Measuring background and systemic risks in finance

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Abstract

This article refines, builds on, and extends SRISK methodology recently proposed in the literature. The refinement is to define SRISK in terms of a put on the Basel shortfall. This is built on by defining the background and systemic stress of a firm as unconditional and departure from unconditional expectation of the put, the latter when a hypothetical systemic stress is applied. Systemic stress is defined in terms of a random variable and can take on variety of forms including alternative scenarios in usual stress testing as well stress driven by the interaction of variables. Stressor random variables are chosen by the practitioner. Stressed expectations are linear, a sector systemic stress is naturally defined as linear in the firm specific systemic stress. Application is made to Australian financial data.

JEL classification C15; E53; G21 (Needs to be filled out)

1. Background

Recent literature in the area of quantitative systemic risk or stress measurement includes Adrian and Brunnermeier (2011), Acharya et al. (2012), Acharya et al. (2012), Brownlees and Engle (2010) and Brownlees and Engle (2015). This study aims to extend and apply these techniques particularly in relation to the entities regulated by the Australian Prudential Regulation Authority (APRA). Thus our broad aim is to develop, implement and bring to bear recent developments in systemic risk measurement and stress testing on the aims of APRA and the CIFR targeted research areas. (This needs much more commentary).

This study refines and develops recently proposed methods and applies the same to publicly available daily financial data for the eight Australian banks detailed in Table 1 spanning the period from 3 April 2000 through to 1 December 2014. The data is described in detail in Appendix E. Prices, adjusted for dividends are plotted in Figure 1. Prices are combined with number of shares to derive total equity. Total debt for each firm at each day is also collected. (Should these be plotted?).

Remaining sections are structured as follows. §2 and §3 define current and future capital shortfall similarly as Brownlees and Engle (2015) under a simplistic Basel II regulatory framework. The definition of systemic risk used by Brownlees and Engle (2015) is discussed in §4, and is refined, extended and illustrated §5 to §9. §10 discusses specific stress examples which extend beyond Brownlees and Engle (2015). Applications to Australian bank data are discussed in §11 and §12. §13 and §14 discuss how stresses are combined across firms with and without allowance for merging or diversification. Generalised stressed are outlined in §15. §16 offers an alternative modeling approach, based on assets. §17 summarises differences between proposed stress measurement in this paper with that of Brownlees and Engle (2015). §18 concludes.

Table 1: Major and minor Australian banks

major		minor	
cba	Commonwealth Bank	mqg	Macquarie
anz	Australia & New Zealand	boq	Bank of Queensland
nab	National Australia	ben	Bendigo and Adelaide
wbc	Westpac	aba	Auswide

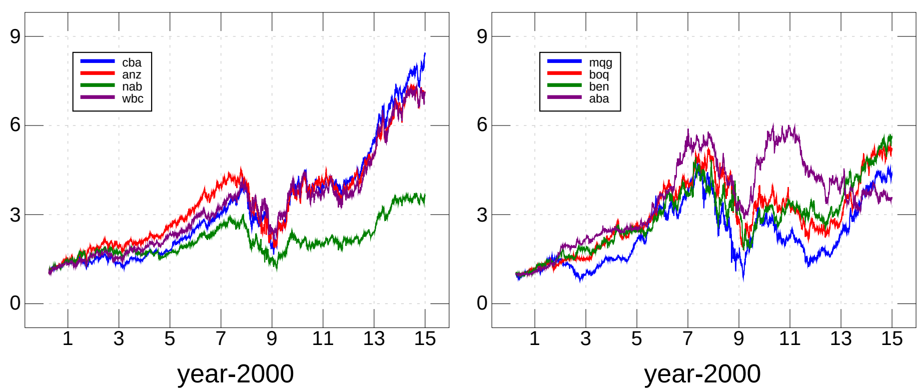

Figure 1: Stock prices of four major (left panel) and four minor (right panel) Australian banks from early 2000 through to the end of 2014. All prices normalised to start at 1.

2. Capital shortfall and leverage

If d_{it} and w_{it} are the debt and equity of firm i at time t respectively and κ is the prudential requirement, then capital shortfall at time t is

$$\kappa(d_{it} + w_{it}) - w_{it} = \kappa d_{it} - (1-\kappa)w_{it} = \kappa d_{it}\left(1 - \mathrm{e}^{-\ell_{it}}\right) \ , \tag{1}$$

where

$$\ell_{it} \equiv \ln \frac{\kappa d_{it}}{(1-\kappa) w_{it}} = \ln \frac{d_{it}}{w_{it}} + \mathrm{lgt}(\kappa) \ , \qquad \mathrm{lgt}(\kappa) \equiv \ln \frac{\kappa}{1-\kappa} \ .$$

The quantity ℓ_{it} is the adjusted log–leverage of firm i at time t and $\ell_{it} > 0$ implies capital shortfall in (1) is positive. The parameter κ is the proportion of assets $d_{it} + w_{it}$ excluded from capital calculations, and higher κ leads to higher capital shortfall. Assume[1] $\kappa = 0.08$ under Basel II implying $\mathrm{lgt}(\kappa) = -2.44$ and ℓ_{it} is the actual log–leverage minus 2.44. The firm is in a Basel breach if actual log leverage is above 2.44, otherwise it is Basel compliant.

Figure 2 displays Basel log–leverages (ℓ_{it} for $\kappa = 0.08$) for the four major and four minor Australian banks listed in Table 1 on the first trading day of each month from January 2003 through to December 2014. Note most banks are Basel compliant up to about 2008, entering into Basel breaches from 2009 as a result of the global financial crisis. Prolonged Basel breaches are experienced for particular banks, notably NAB.

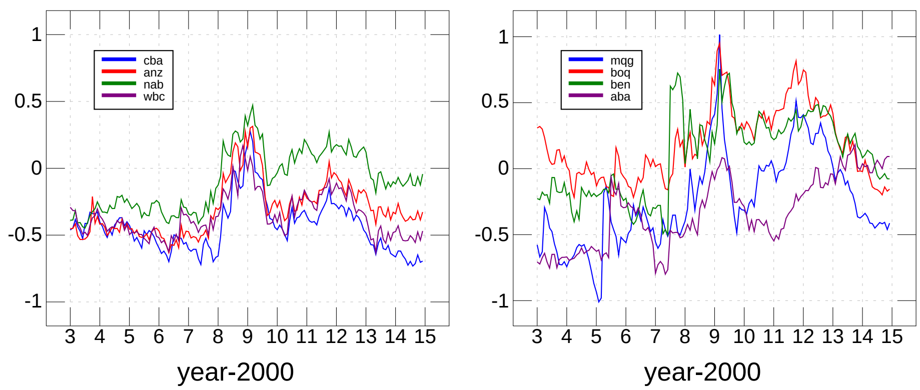

Figure 2: Adjusted or Basel log–leverages for four major (left panel) and four minor (right panel) Australian banks from the beginning of 2003 through to end of 2014. All banks contravene the threshold (corresponding to the zero horizontal) around the start of 2009.

3. Future capital shortfall

Financial institutions and regulators are concerned with future Basel compliance and the likelihood of a future Basel breach. Future compliance depends

[1]See for example Brownlees and Engle (2015). The value of κ can and is varied to test for sensitivity etc.

on the future return on equity. Consider firm i at time $t+h$ where $h>0$. Then[2]

$$\ell_{i,t+h} = \ln \frac{d_{i,t+h}}{w_{i,t+h}} + \mathrm{lgt}(\kappa) = \ell_{it} - \nu_{it} \ , \qquad \nu_{it} \equiv \ln \frac{w_{i,t+h}}{w_{it}} \ ,$$

where it is assumed $d_{i,t+h} = d_{it}$. Here ν_{it} is the log return on equity over $(t, t+h)$ and decreases adjusted log–leverage by the same amount. The future return is unknown at time t but its probability distribution is assumed to be well understood.

