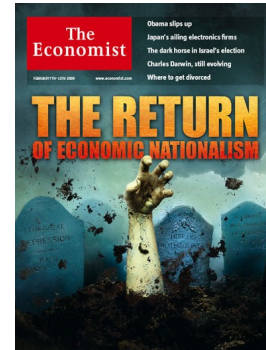


# *“Did a Mathematical Formula Really Blow up Wall Street?”*

Professor Paul Embrechts  
Department of Mathematics  
ETH Zurich



...

Embrechts, P., Resnick, S., Samorodnitsky, G.: **Living on the Edge**. *RISK*, January 1998, 96-100.

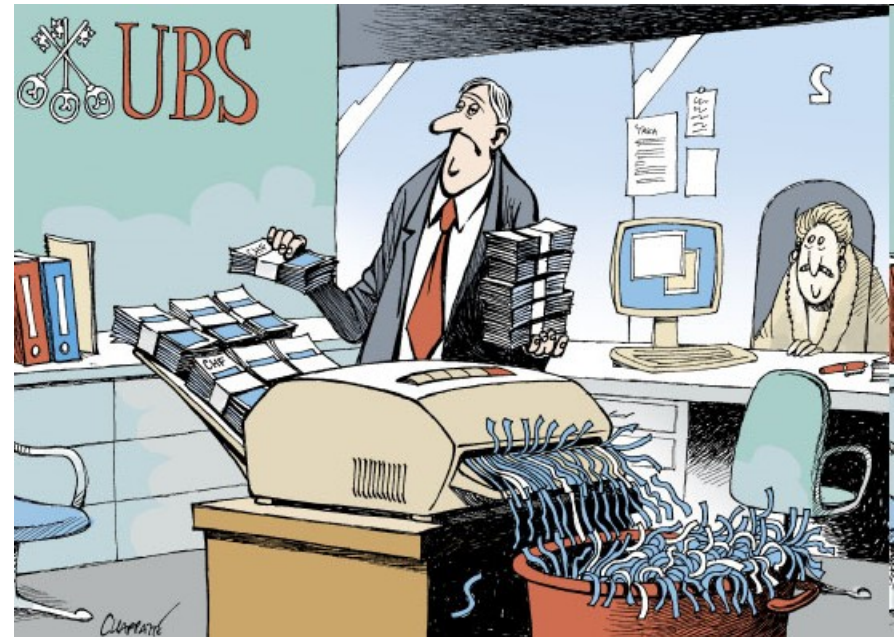
Chavez-Demoulin, V., Embrechts, P.: **Revisiting the edge, ten years on**. To appear in *Communications in Statistics - Theory and Methods* (2008/9)

Some covers and cartoons from The Economist

*Humpty Dumpty sat on a wall,  
Humpty Dumpty had a great fall,  
All the King's horses and all the King's men,  
Couldn't put Humpty together again.*



Cartoonists are having fun:



# Recipe for Disaster: The Formula That Killed Wall Street

By Felix Salmon 23 February, 2009  
Wired Magazine

$$\Pr[T_A < 1, T_B < 1] = \Phi_2(\Phi^{-1}(F_A(1)), \Phi^{-1}(F_B(1)), \gamma)$$



# David X. Li (2000)

## On Default Correlation: A Copula Function Approach,

### Journal of Fixed Income 9:43-54

- This paper studies the problem of default correlation. We first introduce a random variable called "time-until- default" to denote **the survival time of each defaultable entity** or financial instrument, and define the default correlation between two credit risks as the correlation coefficient between their survival times. Then we argue why **a copula function approach should be used to specify the joint distribution of survival times after marginal distributions of survival times are derived from market information**, such as risky bond prices or asset swap spreads. The definition and some basic properties of copula functions are given. We show that the current CreditMetrics approach to default correlation through asset correlation is equivalent to **using a normal copula function**. Finally, we give some numerical examples to illustrate the use of copula functions in the valuation of some credit derivatives, such as **credit default swaps** and first-to-default contracts.
- **April 1, 2000** (sic)

