Monitoring risk in the financial system using time series methods

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Abstract

SRISK methodology recently proposed in the literature is refined and extended. The refinement is to define systemic risk using a formalised stress testing framework including a stress function. Baseline risk and the stress risk are in terms of the ordinary and stressed expectation. Stressed expectation is expectation computed under a hypothetical stress, modelled with the stress function and scenarios. Systemic stress is defined in terms of a stress function and systemic scenarios impacting on a number of firms or financial entities. Stress functions are chosen by the practitioner and typically exaggerate undesirable extreme outcomes. Properties and characterisations of stress and stress related quantities are displayed and explored. Application is made to the study of the stability of Australian banks using daily time series data.

Keywords: Capital shortfall, baseline risk, stress testing, stressed expectation, stress diversification.

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1 Introduction

Monitoring stresses and systemic risk in the financial system is a key function of macro–prudential regulation. Central banks and other financial regulatory bodies such as the US Financial Stability Oversight Council, the European Systemic Risk Board and the Australian Prudential Regulation Authority (APRA) are concerned with developing and applying early warning quantitative measures to warn of potential episodes of heightened systemic risk. Systemic risk has many dimensions. For example, in their influential review paper, Bisias et al. (2012) present a taxonomy of six categories of systemic risk comprising 31 quantitative risk measures. While attempts to gain consensus on a theoretical definition have proved elusive, several empirical measures of systemic risk have been developed and applied with success. An important example is the SRISK measure due to Brownlees and Engle (2015) – an index that measures the expected capital shortfall of a financial institution conditional on a large and lengthy decline in equity markets. Brownlees and Engle (2015) show that their SRISK based measures are able to identify systemically important financial firms and are informative about the impact of financial crises on future macro-economic conditions.

This paper refines and extends the SRISK model of Brownlees and Engle (2015) in key ways. First, their SRISK measure is refined as a put on the expected shortfall. Second, the ordinary expectation of is defined as the baseline risk (BRISK) associated with the put. Using a stress function the so–called stressed expectation is used to characterise the expectation given a stress. Stressed expectation can be similar to the Brownlees and Engle (2015) conditional expectation given a severe market downturn. However our generalised methodology permits a far more flexible definition of stress and a wider class of "stressors" including stress defined in terms of a number of extreme outcomes. The difference between the stressed and ordinary expectation is defined as the risk associated with the stress, called PRISK where PSI refers the function modelling the stress.

BRISK and PRISK capture different aspects of stress on a firm. BRISK is a function of existing volatility and leverage. As these factors increase, BRISK rises and this makes the firm more susceptible to PRISK, regardless of the stress event itself. Properties of PRISK are explored.

Brownlees and Engle (2015) define stress by conditioning on an absolute cutoff of arbitrary size and length. Our development suggests it is also helpful to specify conditional behaviour in percentile terms relative to current market conditions. This yields a sharper focus on the incremental impact of further stress and permits a more incisive view of the implications from SRISK numbers. These extensions provide additional insights into the sources and economic nature of for example systemic risk. The measures provide potentially new and improved ways of anticipating episodes of heightened systemic risk. Their practical use can be expected to help regulators monitor risk including systemic risk and contagion in timely fashion, facilitating remedial action where necessary.

While our measures are designed for use in all financial systems, it is of particular interest to test them using the Australian setting for at least two reasons. First, the Australian financial system is dominated by only four banks. In Australia there are four major banks that hold 78% of the total assets of all Authorised Deposit-taking Institutions in Australia.[1] It is therefore obvious that the four majors are systemically important and the remaining ADIs are not so unless stress in one of these minors somehow creates financial contagion to the majors, in an unforeseeable way. Second, the Australian banking system proved to be resilient through the recent global financial crisis (GFC), implying that systemic risk did not reach the extreme levels experienced by other countries where bank failures occurred. Both these features of the Australian system suggest that it may be difficult to find useful additional information about systemic risk by refining quantitative measures like SRISK. Since we are able to show that our measures are informative about systemic risk in Australia, we conjecture that they will provide additional insights elsewhere.

To demonstrate the nature and usefulness of our measures, we use publicly available daily financial data for the eight Australian banks detailed in Table 1 spanning the period from 3 April 2000 through to 1 December 2014. The data is described in detail in Appendix B. Prices, adjusted for dividends are plotted in Figure 1. Prices are combined with number of shares to derive total equity. Total debt for each firm is also collected from annual accounting reports.

Remaining sections are structured as follows. Section 2 briefly discusses related litera-

[1]Based on balances at 31 December 2014 disclosed in APRA's Quarterly Authorised Deposit-taking Institution (ADI) Performance Report for March 2015.

ture. Section 3 and 4 define current and future capital shortfall similarly as Brownlees and Engle (2015) under a simple Basel II regulatory framework. The definition of systemic risk used by Brownlees and Engle (2015) is discussed in Section 5, and is refined and extended in Section 6. Applications to Australian bank data are discussed in Section 7 and Section 8. Section 9 discusses the aggregation of stresses across firms with and without allowance for merging or diversification. Section 10 displays stress calculations as they would proceed in real time. Section 11 offers an alternative modelling approach, based on assets. Section 12 concludes.

Table 1: Major and minor Australian banks

major		minor	
CBA	Commonwealth Bank	MQG	Macquarie
ANZ	Australia & New Zealand	BOQ	Bank of Queensland
NAB	National Australia	BEN	Bendigo and Adelaide
WBC	Westpac	ABA	Auswide

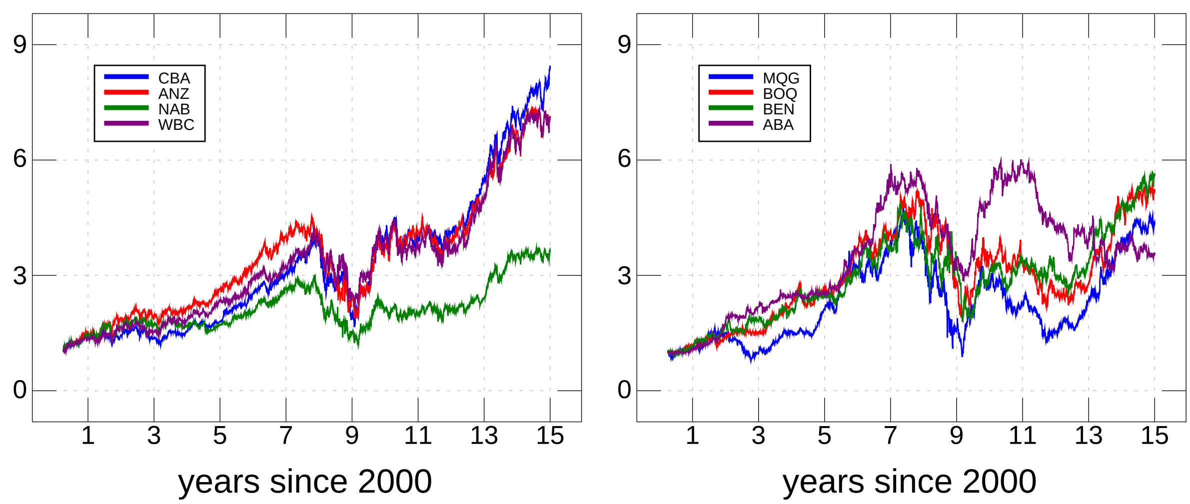

Figure 1: Daily stock prices of four major (left panel) and four minor (right panel) Australian banks 2000–2014. Price series normalised to start at 1.

