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An Evaluation of Black-Litterman, Mean Variance and Naïve Diversification Approaches

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Abstract.

The Black-Litterman (BL) model aims to enhance asset allocation decisions by overcoming the weaknesses of standard mean-variance (MV) portfolio optimization. In this study we implement the BL model in a multi-asset portfolio context. Using an investment universe of global stock indices, bonds, and commodities, we empirically test the out-of-sample portfolio performance of BL optimized portfolios and compare the results to mean-variance (MV), minimum-variance, and naïve diversified portfolios (1/N-rule) for the period from January 1993 to December 2011. We find that BL optimized portfolios perform better than MV and naïve diversified portfolios in terms of out-of-sample Sharpe ratios even after controlling for different levels of risk aversion, realistic investment constraints, and transaction costs. Interestingly, the BL approach is well suited to alleviate most of the shortcomings of MV optimization. The resulting portfolios are less risky, are more diversified across asset classes, and have less extreme asset allocations. Sensitivity analyses indicate that the outperformance of the BL model is due to the consideration of the reliability of return estimates and a lower portfolio turnover.

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1. INTRODUCTION

The traditional mean-variance (MV) optimization (Markowitz, 1952) has played a prominent role in modern financial theory for many decades. It provides the investor with the optimal asset allocation if future asset returns are known and if portfolio risk and return are the only relevant parameters. In practical applications estimation errors in the input parameters, corner solutions, and high transaction costs resulting from extreme portfolio reallocations often result in a poor out-of-sample portfolio performance. Practitioners frequently try to cope with these problems by implementing constraints on the portfolio weights and turnover. Black and Litterman (1992) propose an alternative approach to deal with the shortcomings of MV and to improve portfolio performance. Their approach has gained increasing attraction among practitioners. Surprisingly, the academic literature has paid little attention to this model so far. One advantage of the Black-Litterman (BL) model is that it combines two sources of information - 'subjective' and 'implied' return estimates - thereby reducing the sensitivity of portfolio weights. In contrast to MV optimization, investors have the choice either to provide return estimates for each asset or to stay neutral for some assets they feel uncomfortable in providing reliable future return estimates. Moreover, the reliability of each return estimate can be incorporated enabling investors to distinguish between qualified estimates and pure guesses.

So far the academic literature provides little empirical evidence analyzing the performance of the BL model in an out-of-sample setting. Although several studies investigate the rationale of the BL model and apply it to calculate efficient frontiers, there exists no evidence that in an out-of-sample optimization the BL model generates a superior portfolio performance relative to MV, minimum-variance, or naïve diversified benchmark portfolios. We contribute to the literature by testing the BL model empirically in that we conduct out-of-sample multi-asset portfolio optimizations for the period from January 1993 to December 2011. We implement BL, MV, minimum-variance as well as naïve diversification approaches and evaluate the respective portfolio performance results. The main research question is whether the BL model is able to alleviate the problems of MV optimization and whether it generates a superior out-of-sample portfolio performance. We compute several performance measures and evaluate the BL portfolios in comparison to MV, minimum-variance, and naïve diversified benchmark portfolios. In addition, the literature is extended by implementing the BL model on multi-asset portfolios including global stocks, bonds, and commodities rather than stock-only portfolios.

Our empirical results offer new insights in several dimensions. We find that the BL model can successfully be applied to multi-asset portfolio optimization by using strategic weights for the different asset classes. BL optimized portfolios exhibit a consistently higher out-of-sample portfolio performance, they include a larger number of assets than MV optimized portfolios, and they are better diversified across asset classes. The superior out-of-sample performance of the BL model is due to the additional information on the reliability of return estimates ('views') and a lower portfolio turnover. In an analysis of sub-periods we find that the BL model outperforms MV and naïve diversified portfolios particularly in recessionary periods and that BL-optimized portfolios exhibit lower turnover and superior diversification properties. Finally, our results are robust to variations of the input parameters.

The remainder of the paper is organized as follows. In section 2 we review the literature on MV optimization, naïve diversification rules, and the BL approach. In section 3 the methodology of the BL model and the employed performance measures are described. The data and some descriptive statistics are provided in section 4. Our empirical results are presented and discussed in section 5. Section 6 concludes.

2. LITERATURE REVIEW

2.1. Mean-Variance-Optimization

The traditional mean-variance (MV) optimization (Markowitz, 1952) is widely employed in the academic literature. A number of studies use MV optimization to analyze the diversification benefits of an additional asset class in a multi asset portfolio context. These studies usually employ mean-variance spanning tests and analyze the diversification benefits of emerging equity markets (Bekaert and Urias, 1996; Roon, Nijman and Werker, 2001), real estate (Chiang and Lee, 2007), small cap stocks (Petrella, 2005), microfinance funds (Galema, Lensink, Spierdijk, 2011), commodities (Daskalaki, Skiadopoulos (2011) or hedge funds (Bessler, Holler, Kurmann, 2012). These studies provide evidence that investing in additional asset classes may be attractive for investors seeking to improve their risk-return profile.

In these approaches the portfolios' expected risk and returns are estimated relying on historical data. These historical estimates, however, are subject to a substantial level of uncertainty. Hence, the expectations on the portfolios risk-return structure may not materialize ex post. In fact, an early study by Jobson and Korkie (1981a) highlights the large estimation errors in using sample estimates and the poor out-of-sample performances of MV strategies. Estimation errors of returns, however, are much more critical than those of the covariance matrix since their effect on the optimized portfolio weights is about ten times larger (Chopra and Ziemba, 1993). In the MV optimization framework, assets with the highest estimation errors tend to obtain the highest portfolio weights, resulting in 'estimation error maximization' (Michaud, 1989). Additionally, the MV approach tends to generate extreme portfolio allocations and low levels of diversification across asset classes (Broadie, 1993), i.e. the optimized portfolios are usually corner solutions. Furthermore, mean-variance optimal portfolio weights are highly sensitive to changes in the input parameters which results in radical portfolio reallocations even for small variations in expected return estimates (Best and Grauer, 1991).

A number of authors propose variations and extensions of the MV approach trying to cope with these shortcomings. These approaches range from imposing portfolio constraints (Frost and Savarino, 1988; Jagannathan and Ma, 2003; Bessler, Holler and Kurmann, 2012) to the use of factor models (Chan, Karceski and Lakonishok, 1999) and Bayesian methods for estimating the MV input parameters. Prominent Bayesian approaches include the Bayes-Stein shrinkage estimation (Jorion, 1985 and 1986), which shrinks the return estimates from the sample mean returns towards the minimum variance portfolio return, and approaches proposed by Pastor (2000) and Pastor and Stambaugh (2000) which builds on the prior belief in an asset-pricing model such as the CAPM or a multi-factor model, e.g. the three-factor Fama and French model (1993). However, to evaluate the contribution of these extensions in a more realistic environment, the performance of the optimization rules has to be evaluated in an out-of-sample setting in comparison to adequate benchmark portfolios. Prominent benchmark portfolios and popular investment strategies include naïve diversified portfolios such as the 1/N-portfolio, which is discussed in the next section.

2.2. Naïve Diversification: 1/N

Naïve diversified portfolios are usually based on a simple asset allocation strategy such as the 1/N-rule, which suggests to split the wealth uniformly between the available investment opportunities. In an early study Jobson and Korkie (1980) find that the 'naïve formation rules such as the equal weight rule can outperform the Markowitz rule.' Duchin and Levy (2009) provide a comparison of the 1/N rule with the Markowitz mean-variance optimization using 30 Fama-French industry portfolios for the period from 1991-2007. They conclude that the 1/N strategy outperforms MV optimization for individual small portfolios, but that for large portfolios (i.e. institutional investors) the Markowitz strategy provides superior results in an out-of-sample framework. Another recent study by DeMiguel, Garlappi and Uppal (2009) analyzes whether MV strategies and the various variations adopted in the literature outperform a naïve diversified 1/N portfolio across a wide range of different asset allocation datasets. Using several performance measures DeMiguel, Garlappi and Uppal (2009) find that none of the different MV strategies is able to consistently outperform the naïve equal weighted benchmark (1/N) in an out-of-sample application. However, the analysis by Kirby and Ostdiek (2012) suggests that the results of DeMiguel, Garlappi and Uppal (2009) are largely driven by their research design and their choice of asset allocation datasets. Nevertheless, the results of Kirby and Ostdiek (2012) reveal that a high turnover mostly erodes the benefits of MV optimization when transaction costs are included. These findings might explain why the naïve diversification approach experiences an increasing interest among academicians and practitioners alike. Benartzi and Thaler (2001) provide evidence that the 1/N-rule is also a popular strategy for private investors. More than a third of the analyzed direct contribution plan participants allocate their assets equally among the investment options offered in these plans. Pflug, Pichler and Wozabal (2012) suggest that the 1/N approach is the optimal investment strategy even under high model ambiguity. They demonstrate numerically that MV optimized portfolios convergence to the uniform portfolio if model uncertainty increases.

2.3. The Black-Litterman Approach

The Black-Litterman model (1992) was developed more than twenty years ago in a professional asset management environment and since then experiences an increasing attention among quantitative portfolio managers (Satchell and Scowcroft, 2000; Jones, Lim and Zangari, 2007). In the academic literature several authors analyze the rationale of the BL model and provide examples for applying the methodology (Satchell and Scowcroft, 2000; Lee, 2000; Drobetz, 2001; Idzerek, 2005). For example, Meucci (2006) proposes an extension of the BL model to non-normally distributed markets, whereas Herold (2005) provides an alternative approach to compute implied returns. Chiarawongse et al. (2012) propose an extension for incorporating qualitative views in the form of linear inequalities. An overview of the BL model and its extensions are provided by Walters (2011) and Meucci (2010).