(The Gauss-copula)



David Li 8 years later



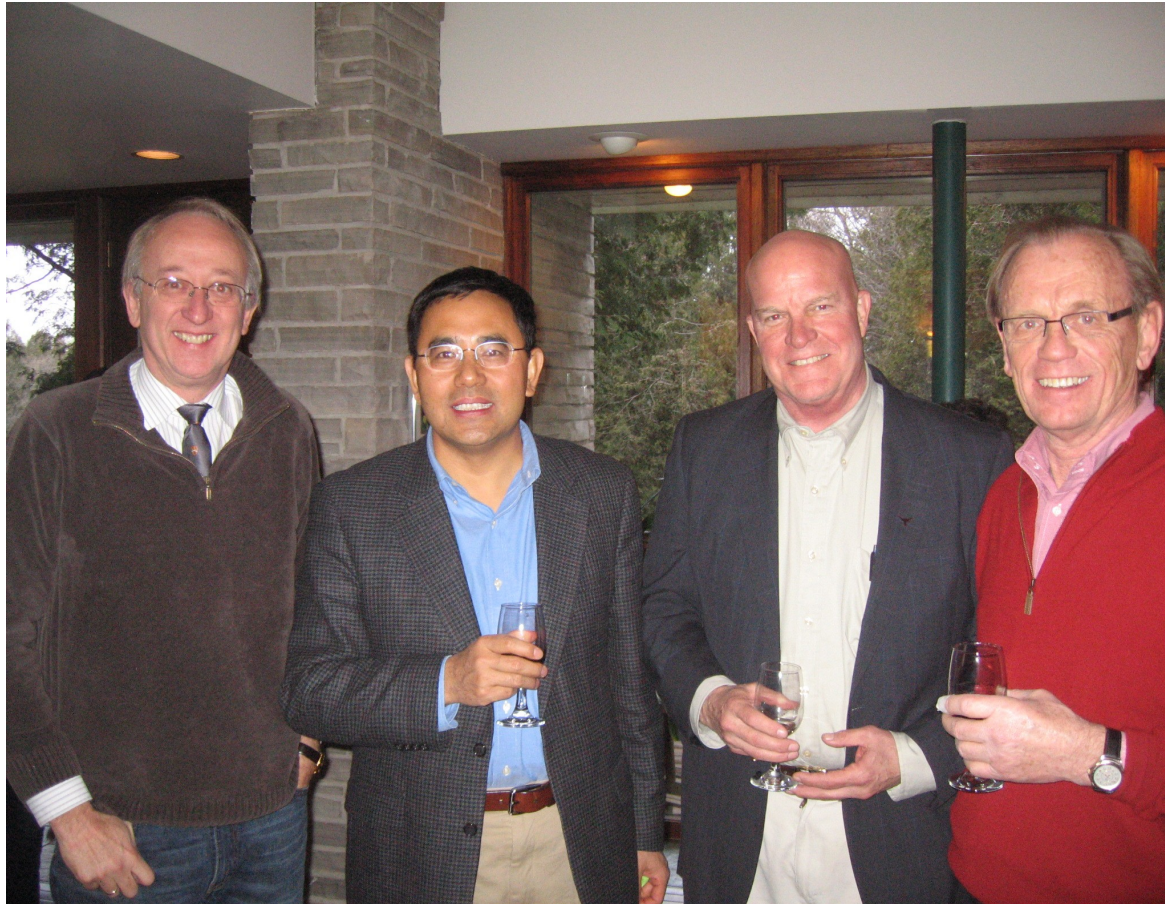









PE, **David Li**, Pat Brockett and Harry Panjer at Harry's retirement party at the University of Waterloo April 11, 2008



The **popular press** is full of statements like:

- From risk-free return to **return-free risk**
- Mark-to-market, mark-to-model, **mark-to-myth**
- Here's what killed your 401(k)
- **Mea Copula**
- Anything that relies on correlation is **charlatanism** (N.N.Taleb)  A story: ...
- Double defeat for Wall Street and Mathematics
- **Rather than common sense, financial mathematics was ruling**
- Etc ...

