Measuring background risk and systemic risk in finance

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Abstract

This article refines, builds on, and extends SRISK methodology recently proposed in the literature. The refinement is to define SRISK in terms of a put on the Basel shortfall. This is built on by defining the background and systemic stress of a firm as unconditional and departure from unconditional expectation of the put, the latter when a hypothetical systemic stress is applied. Systemic stress is defined in terms of a random variable and can take on variety of forms including alternative scenarios in usual stress testing as well stress driven by the interaction of variables. Stressor random variables are chosen by the practitioner. Stressed expectations are linear, a sector systemic stress is naturally defined as linear in the firm specific systemic stress. Application is made to Australian financial data.

JEL classification C15; E53; G21 (Needs to be filled out)

1. Background

Recent literature in the area of quantitative systemic stress measurement includes Adrian and Brunnermeier (2011), Acharya et al. (2012), Acharya et al. (2012), Brownlees and Engle (2010) and Brownlees and Engle (2015). The proposed research aims to extend and apply these techniques particularly in relation to the entities regulated by APRA. Thus our broad aim is to develop, implement and bring to bear recent developments in stress testing on the aims of APRA and the CIFR targeted research areas. (This needs much more commentary).

This study refines and develops recently proposed methods and applies the same to publicly available daily financial data for the eight Australian banks detailed in Table 1 spanning the period from 3 April 2000 through to 1 December 2014. The data is described in detail in Appendix E. Prices, adjusted for dividends are plotted in Figure 1. Prices are combined with number of shares to derive total equity. Total debt for each firm at each day is also collected. (Should these be plotted?).

Table 1: Major and minor Australian banks

major		minor	
cba	Commonwealth Bank	mqg	Macquarie
anz	Australia & New Zealand	boq	Bank of Queensland
nab	National Australia	ben	Bendigo and Adelaide
wbc	Westpac	aba	Auswide

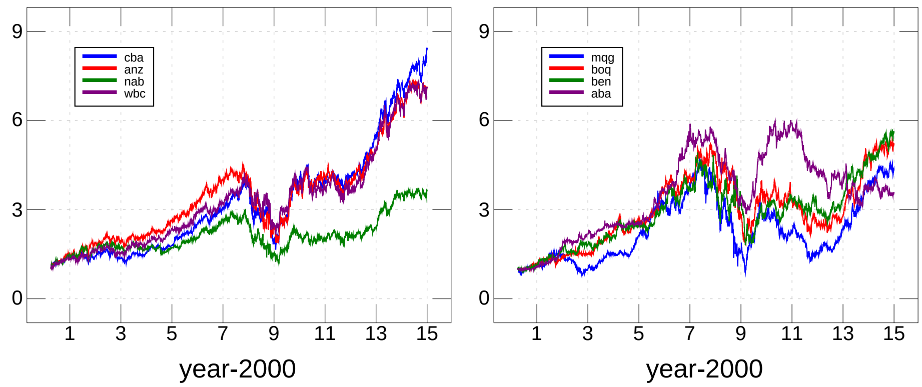

Figure 1: Stock prices of four major (left panel) and four minor (right panel) Australian banks from early 2000 through to the end of 2014. All prices normalised to start at 1.

2. Capital shortfall and leverage

If d_{it} and w_{it} are the debt and equity of a firm i at time t and κ is the capital requirement, then the capital shortfall at time t is

$$\kappa(d_{it} + w_{it}) - w_{it} = \kappa d_{it} - (1-\kappa)w_{it} = \kappa d_{it}\left(1 - \mathrm{e}^{-\ell_{it}}\right) \ , \tag{1}$$

where

$$\ell_{it} \equiv \ln \frac{\kappa d_{it}}{(1-\kappa)w_{it}} = \ln \frac{d_{it}}{w_{it}} + \mathrm{lgt}(\kappa) \ , \qquad \mathrm{lgt}(\kappa) \equiv \ln \frac{\kappa}{1-\kappa} \ .$$

The quantity ℓ_{it} is called adjusted log–leverage and $\ell_{it} > 0$ implies capital shortfall (1) is positive. Basel II assumes[1] $\kappa = 0.08$ implying $\mathrm{lgt}(\kappa) = -2.44$ and ℓ_{it} is the actual log–leverage minus 2.44. As the actual log leverage of a firm increases to 2.44, the firm approaches a Basel breach.

Figure 2 displays Basel log–leverages for the four major and four minor Australian banks listed in Table 1 on the first trading day of each month from January 2003 through to December 2014. Note most banks are Basel compliant up to about 2008.

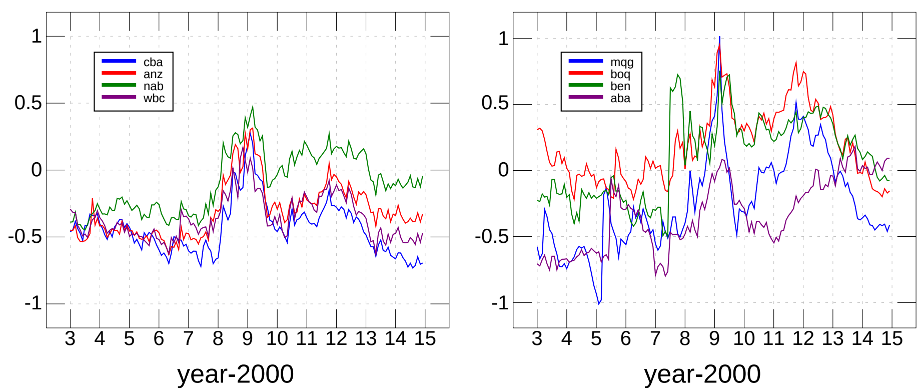

Figure 2: Adjusted log–leverages for four major (left panel) and four minor (right panel) Australian banks from the beginning of 2003 through to end of 2014. All banks contravene the threshold (corresponding to the zero horizontal) around the start of 2009.

3. Future capital shortfall

Financial institutions and regulators are concerned with future Basel compliance and in assessing the likelihood of a future Basel breach. Future compliance

[1] See for example Brownlees and Engle (2015). The value of κ can and is varied to test for sensitivity etc.

depends on the future return on equity. Consider firm i at time $t+h$ where $h>0$. Then[2]

$$\ell_{i,t+h} = \ln \frac{d_{i,t+h}}{w_{i,t+h}} + \mathrm{lgt}(\kappa) = \ell_{it} - \nu_{it} \ , \qquad \nu_{it} \equiv \ln \frac{w_{i,t+h}}{w_{it}} \ ,$$

where it is assumed $d_{i,t+h} = d_{it}$. Hence ν_{it} is the log return on equity over $(t, t+h)$. The future return is unknown at time t but its distribution may be well understood.

The capital shortfall at time $t+h$ is,

$$\kappa d_{it} |1 - \mathrm{e}^{\nu_{it} - \ell_{it}}|^+ \ , \qquad |x|^+ \equiv \max(0, x) \ . \tag{2}$$

Thus shortfall (2) is a put on the return $\mathrm{e}^{\nu_{it}-\ell_{it}}$ with strike 1. The Basel limit is reached at $t+h$ if $\nu_{it} = \ell_{it}$ and breached if $\nu_{it} < \ell_{it}$. A breach is unlikely if ℓ_{it} is low i.e. either the log–leverage is low, or the constant κ in the calculation of ℓ_{it} is low.