The capital shortfall at time $t+h$, ignoring the surplus portion, is,

$$\kappa d_{it} \left|1 - \mathrm{e}^{-\ell_{i,t+h}}\right|^{+} = \kappa d_{it} \left|1 - \mathrm{e}^{\nu_{it} - \ell_{it}}\right|^{+} \ , \qquad |x|^{+} \equiv \max(0, x) \ . \tag{2}$$

Thus shortfall (2) is a κd_{it} multiplied by a put on the return $\mathrm{e}^{\nu_{it}-\ell_{it}}$ with strike 1. The Basel limit is reached at $t+h$ if $\nu_{it} = \ell_{it}$ and breached if $\nu_{it} < \ell_{it}$. A breach is unlikely if ℓ_{it} is low i.e. either the log–leverage or prudential requirement κ is low. This paper models future capital shortfall using log return on equity ν_{it} as it is typically available publicly. An alternative approach models future asset values $d_{i,t+h} + w_{i,t+h}$ in turn implying equity returns, and is briefly discussed in §16. Results in this paper generally hold for any modeling approach.

If there is a breach, $\nu_{it} < \ell_{it}$, then the amount in (2) makes up for the shortfall. This amount may be cold comfort to regulators as it leaves the firm in a precarious position, teetering on the edge of non–compliance. Firms generally maintain a positive capital surplus to avoid breaches when unexpected shocks occur.

Default put options similar to (2) have been discussed in the insurance literature as critical to an evaluation of a firm: see for example Merton (1977), Doherty and Garven (1986), Cummins (1988), Myers and Read (2001) and Sherris (2006).

To illustrate the setup in (2), Figure 11 displays two snapshot outputs from projections generated as described in Appendix A: 6000 simulated one month ahead projected returns for CBA and the market, on two dates: the first trading day in January 2009 and the first trading day in December 2014. Both panels use the same horizontal and vertical scales. Thus based on the available data at those two points in time, policy makers and regulators faced entirely different projections based on the same time series model. The scatter of dots are the forward simulations. Note there is no hindsight bias as the model at each time point is based on the available data to that point in time. The red horizontal line in each panel indicates the Basel limit: any firm return less than the limit would constitute a Basel breach. In the left panel there is little confidence that the Basel limit will be reached since it requires a return on equity approaching 20% – with confidence less than xxx the Basel standard will be breached. In the right panel there is no chance of a Basel breach under any of the projected return on equity view – the bank is "safe."

[2] In the further development e^{ν} is the continuously compounded return (including principal) and ν the rate of return or log return.

The black dots in each panel indicate the actual outcome after one month. In the left panel the outcome is a decline in both the market and CBA stock price. In the right panel the returns are xxx and xxx. Thus at the beginning of January 2009 CBA was projected to be far from compliance in a month while in December 2014 the one month projection is of complete compliance.

The two panels display very different volatility and slopes. In January 2009 both CBA and market return distributions are highly volatile and correlated. The correlation remains strong in December 2014 but with less return volatility. These snapshots are used to compute background and systemic stresses shown in subsequent sections.

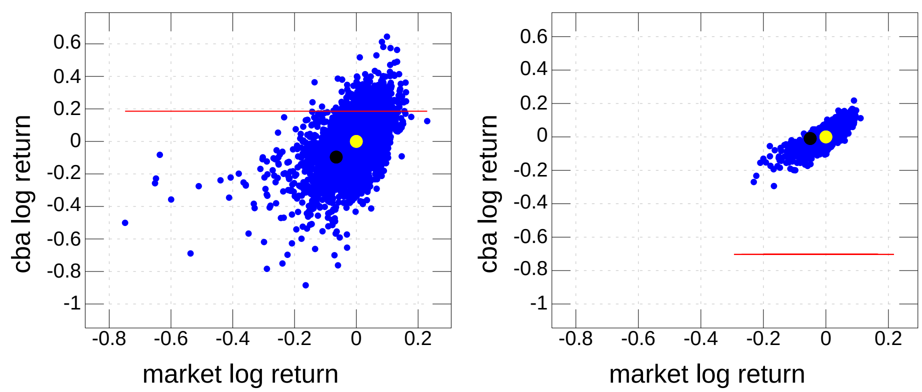

Figure 3: Forecast bivariate distribution of one month forward rates of return for CBA (vertical axis) and the market rate of return (horizontal axis) at start of January 2009 (left panel) and December 2014 (right panel). Note scales in both panels are the same, the relatively large volatility in the left panel, and the left skew in the marginal distributions. Yellow and black dots in each panel indicate origin (0,0) and actual forward rate of return, respectively. Red lines indicate Basel limit – any firm return below the limit indicates a Basel breach given the bank leverage on the applicable date.

4. Systemic risk – SRISK

Brownlees and Engle (2015) defines systemic risk (SRISK) for a group of firms at time t as

$$\mathrm{SRISK}_t \equiv \kappa \sum_i d_{it} \left|1 - \mathrm{E}_\phi(\mathrm{e}^{\nu_{it} - \ell_{it}})\right|^+ \ , \tag{3}$$

where E_ϕ denotes expectation given a major general market downturn. Hence the expected capital shortfall (allowing surplus offset) is computed from (1) assuming a market downturn and added across all firms. Firms having a expected capital surplus are ignored. The systemic risk of firm i at time t is its proportionate contribution to (3):

$$\text{SRISK}_{it} \equiv \pi_{it} \frac{|1 - \mathrm{E}_\phi(\mathrm{e}^{\nu_{it}-\ell_{it}})|^+}{\mathcal{E}_d\{|1 - \mathrm{E}_\phi(\mathrm{e}^{\nu_{it}-\ell_{it}})|^+\}} \ , \qquad \pi_{it} \equiv \frac{d_{it}}{d_t} \ , \qquad d_t \equiv \sum_i d_{it} \ , \quad (4)$$

where $\mathcal{E}_d$ denotes weighted averaging across firms i using weights π_{it}. Large SRISK_{it} indicates firm i is systemically important: it holds a high proportion of the total debt, and it is likely to heavily breach the Basel capital requirement compared to remaining other firms. Expression (4) depends on κ through each of the ℓ_{it}.

Given a general market downturn, put values $|1 - \mathrm{E}_\phi(\mathrm{e}^{\nu_{it}-\ell_{it}})|^+$ in (4) are known at time t: there is no uncertainty except for possible uncertainty in estimating the conditional expected value.

Using the Jensen's inequality,

$$\left|1 - \mathrm{E}_\phi(\mathrm{e}^{\nu_{it}-\ell_{it}})\right|^+ \leq \mathrm{E}_\phi\left(\left|1 - \mathrm{e}^{\nu_{it}-\ell_{it}}\right|^+\right) \ ,$$

with equality if $\nu_{it} - \ell_{it}$ is concentrated on the negative axis. If $\nu_{it} > \ell_{it}$ has positive probability then the inequality is strict. Further SRISK_{it} in (4) is zero whenever

$$\mathrm{E}_\phi(\mathrm{e}^{\nu_{it}}) \geq \mathrm{e}^{\ell_{it}} \ .$$

This inequality typically holds even if the expectation conditions on an extreme market downturn. The above two inequalities suggest SRISK is not risk sensitive and leads to refinements discussed in the next section.