2 Literature review

Many papers in the finance and economics literature are devoted to the development and application of a variety of quantitative measures of systemic risk or financial stress. Bisias et al. (2012) provide an instructive and qualitative survey of numerous measures in use. Giglio et al. (2015) supply empirical evidence on the ability of many of these measures, both individually and collectively, to provide early warning signals of deterioration in macroeconomic conditions. In this section, we provide a selective overview of those measures most closely related to SRISK and its relevance for assessing systemic risk in the financial sector.

The genesis of SRISK lies in a series of papers including Acharya et al. (2012), Acharya et al. (2012) and Brownlees and Engle (2010). In these papers, SRISK is defined as the expected capital shortfall of a financial firm, conditional on a crisis. A crisis is deemed to occur whenever a relevant market index suffers a significant decline over a chosen horizon. SRISK measures require an estimate of the long run marginal expected shortfall (LRMES) typically obtained by simulation methods.

Brownlees and Engle (2015) further develop the empirical methodology to construct SRISK measures. SRISK depends on the firm's size, leverage and its LRMES. The LRMES is obtained by assuming a dynamic process for the joint distribution of firm and market returns. Brownlees and Engle (2015) use the standard GARCH-DCC model of Engle (2002) with threshold ARCH volatilities on the basis that this represents a good trade-off between model complexity and prediction accuracy. The LRMES is computed as the Monte Carlo average of simulated multi-period returns, conditional on the return being worse than the cut-off level chosen to identify a financial crisis. Using a sample of large US financial firms, Brownlees and Engle (2015) show the practical usefulness of their measure in three ways: (i) SRISK rankings identify systemically risky US banks during the GFC; (ii) pre-crisis SRISK helps predict capital injections by the Fed Reserve; (iii) aggregate SRISK provides early warning of declines in industrial production and higher unemployment.

Engle et al. (2015) extend the model in Brownlees and Engle (2015) to incorporate global, country and industry effects. This extended model makes use of Engle (2014)'s dynamic conditional beta model. Their empirical methods are able to provide information about the relative systemic importance of industry type and country identity. For instance, they find that systemic risk in Europe during the GFC was predominantly due to banks while France and the UK recorded the largest country-level SRISK numbers.

Adrian and Brunnermeier (2011) propose an alternative measure of systemic risk that emphasises the co-dependence of financial firms and the importance of risk spillovers within the financial sector. They introduce CoVaR_i which computes the Value-at-Risk (VaR) of the entire financial system, conditional on institution i being in distress. Distress is defined as the state where the institution is exactly at its VaR level. Girardi and Ergün (2013) improve CoVaR by conditioning on the institution being at most at its VaR. This generalisation is useful as it takes into account the severity of tail losses and facilitates back-testing of CoVaR.

Acharya et al. (2012) relate SRISK and CoVaR and demonstrate that assuming the joint distribution of returns is conditionally normal, SRISK is a more complete measure. Benoit et al. (2013) show how several popular systemic risk measures including MES, CoVaR and SRISK are different transformations of market risk measures and derive those conditions under which they provide similar rankings of systemically important financial institutions.

Empirical estimates of SRISK requires a model for the joint distribution of returns of individual firms and the market. Many of these papers use Engle (2002)'s GARCH-DCC model aiming to capture the time-varying dynamics of the return (co)variances in a feasible manner. While we also use this model in our empirical analysis, we emphasize that our approach can utilise any appropriate model for simulating the joint distribution of firm and market returns going forward.

3 Capital shortfall and leverage

If d and w are the debt and equity, respectively, of a particular firm at a particular point of time, and k is the prudential requirement, then as defined Brownlees and Engle (2015), the capital shortfall at that time is

$$k(d+w) - w = kd\left(1 - \frac{1-k}{kL}\right) = kd\left(1 - \mathrm{e}^{-\ell}\right) \ , \tag{1}$$

where

$$L \equiv \frac{d}{w} \ , \qquad \ell \equiv \ln\left(\frac{kL}{1-k}\right) = \mathrm{logit}(k) + \ln(L) \ .$$

The quantity ℓ is called the adjusted log–leverage of firm and $\ell > 0$ implies, for the given k, capital shortfall is positive. The parameter k is the proportion of assets $d + w$ excluded from capital calculations, and higher k leads to higher capital shortfall.

The logit(k) enters ℓ as an additive constant and has minor role in the technical development and can be varied to test for senstivity etc. Assume[2] $k = 0.08$ as under Basel II implying logit$(k) = -2.44$ and the adjusted log–leverage ℓ is the actual log–leverage minus 2.44. At this k, if $S > 0$ there is positive capital shortfall and the firm or financial institution is said be in "Basel default" or a "Basel breach" has occurred. If $S < 0$ there is capital surplus and the firm is said to be "Basel compliant."

The definition of capital shortfall in (1) is restrictive if not simplistic. It is used here to conform to the previous literature and provide an easily accessible platform to the key results of this paper, without becoming entangled in precise and perhaps more realistic definitions of shortfall. Indeed Section 11 deals with a more realistic and intricate definition.

Figure 2 displays adjusted log–leverages ℓ with $k = 0.08$ for the four major and four minor Australian banks listed in Table 1 on the first trading day of each month from January 2003 through to December 2014. Note most banks are Basel compliant up to about 2008, entering into Basel breaches from 2009 as a result of the GFC. Prolonged Basel breaches are experienced for particular banks, notably NAB.

4 Future capital shortfall

Financial institutions and regulators are concerned with future shortfall. Future shortfall depends on the future return on equity. If r is the future return on equity over say a month then the equity in one month's time is $w\mathrm{e}^r$ and future shortfall is

$$S \equiv kd\left(1 - \frac{1-k}{kL\mathrm{e}^{-r}}\right) = kd(1 - \mathrm{e}^{r-\ell}) \ . \tag{2}$$

This assumes debt d stays constant over the month. The future return r is unknown but its probability distribution may be modelled and relatively well understood.

[2] See for example Brownlees and Engle (2015).