If there is a breach, $\nu_{it} < \ell_{it}$, then the amount in (2) makes up for the shortfall. This amount may be cold comfort to regulators as it leaves the firm in a precarious position, teetering on the edge of non–compliance.

Default put options similar to (2) have been discussed in the insurance literature as critical to an evaluation of a firm: see for example Merton (1977), Doherty and Garven (1986), Cummins (1988), Myers and Read (2001) and Sherris (2006).

To illustrate the setup, Figure 3 displays two snapshot outputs from projections generated as described in Appendix A: 6000 simulated one month ahead projected returns for CBA and the market, on two dates: the first trading day in January 2009 and the first trading day in December 2014. Both panels use the same horizontal and vertical scales. Thus based on the available data at those two points in time, policy makers and regulators faced entirely different projections based on the same time series model. The scatter of dots are the forward simulations. Note there is no lookahead bias as the model at each time point is based on the available data to that point in time. The red horizontal line in each panel indicates the projected Basel limit: any firm return less than the limit would constitute a Basel breach. In the left panel there is little confidence that the Basel limit will be reached since it requires a return on equity approaching 20% – with confidence less than xxx the Basel standard will be breached. In the right panel there is no chance of a Basel breach under any of the projected return on equity view – the bank is "safe."

The black dots in each panel indicate the actual outcome after one month. In the left panel the outcome is a decline in both the market and stock price. In the right panel the returns are xxx and xxx. Thus at the beginning of January 2009 the CBA bank was projected to be far from compliance in one month's time while in December 2015 the one month projection is of complete compliance.

[2] In the further development e^{ν} is the continuously compounded return and ν the rate of return or log return.

The two panels display very different volatility and slopes. These snapshots are used to compute systemic stress.

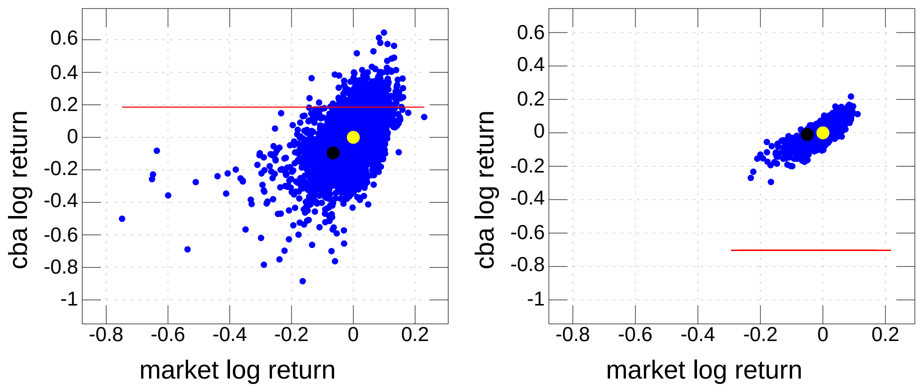

Figure 3: Forecast bivariate distribution of one month forward rates of return for CBA (vertical axis) and the market rate of return (horizontal axis) at start of January 2009 (left panel) and December 2014 (right panel). Note scales in both panels are the same, the relatively large volatility in the left panel, and the left skew in the marginal distributions. Yellow and black dot in each panel indicate origin (0,0) and realized forward rate of return, respectively. Red lines indicate Basel limit – any firm return below the limit indicates a Basel breach given the bank leverage on the applicable date.

4. Systemic risk – SRISK

Brownlees and Engle (2015) define systemic risk (SRISK) for a group of firms at time t as

$$\mathrm{SRISK}_t \equiv \kappa \sum_i d_{it} |1 - \mathrm{E}_\phi(\mathrm{e}^{\nu_{it} - \ell_{it}})|^+ \ , \tag{3}$$

where E_ϕ denotes conditional expectation given a major general market downturn. The proportionate contribution of firm i to (3) is defined as the systemic risk in firm i

$$\mathrm{SRISK}_{it} \equiv \frac{\pi_{it} |1 - \mathrm{E}_\phi(\mathrm{e}^{\nu_{it} - \ell_{it}})|^+}{\mathcal{E}_d\{|1 - \mathrm{E}_\phi(\mathrm{e}^{\nu_{it} - \ell_{it}})|^+\}} \ , \qquad \pi_{it} \equiv \frac{d_{it}}{d_t} \ , \qquad d_t \equiv \sum_i d_{it} \ , \tag{4}$$

where $\mathcal{E}_d$ denotes debt weighted averaging at time t, weighted averaging across firms i using weights π_{it}. Large SRISK_{it} indicates firm i is systemically important: either because it holds a high proportion of the total debt, or it is unusually likely to breach the Basel capital requirement. Expression (4) depends on κ through each of the ℓ_{it}.

Given a general market downturn, the values of the puts $|1-\mathrm{E}_\phi(\mathrm{e}^{\nu_{it} - \ell_{it}})|^+$ in (4) are known at time t: there is no uncertainty except for possible uncertainty in estimating the conditional expected value.

Since

$$|1 - \mathrm{E}_\phi(\mathrm{e}^{\nu_{it}-\ell_{it}})|^+ \leq \mathrm{E}_\phi(|1 - \mathrm{e}^{\nu_{it}-\ell_{it}}|^+) \ ,$$

with equality if $\nu_{it} - \ell_{it}$ is concentrated on the negative axis. If $\nu_{it} > \ell_{it}$ has positive probability then the inequality is strict. Further SRISK_{it} in (4) is zero whenever

$$\mathrm{E}_\phi(\mathrm{e}^{\nu_{it}}) \leq \mathrm{e}^{\ell_{it}} \ .$$

This inequality typically holds even if the expectation conditions on an extreme market downturn.

5. Improved systemic risk measurement

Following from (2) define[3] the Basel put and Basel risk of firm i at time t as

$$p_{it} \equiv |1 - \mathrm{e}^{\nu_{it}-\ell_{it}}|^+ \ , \qquad q_{it} \equiv \mathrm{P}(\nu_{it} \leq \ell_{it}) = \frac{\mathrm{E}(p_{it})}{\mathrm{E}(p_{it}|\nu_{it} \leq \ell_{it})} \ , \tag{5}$$

respectively, where E denotes unconditional expectation. Note $0 \leq p_{it} \leq 1$ with $p_{it} \to 0$ or 1 as $\nu_{it} \to \pm\infty$. Further the Basel risk q_{it} increases as the distribution of ν_{it} moves to the left. Both $\mathrm{E}(p_{it})$ and q_{it} are thought of as probabilities of default with the former reflecting the whole distribution of ν_{it} given there is a default $\nu_{it} \leq \ell_{it}$.

The total Basel put value for firm i is

$$\kappa d_{it}\mathrm{E}(p_{it}) = \kappa d_{it} q_{it} \mathrm{E}(p_{it}|\nu_{it} \leq \ell_{it}) \ ,$$

the cost of insuring firm i does not breach the Basel standard. In practice it may be appropriate to discount p_{it} in (5) by the interest rate over the period $(t, t+h)$ and value the put with risk neutral rather than normal expectation.

The put p_{it} is tradable since, given κ, it relies only on the present leverage and the future return ν_{it} on equity w_{it}. Market participants can value the put in the same way as any other put and use the contract to diversify risk. Firms can buy puts in the market to hedge Basel default risk.

However regulators will value the puts p_{it} using a stressed, rather than risk neutral, expectation. Stressed expectations correspond closely to conditional expectation given a system wide financial shock. Regulators are concerned with the possibility of Basel breaches if such shocks occur and have an interest in monitoring stressed expectations. In the extreme regulators can charge firms all or a proportion of their stressed put values if there are implicit government bailout guarantees.

