

# Measuring systematic risk

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## Abstract

This project firstly aims to assess the level of systemic risk in the Australian economy and in the ADI, insurance and superannuation industries, and identify potential domestic and external shocks to the Australian economy, and potential methods to protect against or remediate these shocks.

(2) Assess the effectiveness of stress testing in group structures as a means of enhancing the resilience of the group, and entities within the group, to financial and economic shocks and limiting intra-group contagion.

(3) Consider how APRA and industry could enhance their stress testing procedures and learn more from the results of this testing.

SYSTEMIC RISK Understanding and managing systemic risk Specific systemic risks Transmission of systemic risk through the global financial system

FINANCIAL MARKET DEVELOPMENTS Market quality measurements Market Innovation, complexity and integrity (products, services, technologies, markets, payment systems, counterparty risk, outsourcing) International capital markets (risks and benefits in flows, diversification and different regulatory frameworks) Financial markets integration

MARKET AND REGULATORY PERFORMANCE (encompassing regulation in all its forms and modes e.g. legislation, self-regulation codes, regulation by the market etc.). Informed regulation Regulatory system architecture Regulatory performance and reform (e.g. assessment of performance, enforcement experience, prudential and accounting standards) Superannuation / managed funds Effective measures for financial consumer/investor protection Development of new markets (e.g. corporate bonds, carbon emission and other environmentally motivated markets)

*Keywords:* Stress testing; systemic risk

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## 1. Introduction

it is sensible to regulate financial institutions whose failure is likely to have major impacts on the financial and real sectors of the economy.

for instance, regulate them to reduce their risk, and consequently the probability that taxpayers will face this choice.

The definition, however, misses a key feature of systemic risk. Systemic risk should not be described in terms of a financial firms failure per se but in the context of a firms overall contribution to system?wide failure. The intuition is straightforward. When only an individual financial firms capital is low, the firm can no longer financially intermediate. This has minimal consequences though because other financial firms can fill in for the failed firms void. When capital is low in the aggregate, however, it is not possible for other financial firms to step into the breach. This breakdown in aggregate financial intermediation is the reason there are severe consequences for the broader economy.

## 2. Two variables

Suppose two uniform random variables  $u$  and  $v$  on  $[0, 1]$ . Define  $u^*$  such that

$$P(u \leq u^* | v > q) = q, \quad 0 < q < 1. \quad (1)$$

Then  $u^*$  is the  $\text{var}_q$  of  $u$  given  $v > q$ . Denote  $u^*$  as  $Q(u|v > q)$  then from (1)

$$q = \frac{P(u \leq u^*, v > q)}{1 - q} = \frac{u^* - C(u^*, q)}{1 - q},$$

where  $C(u, v)$  is the joint distribution (copula) of  $u$  and  $v$ . Rearranging yields

$$u^* = q(1 - q) + C(u^*, q).$$

In terms of  $u^*$  define the ripple effect of  $v$  on  $u$  as

$$r_{uv} \equiv \frac{u^* - q}{q(1 - q)}. \quad (2)$$

Thus  $r_{uv}$  is the proportional change in  $\text{var}_q$  of  $u$  divided by  $1 - q$ , given the extra information  $v > q$ . Then  $1 - 1/(1 - q) \leq r_{uv} \leq 1$  with  $r_{uv} = 0$  if  $u$  and  $v$  are independent:  $C(u, v) = uv$ , and  $r_{uv} = 1$  if  $u$  and  $v$  are perfectly dependent:  $C(u, v) = \min(u, v)$ . Further  $r_{uv} = 1 - 1/(1 - q)$  if  $u$  and  $v$  are perfectly negatively dependent:  $C(u, v) = \min(u, 1 - v)$ .

The measure  $r_{uv}$  is quantifies the effect on the  $\text{var}_q$  of  $u$  if  $v$  is in breach of its  $\text{var}_q$ . The presentation is somewhat analogous to Adrian and Brunnermeier (2011). The above definition appears superior on a number of grounds:

- $r_{uv}$  is divorced from actual scales (good if you believe joint action is best modelled with copulas). Hence invariant to any monotonic transformation of scale. (This may help in empirical modelling by taking say logs etc).
- Doesn't suffer from  $v = q$  conditioning problem. i.e.  $v$  in distress implies  $v > q$ , not  $v = q$ .

