Home Assignment Financial Engineering

Pieter Pijls
Master in Financial and Actuarial Engineering
Faculty of Business and Economics, KU Leuven

Introduction

This paper describes the design of a Barrier Reverse Convertible (BRC). The BRC is linked to a single stock, namely Microsoft Corporation with ticker MSFT. Figure 1 shows the stock price of MSFT on 1 May 2017. The maturity date for this financial product is Wednesday 1 May 2019. The starting date which is considered as $t = 0$ is Tuesday 1 May 2018. In general, the payoff of the BRC is determined as follows. For an investment N at time $t = 0$, the investor receives enhanced coupons during the lifetime of the BRC. At maturity he either receives 100% of N or a predetermined number of MSFT shares. All calculations are executed in Matlab.

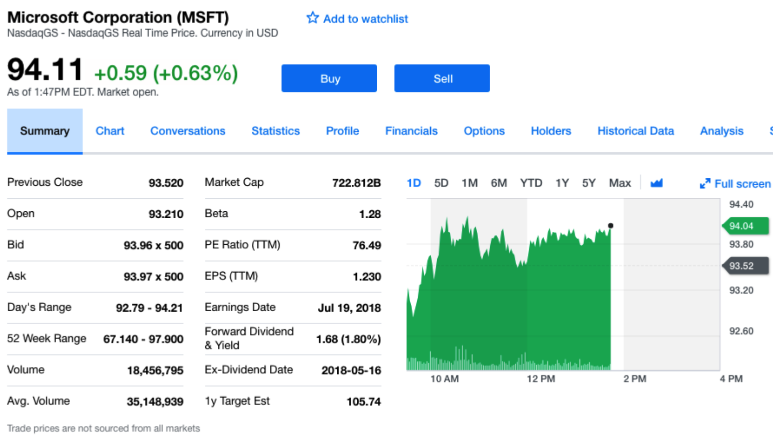

Figure 1: Microsoft Corporation Stock Characteristics on 1 May 2018, source: Yahoo Finance

The first section consist of a brief descriptive part of the BRC. We elaborate why the product is attractive for investors. The second section describes in more details the technicalities and the profitability of selling a BRC from the bank perspective. In the final section we determine the delta hedging strategy that could be put on at the initial valuation day.

1 Descriptive Part

1.1 Product Structure

BRCs are coupon bearing notes, structured to provide enhanced yield while participating in certain equity risk. The yield on the BRC is directly linked to the underlying stock performance. Suppose the initial investment N is $1,000$ USD. The annualized coupon rate is equal to 9.7% and are paid monthly. Every month the investor receives a coupon equal to 8.5 dollar.

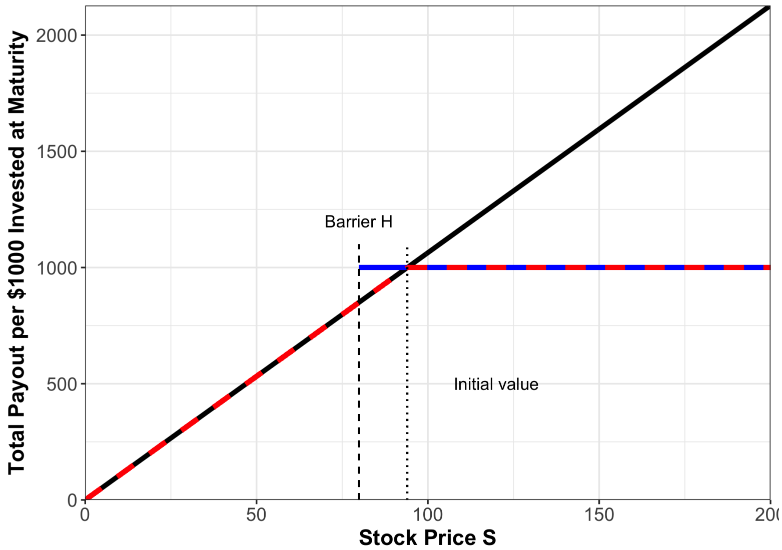

Figure 2: Payout at maturity T with Scenario 1 (blue), Scenario 2 and 3 (red dashed line) and underlying asset (black line) and initial value S_0 and Barrier $H = 0.85S_0$

At maturity, the investor receives (in addition to the stated coupon payments) either 100% of the initial amount N (cash delivery) or 10.64 (N/S_0) MSFT shares (Physical Delivery) with $S_0 = 94.01$. Investors should believe in the underlying company as they possibly end up with MSFT shares. In which scenarios the investor ends depend on whether the MSFT stock S crosses a barrier H. The barrier H is set at 79.91 USD which is 15% below the initial stock price S_0. In total, there exist three possible scenarios at maturity.