[3]Alternative terminology is the per unit capital shortfall put and the per unit expected capital shortfall. For shortness we prefer the adjective Basel. Note "risk" here is not a probability but rather a measure of expected capital shortfall.

6. Stressed expectation

In the above development E is unstressed expectation: that is expectation that does not assume a stressful scenario. A stressed expectation, denoted E_ϕ, is a linear combination of expectations, each assuming some level of stress

$$\mathrm{E}_\phi(p_{it}) \equiv \mathrm{E}\{\phi \mathrm{E}(p_{it}|\phi)\} = \mathrm{E}(\phi p_{it}) = \mathrm{E}(p_{it}) + \mathrm{cov}\{\mathrm{E}(p_{it}|\phi), \phi\} \ . \qquad (6)$$

Here $\phi \geq 0$ with $\mathrm{E}(\phi) = 1$ is a stress factor or "stressor," defining events impacting p_{it} and serving as a basis for stress testing. Thus stress ϕ changes the expected value $\mathrm{E}(p_{it})$ by the covariance between the conditional expectation $\mathrm{E}(p_{it}|\phi)$ and the stressor ϕ.

The conditioning variable ϕ downplays or highlight different scenarios. The conditioning events are scenarios of interest and probabilistic weighting is according to the level of interest. The first equality in (6) from the definition of conditional probability (Whittle, 2000). The second equality follows from

$$\mathrm{cov}(p_{it}, \phi) = \mathrm{cov}\{\mathrm{E}(p_{it}|\phi), \phi\} \ .$$

The covariance is zero whenever p_{it} is independent of ϕ which occurs if $\phi = 1$.

The stressed expectation (6) is the mean of p_{it} using the density

$$\int \phi f(p_{it}, \phi) \mathrm{d}\phi = \mathrm{E}(\phi|p_{it}) f(p_{it}) \ , \qquad \int \phi f(p_{it}, \phi) \mathrm{d}\phi \mathrm{d}p_{it} = \mathrm{E}(\phi) = 1 \ \ , \qquad (7)$$

where f denotes density. Any outcome p_{it} such that $\mathrm{E}(\phi|p_{it})$ is large is amplified and vice versa. To a Bayesian the density in the left expression of (7) is a posterior density given a prior $f(p_{it})$ and "likelihood" $\mathrm{E}(\phi|p_{it})$: more importance is given to outcomes p_{it} where ϕ is expected to be large.

7. Background stress versus systemic stress

The right hand side of (6) splits the stressed expectation $\mathrm{E}_\phi(p_{it})$ into two components:

$$\mathrm{E}_\phi(p_{it}) = \mu_{it} + s_{it} \ , \qquad \mu_{it} \equiv \mathrm{E}(p_{it}) = q_{it} \mathrm{E}(p_{it}|\nu_{it} \leq \ell_{it}) \ , \qquad (8)$$

$$s_{it} \equiv \mathrm{cov}\{\mathrm{E}(p_{it}|\phi), \phi\} \ , \qquad q_{it} = \mathrm{P}(\nu_{it} \leq \ell_i t) \ ,$$

called the background stress and systemic stress, respectively, in firm i at time t. Since both $\mathrm{E}_\phi(p_{it})$ and $\mathrm{E}(p_{it})$ lie between 0 and 1 it follows both μ_{it} and s_{it} lie between 0 and 1 and are loosely interpreted as probabilities. Further note

$$q_{it} \mathrm{E}(p_{it}|\nu_{it} \leq \ell_{it}) \approx q_{it} \mathrm{E}(\ell_{it} - \nu_{it}|\nu_{it} \leq \ell_{it}) \ .$$

and hence background risk decomposes into

$$\mu_{it} \approx q_{it}\ell_{it} - \mathrm{E}(|\nu_{it} - \ell_{it}|^+)$$

(Weihao, do you think it is worthwhile to decompose the s_{it} analogously in terms of a probability time a tail event?)

The term μ_{it} is called the background stress in firm i and reflects the log–leverage and the return volatility of firm i at time t. Background stress is unrelated to the stress factor ϕ and based on the unstressed distribution of the forward return ν_{it} and depends on the current view of the likely future progression of returns including their mean and volatility. (Have to fill out discussion with how mu_{it} measures the general uncertainty building up in the system with all uncertainty due to monitoring the system up to time t.) Background stress is unrelated to the stress factor ϕ. Further

$$\mu_{it} = q_{it}\mathrm{E}(p_{it}|\nu_{it} \le \ell_{it}) \ , \tag{9}$$

where q_{it} is the Basel risk for firm i in the upcoming month, while the second factor is the expected default size (on a log scale) if default occurs. The probability q_{it} depends on the mean and volatility of daily returns at time t.

(*** unformed idea) We can similarly decompose (is this useful? – Weihao can you look at this?)

$$s_{it} = q_{\phi it}\mathrm{E}_\phi(p_{it}|\nu_{it} \le \ell_{it}) - q_{it}\mathrm{E}(p_{it}|\nu_{it} \le \ell_{it})$$

Firm i is at Basel default if $\ell_{it} = 0$ and then $q_{it} \approx 1/2$. As ℓ_{it} increases from zero, one month ahead Basel default status becomes more certain since $\nu_{it} \le \ell_{it}$ becomes more certain and q_{it} increases to 1.

8. Systemic beta's

The second, systemic stress component s_{it} in (8), captures the financial impact of a hypothetical system wide financial shock. The stress ϕ is systemic as it is expected to affect all firms i. Further s_{it} is the estimated increase in put values on account of ϕ. Examples of stress factors are given in §10 and §15.

Writing σ_ϕ as the standard deviation of ϕ, define the systemic beta of firm i with respect to stress ϕ as

$$\beta_{it} \equiv \frac{s_{it}}{\sigma_\phi} \tag{10}$$

where σ_ϕ is the standard deviation of ϕ. Thus β_{it} is the systemic stress component in (6) standardised by σ_ϕ and

$$\mathrm{E}_\phi(p_{it}) = \mu_{it} + \beta_{it}\sigma_\phi \ , \qquad \mathrm{E}(p_{it}|\phi) \approx \mu_{it} + \beta_{it}\frac{\phi - 1}{\sigma_\phi} \ . \tag{11}$$

The second, approximate, relationship is suggested from $\mathrm{cov}(p_{it}, \phi)/\sigma_\phi^2$ analogous to the regression coefficient of p_{it} on ϕ.

The expressions in (11) suggests two ways of thinking about the systemic beta's β_{it}. First, the left hand side equation shows systemic stress β_{it} serves to shift the mean $\mathrm{E}(p_{it})$ of put prices by, on average, σ_ϕ. The stress from ϕ is captured with $\sigma_\phi\beta_{it}$, the second term on the right of the first expression. This term is called systemic stress and represents the addition to expected put prices if stress ϕ materialises.

Systemic beta's are stated in terms of additions to the normal put price $\mathrm{E}(p_{it})$. Thus in an increasingly dire financial situation $\mathrm{E}(p_{it})$ will increase. However β_{it} and the systemic stress measures the change in current put values under further ϕ–stress. This varies from Brownlees and Engle (2015) where SRISK_i does not distinguish between the current, possibly high, put prices $\mathrm{E}(p_{it})$ and the potential increment due to potential stress. Thus in an increasingly dire financial situation put prices are liable to increase on account of decreasing expected returns and increasing volatility. However our definition of systemic stress using systemic beta's measures the further effect on account of potential extra stress imposed onto the system over and above the already existing stress. Systemic beta's capture "marginal" effects.

