Expression of Interest

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1 Legal entity

If APRA agrees to the proposal as set out in this expression of interest, the legal entity with which it would be entering into a contract is Macquarie University.

2 Institutional support

This Expression of Interest is fully supported by Macquarie University. The University Research Office, attached to the Office of the Deputy Vice-Chancellor (Research), has the responsibility for administering all aspects of the University's research activities. This office has extensive experience with industry collaborations and ARC Linkage projects. In addition, the Division of Economic and Financial Studies, which includes The Applied Finance Centre, and the departments of Accounting and Finance, Actuarial Studies, Business, Economics and Statistics has a dedicated Research Officer to specifically maintain contact follow-up and other administrative aspects of the APRS-researcher relationship and the project contract.

2.1 Related funding from Macquarie University

Macquarie University will consider matching financial support for the proposed project under our External Collaborative Grant Scheme (ECGS). A successful ECGS grant application will match industry cash contributions dollar–for–dollar, to a maximum of $50,000, for a 12-month project. ECGS's are not a competitive grant. Projects deemed as genuine collaborations are funded. The Macquarie University Research Office guarantees a response on an application within six weeks of recorded submission. ECGS project funding allows partnerships to demonstrate a successful track record of collaboration prior to application for an ARC Linkage Grant.

3 Project title

The aggregation of outstanding claims liabilities and reserves and determination of capital requirements for general insurance business.

4 Aims and background to the project

Accurate forecasts of insurance company liabilities are of importance to a number of sectors of society. Sufficient reserves are essential to safeguard policyholder's interests. Accurate reserving is important to regulators so that they may take timely action in case of adverse developments. Accurate reserving is important in informing financial markets of the true financial state of insurance companies: the potential hazards they face and the profits they are accruing. Accurate reserving is important to the Boards of insurance companies so that they can make timely and properly informed decisions about company policies, profits and opportunities.

4.1 Background

Our research relates to the capital requirements for general insurance companies. At present these companies are required to have capital in respect of their outstanding claims liability. The capital requirement is called the "outstanding claims provision".

Actuarial and statistical methods exist to compute the expected value of the outstanding claims liability in respect of an individual classes of insurance (e.g. ctp, workers compensation etc).

The methods for quantifying the uncertainty in the liability are less well developed. Based on some measure of the uncertainty, the insurer should hold a reserve that contains a safety margin above the expected value of the liability. APRA presently requires insurers to hold capital equal to the 75th percentile of the distribution of outstanding claims liability. Methods for computing this 75th percentile are not well developed.

The overall reserve for an insurer is based on the aggregate liability across all types of insurance business sold by the firm. Methods for combining the reserves for separate lines of business into an overall reserve are even less well developed.

4.2 Aims

The proposed research will develop innovative yet practical methodologies for modelling dependence between different lines or portfolios of insurance. This applied research will enhance the development of prudential policy, reserving techniques and management of regulated insurance entities.

Our research is about development of new and better methods for aggregating reserve estimates and safety margins from separate classes of insurance business into an overall reserve and overall safety margin. We intend to explore the use of "copulas" for the purpose of measuring the extent of "dependence" in the "tail" of the distributions of the liability for different classes of insurance business, and then investigate how to estimate the quantiles (percentiles) of the distribution of the aggregate liability.

Another aspect of our research is to review current practice for the determination of the capital requirement and safety margin of the overall aggregate liability for outstanding claims.

5 Significance of the project

Paragraph 1.2 of the REOI Number 2005-6/6 says

> "APRA is particularly interested in supporting applied research that will enhance the development of prudential policy, techniques and management of regulated financial institutions."

Our proposed research project falls into this category. The project is important to the prudential regulation of general insurance companies. It will produce a better basis for determining the capital adequacy of general insurers. This impacts on

- Reserve margins;
- Effective monitoring and challenge;
- Active capital management;
- Risk governance; and
- Disclosure

This project is highly relevant to the following APRA priority research areas as identified in REOI Number 2005-6/6

- 1.3(a) "better ways to calculate capital requirements.
- 1.4(a) "provisioning calculations in the general insurance industry"
- 1.4(e) "analysis of risk management processes and methodologies for measuring ... insurance risk."
- "simulation models that can be used to forecast or stress test the financial position and performance of financial institutions."

