Statistics for Finance and Insurance Assignment 2: Portfolio Analysis

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Introduction

In this assignment different portfolios are constructed with 10 individiual stocks. First, we investigate the stock return data for the period 1999-2016. Next, we construct the minimum variance, tangency and minimum risk contribution portfolio using the first 126 observations. Finally, we investigate the three portfolios with weekly rebalaning and a rolling window of 126 and 63 observations.

Data Exploration

We explore the data by investigating the marginal distribution of returns of each stock, and by looking at the inter-dependence of the returns. First, we create 10 QQ-plots to investigate if stocks returns are normally distributed. Figure 1 illustrates that te stock returns are not normally distributed because the obsevartions are not on the linear line. In addition the QQ-plots also illustrates that the distributions of the dail stock returns are heavy tailed and contain some outliers. This can be observerd from the deviations from the linear line which become larger in the more extreme theoretical quantiles.

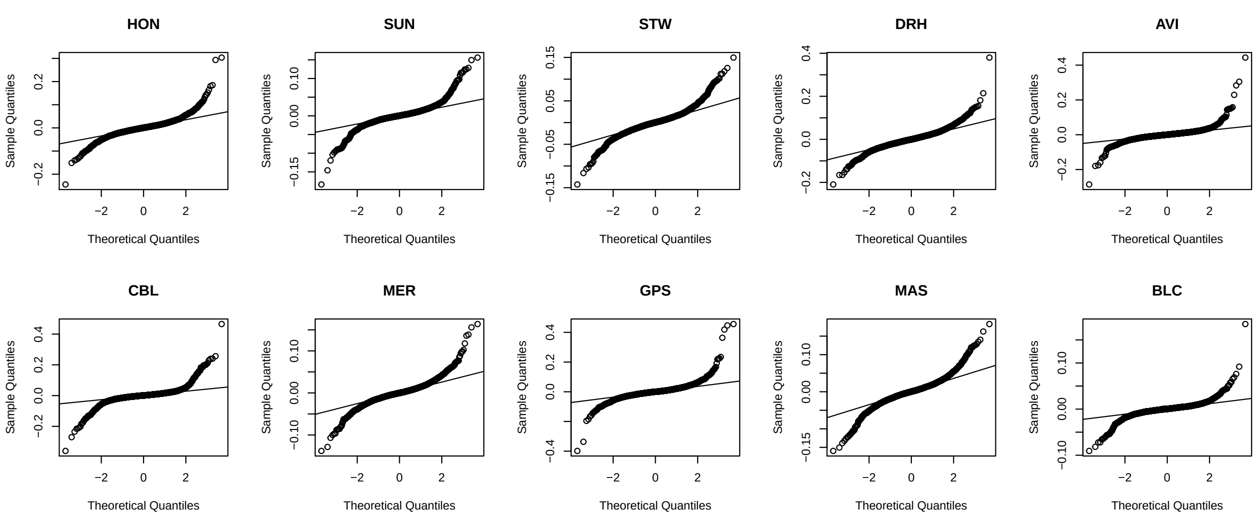

Figure 1: QQ-plots Daily Stock Returns

Figure 2 shows the daily stock returns from 1999 to 2016. The figure illustrates that stock returns are non-stationary as the variance is not constant over time (e.g. Financial crisis 2008-2009 we observe a high volatility in returns). A stationary process has the property that the mean, variance and autocorrelation structure do not change over time. The stock with ticker MER (blue dots) clearly does not satisfy these

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properties as the variance is changing over time. Also, the figure shows that the stock returns are positive dependent. They tend to move in group in times of financial stress.