- Is  $\text{var}_q$  oriented in that it shows the revised  $q$ -safety demand on  $u$ , when  $v$  becomes  $q$ -distressed.
- Given  $u^*$  it follows

$$r_{uv} = \frac{C(u^*, q)/q - q}{1 - q} = \frac{C(u^*, q) - q^2}{q(1 - q)} \quad (3)$$

Note  $C(u^*, q)/q = P(u \leq u^* | v \leq q)$  and hence

$$r_{uv} = \frac{E(u \leq u^* | v \leq q) - E(u \leq q)}{E(u > q)}, \quad q = E(u \leq u^* | v > q)$$

- Don't like the fact that  $\min(r)$  is not -1. Can this be “fixed” or rationalised.

Suppose  $F_x$  and  $F_y$  are the marginal distributions of  $x$  and  $y$  with  $x = F_x^-(u)$  and  $y = F_y^-(v)$ . Then the ripple effect of  $y$  on  $x$  is the change in  $\text{var}_q(x)$  given  $y$  is  $q$ -distressed:  $v > q$ :

$$r_{xy} \equiv F_x^-(u^*) - F_x^-(q) = F_x^-\{q + r_{uv}q(1 - q)\} - F_x^-(q) \quad (4)$$

$$\approx r_{uv} \frac{q(1 - q)}{q'} = \frac{qr_{uv}}{\lambda} \quad (5)$$

where  $q = F_x(c)$ ,  $'$  denotes differentiation and  $\lambda$  is the hazard of  $x$  at  $x = c$ , the  $\text{var}_q$  of  $x$ . If  $F_x$  is linear then the approximation is exact. Hence it is of interest to scale  $x$  such that  $F_x^-$  is linear.

The ripple effect  $r_{xy}$  depends on:

- The  $q$  level. As  $q \rightarrow 1$ , other things equal,  $r_{xy} \rightarrow r_{uv}/\lambda$ .
- “correlation”  $r_{uv}$  as determined by the copula in (2).
- Hazard of  $x$  at  $x = c$ . Small hazard implies  $r_{xy}$  is large.

In practice  $q \approx 1$  implying  $r_{xy}$  is effectively equal to  $r_{uv}/\lambda$ . If  $F_x$  is tail exponential with mean  $\mu$  then  $1/\lambda = \mu$  and  $r_{xy} = \mu qr_{uv}$ . With the Pareto  $\lambda$  is decreasing which means  $1/\lambda$  is increasing and the impact of a “stress” is larger when  $q$  is larger, given the same  $r_{uv}$ .

Below the hazard is modelled as as a function of explanatory variables  $z$  using the Cox proportional hazards model. The Copula thus stays fixed and

$$r_{xy}(t) = \frac{r_{uv}}{\lambda_x} e^{-z'_t \beta}, \quad \ln r_{xy}(t) = \ln r_{uv} - \ln \lambda_x - z'_t \beta$$

### 3. Estimating hazard

The Cox proportional hazards model is the most commonly used model in hazard regression. With this model, the conditional hazard function, given the covariate vector  $z$ , is

$$\lambda_x(z) = \lambda_x e^{\beta' z} ,$$

where  $\beta$  is a vector of regression coefficients, and  $\lambda_x$  the baseline hazard function. No particular shape is assumed for the baseline hazard; it is estimated nonparametrically. The contributions of covariates to the hazard are multiplicative. An accessible introduction to the Cox model is in Klein and Moeschberger (1997).

Since  $r_{uv}$  is invariant to scale it is best to use a scale for  $x$  such that the first order approximation in (5) is robust or accurate. For example if  $x$  is assumed tail-exponential  $F_x(x) = 1 - e^{-\lambda x}$  then

$$P(e^x \leq z) = P(x \leq \ln z) = 1 - e^{-\lambda \ln z} = 1 - z^{-\lambda}$$

is

So for APRA we aim to investigate  $r_{xy}$  in terms of  $r_{uv}$ ,  $q$  and the marginal distributions of the variables. Without much loss of generality we can assume  $r_{uv} \geq 0$ . Copulas are used to model the joint behaviour of variables.