Wow!!!

Even the **Financial Times** joins in:

## **Of couples and copulas** by Sam Jones (April 24, 2009)

In the autumn of 1987, the man who would become **the world's most influential actuary** landed in Canada on a flight from China. He could apply **the broken hearts** maths to **broken companies**.

Li, it seemed, had found **the final piece of a riskmanagement** jigsaw that banks had been slowly piecing together since quants arrived on Wall Street.

Why did no one notice the formula's Achilles heel?



**Johnny Cash and June Carter**



Dear Sir

The article "Of couples and copulas", published on 24 April 2009, suggests that David Li's formula is to blame for the current financial crisis. For me, this is akin to blaming Einstein's  $E=mc^2$  formula for the destruction wreaked by the atomic bomb.

Feeling like a risk manager whose protestations of imminent danger were ignored, I wish to make clear that many well-respected academics have pointed out the limitations of the mathematical tools used in the finance industry, including Li's formula. However, these warnings were either ignored or dismissed with a desultory response: "It's academic".

We hope that we are listened to in the future, rather than being made a convenient scapegoat.

Yours Faithfully,  
Professor Paul Embrechts  
Director of RiskLab  
ETH Zurich



Also Harry Panjer

# Some personal recollections on the issue:

28 March 1999

Columbia-JAFEE Conference on the Mathematics of Finance,  
Columbia University, New York.

10:00-10:45 P. EMBRECHTS (ETH, Zurich):

"Insurance Analytics:  
Actuarial Tools in Financial Risk-Management"

Why relevant?

1. Paper: **P. Embrechts, A. McNeil, D. Straumann (1999)**  
*Correlation and Dependence in Risk Management:  
Properties and Pitfalls*. Preprint RiskLab/ETH Zürich.
2. Coffee break: discussion with David Li.

## Two results from the 1998 RiskLab report

### CORRELATION AND DEPENDENCE IN RISK MANAGEMENT: PROPERTIES AND PITFALLS

PAUL EMBRECHTS, ALEXANDER MCNEIL, AND DANIEL STRAUMANN

Remark 1: See Figure 1 next page

Remark 2: In the above paper it is shown that

Thus the Gaussian copula gives asymptotic independence, provided that  $\rho < 1$ . Regardless of how high a correlation we choose, if we go far enough into the tail, extreme events appear to occur independently in each margin. See Sibuya (1961) or Resnick (1987), Chapter 5, for alternative demonstrations of this fact.

