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Systemic risk measurement: Multivariate GARCH estimation of CoVaR ☆

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ABSTRACT

We modify Adrian and Brunnermeier's (2011) CoVaR, the VaR of the financial system conditional on an institution being in financial distress. We change the definition of financial distress from an institution being exactly at its VaR to being at most at its VaR. This change allows us to consider more severe distress events, to backtest CoVaR, and to improve its consistency (monotonicity) with respect to the dependence parameter. We define the systemic risk contribution of an institution as the change from its CoVaR in its benchmark state (defined as a one-standard deviation event) to its CoVaR under financial distress. We estimate the systemic risk contributions of four financial industry groups consisting of a large number of institutions for the sample period June 2000 to February 2008 and the 12 months prior to the beginning of the crisis. We also investigate the link between institutions' contributions to systemic risk and their characteristics.

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1. Introduction

The recent financial crisis has alerted the public to the fragility of the financial system and systemic risk. Value-at-Risk (VaR), arguably the most widely-used risk measure by financial institutions, has been criticized by many as incapable of capturing the systemic nature of risk since its focus is on an institution in isolation. VaR has been used by regulators as an instrument to determine capital levels that need to be set aside by financial institutions against market risks. However, since VaR considers only the risk that an institution faces when considered in isolation, it is not possible to gauge the risk facing the financial system from an institution's VaR. As a result, recently, there has been considerable interest in alternative risk measures which do not suffer from VaR's shortcoming, namely, its inability to account for the possibly systemic nature of an institution's risk and financial distress.

One of these recent studies is Adrian and Brunnermeier (2011) (referred to as AB henceforth) who introduce a new risk measure: Conditional Value-at-Risk (CoVaR). They define $\text{CoVaR}^{i|j}$ as the VaR of institution i conditional on institution j being in financial distress, which they define as institution j being at its VaR. By conditioning on another institution's financial distress, they aim to go beyond idiosyncratic risk and to capture possible risk spillovers among financial institutions.

While the $\text{CoVaR}^{i|j}$ measure can be computed for any two financial institutions i and j, AB consider the specific case where i is the financial system. In this case CoVaR becomes the VaR of the financial system conditional on institution j being in financial distress, and hence can be used to determine a financial institution's contribution to systemic risk. While it may not be easy to find consensus on the exact definition of systemic risk, the following quote from the Federal Reserve Governor Daniel Tarullo's July 2009 testimony before the Senate Banking, Housing, and Urban Affairs Committee is one definition that many can probably agree on, and also one that CoVaR seems to capture[1]

> "*Financial institutions are systemically important if the failure of the firm to meet its obligations to creditors and customers would have significant adverse consequences for the financial system and the broader economy.*"

Among other recent studies that propose measures to quantify systemic risk are Billio et al. (2012), Zhou (2010), Huang et al. (2009), Segoviano and Goodhart (2009), Acharya et al. (2010),

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[1] http://www.federalreserve.gov/newsevents/testimony/tarullo20090723a.htm.

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Brownlees and Engle (2011), and Allen et al. (2010). Billio et al. (2012) use principal components analysis and Granger-causality tests to propose several econometric measures of systemic risk to capture interconnectedness among the returns of hedge funds, banks, brokers, and insurance companies. Zhou (2010) uses multivariate Extreme Value Theory framework to provide two measures of systemic risk: the systemic impact index and the vulnerability index. The former assesses the risk that an institution imposes on the system and the latter the risk that the system imposes on the institution. Huang et al. (2009) use data on credit default swaps (CDSs) of financial institutions and equity return correlations to model systemic risk as the price of insurance against financial distress. Segoviano and Goodhart (2009) work with CDS data too, and measure institutions' contributions to the distress of the financial system within a multivariate setting. Acharya et al. (2010) use equity returns of financial institutions to calculate Systemic Expected Shortfall (SES) and Marginal Expected Shortfall (MES). MES is an institution's average loss when the financial system is in its left tail, and SES is calculated as the weighted average of the institution's MES and its leverage. While Acharya et al. (2010) calculate time-invariant MES measures, Brownlees and Engle (2011) compute their time-series by using a bivariate GARCH model and non-parametric tail estimators. Finally, Allen et al. (2010) propose a measure of aggregate systemic risk (CATFIN) which, as opposed to the micro-level systemic risk measures such as CoVaR and MES, gauges the macroeconomic effects of systemic risk taking in the banking system as a whole.

When we compare CoVaR with the MES measure of Acharya et al. (2010) and Brownlees and Engle (2011), there is a difference in terms of the conditioning event and the direction; while MES looks at the returns of an institution when the financial system is in distress and experiencing losses, CoVaR does the opposite and looks at the returns of the financial system when an institution is in financial distress. This difference arises not because of the intrinsic properties of the two measures, but because of the usage that has been done of the two. In fact, for both measures it is possible and straightforward to reverse the analysis. In that case CoVaR would correspond to the VaR of an institution conditional on the financial system being in distress, i.e., being at its VaR. This reverse CoVaR would be more in the spirit of MES as it would be measuring the exposure of an institution to the distress of the financial system.[2] However, in the systemic risk definition given in the quote above, it is the failure of an institution that is the cause of distress for the financial system. Therefore, like AB, we only consider CoVaR$^{i|j}$ where i is the financial system, and condition on the financial distress of an institution j.

In this study, we generalize the definition of CoVaR proposed by AB by assuming that the conditioning financial distress event refers to the return of institution j being *at most* at its VaR ($R^j \leqslant \mathrm{VaR}^j$) as opposed to being *exactly* at its VaR ($R^j = \mathrm{VaR}^j$). This change allows us to consider more severe distress events of institution j that are farther in the tail (below its VaR) and to backtest CoVaR estimates using the standard Kupiec (1995) and Christoffersen (1998) tests used for backtesting VaR. In addition, this change also improves the consistency of CoVaR with respect to the dependence parameter; Mainik and Schaanning (2012) show that when financial distress is defined as proposed in our work ($R^j \leqslant \mathrm{VaR}^j$), for a wide range of models, CoVaR has a monotonic relation with the dependence parameter.[3] In other words, as the institution becomes more and more correlated with the financial system, its systemic risk increases. On the other hand, when financial distress is defined as in AB ($R^j = \mathrm{VaR}^j$), CoVaR counter-intuitively starts decreasing after the dependence parameter crosses a threshold. As Mainik and Schaanning argue, in this case CoVaR fails to detect systemic risk where it is most pronounced (high degree of dependence) and financial regulation based on CoVaR with this distress definition could introduce additional instability and set wrong incentives. Lastly, due to the time-varying correlation of the GARCH model, the CoVaR of an institution here has a time-varying exposure to its VaR which, by construction, is not the case in Adrian and Brunnermeier (2011). This feature allows the possible changes over time in the linkage between the institution and the financial system to be captured and incorporated in the systemic risk measure.

We define the systemic risk contribution of an institution as the change from its CoVaR when the institution is in its benchmark state to its CoVaR under financial distress. We define the benchmark state as a one-standard deviation about the mean event. Using daily data from June 2000 to February 2008 for a large number of financial institutions from four industry groups (depositories; insurers; broker-dealers; and others, which are non-depository institutions including government sponsored enterprises), we obtain the industry groups' time-varying CoVaR estimates. We use a three-step procedure to obtain the CoVaR estimates: in the first step we estimate the VaR of each financial institution; in the second step we use the bivariate DCC model of Engle (2002) to estimate the joint distribution of the financial system-institution pair for each institution; and in the third step we numerically solve for CoVaR. To take both skewness and kurtosis into account, we estimate the GARCH models (univariate in the first step and bivariate in the second step) using skewed-*t* as well as the Gaussian distribution. We backtest the CoVaR estimates retrieved from our three-step procedure. We also investigate the link between institutions' characteristics such as size, leverage, and equity beta and their contributions to systemic risk by running panel regressions. Finally, using 12 months of data prior to the beginning of June 2007, we compute each industry group's pre-crisis systemic risk contribution.

Backtesting results show the importance of taking skewness and kurtosis into account as the CoVaR estimates based on the Gaussian distribution fail to satisfy the unconditional coverage property and hence are rejected. CoVaR estimates based on the skewed-*t* distribution, on the other hand, satisfy both the unconditional and conditional coverage properties. During the sample period June 2000 to February 2008, depository institutions were the largest contributors to systemic risk followed by broker-dealers, insurance companies, and non-depository institutions. Unlike in AB, our systemic risk measure and VaR have a weak relation not only in the cross-section but also in the time-series. Thus, the time-series of the systemic risk measure here potentially could have information that is different from the information in the time-series of the institution's VaR. From a regulatory perspective this suggests that monitoring a firm's tail risk may not be sufficient to forecast its systemic risk contribution. Institutions' leverage, size, and beta play an important role in explaining their contributions to systemic risk. Our pre-crisis analysis shows that the systemic risk of all industry groups increased substantially during the 12 month period prior to the beginning of June 2007. Prior to the crisis, while depository institutions were systemically the most risky followed by broker-dealers as in our large sample, the gap was almost closed.

The remainder of the paper is organized as follows: Section 2 formally defines CoVaR and presents the three-step procedure to estimate it. Section 3 describes the maximum likelihood estimation and the backtesting procedure. Section 4 presents the data and estimation results for the June 2000 to February 2008 sample. Section 5 has the results of the subsample analysis using

[2] As AB mention, CoVaR is directional; there is no reason to expect CoVaR of the system conditional on an institution to be the same as CoVaR of the institution conditional on the system. See AB for other properties of CoVaR.

[3] The models they study are when the financial system - institution pair has an elliptical bivariate distribution such as bivariate Gaussian or bivariate-*t*, has bivariate distributions with Gaussian and *t* copulas, and has a bivariate Gumbel copula with *t* marginals.