This is fundamental in the role of APRA and other regulatory bodies responsible for insurance regulation. It is also fundamental to the effective management of an insurance company. Both the regulator and the insurance industry should have a strong interest in developing better methods for the determination of capital, for both regulatory and management purposes.

The actuarial profession also has a stake in this. Current methods for quantifying uncertainty in reserve estimates and combining the uncertainty for different classes of insurance into an overall aggregate reserve and its uncertainty are underdeveloped.

Within insurance companies, the proposed research will strengthen central areas of reinsurance group activity:

- responsible for the corporate underwriting of most non-life lines of business and thus for fundamental issues relating to risk assessment/minimisation, insurance mathematics and claims management
- annually prepares the basis for reserving unknown losses/losses that have been incurred but not reported by the end of the year
- establishes cross-divisional guidelines
- establishes standards for dealing with and reserving losses

- conducts quality assurance for underwriting and loss management in order to maintain and further develop underwriting competence
- integrates mathematical procedures into business processes and develops pricing tools

Fundamental decisions relating to reserving levels and methodologies have to take into account the dependence between different lines of business.

6 Research Approach

Provisions for outstanding claims for Property and Casualty (P&C), are held in respect of various lines of business or entities within an insurance company. A significant part of this is IBNR (Incurred but nor reported), which is typically determined by each insurance company. APRA performs audits on the insurance companies' reserving process. Additionally, plausibility checks need be performed on the reserves of the more prominent companies.

An insurance company can be viewed as an collection of a number m of lines of business. Large insurance companies are now typically collections of different subsidiaries each reporting to a central Board.

Methods exist to compute the expectation of the outstanding claims liability in respect of an individual class of insurance (e.g. ctp, workers commpensation etc).

The methods for quantifying the uncertainty in the liability are less well developed. The overall reserve for an insurer is based on the aggregate across all types of insurance business sold by the firm. Methods for combining the reserves for separate lines of business into an overall reserve are less well developed.

Each line of business or entity is represented by a liability L_j, $j = 1, \ldots, m$ and total liability $L = L_1 + L_2 + \cdots + L_m$. The financial health of the company is is determined by L and prudential standards generally concern themselves with setting standards for L. For example an appropriate standard might be that the company should hold reserves ℓ^* such that $P(L \leq \ell^*) = 0.75$. Thus the distribution of L is required. The distribution of L is is effected by the dependence between the L_j and requires careful analysis as illustrated by the following comments.

- There is no single uniform reserving methodology applied across entities or portfolios. Different schools of thought are being followed as to both the choice of methods and the development of ultimate loss prediction uncertainty ranges.
- Ideally, uncertainty ranges implicit in ℓ^* need to reflect all aspects of uncertainty including
  - Model uncertainty
  - Parameter uncertainty
  - Stochastic error
  - Dependence between different entities lines of business
- For determining the amount of capital needed, some way of estimating the variability / uncertainty in the insurance firm's outstanding claims liability is needed.

We propose to investigate the use of "copulas" to model the "dependence structure" between the outstanding claims liabiliity for different lines of insurance business or subsidieries. We believe this will produce a better, more theoretically valid way to estimate the capital requirement for insurance companies than what is produced using current practice.

Dependence between different lines of business is often measured via correlation. Yet correlation is a very blunt instrument in measuring dependence especially in the "tail" of the distribution. It is the behavior in the upper tail of the distribution of each of the L_j's that is critical to the upper tail behaviour of L.

7 Research Approach

Methods exist to compute the expectation of the outstanding claims liability in respect of an individual class of insurance (e.g. ctp, workers commpensation etc). The methods for quantifying the uncertainty in the liability are less well developed. The overall reserve for an insurer is based on the aggregate across all types of insurance business sold by the firm.