5. Improved systemic risk measurement

Following from (2) define[3] the Basel put payoff and default probability of firm i at time t respectively as

$$p_{it} \equiv \left|1 - \mathrm{e}^{\nu_{it}-\ell_{it}}\right|^+ \ , \qquad q_{it} \equiv \mathrm{P}(\nu_{it} \leq \ell_{it}) = \frac{\mathrm{E}(p_{it})}{\mathrm{E}(p_{it}|\nu_{it} \leq \ell_{it})} \ , \qquad (5)$$

where E denotes expectation without any imposed stress. Note $0 \leq p_{it} \leq 1$ with $p_{it} \to 0$ or 1 as $\nu_{it} \to \pm\infty$. Further q_{it} increases as the distribution of ν_{it} moves to the left. Both $\mathrm{E}(p_{it})$ and q_{it} measure default risk between 0 and 1, with the former reflecting the conditional distribution of ν_{it} below ℓ_{it} and the latter only concerned with the probability mass in the same area.

The monetary Basel put value for firm i is the total expected payout

$$\kappa d_{it}\mathrm{E}(p_{it}) = \kappa d_{it} q_{it} \mathrm{E}(p_{it}|\nu_{it} \leq \ell_{it}) \ ,$$

[3]Alternative terminology is the per unit capital shortfall put and the per unit expected capital shortfall. For shortness we prefer the adjective Basel. Note "risk" here is not a probability but rather a measure of expected capital shortfall.

the expected cost of insuring firm i against a Basel breach. The expression on the right is the product of four terms: the capital requirement κ, firm debt d_{it}, probability of Basel default q_{it}, and the expected payout if a default occurs (against a standardised exercise price of 1). In practice it may be appropriate to discount p_{it} in (5) by the interest rate over the period $(t, t+h)$ and value the put with risk neutral rather than normal expectation.

The put p_{it} is tradable since, given κ, it relies only on the present leverage and the future return ν_{it} on equity w_{it}. Market participants can value the put in the same way as any other put and use the contract to diversify risk. Firms can buy puts in the market to hedge their Basel default risk, or contribute an amount equal to the put value into a bailout pool.

Regulators are inclined to value the puts p_{it} using a stressed, rather than risk neutral, expectation. Stressed expectations correspond closely to conditional expectation given a system wide shock. Regulators are concerned with the possibility and extent of Basel breaches if such shocks occur and have an interest in monitoring stressed expectations across firms and time. In the extreme regulators can charge firms all or a proportion of their stressed put values if there are implicit government bailout guarantees.

6. Stressed expectation

In the above development E is unstressed expectation and does not assume a stressful scenario. A stressed expectation, denoted E_ϕ, is a linear combination of a range of conditional expectations each assuming some level of stress:

$$\mathrm{E}_\phi(p_{it}) \equiv \mathrm{E}\{\phi \mathrm{E}(p_{it}|\phi)\} = \mathrm{E}(\phi p_{it}) = \mathrm{E}(p_{it}) + \mathrm{cov}\{\mathrm{E}(p_{it}|\phi), \phi\} \ . \qquad (6)$$

Here $\phi \geq 0$ with $\mathrm{E}(\phi) = 1$ is a random stress factor or "stressor," with different values re-weighting, and corresponding to, scenarios impacting p_{it}. Thus stress ϕ increases the expected value $\mathrm{E}(p_{it})$ by the covariance between the conditional expectation $\mathrm{E}(p_{it}|\phi)$ and the stressor ϕ. A larger covariance results if higher values of ϕ, corresponding to increasingly stressed scenarios, lead to larger put values. The covariance is zero if p_{it} does not vary with the different scenarios indicated by ϕ, or under the trivial case $\phi = 1$ where scenarios are not re-weighted. The impact of ϕ is formalised in (7) below. Examples of ϕ implying various stress calculations are shown in subsequent sections. Weighted expectations in the form of (6) are discussed in Choo and De Jong (2010) and Furman and Zitikis (2008).

The first equality in (6) follows from the definition of conditional probability (Whittle, 2000). The second equality follows from

$$\mathrm{cov}(p_{it}, \phi) = \mathrm{cov}\{\mathrm{E}(p_{it}|\phi), \phi\} \ .$$

The stressed expectation (6) is the mean of p_{it} using a stressed density

$$\int \phi f(p_{it}, \phi) \mathrm{d}\phi = \mathrm{E}(\phi|p_{it}) f(p_{it}) \ , \qquad \int\int \phi f(p_{it}, \phi) \mathrm{d}\phi \mathrm{d}p_{it} = \mathrm{E}(\phi) = 1 \ \ , \ (7)$$

where f denotes the density of the quantity in parentheses. Hence the likelihood of any outcome p_{it} such that $\mathrm{E}(\phi|p_{it})$ is large is amplified and vice versa. To a Bayesian the density in the left expression of (7) is a posterior density given a prior $f(p_{it})$ and "likelihood" $\mathrm{E}(\phi|p_{it})$: more importance is given to outcomes p_{it} where ϕ is expected to be large.

7. Background stress versus systemic stress

The right hand side of (6) splits the stressed expectation $\mathrm{E}_\phi(p_{it})$ into two components:

$$\mathrm{E}_\phi(p_{it}) = \mu_{it} + s_{it} \ , \qquad \mu_{it} \equiv \mathrm{E}(p_{it}) = q_{it}\mathrm{E}(p_{it}|\nu_{it} \le \ell_{it}) \ , \tag{8}$$

$$s_{it} \equiv \mathrm{cov}\{\mathrm{E}(p_{it}|\phi), \phi\} \ ,$$

called the background stress and systemic stress, respectively, in firm i at time t. Both μ_{it} and s_{it} are computed from the current adjusted log–leverage and the forward return distribution at time t. In addition $0 \le \mu_{it}, \mu_{it} + s_{it} \le 1$ and $s_{it} \ge 0$ typically holds unless firm i benefits from a stress in the system. Applying a Taylor series expansion of the exponential function on p_{it} yields

$$\mathrm{E}(p_{it}|\nu_{it} \le \ell_{it}) \approx \ell_{it} - \mathrm{E}(\nu_{it}|\nu_{it} \le \ell_{it}) \ ,$$

and hence background stress decomposes into

$$\mu_{it} \approx q_{it}\left\{\ell_{it} - \mathrm{E}(\nu_{it}|\nu_{it} \le \ell_{it})\right\} \ ,$$

which is increasing in the Basel default probability q_{it}, adjusted log–leverage ℓ_{it} and the left skewness of ν_{it} as measured by the lower tail conditional expectation. If $\ell_{it} = 0$ and then firm i is at Basel default at time $t+h$ if $\nu_{it} < 0$ with probability $q_{it} \approx 1/2$. As ℓ_{it} increases, one month ahead Basel default becomes more certain since $\nu_{it} \le \ell_{it}$ becomes more certain and q_{it} increases to 1. Background stress is unrelated to the stress factor ϕ and based on the unstressed distribution of the forward return ν_{it}: the current view of the future progression of returns including their mean and volatility.

Similarly decompose systemic stress

$$s_{it} = q_{\phi it}\mathrm{E}_\phi(p_{it}|\nu_{it} \le \ell_{it}) - q_{it}\mathrm{E}(p_{it}|\nu_{it} \le \ell_{it})$$

where $q_{\phi it}$ is the default probability implied from the stressed density in (7). The stress typically increases both the default probability $q_{\phi it}$ and the conditional expected default $\mathrm{E}_\phi(p_{it}|\nu_{it} \le \ell_{it})$ relative to the unstressed quantities q_{it} and $\mathrm{E}(p_{it}|\nu_{it} \le \ell_{it})$. Greater sensitivity of firm i to market wide stresses leads to higher systemic stress s_{it}.

Monitoring μ_{it} across the market provides a leading indication of the level of background stress and anticipates buildups such as shortly before the 2008 global financial crisis. The aggregation of put values and stresses across the market is discussed and illustrated below. An increase in μ_{it} indicates either higher probabilities of defaults or larger capital shortfalls if defaults occur. Introducing systemic stress s_{it} increases stress-sensitivity in two ways. Firstly adverse scenarios are emphasized which would otherwise be less obvious in μ_{it}. Secondly, as ϕ is a market based stressor, system wide defaults and shortfalls resulting from market dependencies are emphasized and revealed.