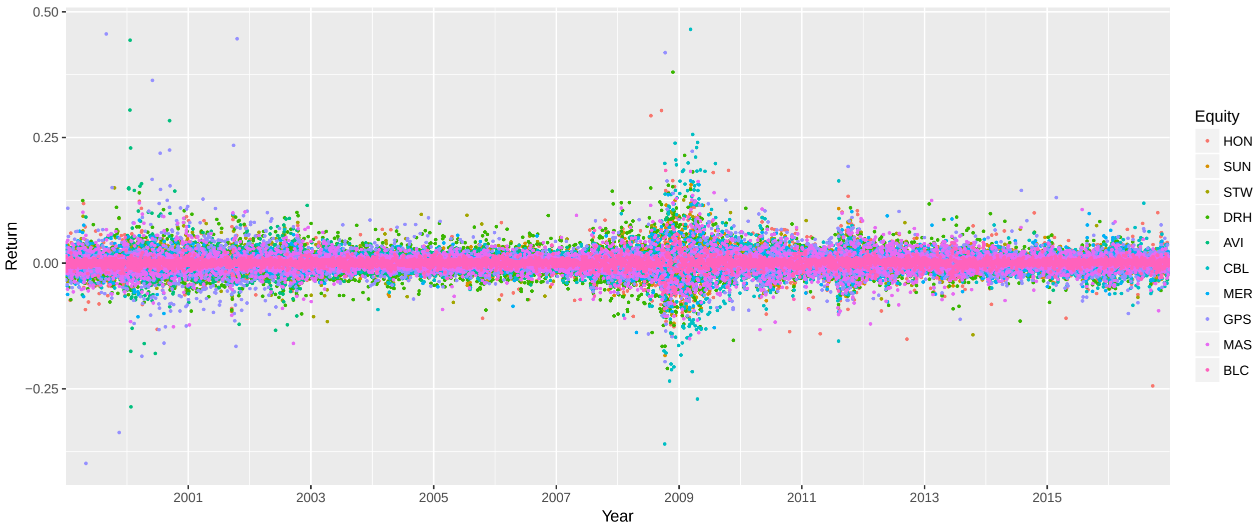

Figure 2: Daily Stock Returns from 1999-2016

Next, we investigate the dependence structure by computing the correlations between the different stock returns. As we already observed from the graph the correlation matrix in table 1 shows that stock returns are positively correlated.

Table 1: Correlation Matrix

	HON	SUN	STW	DRH	AVI	CBL	MER	GPS	MAS	BLC
HON	1.00	0.45	0.47	0.43	0.24	0.41	0.47	0.21	0.47	0.06
SUN	0.45	1.00	0.44	0.44	0.28	0.60	0.44	0.16	0.45	0.13
STW	0.47	0.44	1.00	0.47	0.25	0.43	0.46	0.18	0.57	0.10
DRH	0.43	0.44	0.47	1.00	0.21	0.43	0.39	0.14	0.56	0.10
AVI	0.24	0.28	0.25	0.21	1.00	0.21	0.23	0.10	0.22	0.09
CBL	0.41	0.60	0.43	0.43	0.21	1.00	0.42	0.17	0.45	0.12
MER	0.47	0.44	0.46	0.39	0.23	0.42	1.00	0.20	0.44	0.10
GPS	0.21	0.16	0.18	0.14	0.10	0.17	0.20	1.00	0.18	0.03
MAS	0.47	0.45	0.57	0.56	0.22	0.45	0.44	0.18	1.00	0.06
BLC	0.06	0.13	0.10	0.10	0.09	0.12	0.10	0.03	0.06	1.00

Portfolio construction and optimization

In this section we set up the efficient frontier, the tangency portfolio, the minimum variance portfolio and the minimum risk contribution portfolio based on the first 126 returns of each stock. We assume the risk-free rate $i_r f$ is 3% and that risk is measured as standard deviation of the return. Finally, we compare the expected return μ, standard deviation σ, Sharpe ratio and weights w between the three portfolios. To set up the efficient frontier we need to calculate the covariance matrix $\sum$

