Tensor Starter Kit - 2

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Relevant literature are the following articles: 2010_Oseledets_TTmatrix, 2011_DMRG_schollwock and 2011_Oseledets_TT. Note that a Tensor Train is called a Matrix Product State in physics, and a Tensor Train matrix is called a Matrix Product Operator.

Exercises

Exercise 1

Write a Matlab function `innerprodTT(A,B)` that computes the inner product $\langle \mathcal{A}, \mathcal{B} \rangle$ of two tensors $\mathcal{A}, \mathcal{B}$ that are given as Tensor Trains. Verify your results by computing the Frobenius norm $||\mathcal{A}||_F^2 = \langle \mathcal{A}, \mathcal{A} \rangle$ with your function.

Exercise 2

Write a Matlab function `sitek(A,k)` that brings a given Tensor Train $\mathcal{A}$ into site-k-mixed-canonical form (Definition can be found on page 113 of 2011_DMRG_schollwock). Can you use this function to compute the norm of a Tensor Train with less computations?

Exercise 3

Write a Matlab function `addTT(A,B)` that computes the Tensor Train of $\mathcal{A}+\mathcal{B}$ from the Tensor Trains of $\mathcal{A}$ and $\mathcal{B}$. Verify this function by checking that $\text{addTT}(\mathcal{A}, \mathcal{A})/2 = \mathcal{A}$. Compare the TT-ranks of $\text{addTT}(\mathcal{A}, \mathcal{A})$ with the TT-ranks of $\mathcal{A}$. What do you observe and can you explain your observations?

Exercise 4

Write a Matlab function `B=roundTT(A,epsilon)` that truncates the TT-ranks of the Tensor Train $\mathcal{A}$ such that the relative approximation error $||\mathcal{B} - \mathcal{A}||/||\mathcal{A}||$ is smaller than epsilon. Verify your function by truncating the ranks of $\text{addTT}(\mathcal{A}, \mathcal{A})$ back to the TT-ranks of $\mathcal{A}$.

Exercise 5

Adapt your TT-SVD algorithm such that you get a function `B=TTm(A,dim,epsilon)` that computes a Tensor Train matrix approximation of a given matrix $\mathbf{A}$ for a given tolerance epsilon. The dimensions of each of the TT-cores are specified in the 'dim' argument. How can you do the transpose of a matrix in Tensor Train matrix form?

Exercise 6

Write a Matlab function `b=matrixvec(A,x)` that computes a matrix-vector multiplication $\mathbf{b} = \mathbf{A}\mathbf{x}$ where the matrix $\mathbf{A}$ is given in Tensor Train matrix form and the vector $\mathbf{x}$ is given as a Tensor Train. The resulting vector $\mathbf{b}$ needs to be a Tensor Train. How would you compute the product $\mathbf{x}^T\mathbf{A}$ without modifying your function `matrixvec(A,x)`?