8. Systemic beta's

The second, systemic stress component s_{it} in (8) captures the financial impact of a hypothetical system wide financial shock. The stress ϕ is systemic as it is expected to affect all firms i. Further s_{it} is the estimated increase in put values on account of ϕ. Examples of stress factors are given in §10 and §15.

Writing σ_ϕ as the standard deviation of ϕ, define the systemic beta of firm i with respect to stress ϕ as

$$\beta_{it} \equiv \frac{s_{it}}{\sigma_\phi} = \sigma_{it}\mathrm{cor}(p_{it}, \phi) \tag{9}$$

where σ_{it} is the standard deviation of p_{it}. Thus β_{it} is the systemic stress component in (6) standardised by σ_ϕ and is composed of the variability in the put payoff and its correlation with the stressor. In addition

$$\mathrm{E}_\phi(p_{it}) = \mu_{it} + \beta_{it}\sigma_\phi \ , \qquad \mathrm{E}(p_{it}|\phi) \approx \mu_{it} + \beta_{it}\frac{\phi - 1}{\sigma_\phi} \ . \tag{10}$$

The second, approximate, relationship is suggested from $\mathrm{cov}(p_{it}, \phi)/\sigma_\phi^2$ analogous to the regression coefficient of p_{it} on ϕ.

The expressions in (10) suggests two ways of thinking about the systemic beta's β_{it}. First, the left hand side equation shows systemic stress β_{it} serves to shift the mean $\mathrm{E}(p_{it})$ of put prices by, on average, σ_ϕ. The stress from ϕ is captured with $\sigma_\phi\beta_{it}$, the second term on the right of the first expression. This term is called systemic stress and represents the addition to expected put prices if stress ϕ materialises.

Systemic beta's are stated in terms of additions to the normal put price $\mathrm{E}(p_{it})$. Thus in an increasingly dire financial situation $\mathrm{E}(p_{it})$ will increase. However β_{it} and the systemic stress measures the change in current put values under further ϕ–stress. This varies from Brownlees and Engle (2015) where SRISK_i does not distinguish between the current, possibly high, put prices $\mathrm{E}(p_{it})$ and the potential increment due to potential stress. Thus in an increasingly dire financial situation put prices are liable to increase on account of decreasing expected returns and increasing volatility. However our definition of systemic stress using systemic beta's measures the further effect on account of potential extra stress imposed onto the system over and above the already existing stress. Systemic beta's capture "marginal" effects.

The second interpretation is based on the right hand side approximation in (10) showing the change in the expected Basel put price if σ_ϕ units of stress ϕ are applied. The quantity $(\phi - 1)/\sigma_\phi$ is thought of as a "stress factor" and is akin to the z–score of a normal random variable. The stress factor has mean 0 and standard deviation 1 and is scaled by β_{it} to yield the actual stress impact on p_{it}. Thus β_{it} is thought of a usual finance type "beta" with respect to ϕ. Stress is measured in standardised units. Units of stress take on different meaning depending on ϕ as discussed below. The distribution of ϕ determines the distribution of the stress factor. If the stress factor is normally distributed then a stress effect more than doubling of the put value occurs less than about 2.5% of the time.

The monetary stress of a firm is $d_{it}\beta_{it}$ and represents the change in the monetary value of the Basel put if stress, as captured with ϕ, is applied.

9. Estimating background and systemic stress via simulation

The following computes background Stress μ_{it} and systemic stress s_{it} using simulation. Suppose $(\phi^\omega, p_{it}^\omega)$ are pairs of stress and put values generated from a model: for example ϕ^ω may be zero unless there is a market drop below the α–percentile in which case it is $1/\alpha$. Then

$$\frac{1}{N}\sum_{\omega=1}^{N} p_{it}^\omega \to \mu_{it} \ , \qquad \frac{1}{N}\sum_{\omega=1}^{N} (\phi^\omega - 1) p_{it}^\omega \to s_{it} \ , \qquad N \to \infty \ , \tag{11}$$

where N is the simulation effort. The approximation is increasingly accurate as N becomes large.

10. Stress based on market return

The following formulates stresses as functions of the market return ν_{mt} from time t to $t+h$. The examples generalise and fine–tune the approach of Brownlees and Engle (2015).

10.1. Market return below a percentile threshold

Suppose the stress event is defined as a market return ν_{mt} in the bottom α–tail of the distribution. Then $\phi(u) = 1/\alpha$ for $u < \alpha$ and 0 otherwise (outcomes where ν_{mt} is better than the bottom α–tail is ignored), and

$$\mathrm{E}_\phi(p_{it}) = \frac{1}{\alpha}\int_0^\alpha \mathrm{E}(p_{it}|u_{mt} = u)\mathrm{d}u = \mathrm{E}(p_{it}|u_{mt} < \alpha) = \mathrm{E}(p_{it}|\nu_{mt} < \tau_t) \ ,$$

where u_{mt} is the percentile rank of ν_{mt} and τ_t cuts out α probability in the lower tail of the ν_{mt} distribution: $\mathrm{P}(\nu_{mt} < \tau_t) = \alpha$. In addition background and systemic stresses are calculated from

$$\mu_{it} \approx \frac{1}{N}\sum_{\omega} p_{it}^\omega \ , \qquad \mu_{it} + s_{it} \approx \frac{1}{N/\alpha}\sum_{u_{mt}^\omega < \alpha} p_{it}^\omega \ .$$

The approximations decrease with the simulation effort N. The effective simulation effort for $\mu_{it}+s_{it}$ or s_{it} is N/α and hence small α small requires a large effort.

The threshold τ_t in the stress calculations is $F_{mt}^{-}(\alpha)$ where F_{mt}^{-} is the inverse distribution function of ν_{mt}. The threshold varies across time depending on market volatility, approximately $\sqrt{h}\sigma_{mt}$ where σ_{mt} is current market volatility. Regulators and practitioners are well versed in working with VaR type calculations and hence a varying VaR type cutoff combining market volatility and stress is closely aligned to current practice. In comparison Brownlees and Engle (2015) uses fixed threshold which may be over or under extreme depending on prevailing market volatility.

10.2. Expected worst market return in n identical scenarios

If $\phi(u)=n(1-u)^{n-1}$ then $\phi(u)\mathrm{d}u=\mathrm{d}\{1-(1-u)^n\}$ and

$$\mathrm{E}_\phi(p_{it})\equiv\mathrm{E}\{\phi\mathrm{E}(p_{it}|\phi)\}=\int_0^1\mathrm{E}(p_{it}|u_{mt}=u)\mathrm{d}\{1-(1-u)^n\}\ .$$

If u is the market return percentile then $1-(1-u)^n$ is the distribution of the worst percentile outcome in n identical trials and hence the stressed expectation is that of the expected put price given the worst percentile market return in n identical trials. Further

$$s_{it}\approx\frac{1}{N}\sum_{\omega}\{n(1-u_{mt}^{\omega})^{n-1}-1\}p_{it}^{\omega}\ .$$

The effective simulation effort is N/n. Simulated returns ν_{mt}^{ω} in the upper tail of the distribution have percentiles $u_{mt}^{\omega}\approx 1$ and hence for these market returns $(1-u_{mt}^{\omega})^{n-1}$ is negligable and the associated simulated put p_{it}^{ω} is heavily downweighed.

Note the contrast with the previous example where the bottom α proportion of simulated market returns are selected as the stressed sample. With the current specification for ϕ, every simulated put contributes to the stress computation, albeit with different weights.