Even if several studies analyze the rationale of the BL model, apply it to compute efficient frontiers, and provide model extensions, the academic literature does hardly offer any empirical evidence documenting the performance of the BL model in out-of-sample applications. So far there is no evidence that the BL model generates superior portfolio performance results relative to MV, minimum-variance, or naïve diversified benchmark portfolios. In addition, the literature on the BL model does not provide a satisfying answer how to generate adequate 'subjective' return estimates and how to quantify the reliability of these estimates. Several studies simply assume exogenously given estimates (He and Litterman 1999, Lee 2000, Drobetz 2001, Idzorek 2005) and suggest confidence intervals of the return estimates as a measure of uncertainty (Black and Litterman 1992, Drobetz 2001). As the portfolio performance critically depends on the exogenously assumed estimates, these approaches are hardly capable to evaluate the advantageousness of the BL model in comparison to MV and naïve diversified benchmark portfolios.

3. METHODOLOGY

3.1. The Black-Litterman approach

The BL model combines two sources of information to obtain return estimates: 'neutral' return estimates implied in market weights which are also referred to as 'implied' returns and 'subjective' return estimates that are also referred to as 'views'. 'Implied returns' are derived by market or benchmark weights and are used as a prior. The basic idea is that investors should only depart from the market or benchmark weights if they have reliable information and estimates on future returns, which differ from the implied market or benchmark expectations. 'Implied' returns are derived using the simple assumption that the observed market or benchmark weights of assets are the result of a risk-return optimization. More precisely it is assumed that market participants maximize the utility function U:

$$(1) \quad \max_{\omega} U = \omega^T \Pi_e - \frac{\delta}{2} \omega^T \Sigma \omega$$

where ω is the vector of portfolio weights, Π_e is the vector of implied asset excess returns, Σ is the variance-covariance matrix, and δ is the investor's risk aversion coefficient. Maximizing the unrestricted utility function results in the optimal portfolios weights:

$$(2) \quad \omega^* = (\delta \Sigma)^{-1} \Pi_e$$

Assuming that the observable market weights (ω) are the average optimized portfolio weights of investors, the average excess-return estimates of the market can be calculated as:

$$(3) \quad \Pi_e = \delta \Sigma \omega$$

In the BL framework the vector of implied excess returns (Π_e) is combined with the investor's views expressed in the vector (Q_e*),* incorporating the reliability of each view quantified in the matrix (Ω). To derive the combined return estimates, the original Black-Litterman (1992) paper references the Theil's mixed estimation model (Theil, 1971), while several authors also suggest a Baysian estimation model (Lee, 2000; Drobetz, 2001). Figure 1 illustrates the procedure of the BL approach.

[Figure 1 about here]

We briefly describe the intuition of combining the return estimates following Theil's mixed estimation approach. It is assumed that implied excess returns (Π_e) and subjective views (Q_e*)* are estimators for the mathematically correct excess return estimates (μ_e*)*. Hence, the correct excess return estimates (μ_e) can be written as implied excess return estimates (Π_e) plus an error term (η), where (I) is the identity matrix:

$$(4) \quad \Pi_e = I \cdot \mu_e + \eta \quad \text{with} \quad \eta \sim N(0, \tau \Sigma)$$

The error term (η) is assumed to be normally distributed with a variance proportional to the historic variance-covariance matrix (Σ). The proportional factor (τ) reflects the uncertainty of implied returns.

The subjective excess return estimates (Q_e) can be written as a linear combination with the error term (ε), where (P) is a binary matrix which contains the information for which asset a subjective return estimate is considered.

$$(5) \quad Q_e = P \cdot \mu_e + \varepsilon \quad \text{with} \quad \varepsilon \sim N(0, \Omega)$$

The matrix (Ω) is the covariance matrix of the error terms and represents the reliability of subjective estimates. Implied returns and subjective estimates can be combined as:

$$(6) \quad \begin{bmatrix} \Pi_e \\ \mathrm{Q_e} \end{bmatrix} = \begin{bmatrix} I \\ P \end{bmatrix} \mu + \begin{bmatrix} \eta \\ \varepsilon \end{bmatrix}$$

Applying a generalized least square procedure leads to the estimator of combined excess return estimates which after some simplifications can be written as:

$$(7) \quad \hat{\mu}_{e,\,BL} = \left[(\tau\Sigma)^{-1} + P^T\Omega^{-1}P\right]^{-1}\left[(\tau\Sigma)^{-1}\Pi + P^T\Omega^{-1}Q\right]$$

The resulting return estimate can be interpreted as a weighted average of implied returns and subjective return estimates (Lee 2000) with respect to the correlation structure. The weights are the uncertainty factors of implied returns (τ) and subjective return estimates (Ω), which will be discussed in the following section.

Satchell and Scowscroft (2000) show that the posterior variance covariance matrix is:

$$(8) \quad \Sigma_{BL} = \Sigma + \left[(\tau\Sigma)^{-1} + P^T\Omega^{-1}P\right]^{-1}$$

After computing combined return estimates and the posterior variance-covariance matrix a traditional risk-return optimization is conducted, maximizing the investor's utility.

$$(9) \quad \max_{\omega} U = \omega^T \mu_{e,BL} - \frac{\delta}{2}\omega^T \Sigma_{BL}\, \omega$$

We implement realistic investment constraints, namely a budget restriction, an exclusion of short-selling, an upper bound on the portfolio volatility and solve the maximization problem numerically. The volatility constraint allows us to differentiate between different investor types in terms of their desired portfolio volatility rather than risk aversion coefficients, which are intuitively difficult to quantify. We keep the risk aversion coefficient constant at a level of 2. For MV optimization, we implement the same optimization procedure. The only difference is that for the MV approach the vector of mean historic excess returns ($\bar{\mu}_e$) and the historic variance-covariance-matrix (Σ) are used while in the BL framework combined excess return estimates ($\hat{\mu}_{e,BL.}$) and the posterior variance-covariance-matrix (Σ_{BL}) are plugged in. For the time period from January 1993 to December 2011 we calculate monthly out-of-sample optimized portfolios at every first trading day of the month, using the BL, MV and minimum-variance approach.

3.2. Uncertainty of implied returns

Before implementing the BL model the uncertainty parameter of implied returns (τ) has to be specified. In the literature the values used for (τ) usually range from 0.025 to 0.3

(Black and Litterman, (1992); He and Litterman, (1999); Idzorek, (2005); Drobetz, 2001). For very small values ($\tau \rightarrow 0$) the combined returns converge to implied returns and the optimized portfolio converges to the market portfolio or the respective benchmark. For large values ($\tau \rightarrow \infty$) the combined returns converge to the 'views' and the optimized portfolio converges to the mean-variance portfolio in which the 'views' are the underlying return estimates. The parameter (τ) controls how distinctly the optimized portfolio may depart from the market portfolio or the underlying benchmark. Hence, the parameter (τ) may be calibrated using a desired tracking error. We start with setting the parameter (τ) at a level of 0.1 and analyze variations between 0.025 and 0.30 in a robustness check.

3.3. Uncertainty of 'subjective' return estimates

Furthermore, the uncertainty of 'views' has to be expressed in the matrix (Ω), which is a diagonal matrix comprising the variance of the error terms (ε) of the 'views' on its diagonal. Values offside the diagonal would represent correlations between the error terms of different 'views' which in the BL model are assumed to be zero (Black and Litterman, 1992). If the variance of error terms is large, the uncertainty of the respective 'view' is high. In this case the optimized portfolio weight will be close to the market or benchmark weight.

Drobetz (2001) suggests using confidence intervals for estimating the uncertainty of 'views'. However, mutually estimating returns and their respective confidence intervals might be a challenging task for analyst and might hinder a successful implementation of the BL model. Several authors propose to simply assume the same or a proportional uncertainty for subjective estimates as for implied returns (He and Litterman, 1999; Meucci, 2010) by setting (Ω) as:

$$(10) \quad \Omega = P\,(\tau\Sigma)P^{T}$$

In this approach the resulting combined return estimates are independent of the choice of (τ) but no additional information on the reliability of 'views' is included. We suggest that the out-of sample performance of the BL model should be enhanced if reasonable and time-varying information on the reliability of views is considered. Therefore, we measure the reliability of each view (i) by computing the historic variance of the error terms (ε_i), where ($\varepsilon_{i,t}$) is the difference of the subjective return estimate ($q_{i,t}$) for an asset (i) in month (t) and the realized return ($r_{i,t}$) of asset (i) in month (t). We employ a 12-month moving estimation window to calculate the historic variance of the error terms and employ different window lengths in a robustness check. The idea is that in an uncertain market environment when the last month's return estimates depart strongly from the realized returns the investor should stick closer to the benchmark. On the other hand, in stable market conditions when the last months subjective return estimates were close to the realized returns, we would expect the subjective estimate for the next month to be more reliable.

We analyze the contribution of this historic reliability measure by comparing the out-of-sample portfolio performance of our approach with the assumption of equal uncertainty used by He and Litterman (1999) and Meucci (2010), which substitutes (Ω) according to equation (10).

3.4. Setting strategic weights and constructing benchmark portfolios

To apply the BL model on a multi-asset portfolio we use an investment universe consisting of global stocks, bonds, and commodities. To calculate implied returns we employ strategic weights for all asset classes. This implies that the optimized portfolio weights converge to the strategic weights if the reliability of the 'views' is low. An alternative approach is to employ market weights of the assets to calculate implied returns. However this would be problematic for several reasons. For commodities, market weights would be difficult to measure, while for bonds market weights would be problematic due to their relatively heavy weight in comparison to stocks, which would imply that investors allocate an extreme high proportion of their assets to bonds, if they do not have 'subjective' return estimates or if the reliability of these estimates is low. Since this might not be an adequate assumption for all investors, we rather rely on strategic weights.

