1. Bivariate case

Univariate stressing leads to the stressed put expectation

$$\mathrm{E}\{\phi(r_m)p_{it}\} = \int f(r_m)\phi(r_m)\mathrm{E}(p_{it}|r_m)\mathrm{d}r_m = \int f_\phi(r_m)\mathrm{E}(p_{it}|r_m)\mathrm{d}r_m$$

where f is the original density of a market factor r_m and $f_\phi = f \times \phi$ is the assumed density. Note ϕ may act on r_m or its percentile rank u_m. The intent of ϕ is the enlarge the likelihood of "bad" outcomes of r_m.

Bivariate stressing yields

$$\mathrm{E}\{\phi(r_m, r_n)p_{it}\} = \int\int f(r_m, r_n)\phi(r_m, r_n)\mathrm{E}(p_{it}|r_m, r_n)\mathrm{d}r_m\mathrm{d}r_n$$

$$= \int\int f_\phi(r_m, r_n)\mathrm{E}(p_{it}|r_m, r_n)\mathrm{d}r_m\mathrm{d}r_n$$

where r_n is another market factor and f is now the joint density of (r_m, r_n). The assumed joint density is $f_\phi = f \times \phi$, which again enlarges the joint likelihood of "bad" outcomes of (r_m, r_n). For example f_ϕ may have stronger tail dependence than f, which can be achieved by either selecting a copula which has stronger tail dependence, or setting up ϕ as $\phi(u_m u_n)$ where ϕ is decreasing. Marginal distributions of r_m and r_n may be altered or unchanged. If unchanged then

$$f(r_n) = \int f(r_m, r_n)\mathrm{d}r_m = \int f_\phi(r_m, r_n)\mathrm{d}r_m$$

or

$$1 = \int f(r_m)c(u_m, u_n)\mathrm{d}r_m = \int f(r_m)\phi(r_m, r_n)c(u_m, u_n)\mathrm{d}r_m$$

since $f(r_m, r_n) = f(r_m)f(r_n)c(u_m, u_n)$.

1.1. Copula stressors

Stressing can be based on a vector of variables x with distribution $F(x) = u$. Given a copula $\Phi(u)$ with density $\phi(u)$ consider the stressed expectation

$$\mathrm{E}_\phi(p_{it}) \equiv \int \mathrm{E}(p_{it}|x)\mathrm{d}\Phi\{F(x)\} = \mathrm{E}\{\phi(u)\mathrm{E}(p_{it}|x)\}\ , \qquad (1)$$

The copula $\Phi(u)$ is designed to magnify stressful situations such as where all components of x are abnormally low. If $\Phi(u) = u$ then $\phi(u) = 1$ and $\mathrm{E}_\phi(p_{it}) = \mathrm{E}(p_{it})$.

For example $\Phi\{F(x)\}$ may have stronger tail dependence than F. In the extreme $\Phi(u) = \min_i(u_i)$ making all variables in x comonotonic. The copula $\Phi(u)$ leaves marginals intact and acts as a stressor on the copula $c(u)$ corresponding to F since

$$\mathrm{d}\Phi\{F(x)\} = \phi(u)c(u)\prod_i f_i(x_i)\ ,$$

where the f_i are the marginal densities of F. Further marginal stress is induced by allowing $\Phi(u)$ to have non–uniform marginals in which case

$$\mathrm{d}\Phi(u) = \phi(u) \prod_i \phi_i(u_i) \mathrm{d}u \ ,$$

where $\phi(u)$ has, as before, uniform marginals and the $\phi_i(u_i) \geq 0$ induce non–uniformity in each of the marginals subject to $\mathrm{E}\{\phi_i(u_i)\} = 1$.