### 4. Multivariate case

Suppose  $u$  a vector of uniform random variables with joint distribution  $C(u)$ . Define

$$u^* \equiv Q(u|v > q) \text{ such that } P(u \leq u^* | v > q) = q , \quad 0 < q < 1 .$$

Then

$$q = \frac{P(u \leq u^*, v > q)}{1 - q} = \frac{C(u^*) - C(u^*, q)}{1 - q} ,$$

and rearranging shows

$$C(u^*) = q(1 - q) + C(u^*, q) , \quad \frac{C(u^*) - q}{q} = \frac{C(u^*, q)}{q} - q$$

#### 4.1. Alternate approach

An alternate definition is

$$(1 - q)r_{uv} \equiv \frac{u^* - q}{q} \approx \ln u^* - \ln q , \quad u^* = q\{1 + (1 - q)r_{uv}\} \approx qe^{(1 - q)r_{uv}} .$$

With this definition

$$(1 - q)r_{xy} \approx \frac{(1 - q)r_{uv}}{(\ln q)'} \approx \frac{\ln u^* - \ln q}{(\ln q)'}$$

Note the result below: (not sure if this has any usefulness – note diff is wrt  $q$  not  $x$ )

$$\int_a^b \frac{(\ln q)'}{1-q} dq = \ln \frac{q}{1-q} \Big|_a^b, \quad \int_a^b \frac{1-q}{(\ln q)'} dq = \left[ \frac{x^2}{2} - \frac{x^3}{3} \right]_a^b$$

#### 4.2. Market

Empirical papers seem to focus on the “market”. In this case, say  $v$  is the market and  $u$  a particular industry. Then  $r_{uv}$  indicates the “correlation” of the industry with the market and the exposure industry  $u$  has to the market.

#### 4.3. Many variables

In the vector case define matrix  $R_u$  with entries consisting of the pairwise  $r_{uv}$ . Write

$$R_x \equiv D^{-1} R_u,$$

where  $D$  is a diagonal matrix with entries  $(\ln q)'/(1-q)$ , the slopes (“scores”?) of  $\ln q$  with respect to each of the  $\text{var}_q$  corresponding to  $x$ .

- In essence  $R_u$  captures, in a scale independent fashion, the ripple effects of breaches in each industry on other industries. These scale independent quantities are “scaled” using  $D^{-1}$ , to arrive at  $R_x$ .
- Varying  $q$  indicates the sensitivity of each institution to other institutions at different  $q$ . For example if  $q = 1/2$  then  $R_x$  measures the impact of a “worse than average year” of one institution on the  $\text{var}_{1/2}$  or median of another. Hence constructing  $R_x$  is similar to quantile regression where the explanatory variables are (up to  $(1-q)/(\ln q)'$  the ripple effects on the  $(u, v)$  scale.
- The quantities  $(\ln q)'$  can be determined using quantile regression. i.e. here we are determining the slope of the marginal distribution at quantile  $q$ .
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- $R_u = I$  (independence) implies  $R_x = D^{-1}$ . Thus  $R_x - D^{-1}$  has zero on the diagonal
- Entry  $(i, j)$  indicates the impact, up to  $1-q$ , on the  $\text{var}_q$  of  $i$  of a breach in  $j$ .
- Row  $i$  of  $R_x$  indicates (up to  $q(1-q)$ ) of the ripple effects on  $i$  of breaches elsewhere.
- Column  $j$  of  $R_x$  indicates (up to  $q(1-q)$ ) ripple effects of a breach in  $j$ .
- The vector  $(R_x - D^{-1})1$  where  $1$  is a (column vector) of 1's, is the total  $\text{var}_q$  exposure (up to  $q(1-q)$ ) of each institution of breaches in all other institutions.

- The vector  $(R'_x - D^{-1})1$  where  $'$  indicates transposition, is the sum of all  $\text{var}_q$  impacts of each institution on all other institutions.
- Systemic risk is measured as  $1'(R_x - D^{-1})1$ , that is the sum of all off-diagonal entries of  $R_x$ .
- Maybe systematic risk should be measured as  $1'(R_u - I)1 = 1'R_u1 - p$  where  $p$  is the number of industries.

Problem with the above is that the development is first order. Maybe this is all we can hope for.