1. Scenario 1: The stock price S never crossed the barrier during the lifetime of the product. The product is redeemed in a cash amounting to 100% of the invested nominal N.

2. Scenario 2: The stock price at maturity S_T is lower than at issue S_0 and has crossed the barrier at least once within the lifetime of the product. In this case, the product is redeemed in 10.64 MSFT shares. The number of MSFT shares S

is such that at initiation of the product his amount was equivalent to 100% of the invested nominal N.

3. Scenario 3: The stock price S_T is higher than at issue and has crossed the barrier H at least once. The product is redeemed in cash amounting to 100% of the invested nominal N.

The total pay out at maturity in scenario 1 is equal to 1000 USD and $12 \cdot 8.5 = 102$ USD of the monthly coupons. The stock price S has to fall below $84,72$ USD AND hit the barrier H in order to have a negative return. Notice, you always receive the monthly coupons even if the stock collapses. Figure 2 illustrates the payout at maturity of the BRC per $1,000$ USD invested.

1.2 Motivation

The Barrier Reverse Convertible is a financial product designed for investors which are looking for yield enhancement. We explain three reasons why a BRC is an interesting investment.

1. Although interest rates in the US are increasing because of recent hikes by the Fed most government bonds are trading at very tight levels from a historical perspective. The BRC delivers an attractive annualized coupon rate of 9.7%.

2. In the stock market there exist still a bullish sentiment for the coming months among portfolio managers. Since 2010, the S&P 500 only faced two times a correction of more than 15%. In addition, the MSFT stock price and the technology sector in general are in an upward trend. MSFT shares soared with more than 90% in the last two years. Figure 3 illustrates the total return of MSFT stock outperformed all major indices in the period mid-2013 to 1 May 2018.

3. A BRC allows investors to take advantage of the low volatility in stock markets while receiving higher income streams compared with traditional fixed-income products. The CBOE Volatility Index (VIX) is trading at historical low levels. Notice, the BRC enables investors to take a short volatility position.

For example, the product is interesting for insurance companies. At this moment, insurers and pension funds must look for higher yield investments to make sure to meet future liabilities.

The main market risks of the BRC is a drop in the underlying stock of MSFT. Investors should bear in mind there is no principal protection and can potentially loose the full investment N. The BRC delivers a limited upside while the downside is unlimited. The potential loss is linked to underlying stock performance. Another risk of the BRC is the possibility that the issuer defaults. In that case the investor will not receive his invested amount N and the remaining coupons. An investor could hedge the credit risk

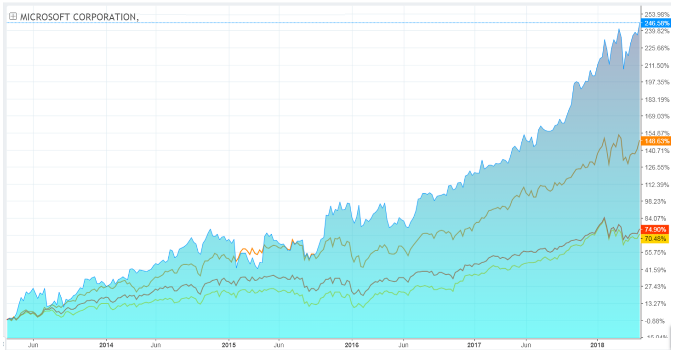

Figure 3: Stock price total return evolution of Microsoft Corporation (blue line), SP500 Index (red line), Dow Jones Index (yellow line) and Nasdaq (orange line) from mid-2013 to 1 May 2018, source: Tradingview

by entering a long position in a Credit Default Swap (CDS). Another important risk is that the BRC is not trading on an exchange but an Over-the-Counter (OTC) market which is less liquid. The liquidity risk will possibly cause losses when the investors sells the BRC before maturity. Finally the BRC does not include the costs of transaction fees, broker commissions and taxes.