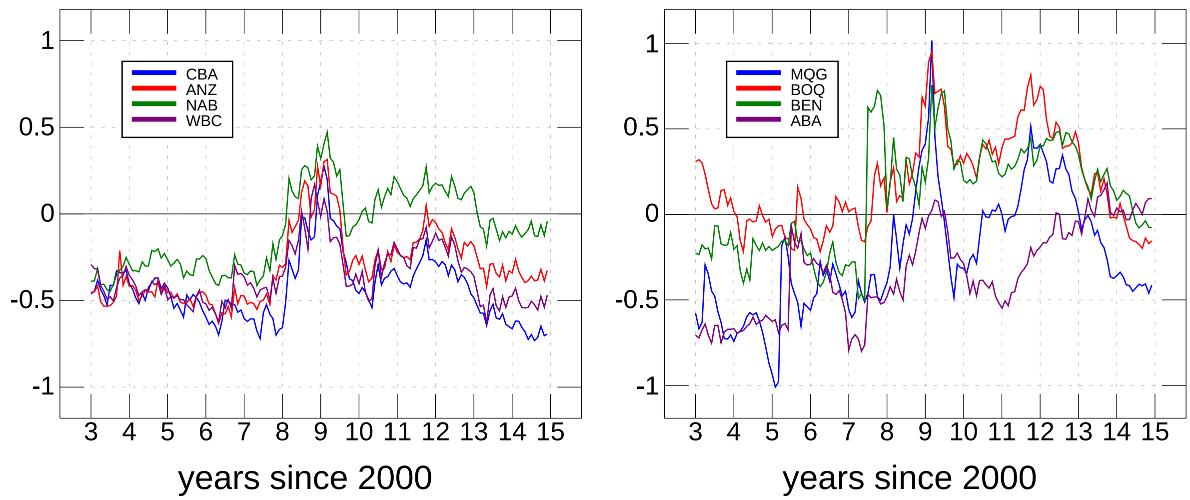

Figure 2: Adjusted log–leverages for four major (left panel) and four minor (right panel) Australian banks from the beginning of 2003 through to end of 2014. All banks contravene the threshold (corresponding to the zero horizontal) around the start of 2009.

The actual capital shortfall is S^+, the positive part of S:

$$S^+ = kd(1 - \mathrm{e}^{r-\ell})^+ \ . \tag{3}$$

Thus S^+ is kd times a put on the return $\mathrm{e}^{r-\ell}$ with strike 1. There is a future shortfall if $r < \ell$. Default put options similar to (3) have been discussed in the insurance literature as critical to an evaluation of a firm: see for example Merton (1977), Doherty and Garven (1986), Cummins (1988), Myers and Read (2001) and Sherris (2006).

To illustrate the setup, Figure 3 displays two snapshot outputs from projections generated as described in Appendix A: 6000 simulated one month ahead projected returns for CBA and the wider market, on two dates: the first trading day in January 2009 and the first trading day in December 2014. The CBA returns are used to calculate the future shortfall per units kd as in (2). Both panels use the same horizontal and vertical scales. Thus based on the available data at those two points in time, policy makers and regulators faced entirely different projections based on the same time series model. The scatter of dots are the forward simulations. Note there is no hindsight bias as the model at each time point is based on the available data to that point in time. In the left panel $S > 0$ for most scenarios. In the right panel $S < 0$ for any conceivable market scenario.

The black dots in each panel indicate the actual outcome after one month. In the left panel the outcome is a decline in both the market and CBA stock price. In the right panel there is slight decline in the stock price, and a more substantial market downturn. Thus at the beginning of January 2009 CBA was projected to be far from compliance in a month while in December 2014 the one month projection is of complete compliance.

The two panels display very different volatility and slopes. In January 2009 both CBA and market return distributions are highly volatile and correlated. The correlation remains strong in December 2014 but with less return volatility. These snapshots are used to compute baseline and systemic stresses shown in subsequent sections.

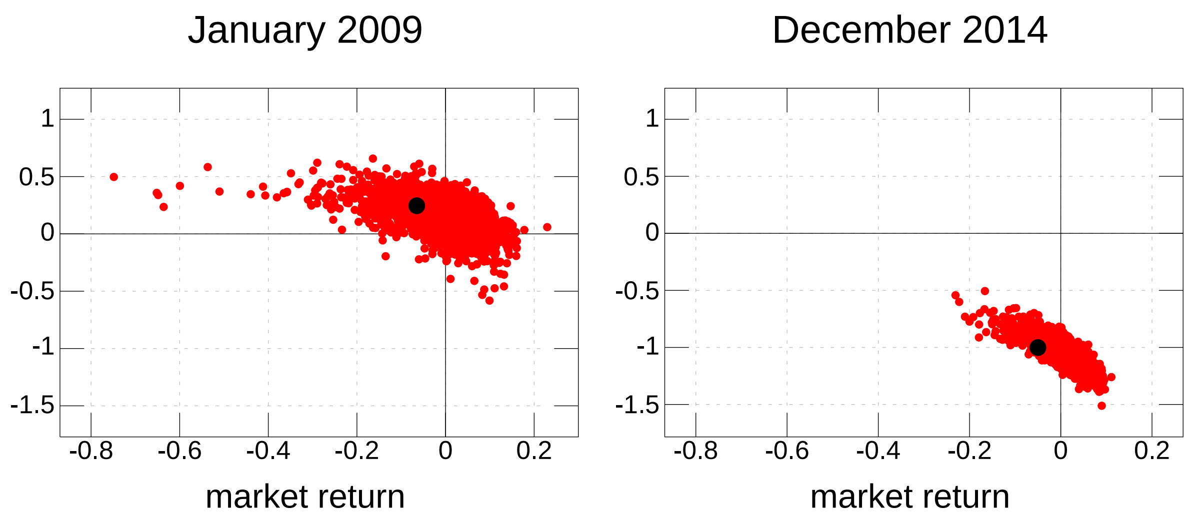

Figure 3: Forecast bivariate distribution of future shortfall S per unit kd for CBA (vertical axis) and market rate of return (horizontal axis) at start of January 2009 (left panel) and December 2014 (right panel). Note scales in both panels are the same, the relatively large volatility in the left panel, and the left skew in the marginal distributions. Black dots indicate actual outcome at the end of the month.

5 Systemic risk SRISK

Brownlees and Engle (2015) defines systemic risk for a group of firms at a particular point of time as

$$\text{SRISK} \equiv \sum_i \left\{\widetilde{\mathrm{E}}(S_i)\right\}^+ \ , \tag{4}$$

where S_i refers to the future shortfall in firm i. Further $\widetilde{\mathrm{E}}$ denotes expectation given a major general market downturn. Hence the expected capital shortfall (allowing surplus offset) is computed from (2) assuming a market downturn and added across all firms. Firms with an expected capital surplus under stress are ignored.

The systemic risk of firm i at a particular point of time is defined by Brownlees and Engle (2015) as the proportionate contribution (4):

$$\text{SRISK}_i \equiv \frac{\{\widetilde{\mathrm{E}}(S_i)\}^+}{\text{SRISK}} \ . \tag{5}$$

Large SRISK_i indicates firm i is systemically important: it holds a high proportion of the total debt, and it is likely to heavily breach the Basel capital requirement compared to remaining other firms. Expression (5) depends on k through each of the adjusted log–leverages ℓ_i. Given a general market downturn, the values $\{\widetilde{\mathrm{E}}(S_i)\}^+$ in (5) are known: there is no uncertainty except for possible uncertainty in estimating the stressed conditional expected value.