The second interpretation is based on the right hand side approximation in (11) showing the change in the expected Basel put price if σ_ϕ units of stress ϕ are applied. The quantity $(\phi - 1)/\sigma_\phi$ is thought of as a "stress factor." The stress factor has mean 0 and standard deviation 1 and is scaled by β_{it} to yield the actual stress effect on p_{it}. Thus β_{it} is thought of a usual finance type "beta" with respect to ϕ. Stress is measured in standardised units. Units of stress take on different meaning depending on ϕ as discussed below. The distribution of ϕ determines the distribution of the stress factor. If the stress factor is normally distributed then a stress effect more than doubling of the put value occurs less than about 2.5% of the time.

The monetary stress of a firm is $d_{it}\beta_{it}$ and represents the change in the monetary value of the Basel put if stress, as captured with ϕ, is applied.

9. Estimating background and systemic stress via simulation

Stress μ_{it} and can be determined using simulation. Suppose $(\phi^\omega, p_{it}^\omega)$ are pairs of stress and put values generated from a model: for example ϕ^ω may be zero unless there is a market drop below the α–percentile in which case it is $1/\alpha$. Then

$$\frac{1}{N}\sum_{\omega=1}^{N} p_{it}^\omega \to \mu_{it} \ , \qquad \frac{1}{N}\sum_{\omega=1}^{N} (\phi^\omega - 1) p_{it}^\omega \to s_{it} \ , \qquad N \to \infty \ , \tag{12}$$

where N is the simulation effort. The approximation is increasingly accurate as N becomes large. More details are given in ...

10. Stress based on market return

The setup is illustrated with examples generalising and fine–tuning the approach of Engle.

10.1. Market return below a percentile threshold

Suppose the stress event is a market return in the bottom α–tail of the market return distribution. Then $\phi(u) = 1/\alpha$ for $u < \alpha$ and 0 otherwise and

$$\mathrm{E}_\phi(p_{it}) = \frac{1}{\alpha}\int_0^\alpha \mathrm{E}(p_{it}|u)\mathrm{d}u = \mathrm{E}(p_{it}|\nu_{mt} < \tau_t) \ .$$

where τ_t cuts out α probability in the lower tail of the ν_{mt} distribution. Further

$$\mu_{it} \approx \frac{1}{N}\sum_\omega p_{it}^\omega \ , \qquad s_{it} \approx \frac{1}{N/\alpha}\sum_{u_{mt}^\omega < 1/\alpha} p_{it}^\omega \ .$$

In the right hand sides $\omega = 1, \ldots, N$ denote N simulations. The approximations decrease with the simulation effort N. The effective simulation effort is N/α and hence α small requires a large effort.

A fixed α cutoff implies the actual threshold depends on the distribution and hence on volatility and time. The actual threshold drop in the market $\tau_t = F_{mt}^-(\alpha)$, where F_{mt}^- is the inverse distribution function of ν_{mt}. The volatility associated with the latter distribution is approximately $\sqrt{h}\sigma_{mt}$ where σ_{mt} is current market volatility. Regulators and practitioners are well versed in working with VaR type calculations and hence a varying VaR type cutoff combining market volatility and stress is closely aligned to current practice.

10.2. Expected worst market return in n identical scenarios

If $\phi(u) = n(1-u)^{n-1}$ then $\phi(u)\mathrm{d}u = \mathrm{d}\{1-(1-u)^n\}$ and

$$\mathrm{E}_\phi(p_{it}) \equiv \mathrm{E}\{\phi \mathrm{E}(p_{it}|\phi)\} = \int_0^1 \mathrm{E}(p_{it}|u)\mathrm{d}\{1-(1-u)^n\} \ .$$

If u is the market return percentile then $1-(1-u)^n$ is the distribution of the worst percentile outcome in n identical trials and hence the stressed expectation is that of the expected put price given the worst percentile market return in n identical trials. Further

$$s_{it} \approx \frac{1}{N}\sum_\omega \{n(1-u_{mt}^\omega)^{n-1} - 1\}p_{it}^\omega \ .$$

The effective simulation effort is N/n. Simulated returns ν_{mt}^ω in the upper tail of the distribution have percentiles $u_{mt}^\omega \approx 1$ and hence for these market returns $(1-u_{mt}^\omega)^{n-1}$ is negligable and the associated simulated put p_{it}^ω is heavily downweighed.

Note the contrast with the previous example where the bottom α proportion of simulated market returns are selected as the stressed sample. With the current specification for ϕ, every simulated put contributes to the stress computation, albeit with different weights.

10.3. Expected worst market return given a tail event

The above two situations can be combined. Suppose $\phi(u) = c(0.05 - u)^{19}$ for $u \leq 0.05$ and 0 otherwise and where c is such that $\phi(u)$ integrates to 1. Then (this needs to be checked)

$$\mathrm{E}(p_{it}) \approx \frac{1}{N/c} \sum_{u^{\omega}_{mt} < 1/20} \left(u^{\omega}_{mt} - \frac{1}{20} \right)^{20} p^{\omega}_{it} \; .$$

Similar to the first example, the stressed sample picks up the bottom 5% of market returns. The bottom 5% of market returns is further stressed by progressively downweighing returns as the percentile approaches 1/20. Large simulation effort N is required for a reasonable approximation since N/c is the effective simulation size and c is small.

11. Simulating future capital shortfall and market return

(maybe this should be merged with material elswhere).

The estimation of systemic beta's requires projections of future capital shortfalls and future stressors. As in Brownlees and Engle (2015) in this article projections are constructed using time series models modelling the forward rates of return ν_{it} for firms i and ν_{mt} for the market. The market return is used as the stressor with different choices of the stress function ϕ modelling different stress scenarios.

The time series models used for projecting are stochastic volatility models based on the GARCH–DCC model of Engle (2002) summarised in Appendix A. The GARCH–DCC framework is only one possible implementation. For example future return scenarios may be constructed in a more ad–hoc manner e.g. judiciously chosen scenarios decided on by regulators or policy makers. The systemic beta framework can be based on any generated future scenarios.

While the above discussion suggests, as obvious after the fact, that in December 2009 the CBA bank was under much stress. However the above calculations do not actually reflect real stress. In terms of the development of the previous sections the actual stress is beta given a stressor function ϕ and stressor variable. In this case the stressor variable is the market return, on the horizontal axis in each panel. Stress occurs if the market return is low. In the left panel low market outcomes are likely to lead to sharply lower CBA bank returns, as compared to the right panel. This is suggested by the slope of the scatter plots: the left slope appears steeper than the right and hence any stress in the market – in the sense of a market downturn is expected to lead to a larger drop in the CBA bank return in the left scenario as compared to the right scenario. The magnitudes of the two responses depends on the stressor function. In essence the stressor function formalises the notion as to what constitutes a "big" drop. If a big drop is the expected worst in 20 similar market scenarios then the stress in the left and right are xxx and xxx, respectively.

12. One month ahead forecast stress in individual financial institutions

Figure 12 displays, in the top panels, the estimated q_{it} one month ahead default probabilities for each major and minor Australian banks based on the projected one month ahead return distribution. The top left panel shows the first inclinations of default arose with NAB in early 2008 followed one month later by ANZ, and a further few months later by WBC and CBA.