Methods for combining the reserves for separate lines of business into an overall reserve are less well developed. We are proposing to do research into the use of "copulas" to model the "dependence structure" between the outstanding claims liabiliity for different lines of insurance business. We believe this will produce a better, more theoretically valid way to estimate the capital requirement for insurance companies than what is produced using current practice.

7.1 Copulas in reserving work

A considerable amount of research has been devoted to the use of copulas for modelling dependence. Suppose just two lines of business L_1 and L_2 with respective distribution function $F_1(\ell_1)$ and $F_2(\ell_2)$. Then any joint distribution $F(\ell_1, \ell_2)$ of L_1 and L_2 can be written as

$$F(\ell_1, \ell_2) = C\{F_1(\ell_1), F_2(\ell_2)\} \ ,$$

where C is a suitably chosen copula function. Different copulas C induce different forms of dependence in the joint distribution $F(\ell_1, \ell_2)$. A further issue is the choice of the copula C and its impact on the joint distribution of $L = L_1 + L_2$.

In practical reserving situations the joint distribution $F(\ell_1, \ell_2)$ is required. This joint distribution determines the distribution of the aggregate liability $L = L_1 + L_2$. Issues arising in this framework are:

- What are appropriate classes of copulas which focus on suitable tail dependence?
- How can the data on individual portfolios or entities be used to provide information about the appropriateness of different copulas?
- What improvements, if any, do copulas provide over and above a straight "correlation" approach to studying dependence between lines of business?

7.2 Stages in the research

7.2.1 Review current best practice

We propose to firstly review current practice used by the regulator, the insurance industry and those professionals who provide advice and consulting services to the industry, in the area of determining the outstanding claims liability, reserve and safety margin included in the reserve.

7.2.2 Investigate the possible application of back testing using simulation and historical simulations

If sufficient relevant data can be obtained, we can construct simulation models to test the adequacy of the reserve estimates. In the banking industry, reserving for market risk can be done using an internal model subject to the model meeting the requirements of the regulator. If this is done the internal model is "back tested" to check whether the reserve established was adequate. One methodology that is used by banks for this purpose is "historical simulation". This is not currently done in the case of general insurance company reserves. It could be done using simulation based models if sufficient data were available.

7.2.3 Investigate the use of copulas to model the dependence between the liability for different classes of insurance business

Methods exist to compute the expectation of the outstanding claims liability in respect of an individual class of insurance (e.g. ctp, workers commpensation etc). The methods for quantifying the uncertainty in the liability are less well developed. The overall reserve for an insurer is based on the aggregate across all types of insurance business sold by the firm.

We are proposing to do research into the use of "copulas" to model the "dependence structure" between the outstanding claims liability for different lines of insurance business. We believe this will produce a better, more theoretically valid way to estimate the capital requirement for insurance companies than what is produced using current practice.

7.2.4 Acquisition of relevant data

In order to apply the approach and to be able to test it, it is necessary to obtain relevant data. The volume of data needed may be quite substantial. Acquisition of this data and storage and retrieval and analysis of it may be a large and time consuming part of the project.

7.2.5 Investigate the effectiveness of this alternative method for setting the reserve and the safety marging using simulation and back testing

Copulas are an alternative method for specifying how to create the joint distribution of several random variables from the individual distributions. In particular it allows for determination of "tail dependence" between the distributions.

There are many different copulas to choose from and hence many ways to create a joint distribution of the claims liability for different classes of insurance. Currently it

is common to make an arbitrary assumption that the joint distribution is multivariate normal, and to make arbitrary assumptions about the correlations and variances involved. These assumptions are often based on subjective judgement without detailed support from statistical evidence.

With any particular joint distribution for the liabilities of different classes of insurance, we need a way to construct the distribution of the aggregate claim amount. This may not be feasible analytically so a monte carlo simulation approach may be required.

Based on historic data, if it is available, we can test the adequacy of reserve estimated obtained using different approaches and different copulas. We shall investigate how to apply the idea of "historical simulation" used in bank value at risk models, to the general insurance liability estimation context.