The above general setup is subject to criticisms and improvements. First, SRISK in (5) can be modified to be based on S_i^+ the positive part of shortfall rather than the positive part of expected shortfall. In many cases $\widetilde{\mathrm{E}}(S_i) = 0$ and insensitive to actual shortfalls. Second, the definition of $\widetilde{\mathrm{E}}$ is restrictive: a market downturn defined by a market downturn greater than 10%. Such a specific absolute downturn is unnecessarily arbitrary. For example the downturn is more easily achieved in a highly volatile environment and hence the stress response takes on different meanings depending on context. Third as shown below the stressed expectation $\widetilde{\mathrm{E}}$ is composed of two parts, so–called baseline risk and change in expectation risk caused by the actual stress event.

6 Improved systemic risk measurement

This section describes a general framework for systemic risk measurement. The framework builds on and aims to improve on the techniques built into f SRISK and SRISK_i in (4) and

(5). The framework assumes a response variable such as capital shortfall S or S^+ for a firm or group of firm subjected to stress. The framework puts "stress testing" on a formal footing.

For notational convenience assume the response of interest is S although this can be replaced with S^+ or some other function of S. The framework is built up from the following parts:

- A stress function $\psi(\omega) \geq 0$ defined on scenarios ω. Scenarios ω can be discrete or continuous. Further $\mathrm{E}(\psi) = 1$ where expectation is across all scenarios. Scenarios ω are systemic if they have potential impact on many firms.

- Baseline risk is defined as the ordinary expectation

$$\mathrm{BRISK} \equiv \mathrm{E}(S) \equiv \sum_{\omega} f(\omega)\mathrm{E}(S|\omega) \ ,$$

  where $f(\omega)$ denotes the "real world" probabilities of different scenarios. If the space of scenarios is continuous then $f(\omega)$ is a density and the sum an integral.

- Stressed expectation is defined in terms of the stress function as

$$\widetilde{\mathrm{E}}(S) \equiv \sum_{\omega} \psi(\omega) f(\omega)\mathrm{E}(S|\omega) = \mathrm{E}\{\psi \mathrm{E}(S|\psi)\} = \mathrm{E}(\psi S) = \mathrm{E}(S) + \mathrm{cov}(S, \psi) \ , \qquad (6)$$

  where the last equality follows since $\mathrm{E}(\psi) = 1$. The import of $\psi(\omega)$ is to change the natural probabilities $f(\omega)$ to "stressed" probabilities $\tilde{f}(\omega) \equiv \psi(\omega) f(\omega)$. Note $\sum_{\omega} \tilde{f}(\omega) = \mathrm{E}(\psi) = 1$.

- ψ–risk is defined as the difference between baseline risk and stressed risk

$$\mathrm{PRISK} \equiv \widetilde{\mathrm{E}}(S) - \mathrm{E}(S) = \mathrm{cov}(S, \psi) \ , \qquad \widetilde{\mathrm{E}}(S) = \mathrm{BRISK} + \mathrm{PRISK} \ .$$

A simple example is where $\psi(\omega) = 0$ except for a single scenario ω where it is equal to $1/f(\omega)$. In this case all probability is transferred to the single scenario ω and $\widetilde{\mathrm{E}}(S) = \mathrm{E}(S|\omega)$. The conditional expectation is often computed with a spreadsheet.

A richer example is where ω is the percentile of a variable such as the overall market return, measured relative to the currently applicable market return distribution. Then

$f(\omega)$ is the uniform density on the unit interval. If

$$\psi(\omega) = n\{1 - (1-\omega)^{n-1}\} \ ,$$

then ψ is the density of the worst percentile outcome in n independent trials. This stress function implies PRISK corresponds to the increase in risk when the general market has its worst outcome in n independent trials. This stress function is used to study stress in Australian banks in Section 8. If $n = 1$ then $\psi(\omega) = 1$, there is no stress, and $\widetilde{\mathrm{E}}(S) = \mathrm{E}(S)$ and PRISK=0.

Another example, close to that implemented in Brownlees and Engle (2015) is where ω is a percentile with $\psi(\omega) = 1/c$ for $\omega < c$ and 0 otherwise. In this case $\widetilde{\mathrm{E}}(S) = \mathrm{E}(S|\psi < c)$ and PRISK is the difference between the conditional tail and ordinary expectation. More general stressed expectations of the form of (6) are discussed in Furman and Zitikis (2008) and Choo and De Jong (2010).

Both BRISK and PRISK are linear and aggregate over firms. For example

$$\mathrm{PRISK}\left(\sum_i S_i\right) = \sum_i \mathrm{PRISK}\left(S_i\right) \ .$$

Linearity is obvious for BRISK and for PRISK follows from the linearity of covariance.

A useful measure of the "danger" associated with a stressor is the volatility of ψ denoted $\sigma_\psi = \sqrt{\mathrm{PRISK}(\psi)}$. For example if $\psi(\omega)$ picks out a single scenario ω then

$$\sigma_\psi = \mathrm{e}^{-\mathrm{logit}\{f(\omega)\}/2} \ ,$$

which is large if the scenario probability $f(\omega)$ is small. This suggest standardising PRISK:

$$\beta_\psi \equiv \frac{\mathrm{PRISK}}{\sigma_\psi} = \frac{\widetilde{\mathrm{E}}(S) - \mathrm{E}(S)}{\sigma_\psi} = \sigma_S \mathrm{cor}(S, \psi) \ .$$

measuring the departure of $\widetilde{\mathrm{E}}(S)$ from $\mathrm{E}(S)$ in units of ψ volatility. Here cor denotes correlation. Standardised PRISK β_ψ facilitates the comparison of stress on S across different stressors and, as the notation suggests, has the interpretation of a regression coefficient:

$$\mathrm{E}(S|\psi = x) \approx \mathrm{E}(S) + \beta_\psi \left(\frac{x-1}{\sigma_\psi}\right) \ .$$

Thus β_ψ measures, in minimum mean square error sense, the change in the expectation of S as stress increases by one standardised unit.

Stress can also be measured in units of S volatility:

$$\frac{\text{PRISK}}{\sigma_S} = \frac{\widetilde{\text{E}}(S) - \text{E}(S)}{\sigma_S} = \sigma_\psi \text{cor}(S, \psi) \ .$$

This measure gauges the extremity of shortfall.

In many financial contexts expected shortfalls may increase due to increased volatility. This is captured with BRISK. PRISK focusses on the stress effects induced with ψ.

BRISK and PRISK are easily computed in a simulation environment given a joint model $f(S, \psi) = f(\psi) f(S|\psi)$ for S and the scenarios ω. If N of simulations $\omega \sim f(\omega)$ and $S(\omega) \sim f(S|\omega)$ are generated and then

$$\text{BRISK} \approx \frac{1}{N} \sum_\omega S(\omega) \ , \qquad \text{PRISK} \approx \frac{1}{N} \sum_\omega \{\psi(\omega) - 1\} S(\omega)$$

If ψ is based on percentile outcomes then the simulated ω are ranked and $\psi(\omega)$ based on the rank of ω. If ω has many components corresponding to percentiles of different variables then $\psi(\omega)$ is a copula density with appropriate induced tail dependence.