Systemic stress is an expected market return equal to the worst outcome in 12 identical months. (Have to expand on this). (Need much more commentary here)

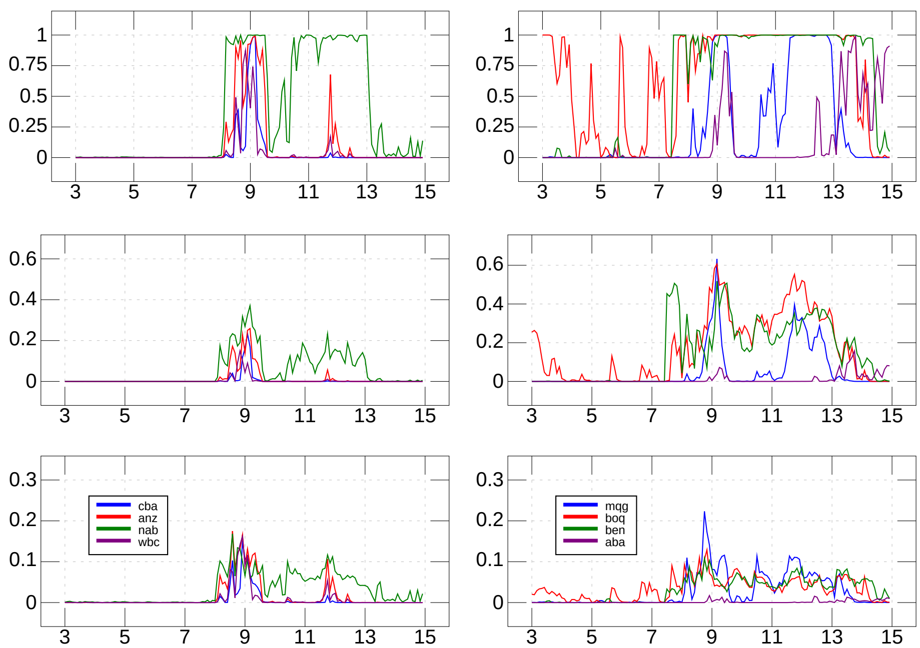

Figure 4: Forecast one month ahead Basel default probability (top panels), background stress (middle panels) and systemic stress (bottom panels) for four major (left panel) and four minor (right panel) banks from the beginning of 2003 through to end of 2014. Note different scale on bottom two rows of panels.

13. Average stress

There are two ways of aggregating stress in the financial sector as a whole. The first is to average, using debt weighted averaging, the put

$$\overline{p}_t \equiv \mathcal{E}_d(p_{it}) \ , \qquad \mathrm{E}_\phi(\overline{p}_t) = \mathcal{E}_d\{\mathrm{E}_\phi(p_{it})\} = \mathcal{E}_d(\mu_{it} + s_{it}) \equiv \overline{\mu}_t + \overline{s}_t \tag{13}$$

The put $\overline{p}_t$ in (13) pays out zero if all firms meet the Basel requirement with payments increasing with the number of Basel breaches, the sizes of the breaches, and the relative debt sizes of the breaching firms. Further $\overline{p}_t > 0$ if and only if one or more firms breach the Basel standard. The relation in (13) follow since E_ϕ and $\mathcal{E}_d$ are linear. (Is $\overline{q}_t$ a useful concept? Weihao can you look at this?)

The monetary value average stress at time t is the debt averaged monetary firm stresses

$$\kappa d_t \overline{\mu}_t = \kappa d_t \mathcal{E}_d(\mu_{it}) = \kappa \sum_i d_{it}\mu_{it} \ , \qquad \kappa d_t \overline{s}_t = \kappa \sum_i d_{it} s_{it} \ .$$

Firm i contribution to for example systemic stress is $\pi_{it}s_{it}/\overline{s}_t$ which, when adding across firms, adds to 1. Other contributions are calculated similarly.

Percentage contributions of firms to background and systemic stress are calculated as

$$\frac{\pi_{it}\mu_{it}}{\overline{\mu}_t} \equiv \frac{\pi_{it}\mu_{it}}{\mathcal{E}_x(\mu_{it})} \ , \qquad \frac{\pi_{it}s_{it}}{\overline{s}_t} \equiv \frac{\pi_{it}s_{it}}{\mathcal{E}_x(s_{it})} \ .$$

where $\pi_{it} \equiv d_{it}/d_t$ is the proportion of total debt held by firm i. A ratio such as $s_{it}/\overline{s}_t$ compares the stress in firm i to average stress but does not signal the importance of that firm to the system as a whole.

14. Total sector stress

The debt weighed average system put $\overline{p}_t \equiv \mathcal{E}_d(p_{it})$ is an average put and does not permit diversification – the sharing of debt and equity across firms. Sharing may not be possible in practice but the concept is useful for assessing the stability of the system as a whole. This leads to the second way of aggregating stress in the financial sector as whole.

Write d_t and w_t are total debt and equity in the financial sector, respectively:

$$d_t \equiv \sum_i d_{it} \ , \qquad w_{it} \equiv \sum_i w_{it} \ .$$

Then the adjusted log leverage for the sector is

$$\ell_t \equiv \ln \frac{d_t}{w_t} + \mathrm{lgt}(\kappa) = \ln \sum_i \frac{w_{it}}{w_t} \left(\frac{d_{it}}{w_{it}} \times \frac{\kappa}{1-\kappa} \right) = \ln \mathcal{E}_w(\mathrm{e}^{\ell_{it}}) \ ,$$

where $\mathcal{E}_w$ denotes equity weighted averaging. A system wide breach occurs at $t+h$ if $\nu_t < \ell_t$ where ν_t is the forward rate of return on total equity w_t:

$$\nu_t = \ln \frac{w_{t+h}}{w_t} = \ln \mathcal{E}_w \left(\frac{w_{i,t+h}}{w_{it}} \right) = \ln \mathcal{E}_w(\mathrm{e}^{\nu_{it}}) \ . \tag{14}$$

In terms of this notation the system Basel put and Basel risk are defined similar to (5)

$$p_t \equiv |1 - \mathrm{e}^{\nu_t - \ell_t}|^+ \leq \overline{p}_t \ , \qquad q_t \equiv \frac{\mathrm{E}(p_t)}{\mathrm{E}(p_t|\nu_t < \ell_t)} = \mathrm{P}(\nu_t \leq \ell_t) \ .$$

The inequality is proved in Appendix C. The put p_t pays 0 at time $t+h$ unless the sector as whole is in Basel default, occurring if $\nu_t < \ell_t$. Moving from $\overline{p}_t$ to p_t allows for diversification: low liquidity in one firm is offset by high liquidity in other firms.

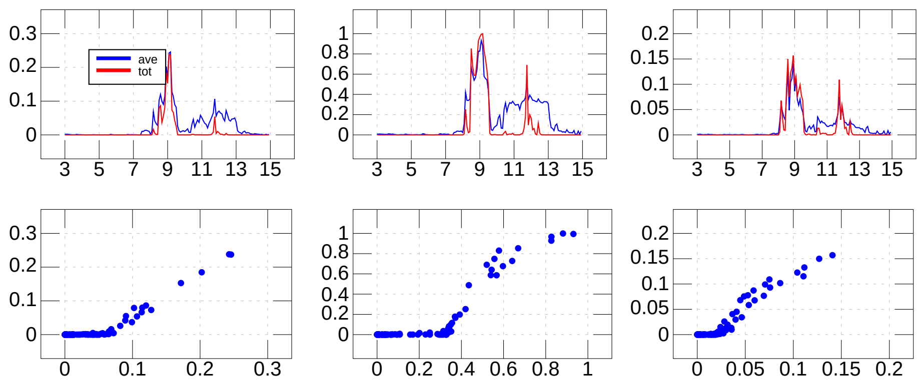

Figure 5: Average stresses and total stresses. Stress measurements are, left to right, μ, q and s, respectively. The top row of panels measure for example (top left panel) $\overline{\mu}_t$ and μ_t over time. In the bottom row of panels specific stresses are measured are against each other: for example μ_t (vertical axis) versus $\overline{\mu}_t$ (horizontal axis). In all cases there is no total stress until there is an appreciable level of average stress. Systemic stress $\overline{s}_t$ appears to be the least diversifiable.