8 Envisaged role of APRA in the project

- To provide data or or give access to data that APRA has which is relevant to the project. Insurance companies submit various returns to APRA concerning the development of claims payments on their various portfolios of insurance liabilities.
- To encourage insurance companies and reinsurers to provide data as well
- To facilitate contact with and input from overseas regulatory agencies in the area of general insurance regulation. We may want to find out about the prudential regulation and capital requirements used by other insurance regulators around the world.
- Give high level input into the framing of research questions in the context of this research
- Provide high level input into specifying the nature of appropriate solutions
- Providing funding for various aspects of the research, such as paying for research assistants, computer technology, specialised software programmers etc
- Providing technical input as and when appropriate.
- To analyse and summarise the data in ways that will facilitate our research we may need additional computer resources and programming support.
- To be the industry partner in an external collaborative research project and grant, preparatory to applying for an ARC Linkage Grant.

This research would have potential application to prudential regulation for other financial institutions (e.g. banks) as well.

9 Project funding from all sources

Source of funding is envisage as follows:

- During the first 12 months, $50,000 from APRA, and

- $50,000 from Macquarie University through its External Collaberative Research Scheme.
- After the first 12 months, moneys from the Australian Research Council (ARC) via their Linkage Grant Scheme.

The following table presents projected destination of funds in the first year.

Destination of funds	Amount
Research support from Dr Glenn Barnet	$
Teaching release for the academics	$
Salaries to research assistants/computer programmers	$
Travel to meet and consult with - regulators - industry experts, academics	$
Meetings with stakeholders	$
Dissemination of the results of the research	$
Acquisition of data from various sources including possibly - financial market data on insurance companies; data held by foreign regulators; insurance company policy and claims databases; APRA itself	$
acquisition of new computer resources to handle the data involved in the project	$
Total	$

10 Benefit of the project to APRA

APRA will obtain the services of Actuarial/Statistical researchers with expertise in the area of insurance reserving. APRA will not have to pay the salaries of these researchers. The university will match the funding provided by APRA on a dollar for dollar basis up to a maximum of $50,000 in the first 12 months. This will provide APRA with research services at significantly reduced cost than if APRA did the research itself. This research will provide APRA with better ways to determine the capital requirement for general insurers. This will reduce the likelihood of insurance company failures and the attendant consequences for the industry and the economy.

This research would have potential application to prudential regulation for other financial institutions (e.g. banks) as well.

11 National benefit

The nation and the taxpayer will benefit from the project by being able a lot more research done with the same amount of money compared to the alternative of doing the research "in house" within APRA. If successful, this research will provide the Government with better ways to determine the capital requirement for general insurers. This would reduce the likelihood of insurance company failures and the attendant consequences for the insurance industry and the economy more widely.

This research would have potential application to prudential regulation for other financial institutions (e.g. banks) as well.

Outcomes of the project will be innovative and practical forecasting methods which will be at the forefront of international research, and more accurate aggregate liability forecasts.

An aim in monitoring liabilities is to ensure that a strong and effective process is in place that above all ensures that balance sheet integrity is maintained at all times and that facilitates sufficiently early action (e.g. via re-pricing, amended underwriting guidelines, active commutations, etc) that responds to reserve developments.

12 Project documentation (including time researchers expect to spend on the project)

13 Timelines and expected deliverables

14 Research record of applicants

14.1 Piet de Jong

14.1.1 Published research

A complete list of my published research is contained in my CV. This part of the application highlights the most important achievements.

An early contribution in the area of claims reserving is DeJongZehnwirth:83a. This is often quoted.