#### 4.4. Joint modelling of many variables

Suppose  $C(u)$  is the copula of a vector  $u$  of uniform random variables. Then

$$\frac{C(u)}{u_i} = P(U \leq u | U_i \leq u_i) = P(U^i \leq u^i | U_i \leq u^i)$$

is the conditional distribution given  $u_i$ . Here  $U^i$  and  $u^i$  indicate  $U$  and  $u$  excluding component  $U_i$  and  $u_i$ , respectively. Suppose  $i = 1$  then

$$\begin{aligned} P(U^1 \leq u^1 | U_1 \leq u^1) &= P(U_2 \leq u_2 | U_1 \leq u_1) P(U_3 \leq u_3 | U_1 \leq u_1, U_2 \leq u_2) \cdots \\ &= C(u_2 | u_1) C(u_3 | u_1, u_2) \cdots \approx C(u_2 | u_1) C(u_3 | u_1) \cdots \end{aligned}$$

The so-called prediction error decomposition.

#### 4.5. Econometric modelling

Econometric modelling thus proceeds as follows:

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### 5. Recent Literature and Methodological Developments

A number of recent papers and technical reports have proposed novel methodologies to measure systematic risk in the financial system. These take an aggregate econometric approach. These papers include:

- Adrian and Brunnermeier (2011)
- Acharya et al. (2012)
- Acharya et al. (2012)
- Brownlees and Engle (2010)

In the next few subsections we review these papers and set them in a context of what may be achieved in an Australian context.

### 5.1. Brownlees and Engle (2010)

This paper defines divides the current value of assets of an institution into fraction  $k$  without risk and  $1 - k$  with risk. These fractions are determined based on current values. If riskless The expectation the value one period ahead is  $kf + (1 - k)E(r)$ . Budget constraint at time  $t = 0$  is

Suppose a firm has wealth  $w$ . It borrows amounts  $f$  and  $g$ , risky and guaranteed debt to arrive at total assets  $w + f + g$  which it allocates to a vector  $x$  of risky assets:  $w + f + g = 1'x$ . Suppose  $w = k(f + g + w)$  where  $0 < k < 1$  where  $k$  measures the leverage.

After 1 period  $f$ ,  $g$  and  $x$  grow to  $f^*$ ,  $g^*$  and  $x^*$ , respectively to yield asset position  $w^* = 1'x^* - f^* - g^* - \phi$  where  $\phi$  is the cost of distress. Hence the capital buffer at the end of the period is

$$k(f^* + g^* + w^*) - w^*$$

Stating that the wealth plus

This paper exploits the two period model developed by Acharya et al. (2012). Let  $f$ ,  $g$  and  $w$  denote risky debt, guaranteed debt and capital, respectively. Risky assets enumerated in vector  $x$ . The equation  $f + gb + w = 1'x$  indicates the allocation of debt and capital to risky assets, where  $b$  is the discount applied to guaranteed debt. One period later the wealth of the institution is  $x'r - f - g - \phi$  where  $r$  is the return vector and  $\phi$  is the cost of distress.

The whole argument then focusses on  $E\{r_{it}|F(r_{mt}) < q\}$

### 5.2. Adrian and Brunnermeier (2011)

Below I summarise the Adrian and Brunnermeier (2011) paper. This approach is clearly less satisfactory than the approach outlined in the previous sections.

Let  $x$  be a vector of losses for financial institutions with components  $x_j$ . Thus negative components in  $x$  indicate profits. For given  $0 < q < 1$  define  $v_i \equiv v(x_i)$  as the value at risk, var, of  $x_i$  such that probability of losses less than  $v_i$  is  $q$ . Typically  $q$  is interpreted as a stress level with  $q \approx 0$  implying no stress while  $q \approx 1$  indicates severe stress. In the presentation below it is understood  $q > 1/2$  is fixed near 1. When an institution is moved from  $\text{var}_{0.5}$ , the median, to stress level  $q$ , the institution is put under stress or “stressed.” Thus “stressing” and “stressed” are actions rather than states of being.