12 months of data prior to June 2007 to compute each industry group's pre-crisis systemic risk contribution. Section 6 concludes.

2. Methodology

Given the returns R_t^i of an institution (or portfolio) i and the confidence level q, $\mathrm{VaR}_{q,t}^i$ is defined as the q-quantile of the return distribution

$$Pr\left(R_t^i \leqslant \mathrm{VaR}_{q,t}^i\right) = q.$$

For example, if $q = 0.05$, $\mathrm{VaR}_{0.05,t}^i$ is the 5*th* quantile of the return distribution. AB define $\mathrm{CoVaR}_{q,t}^{i|j}$ as the VaR of an institution (or portfolio) i conditional on another institution (or portfolio) j being in financial distress, i.e., its return being at its VaR. That is, $\mathrm{CoVaR}_{q,t}^{i|j}$ is implicitly defined by the q-quantile of the conditional distribution

$$Pr\left(R_t^i \leqslant \mathrm{CoVaR}_{q,t}^{i|j} | R_t^j = \mathrm{VaR}_{q,t}^j\right) = q. \tag{1}$$

To be able to obtain time-varying estimates of $\mathrm{VaR}_{q,t}^j$ and $\mathrm{CoVaR}_{q,t}^{i|j}$, AB run, respectively, a q-quantile regression of R_t^j on a set of (lagged) state variables and R_t^i on R_t^j and the same set of (lagged) state variables. After running these regressions they obtain $\mathrm{VaR}_{q,t}^j = a + b\mathrm{M}_{t-1}$ and $\mathrm{CoVaR}_{q,t}^{i|j} = c + d\ \mathrm{VaR}_{q,t}^j + e\mathrm{M}_{t-1}$, where M is a vector of state variables and a, b, c, d, and e are functions of coefficient estimates from their quantile regressions.[4] Hence, even if the correlation between j and i changes over time, the effect of VaR on CoVaR, given by the coefficient estimate d, stays the same. However, when CoVaR is estimated using a GARCH model, the time-series relation between an institution's CoVaR and its VaR becomes time-varying due to the time-varying correlation (see also Benoit et al., 2012). This feature enables us to detect and incorporate in the $\mathrm{CoVaR}_{q,t}^{i|j}$ measurement possible changes over time of the linkage between j and i.

2.1. Definition of CoVaR

In this study we modify the definition of CoVaR proposed by AB. The distress event we condition on is that institution j is *at most* at its VaR, as opposed to being *exactly* at its VaR. Therefore, $\mathrm{CoVaR}_{q,t}^{i|j}$ is now the q-quantile of the following conditional distribution

$$Pr\left(R_t^i \leqslant \mathrm{CoVaR}_{q,t}^{i|j} | R_t^j \leqslant \mathrm{VaR}_{q,t}^j\right) = q. \tag{2}$$

Conditioning on $R_t^j \leqslant \mathrm{VaR}_{q,t}^j$ constitutes a more general case of financial distress for institution j, which allows for more severe losses (farther in the tail), i.e., those beyond $\mathrm{VaR}_{q,t}^j$. Another reason for the change in definition is to facilitate CoVaR backtesting; by changing the definition to Eq. (2), CoVaR estimates can be evaluated similar to VaR estimates using the widely-used Kupiec (1995) and Christoffersen (1998) tests for those time periods in which $R_t^j \leqslant \mathrm{VaR}_{q,t}^j$.

This change in financial distress definition has been shown in Mainik and Schaanning (2012) to have important implications for the consistency of CoVaR measure with respect to the dependence parameter. The authors find that the CoVaR measure proposed in our work is a continuos and increasing function of the dependence parameter between i and j, while the CoVaR measure as proposed in Adrian and Brunnermeier is not.[5]

In Section 3, we describe in detail the steps for estimation and backtesting. In the remainder of the paper we refer to CoVaR as presented in Eq. (2) unless stated otherwise.

2.2. ΔCoVaR and systemic risk contribution

As mentioned above, like AB, we consider $\mathrm{CoVaR}^{i|j}$ where $i \equiv s$ is the financial system (portfolio of all financial institutions), and we condition on the financial distress of an institution j. Hence, $\mathrm{CoVaR}_{q,t}^{s|j}$ is the VaR of the financial system conditional on institution j being in financial distress.

We define the systemic risk contribution of a particular institution j by

$$\Delta\mathrm{CoVaR}_{q,t}^{s|j} = 100 \times (\mathrm{CoVaR}_{q,t}^{s|j} - \mathrm{CoVaR}_{q,t}^{s|b^j})/\mathrm{CoVaR}_{q,t}^{s|b^j},$$

which is the percentage difference of the VaR of the financial system conditional on the distressed state of institution j $\left(R_t^j \leqslant \mathrm{VaR}_{q,t}^j\right)$ from the VaR of the financial system conditional on the *benchmark* state of institution j. We define the benchmark state b^j as a one-standard deviation about the mean event: $\mu_t^j - \sigma_t^j \leqslant R_t^j \leqslant \mu_t^j + \sigma_t^j$, where μ_t^j and σ_t^j are, respectively, the conditional mean and the standard deviation of institution j.[6]

2.3. Three-step procedure

Step 1 First, VaR of each institution j is computed by estimating the following univariate model

$$R_t^j = \mu_t^j + \varepsilon_{j,t},$$

where $\mu_t^j = \alpha_0 + \alpha_1 R_{t-1}^j$; $\varepsilon_{j,t} = z_{j,t}\sigma_{j,t}$, where $z_{j,t}$ is *i.i.d.* with zero mean and unit variance; and the conditional variance has the standard GARCH (1,1) specification

$$\sigma_{j,t}^2 = \beta_0^j + \beta_1^j \varepsilon_{j,t-1}^2 + \beta_2^j \sigma_{j,t-1}^2.$$

Given a distributional assumption for z and, hence, the q-quantile of the estimated conditional distribution, we can compute for each time period the VaR of each institution j (see, among others, Duffie and Pan, 1997, and Giot and Laurent, 2003, for VaR calculation from univariate GARCH models).

Step 2 Next, for each institution j, we estimate a bivariate GARCH model with Engle's (2002) DCC specification for the returns of institution j and the financial system. Let $R_t = (R_t^s, R_t^j)'$, whose joint dynamics is given by

$$R_t = \mu_t + \varepsilon_t,$$
$$\varepsilon_t = \Sigma_t^{1/2} z_t,$$

where Σ_t is the (2×2) conditional covariance matrix of the error term ε_t and μ_t is the (2×1) vector of conditional means. The standardized innovation vector $z_t = \Sigma_t^{-1/2}(R_t - \mu_t)$ is *i.i.d.* with $E(z_t) = 0$ and $Var(z_t) = I_2$. We define D_t to be the (2×2) diagonal matrix with the conditional variances $\sigma_{x,t}^2$ and $\sigma_{y,t}^2$ along the diagonal so that $\{D_{xx}\}_t = \{\Sigma_{xx}\}_t$, $\{D_{yy}\}_t = \{\Sigma_{yy}\}_t$, and $\{D_{xy}\}_t = 0$ for $x, y = s, j$. The conditional variances are modeled as GARCH (1,1)

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[4] In quantile regressions, the extent to which the time-series of CoVaR differs from the time-series of VaR depends on the magnitude of the coefficient vector e. Note that in the case of a single state variable, by solving the two equations for M and equating the two to each other, CoVaR of an institution simply becomes a scaled version of its VaR. AB find a very strong relation in the time-series between their CoVaR measure and VaR. This is not the case for the time-series of our CoVaR measure and VaR (see Section 4.4 below).

[5] In Appendix A, we discuss in more details the two alternative notions of CoVaR and provide a brief comparison of the two when assuming a bivariate Gaussian distribution.

[6] AB consider the *median* event of institution j as the benchmark state, i.e., $R_t^j = median^j$. Also, they consider the simple difference as opposed to percentage difference to calculate ΔCoVaR, i.e., $\Delta\mathrm{CoVaR}_{q,t}^{s|j} = \mathrm{CoVaR}_{q,t}^{s|j} - \mathrm{CoVaR}_{q,t}^{s|b^j}$. In Appendix B, we calculate Adrian and Brunnermeier's ΔCoVaR measure for our sample and compare it with our results reported in Table 3.

$$\sigma^2_{x,t} = \theta^x_0 + \theta^x_1 \varepsilon^2_{x,t-1} + \theta^x_2 \sigma^2_{x,t-1},$$
$$\sigma^2_{y,t} = \theta^y_0 + \theta^y_1 \varepsilon^2_{y,t-1} + \theta^y_2 \sigma^2_{y,t-1},$$

and the conditional covariance $\sigma_{xy,t}$ is

$$\sigma_{xy,t} = \rho_{xy,t}\sqrt{\sigma^2_{x,t}\sigma^2_{y,t}}.$$

Let $C_t = D_t^{-1/2}\Sigma_t D_t^{-1/2} = \{\rho_{xy}\}_t$ be the (2×2) matrix of conditional correlations of ε_t. Following Engle (2002) we specify the conditional correlation matrix as follows

$$C_t = diag(Q_t)^{-1/2} \times Q_t \times diag(Q_t)^{-1/2},$$
$$Q_t = (1 - \delta_1 - \delta_2)\overline{Q} + \delta_1(u_{t-1}u'_{t-1}) + \delta_2 Q_{t-1},$$

where $\overline{Q}$ is the unconditional covariance matrix of $u_t = \{\varepsilon_{x,t}/\sigma_{x,t}\}_{x=s,j}$ and $diag(Q_t)$ is the (2×2) matrix with the diagonal of Q_t on the diagonal and zeros off-diagonal.