Table 2: Coviariance Matrix

	HON	SUN	STW	DRH	AVI	CBL	MER	GPS	MAS	BLC
HON	7.78	0.38	0.66	0.94	0.31	0.38	-0.60	-1.34	1.74	-0.21
SUN	0.38	1.25	0.22	0.52	0.10	0.33	-0.09	0.08	0.22	0.02
STW	0.66	0.22	3.69	0.54	0.10	0.64	-0.02	-0.97	1.72	0.06
DRH	0.94	0.52	0.54	6.95	0.25	0.12	0.73	0.46	2.36	0.12
AVI	0.31	0.10	0.10	0.25	2.80	0.25	0.02	-2.87	0.67	-0.14
CBL	0.38	0.33	0.64	0.12	0.25	1.77	-0.13	-0.57	0.48	0.08
MER	-0.60	-0.09	-0.02	0.73	0.02	-0.13	5.20	0.24	0.36	0.03
GPS	-1.34	0.08	-0.97	0.46	-2.87	-0.57	0.24	18.96	-1.34	0.28
MAS	1.74	0.22	1.72	2.36	0.67	0.48	0.36	-1.34	6.74	-0.04
BLC	-0.21	0.02	0.06	0.12	-0.14	0.08	0.03	0.28	-0.04	0.35

We also have to specify our constraints. We make weights w add up to 100% (full investment constraint), have at least the average return (tartget reward constraint) and no short selling constraint.

$$\sum w_i = 1,\ i = 1, ..., 10$$

$$w_i \geq 0,\ i = 1, ..., 10$$

We computed the the efficient frontier, minimum variance (MV) portfolio, tangency (TAN) portfolio and minimum contribution (MINC) portfolio using quadratic programming (using 100 iterations). Figure 3 illustrates the location of the individual stocks on the "reward-risk space" location. In addition, the efficient frontier, capital market line and the three constructed portfolios are shown. Notice, as we use the no-short selling contraint it is impossible to have an expected return greater than the expected return of HON, the stock with the highest return. In fact, there exists an upper bound on the expected return which is equal to the expected return of HON.

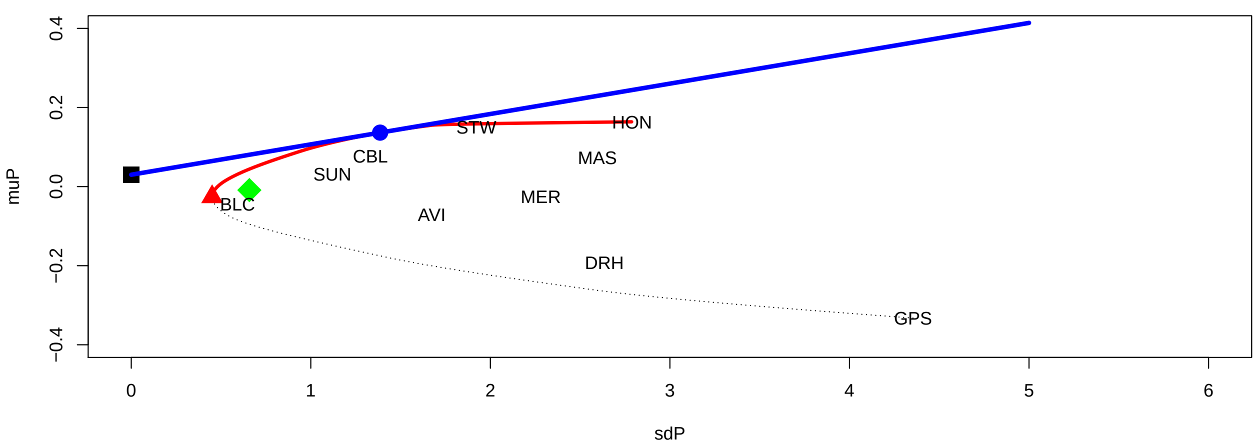

Figure 3: Effcient frontier (red), line (blue) connecting the risk-free asset and tangency portfolio (blue dot), the minimum variance portfolio (red triangle), the minimum risk contribution portfolio (green symbol) and 10 individual stocks

In Table 3 the expected return μ, standard deviance σ and sharpe ratio of the different portfolios are given. The minimum variance portfolio is the one with the smallest standard deviation σ. This follows from the fact that the MVP portfolio is the portfolio with the lowest standard deviation σ on the efficient frontier. The tangency portfolio has the highest expected return μ. The Sharpe ratio is defined as

$$\text{Sharpe ratio} = \frac{\mu - r_f}{\sigma}$$

where $i_r f$ is the risk-free investment equal to 3%. The tangency portfolio is the one with the highest Sharpe ratio. This follows from the fact that the tangency portfolio is the portfolio with the maximum sharpe ratio on the efficient frontier.