10.3. Expected worst market return given a tail event

The above two situations can be combined. Suppose $\phi(u)=c(\alpha-u)^{n-1}$ for $u\leq\alpha$ and 0 otherwise and where c is such that $\phi(u)$ integrates to 1. Then

$$\mu_{it}+s_{it}\approx\frac{1}{N/c}\sum_{u_{mt}^{\omega}<\alpha}(\alpha-u_{mt}^{\omega})^{n-1}\,p_{it}^{\omega}\ .$$

Similar to the first example, the stressed sample picks up the bottom α of market returns. The bottom α of market returns is further stressed by progressively overweighing returns as the percentile approaches 0. Large simulation effort N is required for a reasonable approximation since N/c is the effective simulation size and c is small.

11. Simulating future capital shortfall and market return

The estimation of background stresses and systemic stresses and beta's requires projections of future capital shortfalls. As in Brownlees and Engle (2015), projections in this paper are constructed using time series models of forward rates of return ν_{it} for firm i and ν_{mt} for the market. The market return is used as the stressor with different choices of the stress function ϕ modelling different stress scenarios.

The time series models used are stochastic volatility models based on the GARCH–DCC model of Engle (2002) summarised in Appendix A. The GARCH-DCC model captures prolonged periods of high volatility and correlation in firm and market returns, typical in financial markets. The GARCH–DCC framework is only one possible implementation. For example future return scenarios may be constructed in a more ad–hoc manner e.g. judiciously constructed scenarios by regulators or policymakers. The systemic beta framework can be based on any generated future scenarios.

An earlier illustration shown in Figure 11 is generated from the GARCH–DCC model. In the left panel representing one month forward returns from December 2008, low market returns are likely to lead to sharply lower CBA bank returns, as compared to the right panel showing the same as at January 2014. This is suggested by the slope of the scatter plots: the left slope appears steeper than the right. This implies higher systemic stress in December 2008. In addition CBA and market return volatilities are much higher in the left panel compared to the right, indicating higher background stress in December 2008. Also note that with a stressor function ϕ defined in terms of market return percentiles, the magnitude of a market downturn at a fixed percentile threshold is more significant in the left panel.

12. One month ahead forecast stresses in individual banks

Figure 12 displays, in the top panels, the estimated q_{it} one month ahead default probabilities for each major (left) and minor (right) Australian banks based on the projected one month ahead return distribution fitted using the GARCH–DCC model. The first inclinations of Basel default arose with NAB in early 2008 followed one month later by ANZ, and a further few months later by WBC and CBA. The probabilities subsided shortly after 2009, however NAB was close to default again in 2011 and this sustained till 2013. The volatility in default probabilities is more pronounced for smaller banks. Again note the return distributions are constructed from data only available at that time and hence are not affected by hindsight bias.

The middle two panels in Figure 12 show estimated background stress μ_{it} using the same projected return distributions. A similar pattern as default probabilities is observed, and NAB appears to have the highest put value or background stress per dollar of debt. Smaller banks are subject to higher stress in general after normalising for debt levels. Background stress levels are high across all banks around 2008 during the global financial crisis and less so between 2011 and 2013 (mainly only from NAB).

Bottom panels in Figure 12 show systemic stress s_{it} computed by assuming $\phi(u) = 12u^{11}$: the worst market return in 12 identical months. Systemic stress for each bank generally exhibits similar patterns as background stress μ_{it}. However, importantly, some banks have differing patterns which is an important observation for the regulator since systemic stress indicates sensitivity to market-wide downturns. For example ANZ had similar systemic stress as NAB around 2012 but lower background stress. Hence although ANZ was not obviously in stress during 2012, it would be if a market downturn occurred. Bendigo and BOQ had high background stress levels after 2008, but are overtaken by Macquarie in terms of systemic risk: Macquarie is more likely to suffer in a market downturn whereas Bendigo and BOQ are less likely to be impacted.

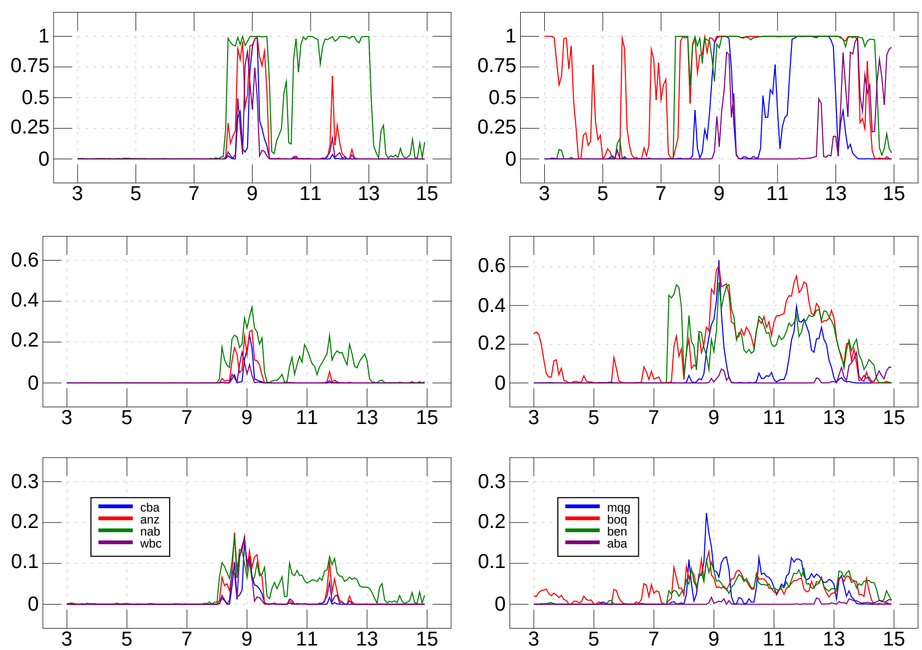

Figure 4: Forecast one month ahead Basel default probability (top panels), background stress (middle panels) and systemic stress (bottom panels) for four major (left panel) and four minor (right panel) banks from the beginning of 2003 through to end of 2014. Note different scale on bottom two rows of panels.

13. Overall market stress

There are two ways of aggregating stress in the financial market as a whole. The first is to apply debt weighted averaging to individual firm stresses, yielding

$$\overline{p}_t \equiv \mathcal{E}_d(p_{it}) \ , \qquad \mathrm{E}_\phi(\overline{p}_t) = \mathcal{E}_d\{\mathrm{E}_\phi(p_{it})\} = \mathcal{E}_d(\mu_{it} + s_{it}) \equiv \overline{\mu}_t + \overline{s}_t \ , \tag{12}$$

where $\overline{\mu}_t = \mathcal{E}_d(\mu_{it})$ and $\overline{s}_t = \mathcal{E}_d(s_{it})$ are overall background and systemic stresses, respectively. The put $\overline{p}_t$ in (12) pays out zero if all firms are Basel compliant with payouts increasing with the number and size of Basel breaches. The relation in (12) follow since E_ϕ and $\mathcal{E}_d$ are linear.

The monetary value of overall background and systemic stresses at time t are, respectively,

$$\kappa d_t \overline{\mu}_t = \kappa d_t \mathcal{E}_d(\mu_{it}) = \kappa \sum_i d_{it}\mu_{it} \ , \qquad \kappa d_t \overline{s}_t = \kappa \sum_i d_{it} s_{it} \ .$$

Percentage contributions of firm i to background and systemic stresses are

$$\frac{\pi_{it}\mu_{it}}{\overline{\mu}_t} \equiv \pi_{it}\frac{\mu_{it}}{\mathcal{E}_d(\mu_{it})} \ , \qquad \frac{\pi_{it}s_{it}}{\overline{s}_t} \equiv \pi_{it}\frac{s_{it}}{\mathcal{E}_d(s_{it})} \ .$$

where $\pi_{it} \equiv d_{it}/d_t$ as before is the proportion of total debt held by firm i. Firm i contributes to a larger portion of overall stress if it holds a large amount of debt and its returns are subject to high downside risk, particularly in a market downturn (for s_{it}). Contributions add to 1 across firms. A ratio such as $s_{it}/\overline{s}_t$, without reflecting firm size, compares the stress in firm i to average stress but does not signal the importance of that firm to the system as a whole.