Step 3 Once we estimate the bivariate density $pdf_t(R^s_t, R^j_t)$ for each $R_t = (R^s_t, R^j_t)'$pair in step 2, in step 3 we proceed to obtain our $\text{CoVaR}^{s|j}_{q,t}$ measure for each financial institution j and time period t. Given the definition of CoVaR in Eq. (2) it follows that

$$Pr\left(R^s_t \leqslant \text{CoVaR}^{s|j}_{q,t} | R^j_t \leqslant \text{VaR}^j_{q,t}\right) = q,$$
$$\frac{Pr\left(R^s_t \leqslant \text{CoVaR}^{s|j}_{q,t}, R^j_t \leqslant \text{VaR}^j_{q,t}\right)}{Pr\left(R^j_t \leqslant \text{VaR}^j_{q,t}\right)} = q.$$

By definition of $\text{VaR}^j_{q,t}, Pr\left(R^j_t \leqslant \text{VaR}^j_{q,t}\right) = q$ so

$$Pr\left(R^s_t \leqslant \text{CoVaR}^{s|j}_{q,t}, R^j_t \leqslant \text{VaR}^j_{q,t}\right) = q^2.$$

If we let $x, y = s, j$, given the $\text{VaR}^j_{q,t}$ estimates obtained in step 1, we can numerically solve the following double integral for $\text{CoVaR}^{s|j}_{q,t}$

$$\int_{-\infty}^{\text{CoVaR}^{s|j}_{q,t}} \int_{-\infty}^{\text{VaR}^j_{q,t}} pdf_t(x, y)dydx = q^2. \tag{3}$$

Fig. 1 has a depiction of $\text{CoVaR}^{s|j}_{q,t}$. Note that the time-varying correlation between R^s_t and R^j_t ensures that the $\text{CoVaR}^{s|j}_{q,t}$ of an institution here has a time-varying exposure to its $\text{VaR}^j_{q,t}$.

In order to compute $\text{CoVaR}^{s|b^j}_{q,t}$ we follow the same three-step procedure. The only difference is the conditioning event which, instead of being $R^j_t \leqslant \text{VaR}^j_{q,t}$, is now the *benchmark* state $\mu^j_t - \sigma^j_t \leqslant R^j_t \leqslant \mu^j_t + \sigma^j_t$. Once we retrieve the marginal probability $Pr(\mu^j_t - \sigma^j_t \leqslant R^j_t \leqslant \mu^j_t + \sigma^j_t) = p^j_t$ for each institution j, $\text{CoVaR}^{s|b^j}_{q,t}$ is defined by the following joint probability

$$Pr\left(R^s_t \leqslant \text{CoVaR}^{s|b^j}_{q,t}, \mu^j_t - \sigma^j_t \leqslant R^j_t \leqslant \mu^j_t + \sigma^j_t\right) = p^j_t q.$$

Similar to our $\text{CoVaR}^{s|j}_{q,t}$ calculation, we numerically solve the following double integral for $\text{CoVaR}^{s|b^j}_{q,t}$

$$\int_{-\infty}^{\text{CoVaR}^{s|b^j}_{q,t}} \int_{\mu^j_t - \sigma^j_t}^{\mu^j_t + \sigma^j_t} pdf_t(x, y)dydx = p^j_t q. \tag{4}$$

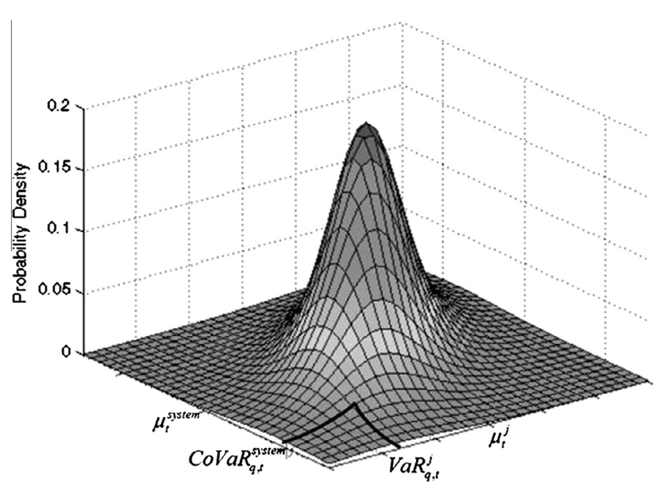

Fig. 1. Graphical representation of CoVaR. *Notes*: The figure depicts, assuming a bivariate Gaussian distribution, the joint density function of the random vector$R_t = (R^s_t,\ R^j_t)$ with means $(\mu^s_t,\ \mu^j_t)$. $\text{CoVaR}^{s|j}_{q,t}$ corresponds to the lower-left limit of the rectangle (whose area equals q^2) with the lower-right limit given by $\text{VaR}^j_{q,t}$.

3. Estimation and backtesting

3.1. Estimation

In this section we discuss the implementation of the three-step procedure outlined above. It is generally agreed that financial series do not follow Gaussian distribution and that the distribution of returns in financial markets exhibit both skewness and (excess) kurtosis. In the VaR estimation literature, many studies find that VaR calculations which take skewness and kurtosis into account are substantially different and more accurate than VaR calculations based on the Gaussian distribution. For instance, Duffie and Pan (1997), Giot and Laurent (2003), and Mittnik and Paolella (2000) show that while Gaussian-based VaR models tend to be rejected, those models that account for asymmetry and fat-tails fail to be rejected and they bring considerable improvements to VaR estimation.

In order to account for both skewness and kurtosis in CoVaR estimation, in Step 1 (2), in addition to the univariate (bivariate) Gaussian distribution, we also consider a univariate (bivariate) skewed-t distribution for each institution j. In Step 1, along with the standard Gaussian distribution, we also model the standardized innovation z as having Hansen's (1994) skewed-t distribution. The density of Hansen's skewed-t distribution is

$$h(z_t|\eta, \lambda) = \begin{cases} bc\left(1 + \frac{1}{\eta-2}\left(\frac{bz_t+a}{1-\lambda}\right)^2\right)^{-(\eta+1)/2} & z_t < -a/b \\ bc\left(1 + \frac{1}{\eta-2}\left(\frac{bz_t+a}{1+\lambda}\right)^2\right)^{-(\eta+1)/2} & z_t \geqslant -a/b \end{cases} \tag{5}$$

where $2 < \eta < \infty$ and $-1 < \lambda < 1$. The constants a, b, and c are given by

$$a = 4\lambda c\left(\frac{\eta-2}{\eta-1}\right), \quad b^2 = 1 + 3\lambda^2 - a^2, \quad c = \frac{\Gamma\left(\frac{\eta+1}{2}\right)}{\sqrt{\pi(\eta-2)}\Gamma\left(\frac{\eta}{2}\right)}.$$

Note that when $\lambda = 0$ and as $\eta \to \infty$, (5) reduces to the standard Gaussian distribution. When $\lambda = 0$ and η is finite, we obtain the standardized symmetric-t distribution.

When we assume that the innovation z follows the Gaussian distribution in Step 1, we carry this assumption to Steps 2 and 3 by modeling the vector $R_t = (R^s_t, R^j_t)'$ using the bivariate Gaussian distribution to estimate the DCC model and then to solve the double integrals in (3) and (4). Similarly, when we assume the skewed-t distribution in Step 1, in Steps 2 and 3 we use the standardized bivariate skewed-t distribution of Bauwens and Laurent (2005) to estimate the DCC model and then to solve the double integrals. The density of this distribution is

$$g(z_t|\upsilon, \xi_1, \xi_2) = c\left(\prod_{i=1}^{2}\frac{2b_i}{\xi_i + \frac{1}{\xi_i}}\right)\left(1 + \frac{z^{*\prime}z^*}{\upsilon-2}\right)^{\frac{\upsilon+2}{2}},$$

where ξ_1 and ξ_2 are the skewness parameters, v is the degrees of freedom parameter, $z^* = \left(z_s^*,\ z_j^*\right)'$, $z_i^* = (b_i z_i + a_i)\xi_i^{I_i}$, and z_i are the standardized innovations for $i = s, j$. The indicator function I_i and the constants a_i, b_i, and c are

$$I_i = \begin{cases} -1 & \text{if } z_i \geqslant \frac{a_i}{b_i} \\ 1 & \text{if } z_i < \frac{a_i}{b_i} \end{cases}$$

and

$$a_i = \frac{\Gamma\left(\frac{v-1}{2}\right)\sqrt{v-2}}{\sqrt{\pi}\Gamma\left(\frac{v}{2}\right)}\left(\xi_i - \frac{1}{\xi_i}\right), \quad b_i^2 = \left(\xi_i + \frac{1}{\xi_i} - 1\right) - a_i^2,$$
$$c = \frac{\Gamma\left(\frac{v+2}{2}\right)}{\pi(v-2)\Gamma\left(\frac{v}{2}\right)}.$$

3.2. Backtesting the newly defined CoVaR

By changing the definition of CoVaR to Eq. (2), its evaluation becomes a straightforward application of the standard Kupiec (1995) and Christoffersen (1998) tests for those time periods in which $R_t^j \leqslant \mathrm{VaR}_{q,t}^j$.[7] Let our sample include N observations with $t = 1,\ldots, N$. For each institution j, once we observe the past ex-ante VaR forecasts, we can compare them with the past ex-post losses and define the "hit sequence" of violations as

$$I_{t+1}^j = \begin{cases} 1 & \text{if } R_{t+1}^j \leqslant \mathrm{VaR}_{q,t+1}^j \\ 0 & \text{if } R_{t+1}^j > \mathrm{VaR}_{q,t+1}^j \end{cases}.$$

The hit sequence returns a 1 if the loss of institution j on that day was larger than its VaR level predicted for that day, and zero otherwise. For the sub-sample, which we assume to have T observations, when institution j is in financial distress, i.e., those days in which $R_{t+1}^j \leqslant \mathrm{VaR}_{q,t+1}^j$, we can construct a second hit sequence which compares the past ex-ante CoVaR forecasts with the past ex-post losses of the financial system

$$I_{t+1}^{s|j} = \begin{cases} 1 & \text{if } R_{t+1}^s \leqslant \mathrm{CoVaR}_{q,t+1}^{s|j} \\ 0 & \text{if } R_{t+1}^s > \mathrm{CoVaR}_{q,t+1}^{s|j} \end{cases}.$$

This second hit sequence has T observations which is equal to the number of violations of the first hit sequence, i.e., number of days in which institution j was in financial distress, and returns a 1 if the loss of the financial system on that day was larger than its predicted CoVaR level, and zero otherwise.