1959





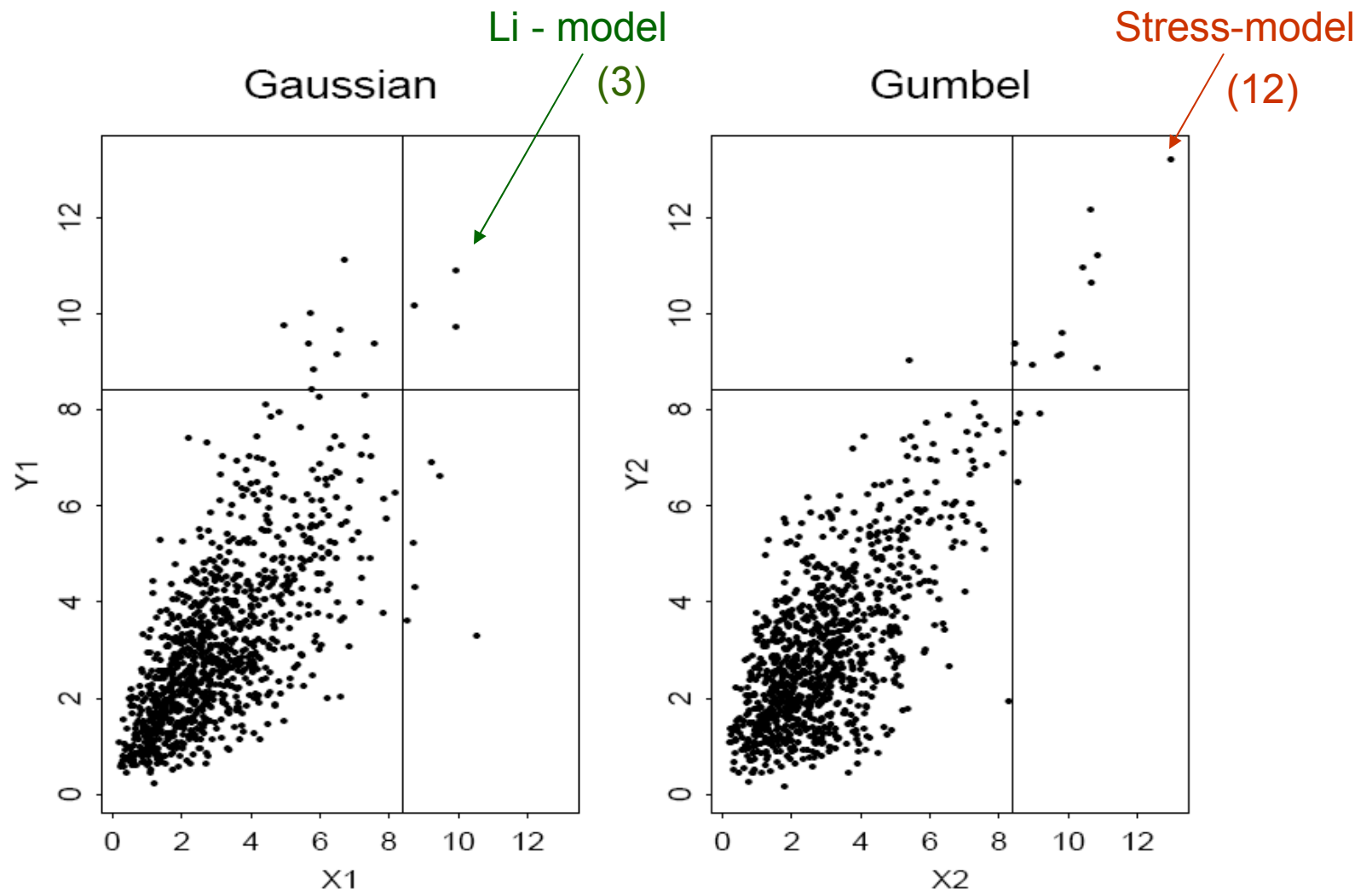


FIGURE 1. 1000 random variates from two distributions with identical  $\text{Gamma}(3,1)$  marginal distributions and identical correlation  $\rho = 0.7$ , but *different* dependence structures.

There were however several **early warnings**

Embrechts, P. et al. (2001): An academic response to Basel II. Financial Markets Group, London School of Economics. (Mailed to the **Basel Committee**)

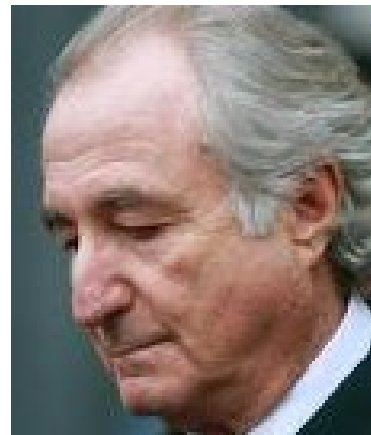
(**Critical on VaR, procyclicality, systemic risk**)

Markopolos, H. (2005): The world's largest hedge fund is a fraud. (Mailed to the **SEC**)

(**Madoff** runs a **Ponzi** scheme)

Charles Ponzi  
1910

Bernard Madoff



The Gauss-copula model had an earlier problem  
but many forgot!