Similar to (11), total stress is decomposed as

$$\mathrm{E}_\phi(p_t) = \mu_t + s_t \ , \qquad q_t = \mathrm{P}(\nu_t < \ell_t) \ , \qquad s_t \equiv \mathrm{cov}\{\mathrm{E}(p_t|\phi), \phi\} \ . \qquad (15)$$

termed the sector background and systemic stress, respectively. Further q_t is the probability of a sector Basel default with the second factor the expected size (on a log scale) of the default if there is a Basel default.

System resilience aims to answer the question whether the system as a whole can absorb shocks. The capacity to absorb relies on implicit merging. When two firms merge the leverage of the resulting firm is less than that of the more highly leveraged firm. The merged firm is more able to withstand return on equity shocks unless negative shocks are more prone for the merged firm. Similarly when many firms merge Basel puts for the conglomerate become less valuable.

Firms do not necessarily merge, even in dire financial circumstances. Hence the mergers spoken of here are hypothetical. From a regulators perspective, what would happen under a merger of all firms (or perhaps a group of firms) is nevertheless of interest. A system as as a whole near Basel non–compliance is more threatening that one where a few firms are near Basel non–compliant but the system itself is well away from non–compliance.

System compliance is partially monitored with the system put $p_t = \mathcal{E}_d(p_{it})$ and its stressed expectation $\mathrm{E}_\phi(p_t)$. The system put however simply monitors a debt weighted average of puts. It gives no guidance as to how much risk can be diversified away. Basing a put on system wide aggregated debt and equity and comparing the same to p_t gives at least some guidance.

Possible additional issues to be discussed here

- Usually $q_t < \overline{q}_t$ reflecting diversification benefits. However this inequality is not guaranteed since ???

$$\mathcal{E}_d\{\mathrm{P}(\nu_{it} \leq \ell_{it})\} \neq \mathrm{P}\{\mathcal{E}_d(\nu_{it}) \leq \mathcal{E}_d(\ell_{it})\}$$

- Comparing say $\overline{s}_t/s_t$ or $(\overline{\mu}_t + \overline{s}_t/(\mu_t + s_t)$ etc. What do each of these different measures illustrate? (System robustness)

- Percentage measures such as $\pi_{it}s_{it}/s_t$, or $\pi_{it}s_{it}/(s_t + \mu_t)$.

- System robustness such as

15. Further generalised stress functions

Practical results for the construction of general stress functions ϕ are contained in the next three subsections.

15.1. Stress functions as weighted linear combinations of tail events

Brownlees and Engle (2015) define a systemic event as a market based downturn greater than a certain threshold and a systemic event depends on the chosen threshold. This arbitrariness is partially sidestepped by choosing a decreasing function $\phi = \phi(u_{mt})$ on the unit interval and noting

$$\mathrm{E}\{\phi \mathrm{E}(p_{it}|\phi)\} = -\mathrm{E}\{u\phi'(u)\mathrm{E}(p_{it}|u_{mt} \leq u)\} \ . \tag{16}$$

The equivalence of the left and right hand sides of (16) follows from

$$\int_0^1 \phi'(u) \int_0^u \mathrm{E}(\nu_{it}|u_{mt})\mathrm{d}u_{mt}\mathrm{d}u = \int_0^1 \mathrm{E}(\nu_{it}|u_{mt}) \int_{u_{mt}}^1 \phi'(u)\mathrm{d}u\mathrm{d}u_{mt} \ ,$$

provided $\phi(1) = 0$.

Thus stressed expectations, stressed with a decreasing function of the market return, is equivalent to taking a weighted average of conditional lower tail expectations. This result is used to circumvent an explicit choice for the market threshold, replacing it with $\phi(u)$, an implicit mixture of thresholds.

15.2. Scenario based stress testing

Stress events captured with ϕ and used in (10) can be defined with respect any events, including a discrete number of scenarios labelled $k = 1, 2, \ldots$. In this case

$$\mathrm{E}_\phi(p_{it}) = \sum_k \pi_k \phi_k \mathrm{E}(p_{it}|k) \ , \qquad \sum_k \pi_k \phi_k = \mathrm{E}(\phi) = 1,$$

where ϕ_k is the weight assigned to scenario κ. The $\pi_k \phi_k \geq 0$ are modified probabilities which weigh different scenarios according to level of interest. For example in standard stress testing the ϕ_k are chosen such that significant weight is given to scenarios κ causing high firm distress. The "real world" probabilities π_k are usually ignored by simply choosing $\pi_k \phi_k$ to be of required magnitude.

In the discrete case s_{it} compares the expected value under an average of distress scenarios to the average without distress. The quantity σ_ϕ is now of limited relevance, signalling the volatility in the distress probabilities.

15.3. Copula stress functions

Stress can be based on a vector of variables x with distribution $F(x) = u$ and marginal densities $f(x_j)$. Suppose $c(u)$ is the copula density of x implying the density of x is $c(u)\prod_j f(x_j)$. If $\phi(u)$ is a copula density then $\mathrm{E}\{\phi(u)\} = 1$ and consider

$$\mathrm{E}_\phi(p_{it}) \equiv \int \mathrm{E}(p_{it}|x)\phi(u)c(u)\prod_j \{f(x_j)\}\mathrm{d}x \ . \tag{17}$$

If $\phi(u) = 1$ then $\mathrm{E}_\phi(p_{it}) = \mathrm{E}(p_{it})$, the ordinary expectation. The density $\phi(u)$ magnifies potentially stressful situation such as where all component of x are highly tail dependent.

The copula density stressor $\phi(u)$ can be combined with marginal stressors written as functions of x_j or percentiles u_j. If the latter then $\phi(u)\prod \phi(u_j)$ is the total stressor, corresponding to a density on the unit hypercube with non–uniform marginals $\phi(u_j) \neq 1$.

Copula stress effects are simulated as follows. Suppose x is a vector ν_{mt} of market factors over the period $(t, t+h)$. Simulations of a joint model are used to derive vectors $(p_{it}^\omega, \nu_{mt}^\omega)$, $\omega = 1, \ldots, N$. The ν_{mt}^ω are converted to percentiles u^ω and scalars

$$\phi^\omega \equiv \phi(u^\omega)\prod_j \phi(u_j^\omega) \ , \qquad \omega = 1, \ldots, N \ .$$

The stress beta is then estimated as in (12).