Most of my published research in the last 20 years has centred on aspects of forecasting. The contributions are interesting and unique because they permit and facilitate full exploitation and implementation of Kalman filtering and smoothing in practical forecasting and monitoring situations. My theoretical contributions are rooted in practical problems: designing computational methods for Kalman filtering and smoothing and related estimation technologies. By practical I mean being able to, without further ado, fully apply methods to actual data using a computer and avoiding pitfalls. Implementation issues include unknown but necessary starting point [De JongDe Jong1991], singularities and inefficiencies in smoothing [De JongDe Jong1989], diagnostics [De Jong and PenzerDe Jong and Penzer unequal data spacing [De Jong and MazziDe Jong and Mazzi2001], connection to the widely known but more specialized Reinsch algorithm [Brown and de JongBrown and de Jong2001], application to ARIMA modeling [DeJong and PenzerDeJong and Penzer2004] and integration with the MCMC estimation paradigm [De Jong and ShephardDe Jong and Shephard1995] [De JongDe Jong1997]

Extensions and avoiding pitfalls are not just sophistry but critical for practical use. Pitfalls appear in simple cases such as random walks, long term dependence, vector and unequally spaced data, or data whose form is nonstandard such as the runoff triangle data of emerging insurance liabilities. The above articles since 1995 contain important practical applications to data such as Nile river flow data and electricity consumption. DeJong:2004 on the problem of insurance reserving is an initial step in harnessing the power of the methodologies in a practical and important actuarial setting. DeJongTickle:2004 implement and tailor methods for demographic forecasting. This last article is the seed for the current thrust of research proposed under this research application.

14.1.2 Consulting experience on related tasks

Extensive international consulting on all aspects of forecasting. Examples:

- Energy modelling and forecasting for California State regulators US product liability ($US500m) reserving for sales of defective building product
- Australian Oil & Gas Supplier - Estimating losses due to well malfunction
- State Insurance Body - Evaluate impact of changed claims classification
- Major Australian Electricity Retailer - Forecast half hourly electricity loads
- NZ Energy Company - Risk management across different electricity nodes
- State Department of Justice - Design and analyze health survey of prisoners
- Residential Price Agency - Critique statistical models to forecast house prices
- Australian mortgage provider - Modelling the propensity to churn mortgage
- Gas Authority - Set forecasting limits for daily gas consumption forecasting

14.2 Timothy Kyng

15 The skills and experience of the respondents

16 The resources of the respondent to conduct the project

Macquarie University has the largest actuarial studies department of any university in Australia. We also have the largest statistics department. In addition we have staff with expertise and experience in the areas of statistical science and actuarial methods applied to general insurance reserving.

We have experience in research and consulting on statistical and actuarial projects and computer resources adequate for the task involved.

17 the respondent's methodology and ability to make recommendations to APRA's management supported by analysis

18 Miscellaneous stuff

18.1 What do we propose to do that will address APRA's priorities?

18.2 Measures of achievement

(specific, measureable, achieveable, results oriented and time based)

Develop specific measures that will demonstrate a positive difference has been made in the priority areas.

We envisage a number of stages

- risk management tool
- trialling period putting tools to use measure of the success of the
- This proposed form of evaluating the project to go into the bid

19 Key Issues for reserving

19.1 Reserve Margins

- What "risk-margin" should be added to the best estimate liabilities?
- What level of reserve margins should be established?
- How do we ensure that margins adequately capture all sources of uncertainty?
- How do we minimize model error (selection of reserving method that is not suited to the claims data) or "past to future extrapolation" errors?
- Should these be part of the risk margin?
- Should reserve margins vary over the cycle?
- How does one encourage the use of "stress testing" using non-actuarial or statistical techniques?
- What is the optimal way of incorporating the results of these stress tests into the risk margin?
- What are the criteria to be used to increase or release margins?
- What is the optimal policy for reserve release for favorable developments, particularly in respect of long tail lines of business?