7 Forward shortfall and return simulations

The estimation of BRISK and PRISK requires simulated future capital shortfalls. As in Brownlees and Engle (2015), projections in this paper are constructed using time series models of forward rates of return for each bank i and the market rate of return. The market return is used as the stressor with different choices of the stress function ψ modelling different stress scenarios.

The time series models used are stochastic volatility models based on the GARCH–DCC model of Engle (2002) summarised in Appendix A. The GARCH-DCC model captures prolonged periods of high volatility and correlation in firm and market returns, typical in financial markets. The GARCH–DCC framework is one possible implementation. For example future return scenarios may be constructed in a more ad–hoc manner e.g. judiciously constructed scenarios by regulators or policymakers. The framework set out in Section 6 can be based on any generated future scenarios.

Figure 3 displays simulations from a joint distribution generated from the GARCH–DCC model estimated from daily returns. The left panel utilises one month forward simulated returns at the start of January 2008. The market returns are on the x–axis while the shortfall S corresponding to associated CBA return are on the y–axis. Low market returns are likely to lead to sharply higher CBA shortfall, as compared to the right panel showing the same as at December 2014. This is suggested by the slope of the scatter plots: the left slope appears steeper than the right. This implies higher systemic stress in December 2008. In addition S volatilities are much higher in the left panel compared to the right, indicating higher baseline stress in December 2008. Also note that with a stressor function ψ defined in terms of market return percentiles, the magnitude of a market downturn at a fixed percentile threshold is more significant in the left panel.

8 One month ahead forecast risk in individual banks

Figure 4 displays, in the top panels, the estimated BRISK corresponding to S^+ for each major (left) and minor (right) Australian banks based on the projected one month ahead return distribution fitted using the GARCH–DCC model. Note the return distributions are constructed from data only available at that time and hence are not affected by look–ahead bias. The first inclinations of increasing BRISK with NAB in early 2008 followed one month later by ANZ, and a further few months later by WBC and CBA. BRISK subsided shortly after 2009, however BRISK for NAB rose again in 2011 and this sustained till 2013. In general smaller banks are subject to higher BRISK after normalising for debt levels.

Bottom panels in Figure 4 show PRISK computed by assuming the worst market return in 12 identical months. PRISK for each bank generally exhibits similar patterns as BRISK. However, importantly, some banks have differing patterns which is an important observation for the regulator since PRISK indicates sensitivity to market-wide downturns. For example ANZ had similar PRISK stress as NAB around 2012 but lower BRISK. Hence although ANZ was not obviously in stress during 2012, it would be if a market downturn occurred. Bendigo and BOQ had high BRISK levels after 2008, but are overtaken by Macquarie in terms of PRISK: Macquarie is more likely to suffer in a market downturn whereas Bendigo and BOQ are less likely to be impacted.

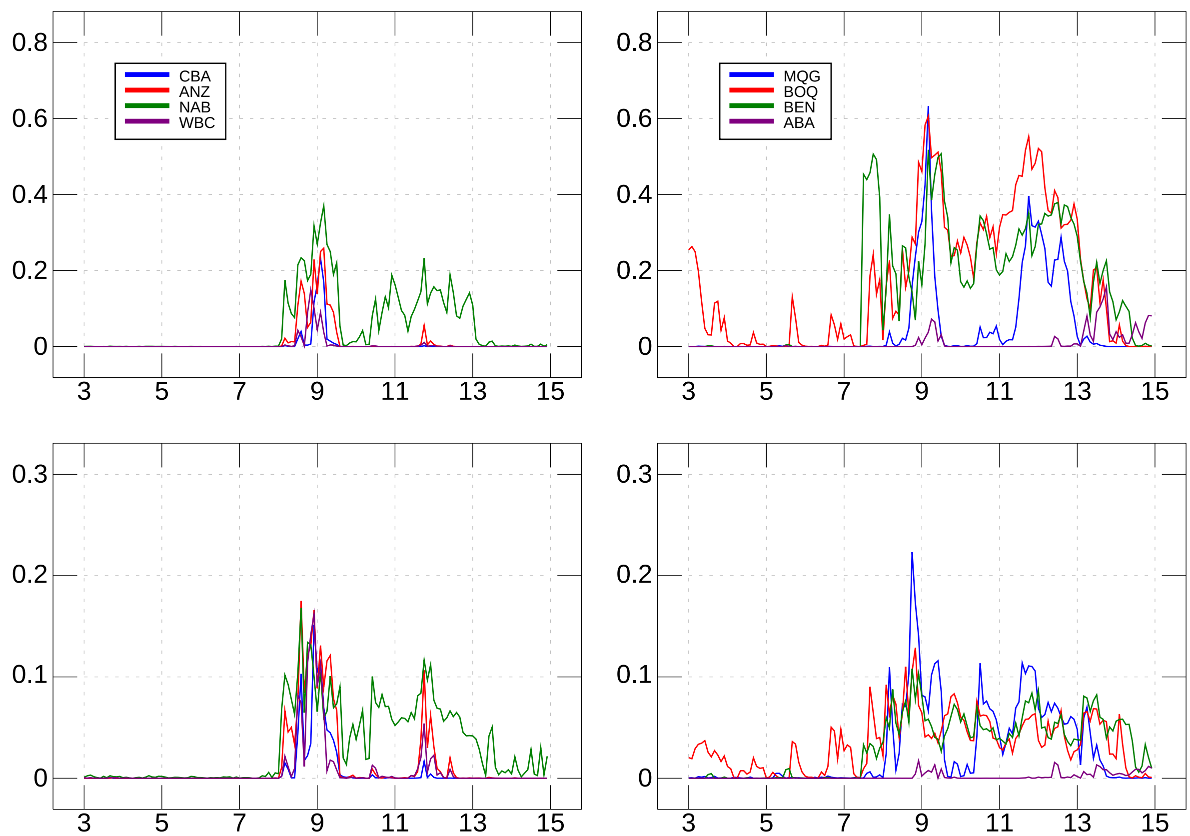

Figure 4: Forecast one month ahead BRISK (top panels) and PRISK (bottom panels) based on S^+ for major (left panel) and minor (right panel) banks 2003–2014. Note different y–scale on top and bottom panels.

9 Aggregate market stress

Brownlees and Engle (2015) defines systemic risk in terms of $\widetilde{\mathrm{E}}(S)$. However a stressed expectation may be high simply on account of higher than normal volatility or leverage. Hence Brownlees and Engle (2015) combines both baseline stress, due to high volatility or leverage, and the imposed system stress. However it is important to understand the mix of the two different stresses.

Two practical alternatives to SRISK for aggregating risk in financial institutions are set out Table 2. The first row of the table reproduces the defintion of SRISK in (5) and decomposes the same into BRISK and PRISK. It is apparent that SRISK measures a mix of BRISK and PRISK. Further risks only arise if there is an "on average" shortfall and the measure is relatively insensitive to tail events .