We account for three different investor types - a 'conservative', a 'moderate' and an offensive' one - and set different strategic weights for bonds, commodities, and stocks for each type. To determine the strategic weight of commodities we rely on the results of earlier studies. Anson (1999) who analyzes US stocks and bonds for the period from 1974 to 1997 suggests that a moderate investor should allocate around 15% to commodities, while a less risk-averse investor should allocate over 20% to commodities. Erb and Harvey (2006) derive an optimal portfolio weight for commodities of 18%. Based on these results we set the strategic weight for commodities to 5%, 15% and 25% for the conservative, moderate and offensive investor clienteles, respectively. Furthermore, we assume that the conservative investor allocates strategically 80% in bonds, which might be, for instance, realistic for pension funds. For the offensive investor we assume that he only invests 10% in bonds strategically, but he might employ higher bond weights in case of stock market downturns or high market uncertainty using bonds as a safe haven. The moderate investor type is assumed to exhibit a strategic weight of bonds of 45% which is right in between the offensive and conservative bond weights. The strategic weights for the different investor types are summarized in table 1. These strategic weights are not only used to compute implied returns as input for the BL model but to construct a naïve diversified benchmark for each investor type as well. This benchmark statically invests in all assets using the strategic weights shown in table 1 (BM I st.w.). A second benchmark (BM II 1/N) follows the simple 1/N-rule in which all considered assets are equally weighted. Both benchmark portfolios, as well as all optimized portfolios are rebalanced at every first trading day of each month.

[Table 1 about here]

We compute 'implicit' return estimates for each asset and for each investor type according to equation (3). To derive the maximum allowed portfolio risk used as optimization constraint we rely on historic benchmark volatilities before the evaluation period from January 1988 to December 1992 and add a premium to allow for some reasonable deviation from the benchmark. More precisely, we assume maximum desired portfolio volatilities for the 'conservative', 'moderate', and 'offensive' investor clienteles of 5%, 10%, and 15% p.a., respectively (see table 1), which we assume to stay constant over time.

3.5. Subjective return estimates

To successfully implement the BL model the determination of 'subjective' return estimates is crucial. To be able to compare the results of BL and MV optimization the same return estimates have to be used in both approaches. Since a common approach in MV optimization is to use mean historic returns as return estimates, we implement this simple approach in the BL framework as well, using a moving estimation window of 12 month in the base case and analyzing different estimation windows in a robustness check. However, the BL approach additionally considers the reliability of the 'subjective' estimates as additional input parameter. As mentioned above we measure the reliability of historic mean return estimates as the variance of the historic differences of forecasted and realized returns, using a 12-months moving estimation window in the base case and analyzing various window sizes in a robustness check. Hence, the historic return estimate of an asset is assumed to be more reliable if the historic error of forecasted returns is lower.

3.6. Performance measures

We calculate several performance measures to evaluate the optimized portfolios. First, we compute the moments of the net portfolio returns (after transaction costs) for each optimization strategy (i). Further, we compute the out-of-sample net Sharpe ratio as the fraction of the out-of-sample mean net excess-return (mean return after transaction costs less risk-free rate) divided by the standard deviation of out-of-sample net returns.

$$(13) \qquad \hat{\mathrm{S}}\mathrm{R}_{\mathrm{Net,i}} = \frac{\overline{R}_{Net,i} - \overline{R}_f}{\hat{\sigma}_{Net,i}}$$

We use the two-sample statistic for comparing Sharpe ratios as proposed by Opdyke (2007) to test if the difference in Sharpe ratios of two portfolios is significant. In contrast to earlier Sharpe ratio tests as proposed by Jobson and Korkie (1981b) or Lo (2002) this test can be applied under very general conditions – stationary and ergodic returns. Most importantly for our analysis the test permits auto-correlated and non-normal distributed returns and allows for a likely high correlation between the portfolio returns of different strategies.

As a further risk measure besides volatility, we compute the maximum drawdown, as proposed by Grossman and Zhou (1993), which reflects the maximum accumulated loss that an investor may suffer in the worst case during the whole investment period from 2000 to 2011. An advantage of the MDD measure is that it does not draw any assumption on the return distribution. We compute the percentage maximum drawdown (MDD) of strategy (i) as:

$$(14) \qquad \mathrm{MDD}_{\mathrm{i}} = \mathrm{Max}_{i,\tau \in (0,\mathrm{T})}\left[\mathrm{Max}_{\mathrm{i},\,\mathrm{t} \in (0,\tau)}\left(\frac{\mathrm{P}_{\mathrm{i,t}} - \mathrm{P}_{i,\tau}}{P_{i,t}}\right)\right]$$

where ($P_{i,t}$) is the price of portfolio (i) at time (t), when the portfolio is bought and ($P_{i,\tau}$) is the price of portfolio (i) at time (τ), when the portfolio is sold.

Further, we compute the portfolio turnover, in line with Daskalaki, Skiadopoulos (2011), and DeMiguel et al. (2009), which quantifies the amount of trading required to implement a certain strategy. The portfolio turnover (PT_i) of strategy (i) is the average absolute change of the portfolio weights (ω) over the T rebalancing points in time and across the N assets:

(15) $$PT_i = \frac{1}{T}\sum_{t=1}^{T}\sum_{j=1}^{N}\left(\left|\omega_{i,j,t+1} - \omega_{i,j,t+}\right|\right)$$

We account for trading costs by assuming proportional transaction costs of 30 basis points of the transaction volume in the base case and compute the percentage trading costs generated by a certain strategy. We investigate the impact of differing trading costs in the robustness check. Additionally we compute the optimized portfolios tracking error and information ratio by relying on the benchmark I, which is the reference portfolio for BL.

4. DATA

To construct multi-asset portfolios we include global stocks, bonds, and commodities in the investment universe. We use tow geographic MSCI stock indices covering both developed and emerging markets: MSCI World and MSCI Emerging Markets, both denominated in US dollar. Emerging markets usually provide higher stock returns than developed markets which are related to additional risk factors such as illiquidity or institutional and political conditions (Iqbal et al., 2010). Chiou et al. (2009) show that international diversification is beneficial for US investors by reducing portfolio volatility and improving risk-adjusted returns. Their results hold when including investment constraints such as a short-sale constraint and are also evident for time-rolling efficient frontiers and in out-of-sample tests.

Bonds are usually negatively correlated with stocks and are considered as a safe haven during stock market downturns. To ensure their function as a low risk investment we use US Government bonds. Thereby we exclude default and currency risk from the bond investment. We rely on the Bank of America / Merill Lynch US-Government Bond Index (all maturities) to represent investments in bonds. In addition we include the Bank of America / Merill Lynch US High Yield 100 Bond Index in order to add an exposure to default risk factors. This index is expected to yield higher returns than Government bonds, but at the same time to provide a lower volatility than stock indices.

The S&P GSCI Light Energy Index represents the exposure to commodity investments. This diversified commodity index enables investors to participate in price changes in a wide range of commodity markets. While the often used S&P GSCI Index is mainly driven by energy prices, the S&P GSCI Light Energy Index is more balanced across different commodity classes. It reflects the price developments on the future markets for energy (37.4%), agricultural products (31.2%) and livestock (10%) as well as industry metals (14%) and precious metals (7.4%)[1]. Commodities are expected to provide low correlations with the traditional asset classes stocks and bonds since their prices are related to additional risk factors such as weather, geographical conditions or supply constraints. Moreover, as several studies document a positive correlation between commodity returns and future inflation, investments in commodities might be used as a hedge against inflation (e.g. Bodie and Rosanky, 1980; Erb and Harvey, 2006; Gorton and Rouwenhorst, 2006). Several studies find that an inclusion of commodities improves the efficient frontier of stock-bond portfolios (e.g. Satyanarayan and Varangis, 1996; Abanomey and Marthur, 1999; Anson, 1999; Jensen et. al., 2000). However, a recent study of Cheung and Miu (2010) indicates that diversification benefits of commodities are regime-dependent and Daskalaki and Skiadopoulos (2011) report that portfolio improvements of commodities are not present in out-of sample MV optimized portfolios. Since a large strand of the literature finds evidence for the positive role of commodities in portfolio optimization, we include commodities as another asset class in our analysis.

We obtain monthly total return index data for all indices for the time period from January 1988 to December 2011 from Thomson Reuters Datastream. All data is denominated in US dollar. As risk-free rate we use the yield of a three month US T-Bill. Table 2 provides descriptive statistics of the monthly asset returns during the whole evaluation period from January 1993 to December 2011.

[Table 2 about here]

The table shows similar annualized mean returns for stock and bond indices ranging between 6.16% and 8% p.a. The average return of the commodity index is slightly lower than the average risk free rate of 3.12% during the period, resulting in a negative Sharpe ratio. The highest Sharpe Ratio of 0.655 is generated by the US Governmental Bond Index. The maximum drawdowns (MDD) of the assets reveals that the maximal loss an investor could have suffered during the observed period by investing in stocks was between 55.16% and 63.04% of the invested capital. This figure was roughly 60% for commodities, 27.21% for the US High Yield Bond Index, and 5.29% for the US Government Bond Index. The Jarque-Bera statistics is significant for all asset classes. Hence, the assumption of normal distributed

[1] Index weights in parentheses are of 30th of December 2011.

returns has to be rejected for the whole period.

For implementing an out-of-sample portfolio optimization with the BL and MV approach, we estimate the variance-covariance matrix and historic mean returns using rolling estimation windows. Rolling estimation windows provide the advantage that they are more responsive to structural breaks than expanding estimation windows. In the base case we use estimation windows of 36 months for the variance-covariance matrix and 12 month for returns. We choose a longer estimation window for the covariance matrix since we expect the correlation structure to be more stable over time than returns. Different window sizes are considered in a sensitivity analysis. For these shorter observation windows the assumption of normal distributed returns cannot be rejected and the application of the mean-variance framework is reasonable.