September 12, 2005

**How a Formula Ignited Market  
That Burned Some Big Investors**



## Some replies by researchers:

- (L.C.G. Rogers) The problem is not that mathematics was **used** by the banking industry, the problem was that it was **abused** by the banking industry. Quants were instructed to build models which fitted the market prices. Now if the market prices were way out of line, the calibrated models would just faithfully reproduce those wacky values, **and the bad prices get reinforced by an overlay of scientific respectability!**

And Rogers continues:

- The standard models which were used for a long time before being rightfully discredited by (some) academics and the more thoughtful practitioners were from the start a complete fudge; so you had garbage prices being underpinned by garbage modelling.
- (M.H.A. Davis) The whole industry was stuck in a classic positive feedback loop which no party could (P.E. “wanted to”) walk away from.

Unfortunately only very few!

Indeed only some!

## The Turner Review

### A regulatory response to the global banking crisis

March 2009, FSA, London (126 pages)



#### 1.1 (iv) **Misplaced reliance on sophisticated maths**

There are, however, **fundamental questions about The validity of VAR as a measure of risk** (see Section 1.4 (ii) below). And the use of VAR measures based on relatively short periods of historical observation (e.g. 12 months) introduced dangerous **procyclicality** into the assessment of trading-book risk for the reasons set out in Box 1A (deficiencies of VAR).

The very **complexity of the mathematics** used to measure and manage risk, moreover, made it increasingly difficult for top management and boards to assess and exercise judgement over the risks being taken. **Mathematical sophistication** ended up not containing risk, but **providing false assurance** that other prima facie indicators of increasing risk (e.g. rapid credit extension and balance sheet growth) could be safely ignored.

#### 1.1 (v) **Hard-wired procyclicality: ...**

## 1.4 (iii) **Misplaced reliance on sophisticated maths: fixable deficiencies or inherent limitations?**

Four categories of problem can be distinguished:

- **Short observation periods**
- **Non-normal distributions**
- **Systemic versus idiosyncratic risk**
- **Non-independence of future events; distinguishing **risk** and **uncertainty****

Frank H. Knight, 1921



This is the main reason why we  
make a difference between  
Model **Risk** and Model **Uncertainty**.



# Supervisory guidance for assessing banks' financial instrument fair value practices

April 2009, Basel Committee on Banking Supervision

- Principle 8: Supervisors expect bank valuation and risk measurement systems to systematically **recognise and account for valuation uncertainty**. In particular, valuation processes and methodologies should produce **an explicit assessment of uncertainty** related to the assignment of value for all instruments or portfolios. When appropriate this may simply be a statement that uncertainty for a particular set of exposures is very small. **While qualitative assessments are a useful starting point, it is desirable** that banks develop methodologies that provide, to the extent possible, **quantitative assessments**. These methodologies may gauge the sensitivity of value to the **use of alternative models and modelling assumptions** (when applicable), to the use of alternative values for key input parameters to the pricing process, and to alternative scenarios to the presumed availability of counterparties. The extent of this analysis should be commensurate to the importance of the specific exposure for the overall solvency of the institution.

So back to the question:

## “Did a Mathematical Formula Really Blow Up Wall Street?”

- A **YES** would be nice for Hollywood ...
- However, we **all** are to blame:
  - Greed, incentives
  - Product opaqueness
  - Political shortsightedness
  - Regulatory failure
  - Systemic failure of academic economics
  - Rating agencies
  - Overall academic distance to reality
  - etc, etc, etc ...
- If only we could hide all of this behind a mathematical formula ... if only ...

# A message for my students

New generations of students will have to use the tools and techniques of QRM **wisely** in a world where the rules of the game will have been changed.

Always be **scientifically critical**, as well as **socially honest**, adhere to the **highest ethical principles**, especially in the face of **temptation** ... which will come!

And on the boundedness of our knowledge:

There are more things in heaven and  
earth, Horatio, than are dreamt of in your  
philosophy!

William Shakespeare  
(Hamlet I.v. 166)