Stochastic Finance in Insurance (Part 1/ Prof. Pierre Devolder)

Tine Huybrechts
Pieter Pijls
Master in Financial and Actuarial Engineering
Faculty of Business and Economics, KU Leuven

Introduction

In this assignment we illustrate the fair valuation and premium calculation of six insurance products with various participation schemes in life insurance. Our main source are the slides presented by prof. Dr. Pierre Devolder during the first part of the course Stochastic Finance. We also made use of some additional sources: Devolder and Azizieh (2013); Devolder and Dominguez-Fabian (2005). All simulations and calculations are executed in R.

Question 1

The product issued is a pure endowment insurance with guaranteed interest rate and without participation for a capital in case of life of 100,000 euro (C) (single premium). On the one hand, the liability of the insured exists of a single premium at the start of the contract. On the other hand, the liability of the insurer towards the insureds exists of paying 100,000 euro at the end of the contract in case the insured is still alive, no bonus is paid out.
We calculate the premium as follows:

$$\text{Premium} = C * E_Q\{\exp(-\int_0^T \mu_{x+s}(s)ds)\} * v^T \tag{2}$$

with $v = \frac{1}{1+i}$. The value of the technical interest rate i is 1% and the according premium is equal to $45,135.26$ euro (for $T = 30$, the premium is equal to $20,016.16$ euro). We created 1,000 random paths of the mortality intensity μ with 2,000 time steps over 20 years ($dt = 0.01$) via sampling from the standard normal distribution. Z_m is a standard normally distributed random variable. Multiplied with the square of the timestep this simulates the brownian motion. We calculate the random paths of the mortality intensity as follows:

$$\delta\mu_{35+t}(t) = 0.08 * \mu_{35+t}(t) * \delta t + 0.008 * Z_m * \sqrt{\delta t}, \tag{3}$$

with

$$\mu_{35+t} = |\mu_{35+t-\delta t} + \delta\mu_{35+t}|, \quad \text{for} \quad t \in \{0.01, 20\}. \tag{4}$$

The mortality intensity follows the Vasicek model (Vasicek, 1977). We notice that the speed of mean reversion is negative which causes the model to expand tremendously. The instantaneous volatility is rather big leading to a negative mortality intensity. To overcome the problem of having negative mortality intensity, we force it to be positive by taking the absolute value (see equation 4). To calculate the expected value of the exponential of the integral of the short term interest rate, $r(t)$, we create 1,000 random paths following the same method as for the mortality intensity (see equation 3) except from the fact that we do not take the absolute value this time. The short term interest rate also follows the Vasicek model. We sample from a second r.v. Z_r. Remark that Z_m and Z_r are independent. We calculate the random paths of the short term interest rate as follows:

$$\delta r(t) = 0.20 * (0.02 - r(t)) * \delta t + 0.012 * Z_r * \sqrt{\delta t}, \tag{5}$$

with

$$r(t) = r(t - \delta t) + \delta r(t), \quad \text{for} \quad t \in \{0.01, 20\}. \tag{6}$$

The fair value is calculated as follows:

$$FV(0) = E_Q\{\ C * \exp(-\int_0^T r(s)ds) * \exp(-\int_0^T \mu_{x+s}(s)ds)\}. \tag{7}$$

The fair value is equal to $39,514.11$ euro (for $T = 30$, the fair value equals $15,970.2$ euro). We obtain 1.66% as the value of the technical interest rate to obtain an equilibrated contract by putting the fair value equal to the premium.

Question 2

In this question we consider a pure capitalization bond with guaranteed interest rate and with a terminal bonus for a capital of $100,000$ euro (C) (single premium). As bonus, the insurer distributes 85% (β) of the final surplus resulting from the investment in fund F. On the one hand, the liability of the insured exists of a single premium at the start of the contract. On the other hand, the liability of the insurer towards the insured exists of paying $100,000$ euro at the end of the contract plus a bonus in case the premium we invested in fund F amounts more than the guarantee. We consider a pure capitalization bond so we do not consider mortality in here.
We calculate the premium as follows:

$$\text{Premium} = C * \left(\frac{1}{1+i}\right)^T. \tag{8}$$

The fair value can be split into two parts: the fair value of the guarantee and the fair value of the terminal bonus. We calculate these values as follows and the total fair value is equal to the sum of these components:

$$FV_{guar}(0) = E_Q\{\ C * \exp(-\int_0^T r(s)ds) * (1+i)^T\}, \tag{9}$$

$$FV_{TB}(0) = E_Q\{\ \beta * \text{premium} * \exp(-\int_0^T r(s)ds) * \max(0, (S_F(T) - (1+i)^T))\}. \tag{10}$$

with β as the participation rate equal to 0.85. $S_F(T)$ is the value of fund F after 20 years. We simulated this in the same way as we did for the short term interest rate (see equation 5). We include the correlation between the short term interest rate and the fund F by correlating the normal values. For the short term interest rate we used the standard normal r.v. Z_r. For the fund F we will create standard normal r.v. $Z_F* = \rho * Z_r + \sqrt{1-\rho^2} * Z_F$, with Z_F a standard normal r.v.. This procedure incorporates the correlation structure between fund F and the short term interest rate which is equal to -0.5.

When T is equal to 20 years, the premium amounts $81,954.45$ euro and the total fair value is equal to $140,207.8$ euro. If T is equal to 30 years, the premium amounts $74,192.29$ euro and the total fair value is equal to $158,538$ euro. We obtain -1.52% as the value of the technical interest rate to obtain an equilibrated contract by putting the fair value equal to the premium.