14. Overall stress after diversification

The debt weighed average system put $\overline{p}_t \equiv \mathcal{E}_d(p_{it})$ is an average put and does not permit diversification – the sharing of debt and equity across firms. Sharing may not be possible in practice but the concept is useful for assessing the resilience of the system as a whole. This leads to the second way of aggregating stresses across firms.

Write d_t and w_t as total debt and equity in the market, respectively:

$$d_t \equiv \sum_i d_{it} \ , \qquad w_{it} \equiv \sum_i w_{it} \ .$$

Then the adjusted log–leverage for the market overall is

$$\ell_t \equiv \ln \frac{d_t}{w_t} + \mathrm{lgt}(\kappa) = \ln \sum_i \frac{w_{it}}{w_t}\left(\frac{d_{it}}{w_{it}} \times \frac{\kappa}{1-\kappa}\right) = \ln \mathcal{E}_w(\mathrm{e}^{\ell_{it}}) \ ,$$

where $\mathcal{E}_w$ denotes equity weighted averaging. A system wide breach occurs at $t+h$ if $\nu_t < \ell_t$ where ν_t is the forward rate of return on total equity w_t:

$$\nu_t = \ln \frac{w_{t+h}}{w_t} = \ln \mathcal{E}_w\left(\frac{w_{i,t+h}}{w_{it}}\right) = \ln \mathcal{E}_w(\mathrm{e}^{\nu_{it}}) \ . \tag{13}$$

In terms of this notation the market Basel put and Basel risk are defined similar to (5)

$$p_t \equiv \left|1 - \mathrm{e}^{\nu_t - \ell_t}\right|^+ \leq \overline{p}_t \ , \qquad q_t \equiv \frac{\mathrm{E}(p_t)}{\mathrm{E}(p_t|\nu_t < \ell_t)} = \mathrm{P}(\nu_t \leq \ell_t) \ .$$

The inequality is proved in Appendix C. The put p_t pays 0 at time $t+h$ unless the sector as whole is in Basel default, occurring if $\nu_t < \ell_t$. Moving from $\overline{p}_t$ to p_t allows for diversification: low liquidity in one firm is offset by high liquidity in other firms.

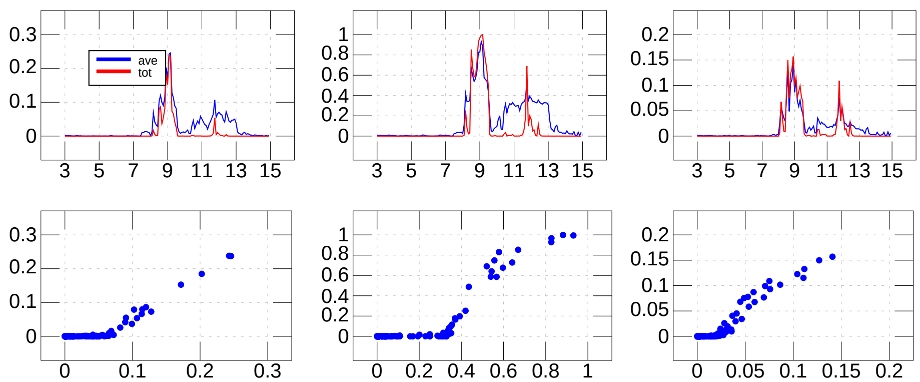

Figure 5: Average stresses and total stresses. Stress measurements are, left to right, μ, q and s, respectively. The top row of panels measure for example (top left panel) $\overline{\mu}_t$ and μ_t over time. In the bottom row of panels specific stresses are measured are against each other: for example μ_t (vertical axis) versus $\overline{\mu}_t$ (horizontal axis). In all cases there is no total stress until there is an appreciable level of average stress. Systemic stress $\overline{s}_t$ appears to be the least diversifiable.

Similar to (10), total stress is decomposed as

$$\mathrm{E}_\phi(p_t) = \mu_t + s_t \ , \qquad q_t = \mathrm{P}(\nu_t < \ell_t) \ , \qquad s_t \equiv \mathrm{cov}\{\mathrm{E}(p_t|\phi), \phi\} \ . \tag{14}$$

termed the diversified overall background and systemic stress, respectively. Further q_t is the probability of a market Basel default with the second factor the expected size of the default if one occurs.

System resilience aims to answer the question of whether the system as a whole can absorb shocks. The capacity to absorb relies on implicit merging. When two firms merge the leverage of the resulting firm is less than that of the more highly leveraged firm. The merged firm is more able to withstand return on equity shocks unless negative shocks are more prone for the merged firm. Similarly when many firms merge Basel puts for the conglomerate become less valuable.

Firms do not necessarily merge, even in dire financial circumstances. Hence the mergers spoken of here are hypothetical. From a regulators perspective, what would happen under a merger of all firms (or perhaps a group of firms) is nevertheless of interest. A system as as a whole near Basel breach is more threatening that one where a few firms are near Basel breach but the system as a whole is strongly Basel compliant.

System compliance is partially monitored with the system put $\overline{p}_t = \mathcal{E}_d(p_{it})$ and its stressed expectation $\mathrm{E}_\phi(\overline{p}_t)$. However both quantities are before diversification and gives no indication of system resilience and as to how much risk can be diversified away. Basing a put on system wide aggregated debt and equity and comparing the same to p_t gives at least some guidance.

Figure 14 compares background and systemic stresses before and after diversification. As expected, stresses reduce after diversification. The diversification is most apparent for small levels of stresses where they are close to zero. The diversification disappears when stresses reach a certain level, with points in bottom panels of Figure 14 following the 45° line. The extent of resilience or diversification over time can also be measured using the ratios $\mu_t/\overline{\mu}_t$ and $(\mu_t + s_t/(\overline{\mu}_t + \overline{s}_t) \le 1$. As diversification or resilience weakens such as during 2008, the ratio increases to 1.

15. Further generalised stress functions

Practical results for the construction of general stress functions ϕ are contained in the next three subsections.

15.1. Stress functions as weighted linear combinations of tail events

Brownlees and Engle (2015) defines a systemic event as a market based downturn greater than a certain threshold and a systemic event depends on the chosen threshold. This arbitrariness is partially sidestepped by choosing a decreasing function $\phi = \phi(u_{mt})$ on the unit interval and noting

$$\mathrm{E}\{\phi \mathrm{E}(p_{it}|\phi)\} = -\int_0^1 v\phi'(v)\mathrm{E}(p_{it}|u_{mt} \le v)\mathrm{d}v \ . \qquad (15)$$

The equivalence of the left and right hand sides of (15) follows from

$$\int_0^1 \phi'(v) \int_0^v \mathrm{E}(p_{it}|u_{mt})\mathrm{d}u_{mt}\mathrm{d}v = \int_0^1 \mathrm{E}(p_{it}|u_{mt}) \int_{u_{mt}}^1 \phi'(v)\mathrm{d}v\mathrm{d}u_{mt} \ ,$$

provided $\phi(1) = 0$.

Thus stressed expectations, stressed with a decreasing function of the market return, is equivalent to taking a weighted average of conditional lower tail expectations. This result is used to circumvent an explicit choice for the market threshold, replacing it with $\phi(u)$, an implicit mixture of thresholds.