3.2.1. Unconditional coverage property

The $\mathrm{CoVaR}_{q,t}^{s|j}$ measure satisfies the unconditional coverage property if $Pr(I_{t+1}^{s|j} = 1) = q$. The hypothesis to test for the unconditional coverage property is

$$H_0 : E[I_t^{s|j}] \equiv p = q,$$

and we use the likelihood ratio test of Kupiec (1995) which has a χ_1^2 distribution (see Kupiec, 1995, for details).

3.2.2. Conditional coverage property

The $\mathrm{CoVaR}_{q,t}^{s|j}$ measure satisfies the conditional coverage property if $Pr_t(I_{t+1}^{s|j} = 1) = q$. Assume that the hit sequence $I_t^{s|j}$ is dependent over time and that the dependence can be described as a first-order Markov sequence with transition probability matrix

$$P_1 = \begin{bmatrix} 1 - p_{01} & p_{01} \\ 1 - p_{11} & p_{11} \end{bmatrix}$$

The transition probabilities in the above matrix are:

p_{01}: probability that conditional on today being a non-violation $\left(R_{t+1}^s > \mathrm{CoVaR}_{q,t+1}^{s|j}\right)$ next period in which $R_{t+1}^j \leqslant \mathrm{VaR}_{q,t+1}^j$ is a violation $\left(R_{t+1}^s \leqslant \mathrm{CoVaR}_{q,t+1}^{s|j}\right)$.

p_{11}: probability that conditional on today being a violation $\left(R_{t+1}^s \leqslant \mathrm{CoVaR}_{q,t+1}^{s|j}\right)$ next period in which $R_{t+1}^j \leqslant \mathrm{VaR}_{q,t+1}^j$ is also a violation $\left(R_{t+1}^s \leqslant \mathrm{CoVaR}_{q,t+1}^{s|j}\right)$.

$1 - p_{01}$: probability that conditional on today being a non-violation $\left(R_{t+1}^s > \mathrm{CoVaR}_{q,t+1}^{s|j}\right)$ next period in which $R_{t+1}^j \leqslant \mathrm{VaR}_{q,t+1}^j$ is also a non-violation $\left(R_{t+1}^s > \mathrm{CoVaR}_{q,t+1}^{s|j}\right)$.

$1 - p_{11}$: probability that conditional on today being a violation $\left(R_{t+1}^s \leqslant \mathrm{CoVaR}_{q,t+1}^{s|j}\right)$ next period in which $R_{t+1}^j \leqslant \mathrm{VaR}_{q,t+1}^j$ is a non-violation $\left(R_{t+1}^s > \mathrm{CoVaR}_{q,t+1}^{s|j}\right)$.

If the hit sequence $I_t^{s|j}$ satisfies the conditional coverage property, the probability of a violation next period in which $R_{t+1}^j \leqslant \mathrm{VaR}_{q,t+1}^j$ does not depend on the last period in which $R_{t+1}^j \leqslant \mathrm{VaR}_{q,t+1}^j$ being a violation or not. The hypothesis to test for the conditional coverage property is

$$H_0 : E[I_t^{s|j}] \equiv p = p_{01} = p_{11},$$

and we use the likelihood ratio test of Christoffersen (1998) which has a χ_1^2 (see Christoffersen, 1998, for details).

4. Data and results

4.1. Data

We consider 74 of the 102 US financial institutions that Acharya et al. (2010) have in their sample with equity market capitalization in excess of 5bln USD as of end of June 2007.[8] Table 1 lists these financial institutions and their type based on two-digit SIC classification code (Depository Institutions, Securities Dealers and Commodity Brokers, Insurance, and Others). The Dow Jones US Financials Index (DJUSFN) is used as a proxy for the financial system. Sample period is from 6/26/2000 to 2/29/2008, and there are 1930 observations for each institution. VaR and CoVaR measures are computed at the $q = 5\%$ confidence level. Table 2 reports the summary statistics of the returns of the financial system and the financial institutions by group. We obtained our data from the Bloomberg terminal and CRSP database.

4.2. Results

We first estimate $\mathrm{VaR}_{q,t}^j$ for each institution j and time period t. The second and third steps consist of estimating $\mathrm{CoVaR}_{q,t}^{s|j}$ and then $\Delta\mathrm{CoVaR}_{q,t}^{s|j}$ of the financial system for each time period t conditional on the financial distress of institution j. Table 3 reports, for each distributional assumption, the summary statistics of the estimated $\Delta\mathrm{CoVaR}_{q,t}^{s|j}$ series for all 74 institutions (labeled Overall) and for institutions by industry group. In columns 2 and 7 (labeled Std TS), we first compute the standard deviation of the individual

[7] Note that the tests are for CoVaR (which is itself a VaR), not ΔCoVaR. Also, we can test for $\mathrm{CoVaR}^{s|b^j}$ in a similar way; however, given that the conditioning event in the benchmark state is the center of the distribution rather than the tail, this case is less interesting.

[8] We exclude 28 institutions due to the limited length of their return series.

Table 1
Names and classifications of US financial institutions.

Depositories	Others	Insurance	Broker-dealers
BAC Bank of America	ACAS American capital Strategies	AFL AFLA Inc	BSC Bear Stearns
BBT BB & T	AMTD Ameritrade Holding	AIG American International Group	ETFC E-Trade Financial
BK Bank of New York Mellon	AXP American Express	ALL Allstate Corp	GS Goldman sachs
C Citigroup	BEN Franklin Resources Inc	AON AON Corp	LEH Lehman Brothers
CBH Commerce Bankcorp Inc NJ	BLK Blackrock Inc	BRKA Berkshire Hathaway Inc Del	MER Merrill Lynch
CMA Comerica Inc	COF Capital One Financial	BRKB Berkshire Hathaway Inc Del	MS Morgan Stanley
HBAN Huntington Bancshares Inc	EV Eaton Vance Corp	CB Chubb Corp	SCHW Schwab Charles Corp
HCBK Hudson City Bancorp Inc	FNM Federal National Mortgage Assn	CFC Countrywide Financial Corp	TROW T Rowe Price
JPM JP Morgan Chase	FRE Federal Home Loan Mortgage	CINF Cincinnati Financial Corp	
KEY Keycorp New	JNS Janus Cap Group Inc	CNA Can Financial Corp	
MI Marshall Isley	LM Legg Mason Inc	HIG Hartford Financial Svcs Group	
MTB M & T Bank Corp	LUK Leucadia national	HUM Humana Inc	
NCC National City Corp	SEIC Sei Investments Company	L Loews Corp	
NTRS Northern Trust Corp	SLM S L M Corp	LNC Lincoln national Corp	
NYB New York Community Bankcorp	UNP Union Pacific	MBI MBIA Inc	
PBCT People United Financial		MET Metlife Inc	
PNC PNC Financial Services		MMC Marsh and Mclennan Cos Inc	
RF Regions Financials		PGR Progressive Corp OH	
SNV Synovus Financial Corp		SAF Safeco Corp	
SOV Sovereign Bancorp		TMK Torchmark Corp	
STI Suntrust Banks Inc		TRV Travelers Companies Inc	
STT State Street Corp		UNH United Health Group	
UB Unionbancal Corp		UNM Unum Group	
USB US Bancorp Del			
WB Wachovia			
WFC Wells Fargo			
WM Washington Mutual			
ZION Zions Bancorp			

Notes: We consider 74 of the 102 US financial institutions that Acharya et al. (2010) have in their sample with equity market capitalization (as of end of June 2007) in excess of 5bln USD. We exclude 28 institutions due to the limited length of their return series. Although Goldman Sachs has an SIC code of 6282, thus belonging to the Others group, we include it in the Broker-dealers group.

Table 2
Summary statistics for the returns of the financial system and the financial institutions by industry group.

	Mean	Median	St. dev.	Skewness	Kurtosis
DJUSFN	0.017	0.017	1.278	0.175	6.082
Depositories	0.044	0.023	1.690	0.029	8.737
Others	0.060	0.021	2.177	−0.075	12.021
Insurance	0.051	0.015	1.842	0.168	17.637
Broker-dealers	0.044	- 0.002	2.575	0.337	9.776

Notes: Sample period is from 6/26/2000 to 2/29/2008 and sample size is 1930.

$\Delta\text{CoVaR}^{s|j}_{q,t}$ series for an institution j and then average these standard deviations across institutions. In columns 3 and 8 (labeled Std CS), we compute the standard deviation of the means of the individual institutions' $\Delta\text{CoVaR}^{s|j}_{q,t}$ measures. The first measure is a proxy for the volatility of systemic risk contributions over time, while the second is a proxy for the dispersion of average systemic risk contributions.

To understand the numbers in the first and sixth columns of Table 3, consider the number in the first row and first column: assuming Gaussian distribution, financial distress of an institution (when the return of the institution is below its 5% VaR), on average, increases the 5% VaR of the financial system by 95.69% over its VaR when the institution is in its benchmark state. The second column indicates that the average standard deviation of the $\Delta\text{CoVaR}^{s|j}_{q,t}$ time-series is 15.04, while the third column indicates that the standard deviation across average $\Delta\text{CoVaR}^{s|j}_{q,t}$ measures is 0.26. It is clear in Table 3 how the maximum, the minimum, and the average contributions to systemic risk, and also their variance over time and across institutions substantially increase when we employ the skewed-*t* distribution to account for skewness and excess kurtosis of financial returns. For instance, average ΔCoVaR goes from 95.69% to 168.77% and its average standard deviation increases from 15.04 to 21.38. This suggests that a financial institution's distress, on average, results in higher losses and higher degree of volatility in the financial system than predicted under the Gaussian assumption.