Similarly define  $v\{x_i|f(x) = c\}$  the  $\text{var}_q$  in the conditional distribution of  $x_i$  given  $f(x) = c$ . The case  $v(x_i|x_j = v_j)$  is called the  $\text{cvar}_q$  of  $i$  with  $j$ , measuring the  $\text{var}_q$  of  $i$  when  $j$  is at  $\text{var}_q$ . The ripple effect of  $j$  on  $i$  is

$$r_{ij} \equiv v(x_i|x_j = v_j) - v(x_i|x_j = m_j) ,$$

where  $m_j$  is the median of  $x_j$ . Thus  $r_{ij}$  measures the change in  $\text{var}_q$  of  $i$  when  $j$  moves from  $\text{var}_{0.5}$  (average stress) to  $\text{var}_q$  (extreme stress). Generally  $r_{ij} > 0$  and is the “ripple effect” on  $i$  when  $j$  is stressed.

Arrange the  $r_{ij}$  into matrix  $R$ .

- Note  $v(x_i|x_i = v_i)$  and  $v(x_i|x_i = m_i)$  are undefined. Hence the diagonal of  $R$  is undefined: however we set it to zero indicating the ripple effect of an institution on itself is zero.
- Note  $R$  is not generally symmetric: the ripple effect of  $j$  on  $i$  is not generally equal to the ripple effect of  $i$  on  $j$ .
- The sum of all entries  $s \equiv 1'R1$  where  $1$  is a vector of 1's is the sum of all ripple effects called the systemic risk. Thus  $s$  measures the total effect on other institutions of moving each institution from its median to its  $\text{var}_q$  position.
- If  $i$  is independent of  $j$  then  $r_{ij} = 0$ . Hence if  $R = 0$  there is no systemic risk in the system. Zero columns in  $R$  indicate the corresponding institution has no ripple effects on other institutions. Zero rows in  $R$  indicate the corresponding institution is not exposed to other institutions.
- Let  $1$  indicate a column vector of 1's. Then vector  $r \equiv R'1$  is the sum of each column of  $R$  and indicates the sum of all the ripple effects in response to stressing a given institution. Large (negative) entries of  $r$  indicate institutions with a combined major impact on other institutions. Hence  $r$  is called the ripple vector.
- In contrast  $e \equiv R1$  is the vector of row sums. Entries indicate the sum of ripple effects on  $i$  when stressing all other institutions called exposures. If a particular component is zero, then the corresponding institution is not affected by stresses elsewhere in the system. Large components of  $e$  indicate exposed institutions and  $e$  is called the exposure vector.
- $e \circ r$  is the exposure times ripple effect and hence is a second order effect.
- Note  $s \equiv 1'e = 1'r = 1'R1$  the total systemic risk measuring total losses when all  $i$  move from average risk to  $\text{var}_q$ .
- Look at Engle article.
- The above has assumed measurement is relative to the median i.e. the 0.5 quantile. However suppose we measure ripple effects relative to the current state of each institution:  $R_t \equiv R(x_t)$  where  $x_t$  is the vector  $x$  observed at time  $t$ . Then  $R_t$  is the ripple matrix at time  $t$  where ripple effects are measured with respect to the actual percentiles applicable at time  $t$  i.e. entry  $(i, j)$  of  $R_t$  is the change  $\text{var}_q$  of  $i$  when  $j$  moves from its current profitability to its  $\text{var}_q$ .
- Write  $r_t \equiv R_t'1$  and  $e_t = R_t1$ . Hence with  $R_t, r_t$  and  $e_t$  focus on measuring risk, exposure and ripple effects given the actual state percentile state at each time  $t$ .
- Total systemic risk in the system at time  $t$  is  $s_t \equiv 1'R_t1$ . Note  $x_t \rightarrow q$  implies  $s_t \rightarrow 0$ .



- A more general analysis is where

$$r_i \equiv v(x_i|x^i = v^i) - v(x_i|x^i)$$

where  $x^i$  is  $x$  excluding component  $i$ . In this case  $r_i$  measures the effect on  $\text{var}_q$  of  $i$  of stressing all other institutions from their current position to stressed positions.

- Systemic risk is measured relative to  $q$ . This can be varied. Furthermore the risk is measured through a specific risk measure. More generally we can substitute  $\text{var}_\phi$  with say  $v_i \equiv v(x_i) \equiv E\{x_i\phi(u_i)\}$  where  $\phi$  defines a risk measure. Further

$$r_{ijt} \equiv v(x_i|x_j = v_j) - v(x_i|x_j = x_{jt}) ,$$

arranged into the matrix  $R_t$ . Thus  $R_t$  measures the ripple effects on  $i$  of  $j$  where the effect is measured by the expected change in the risk floor of  $x_i$  if  $x_j$  moves from its current value to its risk floor.