2 Technical Part

This section describes the technicalities and the profitability of selling a BRC from the bank perspective. We construct the BRC as a combination of a long position in a zero-coupon bond (ZCB) and a short position in Down-and-In-Barrier Put options (DIBP). We start with a brief presentation of our algorithm to price European vanilla calls and puts under the Heston model. Next, we elaborate on our Monte-Carlo pricer for the exotic structure in the BRC. In this section we determine the price of the DIBP. Furthermore, we value the ZCB and construct the BRC. We also determine the BRC's annualized coupon rate. In the final part the profitability of this financial product is explained.

2.1 Heston Calibration

In this assignment we use the Heston model to price European vanilla call and put options. The parameters $\kappa, \eta, \theta, \rho$ and v_0 of Heston are calibrated we minimize the average relative price error (ARPE) by solving the following optimization problem:

$$\min_{\kappa,\eta,\theta,\rho,v_0} \frac{1}{n} \sum_{i=1}^{n} \frac{|P_i - \pi_i(\kappa,\eta,\theta,\rho,v_0)|}{\pi_i(\kappa,\eta,\theta,\rho,v_0)} \tag{2}$$

The Heston model price is π. The market option price is denoted by P_i. The market prices corresponds with the MSFT European vanilla option prices from the Chicago

Board Options Exchange (CBOE) (opt) on 1 May 2018. We have both the ask and bid prices for the market options prices. The market option price P_i in our algorithm corresponds with the mid price. The mid price takes the average of the ask and bid price. In total we calibrate the parameters on n vanilla options. We only take out-of-the-money (OTM) options because in-the-money (ITM) options are less liquid. Furthermore, we excluded options with a bid price lower as 0.2 USD and options which matures in 0.1 year.

We use the Carr-Madan Formula together with Fast Fourier Transform (FFT) to calculate the Heston model price π. The FFT is a numerical technique to evaluate the integral in the Carr-Madan Formula in an efficient way and gives us extremely fast pricing algorithm. We incorporate Simpson's rule for a more refined weighting for the integral in the Carr-Madan formula. This approximation gives a much more accurate integration. In addition, we make sure the Feller condition $2\kappa\eta \geq \theta^2$ is satisfied by imposing a constraint in the optimization algorithm. In that case, theoretically the volatility process v_t will never hit zero.

We find the following parameters for Heston. Table 1 gives an overview of the Heston parameters values which minimizes the relative deviance from the market prices. The initial value column corresponds with the initial values we incorporate in the code to speed up the algorithm. Finally, we used a lower bound (LB) and upper bound (UB) for the parameters.

Parameter	Description	Value	Initial Value	LB	UB
κ	Spead of mean reversion	5.059	0.25	0	20
η	Level of mean reversion	0.075	1	0	1
θ	Vol-of-vol	-0.533	0.81	0	5
ρ	Correlation vol-stock	0.8243	-0.7	-1	0
v_0	Initial vol	0.0502	0.01	0	1

Table 1: Parameters of Heston Model

Figure 4 illustrates the Heston option prices together with the market prices of the European vanilla options. The horizontal axis corresponds with the strike K of the option. The vertical axis shows the option price. In general, the Heston model delivers an adequate fit to the option prices. We end up with a ARPE of 10.62% [1]. Figure 4 shows that the Heston model gives a good fit to the market prices of C.B.O.E..