Table 2: Measures of aggregate stress

Measure	Expression
SRISK (Brownlees and Engle, 2015)	$\sum_i \{\widetilde{\mathrm{E}}(S_i)\}^+ = \sum_i \{\mathrm{BRISK}(S_i) + \mathrm{PRISK}(S_i)\}^+$
No shortfall diversification: $S^* \equiv \sum_i S_i^+$	$\sum_i \widetilde{\mathrm{E}}(S_i^+) = \mathrm{BRISK}(S^*) + \mathrm{PRISK}(S^*)$
Shortfall diversification: $S \equiv \sum_i S_i$	$\widetilde{\mathrm{E}}\{(\sum_i S_i)^+\} = \mathrm{BRISK}(S^+) + \mathrm{PRISK}(S^+)$

The second aggregation of risk displayed in Table 2 uses S^*, the aggregate sum of positive shortfalls. With S^* shortfall in one firm are not offset by surpluses elsewhere and the measure is sensitive to tail risk. If subscript % denotes percentage contribution to the corresponding total then a direct calculation shows the total risk BRISK + PRISK in S_i^+ as a percentage of the total risk in $S^* \equiv \sum_i S_i^+ \geq S^+$ is

$$\mathrm{PRISK}_{\%}(S_i^+) + \frac{\mathrm{BRISK}(S^*)}{\mathrm{BRISK}(S^*) + \mathrm{PRISK}(S^*)} \left\{\mathrm{BRISK}_{\%}(S_i^+) - \mathrm{PRISK}_{\%}(S_i^+)\right\} , \qquad (7)$$

where the percentage subscripts indicate percentages of the corresponding totals. Thus total risk is dominated by BRISK if the latter is large compared to PRISK as apparent in the time series analyses in Section 8.

The final row measure of Table 2 allows for pooling of shortfall across firms. Since

$S^* \geq S^+$ and both BRISK and PRISK are additive

$$\sum_i \text{BRISK}(S_i^+) = \text{BRISK}(S^*) \geq \text{BRISK}(S^+) \ ,$$

$$\sum_i \text{PRISK}(S_i^+) = \text{PRISK}(S^*) = \text{PRISK}(S^+) + \text{PRISK}(S^* - S^+) \ .$$

The difference $S^* - S^+$ is the "diversifiable" shortfall and PRISK$(S^* - S^+)$ measures the covariance between the diversifiable shortfall and stress ψ.

System resilience aims to answer the question of whether the system as a whole can absorb shocks. The capacity to absorb relies on implicit merging and hence on a measure of shortfall such as S^+. When two firms merge the leverage of the resulting firm is less than that of the more highly leveraged firm. The merged firm is more able to withstand return on equity shocks unless negative shocks are more prone for the merged firm. Similarly when many firms merge puts S^+ for the conglomerate become less valuable.

Firms do not necessarily merge, even in dire financial circumstances. Hence the mergers spoken of here are hypothetical. From a regulators perspective, what would happen under a merger of all firms (or perhaps a group of firms) is nevertheless of interest. A system as as a whole near Basel breach is more threatening that one where a few firms are near Basel breach but the system as a whole is strongly Basel compliant.

10 Monthly monitoring of financial stress

Table 3 contains real time stress calculations on the first trading day of January 2009 and December 2014. As before there is no look ahead bias – calculations on each of the two dates use data available on the first day of the applicable month. The first eight rows in in the body of Table 3 correspond to the eight banks used in this study. The first and second columns in the two halves of the table body contain the Adjusted log–leverage and debt (as a percentage of total sustem debt) for each of the banks.

January 2009 was a time of great stress for all eight banks. Six of the eight banks were in Basel default with positive capital shortfalls as indicated by the adjusted log–leverage column: the two banks not in Basel default were WBC and ABA.

Table 3: Projected one month ahead stress for banks

	January 2009				December 2014			
	Aloglev	Debt	BRISK	PRISK	Aloglev	Debt	BRISK	PRISK
CBA	18.57	24.30	22.99	29.60	-70.34	22.70	0.00	0.00
ANZ	15.93	18.37	14.87	18.89	-38.64	22.10	0.00	0.00
NAB	31.97	25.50	39.98	19.41	-12.45	25.61	50.52	67.13
WBC	-1.55	22.84	5.86	23.94	-53.84	22.03	0.00	0.00
MQG	41.22	5.75	11.02	5.46	-46.10	4.33	0.17	0.12
BOQ	63.30	1.32	3.55	0.99	-17.05	1.33	0.80	1.34
BEN	18.91	1.81	1.72	1.71	-7.63	1.84	25.43	30.71
ABA	-8.12	0.10	0.00	0.00	9.22	0.07	23.07	0.71
Total S^*	18.78	2.42	17.17	8.63	-41.91	3.27	0.03	0.12
Pooled S^+	17.81	2.42	15.28	10.18	-44.22	3.27	0.00	0.00

The main body of table uses shortfall measure is S_i^+. Aloglev is 100 times the adjusted log–leverage. PRISK is with respect to expected worst monthly market return in 12 months. Main body of table displays percentage contributions of each firm to totals displayed in "Total" row. Total row displays 100 times debt weighted adjusted log–leverage, total debt in \$$10^{11}$, and BRISK and PRISK per 0.08×debt and based on S^*. Pooled row displays results with debt and equity pooled across banks and results based on S^+.

Percentage BRISK and PRISK are displayed in the next 2 columns in each half of the table. The baseline stress column indicates most of the baseline stress arises from NAB – almost 40% of the total. The next most baseline stressed bank is CBA with WBC also a substantial contributor. The MQG bank contributes almost double to baseline stress compared to the proportion of total debt it carries. The other three small banks contribute relatively little to baseline stress with BOQ almost 3 times expected on the basis of its debt. PRISK is highest for the CBA, higher than expected on the basis of its debt load and hence CBA was most susceptible to stress from additional general market equity devaluation. All other banks appear have PRISK comparable to their size in terms of debt load with only

NAB being less systemically important. This should be compared to NAB's high baseline stress.

Continuing with January 2009, the final two rows indicate the total amount of stress in the system and it's diversifiability. The second last row labelled "Total" displays, in order, debt weighted adjusted log–leverage, total BRISK and total PRISK. On aggregate Total BRISK is about twice PRISK. Thus there is more danger of increasing capital shortfall due to market volatility as opposed to further stress from further substantial general market devaluation. The final row indicates stresses are not diversifiable: The marginally smaller "diversified" baseline stress is offset by an increase in PRISK.

Stress readings change dramatically when moving to December 2014 – there is virtually no stress in any bank and the small amount of stress in the system is diversifiable. Most BRISK is carried by NAB with lesser contributions by BEN and ABA. All the stress in the minor bank ABA is baseline stress as only NAB and NAB have substantial PRISK contributions. Again, however, it must be emphasised that there is minimal systemic stress in the system. Notice total debt in the banking sector jumps about 35% between the two dates.