- One of the institutions may be the financial system as a whole denoted  $y$ . In this case

$$r_j^* \equiv v\{y|F(x_j) = q\} - v\{y|F(x_j) = 0.5\} \quad (6)$$

measures the exposure of the financial system to institution  $j$ .

- Quantile regression is used to estimate the exposure of the financial system to  $j$ . Consider the regression model  $y_t = q_j + \beta_j x_{jt} + \epsilon_t$ , estimated using quantile regression techniques leads to  $\text{var}_q$  of  $y$  as a function of  $x_j$ . Given  $q_j$  and  $\beta_j$  then (6) becomes

$$r_j^* = q_j + \beta_j v_{jt} - (q_j + \beta_j m_{jt}) = \beta_j (v_{jt} - m_{jt}) .$$

- The next step relates  $\text{var}$  of  $x_j$  to state variables. In this prescription state variables at different points of time become critical

$$y_t = q_j + \beta_j x_{jt} + \gamma_j q_t + \epsilon_t , \quad x_{jt} = a_j + g_j q_t + \eta_t$$

where  $q_t$  is a state variable. Again quantile regression is used and the results from the regression are used to predict the quantile of  $y_t$  given the quantile of  $x_{jt}$ .

In terms of this notation

$$q_j + \beta_j v_{jt} + \gamma_j q_t - (a_j + g_j q_t) =$$

### 5.2.1. Quantile regression of ripple effects

Suppose we have a vector time series of losses for each institution:  $x_t$ ,  $t = 1, \dots, n$ . Then we can regress each component of  $x_t$ , denoted  $x_{it}$  on regressor variables. Using absolute values leads to median regression and a more

generalised penalising function leads to appropriate percentiles. In this project we are interested in the  $q$ -percentiles. Why not just regress each component on losses in other sectors and use a penalty function that tunes to the  $q$  quantile. In particular we solve the following problem

$$\min_{\beta} \sum \rho_q \{x_{it} - f(x_t^i, \beta)\}$$

where  $\rho_q$  is the tilting function appropriate to  $\text{var}_q$  and  $x_t^i$  is vector  $x_t$  excluding component  $i$ , while  $f$  is some function. Given  $\beta$ ,

$$r_{ijt} \equiv v(x_i | x_{jt} = v_{jt}) - v(x_i | x_{jt})$$

Suppose we regress parameters of the distribution of  $x_i$  on external factors. Here  $x_i$  is the loss of  $i$  typically observed over time. This will take the form of a generalised linear regression using one of the extreme tail distributions. We can related the parameters of this distribution to the profitability of all other instructions. This will result in a distribution conditioned on the other variables and a  $\text{var}_q$  value. The ripple effect is then straightforward to estimate.

The regression curve gives the average of the distributions for the different xs. Percentage points or  $\text{var} + q$  can be similarly summarised for a more complete picture. Ordinarily this is not done, and so regression often gives a rather incomplete picture. Just as the mean gives an incomplete picture of a single distribution, so the regression curve gives a corresponding incomplete picture for a set of distributions.

## 6. Measures related to $\text{cvar}_q$

The measure  $\text{cvar}_q$  suffers from a number of difficulties:

- It is couched in terms of  $\text{var}_q$  which contains the scale of the original measurements. It is worthwhile to have a scale independent measures
- As seen below the conditioning  $m_j$  is undesirable as it ... Better to condition on