Table 3
Summary statistics for ΔCoVaR for all institutions and for institutions by industry group.

	Gaussian distribution					Skewed-t distribution				
	Mean (%)	Std TS	Std CS	Max (%)	Min (%)	Mean (%)	Std TS	Std CS	Max (%)	Min (%)
Overall	95.69	15.04	0.26	141.31	29.93	168.77	21.38	0.33	227.80	77.88
Depositories	110.27	17.38	0.21	141.31	68.19	187.75	23.95	0.33	227.80	77.88
Others	83.68	15.31	0.21	122.08	51.87	152.42	22.33	0.22	190.65	121.38
Insurance	78.89	11.92	0.23	110.82	29.93	155.03	18.52	0.31	190.37	90.47
Broker-dealers	115.45	15.32	0.17	130.15	79.83	171.20	18.89	0.18	191.67	140.16

Notes: Sample period is from 6/26/2000 to 2/29/2008 and sample size is 1930. q = 5% for all calculations. Gaussian results refer to the estimation of VaR and CoVaR by univariate and bivariate Gaussian distribution, respectively. Skewed-*t* results refer to the estimation of VaR and CoVaR by Hansen's (1994) univariate and Bauwens and Laurent's (2005) bivariate skewed-*t* distribution, respectively. In columns 2 and 7, we first calculate the standard deviation of individual $\Delta\text{CoVaR}^{s|j}_{q,t}$ time-series for institution j and then average these standard deviations across the institutions within each group. In columns 3 and 8, we calculate the standard deviation of the mean of the individual $\Delta\text{CoVaR}^{s|j}_{q,t}$ measures.

Note that, under the skewed-*t* assumption, the ranking among the top two and bottom two industry groups reverses. While broker-dealers impose the biggest risk on the financial system under the Gaussian distribution followed by depositories, when we account for skewness and excess kurtosis, the ranking is reversed and depositories are the most risky group followed by broker-dealers. These two groups are followed by others and insurers (insurers and others) under the Gaussian (skewed-*t*) assumption.

Our finding under the skewed-*t* distribution about depository institutions being the most risky group is consistent with the results of Billio et al. (2012). The authors suggest that the reason why commercial banks may have been a more important source of risk than other groups is that illiquidity of their assets, coupled with their structure which is not designed to withstand rapid and large losses, make them a natural repository of systemic risk. Moreover, because commercial banks tend to be subject to more strict capital requirements than broker-dealers, in times of high market volatility, they may amplify their losses in an attempt to meet their capital requirements. As Acharya (2009) and Lorenzoni (2008) argue this, in turn, can create a negative endogenous feedback effect with fire-sales externalities of their assets which could have significant implications for the entire financial system.

A result that is consistent across different distributional assumptions is the ranking of the average standard deviation of the ΔCoVaR series and the standard deviation across average ΔCoVaR measures. Under both the Gaussian and skewed-*t* distributions, depositories have the more volatile contribution to systemic risk over time (columns 2 and 7) while broker-dealers are the more homogeneous group with the lowest variation across their average contributions to systemic risk (columns 3 and 8).

4.3. CoVaR backtesting results

We backtest our CoVaR$^{s|j}$ measure under both the Gaussian and skewed-*t* distribution assumptions for the unconditional and conditional coverage properties. The null hypothesis for the unconditional coverage property is whether the average of the violations $\left(R^s_{t+1} \leqslant \text{CoVaR}^{s|j}_{q,t+1}\right)$ is equal to the coverage level q. The null hypothesis for the conditional coverage property is whether the probability of a violation in the next period in which $R^j_{t+1} \leqslant \text{VaR}^j_{q,t+1}$ depends on the last period in which $R^j_{t+1} \leqslant \text{VaR}^j_{q,t+1}$ is a violation or not. The first columns of both panels in Table 4 report, for each distributional assumption, the average test statistics LR_{ucp} and LR_{indp} for all 74 institutions and for institutions within the specific industry group, and the second columns report the average *p*-values.

While the conditional coverage property is satisfied, the results on the left panel of Table 4 show that, under the Gaussian assumption, the unconditional coverage property is rejected at 5% level of significance for all institutions and also by industry groups. This result is in line with most VaR backtest results in the literature under the Gaussian assumption; VaR violations often do not cluster over time and hence satisfy the conditional coverage property. However, VaR measures based on the Gaussian distribution often fail the unconditional coverage property and thus potentially underestimate risk.

The test results for the skewed-*t* distribution are also as expected and in line with most VaR backtest results in the literature; both the unconditional and conditional coverage properties are satisfied for the CoVaR measure based on the skewed-*t* distribution.

4.4. Relation between systemic risk and individual characteristics

In this section we analyze the relation between individual institution characteristics such as its VaR, size, and leverage, and the institution's systemic risk contribution as measured by ΔCoVaR. Fig. 2 has the cross-section plots of systemic risk contributions and institution characteristics. In all plots, VaR and ΔCoVaR are based on the skewed-*t* distribution results.

The upper-left plot in Fig. 2 reveals the weak relation between institutions' risk in isolation (measured by average $\text{VaR}^j_{q,t}$ over time for each institution *j*) and institutions' contribution to systemic risk (measured by average $\Delta\text{CoVaR}^{s|j}_{q,t}$ over time for each institution *j*). From a regulator's perspective, because these two measures seem not to be strongly correlated, capital requirements determined exclusively based on an institution's VaR could differ substantially from capital requirements which consider ΔCoVaR of each institution as well, and might fail to account for the risk that each institution imposes on the financial system.

Given the recent discussions about "too big to fail" and various proposals about the close regulatory scrutiny that these institutions should get, it is of interest to look at the relation between the size of an institution and its contribution to systemic risk. The upper-middle plot in Fig. 2 shows the link between institutions' size (measured in millions of total assets) and institutions' ΔCoVaR. The scatter plot shows the weak relation between the two measures, especially for those institutions whose size is smaller than about 250 billion dollars. However, it appears in the upper-right plot in Fig. 2 that the relation between size and systemic risk contribution is logarithmic as the relation between the log of institutions' size and ΔCoVaR seems to be somewhat positively correlated.

Table 4
Average test statistics and *p*-values for CoVaR unconditional and conditional coverage properties.

		Gaussian distribution			Skewed-t distribution	
		Average	*p* Value		Average	*p* Value
Overall	LR_{ucp}	8.84	0.0210	LR_{ucp}	0.77	0.5371
	LR_{indp}	1.08	0.4606	LR_{indp}	0.31	0.5991
Depositories	LR_{ucp}	9.28	0.0177	LR_{ucp}	0.55	0.6149
	LR_{indp}	1.14	0.4709	LR_{indp}	0.34	0.5749
Others	LR_{ucp}	8.51	0.0149	LR_{ucp}	0.75	0.5159
	LR_{indp}	1.10	0.4506	LR_{indp}	0.28	0.6125
Insurance	LR_{ucp}	9.24	0.0282	LR_{ucp}	1.48	0.3623
	LR_{indp}	1.14	0.4213	LR_{indp}	0.25	0.6608
Broker-dealers	LR_{ucp}	8.57	0.0109	LR_{ucp}	0.22	0.6945
	LR_{indp}	0.82	0.5128	LR_{indp}	0.45	0.5112

Notes: LR_{ucp} is Kupiec's (1995) test statistic for the unconditional coverage property and LR_{indp} is Christoffersen's (1998) test statistic for the conditional coverage property.