Table 3: Summary Statistics Portfolios

	Minimum Variance	Tangency	Minimum Contribution
Expected Return	-0.0259871	0.1364267	-0.0088806
Standard Deviation	0.4501818	1.3861213	0.6577884
Sharpe Ratio	-0.1243656	0.0767802	-0.0591081

In Table 4 the weights w of the different portfolios are given. Notice, the minimum variance portfolio invests in less volatile stock, while the tangency portfolio stock invests in shares with higher expected return μ. The minimum risk contribution portfolio is quite similar to the minimum variance portfolio as it also invest in stocks with a low standard deviation σ.

Table 4: Portfolios Weights

	HON	SUN	STW	DRH	AVI	CBL	MER	GPS	MAS	BLC
Minimum Variance	0.03	0.12	0.03	0.00	0.11	0.04	0.04	0.02	0.00	0.60
Tangency	0.26	0.00	0.50	0.00	0.00	0.24	0.00	0.00	0.00	0.00
Minimum Contribution	0.07	0.15	0.08	0.06	0.14	0.13	0.09	0.07	0.06	0.16

The portfolio weights from the previous section are based on the first 126 returns of each stock. This gives us an estimate of the expected portfolio return μ for the next day. After 5 trading days (a week) we can compute the realized weekly portfolio returns. We consider the following rebalancing strategy. First, we estimate the weights w of the tangency, the minimum variance and the minimum risk contribution portfolio using the most recent 126 daily returns and rebalance the respective portfolio weights w at the close of every Friday. Notice, when Friday the exchange is not open we will consider Thursday as the rebalancing day. Again, we don't allow short sales and assume that the risk-free rate remains fixed at 3%.

Starting with an inital wealth of USD 1000 we plot the evolution of the portfolio returns over time. If one would invested USD 1000 in 1999 he would have around USD 5000 in 2016 when he invested in the minimum variance portfolio, USD 6500 when he invested the tangency portfolio and USD 8000 in the minimum contribution portfolio.

Figure 4: Portfolio wealth evolution starting with an inital wealth of 1000$ with a rolling window with six months

In Table 5 the summary statistics are shown. We can draw the following conclusions on these statistics. The expected return of the tangecy portfolio is the highest. Next, the standard deviation of the minimum variance portfolio is the lowest as expected. The information ratio (IR) measures the expected return per unit of risk. In this case the risk is measured as the standard deviation σ. The minimum variance earned an average return of 4.7% per unit of standard deviation σ.

The Value-at-Risk (VaR) indicators of 1% and 5% show that tangency portfolio is the most risky. We can interprete this as follows. For the minimum variance portfolio the likelihood of having a daily return lower as -1.18% and -2.55% is respecitively 5% and 1%. Notice, this risk indicator is based on historical data.

Finally, the portfolio turnover measures how frequently assets within a portfolio are bought and sold. Table 5 shows that the tangency portfolio is the most active portfolio as it has the highest asset turnover. On the other hand, the minimum variance portfolio has bought and sold the least assets. Notice, for an individual investor this can be really important statistic as transactions costs can have a big impact on the net portolio return.

Table 5: Summary Statistics Portfolios (rebalancing with six months)

	Minimum Variance	Tangency	Minimum Contribution
Cumulative Return	5.4560175	6.8013320	8.3609678
Expected Return	0.0004261	0.0006161	0.0005733
Standard Deviation	0.0090308	0.0190285	0.0134894
Information Ratio	0.0471833	0.0323797	0.0425017
Value-at-Risk 5%	-0.0118441	-0.0263850	-0.0182664
Value-at-Risk 1%	-0.0254512	-0.0562030	-0.0388552
Asset Turnover	0.0712016	0.3996964	0.1357903

Rolling window with three months

Finally, we repeat the same exercise with a rolling window of three months (63 days instead 126). The minimum variance and minimum contribution portfolio end up with higher cumulative returns as with a rolling window of six months. This could be explained by the fact that rebalancing each three months allows for more flexibility. Therefore, these portfolios takes more recent data into account.

