note that

$$\frac{dQ(x_i)}{f_j dx_j} = \frac{dQ(x_i)}{f\{Q(x_j)\}dx_j} = \frac{dQ(x_i)dQ(x_j)}{dF\{Q(x_j)\}dx_j} = \frac{dQ(x_i)/dx_j}{dF\{Q(x_j)\}/dQ(x_j)}$$

Using the usual rules of calculus

$$\frac{dQ(x_i)}{dQ(x_j)} = \frac{dQ(x_i)}{du_i} \frac{du_i}{du_j} \frac{du_j}{dQ(u_j)} = \frac{dQ(x_i)}{du_i} \frac{du_i}{du_j} \frac{1}{q(1-q)}$$

Now

$$\begin{aligned} \frac{dQ(x_i)}{du_i} &= \frac{F^-(q_i)}{u_j}, & \frac{u_i}{u_j} &= q_{ij} \\ \frac{dQ(x_j)}{dx_j} &= f(x_j) \frac{dQ(x_j)}{du_j} & \Rightarrow & \frac{dQ(x_j)}{dQ(x_i)} \\ \frac{dq}{du_j} &= \frac{dF\{Q(x_i)\}}{du_j} = f_i \frac{dQ(x_i)}{du_j} & \Rightarrow & \end{aligned}$$

4.6. Generalisations

Can the above be generalised to other risk measures? Suppose $Q(x) \equiv E\{x\phi(u)\}$ where ϕ is a given function which acts componentwise and $x\phi(u)$ denotes componentwise multiplication. Then the analogue of (1) is

$$\frac{dQ(x)}{dx_j} \equiv Q\{x|x_j > Q_j(x)\} - Q(x), \quad (6)$$

where stress is $x_j > Q_j(x)$. In this case we assume $Q(x)$ is again linear when we cut the left tail off (how to formalise exactly – maybe linearity is more appropriate in other risk measures)

5. Financial sensitivity and contagion

The matrix S of sensitivities can be decomposed with the singular value decomposition

$$S = UDV' = s_1 c'_1 + s_2 c'_2 + \cdots + s_p c'_p.$$

Here D a diagonal matrix of singular values, arranged in decreasing order along the diagonal. So the best thing is to compare

$$Q = s_{i1}c_{j1} + \cdots + s_{ip}c_{jp}$$

If $Q = I$ then $S = C = I$. If $Q = 11'$, a matrix of ones, then $S = \sqrt{p}(1, 0)$ and $C = (c, 0)$ both have rank 1: here s and c are the leading columns of S and C , respectively and 0 a matrix of zeros.

The approximation $Q \approx sc'$ is exact if Q has rank 1 implying $Q = 11'$. The approximation $Q \approx sc'$ is inadequate if $Q = I$. Inadequacy can be judged from

the largest singular value in D , as a proportion of p , the number of variables or rows in Q .

$$Q_* \equiv \frac{1}{p-1}(Q - I) = UDV', \quad s \equiv Q_*1, \quad c \equiv Q'_*1.$$

$$Q = I + s1' + UDV', \quad s \equiv \frac{1}{p-1}(Q - I)1$$

Then s is the vector of average VaR_q sensitivity of each variable to all others. Further c is the average VaR_q impact of each variable on all others. The matrices U , D and V define the singular value (svd) decomposition of Q_* , arranged so that diagonal matrix D has the singular values on the diagonal in descending order. If $b = d_1 u_1$ and $c = v_1$ where u_1 , d_1 and v_1 are the first column, top diagonal entry, and first column of U , D and V respectively, then for two variables $y \neq x$ in Q ,

$$\Delta_x q_y = s_y + b_y c_x + \epsilon_{yx}$$

This states that percentage sensitivities are, apart from the “error” ϵ_{yx} , an average sensitivity plus a scaled response to the contagious effect of the x variable. The contagious effects contained in c are estimated by maximising the explanation of Q .

The vector $1'Q$ sums the changes in VaR_q when each of the column variables is stressed, and writes this as a proportion of the change in the variable being stressed. These proportional sums, subtracting 1 and divided by $p - 1$ where p is the number of variables, measures the average contagion of each variable on all others: $c' = (p - 1)^{-1}1'(Q - I)$ or $c = (p - 1)^{-1}(Q - I)'1$

Alternatively the vector $Q1$ sums the changes in VaR_q of each row variable when all the column variables are stressed. Again it is appropriate to remove the effect of a variable on itself and consider the average over the remaining variables: $s \equiv (p - 1)^{-1}(Q - I)1$. If $Q_* \equiv (p - 1)^{-1}(Q - I)$ then $s = Q_*1$ and $c = Q'_*1$ are the vectors of sensitivities and contagions, respectively. If $Q = I$ then $s = c = 0$. If $Q = 11'$ then $s = c = 1$.

6. Systemic risk and causal chains

A rank one approximation to the matrix Q is $Q \approx sc'$. Vector c is an index of the contagious impact of each of variables on the others while s measures the sensitivity of each variable to each of the others. The vectors s and c are derived from the singular value decomposition $Q = UDV'$ where $U'U = V'V = 1$ and where s and c are the first column of UD and V respectively, assuming the svd is organised so that the singular values in the diagonal matrix D are organised from largest to smallest. The appropriateness of the summarisation sc' is measured with $\text{tr}(Q - sc')$.

If the variables are independent then $Q = I$ and both s and c equal a column of the identity matrix with sc' a matrix of 0's except in a single diagonal position where it is 1. Then all but one variable has a contagion effect and only that variable is sensitive to the contagion provided by the variable.

If the variables are comonotonic then $Q = 11'$ and $s = 1$ and $c = 1$ where 1 denotes a vector of ones. Thus the rank 1 approximation is exact and each variable is equally contagious and equally sensitive.

Note that

$$Q^n \approx (s'c)^{n-1}sc' = ,$$

If the random variables are independent the $Q - I = 0$ and $a = b = k = 0$ and there is no error in the first order svd approximation. If the random variables are comonotonic then $Q = 11'$ and $Q - I$ has ones everywhere except on the diagonal where it is zero. The vector of row means is then $p^{-1}(p - 1)1$

Systemic risk in the system is measured with $b'k = d_1(u_1'v_1)$. In the case of comonotonic random variables $Q = 11'$, $a = 1$ and x , where p is the number of variables, $d_1 = 1$.

Furthermore we may define quantities such as $u^- \equiv \text{VaR}_q(v|u \leq q)$ measuring the impact of a non distressed state in v . For brevity we do not dwell on these constructs in this writeup although the ramifications and potential uses of these constructs will be investigated in the research.

7. Literature

We propose a measure for systemic risk: CoVaR, the value at risk (VaR) of financial institutions conditional on other institutions being in distress. We define an institution's (marginal) contribution to systemic risk as the difference between CoVaR and the financial system's VaR. From our estimates of CoVaR for characteristic-sorted portfolios of publicly traded financial institutions, we quantify the extent to which characteristics such as leverage, size, and maturity mismatch predict systemic risk contribution. We argue for macro-prudential regulation based on the degree to which such characteristics forecast systemic risk contribution.

8. Econometric implementation

The above development sets out our proposed broad framework for linking bivariate copulas and marginals to external variables and shocks study the impact of the same on stresses within the system and the contagious effects of crises. Proposed econometric analysis will implement and extend Brownlees and Engle (2010).

9. Data

We will employ publicly available data as published by APRA and other regulators.

References

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