To respond to these issues we define  $u_i \equiv F_i(x_i)$ , the percentile of  $x_i$ , uniformly distributed on  $(0, 1)$ . Note that

$$r_{uv} = F_{u|v=q}^-(q) - F_{u|v=1/2}^-(q)$$

However redefine things as

$$r_{uv} = C_{u|v>q}^-(q) - C_u^-(q) = \mathbb{Q}(u|v > q) - q$$

Note  $-1 \leq r_{uv} \leq 1$ . If  $C(u, v) = uv$  (independence) then  $r_{uv} = 0$ . If  $u = v$  (perfect dependence) then  $C(u, v) = \min(u, v)$  and

$$r_{uv} = \frac{1+q}{2} - q = \frac{1-q}{2}$$

Note this does not equal 1 indicating that moving  $v$  into stress implies the  $q$ -stress of  $u$  moves from  $q$  to  $(1 + q)/2$  which is a small amount if  $q$  is near 1. Hence  $q$  stress

$$\frac{r_{uv}}{1 - r_{uv}} = \frac{(1 - q)/2}{1 - (1 - q)/2} = \frac{1 - q}{1 + q}$$

## 7. Percentile rank gap

Let  $(u, v)$  represent percentile ranks of two random quantities, thus  $(u, v)$  is multivariate uniform. In this paper “uniform” refers to the uniform distribution over the unit interval unless stated otherwise. Assume the joint distribution of  $(u, v)$  is exchangeable. Define the “ $q$ -percentile rank gap” between  $u$  and  $v$  as

$$\rho_q \equiv \frac{E(v|u > q) - E(v|u \leq q)}{E(u|u > q) - E(u|u \leq q)} = \frac{E(v|u > q) - E(v|u \leq q)}{1 - q} \quad (7)$$

In other words

$$E(v|u > q) = E(v|u \leq q) + (1 - q)\rho_q$$

But

$$1/2 = (1 - q)E(v|u > q) + qE(v|u \leq q)$$

implying

$$E(v|u \leq q) = \frac{1/2 - (1 - q)E(v|u > q)}{q}$$

Substituting yields

$$E(v|u > q) = \frac{1/2 - (1 - q)E(v|u > q)}{q} + (1 - q)\rho_q$$

or

$$\frac{qE(v|u > q) + (1 - q)E(v|u > q)}{q} = \frac{1/2}{q} + (1 - q)\rho_q$$

or

$$E(v|u > q) = 1/2 + q(1 - q)\rho_q$$

where  $E$  calculates expectations under the joint distribution of  $(u, v)$ . Hence  $q$ -percentile rank gap  $\rho_q$  is the scaled difference between upper and lower conditional tail expectations of  $v$  with respect to  $u$ , using threshold  $q$ . Since  $u$  is uniform and the joint distribution of  $(u, v)$  is exchangeable,

$$\rho_q = 2 \{E(v|u > q) - E(v|u \leq q)\} = 2 \{E(u|v > q) - E(u|v \leq q)\} .$$

Comparing to CoVaR the analogy is

$$r_{ij} \equiv E(u_i|u_j = q) - E(u_i|u_j = 0.5) ,$$

### 7.0.2. Generalized linear model distributional approach

#### 7.1. Brownlees and Engle (2010)

The method computes SRISK, defined as the capital a firm is expected to need given a financial crisis:

$$\text{SRISK}_{it} = E(\text{CS}_{it} | \text{Crisis}_t)$$

(2) This can be estimated with a bivariate daily time series model of equity returns on firm  $i$  and on a broad market index (which could be just the financial sector).

## 8. APRA's Research Interests

APRA is providing the following information to assist researchers, research institutions, and funding bodies to better understand its research interests.

### 8.1. APRA's role and mission

APRA's main role is prudential supervision and regulation of Australian banks, building societies, credit unions, life insurers, general insurers, friendly societies, superannuation funds and other relevant entities. In this role, APRA seeks to ensure that, under all reasonable circumstances, regulated institutions meet their financial promises within a stable, efficient and competitive financial system. Through this work, APRA also promotes financial system stability in Australia. APRA has two auxiliary missions: it is a national statistical agency for the Australian financial sector and it administers the Financial Claims Schemes in the authorised deposit-taking institution (ADI) and general insurance industries.

### 8.2. Specific research interests

APRA is interested in research that might support its mission. Research on the economics of APRA-regulated industries, and on the effectiveness or otherwise of regulation, are of general interest. However, such research is not often directly relevant to APRA. The following specific areas are likely to have more useful application for APRA.