Table 3 provides evidence on potential diversification benefits in terms of pair-wise correlation coefficients. Over the entire period the diversification benefits across stock indices are limited. The correlation between the MSCI World and MSCI Emerging Markets is highly significant and larger than 0.8 indicating a strong co-movement of developed and emerging market stocks. While the US High Yield Bond Index and Commodities provide a slightly larger diversification effect with correlation coefficients ranging between 0.35 and 0.65, the highest diversification potential is provided by the US Government Bond Index, which is reflected in negative correlation coefficients. Consequently, we expect to find significant portfolio benefits by applying the BL and MV frameworks on a multi-asset portfolio, including bonds and commodities, rather than on a stock-only portfolio.

[Table 3 about here]

5. EMPIRICAL RESULTS

5.1. Results for the base case

To analyze whether the BL mixed estimation approach enhances historic return estimates we compute monthly out-of-sample estimation errors (mean squared errors) of forecasted returns. The results are presented in table 4. BL forecasts are derived by combining implied returns with ‘views’. Implied returns are computed using the strategic weights presented in table 1. Within the asset class stocks the strategic weights are assumed to be 25% for emerging markets and 75% for developed markets. Accordingly, within the asset class bonds the strategic weights are set 75% for Governmental Bond Index and 25% for the High

Yield Bond Index. The same asset weights are used to construct the naïve diversified Bond Index Benchmark I. 'Views' are historic mean returns using a 12-month rolling estimation window. The results reveal that BL return forecasts exhibit lower estimation errors in comparison to simple historic means, which would be used in traditional MV optimization.

[Table 4 about here]

To analyze whether the improved return forecasts translate to a significant outperformance of BL optimized portfolios in comparison to MV, we compute out-of sample BL, MV and minimum variance optimized portfolios and two naïve diversified benchmark portfolios. Table 5 summarizes the empirical results for the evaluation period from January 1993 to December 2011. We compute sample moments of portfolio returns and portfolio performance measures for the three investor types 'conservative', 'moderate' and 'offensive'. All data is computed net of transaction costs. For both the BL and MV optimization we use historic mean returns as (subjective) return estimates. Benchmarks I is computed according to the asset weights in table 1. Benchmark II is an equally weighted (1/N) portfolio in which all five asset classes obtain a portfolio weight of 20%. All portfolios are rebalanced at the first trading day of each month. The results reveal that BL optimized portfolios exhibit a better performance in terms of net Sharpe ratio than MV, minimum-variance and both benchmark portfolios for all investor types. For the moderate and offensive investors the outperformance of the BL portfolio in comparison to MV is significant. The insignificant result for the conservative investor is not surprising since both BL and MV optimization converge to the minimum-variance portfolio for high risk aversions. Both risk measures volatility and maximum drawdown (MDD) indicate a consistently lower risk of the BL optimized portfolios in comparison to MV, independently of the investor type. The lower risk is also reflected in a lower absolute sample skewness of BL portfolio returns in comparison to all other portfolios.

The average portfolio turnover is an indicator for the amount of trading and, hence, transaction costs generated by implementing a certain optimization strategy. However, transaction costs are already priced in the net return, maximum drawdown and Sharpe ratio measure. The results show that for all investor types the BL approach exhibits a lower portfolio turnover and, therefore, lower transaction costs and less extreme reallocations of the optimized portfolios in comparison to the MV approach. The average number of assets in the optimized portfolio as an indicator for the diversification of the optimized portfolios across asset-classes is on average higher for BL portfolios than for MV portfolios. Consequently, BL portfolios tend to be better diversified across asset classes.

[Table 5 about here]

Figure 2 shows the optimized portfolio weights during the period 1993-2011 for BL and MV optimization for the three investor types. In line with the turnover and diversification measures, the figure reveals less extreme portfolio reallocations and a higher benchmark orientation of the BL optimized portfolios in comparison to MV.

[Figure 2 about here]

5.2. Sensitivity Analyses

Next, we perform various sensitivity analyses to check if our results are robust to changes in the input data. To test whether the outperformance of the BL approach is responsive to the optimization constraint 'maximum allowed portfolio volatility', presented in table 1, we vary the maximum allowed portfolio volatility, while keeping the strategic weights of stocks, bonds and commodities constant at the level of 40%, 45% and 15%, respectively. Panel I of table 6 shows that the BL approach generates consistently and significantly higher net Sharpe ratios than MV optimization and both benchmark portfolios for all considered volatility constraints. Additionally, we find lower portfolio risks reflected in a lower maximum drawdown (MDD), a higher degree of diversification across asset classes expressed in a larger number of assets in the optimized portfolio, a lower portfolio turnover and a higher information ratio for BL optimized portfolios in comparison to MV portfolios.

In panels II and III of table 6, we present the results for different estimation windows for the variance-covariance matrix and the return estimates. As before, we find consistently higher net Sharpe ratios for the BL approach in comparison to MV optimization and both benchmark portfolios. The results get insignificant for too short or too long estimation windows for the return estimates. For short estimation windows the portfolio turnover increases dramatically, resulting in immense transaction costs and a relatively lower outperformance in comparison to the naïve diversified benchmark portfolios. For long estimation windows of 18 month and more the responsiveness to structural breaks such as a stock market downturn is lower, resulting in a lower out-of-sample Sharpe ratio. Additionally an analysis of the auto correlation functions of the asset returns shows that returns are significantly correlated with the last months returns only, while returns with a lag larger than 12 month do not have almost any explanatory power.

Additionally, we observe a better performance of BL optimized portfolios in terms of maximum drawdown, diversification, turnover, and information ratios. Based on these results, we identify an optimal estimation window of 36 to 48 month for the covariance matrix and 12 month for the return estimates. The insignificant outperformance of the BL model for too long and too short return estimation windows highlights the importance of accurate and responsive return estimates. However, further research is required to analyze the performance of the BL approach when using alternative return estimates. This issue will be addressed in a further study.

[Table 6 about here]

To derive additional insights for explaining the outperformance of the BL approach we conduct a further sensitivity analysis, varying the BL parameters (τ) and (Ω). The results are shown in table 7. To analyze the contribution of the uncertainty measure (Ω) of views, we alternatively apply the approach used by He and Litterman (1999) and Meucci (2010) and substitute (Ω) according to equation (10). In this case we simply assume the same uncertainty for 'views' as for 'implied' returns. Hence, no additional information on the reliability of 'views' is considered. The results show that this approach leads to an almost complete disappearance of the outperformance of the BL model in comparison to MV. Therefore, we infer that a major part of the outperformance of the BL model can be explained by the consideration of additional information on the reliability of return estimates. This leads to an investment close to the benchmark or market portfolio in case of uncertain market conditions and high deviations from the benchmark or market portfolio, when markets are more stable and return forecast errors are low.

[Table 7 about here]

Next, we vary the parameter (τ) on a range from 0.025 to 0.3 which captures most of the documented approaches in the literature (Black and Litterman, 1992: He and Litterman, 1999; Idzorek, 2005; Drobetz, 2001). The results show that the BL model outperforms MV optimization for all considered values of (τ). This is not only true for the out-of-sample Sharpe ratios but also for the risk measure "maximum drawdown" (MDD), the diversification measure "average number of assets", the portfolio turnover, and the information ratio measures. Further, we observe that the BL portfolio's deviation from the benchmark declines with lower values of (τ) which is reflected in lower tracking errors for lower (τ)-values. This is in line with the interpretation of (τ) as an uncertainty measure of implied returns and confirms its function to control the desired deviation from the benchmark or market portfolio. Further, we find that for our sample a (τ)-value of 0.05 is superior to all other analyzed cases. As expected, for tiny values of (τ), close to zero, we find that the unconstrained optimized BL portfolio equals exactly the benchmark portfolio. On the other hand, for infinite large values of (τ), the optimized BL portfolio is equivalent to the MV optimized portfolio. A further variation of (Ω) reveals that the outperformance of the BL portfolio is robust to changes in the estimation window of (Ω) for all considered cases from 6 to 36 months.

To investigate the impact of the assumed level of transaction costs, we vary the variable transaction costs from 5 to 50 basis points. The net Sharpe ratio measures for BL, MV and the two benchmark portfolios are reported in table 8. Again, the results reveal that the BL portfolio performs significantly better than the MV approach and better than both naïve diversified benchmark portfolios for all considered levels of transaction costs. However, for large transaction costs of 50 basis points or more, the lower portfolio turnover of both benchmark portfolios gets more pronounced leading to a lower level of significance of their relative underperformance to the BL portfolio. For low levels of transaction costs the significance level of the outperformance of the BL portfolio relative to the MV portfolio is lower, illustrating that the lower level of portfolio turnover relatively to MV is an additional driver of the outperformance of the BL approach.

[Table 8 about here]

Finally, we analyze the impact of the chosen reference portfolio for the BL portfolio performance. So far, we applied the strategic weights for stocks, bonds, and commodities as presented in table 1 (BL-st.w.). Now, we consider two alternative reference portfolios. The first is a naïve diversified portfolio in which all the five considered assets obtain the same strategic weight of 20% (BL-1/N). The second approach uses the minimum variance portfolio as reference portfolio (BL-MinVar). This approach is particularly reasonable for a conservative investor, since it implies that investors hold the minimum variance portfolio if they do not have information on future returns or the reliability of return forecasts (‘views’) is low. The results for different reference portfolios are presented in table 9. The results reveal that for all considered reference portfolios the BL optimization leads to consistently better portfolio performances in comparison to MV and both benchmark portfolios and all investor types. Not surprisingly, we find that for the conservative investor type the BL-MinVar approach performs slightly better than the BL-strategic-weights approach in terms of net

Sharpe Ratio and portfolio volatility. For offensive investors, however, the BL model with strategic weights as reference portfolio performs marginally better.