16. Risk weighted asset methodology

The risk weighted asset (RWA) methodology (references ???) for determining capital adequacy is to value assets and subtract debt to arrive at capital shortfall. The capital shortfall is then, similar to (1),

$$\kappa d_{it} - (1-\kappa)\sum_j \mathrm{e}^{-r_{ijt}} a_{ijt} = \kappa d_{it} - (1-\kappa) a_{it} \mathcal{E}_a(\mathrm{e}^{-r_{ijt}}) \ , \qquad a_{it} = \sum_j a_{ijt} \ . \tag{18}$$

Here a_{ijt} is the value of firm i's asset j at time t. Further the $r_{ijt} \geq 0$ are risk weights and $\mathcal{E}_a$ denotes an asset weighted average using firm i's asset values at time at time t. If $r_{ijt} = 0$ in (18) then asset j is valued at its present value.

Higher weights reflect more risky assets and potential for asset devaluation. The RWA methodology is a standard approach to capital shortfall calculations.

The RWA calculations can be fit into the present framework. If ν_{ijt} is the forward return on firm i class j asset at time t then a put on the projected capital shortage at time $t+h$ is, per unit κd_{it},

$$p_{it} \equiv \left|1 - \mathrm{e}^{-\mathrm{lgt}\kappa - L_{it}} \mathcal{E}_a(\mathrm{e}^{\nu_{ijt}})\right|^+ \ , \qquad L_{it} \equiv \ln \frac{d_{it}}{a_{it}} \ . \tag{19}$$

Thus $\mathrm{e}^{L_{it}}$ is the often used alternative leverage definition: dividing debt by total assets. Similar to before $0 \leq p_{it} \leq 1$ is a measure of stress on firm i at time t with expected stress decomposed as before

$$\mathrm{E}_\phi(p_{it}) = \mathrm{E}(p_{it}) + \mathrm{cov}\{\mathrm{E}(p_{it}|\phi), \phi\} = \mu_{it} + s_{it} \ ,$$

where μ_{it} is the background stress and s_{it} is the systemic stress, i.e. the result of systemic event modelled with ϕ.

To calculate asset weights w_{ijt}, a sample of N forward returns ν_{ijt}^ω and stress factors ϕ^ω are simulated using an appropriate model such as the GARCH–DCC model. The returns ν_{ijt}^ω are used to derive $\mathcal{E}_a(\mathrm{e}^{\nu_{ijt}^\omega})$ and in turn, using (19), p_{it}^ω which are multiplied by ϕ^ω to arrive at stressed put prices $\phi^\omega p_{it}^\omega$. As $N \to \infty$,

$$\frac{1}{N}\sum_\omega \phi^\omega p_{it}^\omega \to \mathrm{E}_\phi(p_{it}) \ , \qquad \mathcal{E}_a\left(\frac{1}{N}\sum_\omega \phi^\omega \mathrm{e}^{\nu_{ijt}^\omega}\right) \to \mathrm{E}_\phi\{\mathcal{E}_a(\mathrm{e}^{\nu_{ijt}})\} \ . \tag{20}$$

Note that

$$\mathrm{E}_\phi(p_{it}) \neq \left|1 - \mathrm{e}^{-\mathrm{lgt}\kappa - L_{it}} \mathrm{E}_\phi\{\mathcal{E}_a(\mathrm{e}^{\nu_{ijt}})\}\right|^+ \ ,$$

and the stressed average return

$$\mathrm{E}_\phi\{\mathcal{E}_a(\mathrm{e}^{\nu_{ijt}})\} = \mathcal{E}_a\{\mathrm{E}_\phi(\mathrm{e}^{\nu_{ijt}})\} \approx \mathcal{E}_a\left(\frac{1}{N}\sum_\omega \phi^\omega \mathrm{e}^{\nu_{ijt}^\omega}\right) \ ,$$

is cannot be used directly in the calculation of the stress in the firm.

Easily disstressed assets j have a return distribution sensitive to stress ϕ and under stress, $\mathrm{e}^{\nu_{ijt}}$ is likely to be much less than 1. Assets whose return $\mathrm{e}^{\nu_{ijt}}$ tends to be much less than 1 when stress ϕ is high, have high risk and low weight. An asset with no volatility, $\nu_{ijt} \equiv 0$, has risk weight 1 since, on average, ϕ^ω is 1.

17. Application to Australian financial institutions

(have to merge this with other stuff elsewhere)

At each of the first trading day in the months from January 2003 through to December 2014, all prior returns are used estimate a TARCH-DCC model described below. For each first trading day of the month, the fitted model is then used to simulate the forward return distribution over the next 22 trading days, corresponding to approximately one month. The details of the fitted models are described in the next subsection.

17.1. Forward return simulation

The forward simulations are implemented similar to Brownlees and Engle (2015). Given the latest available volatility and correlation estimates the filter recursions are moved forward in time using innovations randomly chosen from past standardised innovations. Thus the innovations are chosen to have the same marginal distributions as applicable in the past. The random choices are such that the same market return innovations are used in each bivariate analysis.

18. Background and systemic stress compared to SRISK

The methodology developed and employed in this article departs in two respects from the SRISK methodology set out in Brownlees and Engle (2015). This section examines the force of these differences.

18.1. Put value versus put on expected value

The SRISK methodology described in §4, in particular formula (4), is based on the put

$$|1 - \mathrm{E}_\phi(\mathrm{e}^{\nu_{it}-\ell_{it}})|^+ \le \mathrm{E}_\phi(|1 - \mathrm{e}^{\nu_{it}-\ell_{it}}|^+) \equiv \mu_{it} + s_{it} \ , \tag{21}$$

where stressed expectation E_ϕ in the Brownlees and Engle (2015) article is conditional expectation given a major market downturn. The put in the first expression is evaluated after the expectation E_ϕ and checks, in essence whether in a significant market downturn, the expected h period ahead return ν_{it} exceeds the adjusted log–leverage. If this holds true there may still be substantial risk of a Basel breach, depending on the volatility of the return. Volatility in ν_{it} is ignored in SRISK_{it} other than through the usual adjustment on account of continuous compounding. In particular increased volatility due to stress is ignored.

With $\mu_{it}+s_{it}$, volatility is taken into consideration. Other things equal, high volatility leads to higher put values since breach size is taken into consideration.

Note further we split up $\mu_{it} = q_{it}\mathrm{E}(p_{it}|\nu_{it} > \ell_{it})$. (Can do analogous split up on s_{it} ????)

(Should we compare $\mu_{it} + s_{it}$ to the left hand side of (21) using graphs???)

18.2. Systemic stress versus background stress

Brownlees and Engle (2015) use E_ϕ as in (21). A stressed expectation may be high simply on account of put prices being high, again reflecting higher than normal volatility. Stress measured with E_ϕ picks up both background stress, due to high volatility, and the extra stress imparted by the stressing variable.

To gain insight into the relative sizes of these two components, consider the proportion of the stressed put price due to a possible future stress scenario

$$\frac{\mathrm{E}_\phi(p_{it}) - \mathrm{E}(p_{it})}{\mathrm{E}_\phi(p_{it})} = \frac{\sigma_\phi \beta_{it}}{\mu_{it} + \sigma_\phi \beta_{it}} \ .$$

The denominator is total stress in firm i, made up of two components: the volatility stress $\mu_{it} = \mathrm{E}(p_{it})$ and the systemic stress $\sigma_\phi \beta_{it} = \mathrm{E}_\phi(p_i t) - \mathrm{E}(p_{it})$. Volatility stress arises on account of high volatility in the markets causing high put prices. Systemic stress is the additional stress caused by a potential bad systemic event.