15.2. Scenario based stress testing

Stress events captured with ϕ and used in (9) can be defined with respect any events, including a discrete number of scenarios labelled $k = 1, 2, \ldots$. In this case

$$\mathrm{E}_\phi(p_{it}) = \sum_k \pi_k \phi_k \mathrm{E}(p_{it}|k) \ , \qquad \sum_k \pi_k \phi_k = \mathrm{E}(\phi) = 1,$$

where ϕ_k is the weight assigned to scenario k and π_k here is the original probability of scenario k. The $\pi_k \phi_k \geq 0$ are modified probabilities which weigh different scenarios according to level of interest. For example in standard stress testing the ϕ_k are chosen such that significant weight is given to scenarios causing high firm distress. The "real world" probabilities π_k are usually ignored by simply choosing $\pi_k \phi_k$ to be of required magnitude.

In the discrete case s_{it} compares the expected value under an average of distress scenarios to the average without distress. The quantity σ_ϕ is now of limited relevance, signalling the volatility in the distress probabilities.

15.3. Copula stress functions

Stress can be based on a vector of variables x with marginal distributions F and marginal densities f. The vector of percentile ranks corresponding to x is $u = F(x)$. Suppose c is the copula density of x implying the joint density of x is $c(u) \prod_j f(x_j)$. If $\phi(u)$ is a copula stress density where $\mathrm{E}\{\phi(u)\} = 1$ then the stressed expectation of the put is

$$\mathrm{E}_\phi(p_{it}) \equiv \int \mathrm{E}(p_{it}|x)\phi(u)c(u) \prod_j \{f(x_j)\} \mathrm{d}x \ . \tag{16}$$

If $\phi(u) = 1$ then $\mathrm{E}_\phi(p_{it}) = \mathrm{E}(p_{it})$, the ordinary expectation. The density $\phi(u)$ magnifies potentially stressful situation such as where all components of x are distressed (either high or low depending on each component of x).

The copula density stressor $\phi(u)$ can be combined with marginal stressors written as functions of x_j or percentiles u_j. If the latter then $\phi(u) \prod_j \psi_j(u_j)$ is the total stressor which is applied to $\mathrm{E}(p_{it}|x)$ as above.

Copula stress effects are simulated as follows. Suppose x is a vector ν_{mt} of market factors over the period $(t, t+h)$. Simulations of a joint model are used to compute vectors $(p_{it}^\omega, \nu_{mt}^\omega)$, $\omega = 1, \ldots, N$. The ν_{mt}^ω are converted to percentiles u^ω and scalars

$$\phi^\omega \equiv \phi(u^\omega) \prod_j \psi_j(u_j^\omega) \ , \qquad \omega = 1, \ldots, N \ ,$$

are calculated. The systemic stress and beta are then estimated as in (11).

16. Modelling assets to determine capital shortfall

An alternative method to determine capital adequacy is to value assets separately, apply the prudential requirement and subtract debt to arrive at capital shortfall. The capital shortfall at time $t+h$ is then, similar to (1),

$$d_{it} - (1-\kappa)\sum_j \mathrm{e}^{\nu_{ijt}} a_{ijt} = d_{it} - (1-\kappa) a_{it} \mathcal{E}_a(\mathrm{e}^{\nu_{ijt}}) \ , \qquad a_{it} = \sum_j a_{ijt} \ . \quad (17)$$

Here a_{ijt} is the value of firm i's asset j at time t and has forward log–return ν_{ijt}. Further $\mathcal{E}_a$ denotes an asset weighted average using firm i's asset values at time t. The default put is, per unit κd_{it},

$$p_{it} \equiv \left| k^{-1} - \mathrm{e}^{-\mathrm{lgt}\kappa - L_{it}} \mathcal{E}_a(\mathrm{e}^{\nu_{ijt}}) \right|^+ \ , \qquad L_{it} \equiv \ln \frac{d_{it}}{a_{it}} \ . \quad (18)$$

Thus $\mathrm{e}^{L_{it}}$ is the often used alternative leverage definition: dividing debt by total assets. Similar to before $0 \le p_{it} \le 1$ is a measure of stress on firm i at time t with total stressed expectation decomposed as before:

$$\mathrm{E}_\phi(p_{it}) = \mathrm{E}(p_{it}) + \mathrm{cov}\{\mathrm{E}(p_{it}|\phi), \phi\} = \mu_{it} + s_{it} \ ,$$

where μ_{it} is the background stress and s_{it} is the systemic stress, i.e. the result of systemic event modelled with ϕ.

To calculate put values or stresses as before, a sample of N forward returns ν^ω_{ijt} and stress factors ϕ^ω are simulated using an appropriate model such as the GARCH–DCC model. The returns ν^ω_{ijt} are used to derive $\mathcal{E}_a(\mathrm{e}^{\nu^\omega_{ijt}})$ and in turn, using (18), p^ω_{it} which are multiplied by ϕ^ω to arrive at stressed put prices $\phi^\omega p^\omega_{it}$. The calculations converge as $N \to \infty$:

$$\frac{1}{N}\sum_\omega \phi^\omega p^\omega_{it} \to \mathrm{E}_\phi(p_{it}) \ , \qquad \mathcal{E}_a\left(\frac{1}{N}\sum_\omega \phi^\omega \mathrm{e}^{\nu^\omega_{ijt}}\right) \to \mathrm{E}_\phi\{\mathcal{E}_a(\mathrm{e}^{\nu_{ijt}})\} \ . \quad (19)$$

Note that

$$\mathrm{E}_\phi(p_{it}) \ne \left| k^{-1} - \mathrm{e}^{-\mathrm{lgt}\kappa - L_{it}} \mathrm{E}_\phi\{\mathcal{E}_a(\mathrm{e}^{\nu_{ijt}})\} \right|^+ \ .$$

The stressed average return

$$\mathrm{E}_\phi\{\mathcal{E}_a(\mathrm{e}^{\nu_{ijt}})\} = \mathcal{E}_a\{\mathrm{E}_\phi(\mathrm{e}^{\nu_{ijt}})\} \approx \mathcal{E}_a\left(\frac{1}{N}\sum_\omega \phi^\omega \mathrm{e}^{\nu^\omega_{ijt}}\right) \ ,$$

cannot be used directly in the calculation of firm stress.

An easily stressed asset j has a return distribution sensitive to stresses or changes in values of ϕ. Under stress, $\mathrm{e}^{\nu_{ijt}}$ is likely to be much less than 1 thus increasing the capital shortfall. This asset increases the stressed expectation of p_{it} particularly when a_{ijt} contributes to large portion of a_{it}.

17. Background and systemic stress compared to SRISK

The methodology developed and employed in this article departs in three respects from the SRISK methodology set out in Brownlees and Engle (2015). This section examines the force of these differences.

17.1. Put value versus put on expected value

The SRISK methodology described in §4, in particular (4), is based on a put on the expected shortfall:

$$|1 - \mathrm{E}_\phi(\mathrm{e}^{\nu_{it}-\ell_{it}})|^+ \leq \mathrm{E}_\phi(|1 - \mathrm{e}^{\nu_{it}-\ell_{it}}|^+) \equiv \mu_{it} + s_{it} \ , \tag{20}$$

where stressed expectation E_ϕ in Brownlees and Engle (2015) is conditional expectation given a major market downturn. The put is evaluated after the expectation E_ϕ and checks, in essence whether in a significant market downturn, the expected h period ahead return ν_{it} exceeds the adjusted log–leverage. If this holds true there may still be substantial risk of a Basel breach, depending on the volatility of the return. Volatility in ν_{it} is ignored in SRISK_{it} other than through the usual adjustment on account of continuous compounding. In particular increased volatility due to stress is ignored.