[Table 9 about here]

5.3. Performance of optimized portfolios in different market environments

To examine the performance of the BL, MV and minimum variance portfolios in different market environments, we separate the total time period between 1993 and 2011 into several sub-periods. We determine expansionary and recessionary sub-periods on an ex ante basis following the approach proposed by Jensen and Mercer (2003). This approach builds on the monetary cycle defined as the first change of the short-term interest rate by the central bank that runs counter to the previous trend. In line with Bessler et al. (2012) we rely on changes in the federal funds target rate augmented by signals originating from the stock market. As in Bessler et al. (2012) it is assumed that the stock market signals a change in the business cycle if the 24-months moving average of the MSCI World is crossed by the actual index from below (expansionary state) or above (recessionary state). For the transition from one state to another it is required that both instruments, the monetary policy as well as the stock market, provide a consistent signal, thereby reducing the probability of incorrect signals. Figure 3 shows the definition of sub-periods as well as the monetary policy and stock market signals, where shaded areas denote down markets/recessionary periods.

The first sub-period ranges from January 1993 to January 2001 and covers a number of events such as the Asian crisis, the Russian default, and the build-up of the technology bubble. This period comprises 96 months and can be characterized as 'expanding' with increasing values in developed stock markets and relatively high interest rates with a T-Bill yielding on average 4.77% p.a.. The second sub-period between February 2001 and June 2004 covers the end of the new economy bubble and the subsequent rebound of international stock markets. The second period comprises 41 months and can be characterized as 'recessionary' with bearish stock markets and an average risk free rate of 1.85% p.a. (average yield of a 3-month T-Bill). The third sub-period from July 2004 to February 2008 comprises 48 months and covers bullish stock markets and high interest rates. The average risk free rate in this period is 3.67% p.a.. The final sub-period from March 2008 to December 2011 includes 47 months and incorporates the recent financial crisis that led to significant declines in the values of equities and alternative asset classes, such as commodities and Hedge Funds. The average risk free rate in the fourth period was 0.37% p.a..

Table 10 summarizes the performance measures of the out of sample optimized portfolios for the four sub-periods for the moderate investor type. We analyzed the sub-periods for the conservative and offensive investor clienteles as well. Since the results do not change qualitatively, we only report the performance measures for the moderate investor type for the sake of clarity and simplicity. In the BL and MV optimized portfolios the maximum expected volatility is constrained to 10% p.a.. For the BL optimization the strategic weights are set according to table 1: 45% for bonds, 15% for commodities and 40% for stocks. These weights are also used to compute the naïve diversified benchmark I. In benchmark II all assets are equally weighted (1/N). All portfolios are rebalanced at the first trading day of every month.

For both recessionary sub-periods we find significantly higher Sharpe ratios for the BL optimized portfolios in comparison to MV and a better performance in comparison to both naïve diversified benchmark portfolios as well. In both expansionary periods we find a relatively smaller and insignificant outperformance of BL in comparison to MV. In the third sub-sample, which covers the bullish stock markets between July 2004 and February 2008 we find that the naïve diversified portfolios outperform both BL and MV optimized portfolios. However the difference in Sharpe ratios is insignificant.

Consistently with the analysis for the full sample, we find that for all sub-periods BL optimized portfolios are less risky than MV and both naïve diversified portfolios which is indicated by a lower maximum drawdown. Furthermore, the analysis of the portfolio turnover reveals consistently lower turnovers and, hence, lower transaction costs for BL compared to MV for all sub-periods. Additionally, we find that for all sub-periods BL portfolios are better diversified across asset classes than MV portfolios, which is indicated by a higher average number of assets in the optimized portfolios.

Overall, we find that the BL model outperforms MV and naïve diversified portfolios particularly in recessionary periods. In expansionary periods the outperformance in comparison to MV is insignificant and naïve diversified portfolios even perform better in one sub-period. In terms of the maximum drawdown, portfolio turnover and portfolio diversification the results for all sub-periods are consistent with the results for the full sample.

[Table 10 about here]

6. CONCLUSION

We analyze the out-of-sample performance of BL optimized portfolios in comparison to MV, minimum variance and naïve diversified benchmark portfolios using multi-asset portfolios. To ensure the comparability of MV and BL optimization, we use the same historic return estimates in both approaches. While in the MV approach the historic estimates are directly used in the optimization condition, in the BL approach the estimates are first combined with implied returns, considering the reliability of each historic estimate.

Our empirical results contribute to the literature in several ways. First, we find that the BL model can successfully be applied on multi-asset portfolios, rather than to stock-only portfolios, by using strategic weights for the different asset classes or using the minimum variance portfolio as reference portfolio. For the period from January 1993 to December 2011, we find that BL optimized portfolios exhibit consistently higher out-of-sample portfolio performances in terms of net Sharpe ratios and the risk measure 'maximum drawdown' in comparison to MV, minimum variance and two benchmark portfolios. While in line with DeMiguel et al. (2009), the MV approach fails in most cases to significantly outperform a naïve equally weighted (1/N) benchmark, the BL model significantly outperforms both considered static benchmark portfolios in almost all considered cases. Moreover, BL optimized portfolios include, on average, a larger number of assets than MV optimized portfolios and, therefore, are better diversified across asset classes. A further sensitivity analysis reveals that the out-of-sample outperformance of the BL model is driven by the consideration of additional information on the reliability of return estimates ('views') and by a lower portfolio turnover.

Furthermore we separate the full sample from January 1993 to December 2011 into four sub-periods based on the monetary cycle and stock market signals, following the approach of Bessler et al. (2012). We find that the BL model outperforms MV and naïve diversified portfolios particularly in recessionary periods. The other benefits of the BL optimized portfolios such as the lower maximum drawdowns, the lower portfolio turnover and the higher portfolio diversification could be observed for all sub-periods consistently with the results for the full sample.

Finally, we find that for conservative investors using the minimum variance portfolio as reference portfolio in the BL approach performs slightly better in terms of net Sharpe ratios than setting strategic weights. Our results are robust to all considered variations of the input parameters. However, further research is required to evaluate the BL portfolio performance for other than historic return estimates. We will address this research question in another study.

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Table 1: Strategic Weights and Benchmark Portfolios

Investor type	Benchmark portfolio weights: Bonds	Commodities	Stocks	Historic volatility of benchmark portfolio	Optimization constraint: max. portfolio volatility
conservative	80%	5%	15%	4.58%	5.00% p.a.
moderate	45%	15%	40%	6.66%	10.00% p.a.
offensive	10%	25%	65%	9.72%	15.00% p.a.

This table provides the strategic weights for the three analyzed investor types: conservative, moderate and offensive, which are used to compute implied return estimates. Within the asset class stocks and bonds emerging market stocks and high yield bonds obtain a strategic weight of 25% while developed market stocks and government bonds obtain a strategic weight of 75%. We assume that the three investor types prefer a maximum expected portfolio volatility of 5%, 10% and 15%, respectively. We compute two alternative benchmark portfolios. Benchmark I is computed using the strategic weights presented in this table. Benchmark II is a naïve diversified 1/N benchmark, in which all five asset classes obtain a portfolio weight of 20%.

Table 2: Descriptive statistics of asset returns (January 1993- December 2011)

	MSCI World	MSCI Emerging Markets	US Gov. Bondindex	US High Yield Bondindex	S&P GSCI Light Energy
Mean Return p.a.	6,71%	8,00%	6,16%	7,31%	2,66%
SD p.a.	16,61%	25,40%	4,64%	8,44%	16,58%
Skewness	-0,897	-0,934	-0,037	-1,467	-1,228
Kurtosis	5,337	5,928	4,259	10,444	8,447
Sharpe Ratio	0,216	0,192	0,655	0,496	-0,028
MDD	55,16%	63,04%	5,29%	27,21%	59,95%
JB	82.47***	114.61***	15.10***	608.24***	339.20***
Observations	228	228	228	228	228

This table provides sample moments, Sharpe ratios, Maximum Drawdown and Jarque-Bera statistics of the eight asset classes considered in the empirical analysis. The time period covers the months from January 1993 to December 2011. 'Mean Return p.a.' denotes annualized time-series mean of monthly returns while 'SD p.a.' denotes the associated annualized standard deviation. 'Skewness' and 'Kurtosis' represent the third and fourth moment of the return distribution. 'Sharpe Ratio' shows the annualized Sharpe ratios of the respective asset classes using the average 1993 -2011 risk-free interest rate of 3.12% per year. MDD shows the maximum drawdown of the respective asset class during the period from January 1993 to December 2011 and 'JB' is the Jarque-Bera statistic for testing normality of returns. ***, **, * indicate statistical significance at the 1%, 5%, and 10% level, respectively.

Table 3: Correlation matrix of asset classes (January 1993- December 2011)

	MSCI World	MSCI Emerging Markets	US Gov. Bondindex	US High Yield Bondindex	S&P GSCI Light Energy
MSCI World	1				
MSCI Emerging Markets	0.811***	1			
US Gov. Bondindex	-0.182***	-0,221***	1		
US High Yield Bondindex	0.652***	0.619***	-0.069	1	
S&P GSCI Light Energy	0.470***	0.463***	-0.128**	0.353***	1

This table provides the correlation matrix for the asset classes considered in the analysis over the time period January 1993 to December 2011. ***, **, * indicate values significantly different from 0 at the 1%, 5%, and 10% level, respectively.

Table 4: Mean squared error of monthly out-of-sample forecasted returns

	Black Litterman mixed estimation			View
	conservative	moderate	offensive	12 month mean
MSCI World	0.227%	0.227%	0.227%	0.245%
MSCI Emerging Markets	0.533%	0.532%	0.532%	0.583%
US Gov. Bondindex	0.018%	0.018%	0.018%	0.019%
US High Yield Bondindex	0.060%	0.059%	0.059%	0.063%
S&P GSCI Light Energy	0.226%	0.227%	0.227%	0.255%

This table documents mean-squared errors (MSE) of monthly forecasted returns. Forecasts are computed using the BL mixed estimation approach (combining 12-month means as “views” with implied returns) and the views as used in MV.