19.2 Effective Monitoring and Challenge

- The optimal means of incorporating rate change monitoring (in primary and reinsurance) into the reserving process and reserve challenge process
- The optimal linkage from reserve development through to the setting of the unexpired risk reserve through to new business pricing
- The optimal way of monitoring non-pricing information in the reserve development process
- The optimal means of sense checking implied claims inflation and superimposed inflation levels against exogenous data in reserving and reserve challenge process
- Monitoring of the challenging degree of conservatism in assumed tail developments for long tail business
- The optimal division of responsibilities between the "reserving " process and the "reserve challenge" process in the overall area of monitoring "
- Monitoring of the changing degree of conservatism in assumed tail developments for long tail business
- The optimal way of presenting "reserve" checks to company Boards and regulators
- The optimal policies around data segmentation principles and data reconciliation checks that must be performed prior to the reserving and reserve challenge process
- The need for transparency around decisions made by the chief reserving actuary in data segmentation (lines of business and sub-portfolios as well as accident year vs. underwriting year segmentations and the challenge of building up a Group measure using different legal entity approaches and practices)
- The need for transparency around decisions made regarding the chosen "forecasting model" for that data.
- The optimal way of monitoring the stability of "correlations" in the movements in reserves across different segments as well as correlations with investment gains and losses
- The increasing rating agency sophistication around "reserve challenges" and their expectations with respect to the role of the group risk management function
- The optimal way of using available external indicators (such as the impact of the cycle and peer comparisons) for "challenging" reserve estimated using standard actuarial techniques
- The suite of external benchmarks that ought to be developed and the limitations of each measure (e.g. premiums to reserves, survival ratios, reserves as a % of ultimates, IBNR as % of total reserves, etc)
- Benefits of independent and timely monitoring of reported cedant movements in reserves vis a vis those cedant's externally published disclosures

- Accumulation control - the need to monitor accumulated unfunded limit (similar to banking perspective where we know par outstanding) and linkage to commutation strategy (or any other exit strategy)
- Use of claims experts in the challenge process

19.3 Active Capital Management

- Optimal way of intra-group optimization of reserve risk mitigation (e.g. via LPTs)
- Linkage between best estimate reserves and capital requirements
- Ring fencing of portfolios using the best features of the "good bank/bad bank" concept
- tailored "VBM" styled targets for claims run-off units that focus on explicit incentives for running off at below carried reserves and/or ahead of expected settlement pattern.

19.4 Risk Governance

- The legal ramifications of a reserve challenge and the Board's obligations to make "ad-hoc" announcements outside of the regular reserve review process
- The challenge of balancing the need to have effective "reserve challenge" procedures in place with the need to only have one Group held view at any time on the reserves that ought to be carried
- What roles should the Board Risk Committee play when there is an unresolved dispute between the Chief Reserving Actuary and the Group CRO? What ought to be the procedures during each phase of the resolution of the dispute(from information gathering, to analysis, to exchange of views, to external review, etc)? How does the Board ensure balance sheet integrity is maintained at all times during the resolution of such disputes?
- What is the value of a periodic, independent, external review? What ought to be the policy around the commissioning of such reviews? Should this be a purely compliance driven policy or is there a role for a more proactive approach that represents stronger risk governance? Who should have the right to call for an independent (but extraordinary) review?
- Implications of Sarbanes Oxley on US listed insurers and implications for Munich Re.
- Resolving conflicts of interest via a formalized "challenge process" " Reporting lines of Decentralized Reserving functions (managerial & functional)
- The possible inappropriate selection of actuarial techniques for some lines of business and the limited transparency of such "technical decisions"

19.5 Disclosure

- Formalized internal reporting process around a key risk for the Group " Increased need for disclosure of ranges of reserves and analysts' understanding of such ranges. That is, what do we mean by "range of estimates"? What is disclosed about the margin? Where is the balance between transparency, consistency, and avoidance of misunderstanding given the market practices of our peers

- Communication of results to external stakeholders - Rating agencies, regulators and investors/analysts. The development of a standard package / presentation format that external stakeholders can expect to receive on a regular basis.

20 Previous stuff - Forecasting claims for General Insurance companies

This is research is into the important problem of general insurance reserving. Current methods for calculating reserves tend to be ad-hoc rather than statistical model based. I have developed statistical models that capitalize on the time series aspect of the problem and which facilitate forecasting and the important problem of evaluating the accuracy of the forecasts and hence the safety margin in reserves. Methods have been programmed but need testing, further development and interfacing into a spreadsheet environment prior to presentation to professional actuaries and academic conferences. The research is jointly being carried out with the actuarial arm of Deloitte, Sydney.

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