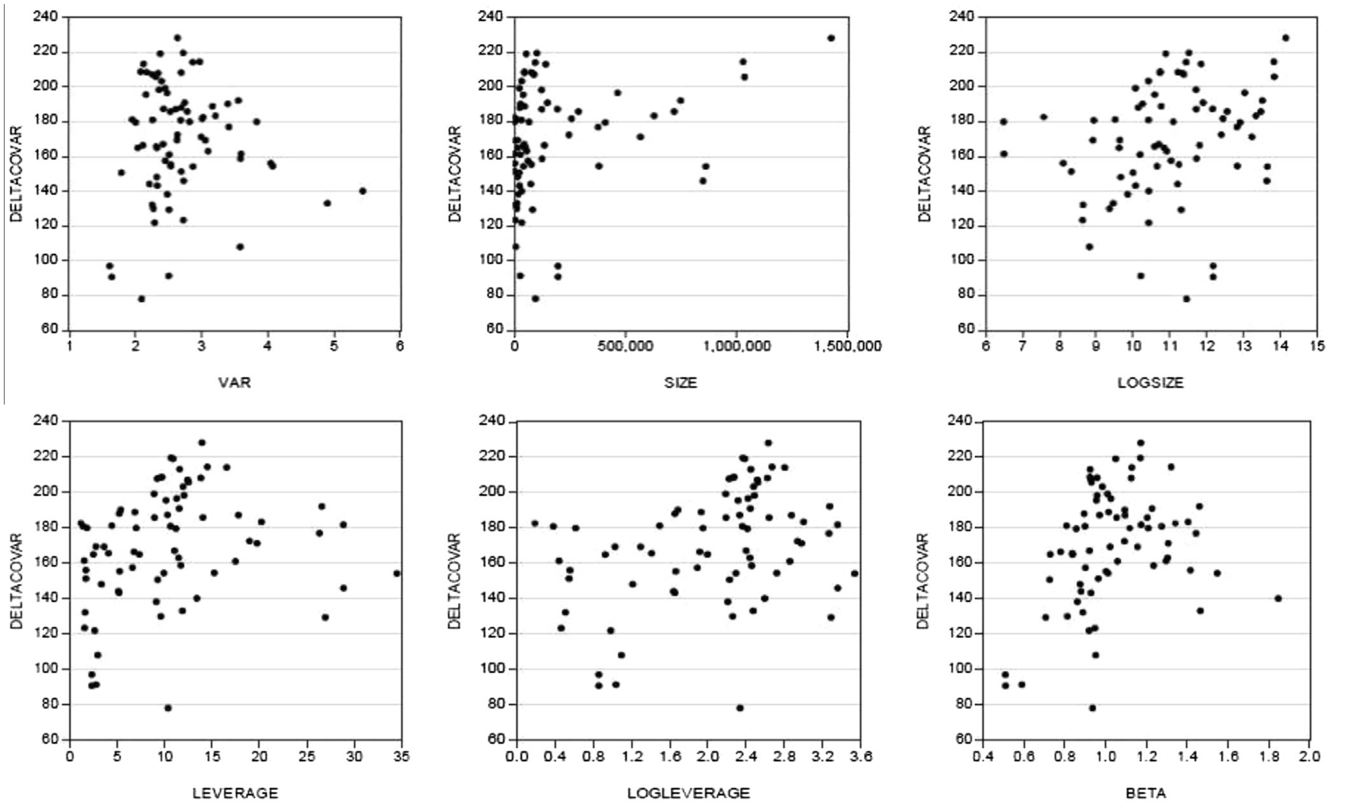

Fig. 2. Cross-section relation between financial institutions' characteristics and contributions to systemic risk. *Notes*: The figure reports the cross-section plots of contributions to systemic risk (measured by average ΔCoVaR) and institutions' risk in isolation (measured by average VaR), institutions' size (in millions of total assets), institutions' leverage (average ratio of total assets to book equity) and institutions' beta.

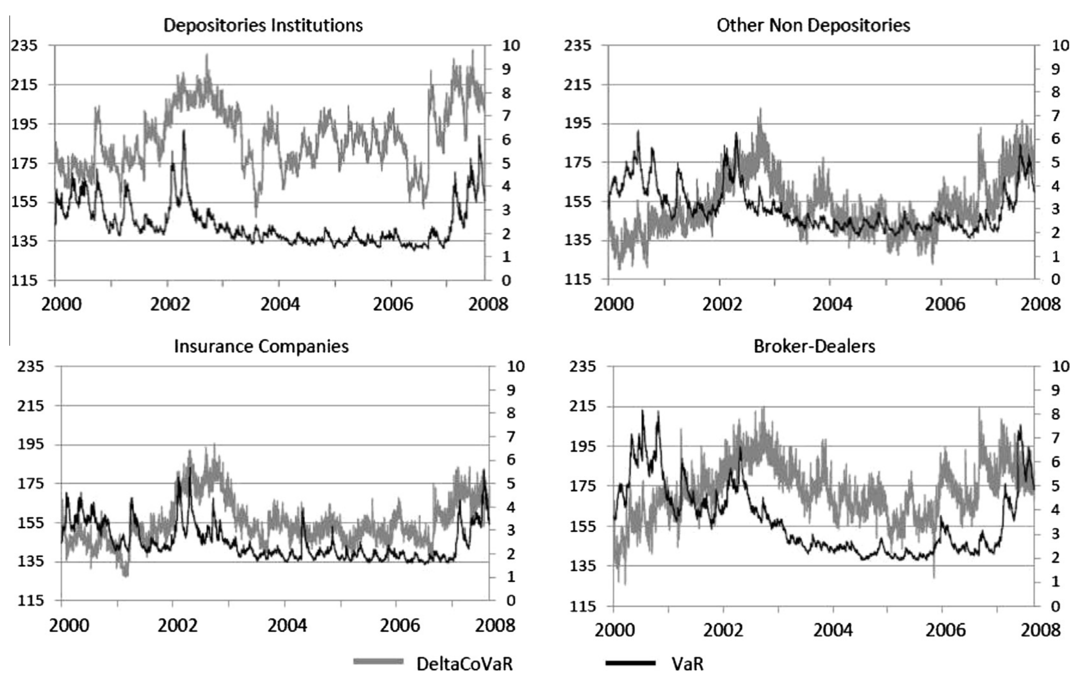

Fig. 3. Time-series plots of ΔCoVaR and VaR. *Notes*: The figure shows the time-series plots of daily ΔCoVaR and VaR measures aggregated by industry group. ΔCoVaR series are plotted on the left axis and VaR series on the right axis.

The relation between leverage (measured by the average ratio of total assets to book equity) and systemic risk contribution seems to be similar to the relation between the latter and size; the lower-left scatter plot in Fig. 2 between leverage and risk contribution shows a weak relation whereas the logarithm of leverage in the lower-middle plot seems to be somewhat positively correlated with ΔCoVaR.

Finally, the lower-right scatter plot in Fig. 2 shows the cross-section relation between ΔCoVaR and the beta of the financial institutions; the beta seems to be more correlated with systemic risk contribution than log-size and log-leverage.

AB find that while their ΔCoVaR and VaR have only a weak relation in the cross-section (their Fig. 2), they report a very strong relation between the two measures in the time-series (their Fig. 3). Thus, for a given financial institution, ΔCoVaR in AB brings limited added-value over VaR to forecast systemic risk. The upper-left plot in Fig. 2 seems to confirm the weak relation between the two risk measures in the cross-section. Fig. 3, on the other hand, has the time plot of ΔCoVaR and VaR aggregated by industry groups. Even though there seems to be a stronger relation in the time-series than in the cross-section, this relation does not appear to be as strong as AB's finding. We conclude, therefore, that our ΔCoVaR measure and VaR are not strongly related not only in the cross-section but also in the time-series. The time-series of the systemic risk measure here potentially could have information that is different from the information in the time-series of the VaR of the institution. Hence, for a given financial institution, monitoring its tail risk may not be sufficient to forecast its systemic risk contribution.

To study in more detail the time-series relation between ΔCoVaR and institution characteristics, we now turn to a panel regression analysis. In order to investigate the relation between systemic risk contribution and institution characteristics over time, we regress the quarterly-aggregated ΔCoVaR on a set of institution characteristics. We define the quarterly average of the daily $\Delta\text{CoVaR}^{s|j}_{q,t}$ and $\text{VaR}^{j}_{q,t}$ measures as

$$\Delta\text{CoVaR}^{s|j}_{q,r} = \frac{1}{T_r}\sum_{t\in T_r}\Delta\text{CoVaR}^{s|j}_{q,t}, \text{VaR}^{j}_{q,r} = \frac{1}{T_r}\sum_{t\in T_r}\text{VaR}^{j}_{q,t},$$

where T_r denotes the set of periods t in quarter r. The explanatory variables we use are quarterly measures of VaR, size, leverage, institutions' beta, industry group dummies, and time effects. The results in Table 3 suggest that different industry groups can have different ΔCoVaR risk measures. Industry group dummies allow us to control for these differences. We also allow leverage to have an asymmetric effect by including the interaction of leverage with a dummy indicator of negative quarterly return on the market. Table 5 reports the estimation results including either $\text{VaR}^{j}_{q,r}$ or log($\text{VaR}^{j}_{q,r}$) as an explanatory variable.

All three industry group dummies are significant in both regressions in line with the results in Table 3 that different industry groups have different systemic risk contributions. Neither VaR nor log (VaR) is statistically significant (in the second regression the coefficient estimate on log (VaR) has a p-value of 0.1011). These results confirm the time-series plots in Fig. 3 in that VaR is only weakly related to our ΔCoVaR measure.[9]

The effect of log (size) on ΔCoVaR is positive and statistically significant, which suggests that bigger institutions impose more of a systemic risk. The effect of leverage seems to be asymmetric in that it significantly increases systemic risk contribution during down markets but does not seem to have an effect during up markets. Finally, the coefficient estimate on beta is positive and statistically significant, suggesting that the higher is the beta of an institution, the higher is its systemic risk contribution.

Overall, our results in Table 5 are in line with other studies. We find that, similar to AB and Acharya et al. (2010), leverage, size, and equity beta are all important in explaining institutions' contributions to systemic risk. As mentioned above, while AB find that VaR and their ΔCoVaR measures have only a weak relation in the cross-section but a very strong relation in the time-series, our results suggest that the relation between VaR and our ΔCoVaR measures is weak in the time-series as well as in the cross-section.

Table 5
Determinants of systemic risk.

Dependent variable	DeltaCoVaR	
	(1)	(2)
Constant	122.957***	111.237***
	(9.048)	(8.288)
Insurance	−27.046***	−28.208***
	(-7.483)	(-7.824)
Others	−22.348***	−24.545***
	(-4.113)	(-4.501)
Broker-dealers	−22.416***	−24.866***
	(-5.690)	(-6.103)
VaR	−0.737	
	(-0.609)	
Log (VaR)		7.099
		(1.640)
Log (size)	2.819*	3.277**
	(1.875)	(2.238)
Log (leverage)	3.953	3.062
	(1.137)	(0.898)
Beta	13.883***	11.546***
	(5.793)	(4.907)
Log (leverage) x$I^{-}_{s,r}$	1.455***	1.055**
	(3.269)	(2.301)
Adj.R^2	27.932%	28.129%

Notes: Number of observations is 2294. t-statistics reported in parentheses are based on the Newey-West standard errors allowing for up to five periods of autocorrelation. Both regressions include time effects.
* Significance at the 10% level.
** Significance at the 5% level.
*** Significance at the 1% level.