[1] To reproduce these results use the random seed rng(0387948) in Matlab

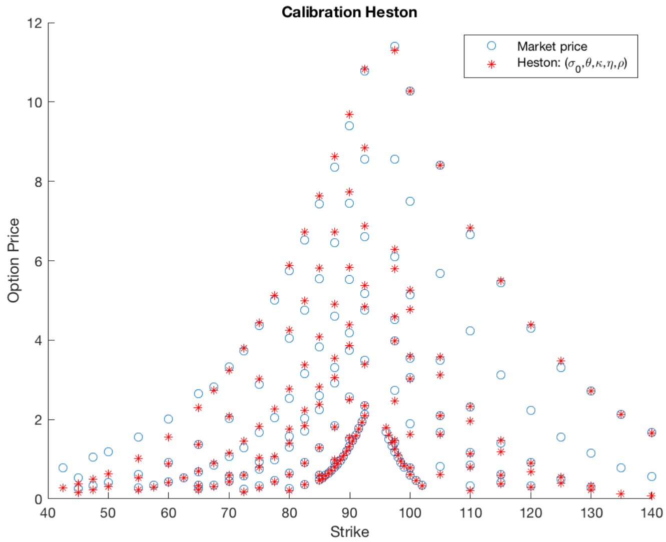

Figure 4: Calibration for the Heston model, source: C.B.O.E.

2.2 Pricing DIBP

In this section we present our algorithm to value the DIBP option. Table 2 gives an overview of the characteristics of the DIBP we will value

Initial Date	1 May 2018
Maturity Date	1 May 2019 (T=1)
Barrier H	$0.85S_0$= 79.91 USD
Strike K	S_0=94.01 USD

Table 2: Characteristics Down-and-In-Barrier Put Option

In order to price the DIBP we create stochastic scenarios under the Heston model. In total we simulate $100,000$ scenarios of the volatility v and MSFT stock price S. The step length in the processes correspond with one day. In total we have 252 days because we assume that every year there exist 252 trading days on average and $T = 1$. When we use the Euler Scheme stochastic process of v is calculated as:

$$v_{t+1} = |v_t + (\kappa(\eta - v_t))|\mathrm{dt} + \theta\sqrt{v_t}\sqrt{\mathrm{dt}}\epsilon_2, \quad \text{for} \quad t \in \{0, 252\}. \tag{3}$$

Due to the Feller condition the volatility process will never hit zero in theory. However, in practice this problem can still occur because of the discretization errors in the MC simulations. We avoid negative volatility by taking the absolute value. The stochastic process of the stock price S using the Euler Scheme is calculated as:

$$S_{t+1} = S_t \exp((r - q - \frac{1}{2}v_t)\mathrm{dt} + \sqrt{v_t\mathrm{dt}}\epsilon_1) \tag{4}$$

Another problem that possibly occurs are negative stock prices. Rouah (3) avoid negative stock prices S by using a a geometric Brownian motion in the stochastic process. The dividend yield is equal to 1.6%. We estimate dividend yield q from the Pull-Call Parity which states $\exp(-qT)S_0 - \exp(-rT)K + EP(K,T) - EC(K,T) = 0$. We use an European call EC and put option EP with strike K of 95 USD and maturity T at 21 June 2019. We use the mid price of an European call and put option on the same underlying dividend paying stock. The current stock price is S_0 and the options have the same time to maturity T and the same strike price K.

In the stochastic process of v and S we have ϵ_1 and ϵ_2 which are standardized Gaussian variables with correlation ρ. The correlation ρ is equal to -0.53 which is the optimal parameter under the Heston model. In order to calculate ϵ_1 and ϵ_2 we simulate independent standard normal numbers ϵ and ϵ^*. ϵ_1 and ϵ_2 are defined by:

$$\epsilon_1 = \epsilon \tag{5}$$

$$\epsilon_2 = \rho\epsilon + \sqrt{1+\rho^2}\epsilon^* \tag{6}$$

The Monte-Carlo simulation procedure goes as follows:

1. The time interval T is divided in 252 steps such that the interval time $\mathrm{dt} = \frac{1}{252}$;
2. We sample a random path for the stock price of Microsoft Corp. S. Notice, the DIBP option is dependent on S;
3. We calculate the present value (PV) of the DIBP for this particular path;
4. We repeat step 1 and 2 to obtain M sample PVs for the DIBP option;
5. We compute the mean of the M sample values to obtain an estimate of the PV of the DIBP option.