With the proposed $\mu_{it} + s_{it}$ where the stressed expectation is performed on the put payoff as opposed to the other way round, return volatility is taken into consideration as for any other puts in general. Other things equal, higher volatility leads to higher put values since breach size is taken into consideration. An explicit decomposition is discussed in §5. In addition the calculation by Brownlees and Engle (2015) has a probability mass at the zero value whereas the proposed calculation yields a continuous range of values unless a breach is highly unlikely.

17.2. Systemic stress versus background stress

Brownlees and Engle (2015) defines systemic risk as the entire left of (20). However a stressed expectation may be high simply on account of put prices already being high before stressing, reflecting higher than normal volatility or leverage. Hence Brownlees and Engle (2015) combines both background stress, due to high volatility or leverage, and the imposed system stress. However it is important to understand the mix of the two different stresses. Figure 6 plots systemic stress versus background stress for the eight Australian banks of Table 1. Generally background stress is higher than systemic stress, although the mix differs between banks and the extent of stress. For example compared to the smaller Australian banks, bigger banks have a larger proportion of systemic stress relative to background stress. In addition Macquarie is more systemically risky amongst the smaller banks.

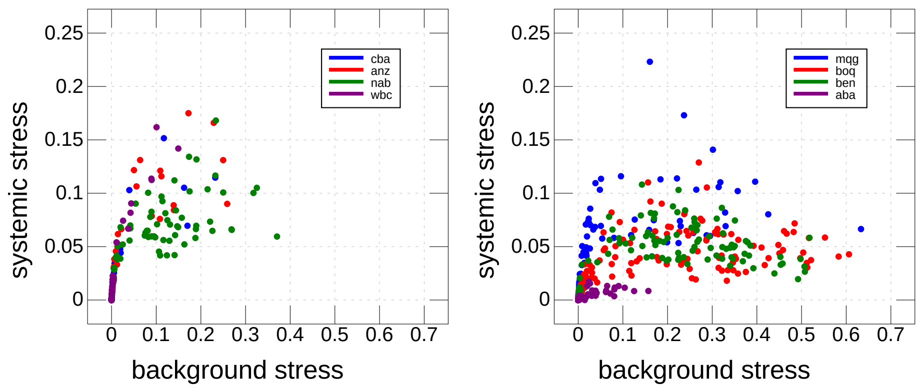

Figure 6: Systemic, s_{it}, versus background μ_{it} for four major banks (left panel) and four minor banks (right panel) for 144 months.

17.3. Flexible stress functions

The further modification made to the SRISK methodology of Brownlees and Engle (2015) is to generalise the "cutoff" functional form for stressing. With the latter, stressed expectations are conditional expectations given a specified cutoff point. For SRISK in Brownlees and Engle (2015) the cutoff is in absolute terms: a 10% fall in the market although this can varied. This cutoff does not allow for market volatility: a 10% reduction is increasingly plausible as volatility increases. In addition SRISK does not make judgement as to the likelihood of the conditioning stress event. Thus SRISK mixes in various sources of risk in an imprecise manner and it is difficult to determine the implications of SRISK readings.

This argues for specifying conditional behaviour in percentile terms relative to the then applicable market conditions, as in done in this paper. After all, current market volatility is known and there is a good reason to disentangle current market conditions from the evaluation of what is likely to happen if further stress arrives in the system.

Stresses are further generalised in this paper with generalized univariate and multivariate forms of ϕ, as well as discrete forms of ϕ which are aligned with the widespread approach of stress testing to measure individual firm and systemic risk by APRA and other prudential regulators.

18. Conclusion

This paper develops an approach to measure systemic risk using the concept of stressed expectations are which akin to stress testing widely used in practice. Systemic stress adds to background stress which is the value of default puts on firms. Fitting time series to Australian bank data identifies banks previously in distress and those with high contribution to systemic risk.

A. GARCH–DCC model

Denote the daily (log) return for firm i at time t as

$$\delta_{it} = \mu_i + \sigma_{it}\epsilon_{it} \ , \qquad \epsilon_{it} \sim (0,1) \ , \tag{A.1}$$

The volatility σ_{it} is modelled as

$$\sigma^2_{i,t+1} = \omega + \sigma^2_{it}\{\beta + (\alpha + \gamma\epsilon^-_{it})\epsilon^2_{it}\} \ , \qquad \epsilon^-_{it} \equiv I(\epsilon_{it} < 0) = I(\delta_{it} < \mu) \ . \tag{A.2}$$

where I denotes the indicator function. Hence the response of $\sigma^2_{i,t+1}$ to ϵ^2_{it} is increased by γ if the rate of return is below the average μ_i, compared to the response if $\delta_{it} > \mu_i$. Equations (A.1) and (A.2) defined a simple threshold GARCH model: called the TARCH(1,1). In (A.1) the mean μ_i does not vary with time t and in (A.2) it is assumed the terms σ^2_{it} and ϵ_{it} in the right hand side of (A.2) are sufficient to structure the dynamics of volatility.

The model defined by (A.1) and (A.2) is estimated for each security i jointly with a similar model for the market, $i = m$. The correlations between securities and the market are modelled using positive definite recursions (Engle, 2002)

$$(Q_{i,t+1} - S) = \alpha(\eta_{it}\eta'_{it} - S) + \beta(Q_{it} - S) \ , \qquad \eta_{it} \equiv (\epsilon_{it}, \epsilon_{mt})'.$$

Correlations ρ_{it} recovered from the Q_{it} are used as the correlation between ϵ_{it} and ϵ_{mt}.

B. Computations

All model fits in this paper have been performed with the R language (R Development Core Team, 2008) and in particular the rmgarch package described by Ghalanos (2012). All other calculations were performed using the J language (Iverson, 2003).

In the forward simulated forward projections the innovations are chosen randomly from past innovations. These past innovations are chosen consistently: at a particular t either all or none of ϵ_{it} are chosen.

C. Proof of system put is less than debt weighted average put

$$1 - \mathrm{e}^{-\ell_{it}} = \frac{\kappa d_{it} - (1-\kappa)w_{it}}{\kappa d_{it}} \ , \qquad \mathcal{E}_d(1 - \mathrm{e}^{-\ell_{it}}) = \frac{\kappa d_t - (1-\kappa)w_t}{\kappa d_t} = 1 - \mathrm{e}^{-\ell_t} \ ,$$

and similarly if $-\ell_{it}$ is replaced by $\nu_{it} - \ell_{it}$. Hence

$$p_t \equiv |1 - \mathrm{e}^{-\ell_t}|^+ \le \mathcal{E}_d(|1 - \mathrm{e}^{-\ell_{it}}|^+) \equiv \overline{p}_t \ .$$

D. Use of more general puts

Consider the put $p_{it}(1+cp_{it}) = p_{it} + cp_{it}^2$ for some constant $c \geq 0$. This put is zero if $p_{it} = 0$ and has slope $1 + 2cp_{it}$ for $p_{it} > 0$ and $p_{it}(1+cp_{it}) > p_{it}$ if the put is positive. Larger slopes c imply greater payoff.

The put $p_{it} + cp_{it}^2$ imposes a higher cost structure on defaults. For every extra dollar of default the cost increases by $1 + 2cp_{it}$. Further the stressed put value is

$$\mathrm{E}(p_{it})\left\{1 + c\mathrm{E}(p_{it})\right\} + c\left\{\mathrm{cov}(p_{it})\right\} + \mathrm{cov}(p_{it}, \phi) + c\left\{\mathrm{cov}(p_{it}^2, \phi)\right\}$$

$$= \mathrm{E}(p_{it})\left\{1 + c\mathrm{E}(p_{it})\right\} + c\left\{\mathrm{cov}(p_{it})\right\} + \mathrm{cov}(p_{it} + cp_{it}^2, \phi) \ .$$

The first three terms in the final expression are unrelated to stress ϕ: stress only impacts the last term.

Can easily determine all terms via simulation.

E. Data sources

(Geoff – can you fill out?)

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