Table 5: Empirical Results for the full sample (evaluation period 1993-2011):

Investor type: conservative	Black-Litterman	Mean-Variance	Minimum Variance	BM I (st.w.)	BM II (1/N)
Net mean return p.a.	7.81%	7.29%	6.09%	6.29%	6.08%
Volatility p.a.	5.16%	6.08%	4.17%	4.86%	11.08%
Skewness	-0.23	-1.14	-0.48	-1.11	-1.48
Kurtosis	3.70	8.30	4.26	8.11	9.41
Jarque-Bera	6.63**	316.14***	23.77***	294.52***	473.16***
Net Sharpe Ratio	0.91††	0.68†	0.71	0.65	0.27
Net MDD	5.99%	9.41%	7.45%	13.99%	40.59%
Avrg. number of assets	3.32	2.69	3.71	8.00	8.00
Avrg. turnover p.a.	2.13	2.67	0.77	0.18	0.29
Tracking Error	1.16%	1.37%	0.73%	/	/
Information Ratio	1.31	0.73	-0.28	/	/
Obs.	228	228	228	228	228

Investor type: moderate	Black-Litterman	Mean-Variance	Minimum Variance	BM I (st.w.)	BM II (1/N)
Net mean return p.a.	9.58%	8.21%	/	6.03%	6.08%
Volatility p.a.	8.65%	10.37%	/	9.16%	11.08%
Skewness	-0.53	-0.72	/	-1.37	-1.48
Kurtosis	4.98	5.62	/	8.84	9.41
Jarque-Bera	47.83***	85.14***	/	395.82***	473.16***
Net Sharpe Ratio	0.75$^{**'\#\#'\dagger\dagger}$	0.49	/	0.32	0.27
Net MDD	9.46%	16.18%	/	35.10%	40.59%
Avrg. number of assets	3.26	2.16	/	8.00	8.00
Avrg. turnover p.a.	3.32	4.44	/	0.27	0.29
Tracking Error	2.03%	2.50%	/	/	/
Information Ratio	1.75	0.86	/	/	/
Obs.	228	228	228	228	228

Investor type: offensive	Black-Litterman	Mean-Variance	Minimum Variance	BM I (st.w.)	BM II (1/N)
Net mean return p.a.	11.72%	10.35%	/	5.81%	6.08%
Volatility p.a.	11.68%	13.62%	/	14.27%	11.08%
Skewness	-0.34	-0.42	/	-1.26	-1.48
Kurtosis	5.17	5.17	/	8.01	9.41
Jarque-Bera	49.16***	51.14***	/	299.35***	473.16***
Net Sharpe Ratio	0.74$^{*'\#\#\#'\dagger\dagger}$	0.53$^{\#}$	/	0.19	0.27
Net MDD	14.39%	24.59%	/	50.99%	40.59%
Avrg. number of assets	3.12	1.72	/	8.00	8.00
Avrg. turnover p.a.	3.62	4.61	/	0.25	0.29
Tracking Error	2.63%	3.52%	/	/	/
Information Ratio	2.25	1.27	/	/	/
Obs.	228	228	228	228	228

This table reports the portfolio performance measures for the full sample from 1993-2011 in the base case. In the BL and MV approach (subjective) return estimates are mean historic returns using a rolling estimation window of 12-month. The variance-covariance matrix is calculated using a 36-month moving estimation window. In the BL model the parameter (τ) is set equal to 0.1, (Ω) is computed using the variance of historic return forecast errors (12-month rolling estimation window). In the BL and MV optimized portfolios the maximum expected volatility is constrained to 5%, 10%, and 15% for the conservative, moderate, and offensive investor type, respectively. Benchmark I is computed using the asset weights presented in table 1. In Benchmark II all assets are equally weighted (1/N). Benchmark I is the reference portfolio for BL and is used to compute tracking errors and information ratios. All portfolios are rebalanced at the first trading day of every month. * / ** / ***, ($^{\#}$/$^{\#\#}$/$^{\#\#\#}$), [†/††/†††] represents a significant higher Sharpe Ratio compared to Mean-Variance, (Benchmark I), [Benchmark II] at the 10%- / 5%- / 1%-level, respectively.

Table 6: Robustness Check I

(I) Variation of maximum allowed portfolio volatility

Maximum Volatility p.a.	5,00%		7,50%		10%		15%		20%		Benchmark	
	BL	MV	BL	MV	BL	MV	BL	MV	BL	MV	I (st.w.)	II (1/N)
Net Sharpe Ratio	$0.92^{*'\#\#'\dagger\dagger\dagger}$	$0.68^{\#'\dagger}$	$0.80^{**'\#\#'\dagger\dagger}$	0.54	$0.75^{**'\#\#'\dagger\dagger}$	0.49	$0.75^{*'\#\#'\dagger\dagger}$	0.53	$0.74^{*'\#\#'\dagger\dagger}$	0.49	0.32	0.27
Net MDD	5.88%	9.41%	7.58%	11.54%	9.46%	16.18%	11.18%	24.59%	16.41%	27.98%	35.10%	40.59%
Avrg. number of assets	3.64	2.69	3.45	2.41	3.26	2.16	3.00	1.71	2.76	1.35	8.00	8.00
Avrg. turnover p.a.	2.00	2.67	2.80	3.79	3.32	4.44	3.91	4.61	4.07	4.36	0.27	0.29
Information Ratio	1.06	0.67	1.50	0.76	1.75	0.86	2.26	1.46	2.36	1.39	/	/

(II) Variation of estimation window for variance-covariance matrix

Estimation window # month	12		24		36		48		60		Benchmark	
	BL	MV	BL	MV	BL	MV	BL	MV	BL	MV	I (st.w.)	II (1/N)
Net Sharpe Ratio	$0.63^{'\#,\dagger}$	0.56	$0.73^{**'\#\#'\dagger\dagger}$	0.48	$0.75^{**'\#\#'\dagger\dagger}$	0.49	$0.75^{**'\#\#'\dagger\dagger}$	0.46	$0.73^{*'\#'\dagger\dagger}$	0.47	0.32	0.27
Net MDD	16.13%	18.28%	14.58%	15.73%	9.46%	16.18%	8.97%	16.99%	8.76%	18.26%	35.10%	40.59%
Avrg. number of assets	3.04	1.90	3.14	2.09	3.26	2.16	3.38	2.24	3.41	2.29	8.00	8.00
Avrg. turnover p.a.	4.34	4.63	3.56	4.33	3.32	4.44	3.48	4.60	3.27	4.59	0.27	0.29
Information Ratio	1.65	1.37	1.94	0.90	1.75	0.86	1.59	0.76	1.45	0.76	/	/

(III) Variation of estimation window for historic return estimatesx

Estimation window # month	1		6		12		18		24		Benchmark	
	BL	MV	BL	MV	BL	MV	BL	MV	BL	MV	I (st.w.)	II (1/N)
Net Sharpe Ratio	0.45	0.23	0.57	0.32	$0.75^{**'\#\#'\dagger\dagger}$	0.49	$0.56^{\dagger}$	0.33	0.51	0.29	0.32	0.27
Net MDD	14.48%	22.69%	15.65%	24.93%	9.46%	16.18%	17.45%	20.58%	23.99%	32.08%	35.10%	40.59%
Avrg. number of assets	2.97	2.01	3.1	2.15	3.26	2.16	3.38	2.13	3.38	2.12	8.00	8.00
Avrg. turnover p.a.	12.30	15.29	4.76	6.41	3.32	4.44	2.92	4.07	2.49	3.42	0.27	0.29
Information Ratio	0.46	-0.24	1.01	0.17	1.75	0.86	1.15	0.18	0.95	0.04	/	/

Base Case: Moderate investor maximum portfolio volatility 10% p.a.. Strategic weights: Bonds: 45%, Commodities: 15%; Stocks: 40%. * / ** / ***, ($^{\#}/^{\#\#}/^{\#\#\#}$), [$^{\dagger}/^{\dagger\dagger}/^{\dagger\dagger\dagger}$] represents a significant higher Sharpe Ratio compared to Mean-Variance, (Benchmark I), [Benchmark II] at the 10%- / 5%- / 1%-level, respectively. Benchmark I is composed of 45% Bonds, 15% Commodities, 40% stocks. In Benchmark II all assets are equally weighted (1/N). Benchmark I is the reference portfolio for BL and is used to compute information ratios.

Table 7: Robustness Check II: Variation of BL model parameters

Variation of parameter (τ)

	MV	Black Litterman						BM I (st.w.)
Parameter τ	(τ → ∞)	0.3	0.15	0.1	0.05	0.025	$\Omega = P(\tau\Sigma)P^T$	(τ → 0)
Net Sharpe Ratio	0.49	$0.67^{\#'\dagger}$	$0.71^{*'\#\#,\dagger\dagger}$	$0.75^{**'\#\#'\dagger\dagger}$	$0.77^{**'\#\#'\dagger\dagger}$	$0.67^{\#\#'\dagger\dagger}$	0.51	0.32
Net MDD	16.18%	10.01%	9.58%	9.46%	10.34%	20.82%	15.74%	35.10%
Avrg. number of assets	2.16	2.86	3.06	3.26	3.87	4.37	2.34	8.00
Avrg. turnover p.a.	4.44	3.88	3.56	3.32	2.79	2.14	4.45	0.27
Tracking Error	2.50%	2.20%	2.12%	2.03%	1.68%	1.20%	2.40%	/
Information Ratio	0.86	1.30	1.58	1.75	2.08	2.10	0.92	/

Variation of estimation window for uncertainty measure of views (Ω)

Estimation window for	MV	Black Litterman						BM I (st.w.)
Ω # month	(Ω → 0)	3	6	12	18	24	36	(Ω → ∞)
Net Sharpe Ratio	0.49	0.52	$0.68^{\#'\dagger}$	$0.75^{**'\#\#'\dagger\dagger}$	$0.73^{*'\#'\dagger\dagger}$	$0.76^{*'\#\#'\dagger\dagger}$	$0.74^{*'\#'\dagger\dagger}$	0.32
Net MDD	16.18%	13.01%	13.66%	9.46%	10.53%	10.64%	11.08%	35.10%
Avrg. number of assets	2.16	2.89	3.15	3.26	3.22	3.26	3.30	8.00
Avrg. turnover p.a.	4.44	6.28	4.28	3.32	3.10	2.98	2.98	0.27
Tracking Error	2.50%	2.17%	2.18%	2.03%	2.07%	2.09%	2.07%	/
Information Ratio	0.86	0.86	1.47	1.75	1.61	1.41	1.33	/

Base Case: Moderate investor maximum portfolio volatility 10% p.a.. Strategic weights: Bonds: 45%, Commodities: 15%; Stocks: 40%. * / ** / ***, ($^{\#/\#\#/\#\#\#}$), [$^{\dagger/\dagger\dagger/\dagger\dagger\dagger}$] represents a significant higher Sharpe Ratio compared to Mean-Variance, (Benchmark I), [Benchmark II] at the 10%- / 5%- / 1%-level, respectively. Benchmark I is composed of 45% Bonds, 15% Commodities, 40% stocks. In Benchmark II all assets are equally weighted (1/N). Benchmark I is the reference portfolio for BL and is used to compute information ratios.