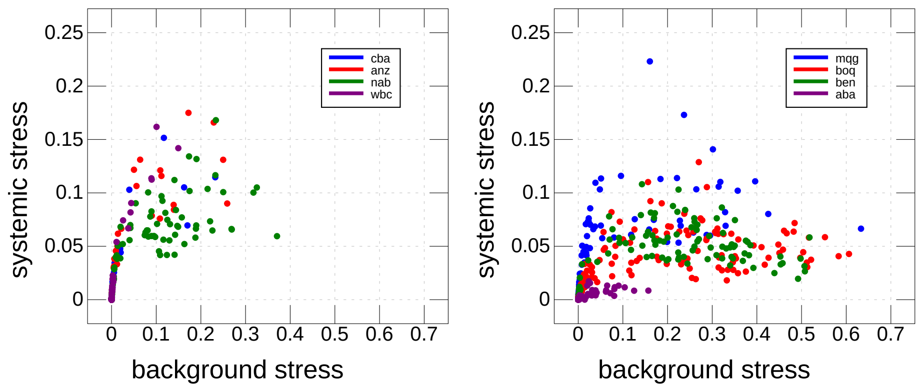

Figure 6: Systemic, s_{it}, versus background μ_{it} for four major banks (left panel) and four minor banks (right panel) for 144 months.

Figure 6 plots systemic stress versus volatility stress for the eight Australian banks of Table 1. Generally volatility stress is higher than systemic stress.

18.3. Flexible stress functions

The further modification made to the SRISK methodology of Brownlees and Engle (2015) is to generalise the "cutoff" functional form for stressing. With the latter, stressed expectations are conditional expectations given a specified cutoff point. In Brownlees and Engle (2015) the cutoff is in absolute terms: a 10% fall in the market although this can varied. The cutoff methodology does not make explicit allowance for market volatility: an absolute 10% reduction is increasingly plausible as volatility increases. However SRISK does not make judgement as to the likelihood of the conditioning stress event. Thus SRISK mixes in various sources of risk in an imprecise manner and it will be difficult to determine the implications of SRISK readings.

This argues for specifying conditional behaviour in percentile terms relative to the then applicable market conditions. After all, current market volatility is known and there is a good reason to disentangle current market conditions from the evaluation of what is likely to happen if further stress arrives in the system.

Say something about the generalized functional forms ϕ as well as the fact that we have firmly embedded and/or aligned the methodology with more ad–hoc approaches to stress testing.

19. Comparing SRISK for different firms (not sure)

Suppose of interest is whether β_{it} varies similarly across firms as τ varies where τ is the threshold, κ, or some other parameter used to compute β_{it}. High correlation implies high systemic risk since firms are simultaneously affected.

Define $\epsilon_{it} \equiv (\phi - 1)p_{it}$ and ϵ_{it} as the vector with components ϵ_{it}. Then $\mathrm{E}(\epsilon_{it}|\tau) = \sigma_\phi(\tau)\beta_{it}(\tau)$, the stress in firm i at the given parameter setting τ and

$$\mathrm{cov}\{\mathrm{E}(\epsilon_{it}|\tau)\} = \mathrm{cov}(\epsilon_t) - \mathrm{E}\{\mathrm{cov}(\epsilon_{it}|\tau)\} \ ,$$

is the covariance between stresses as τ, the stress parameter varies. Thus the covariances and correlations between firms as stress parameters vary can be computed from the covariability between the ϵ_{it} and the ...

A. GARCH–DCC model

Denote the daily (log) return for firm i at time t as

$$\delta_{it} = \mu_i + \sigma_{it}\epsilon_{it} \ , \qquad \epsilon_{it} \sim (0,1) \ , \tag{A.1}$$

The volatility σ_{it} is modelled as

$$\sigma^2_{i,t+1} = \omega + \sigma^2_{it}\{\beta + (\alpha + \gamma\epsilon^-_{it})\epsilon^2_{it}\} \ , \qquad \epsilon^-_{it} \equiv I(\epsilon_{it} < 0) = I(\delta_{it} < \mu) \ . \tag{A.2}$$

where I denotes the indicator function. Hence the response of $\sigma^2_{i,t+1}$ to ϵ^2_{it} is increased by γ if the rate of return is below the average μ_i, compared to the response if $\delta_{it} > \mu_i$. Equations (A.1) and (A.2) defined a simple threshold GARCH model: called the TARCH(1,1). In (A.1) the mean μ_i does not vary with time t and in (A.2) it is assumed the terms σ^2_{it} and ϵ_{it} in the right hand side of (A.2) are sufficient to structure the dynamics of volatility.

The model defined by (A.1) and (A.2) is estimated for each security i jointly with a similar model for the market, $i = m$. The correlations between securities and the market are modelled using positive definite recursions (Engle, 2002)

$$(Q_{i,t+1} - S) = \alpha(\eta_{it}\eta'_{it} - S) + \beta(Q_{it} - S) \ , \qquad \eta_{it} \equiv (\epsilon_{it}, \epsilon_{mt})'.$$

Correlations ρ_{it} recovered from the Q_{it} are used as the correlation between ϵ_{it} and ϵ_{mt}.

B. Computations

All model fits in this paper have been performed with the R language (R Development Core Team, 2008) and in particular the rmgarch package described by Ghalanos (2012). All other calculations were performed using the J language (Iverson, 2003).

In the forward simulated forward projections the innovations are chosen randomly from past innovations. These past innovations are chosen consistently: at a particular t either all or none of ϵ_{it} are chosen.

C. Proof of system put is less than debt weighted average put

$$1 - \mathrm{e}^{-\ell_{it}} = \frac{\kappa d_{it} - (1-\kappa)w_{it}}{\kappa d_{it}} \ , \qquad \mathcal{E}_d(1 - \mathrm{e}^{-\ell_{it}}) = \frac{\kappa d_t - (1-\kappa)w_t}{\kappa d_t} = 1 - \mathrm{e}^{-\ell_t} \ ,$$

and similarly if $-\ell_{it}$ is replaced by $\nu_{it} - \ell_{it}$. Hence

$$p_t \equiv |1 - \mathrm{e}^{-\ell_t}|^+ \leq \mathcal{E}_d(|1 - \mathrm{e}^{-\ell_{it}}|^+) \equiv \overline{p}_t \ .$$

D. Use of more general puts

Consider the put $p_{it}(1 + cp_{it}) = p_{it} + cp_{it}^2$ for some constant $c \geq 0$. This put is zero if $p_{it} = 0$ and has slope $1 + 2cp_{it}$ for $p_{it} > 0$ and $p_{it}(1 + cp_{it}) > p_{it}$ if the put is positive. Larger slopes c imply greater payoff.

The put $p_{it} + cp_{it}^2$ imposes a higher cost structure on defaults. For every extra dollar of default the cost increases by $1 + 2cp_{it}$. Further the stressed put value is

$$\begin{aligned}
&\mathrm{E}(p_{it})\left\{1 + c\mathrm{E}(p_{it})\right\} + c\left\{\mathrm{cov}(p_{it})\right\} + \mathrm{cov}(p_{it}, \phi) + c\left\{\mathrm{cov}(p_{it}^2, \phi)\right\} \\
&= \mathrm{E}(p_{it})\left\{1 + c\mathrm{E}(p_{it})\right\} + c\left\{\mathrm{cov}(p_{it})\right\} + \mathrm{cov}(p_{it} + cp_{it}^2, \phi) \ .
\end{aligned}$$

The first three terms in the final expression are unrelated to stress ϕ: stress only impacts the last term.

Can easily determine all terms via simulation.

E. Data sources

(Geoff – can you fill out?)

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