The fact that the cumulative of the tangency portfolio is lower as the minimum variance portfolio could be counter-intuitive. Notice, the minimum variance solution is constructed with stocks that have low variances and co-variances. Theoretically, we would expect that this should have a low expected return profile in the long run. However, in contradiction to modern portfolio theory it turns out that stocks that have low-volatility or low-beta experience higher returns than high-volatility or high-beta stocks. This is well-documented in the current literature as the low-volatility anomaly. This explains the fact that we end up with a higher cumulative return for the minimum variance portfolio compared to the tangency portfolio.

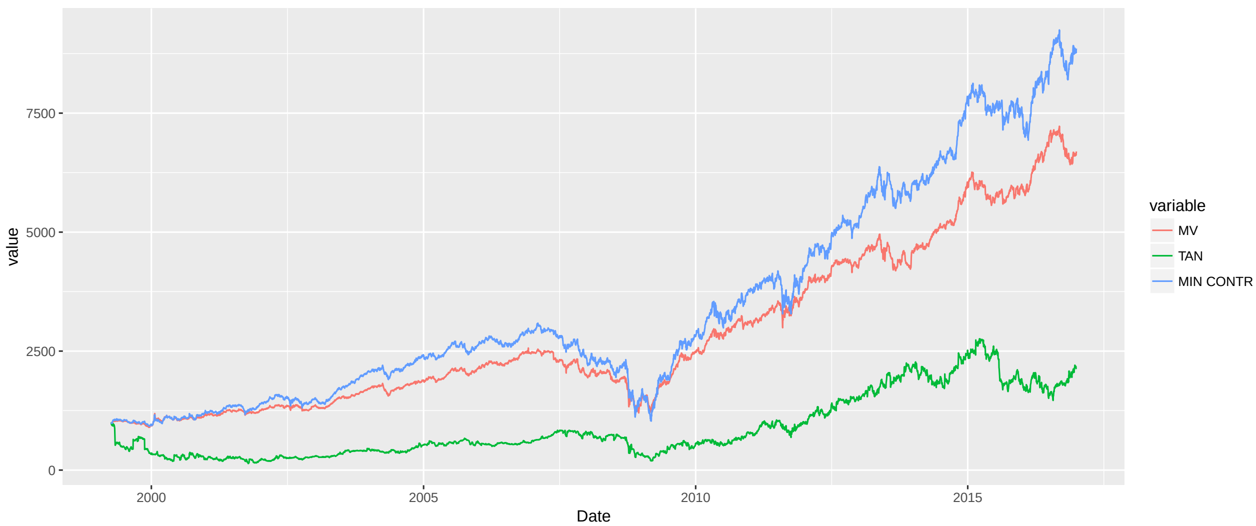

Figure 5: Portfolio wealth evolution starting with an inital wealth of 1000$ with a rolling window with three months

Table 6 gives an overview of the summary statistics of the different portfolios. The cumulative returns of the minimum variance and minimum contribution portfolio is higher, while the cumulative return of the tangency portfolio is lower when we only use three months of data for rebalancing the weights. Notice, the value-at-risk indicator illustrates the tangency risk is more riskier. Finally, the asset turnover statistics are higher for all portfolios.

Table 6: Summary Statistics Portfolios (rebalancing with three months)

	Minimum Variance	Tangency	Minimum Contribution
Cumulative Return	6.6948698	2.1227270	8.8561205
Expected Return	0.0004742	0.0006719	0.0005748
Standard Deviation	0.0098472	0.0320119	0.0131403
Information Ratio	0.0481589	0.0209906	0.0437414
Value-at-Risk 5%	-0.0133924	-0.0399992	-0.0181714
Value-at-Risk 1%	-0.0265320	-0.0805782	-0.0380049
Asset Turnover	0.1325294	0.4888227	0.1501677