Table 8: Robustness Check III: Variation of transaction costs

Variable transaction costs in bp	5	10	20	30	40	50
Net Sharpe Ratio BL	$0.850^{*'\#\#'\dagger\dagger}$	$0.829^{*'\#\#'\dagger\dagger}$	$0.788^{*'\#\#'\dagger\dagger}$	$0.746^{**'\#\#'\dagger\dagger}$	$0.706^{**'\#'\dagger\dagger}$	$0.665^{**'\#'\dagger}$
Net Sharpe Ratio MV	$0.605^{\dagger}$	0.582	0.536	0.490	0.445	0.400
Net Sharpe Ratio BM I (st.w.)	0.325	0.323	0.320	0.317	0.314	0.310
Net Sharpe Ratio BM II (1/N)	0.273	0.272	0.269	0.267	0.264	0.260

Base Case: Moderate investor maximum portfolio volatility 10% p.a.. Strategic weights: Bonds: 45%, Commodities: 15%; Stocks: 40%. * / ** / ***, ($^{\#/\#\#/\#\#\#}$), [$^{\dagger/\dagger\dagger/\dagger\dagger\dagger}$] represents a significant higher Sharpe Ratio compared to Mean-Variance, (Benchmark I), [Benchmark II] at the 10%- / 5%- / 1%-level, respectively. Benchmark I is composed of 45% Bonds, 15% Commodities, 40% stocks. In Benchmark II all assets are equally weighted (1/N). Benchmark I is the reference portfolio for BL and is used to compute information ratios.

Table 9: Robustness Check IV: Alternative Reference portfolios

		BL-st.w.	BL-1/N	BL-MinVar	MV	MinVar	BM I (st.w.)	BM II (1/N)
conservative	Net mean return p.a.	7.81%	7.79%	7.86%	7.29%	6.09%	6.29%	6.08%
	Volatility p.a.	5.16%	5.22%	5.13%	6.08%	4.17%	4.86%	11.08%
	Net Sharpe Ratio	$0.91^{\dagger\dagger}$	$0.89^{\dagger\dagger}$	$0.92^{\dagger\dagger}$	$0.68^{\dagger}$	0.71	0.65	0.27
	Net MDD	5.99%	6.07%	6.17%	9.41%	7.45%	13.99%	40.59%
	Avrg. number of assets	3.32	3.79	3.20	2.69	3.71	8.00	8.00
	Avrg. turnover p.a.	2.13	2.06	2.11	2.67	0.77	0.18	0.29
moderate	Net mean return p.a.	9.58%	9.51%	9.28%	8.21%	/	6.03%	6.08%
	Volatility p.a.	8.65%	8.87%	8.05%	10.37%	/	9.16%	11.08%
	Net Sharpe Ratio	$0.75^{**'\#\#'\dagger\dagger}$	$0.72^{*'\#'\dagger\dagger}$	$0.76^{**'\#'\dagger\dagger}$	0.49	/	0.32	0.27
	Net MDD	9.46%	11.02%	9.21%	16.18%	/	35.10%	40.59%
	Avrg. number of assets	3.26	3.48	2.86	2.16	/	8.00	8.00
	Avrg. turnover p.a.	3.32	3.29	3.27	4.44	/	0.27	0.29
offensive	Net mean return p.a.	11.72%	11.32%	10.37%	10.35%	/	5.81%	6.08%
	Volatility p.a.	11.68%	11.30%	9.85%	13.62%	/	14.27%	11.08%
	Net Sharpe Ratio	$0.74^{*'\#\#\#'\dagger\dagger}$	$0.73^{*'\#\#'\dagger\dagger}$	$0.73^{'\#\#'\dagger\dagger}$	$0.53^{\#}$	/	0.19	0.27
	Net MDD	14.39%	11.67%	12.10%	24.59%	/	50.99%	40.59%
	Avrg. number of assets	3.12	3.21	2.63	1.72	/	8.00	8.00
	Avrg. turnover p.a.	3.62	3.83	3.88	4.61	/	0.25	0.29

This table reports portfolio performance measures for the full sample from 1993-2011 using different reference portfolios in the BL approach. In the BL and MV approaches (subjective) return estimates are mean historic returns using a rolling estimation window of 12-month. The variance-covariance matrix is calculated using a 36-month moving estimation window. In the BL model the parameter (τ) is set equal to 0.1, (Ω) is computed using the variance of historic return forecast errors (12-month rolling estimation window). Benchmark I is computed using the asset weights presented in table 1. In Benchmark II all assets are equally weighted (1/N). Benchmark I is the reference portfolio for BL-st.w. and is used to compute BL-st.w. and MV information ratios. Benchmark II is the reference portfolio for BL-1/N and is used to compute BL-1/N information ratios. In the BL-MinVar approach the reference portfolio is the minimum variance portfolio. All portfolios are rebalanced at the first trading day of every month. * / ** / ***, ($^{\#}$/$^{\#\#}$/$^{\#\#\#}$), [†/††/†††] represents a significant higher Sharpe Ratio compared to Mean-Variance, (Benchmark I), [Benchmark II] at the 10%- / 5%- / 1%-level, respectively.

Table 10: Robustness Check V: Analysis of Sub-Periods

January 1993-January 2001	Black-Litterman	Mean-Variance	BM I (st.w.)	BM II (1/N)
Net mean return p.a.	10.25%	10.41%	7.86%	6.74%
Volatility p.a.	9.16%	11.08%	6.90%	8.30%
Net Sharpe Ratio	0.60	0.51	0.45	0.24
Net MDD	9.46%	12.93%	15.32%	19.41%
Avrg. number of assets	2.95	2.09	8.00	8.00
Avrg. turnover p.a.	4.16	5.48	0.22	0.26
Avrg risk free rate	4.77%			
Obs.	96			
February 2001-June 2004	Black-Litterman	Mean-Variance	BM I (st.w.)	BM II (1/N)
Net mean return p.a.	7.77%	5.01%	3.71%	5.19%
Volatility p.a.	6.40%	8.05%	8.32%	10.00%
Net Sharpe Ratio	0.92*	0.39	0.22	0.33
Net MDD	6.52%	10.67%	14.19%	17.37%
Avrg. number of assets	2.76	1.90	8.00	8.00
Avrg. turnover p.a.	2.67	4.42	0.30	0.31
Avrg risk free rate	1.85%			
Obs.	41			
July 2004 - February 2008	Black-Litterman	Mean-Variance	BM I (st.w.)	BM II (1/N)
Net mean return p.a.	13.81%	11.97%	10.51%	12.44%
Volatility p.a.	9.41%	11.35%	5.08%	6.66%
Net Sharpe Ratio	1.08	0.73	1.35	1.32
Net MDD	6.63%	10.48%	34.14%	39.57%
Avrg. number of assets	3.84	2.59	8.00	8.00
Avrg. turnover p.a.	3.45	4.20	0.22	0.25
Avrg risk free rate	3.67%			
Obs.	44			
March 2008 - December 2011	Black-Litterman	Mean-Variance	BM I (st.w.)	BM II (1/N)
Net mean return p.a.	5.84%	3.00%	0.14%	-0.43%
Volatility p.a.	8.63%	9.75%	15.04%	18.10%
Net Sharpe Ratio	0.63*	0.27	-0.01	-0.04
Net MDD	9.34%	16.18%	35.10%	40.59%
Avrg. number of assets	3.81	2.13	8.00	8.00
Avrg. turnover p.a.	2.02	2.57	0.39	0.38
Avrg risk free rate	0.37%			
Obs.	47			

This table reports portfolio performance measures for the four sub-periods from 1993-2011 for the moderate investor type. In the BL and MV approach (subjective) return estimates are mean historic returns using a rolling estimation window of 12-month. The variance-covariance matrix is calculated using a 36-month moving estimation window. In the BL model the parameter (τ) is set equal to 0.1, (Ω) is computed using the variance of historic return forecast errors (12-month rolling estimation window). In the BL and MV optimized portfolios the maximum expected volatility is constrained to 10% p.a.. For the BL optimization the strategic weights are set according to table 1: 45% for bonds, 15% for commodities and 40% for stocks. These weights are used to compute the naïve diversified benchmark I, as well. In benchmark II all assets are equally weighted (1/N). All portfolios are rebalanced at the first trading day of every month. * / ** / ***, (#/##/###), [†/††/†††] represents a significant higher Sharpe Ratio compared to Mean-Variance, (Benchmark I), [Benchmark II] at the 10%- / 5%- / 1%-level, respectively.

