1. **Weihao's observations**

- How to standardise $\mathrm{cov}(p_{it}, \phi)$ – still thinking but inclined to standardise by σ_ϕ instead of σ_ϕ^2.

  pdj – I agree. Then one is inclined to write

  $$p_{it} \approx \mu_{it} + \beta_{it} \frac{\phi - 1}{\sigma_\phi} \ ,$$

  and hence β_{it} is the response to a one–unit increase in the standardised stress factor $(\phi - 1)/\sigma_\phi$. Hence to my mind it argues the stress factor $(\phi - 1)/\sigma_\phi$ is core and needs concrete characterisation.

- Capital shortfall at time $i+h$ is the difference between debt and admissible assets

  $$d_{it} - (1-k)(w_{i,t+h} + d_{it}) \ , \qquad w_{i,t+h} = w_{it}\mathrm{e}^{\nu_{it}}$$

  where k is the non-admissible factor and ν_{it} is the return on equity. Capital shortfall at time $i+h$ can be rewritten as

  $$d_{it} - (1 - k_{i,t+h})(w_{it} + d_{it}) \ , \qquad k_{i,t+h} \equiv 1 - (1-k)\frac{w_{it}\mathrm{e}^{\nu_{it}} + d_{it}}{w_{it} + d_{it}}$$

  where assets are constant, similar to debt. However the non-admissible or impairment factor $k_{i,t+h}$ varies inversely with ν_{it}. One can either forecast capital shortfall by modeling ν_{it} or $k_{i,t+h}$, and this paper adopts the former due to available data on equity returns.

  pdj – Another way to look at this is to write

  $$\frac{1 - k_{i,t+h}}{1-k} = \left(1 - \frac{d_{it}}{a_{it}}\right)\mathrm{e}^{\nu_{it}} + \frac{d_{it}}{a_{it}}$$

  Can this be exploited? – note debts to assets is the odds of debt to equity

- Artzner asserts that any coherent risk measure is the supremum of the expected capital shortfall in a collection of generalised scenarios or probability measures $\mathcal{P}$ on states of the world, i.e.

  $$\sup\left\{\mathrm{E}_{\mathbb{P}}(p_{it}) | \mathbb{P} \in \mathcal{P}\right\}$$

  pdj– this seems a useful background result allowing us to argue the generality of our approach. Not sure how to best write up/connect to our presentation.

- $\tilde{\mathrm{E}}(p_{it}) = \mathrm{E}(p_{it}) + \mathrm{cov}\{\mathrm{E}(p_{it}|\phi), \phi\}$

  pdj– The implication here is that one can take each given scenario (ie possible outcome of ϕ) and just consider how expected put prices for each scenario covary with scenario:

  $$\beta_{it} = \mathrm{cov}\{p_{it}(\phi^*), \phi^*\} \ , \qquad \phi^* \equiv \frac{\phi - 1}{\sigma_\phi} \ , \qquad p_{it}(\phi^*) \equiv \mathrm{E}(p_{it}|\phi^*) \approx \mu_{it} + \beta_{it}\phi^*$$

  This seems useful to me.

- Write the put payout as

$$p_{it} = k\left|1 - \mathrm{e}^{\nu_{it}^*}\right|^+ = k\int_0^\infty (1 - \mathrm{e}^{\nu_{it}^*} > s)\mathrm{d}s = k\int_0^\infty (\nu_{it}^* < \ln(1-s))\mathrm{d}s$$

$$= k\int_{-\infty}^0 (\nu_{it}^* < s)\mathrm{e}^s\mathrm{d}s$$

and hence the put covariance is

$$\mathrm{cov}(p_{it}, \phi) = \mathrm{cov}\left\{k\int_{-\infty}^0 (\nu_{it}^* < s)\mathrm{e}^s\mathrm{d}s, \phi\right\} = k\int_{-\infty}^0 \mathrm{cov}\left\{(\nu_{it}^* < s), \phi\right\}\mathrm{e}^s\mathrm{d}s\ .$$

The covariance in the final integral is between a capital shortfall scenario $(\nu_{it}^* < s)$ and the stress factor.

pdj – can you use the expected stress factor trick here? Still thinking about the implications of above.