The two top panels of Figure 5 track BRISK and PRISK for the banking sector as a whole through time. There is virtually no risk in the system till late 2007 and the early warnings show a buildup in BRISK in S^* compared to S^+. Thus initially all BRISK is diversifiable suggesting BRISK is building up in a few banks. This pattern of diversifiability in BRISK is generally maintained except at the peak of the crisis at the beginning of 2009 and in late 2011. This pattern of BRISK buildup is evident in the bottom panel displaying all BRISK is, for small levels of risk initially absorbable – that is present in S^* but not present in S^+.

The top right panels in Figure 5 display the time series behaviour of PRISK. Generally PRISK is less than BRISK. Further, in situations of elevated risk, PRISK in S^+ exceeds that in S^*f indicating systemic stress from a general market downturn is greater in the system as a whole than its constituent parts. Again, however, as displayed in the bottom right panel, PRISK initially builds up in S^*, and only later manifests itself in S^+. However the buildup is rapid so that with significant PRISK the level in S^+ exceeds that in S^*.

Figure 6 gives a picture of the relative magnitudes of BRISK and PRISK in each of the eight banks. For the major banks NAB stands out as, having BRISK more dominant than PRISK and generally having heightened levels of BRISK. ANZ appears to have relatively higher levels of PRISK. For the minors, MQG has persistently higher levels of PRISK with BOQ and BEN comparable ratios and ABA having virtually no PRISK compares to BRISK at any level of the latter.

11 Risk weighted asset methodology

This section displays the application of the current stress framework to the risk weighting asset (RWA) methodology often used in determining capital shortfall. RWA in effect assesses future stressed assets, as opposed to, with SRISK, future equity. The same stress methodology can be used to stress assets.

Given debt d and equity w for a firm at a particular point of time, assets are

$$a \equiv d + w = \sum_j a_j \ ,$$

where a_j denotes the value of assets in asset class j. In terms of current assets a, future shortfall is, equivalent to (2),

$$S \equiv d - \alpha \mathrm{A}(\mathrm{e}^{r_j}) \ , \qquad \alpha \equiv (1-k)a \ , \qquad \mathrm{A}(\mathrm{e}^{r_j}) \equiv \sum_j \frac{a_j}{a} \mathrm{e}^{r_j} \ . \tag{8}$$

Here r_j is the (uncertain) log–return on assets a_j and A denotes weighted averaging using weights a_j, the current value of assets in class j.

The expected shortfall is

$$\mathrm{E}(S) = d - \alpha \mathrm{E}\{\mathrm{A}(\mathrm{e}^{r_j})\} = d - \alpha \mathrm{A}\{\mathrm{E}(\mathrm{e}^{r_j})\} \ . \tag{9}$$

With the risk weighting methodology the weights a_j/a in A are replaced by $a_j/(a\sigma_j)$ where $\sigma_j > 0$ is a "risk" weight. Large risk weights σ_j are assigned to assets classes j deemed risky: assets are downweighed according to perceived "riskiness."

The methodology set out in previous sections can be used to formalise the selection of weights σ_j. In particular $\widetilde{\mathrm{E}}$ based on an appropriate risk function ψ replaces E in (9) and

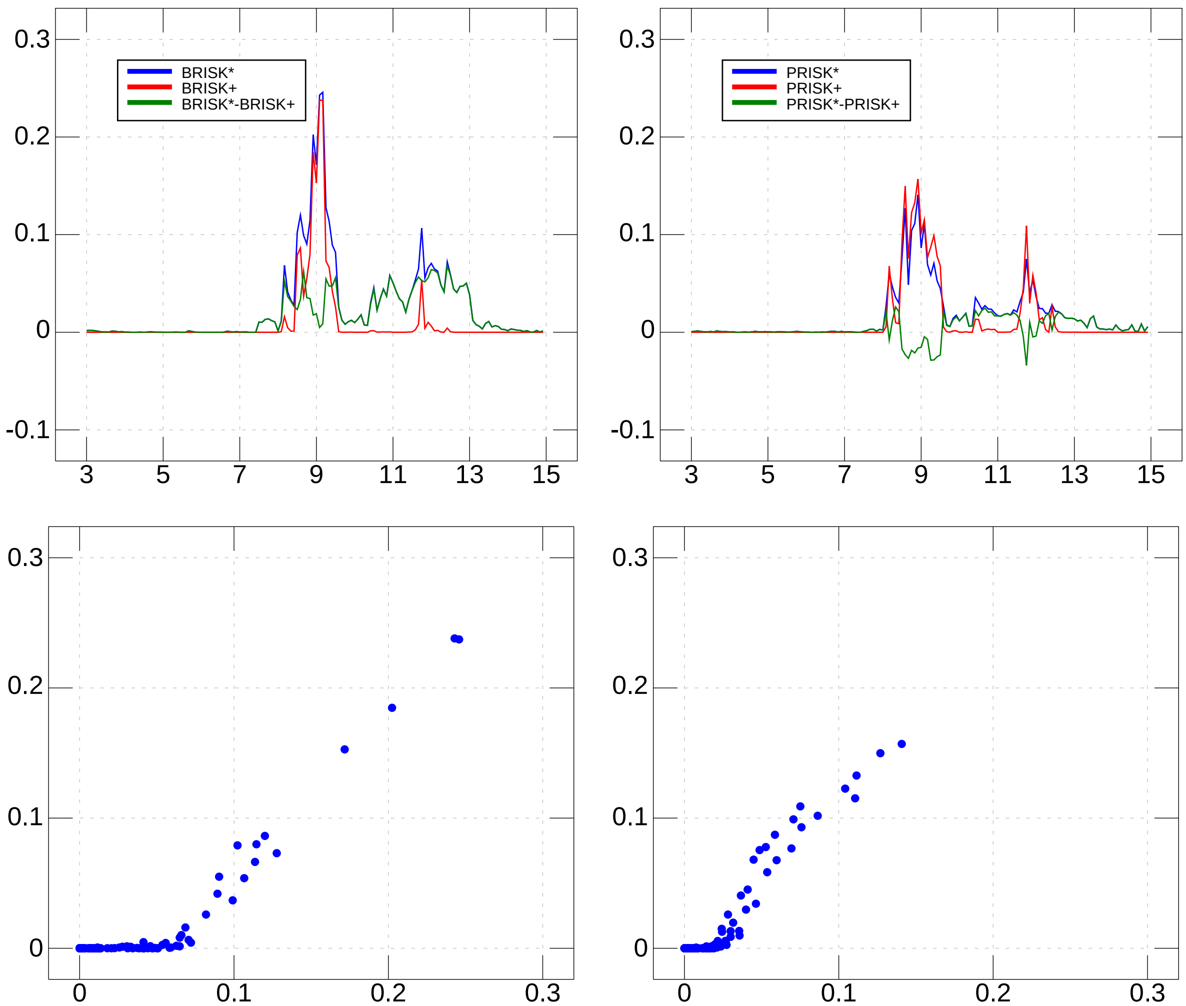

Figure 5: Risk based on S^* and S^+. Results for BRISK and PRISK in left and right panels, respectively. The top panels track results from the beginning of 2003 through to the end of 2014. Bottom panels plot two measures in the corresponding top panel against each other: On the left BRISK(S^+) on y–axis versus BRISK(S^*) on x–axis, and similarly for PRISK in the bottom right panel.

