1. Regression dependence model

The regression dependence model is

$$z_t = \alpha_t + \beta q_t + s\epsilon_t \ .$$

Then

$$z_t - z_{t-1} = (\alpha_t - \alpha_{t-1}) + \beta(q_t - q_{t-1}) + s(\epsilon_t - \epsilon_{t-1}) \ ,$$

yielding the variance breakdown at $[t, t+1]$:

$$V_t^z = V_t^r + V_t^\epsilon$$

where V_t^z, V_t^r and V_t^ϵ are total, regression and error sum of squares at $[t, t+1]$, defined as

$$V_t^z \equiv \mathrm{E}\left\{(z_t - z_{t-1})^2\right\} \ , \qquad V_t^\epsilon \equiv \mathrm{E}\left\{(\epsilon_t - \epsilon_{t-1})\right\}^2 \ ,$$

$$V_t^r \equiv \mathrm{E}\left\{(\alpha_t - \alpha_{t-1})^2\right\} + \beta^2(q_t - q_{t-1})^2 + 2\beta(q_t - q_{t-1})\mathrm{E}(\alpha_t - \alpha_{t-1}) \ .$$

A measure of "local dependence" at $[t, t+1]$ is the proportion of total variation explained by regression as opposed to error:

$$\rho_t \equiv \frac{V_t^r}{V_t^z} \ ,$$

which is higher if α_t and q_t "move faster" and "together" relative to ϵ_t.

Questions:

- Does specifying ρ_t for all t lead to a specification of α_t?
- Should s vary with t? Otherwise how do we model the case of say weak lower tail dependence, since s will need to be large for small t?
- Is it possible to fit $z_t = \alpha_t + \beta q_t + s_t\epsilon_t$ where s_t is unknown and varies smoothly, similar to α_t? Maybe s_t varies inversely with α_t.
- Maybe should try fitting the model to data with asymmetric dependence and see if it fits properly.

2. Factor copula model

A factor copula model is

$$z_t = \phi(q_t) + s\epsilon_t$$

where ϕ is a transformation and $\phi(q_t)$ corresponds to $\alpha_t + \beta q_t$ in the regression above.

The first order difference is

$$z_t - z_{t-1} = \phi(q_t) - \phi(q_{t-1}) + (\epsilon_t - \epsilon_{t-1}) \sim \phi'(q_t)(q_t - q_{t-1}) + (\epsilon_t - \epsilon_{t-1}) \ .$$

hence the variance decomposition at $[t, t+1]$ is

$$\mathrm{E}\left\{(z_t - z_{t-1})^2\right\} = \left\{\phi(q_t) - \phi(q_{t-1})\right\}^2 + \mathrm{E}\left\{(\epsilon_t - \epsilon_{t-1})\right\}^2 \ .$$

The issue here is that the marginal distribution of z_t is going to be "out of control." Maybe s also needs to vary with t?

References