1. Weihao's approach

Univariate stressing leads to the stressed put expectation

$$\mathrm{E}\{\phi(r_m)p_{it}\} = \int f(r_m)\phi(r_m)\mathrm{E}(p_{it}|r_m)\mathrm{d}r_m = \int f_\phi(r_m)\mathrm{E}(p_{it}|r_m)\mathrm{d}r_m$$

where f is the original density of a market factor r_m and $f_\phi = f \times \phi$ is the assumed density. Note ϕ may act on r_m or its percentile rank u_m. The intent of ϕ is the enlarge the likelihood of "bad" outcomes of r_m.

Bivariate stressing yields

$$\mathrm{E}\{\phi(r_m, r_n)p_{it}\} = \int\int f(r_m, r_n)\phi(r_m, r_n)\mathrm{E}(p_{it}|r_m, r_n)\mathrm{d}r_m\mathrm{d}r_n$$

$$= \int\int f_\phi(r_m, r_n)\mathrm{E}(p_{it}|r_m, r_n)\mathrm{d}r_m\mathrm{d}r_n$$

where r_n is another market factor and f is now the joint density of (r_m, r_n). The assumed joint density is $f_\phi = f \times \phi$, which again enlarges the joint likelihood of "bad" outcomes of (r_m, r_n). For example f_ϕ may have stronger tail dependence than f, which can be achieved by either selecting a copula which has stronger tail dependence, or setting up ϕ as $\phi(u_m u_n)$ where ϕ is decreasing. Marginal distributions of r_m and r_n may be altered or unchanged. If unchanged then

$$f(r_n) = \int f(r_m, r_n)\mathrm{d}r_m = \int f_\phi(r_m, r_n)\mathrm{d}r_m$$

or

$$1 = \int f(r_m)c(u_m, u_n)\mathrm{d}r_m = \int f(r_m)\phi(r_m, r_n)c(u_m, u_n)\mathrm{d}r_m$$

since $f(r_m, r_n) = f(r_m)f(r_n)c(u_m, u_n)$.

1.1. Copula stressors

Stressing can be based on a vector of variables x with distribution $F(x) = u$ and marginal densities $f_j(x_j)$. Suppose $c(u)$ is the copula density of x implying the density of x is $c(u)\prod_j f_j(x_j)$. If $\Phi(u)$ is another copula with density $\phi(u)$ consider

$$\mathrm{E}_\phi(p_{it}) \equiv \int \mathrm{E}(p_{it}|x)\phi(u)c(u)\prod_j\{f_j(x_j)\}\mathrm{d}x \ , \tag{1}$$

If $\phi(u) = 1$ then $\mathrm{E}_\phi(p_{it}) = \mathrm{E}(p_{it})$, the ordinary expectation. The density $\phi(u)$ is designed to magnify potentially stressful situation such as where all component of x are highly tail dependent. In the extreme case $\Phi(u) = \min_j(u_j)$ making the variables in x comonotonic.

The copula density stressor $\phi(u)$ can be combined with marginal stressors ϕ_j written as functions of x_j or percentiles u_j. If the latter then $\phi(u)\prod_j \phi_j(u_j)$ is the total stressor, corresponding to a density on the unit hypercube with non–uniform marginals if $\phi_j(u_j) \neq 1$.

Copula stress effect is simulated as follows. Suppose x is a vector ν_{mt} of market factors over the period $(t, t+h)$. Simulations are to derive the vectors (p^s_{it}, ν^s_{mt}), $s = 1, \ldots, S$. The ν^s_{mt} are converted to percentiles u^s and scalars

$$\phi^s \equiv \phi(u^s) \prod_j \phi_j(u^s_j) \ , \qquad s = 1, \ldots, S \ .$$

The stress effect is then estimated as before using the equations

$$\beta_{it} \approx \frac{1}{\sigma_\phi S} \sum_s (\phi^s - 1)\, p^s_{it} \ , \qquad \sigma^2_\phi \approx \frac{1}{S} \sum_s (\phi^s)^2 - 1 \ .$$