

## Monitoring systemic risk

Brownlees and Engle (2015) define systemic risk<sup>1</sup> for a group of firms in terms of the expected future capital requirement  $S_i$  for each firm  $i$  in the group:

$$\text{SRISK} = \sum_i \pi_i \{\mathcal{E}(S_i)\}^+ = D[\{\mathcal{E}(S_i)\}^+] , \quad \pi_i = \frac{d_i}{d} , \quad d = \sum_i d_i , \quad (1)$$

where  $S_i = 1 - e^{r_i - \ell_i}$  is the shortfall for firm  $i$ . Here  $\mathcal{E}$  denotes expectation given a major general market downturn and  $D$  is debt weighted averaging. Further SRISK is expressed in units  $kd$  and  $0 \leq \text{SRISK} \leq 1$  with SRISK near 1 indicating extreme financial distress for all firms. SRISK is based on the expected capital requirement and firms with an expected capital surplus under stress are ignored. The expression (1) slightly modifies the definition of Brownlees and Engle (2015) by dividing their SRISK expression by  $kd$ .

The systemic risk contribution of firm  $i$  is (Brownlees and Engle, 2015)

$$\text{SRISK}_i = \frac{\pi_i \{\mathcal{E}(S_i)\}^+}{\text{SRISK}} . \quad (2)$$

Note  $0 \leq \text{SRISK}_i \leq 1$  with larger values indicating firm  $i$  is systemically important: it holds a high proportion  $\pi_i$  of the total debt, and/or is likely to heavily breach, compared to other firms, the Basel capital requirement under the stress event implicit in the calculation of  $\mathcal{E}$ . Expression (2) depends on  $k$  through each of the adjusted log-leverages  $\ell_i$ .

In this article we propose four modifications to SRISK and  $\text{SRISK}_i$  defined in (1) and (2). Each of these modifications is designed to improve stress monitoring. The proposed modifications to SRISK are shown, theoretically and empirically, to have more sensitivity and specificity. Sensitive, in the sense of more likely detect firms likely to face financial distress. Specific in the sense of minimising false alarms. The modifications are:

1. Replacing  $\{\mathcal{E}(S_i)\}^+$  in (1) and (2) by  $\mathcal{E}(S_i^+) \geq \{\mathcal{E}(S_i)\}^+$ . The justification is that typically  $\mathcal{E}(S_i) = 0$  and hence (1) is not affected by firms that have zero expected capital requirement even though in individual instances that capital requirement may be large. Using  $\mathcal{E}(S_i^+)$  instead of  $\mathcal{E}(S_i)$  implies each firm counts with degree depending on the its proportionate contribution  $\pi_i$  to total debt and the expected value of a put on its shortfall.
2. Using more general definitions of  $\mathcal{E}$  to encompass more flexible stress specifications. For example modelling stress as a 10% market downturn, while pertinent and interesting, is obviously arbitrary. This paper posits more flexible and robust specification of stressed expectation  $\mathcal{E}$ .

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<sup>1</sup>In contrast to Brownlees and Engle (2015) we factor out the scale factor  $kd$  making for a more transparent 0 to 1 scale for SRISK.

3. Baseline risk is defined as  $0 \leq E(S_i^+) \leq 1$ . The stress index is the proportionate increase in baseline risk when stress  $\psi$  is applied:

$$s_i = \frac{\mathcal{E}(S_i^+) - E(S_i^+)}{E(S_i^+)} .$$

Thus  $1 + s_i$  is a risk ratio, comparing the default put option price under stress and baseline conditions.

4. Total diversifiable baseline risk is  $0 \leq E(S^+) \leq 1$  where, as before,  $S = D(S_i)$ , total shortfall per unit  $kd$  where  $d$  is total debt. The diversifiable stress index is

$$s = \frac{\mathcal{E}(S^+) - E(S^+)}{E(S^+)} .$$

5. Total non-diversifiable baseline risk is  $0 \leq E\{D(S_i^+)\} \leq 1$ . The risk is non-diversifiable since shortfall  $S_i > 0$  in one firm cannot be offset by surplus in other firms. Total non-diversifiable stress risk is

$$s^* = \frac{\mathcal{E}\{D(S_i^+)\} - E\{D(S_i^+)\}}{E\{D(S_i^+)\}} = \frac{D\{s_i E(S_i^+)\}}{D\{E(S_i^+)\}} ,$$

a weighted average of firm specific stress risks  $s_i$  using weights  $\pi_i E(S_i^+)$ . A firm figures highly in overall stress risk if its stress index is large, carries a significant portion of debt, and its baseline risk is high.

6. A measure of the absorbability of defaults in single firms is

$$\alpha = \frac{s^* - s}{s} \geq 0 .$$

If  $\alpha = 0$  then  $s^* = s$  and shortfalls in individual firms translate directly into a shortfall for the system as a whole. Conversely a large value of  $\alpha$  indicates stress in single firms can be absorbed elsewhere. The reciprocal  $1/\alpha$  is a measure of contagion.

## References

Brownlees, C. T. and R. F. Engle (2015). SRISK: A conditional capital shortfall index for systemic risk measurement. FRB of New York Staff Report No. 348. Available at SSRN: <http://ssrn.com/abstract=1611229> or <http://dx.doi.org/10.2139/ssrn.1611229>.