Question 3

In this question we work with a pure endowment insurance with guaranteed interest rate and with a terminal bonus for a capital of $100,000$ euro (C) (single premium) (distribution of 85% (β) only of the mortality result). On the one hand, the liability of the insured exists of a single premium at the start of the contract. On the other hand, the liability of the insurer towards the insured exists of paying 100,000 euro at the end of the contract in case the insured is still alive, and an extra bonus in case to correct for the fact that the life table does not match reality entirely.

We compute both the premium and the fair value of the contract for a technical interest rate of 1% and compute the value of the technical interest rate to obtain an equilibrated contract, for a duration of 20 or 30 years. The definition of the premium is the same as in Question 1. For $T = 20$ years, the premium is equal to $81,954.45$ euro. The fair value is split into two parts: the fair value of the guarantee and the fair value of the terminal bonus and defined as:

$$FV_{guar}(0) = E_Q\{\ C * \exp(-\int_0^T \mu_{x+s}(s)ds) * \exp(-\int_0^T r(s)ds)\}, \tag{11}$$

$$FV_{TB}(0) = E_Q\{\ \beta * C * \exp(-\int_0^T r(s) + \mu_{x+s}(s)ds) * \max(0, \frac{{}_Tp_x}{\exp(-\int_0^T \mu_{x+s}(s)ds)} - 1)\}. \tag{12}$$

where ${}_Tp_x$ is defined as:

$$ {}_Tp_x = \frac{l_{(x+T)}}{l_x} = s^T * g^{(c^{(x+T)} - c^x)} \tag{13} $$

with initial age $x = 35$ year, maturity of the contract $T = 20$ years, and first order actuarial bases: technical rate, $i = 1\%$, technical life table: l_x determined by the Makeham constants, $s = 0.999441704$, $g = 0.999733441$ and $c = 1.101077536$. The parameter values to calculate ${}_Tp_x$ are from Devolder and Azizieh (2013). The fair value is equal to 95,818 euro. The fair value of the terminal bonus is 8,090 euro. The fair value of the guarantee is 87,728 euro. We obtain 2.28% as the value of the technical interest rate to obtain an equilibrated contract by putting the fair value equal to the premium.

Question 4

In this question we calculate the premium and fair value for of a pure endowment insurance with guaranteed interest rate and with a terminal bonus for a capital of $100,000$ euro (single premium) (distribution of 85% only of the financial result; investment in the fund G). Only the financial profits are taken into account to decide whether or not to pay out a bonus. This will correct the fact that the technical guaranteed interest rate i will not match reality.

We assume a technical interest rate of 1% and compute the value of the technical interest rate to obtain an equilibrated contract, for a duration of 20 or 30 years. The definition of the premium is the same as in question 1. The premium is equal to 45,135.26 euro. The fair value is defined as:

$$ FV_{guar}(0) = E_Q\{\ C * \exp(-\int_0^T \mu_{x+s}(s)ds) * \exp(-\int_0^T r(s)ds)\}, \tag{14} $$

$$ FV_{TB}(0) = E_Q\{\ \beta * C * \exp(-\int_0^T r(s) + \mu_{x+s}(s)ds) * \max(0, \frac{G(T)}{(1+i)^T} - 1\}. \tag{15} $$

The fair value is equal to 63,676.46 euro. The fair value of the terminal bonus is 24,162 euro. The fair value of the guarantee is 39,514 euro. We obtain -1.96% as the value of the technical interest rate to obtain an equilibrated contract by putting the fair value equal to the premium.

Question 5

In this question we compute the fair value of a unit linked insurance with an exogenous guarantee to recuperate in case of life at maturity. The policyholder receives at least the initial amount invested in the fund G equal to $50,000$ euro (C). Again the duration of the contract is 20 years (T). The FV is defined as:

$$ FV(0) = E_Q\{\ C * \max(1, G(T)) * \exp(-\int_0^T r(s)ds) * \exp(-\int_0^T \mu_{x+s}(s)ds)\}. \tag{16} $$

The fair value for this specific contract is equal to 102.240,1 euro.

Question 6

In this question we compute the fair value a unit linked insurance without mortality for an initial amount invested in the fund G equal to 50,000 euro (C). The product also contains a switching option to the fund F only at maturity. Again the duration of the contract is 20 years. The FV is defined as:

$$FV(0) = E_Q\{\ C * \max(G(T), F(T)) * \exp(-\int_0^T r(s)ds)\}. \tag{17}$$

The fair value for this specific contract is equal to 188,220.2 euro.

Literature

Devolder, P. and Azizieh, C. (2013). Margrabe option and life insurance with participation. *BULLETIN FRANCAIS D'ACTUARIAT*, 13(26):47–77.

Devolder, P. and Dominguez-Fabian, I. (2005). Fair valuation of various participation schemes in life insurance. *ASTIN BULLETIN*, 35(1):275–297.

Vasicek, O. (1977). An equilibrium characterization of the term structure. *Journal of Financial Economics*, (5):177–188.

FACULTY OF BUSINESS AND ECONOMICS
Naamsestraat 69 bus 3500
3000 LEUVEN, BELGIË
tel. + 32 16 32 66 12
fax + 32 16 32 67 91
info@econ.kuleuven.be
www.econ.kuleuven.be