Figure 5 illustrate 10 random paths for the underlying stock price of Microsoft S. The present value of the DIBP for a particular path S_i is calculated as:

$$DIBP_i = \exp\left(-r_T \cdot T\right) \cdot \max\left(\frac{H - \min(S_{t,i}, 0 \leq t \leq T)}{|H - \min(S_{t,i}, 0 \leq t \leq T)|}, 0\right) \cdot \max\left(K - S_{T,i}, 0\right) \tag{7}$$

Taking the average of all random paths i an individual DIBP option is worth 8.62 USD using this procedure. The number of DIBP options that we sell is equal to $N/S_0 = 10.64$. This is the maximum number of stocks S you can acquire with the initial amount N of the investor. In total we receive $91,75$ USD when we sell 10.64 DIBP options.

An even better approach to value the DIBP option is to take the time steps dt smaller when the stock price S_t is close to the barrier H. This is because crossing the barrier

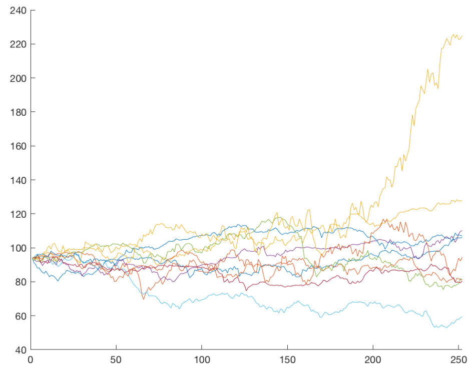

Figure 5: 10 random paths of the MSFT stock price S_t

H is crucial for the payoff of the DIBP. However, we decide to take daily observations of S to decrease the computational time. There exist a trade-off between accuracy and computational time. Another improvement could be to include jumps in the stock price process S. Heston incorporates stochastic volatility in an tractable and intuitive way. However the pure Heston model does not include jumps. One could also try to calibrate on the volatility instead of the market prices of the options.

2.3 Zero Coupon Bond

The ZCB is worth 977.89 USD on 1 May 2018 with maturity in one year. To value the ZCB we use the interest rates from Daily Yield Curve Rates of the US Treasury Bills (int) on 1 May 2018. To find the interesting rate corresponding to the ZCB maturity we use cubic spline interpolation [2]. The corresponding annualized interest rate of the ZCB is 2.26%. Notice, we bear the credit risk the counter-party of the ZCB will default.

2.4 Product Construction

In total we receive the initial investment $N = 1,000$ USD from the client. Furthermore, we sell DIBP options for a total amount of $91,75$ USD. We invest 977.89 in an ZCB which will grow to $1,000$ in one year. Now we have 113.85 USD left to pay out coupons to the investor of the BRC. The maximum annualized coupon rate we can give without losing money is equal to 10.7%. We decide to give the investors in the BRC a monthly coupon of 8.5 USD or an annualized coupon rate of 9.71% to have a certain profit margin. The PV of all coupons is equal to 99.96 USD. Therefore the fair value/price of the BRC is in fact equal to $977.89 - 92.24 + 99.96 = 986.12$ USD. We make a profit of 13.88 USD for every $1,000$ USD invested in the BRC or a profit margin of 1.39%.

[2]This is done by using the function `spline` in Matlab

3 Hedging

In this section we present a delta hedging strategy that could be put on at the initial valuation day (1 May 2018). We assume we take a short position in a BRC with an initial amount of $N = 1,000,000$ USD. This can be interpreted as follows. Essentially we buy DIBP options from the investor. Therefore we have a long position in the DIBP options. The Greek Δ for the BRC under the Heston model is calculated. Finally we specify for the Delta hedge the exact hedging position in the related hedging instruments.

In order to delta hedge the BRC we need to calculate the delta of every product involved in the structured product. We constructed the BRC as a combination of a short position in a ZCB and a long position in DIBP options. The Δ of the ZCB is zero because it is not dependent on the underlying stock price S of Microsoft. On the other hand, the DIBP option price is dependent on S. The Δ of the DIBP is calculated using the following procedure.