5. Systemic risk contribution prior to the 2007–2008 crisis

In this section, we calculate our ΔCoVaR risk measure using a pre-crisis sample. Similar to Acharya et al. (2010), we use the 12 month period prior to the beginning of June 2007 as the pre-crisis sample (251 daily observations from June 1, 2006 to May 31, 2007). As we discuss above, results in Table 4 suggest that CoVaR estimates based on the Gaussian distribution fail the unconditional coverage property whereas the estimates based on the skewed-t distribution satisfy both the conditional and unconditional coverage properties. Therefore, in this section we only use the skewed-t distribution to compute the ΔCoVaR measure to determine each industry group's pre-crisis systemic risk contribution.

The pre-crisis sample results are presented in Table 6. Prior to the crisis, risk contributions overall and by each industry group are significantly higher than the full-sample contributions reported

Table 6
Summary statistics for ΔCoVaR for all institutions and for institutions by industry group during June 2006 to June 2007.

	Mean (%)	Std. TS	Std. CS	Max (%)	Min (%)
Overall	203.36	22.52	0.47	306.07	89.82
Depositories	229.05	27.80	0.41	306.07	148.26
Others	203.09	22.03	0.45	277.40	126.45
Insurance	167.27	17.45	0.38	227.12	89.82
Broker-dealers	224.65	22.20	0.17	257.48	205.14

Notes: Sample period is from 6/1/2006 to 5/31/2007 and sample size is 251. q = 5% for all calculations. Results are based on Hansen's (1994) univariate and Bauwens and Laurent's (2005) bivariate skewed-t distribution for VaR and CoVaR calculations, respectively. In column 2, we first calculate the standard deviation of individual $\Delta\text{CoVaR}^{s|j}_{q,t}$ time-series for institution j and then average these standard deviations across the institutions within each group. In column 3, we calculate the standard deviation of the mean of the individual $\Delta\text{CoVaR}^{s|j}_{q,t}$ measures.

[9] Time-series plots in Fig. 3 seem to suggest that VaR and ΔCoVaR measures might be more closely related after 2002. In our panel regressions we also interacted VaR and log (VaR) with a dummy which takes the value one after 2002. Those interaction terms were not statistically significant either. These regression results are not reported; however, they are available upon request.

in the right panel of Table 3. Note that while the depositories are still the most risky group prior to the crisis as they are in the full-sample, broker-dealers closed the gap almost entirely and caught up with depository institutions in terms of their systemic risk contribution. When we compare column 3 of Table 6 with column 8 of Table 3, we see that while the variation across the average contributions to systemic risk is substantially higher for depositories, others, and insurers, broker-dealers appear to be much more homogeneous in terms of their systemic risk contribution than any other industry group prior to the crisis.

The pre-crisis ranking of industry groups in Table 6 based on the ΔCoVaR measure is in line with Acharya et al. (2010) in that their results also suggest that prior to the crisis broker-dealers are among the highest contributors to systemic risk and insurers are among the lowest. The results in Table 6 are also in line with the pre-crisis findings of Billio et al. (2012) in that banks appear to be more systemically risky than other financial industry groups.

6. Conclusion

The recent financial crisis has raised the public's, and in particular the regulators', awareness of systemic risk. Value-at-Risk (VaR), arguably the most widely-used risk measure by the industry and regulators, has been criticized as incapable of measuring the systemic risk contribution of financial institutions since it focuses on an institution in isolation. There seems to be overall consensus on the need for better risk measures with a systemic focus.

In this study, we build on and generalize a recently proposed systemic risk measure by Adrian and Brunnermeier (2011): Conditional Value-at-Risk (CoVaR). CoVaR is the VaR of the financial system when an institution is in financial distress. We define financial distress as the return of the institution being *at most* at its VaR as opposed to being *exactly* at its VaR as proposed by the original study. This change allows us to consider more severe distress events (those beyond the institution's VaR and farther in the tail) and facilitates backtesting of the CoVaR measure using standard tests. In addition, Mainik and Schaanning (2012) find that, compared to Adrian and Brunnermeier's risk measure, CoVaR as defined in our work has more favorable properties that guarantee its continuity and monotonicity with respect to the dependence parameter; thus, it seems to be a wise choice for calculating systemic risk and setting regulations. Due to the time-varying correlation of the GARCH model employed in the estimation process, the CoVaR of an institution here has a time-varying exposure to its VaR which, by construction, is not the case in Adrian and Brunnermeier (2011). This feature enables us to detect and incorporate in the systemic risk measurement possible changes over time in the linkage between the institution and the financial system.

We define the systemic risk contribution of an institution – ΔCoVaR – as the change from its CoVaR in its benchmark state to its CoVaR under financial distress. We define the benchmark state as a one-standard deviation event. We estimate CoVaR via a three-step procedure using univariate and bivariate GARCH models. For both the univariate and bivariate models, we take skewness and excess kurtosis into account by estimating skewed-*t* distributions in addition to the Gaussian distribution. We estimate the systemic risk contribution of four industry groups (depositories; broker-dealers; insurers; and others, which are non-depository institutions including government sponsored enterprises) consisting of a total of 74 major US financial institutions.

Backtesting results show that CoVaR estimates based on the Gaussian distribution are rejected whereas the estimates based on skewed-*t* distributions cannot be rejected, showing the importance of taking skewness and excess kurtosis into account in financial modeling.

Our systemic risk measure ΔCoVaR calculations show that during the sample period June 2000 to February 2008, depository institutions were the largest contributors to systemic risk, followed by broker-dealers, insurance companies, and non-depository institutions. Our result about the depository institutions is in line with Billio et al. (2012) who find that banks may be more central to systemic risk than other financial industry groups.

We also investigate the relation between institutions' individual characteristics and systemic risk contributions. Adrian and Brunnermeier (2011) find that while institutions' VaR and their ΔCoVaR have only a weak relation in the cross section, they find a very strong relation between the two in the time-series. Our results suggest that VaR and our ΔCoVaR are weakly related in the time-series as well as in the cross-section. These findings have important implications for regulatory policies; on the one hand, the weak relation that both Adrian and Brunnermeier and this study find in the cross-section suggests that capital requirements determined exclusively based on an institution's VaR could differ substantially from capital requirements which consider ΔCoVaR of each institution. On the other hand, the strong relation that Adrian and Brunnermeier find in the time-series of VaR and their ΔCoVaR implies that, for a given financial institution, ΔCoVaR brings limited added-value over VaR to forecast systemic risk. The weak link between the time-series of the two risk measures documented in this study, however, suggests that the time-series of ΔCoVaR here potentially could have information that is different from the information in the time-series of the VaR of the institution. Thus, monitoring a firm's tail risk in isolation is not sufficient to determine its systemic risk contribution. When investigating other individual characteristics of institutions, similar to Adrian and Brunnermeier (2011) and Acharya et al. (2010), we find leverage, size, and equity beta to be important in explaining institutions' contributions to systemic risk.

Finally, using 12 months of data prior to the beginning of June 2007, we also compute industry groups' pre-crisis ΔCoVaR. Systemic risk of all industry groups increased substantially prior to the crisis. While depository institutions were still systemically the most risky followed by broker-dealers as in our large sample, prior to the crisis, the gap was almost closed. In addition, on the verge of the crisis, broker-dealers appeared to be more homogeneous in terms of their systemic risk contribution than other industry groups.

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Appendix A.

The main methodological differences between Adrian and Brunnermeier (2011) and this study are: (*i*) how financial distress is defined in CoVaR $\left(R_t^j = \mathrm{VaR}_{q,t}^j \text{ vs. } R_t^j \leqslant \mathrm{VaR}_{q,t}^j\right)$ and (*ii*) how CoVaR is estimated (quantile regressions vs. GARCH models). Their focus is on defining financial distress as $R_t^j = \mathrm{VaR}_{q,t}^j$ and estimating CoVaR using quantile regressions whereas our focus is on defining financial distress as $R_t^j \leqslant \mathrm{VaR}_{q,t}^j$ and estimating CoVaR using GARCH models.

Generally speaking, CoVaR is intended to capture the tendency of distress in an institution to be associated with distress in the financial system. The ability of Adrian and Brunnermeier's CoVaR to describe this tendency strongly depends on how well $F_{Y|X=\mathrm{VaR}_q(X)}$ approximates $F_{Y|X=x}$ for $x \leqslant \mathrm{VaR}_q(X)$, with $X,Y = R^j,R^s$. As shown in Mainik and Schaanning (2012), this approximation fails even in very basic models and it typically underestimates the contagion from R^j to R^s.

To get an idea about how the two systemic risk measures with different financial distress definitions compare, in this Appendix we plot Adrian and Brunnermeier's CoVaR and ΔCoVaR and ours under the assumption of bivariate Gaussian distribution for different correlation values. Their CoVaR, under the assumption of bivariate Gaussian distribution, has a closed-form solution and is defined as $\mathrm{CoVaR}^{s|j}_{pq,t} = \Phi^{-1}(q)\sigma^s_t\sqrt{1-\rho^2_{sj,t}} + \Phi^{-1}(p)\rho_{sj,t}\sigma^s_t$, where $\rho_{sj,t}$ is the correlation coefficient between the financial system and institution j and p is the confidence level for the VaR of the institution.

We calculate both risk measures at $q = p = 5\%$ confidence level. We set the means equal to zero and $\sigma^s_t = 1$ (and $\sigma^j_t = 1$ for our measures). Fig. 4 has the CoVaR and ΔCoVaR plots. In the plots, measures of Adrian and Brunnermeier are denoted by AB. As mentioned in the text, Adrian and Brunnermeier calculate ΔCoVaR as the simple difference. In the right plot of Fig. 4, all ΔCoVaR plots are based on simple difference. We also calculate our CoVaR and ΔCoVaR measures using the skewed-t distribution by setting $\xi_1 = \xi_2 = 0$ and $\nu = 5$.