#### 8.2.1. Stress testing

APRA has conducted formal stress testing for more than a decade and has found it a very useful supervisory tool. APRA is interested in research that assists it and industry better define and learn from stress tests. Specific areas of interest include:

- procedures for developing macroeconomic and industry stress scenarios that provide robust but reasonable stresses;
- the mathematics of stress testing (e.g. properly accounting for random variables such as default rates and collateral values);

- understanding second-round stress effects (e.g. if the banking sector is stressed, does the resultant credit tightening create second-order stress?);
- stress testing insurance portfolios for both catastrophic and gradual changes in loss rates;
- stress testing insurance and superannuation portfolios for investment loss; and
- stress testing ADIs and superannuation funds for liquidity shocks.

Many of APRA's prudential capital requirements in the ADI and insurance industries rely upon de facto stress tests underpinning the capital calculations. APRA is interested in research on the efficacy of these capital models.

#### *8.2.2. Operational risk*

Operational risk is relatively under-researched compared to investment risk, credit risk and traded markets risk. For most APRA-regulated institutions, operational risk has the characteristic that a very high proportion of loss events are tractable to estimate and capitalise, but most losses come from a small number of high value, and hard-to-predict, operational failures. APRA is interested in research that would increase its ability to:

- identify firm characteristics that are associated with increased incidence of large operational losses;
- require appropriate capital for large and unpredictable losses; and
- identify industry or economic circumstances that are associated with increased incidence of operational losses.

APRA has specifically encouraged the general and life insurance industries to increase their understanding of the drivers of operational losses, and this is an emerging issue for superannuation. Some insurance risk is captured through stress testing, as noted above. APRA has encouraged the general and life insurance industries to develop a deeper understanding of risk margins in insurance valuations. The general insurance industry and associated service providers have undertaken considerable applied work in catastrophe modelling. APRA is interested in seeing this area investigated in a rigorous way through more academic research and more research specifically focussed upon Australasian hazards, both known and potential. APRA has supported and partially funded research in Australian demographics, particularly morbidity and mortality. Further research in this area would be directly relevant to the life insurance and superannuation industries, and potentially to the ADI industry.

### *8.2.3. Superannuation*

APRA is interested in research that clarifies how current and prospective superannuation fund members might effectively select trustees, funds and investment options, given the data available in Australia. Similarly, research that assists fund members and other parties assess relative and absolute performance, both retrospectively and prospectively, is of interest. APRA is interested in research on the extent to which the aggregate investment decisions of superannuation fund trustees might impact upon financial system stability in Australia. In particular, would the relatively high asset allocation to equities create procyclical wealth or other effects during a serious economic downturn?

### *8.2.4. Financial system stability*

APRA is interested in research oriented to better characterising the level of systemic risk in the Australian economy and in the ADI, insurance and superannuation industries. APRA is also interested in research that identifies plausible domestic and external shocks to the Australian economy, and potential methods to protect against or remediate these shocks. APRA sees less value in research oriented to determining the systemic risk associated with individual financial institutions, compared with research on risk across the whole system.

### *8.2.5. Statistics*

APRAs statistical publications and public databases are available to any user, including researchers. APRA is moving to make more of its statistical data publicly available. Researchers may request assistance from APRA, via [statistics@apra.gov.au](mailto:statistics@apra.gov.au), to discuss the publicly available material or, in some circumstances, to seek non-public material from APRAs collections. APRA will only provide non-public material when it is satisfied that its confidentiality and public use requirements will be satisfied and that the effort deployed by APRA is justified by the value of the research to APRAs role and mission. From time to time, APRA revises its statistical collections and publications. APRA welcomes suggestions from researchers on what data should be collected and how such data might be published.

## **9. Further stuff**

Stress testing is used in the context of the measurement of bank capital adequacy and reserving for market risk, in conjunction with Value at Risk. Value at Risk is essentially an estimate of the quantile of the distribution of the change in value of a portfolio of financial instruments. In order to do a VaR calculation, you need to specify a joint distribution for the instruments in the portfolio.

Stress testing involves estimating how a portfolio would perform under some “extreme market movements”. It is a form of scenario analysis. It is a way of taking into account extreme events that do occur from time to time but which are virtually impossible according to the probability distributions assumed for

the variables that impact on the portfolio value. For example a 5 standard deviation daily move in a market index (e.g. the s&p 500 stock market index) is one such extreme event. Under the assumption of a normal distribution this would happen about once every 7000 years, but in practice such movements are observed once or twice every 10 years.

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