1. The time interval T is divided in N steps such that the interval time $\mathrm{dt} = \frac{T}{N}$;
2. We sample a random path for the stock price of Microsoft Corp. S. The staring value is equal to S_0.
3. We calculate the present value (PV) of the DIBP for this particular path;
4. We repeat step 1 and 2 to obtain M sample PVs for the DIBP option for a specific S_0;
5. We compute the mean of the M sample values to obtain an estimate of the PV of the DIBP option.
6. We repeat all steps using a different staring value S_0 of the MSFT stock price to obtain the DIBP option price dependent on S_0.

We approximate the Δ of the DIBP option with $\mathrm{dS} = 0.01$ as:

$$\Delta \approx \frac{DIBP(S_0 + \mathrm{dS}, K, H, T) - DIBP(S_0, K, H, T)}{\mathrm{dS}} \tag{8}$$

The Δ of the DIBP option is equal to -0.4237 if $S_0 = 94.01$ USD.[3] When N is 1 mio USD we create a long position in $N/S_0 = 10,637$ DIBP options. We multiply the Δ of the DIBP with the number of DIBP options we bought from the investor. Because we are long the DIBP options we buy $10,637 \cdot 0.4237 = 4,507$ stocks of Microsoft. Table 3 gives an overview of the situation where the stock price of Microsoft Corp. increases to 95 USD.

If the stock price S increases we make a loss on our DIBP portfolio and a profit on our stock portfolio. The DIBP is equal to 8.62 USD when S_0 is 94.01 USD. When the initial

[3] To reproduce the Δ use the random seed rng(0387948) in Matlab

S	DIBP Portfolio	DIBP CF	Stock Portfolio	Stock CF	Total CF
94.01	$8.62 \cdot 10,637 = 91,691$	/	$94.01 \cdot 4,507 = 423,703$	/	/
95	$8.24 \cdot 10,637 = 87,649$	+ 4,042	$95 \cdot 4,507 = 428,165$	4,462	420

Table 3: Delta Hedge Strategy at Initial Date with N =1 mio USD

stock price S_0 is 95 USD the DIBP is worth 8.25 USD. The impact of the stock price S on our BRC is almost neutralized using this delta hedge strategy (impact of 0.0042% when $N = 1$mio USD). When the stock price S changes one need to adjust the number of stocks because the Δ of the DIBP option has changed. Figure 6 illustrate the value of Δ when the stock price of MSFT S changes. We observe that Δ below the barrier H is the same as a standard put because the DIBP has knocked in. The Δ is always negative and approaches zero for large stock prices because the probability of a knock-in decreases.

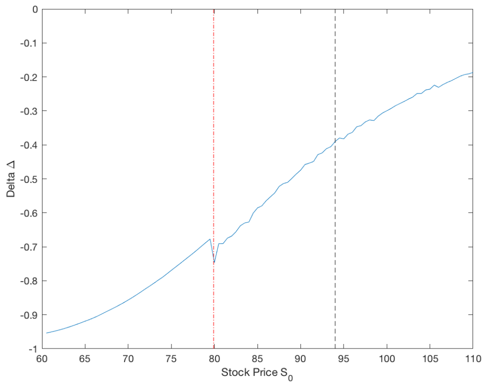

Figure 6: Δ given the MSFT stock price S, black dashed line corresponds with $S_0 = 94.01$ USD, red dashed line corresponds with the barrier $H = 79.91$ USD

In the normal case we described in the previous sections we sold the BRC to an investor. To create the payoff of a BRC we took a long position in a ZCB and sold DIBP options. The investor received an increased coupon rate from us. In this case we do not need to hedge. This is because the possible loss on our DIBP portfolio is shifted to the client. Suppose the MSFT stock decreases to 80 USD at maturity T. We face a loss of $(94,01 - 80) \cdot 10,64 = 149$ USD. In this case we only need to deliver the client $N/S_0 = 10,64$ MSFT stocks which is worth $10.64 \cdot 80 = 851$ USD instead of $N = 1,000$ USD. We make a loss of 149 USD on the DIBP portfolio but we also have to pay 149 USD less to the investor at maturity T.

Notice, another approach could be to find the Greeks of the DIBP under the Black-Scholes model. Rubinstein et al. (4) gives an analytical solution of the DIBP option price under the Black-Scholes setting.

Literature

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