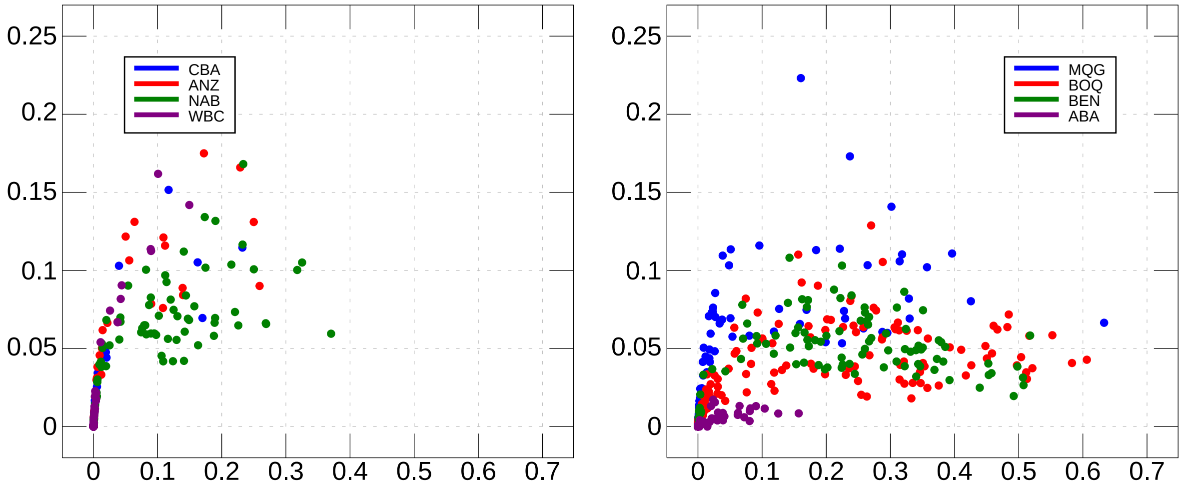

Figure 6: PRISK (y–axis) versus BRISK (x–axis) for major banks (left panel) and minor banks (right panel) for 144 months 2003–2014

leads to BRISK and PRISK. In particular

$$\mathrm{PRISK}(S_i) = -\alpha \times \mathrm{PRISK}\{\mathrm{A}_i(\mathrm{e}^{r_j}\} = -\alpha \times \mathrm{A}_i\{\mathrm{PRISK}(\mathrm{e}^{r_j})\} ,$$

with a similar result for BRISK. Here S_i is the shortfall for firm i and A_i utilises the asset weights of firm i.

If $\mathrm{PRISK}(\mathrm{e}^{r_j}) \approx \mathrm{PRISK}(r_j)$ is negative and asset j is important then $\mathrm{PRISK}(S_i)$ is large. Hence risky assets increase the PRISK associated with the firm and implicitly increases the imputed shortfall. Note $\mathrm{PRISK}(\mathrm{e}^{r_j})$ is independent of i: different firms combine these via firm specific asset weighting A_i. Thus a model is constructed for each asset class j rather than, as in the previous sections, each equity i. Returns r_j and stresses ψ are jointly simulated from a joint density $f(r_j, \psi) = f(\psi)f(r_j|\psi)$ and simulated returns are used to compute the shortfall (8) and, in turn BRISK and PRISK.

12 Conclusion

This paper presents a consistent methodology for stress testing and the measurement of risk using the concept of stressed expectations. Stressed expectation formalises stress testing widely used in practice. Stresses are introduced using a stress function and systemic stress uses a stress function based on system wide variables. Methods and concepts are applied to publicly available time series of Australian bank data to monitor stress in Australian banks over time. The methods identity banks in distress and those with high contribution to systemic stress.

SUPPLEMENTARY MATERIAL

A GARCH–DCC model

Denote the daily (log) return for firm i at time t as

$$\delta_{it} = \mu_i + \sigma_{it}\epsilon_{it} \ , \qquad \epsilon_{it} \sim (0,1) \ . \tag{10}$$

The volatility σ_{it} is modelled as

$$\sigma^2_{i,t+1} = \omega + \sigma^2_{it}\{\beta + (\alpha + \gamma\epsilon^-_{it})\epsilon^2_{it}\} \ , \qquad \epsilon^-_{it} \equiv I(\epsilon_{it} < 0) = I(\delta_{it} < \mu) \ , \tag{11}$$

where I denotes the indicator function. Hence the response of $\sigma^2_{i,t+1}$ to ϵ^2_{it} is increased by γ if the rate of return is below the average μ_i, compared to the response if $\delta_{it} > \mu_i$. Equations (10) and (11) defined a simple threshold GARCH model: called the TARCH(1,1). In (10) the mean μ_i does not vary with time t and in (11) it is assumed the terms σ^2_{it} and ϵ_{it} in the right hand side of (11) are sufficient to structure the dynamics of volatility and contemporaneous correlation.

The model defined by (10) and (11) is estimated for each security i jointly with a similar model for the market, $i = m$. Correlation between security i and the market m are implicitly modelled using positive definite recursions (Engle, 2002)

$$(Q_{i,t+1} - S) = \alpha(\eta_{it}\eta'_{it} - S) + \beta(Q_{it} - S) \ , \qquad \eta_{it} \equiv (\epsilon_{it}, \epsilon_{mt})'.$$

The correlation defined by Q_{it} is used as the correlation between ϵ_{it} and ϵ_{mt}.

Table 4: DataStream codes

Company codes	
A:ANZX	Australia and New Zealand Banking Group Limited
A:ABAX	Auswide Bank Limited
A:BOQX	Bank of Queensland Limited
A:BENX	Bendigo and Adelaide Bank Limited
A:CBAX	Commonwealth Bank of Australia
A:MQGX	Macquarie Bank Limited
A:NABX	National Australia Bank Limited
A:WBCX	Westpac Banking Corporation.
Datatype codes	
NOSH	Number of Shares
P	Price
WC02999	Total Assets
WC03501	Shareholders Equity
RZ	Total return index

B Data sources

Data are sourced from DataStream using the company and datatype codes as set out in Table 4.

Firm equity w is Number of Shares multiplied by Price. Firm debt d is Total Assets minus Shareholders Equity. Stock returns are computed from the Total Return Index including paid dividends.

C Computations

All model fits in this paper are performed using the R language (R Development Core Team, 2008) and in particular the rmgarch package described by Ghalanos (2012). All other calculations including simulations are implemented in the J language (Iverson, 2003).

In the forward simulated forward projections the innovations are chosen randomly from past innovations. These past innovations are chosen consistently: at a particular t either all or none of ϵ_{it} are chosen.

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