As one would expect from the difference in the definitions of financial distress ($R^j_t = \mathrm{VaR}^j_{q,t}$ vs. $R^j_t \leqslant \mathrm{VaR}^j_{q,t}$), Adrian and Brunnermeier's CoVaR in the left plot seems to be a lower bound (in absolute value) for our systemic risk measure. Note that since zero correlation implies independence in the case of bivariate Gaussian distribution, CoVaR becomes the (unconditional) VaR of the system and thus both CoVaR measures are -1.645 in the left plot and ΔCoVaR measures are zero in the right plot. In the case of bivariate-t distribution, because zero correlation does not necessarily imply independence, ΔCoVaR is not zero when $\rho_{sj,t} = 0$. This result, along with the ability to capture heavy tails and asymmetries, shows the importance of being able to estimate the systemic risk measures with the skewed-t distribution without having to assume Gaussian distribution (which we reject in our backtests).

In the left plot of Fig. 4, as the correlation increases, Adrian and Brunnermeier's CoVaR increases at a decreasing rate. When the correlation is above about 0.7, their risk measure starts to decrease in absolute value reaching asymptotically the (unconditional) VaR level -1.645 (in general, as $\rho_{sj,t} \to 1$, Adrian and Brunnermeier's $\mathrm{CoVaR}^{s|j}_{pq,t}$ reduces to $\Phi^{-1}(p)\sigma^s_t$). This plot shows that while CoVaR proposed in this study is an increasing function of the correlation parameter, Adrian and Brunnermeier's CoVaR counter-intuitively starts decreasing at high dependence levels, thus failing to detect systemic risk where it is more pronounced.

In the right plot of Fig. 4 Adrian and Brunnermeier's ΔCoVaR, which is based on the simple difference and computed as $\Phi^{-1}(q)\rho_{sj,t}\sigma^s_t$, seems to be a lower bound (in absolute value) for our ΔCoVaR similar to CoVaR in the left plot. Notice that in the right plot, both our and Adrian and Brunnermeier's ΔCoVaR are an increasing function of the correlation parameter.

Unfortunately Mainik and Schaanning (2012) show that the monotonicity of Adrian and Brunnermeier's ΔCoVaR with respect to $\rho_{sj,t}$ illustrated above is a special property of the bivariate Gaussian model. They derive representations of CoVaR and ΔCoVaR measures based on the two alternative notions of financial distress $R^j_t = \mathrm{VaR}^j_{q,t}$ vs. $R^j_t \leqslant \mathrm{VaR}^j_{q,t}$, study their general dependence consistency, and compare their performance in several stochastic models. Their analysis shows that, for instance, in the bivariate-t model and in the model with t marginals and a Gumbel copula, when conditioning on $R^j_t = \mathrm{VaR}^j_{q,t}$, not only CoVaR, but also ΔCoVaR does not monotonically increase with respect to the dependence parameter, and detects lower systemic risk where dependence is more pronounced. For all the models that they consider, they find that the distress definition $R^j_t \leqslant \mathrm{VaR}^j_{q,t}$ proposed here leads to CoVaR and ΔCoVaR measures which, as one would expect, are both increasing and continuous functions of the dependence parameter. The two authors conclude that the applicability of Adrian and Brunnermeier's ΔCoVaR measure is problematic due to its non-monotonicity with respect to the dependence parameter and it is essentially restricted to the bivariate Gaussian distribution setting (where it is superfluous because it carries essentially the same information as the CAPM beta, which is equal to $\rho_{sj,t}\sigma^s_t/\sigma^j_t$). Financial regulation based on CoVaR as defined in Adrian and Brunnermeier, therefore, has the potential to introduce instability and to set the wrong incentives. Overall, conditioning on the event $R^j_t \leqslant \mathrm{VaR}^j_{q,t}$ seems to have a more meaningful and practical interpretation and leads to a risk measure that appears to have more favorable theoretical and practical properties.

Appendix B

While Adrian and Brunnermeier (2011) advocate and use quantile regressions to estimate their CoVaR, they also briefly discuss

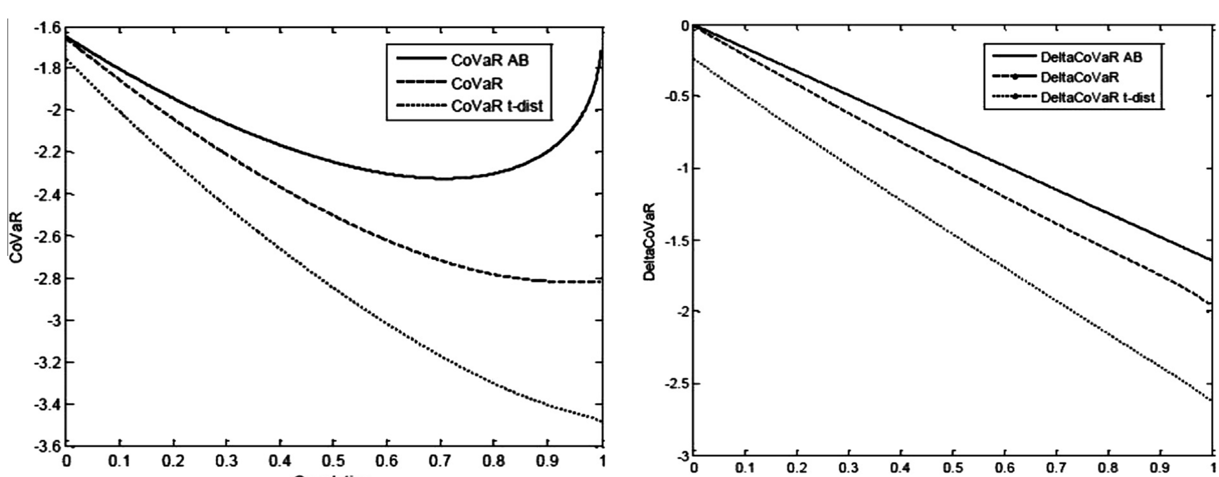

Fig. 4. CoVaR and ΔCoVaR plots with different financial distress definitions. *Notes*: The figure shows the plots of CoVaR and ΔCoVaR measures with different financial distress definitions under the assumption of Gaussian or Skewed-t distribution. The right plot depicts ΔCoVaR computed as simple difference. AB denotes the risk measure with Adrian and Brunnermeier's (2011) financial distress definition.

Table 7
Comparison of ΔCoVaR measures.

Distress definition	$R_t^j = \mathrm{VaR}_{q,t}^j$		$R_t^j \leqslant \mathrm{VaR}_{q,t}^j$	
	AB (quantile) (%)	AB (GARCH) (%)	Gaussian distribution (%)	Skewed-t distribution (%)
Overall	80.46	88.34	95.69	168.77
Depositories	96.82	105.38	110.27	187.75
Others	69.31	74.18	83.68	152.42
Insurance	60.77	69.01	78.89	155.03
Broker-dealers	100.69	110.82	115.45	171.20

Notes: The table reports the average ΔCoVaR measures. The columns denoted AB report the risk measure with Adrian and Brunnermeier's (2011) financial distress definition $\left(R_t^j = \mathrm{VaR}_{q,t}^j\right)$ computed using quantile regressions or a GARCH model with bivariate Gaussian distribution. Last two columns report our ΔCoVaR results fromTable 3. Sample period is from 6/26/2000 to 2/29/2008 and sample size is 1930. $q = 5\%$ for all calculations.

estimating CoVaR as defined in Eq. (1) using a bivariate GARCH model for the special case when the joint distribution of R_t^s and R_t^j follow a bivariate Gaussian distribution. Their CoVaR under the assumption of bivariate Gaussian distribution has a closed-form solution and is defined as $\mathrm{CoVaR}_{pq,t}^{s|j} = \Phi^{-1}(q)\sigma_t^s\sqrt{1-\rho_{sj,t}^2} + \Phi^{-1}(p)\rho_{sj,t}\sigma_t^s$, where $\rho_{sj,t}$ is the correlation coefficient between the financial system and institution j and p is the confidence level for the VaR of the institution.

In this Appendix, we calculate Adrian and Brunnermeier's ΔCoVaR for our sample using both quantile regressions and a GARCH model with bivariate Gaussian distribution and compare with our results.

As mentioned in the text, to be able to obtain time-varying VaR and CoVaR, they include (lagged) state variables in their quantile regressions. We collect the same seven state variables that they consider for our sample period. These state variables are: (i) VIX; (ii) liquidity spread (difference between the 3-month repo rate and the 3-month T-bill rate); (iii) change in the 3-month T-bill rate; (iv) change in the slope of the yield curve (measured by the yield spread between the ten-year Treasury rate and the 3-month T-bill rate); (v) change in the credit spread between BAA-rated bonds and the ten-year Treasury rate; (vi) return on the market (CRSP value-weighted index); and (vii) real-estate sector return (proxied by a REIT index).

In Table 7, similar to Table 3, we report the average ΔCoVaR, calculated at $q = p = 5\%$ confidence level, for all 74 institutions (labeled Overall) and for institutions by industry group. The columns denoted AB report the results from quantile regressions and GARCH model with bivariate Gaussian distribution with financial distress definition $R_t^j = \mathrm{VaR}_{q,t}^j$. For convenience, our results from Table 3 are also reported in the last two columns. In Table 7, to be able to make the comparison with our results, Adrian and Brunnermeier's ΔCoVaR is based on percentage difference.[10]

Regardless of the estimation methodology (quantile regressions or GARCH), when financial distress is defined as in Adrian and Brunnermeier, the ranking of the industry groups in terms of their systemic risk contribution is the same; broker-dealers are the most risky group followed by depositories, others, and insurers. Notice that this ranking is exactly the same as ours under the Gaussian distribution assumption, which is rejected in our backtests. Our results with skewed-t distribution (supported by the backtesting analysis), on the other hand, designate depository institutions as systemically most important, followed by broker-dealers, insurers, and others. The difference in these rankings would have important implications for regulatory and risk management purposes.

[10] Results based on simple difference are qualitatively the same and available upon request.

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