Figure 1: The Procedure of the Black-Litterman Approach (Idzorek 2005)

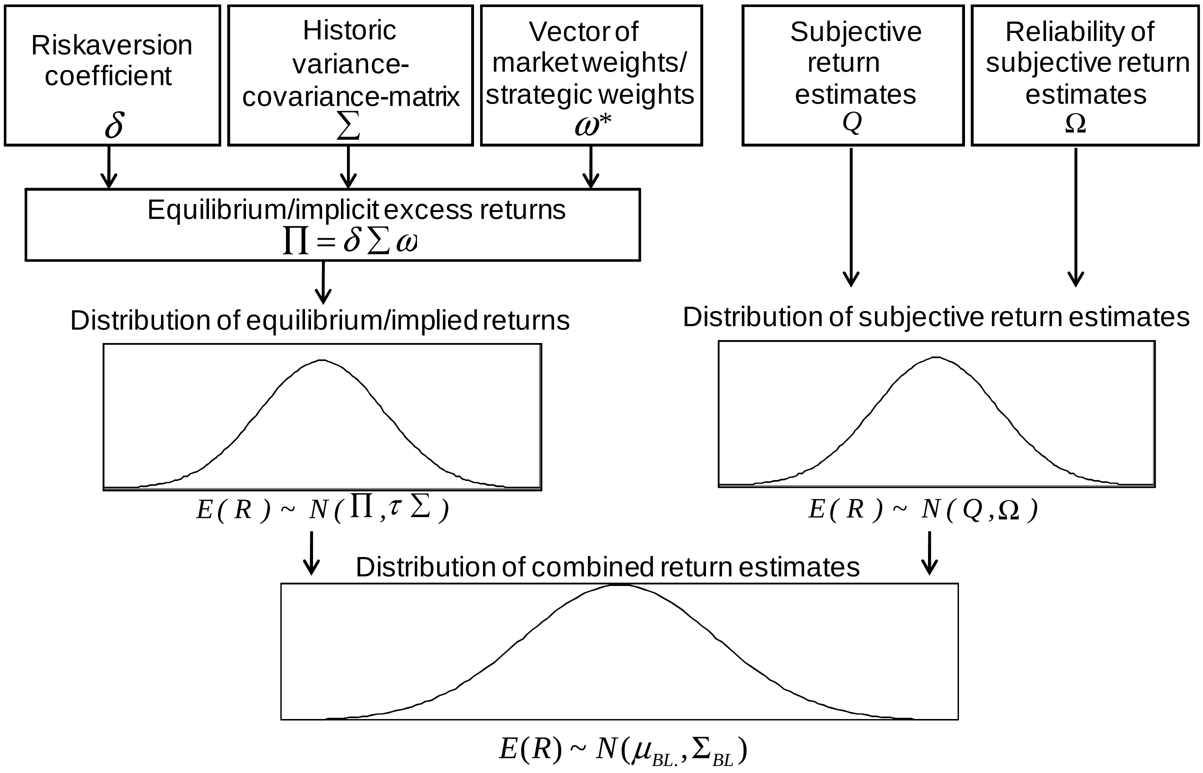
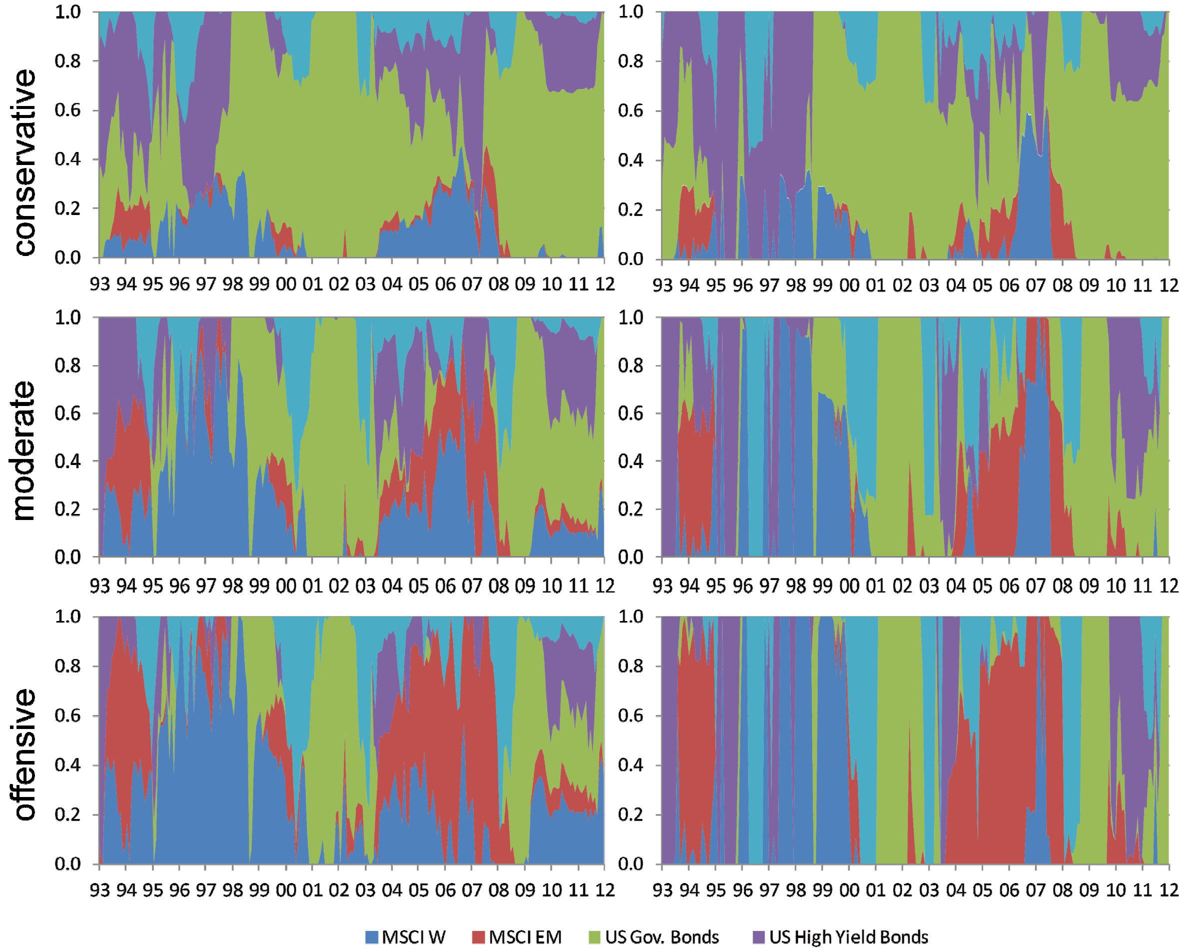
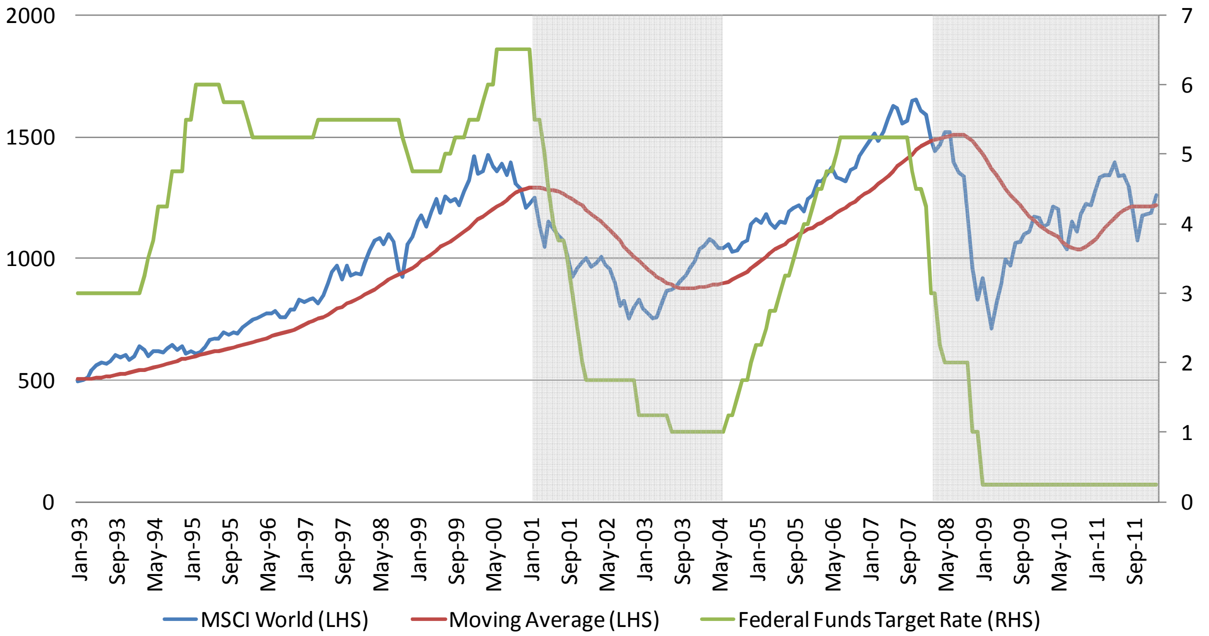


Figure 2: BL and MV optimized portfolio weights (base case)



This figure reports the optimized portfolio weights for the full sample from 1993-2011 in the base case. In the BL and MV approach (subjective) return estimates are mean historic returns using a rolling estimation window of 12-month. The variance-covariance matrix is calculated using a 36-month moving estimation window. In the BL model the parameter (τ) is set equal to 0.1, (Ω) is computed using the variance of historic return forecast errors (12-month rolling estimation window). In the BL and MV optimized portfolios the maximum expected volatility is constrained to 5%, 10%, and 15% for the conservative, moderate, and offensive investor type, respectively. All portfolios are rebalanced at the first trading day of every month.

Figure 3: Definition of sub-periods



The figure shows the definition of individual sub-periods conditional on monetary policy signals as well as stock market signals. Shaded areas denote down markets/recessionary periods.